Title: Factorization and Global symmetry in holography

Speakers: Lorenzo Di Pietro

Series: Quantum Fields and Strings

Date: May 10, 2022 - 2:00 PM

URL: https://pirsa.org/22050055

Abstract: Based on 2203.09537. In the context of toy models of holography arising from 3d Chern-Simons theory, I will describe an approach in which, rather than summing over bulk geometries, one gauges a one-form global symmetry of the bulk theory. This ensures that the bulk theory has no global symmetries, and it makes the partition function on spacetimes with boundaries coincide with that of a modular-invariant 2d CFT on the boundary. In particular, on wormhole geometries one finds a factorized answer for the partition function.

Zoom Link: https://pitp.zoom.us/j/91694973383?pwd=VWFCZWxtdy9GczUrN1JFc2xMVUhmdz09

FACTORIZATION & GLOBAL SYMMETRIES IN HOLOGRAPHY

Lorenzo Di Pietro
- Università di Trieste -

PERIMETER INSTITUTE - 10/05/2022

Based on: 2203.09537
w/ F. Benini, C. Copetti

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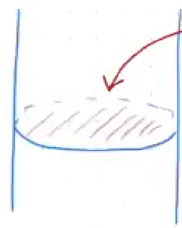
HOLOGRAPHY

QFT on M_d $\approx$ Quantum gravity with M_d asymptotics



Recently, solvable low dimensional examples:

QFT $\Longrightarrow$ ensemble average of QFTs

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Examples:

- Jackiw-Teitelboim (JT) gravity:

→ low energy of Sachdev-Ye-Kitaev (SYK) model

$$H \sim \sum J_{ijkl} \psi^i \psi^j \psi^k \psi^l$$ random coupling

[Sachdev, Ye; Kitaev; Maldacena, Stanford; Jensen]

→ path integral + sum over geometries [Saad, Shenker, Stanford]
= QM with H random matrix

- Chern-Simons + "topological gravity"

[Afkhami-Jeddi, Cohn, Hartman, Tajdini; Maloney, Witten]

$$\sum_{\text{topologies}} \left(u(1)^{2D} \, CS \right) = \int_{\substack{\text{Narain} \\ \text{moduli space}}} d\mu \left(\begin{matrix} D \text{ compact} \\ \text{free bosons} \end{matrix} \right)$$

QUESTIONS

- When averages?

$\langle QFT \rangle \neq QFT$ e.g.

lack of factorization

$$\langle Z[\mathcal{O} \sqcup \mathcal{O}] \rangle \neq \langle Z[\mathcal{O}] \rangle \langle Z[\mathcal{O}] \rangle$$

In the dual, due to wormholes 

[Maldacena, Maoz]

- What is "lacking" on the gravity side?

- Can we modify / complete the gravity side to get a QFT?

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OUR WORK

An answer to these questions in the context of a toy model for holography:

Chern-Simons w/ compact gauge group $\Longleftrightarrow$ Rational Conformal Field Theory (RCFT)

$G_k \times G_{-k}$

$(G_k)_L \times (G_k)_R$ chiral algebra

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→ Unwanted feature of CS as a "quantum gravity" theory: global symmetry

[Seiberg, Banks; Harlow, Ooguri]

→ Getting rid of it by gauging gives a modular invariant dual, w/out need to sum over topologies

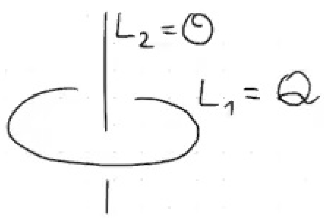
→ Turns CS/RCFT correspondence into full holographic duality

