Title: Infrared finite scattering in QFT & quantum gravity - Kartik Prabhu, UCSB

Speakers:

Series: Strong Gravity

Date: May 12, 2022 - 1:00 PM

URL: https://pirsa.org/22050056

Abstract: The "infrared problem" is the generic emission of an infinite number of low-frequency quanta in any scattering process with massless fields. The "out" state contains an infinite number of such quanta which implies that it does not lie in the standard Fock representation. Consequently, the standard S-matrix is undefined as a map between "in" and "out" states in the standard Fock space. This fact is due to the existence of a low-frequency tail of the radiation field i.e. the memory effect. In massive QED, the Faddeev-Kulish representations have been argued to yield an I.R. finite S-matrix. We clarify the "preferred " status of such representations as eigenstates of the conserved "large gauge charge" at spatial infinity. We prove a "No-Go" theorem for the existence of a suitable Hilbert space analogously constructed in massless QED, QCD, linearized quantum gravity with massive/massless sources, and in full quantum gravity. We then suggest an "infrared-finite" formulation of scattering theory in terms of correlation functions without any a priori choice of "in/out" Hilbert spaces.

Zoom Link: https://pitp.zoom.us/j/91774106578?pwd=T0tFS1BIRkNRNllYdGNMWFIyUHBDZz09

INFRARED FINITE SCATTERING

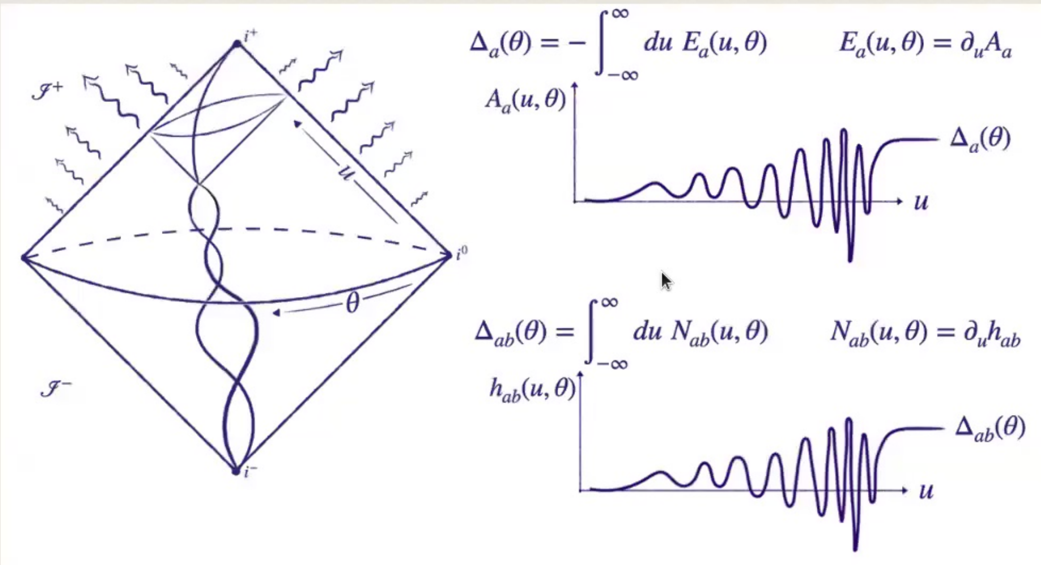
IN QFT & QUANTUM GRAVITY

Kartik Prabhu
(with Gautam Satishchandran and Robert M. Wald) 2022

SCATTERING: ASYMPTOTIC FLATNESS

1. generic scattering of particles or fields from past to future infinity

2. The radiation field has a long time tail or a zero frequency part



MASSIVE QED: ASYMPTOTIC FIELDS

Massive charged scalar field φ coupled to electromagnetism

- Massive scalar asymptotics in the limit to $i^{\pm}$ described by $b(p), b^{\dagger}(p), c(p), c^{\dagger}(p)$, where p lies on a unit hyperboloid. *(usual Fourier description)*

- EM vector potential near $\mathscr{I}^{\pm}$:
  $A_a \sim A_A(u,\theta) + O(1/r)$
  Radiative field: $E_A = \partial_u A_A$

- EM memory:
  $\Delta_A(\theta) \sim A_A(u \to +\infty) - A_A(u \to -\infty) = -\int du E_A(u,\theta)$