ABELIAN CASE

$U(1)_K \times U(1)_{-K}$ Chern-Simons, $K = 2n > 0$

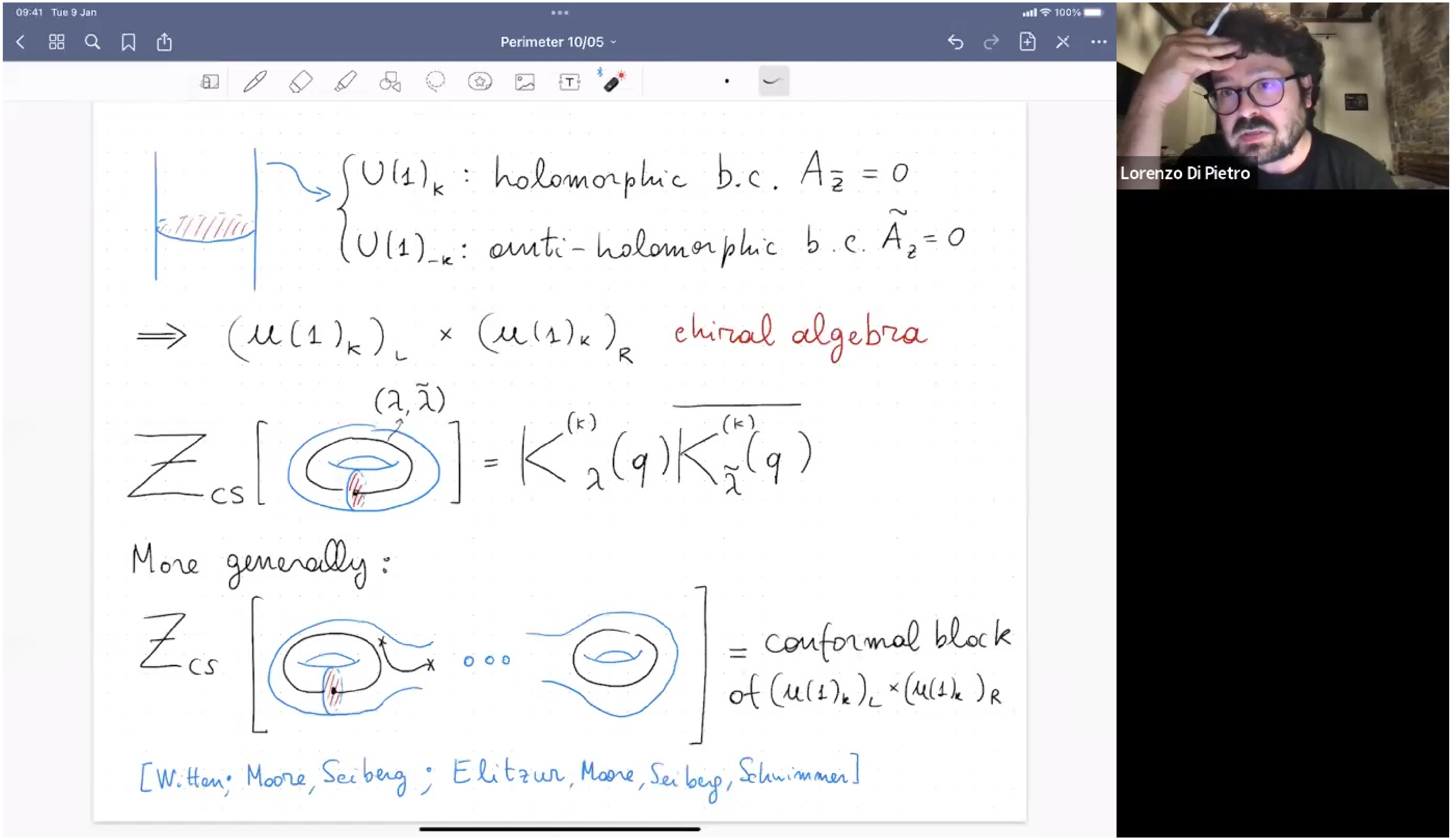
$$S_{CS} = \frac{K}{4\pi} \int (A\,dA - \tilde{A}\,d\tilde{A})$$

Lines : $\Big|_{(\lambda, \tilde{\lambda})}$ $\quad \lambda, \tilde{\lambda} = 0, \dots, K-1$



$$= e^{2\pi i \frac{\lambda_1 \lambda_2 - \tilde{\lambda}_1 \tilde{\lambda}_2}{K}} \Big|^{O}$$

1- form symmetry $\mathbb{Z}_K^{[1]} \times \mathbb{Z}_K^{[1]}$ [Gaiotto, Kapustin, Seiberg, Willet]



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$U(1)_k$: holomorphic b.c. $A_{\bar z} = 0$
$U(1)_{-k}$: anti-holomorphic b.c. $\tilde{A}_z = 0$
$\Longrightarrow (u(1)_k)_L \times (u(1)_k)_R$ chiral algebra
$(\lambda, \tilde\lambda)$
$Z_{cs}[\ ] = K^{(k)}_\lambda(q) \overline{K^{(k)}_{\tilde\lambda}(q)}$
More generally:
$Z_{cs}[\ \cdots\ ]$ = conformal block of $(u(1)_k)_L \times (u(1)_k)_R$
[Witten; Moore, Seiberg; Elitzur, Moore, Seiberg, Schwimmer]
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Chern-Simons as "gravity side" of holographic duality

PRO's:

✓ general covariant (topological)

✓ partition function on spaces with bdry has 2d CFT interpretation

CON's:

× the answer is not modular invariant

e.g. $Z_{cs}[\quad] = K_0^{(k)}(q)\,\overline{K_0^{(k)}(q)}$

× global symmetry

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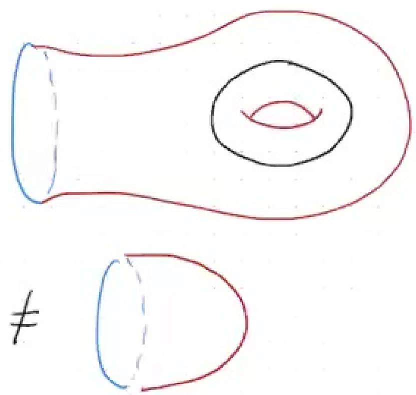
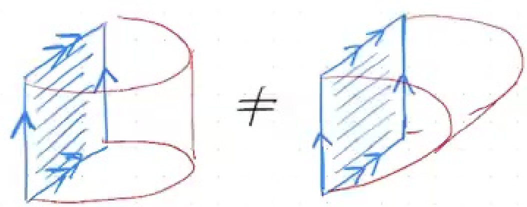
The two con's are related to each other :

Global symmetry $\Rightarrow$ dependence on the topology $\Rightarrow$ lack of modular invariance

Topology detected by the insertion of topological operators

The answer does not depend only on the geometry of the boundary





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A possibility to obtain modular invariance is to sum over topologies. This leads to ensemble average:

$$\langle Z[\bigcirc \sqcup \bigcirc] \rangle =$$

$$\neq \langle Z[\bigcirc] \rangle^2$$

Approach pursued in non-rational case by [Afkhami-Jeddi, Cohn, Hartman, Tajdini; Maloney, Witten]
and in rational case by [Meruliya, Mukhi; same + Singh].

Our approach instead will be to get rid of the global symmetry by gauging

$\Longrightarrow$ we will also lose dependence on topology!

Strongest form of general covariance.

Reminiscent of: [Eberhardt; Mukhametzhanov]

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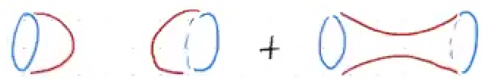
A possibility to obtain modular invariance is to sum over topologies. This leads to ensemble average:

$$\langle Z[\,O \sqcup O\,]\rangle =$$



$$+ \cdots$$

$$\neq \langle Z[\,O\,]\rangle^2$$

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GAUGING 1-FORM SYMMETRY

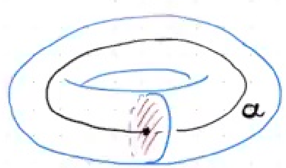
$$\mathbb{Z}_k^{[1]} \times \mathbb{Z}_k^{[1]} \supset A \quad \text{subgroup}$$

Gauging = Summing over bundles $\rightsquigarrow H^2(M, A)$

Poincaré duality:

sum over $H_1(M, A) \approx$ sum over line insertions

$$Z^{\text{1-form gauged}}[M] = \frac{|H_3(M, A)|}{|H_2(M, A)|} \sum_{a \in H_1(M, A)} Z[M; a]$$



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What sub-groups can we gauge?