MASSIVE QED: ASYMPTOTIC FIELDS

Massive charged scalar field φ coupled to electromagnetism

- Massive scalar asymptotics in the limit to $i^{\pm}$ described by $b(p), b^{\dagger}(p), c(p), c^{\dagger}(p)$, where p lies on a unit hyperboloid. *(usual Fourier description)*

- EM vector potential near $\mathscr{I}^{\pm}$:
  $A_a \sim A_A(u, \theta) + O(1/r)$
  Radiative field: $E_A = \partial_u A_A$

- EM memory:
  $\Delta_A(\theta) \sim A_A(u \to +\infty) - A_A(u \to -\infty) = -\int du E_A(u, \theta)$

EM: Fock space

Quantum operator-valued *distribution*: $\mathbf{E}_A(u,\theta)$.

- Vacuum state: $|\omega_0\rangle$ *(Poincaré invariant)*: $\langle\omega_0|\mathbf{E}_{1A}\mathbf{E}_{2B}|\omega_0\rangle \propto \frac{\delta(\theta_1,\theta_2)}{(u_1-u_2-i0^+)^2} q_{AB}$

- one-photon Hilbert space: test "wavefunctions" $s_A(u,\theta)$

$$\|s\|^2 \propto \int d^2\theta \int_0^\infty d\omega\, \omega |s(\omega,\theta)|^2$$

- If wavefunction has memory then $s_A \sim \frac{\Delta_A(\theta)}{\omega}$ and $\|s\|^2$ *diverges logarithmically*. *No states with memory in standard Fock space!*

EM: Memory Hilbert spaces

Use coherent states with memory

- Classical solution $\mathcal{E}_A(u,\theta)$ with memory $\Delta_A(\theta) = -\int du \mathcal{E}_A(u,\theta)$
- $\mathbf{E}_A \mapsto \mathbf{E}_A + \mathcal{E}_A \mathbf{1}$ (*automorphism of the operator algebra*)
- New Hilbert space of states $\mathscr{F}_\Delta$ with memory $\Delta_A(\theta)$
- $\mathscr{F}_\Delta$ is *unitarily-equivalent* to $\mathscr{F}_{\Delta'}$ if and only if $\Delta_A(\theta) = \Delta'_A(\theta)$
- *Uncountably-many* inequivalent Hilbert spaces!

MASSIVE QED: SCATTERING

- EM with no sources: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta)$; antipodal map $\theta \mapsto -\theta$ on $\mathbb{S}^2$
- EM with *classical* source: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta) +$ *classical* current term
- EM with *massive quantum* field: given a $\Delta_A^{in}(\theta)$ *no* definite out-memory!
- If we *pretend* that all states are in the standard Fock space $\mathscr{F}_0$ then S-matrix diverges (*Weinberg soft theorem*)

Idea: build Hilbert spaces using *conserved* quantities, not memory.

Massive QED: asymptotic fields

Massive charged scalar field φ coupled to electromagnetism

- Massive scalar asymptotics in the limit to $i^{\pm}$ described by $b(p), b^{\dagger}(p), c(p), c^{\dagger}(p)$, where p lies on a unit hyperboloid. *(usual Fourier description)*
- EM vector potential near $\mathscr{I}^{\pm}$: $A_a \sim A_A(u,\theta) + O(1/r)$ Radiative field: $E_A = \partial_u A_A$
- EM memory: $\Delta_A(\theta) \sim A_A(u \to +\infty) - A_A(u \to -\infty) = -\int du E_A(u,\theta)$

EM: memory Hilbert spaces

Use coherent states with memory

- Classical solution $\mathcal{E}_A(u,\theta)$ with memory $\Delta_A(\theta) = -\int du \mathcal{E}_A(u,\theta)$
- $\mathbf{E}_A \mapsto \mathbf{E}_A + \mathcal{E}_A \mathbf{1}$ (*automorphism of the operator algebra*)
- New Hilbert space of states $\mathscr{F}_\Delta$ with memory $\Delta_A(\theta)$
- $\mathscr{F}_\Delta$ is *unitarily-equivalent* to $\mathscr{F}_{\Delta'}$ if and only if $\Delta_A(\theta) = \Delta'_A(\theta)$
- *Uncountably-many* inequivalent Hilbert spaces!

EM: Fock space

Quantum operator-valued *distribution*: $\mathbf{E}_A(u,\theta)$.