Anomaly free conditions:

- Integer spins $\Downarrow$
- Mutually transparent $=$) (

Both satisfied by diagonal $\mathbb{Z}_k^{[1]}$: $(\lambda, \tilde{\lambda} = \lambda)$

More generally: $\left(\mathbb{Z}_k^{[1]}\right)_\omega = \quad (\lambda, \tilde{\lambda} = \omega\lambda) \quad , \omega \in \mathbb{Z}_k$

with $\omega^2 = 1 \mod 2k$ $\rightsquigarrow$ automorphism of $\mathbb{Z}_k$

$\#$ lines after gauging $= \dfrac{k^2}{|A|^2} = 1 \Rightarrow$ only $\mathbb{1}$ left

$\Rightarrow$ independence on bulk topology

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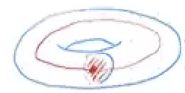
$$T = \frac{U(1)_k \times U(1)_{-k}}{(\mathbb{Z}_k^{[1]})_\omega} \quad \leftarrow \text{ our "quantum gravity" theory}$$

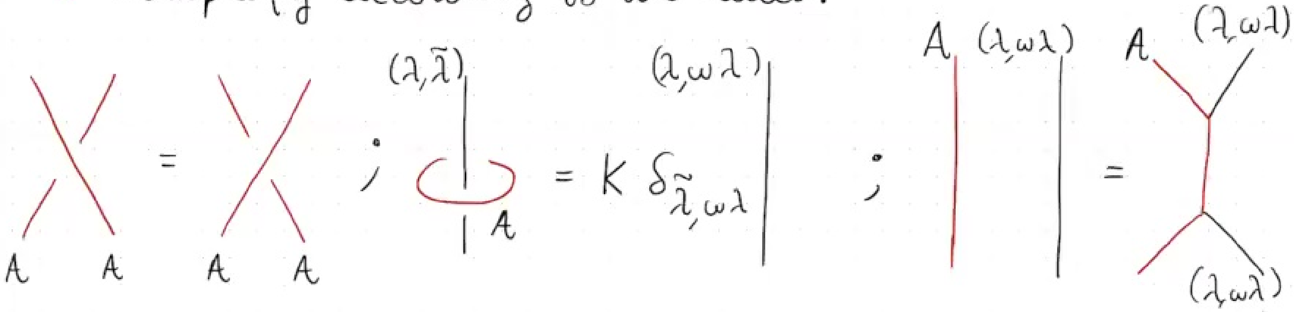
Algorithm to compute holographic observables:

- CS theory on M with $\partial M \neq \emptyset$, sum over insertions of

$$A = \Big| = \sum_{\lambda=0}^{k} \Big|_{(\lambda,\omega\lambda)} \quad \text{along } H_1(M)$$



- Simplify according to the rules:



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CFT DUAL: RATIONAL SCALAR

$$S_{\text{Scalar}} = \frac{1}{8\pi}\int d^2\sigma\, \partial_\mu\varphi\, \partial^\mu\varphi$$

$$\varphi \sim \varphi + 2\pi R$$

Consider $\omega = 1$: diagonal $\mathbb{Z}_k^{[1]}$

$$Z_T[\;\;] = Z_{CS}[\;\;]$$

$$= \sum_{\lambda=0}^{k-1} Z_{CS}[\;\;]^{(\lambda,\lambda)}$$

$$= \sum_{\lambda=0}^{k-1} K_\lambda^{(k)}(q)\, \overline{K_\lambda^{(k)}}(\bar q)$$

$$= Z_{\text{Scalar},\ R=\sqrt{k}}[\;\;]$$

[each term corresponds to a $(u(1)_k)_L \times (u(1)_k)_R$ primary in the scalar CFT:

$$\mathcal{O}_{\lambda,\lambda} = V_\lambda(z)\, \overline{V}_\lambda(\bar z)$$

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This suggests :

$$\left[T = \frac{U(1)_k \times U(1)_{-k}}{(\mathbb{Z}_k^{[1]})_{diag}} \quad \text{w/ holo/anti-holo b.c.} \quad \approx \quad \text{compact scalar with } R = \sqrt{K} \right]$$

More generally: when $K = 2pp'$, p, p' coprime, modular invariant gluing labeled by ω s.t. $\omega^2 = 1 \bmod 2K$

$$\sum_\lambda K_\lambda^{(\kappa)}(q) \overline{K_{\omega\lambda}^{(\kappa)}(q)}$$

Correspond to different rational scalars with same chiral algebra, $R = \sqrt{\frac{2p'}{p}}$.