- Vacuum state: $|\omega_0\rangle$ *(Poincaré invariant)*: $\langle\omega_0|\mathbf{E}_{1A}\mathbf{E}_{2B}|\omega_0\rangle \propto \frac{\delta(\theta_1,\theta_2)}{(u_1-u_2-i0^+)^2} q_{AB}$
- one-photon Hilbert space: test "wavefunctions" $s_A(u,\theta)$

$$\|s\|^2 \propto \int d^2\theta \int_0^\infty d\omega\, \omega |s(\omega,\theta)|^2$$

- If wavefunction has memory then $s_A \sim \frac{\Delta_A(\theta)}{\omega}$ and $\|s\|^2$ *diverges logarithmically*. *No states with memory in standard Fock space!*

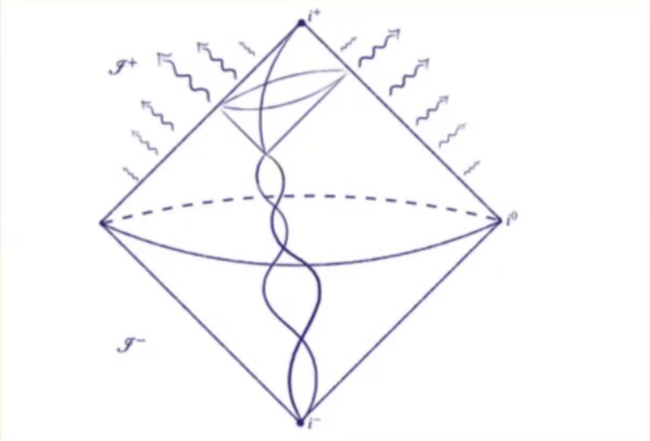
Massive QED: scattering

- EM with no sources: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta)$; antipodal map $\theta \mapsto -\theta$ on $\mathbb{S}^2$
- EM with *classical* source: $\Delta_A^{out}(\theta) = \Delta_A^{in}(-\theta) +$ *classical* current term
- EM with *massive quantum* field: given a $\Delta_A^{in}(\theta)$ *no* definite out-memory!
- If we *pretend* that all states are in the standard Fock space $\mathscr{F}_0$ then S-matrix diverges (*Weinberg soft theorem*)

Idea: build Hilbert spaces using *conserved* quantities, not memory.

CONSERVED CHARGES AT SPATIAL INFINITY

- $\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta) + \mathscr{D}^A \Delta_A(\theta)$
- $\mathcal{Q}^{out}_{i^0}(\theta) = \mathcal{Q}^{in}_{i^0}(-\theta)$



Find Hilbert spaces of definite charge $\mathcal{Q}_{i^0}(\theta)$

Faddeev-Kulish Hilbert space

1. For any massive *(improper)* momentum state $|p_1, \ldots, p_n, q_1, \ldots, q_n\rangle$ the charge $\mathcal{Q}_{i^-}(\theta; \{p_i, q_i\})$ is the sum of boosted Coulomb charges.

2. add *any* EM state $|\psi\rangle \in \mathscr{F}_\Delta$ with memory chosen so that
   $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{Q}_{i^-}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \Delta_A(\theta; \{p_i, q_i\})$

3. Then the *dressed* state
   $\int d^3p_1 \ldots d^3q_n w(\{p_i, q_i\}) |p_1, \ldots, p_n, q_1, \ldots, q_n\rangle \otimes |\psi\rangle_{\Delta(\theta;\{p_i,q_i\})}$ has zero charge at spatial infinity.

4. Faddeev-Kulish Hilbert space $\mathscr{H}^{FK}$ by direct summing over particle-antiparticle number.

Charge conservation ensures that this Hilbert space scatters to itself! So the S-matrix $S : \mathscr{H}^{FK} \to \mathscr{H}^{FK}$ is well-defined.

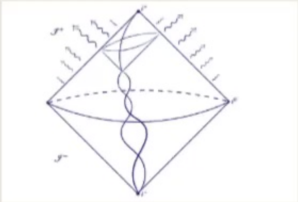
FK Hilbert spaces: issues

- Total charge cannot be dressed away; memory is $\ell \geq 1$; always need equal number of particles and antiparticles *(hide particles behind the moon)*
- Can define FK Hilbert spaces with any value of $\mathcal{Q}_{i^0}(\theta)$ but not Lorentz-invariant; angular momentum/spin of states is undefined! *("Lorentz is spontaneously broken")*
- Massive fields always come with a radiative photon cloud; can hide the cloud at very low frequencies but always have to send it in. Photon cloud *completely decoheres* the states! (G. Satishchandran and R. M. Wald, to appear)

All these issues can be fixed in massive QED (on-going work with G. Satishchandran)

Conserved charges at spatial infinity

- $\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta) + \mathscr{D}^A \Delta_A(\theta)$
- $\mathcal{Q}^{out}_{i^0}(\theta) = \mathcal{Q}^{in}_{i^0}(-\theta)$



Find Hilbert spaces of definite charge $\mathcal{Q}_{i^0}(\theta)$

FK Hilbert spaces: issues

- Total charge cannot be dressed away; memory is $\ell \geq 1$; always need equal number of particles and antiparticles *(hide particles behind the moon)*
- Can define FK Hilbert spaces with any value of $\mathcal{Q}_{i^0}(\theta)$ but not Lorentz-invariant; angular momentum/spin of states is undefined! *("Lorentz is spontaneously broken")*
- Massive fields always come with a radiative photon cloud; can hide the cloud at very low frequencies but always have to send it in. Photon cloud *completely decoheres* the states! (G. Satishchandran and R. M. Wald, to appear)

All these issues can be fixed in massive QED (on-going work with G. Satishchandran)

Faddeev-Kulish Hilbert space

1. For any massive *(improper)* momentum state $|p_1, \ldots, p_n, q_1, \ldots, q_n\rangle$ the charge $\mathcal{Q}_{i^-}(\theta; \{p_i, q_i\})$ is the sum of boosted Coulomb charges.
2. add *any* EM state $|\psi\rangle \in \mathscr{F}_\Delta$ with memory chosen so that $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{Q}_{i^-}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \Delta_A(\theta; \{p_i, q_i\})$
3. Then the *dressed* state $\int d^3p_1 \ldots d^3q_n w(\{p_i, q_i\}) |p_1, \ldots, p_n, q_1, \ldots, q_n\rangle \otimes |\psi\rangle_{\Delta(\theta; \{p_i, q_i\})}$ has zero charge at spatial infinity.
4. Faddeev-Kulish Hilbert space $\mathscr{H}^{FK}$ by direct summing over particle-antiparticle number.

Charge conservation ensures that this Hilbert space scatters to itself! So the S-matrix $S : \mathscr{H}^{FK} \to \mathscr{H}^{FK}$ is well-defined.

MASSLESS QED

EM coupled to a *massless quantum* field (same issues with memory)

- For the FK construction: need to solve
  $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{J}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \Delta_A(\theta; \{p_i, q_i\})$
- $\mathcal{J}(\theta; p) = \delta(\theta, \hat{p})$ implies $\Delta_A \sim 1/(\theta - \hat{p})$ *(collinear divergence)*
- In fact, Δ_A is not L^2; *photon dressing has infinite energy!*

LINEARIZED GRAVITY WITH MASSLESS SOURCE

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u, \theta)$

- For the FK construction: need to solve $\mathcal{Q}_{i^0}(\theta)\big|_{\ell \geq 2} = 0 = \mathcal{T}(\theta; \{p_i, q_i\}) + \mathscr{D}^A \mathscr{D}^B \Delta_{AB}(\theta; \{p_i, q_i\})$
- $\mathcal{T}(\theta; p) = \delta(\theta, \hat{p})$ implies $\Delta_{AB} \sim \log(\theta - \hat{p})$ *(collinear divergence)*
- but, Δ_{AB} is L^2; *dressing has finite energy!*
- $\mathcal{Q}_{i^0}(\theta)\big|_{\ell \geq 2} = 0$ is not Lorentz-invariant, so no angular momentum/spin for these states

NONLINEAR GR

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u, \theta)$

$$\mathcal{Q}_{i^0}(\theta) = \int du\, N_{AB}(u, \theta) N^{AB}(u, \theta) + \mathscr{D}^A \mathscr{D}^B \Delta_{AB}(\theta)$$

- The memory Hilbert spaces $\mathscr{F}_\Delta$ still exist and memory is not conserved
- *gravity sources itself*; any kind of dressing will further contribute to the charge

Theorem: The only *Hadamard* state of fixed $\mathcal{Q}_{i^0}(\theta)$ is the *vacuum* with $\mathcal{Q}_{i^0}(\theta) = 0$.

So no non-vacuum state is in *any* FK Hilbert space!

ISSUES: ONE TRUE HILBERT SPACE!