$\Rightarrow$ Gauging $\approx$ prescription for gluing left/right conformal blocks

[Moore, Seiberg ; Fuchs, Runkel, Schweigert ; Kapustin, Saulina ; Hung, Wan]

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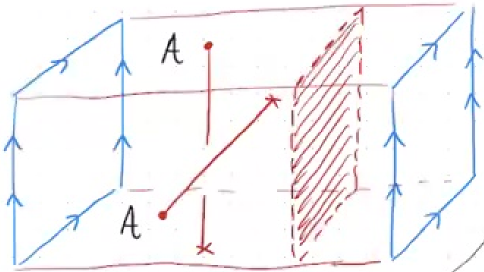
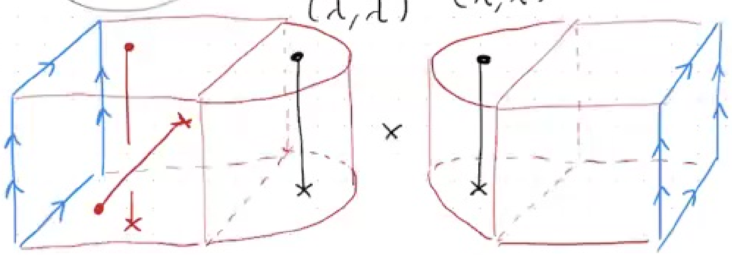
FACTORIZATION ON $T^2 \times I$