- massive QED: FK construction "works"; particle behind the moon, no states without dressing (no coherent superposition!), if non-zero charge then infinite angular momentum/spin
- massless QED: collinear divergences, infinite energy
- Lin. GR with massless source: infinite angular momentum/spin
- Nonlinear GR: no state except vacuum!
- Non-FK constructions also don't work; no way to ensure conservation of charge and be Lorentz-invariant.

Problem: asking for a Hilbert space! Operator algebra is fine. Huge supply of states which are well-defined as correlation functions of operators. *Just cannot force all in/out states into one Hilbert space*

ALGEBRAIC SCATTERING

- In/out states defined as lists of correlation functions $\omega(\mathbf{O}_1, \ldots, \mathbf{O}_n)$
- Any such state lives in some Hilbert space, but *not all states* can be put in a *single Hilbert space*
- Evolution/scattering is given by a \$-map: $\$: \omega^{in} \mapsto \omega^{out}$
- conservation of probability: $\omega^{in}(\mathbf{1}) = 1$ implies $\omega^{out}(\mathbf{1}) = 1$;
  no information loss: \$ maps pure in-states to pure out-states

How to *practically* compute such a \$? *¯_(ツ)_/¯* Usual methods like Feynman diagrams/LSZ reduction do not work; Hawking (1976) has interesting ideas, but still assumes a single Hilbert space

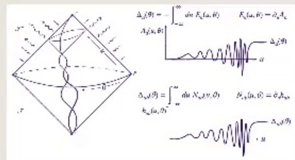
don't take a vacation yet! Or maybe take a vacation and think about it!!

INFRARED FINITE SCATTERING
IN QFT & QUANTUM GRAVITY

Kartik Prabhu
(with Gautam Satishchandran and Robert M. Wald) 2022

SCATTERING: ASYMPTOTIC FLATNESS

1. generic scattering of particles or fields from past to future infinity
2. The radiation field has a long time tail or a zero frequency part



LINEARIZED GRAVITY WITH MASSLESS SOURCE

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u,\theta)$

- For the FK construction: need to solve $\mathcal{Q}_{i^0}(\theta)|_{\ell\geq 2} = 0 = \mathcal{T}(\theta;\{p_i,q_i\}) + \mathscr{D}^A\mathscr{D}^B\Delta_{AB}(\theta;\{p_i,q_i\})$
- $\mathcal{T}(\theta;p) = \delta(\theta,\hat{p})$ implies $\Delta_{AB} \sim \log(\theta-\hat{p})$ *(collinear divergence)*
- but, Δ_{AB} is L^2; *dressing has finite energy!*
- $\mathcal{Q}_{i^0}(\theta)|_{\ell\geq 2} = 0$ is not Lorentz-invariant, so no angular momentum/spin for these states

MASSLESS QED

EM coupled to a *massless quantum* field (same issues with memory)

- For the FK construction: need to solve $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{J}(\theta;\{p_i,q_i\}) + \mathscr{D}^A\Delta_A(\theta;\{p_i,q_i\})$
- $\mathcal{J}(\theta;p) = \delta(\theta,\hat{p})$ implies $\Delta_A \sim 1/(\theta-\hat{p})$ *(collinear divergence)*
- In fact, Δ_A is not L^2; *photon dressing has infinite energy!*

MASSIVE QED: ASYMPTOTIC FIELDS

Massive charged scalar field φ coupled to electromagnetism

- Massive scalar asymptotics in the limit to $i^{\pm}$ described by $b(p), b^{\dagger}(p), c(p), c^{\dagger}(p)$, where p lies on a unit hyperboloid. *(usual Fourier description)*
- EM vector potential near $\mathscr{I}^{\pm}$: $A_a \sim A_A(u,\theta) + O(1/r)$ Radiative field: $E_A = \partial_u A_A$
- EM memory: $\Delta_A(\theta) \sim A_A(u\to+\infty) - A_A(u\to-\infty) = -\int du E_A(u,\theta)$

EM: MEMORY HILBERT SPACES

Use coherent states with memory

- Classical solution $\mathcal{E}_A(u,\theta)$ with memory $\Delta_A(\theta) = -\int du\mathcal{E}_A(u,\theta)$
- $\mathbf{E}_A \mapsto \mathbf{E}_A + \mathcal{E}_A\mathbf{1}$ *(automorphism of the operator algebra)*
- New Hilbert space of states $\mathscr{F}_\Delta$ with memory $\Delta_A(\theta)$
- $\mathscr{F}_\Delta$ is *unitarily-equivalent* to $\mathscr{F}_{\Delta'}$ if and only if $\Delta_A(\theta) = \Delta'_A(\theta)$
- *Uncountably-many* inequivalent Hilbert spaces!

CONSERVED CHARGES AT SPATIAL INFINITY

- $\mathcal{Q}_{i^0}(\theta) = \mathcal{Q}_{i^-}(\theta) + \mathscr{D}^A\Delta_A(\theta)$
- $\mathcal{Q}^{out}_{i^0}(\theta) = \mathcal{Q}^{in}_{i^0}(-\theta)$

Find Hilbert spaces of definite charge $\mathcal{Q}_{i^0}(\theta)$

FK HILBERT SPACES: ISSUES

- Total charge cannot be dressed away; memory is $\ell \geq 1$; always need equal number of particles and antiparticles *(hide particles behind the moon)*
- Can define FK Hilbert spaces with any value of $\mathcal{Q}_{i^0}(\theta)$ but not Lorentz-invariant; angular momentum/spin of states is undefined! *("Lorentz is spontaneously broken")*
- Massive fields always come with a radiative photon cloud; can hide the cloud at very low frequencies but always have to send it in. Photon cloud *completely decoheres* the states! (G. Satishchandran and R. M. Wald, to appear)

All these issues can be fixed in massive QED (on-going work with G. Satishchandran)

EM: FOCK SPACE

Quantum operator-valued *distribution*: $\mathbf{E}_A(u,\theta)$.

- Vacuum state: $|\omega_0\rangle$ *(Poincaré invariant)*: $\langle\omega_0|\mathbf{E}_{1A}\mathbf{E}_{2B}|\omega_0\rangle \propto \frac{\delta(\theta_1,\theta_2)}{(u_1-u_2-i0^+)^2}q_{AB}$
- one-photon Hilbert space: test "wavefunctions" $s_A(u,\theta)$
$$\|s\|^2 \propto \int d^2\theta \int_0^\infty d\omega\, \omega|s(\omega,\theta)|^2$$
- If wavefunction has memory then $s_A \sim \frac{\Delta_A(\theta)}{\omega}$ and $\|s\|^2$ *diverges logarithmically. No states with memory in standard Fock space!*

MASSIVE QED: SCATTERING

- EM with no sources: $\Delta^{out}_A(\theta) = \Delta^{in}_A(-\theta)$; antipodal map $\theta \mapsto -\theta$ on $\mathbb{S}^2$
- EM with *classical* source: $\Delta^{out}_A(\theta) = \Delta^{in}_A(-\theta) +$ *classical* current term
- EM with *massive quantum* field: given a $\Delta^{in}_A(\theta)$ *no* definite out-memory!
- If we *pretend* that all states are in the standard Fock space $\mathscr{F}_0$ then S-matrix diverges (*Weinberg soft theorem*)

Idea: build Hilbert spaces using *conserved* quantities, not memory.

FADDEEV-KULISH HILBERT SPACE

1. For any massive *(improper)* momentum state $|p_1,\ldots,p_n,q_1,\ldots,q_n\rangle$ the charge $\mathcal{Q}_{i^-}(\theta;\{p_i,q_i\})$ is the sum of boosted Coulomb charges.
2. add *any* EM state $|\psi\rangle \in \mathscr{F}_\Delta$ with memory chosen so that $\mathcal{Q}_{i^0}(\theta) = 0 = \mathcal{Q}_{i^-}(\theta;\{p_i,q_i\}) + \mathscr{D}^A\Delta_A(\theta;\{p_i,q_i\})$
3. Then the *dressed* state $\int d^3p_1\ldots d^3q_n w(\{p_i,q_i\})|p_1,\ldots,p_n,q_1,\ldots,q_n\rangle \otimes |\psi\rangle_{\Delta(\theta;\{p_i,q_i\})}$ has zero charge at spatial infinity.
4. Faddeev-Kulish Hilbert space $\mathscr{H}^{FK}$ by direct summing over particle-antiparticle number.

Charge conservation ensures that this Hilbert space scatters to itself! So the S-matrix $S : \mathscr{H}^{FK} \to \mathscr{H}^{FK}$ is well-defined.