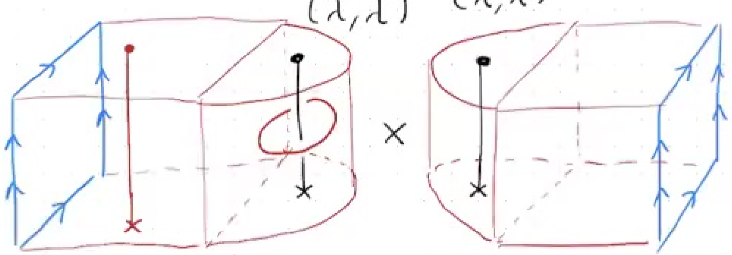


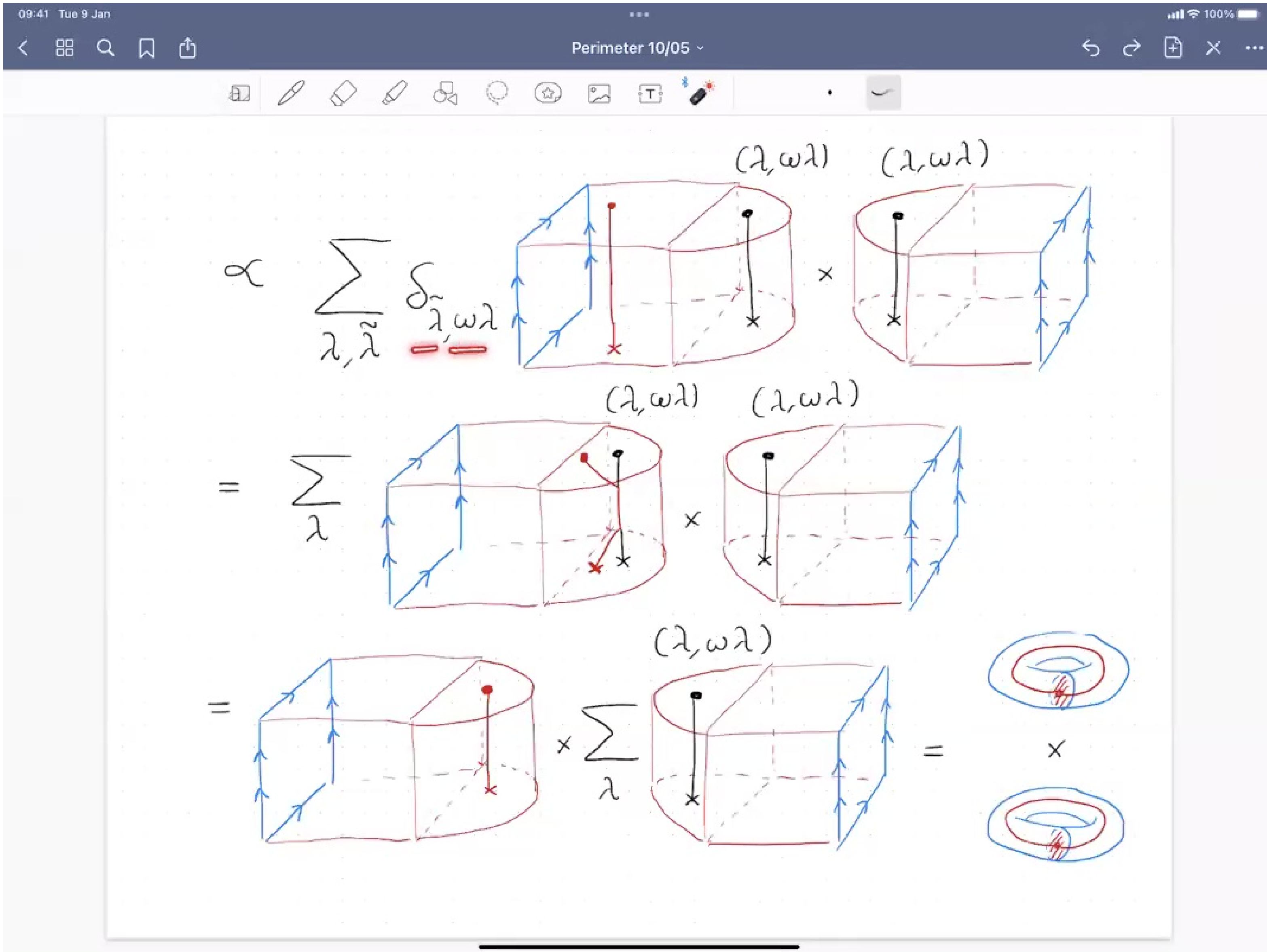
complete set of states in $\mathcal{H}_{cs}[\;\;] = \{\text{Lines}\}$