NONLINEAR GR

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u,\theta)$

$\mathcal{Q}_{i^0}(\theta) = \int du\, N_{AB}(u,\theta)N^{AB}(u,\theta) + \mathscr{D}^A\mathscr{D}^B\Delta_{AB}(\theta)$

- The memory Hilbert spaces $\mathscr{F}_\Delta$ still exist and memory is not conserved
- *gravity sources itself*; any kind of dressing will further contribute to the charge

Theorem: The only *Hadamard* state of fixed $\mathcal{Q}_{i^0}(\theta)$ is the *vacuum* with $\mathcal{Q}_{i^0}(\theta) = 0$.

So no non-vacuum state is in *any* FK Hilbert space!

ISSUES: ONE TRUE HILBERT SPACE?

- massive QED: FK construction "works"; particle behind the moon, no states without dressing (no coherent superposition!), if non-zero charge then infinite angular momentum/spin
- massless QED: collinear divergences, infinite energy
- Lin. GR with massless source: infinite angular momentum/spin
- Nonlinear GR: no state except vacuum!
- Non-FK constructions also don't work; no way to ensure conservation of charge and be Lorentz-invariant.

Problem: asking for a Hilbert space! Operator algebra is fine. Huge supply of states which are well-defined as correlation functions of operators. *Just cannot force all in/out states into one Hilbert space*

ALGEBRAIC SCATTERING

- In/out states defined as lists of correlation functions $\omega(\mathbf{O}_1,\ldots,\mathbf{O}_n)$
- Any such state lives in some Hilbert space, but *not all states* can be put in a *single Hilbert space*
- Evolution/scattering is given by a \$-map: $\$: \omega^{in} \mapsto \omega^{out}$
- conservation of probability: $\omega^{in}(\mathbf{1}) = 1$ implies $\omega^{out}(\mathbf{1}) = 1$; no information loss: \$ maps pure in-states to pure out-states

How to *practically* compute such a \$? ¯_(ツ)_/¯ Usual methods like Feynman diagrams/LSZ reduction do not work; Hawking (1976) has interesting ideas, but still assumes a single Hilbert space

don't take a vacation yet! Or maybe take a vacation and think about it!!

FK Hilbert spaces: issues

- Total charge cannot be dressed away; memory is $\ell \geq 1$; always need equal number of particles and antiparticles *(hide particles behind the moon)*
- Can define FK Hilbert spaces with any value of $\mathcal{Q}_{i^0}(\theta)$ but not Lorentz-invariant; angular momentum/spin of states is undefined! *("Lorentz is spontaneously broken")*
- Massive fields always come with a radiative photon cloud; can hide the cloud at very low frequencies but always have to send it in. Photon cloud *completely decoheres* the states! (G. Satishchandran and R. M. Wald, to appear)

All these issues can be fixed in massive QED (on-going work with G. Satishchandran)

NONLINEAR GR

Radiative field is News N_{AB}; gravitational memory is $\Delta_{AB}(\theta) = \int du N_{AB}(u,\theta)$

$$\mathcal{Q}_{i^0}(\theta) = \int du\, N_{AB}(u,\theta) N^{AB}(u,\theta) + \mathscr{D}^A \mathscr{D}^B \Delta_{AB}(\theta)$$

- The memory Hilbert spaces $\mathscr{F}_\Delta$ still exist and memory is not conserved
- *gravity sources itself*; any kind of dressing will further contribute to the charge

Theorem: The only *Hadamard* state of fixed $\mathcal{Q}_{i^0}(\theta)$ is the *vacuum* with $\mathcal{Q}_{i^0}(\theta) = 0$.

So no non-vacuum state is in *any* FK Hilbert space!

ALGEBRAIC SCATTERING

- In/out states defined as lists of correlation functions $\omega(\mathbf{O}_1, \ldots, \mathbf{O}_n)$
- Any such state lives in some Hilbert space, but *not all states* can be put in a *single Hilbert space*
- Evolution/scattering is given by a $-map: $\$: \omega^{in} \mapsto \omega^{out}$
- conservation of probability: $\omega^{in}(\mathbf{1}) = 1$ implies $\omega^{out}(\mathbf{1}) = 1$;
  no information loss: $ maps pure in-states to pure out-states

How to *practically* compute such a $? ¯_(ツ)_/¯ Usual methods like Feynman diagrams/LSZ reduction do not work; Hawking (1976) has interesting ideas, but still assumes a single Hilbert space

don't take a vacation yet! Or maybe take a vacation and think about it!!