$$= \sum_{\lambda,\tilde{\lambda}}$$



$$= \sum_{\lambda,\tilde{\lambda}}$$

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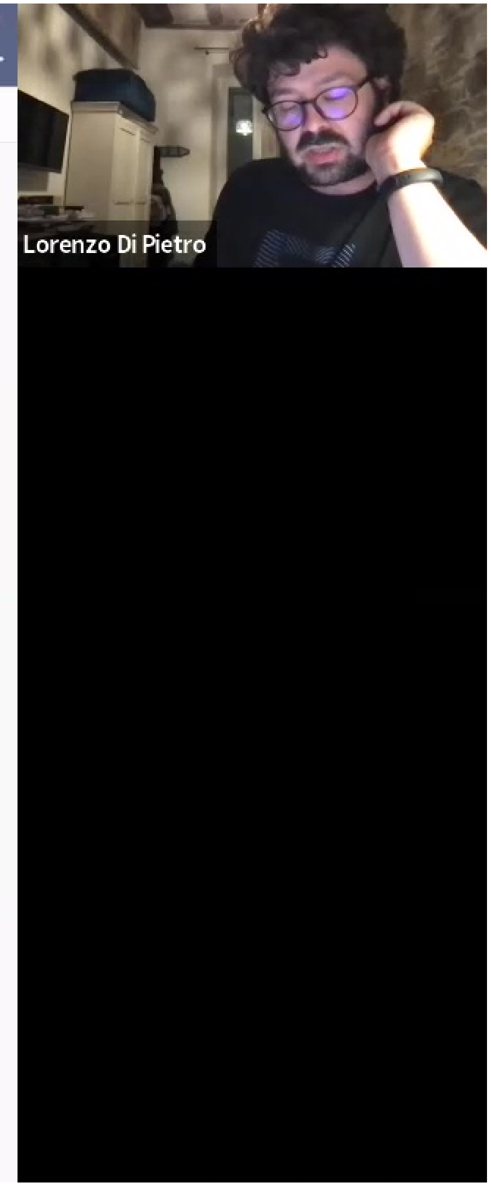
(λ,ωλ) (λ,ωλ)
$\propto \sum_{\lambda,\tilde\lambda} \delta_{\tilde\lambda,\omega\lambda}$ ×
(λ,ωλ) (λ,ωλ)
$= \sum_{\lambda}$ ×
(λ,ωλ)
= × $\sum_{\lambda}$ = ×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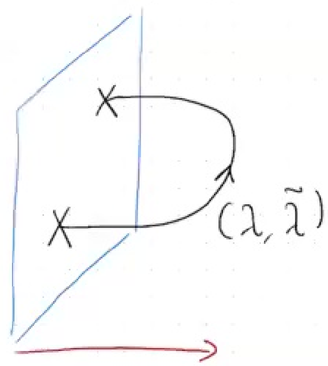
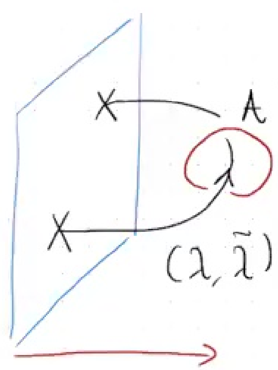
CORRELATION FUNCTIONS



- Before gauging: correlation functions of chiral vertex operators.



$\propto \delta_{\tilde{\lambda}, \omega\lambda}$

- Gauging: projects to physical operators.

  Matches w/ scalar result.

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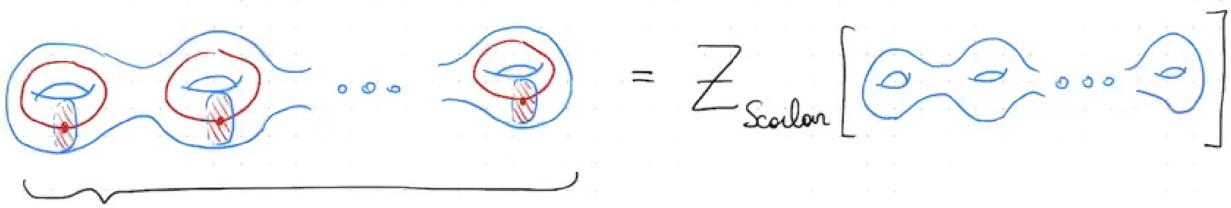
OTHER CHECKS

- Factorization for multi-boundary S^2

- Factorization for multi-boundary T^2

- Higher-genus handle bodies



reproduces the full sum over conformal blocks

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SUMMARY

Holographic duality :

$$\boxed{\begin{array}{ccc} \dfrac{U(1)_k \times U(1)_{-k}}{(\mathbb{Z}_k^{[1]})_\omega} & \approx & \text{Scalar with rational } R^2(k,\omega) \\ \text{on any } M_3 \text{ with } \partial M_3 = M_2 & & \text{on } M_2 \end{array}}$$

Bulk triviality : all the dynamics is due to holo/anti-holo boundary conditions.

Implies factorization. Non-trivial to check using description as gauged CS theory.

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GENERALIZATIONS

- Multiple scalars
- Different L-R chiral algebras
- Non rational compact boson
- Non abelian case ←

NON ABELIAN CASE

$G_k \times G_{-k}$ CS theory w/ G non-abelian gauge group.

holo / anti-holo b.c. $\Rightarrow$ $(\mathfrak{g}_k)_L \times (\mathfrak{g}_k)_R$
chiral algebra

We want an analogous gauging that gives a modular invariant RCFT on the bdry with $(\mathfrak{g}_k)_L \times (\mathfrak{g}_k)_R$ symmetry, e.g. WZW models.

However now the lines do not form a group.

Non-invertible 1-form symmetry

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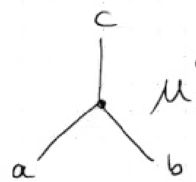
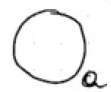
Structure of fusion / braiding of lines described by a modular tensor category (MTC)

→ objects: lines 

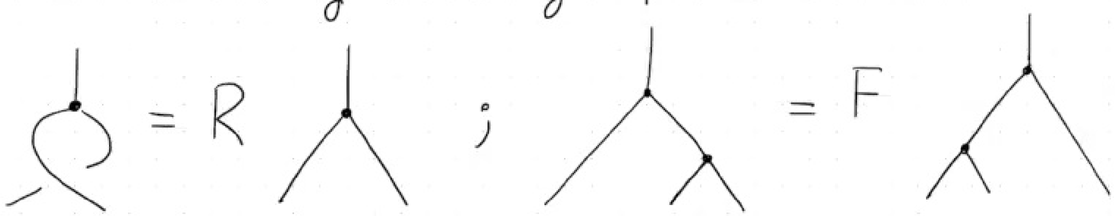
→ morphism: junctions 

$\mu_{ab}^{c} : a \otimes b \to c$

→ trace: quantum dimension 

$= \dim(a)$

Characterized by braiding & fusion matrices



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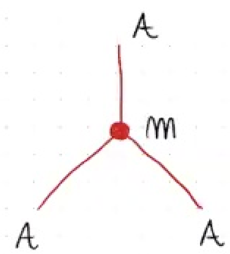
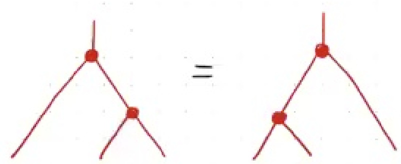
The concept of gauging is replaced by

anyon condensation [Bais, Slingerland; Kong]

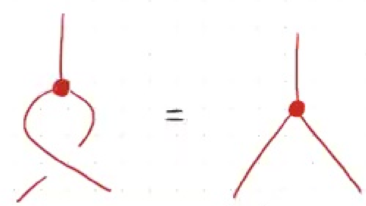
$A = \sum_a Z_a^A \, a$ algebra object (special Frobenius)



$m : A \otimes A \to A$



associative



commutative

"compatibility" of m with R and F

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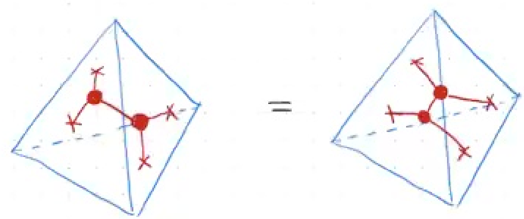
Condensation

- Triangulation of the manifold
- Network of A on the dual edges

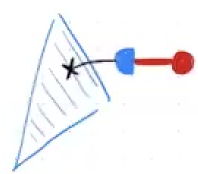


- On boundary faces

"Dirichlet"

$\circ : A \to \mathbb{1}$

trivial line



insertion of a boundary operator

$\blacksquare : A \to a$

projector to simple line $a \in A$

Total dimension of MTC $= \sqrt{\sum_a \dim(a)^2}$

Dimension after condensation $= \dfrac{\text{Dim. before}}{\dim A}$

where $\dim A = \sum_a Z_a^A \dim(a)$.

For our construction we need A such that after condensation $\dim = 1 \Rightarrow$ trivial bulk.

One option always exists in $G_k \times G_{-k}$:

$$A = \sum_{\lambda \in \text{int. rep.}} (\lambda)_L \otimes (\lambda)_R \implies$$ diagonal WZW model on the boundary

Other options $\longrightarrow$ non-diagonal modular invariants

e.g. ADE classification for $SU(2)_k$ [Cappelli, Itzykson, Zuber]

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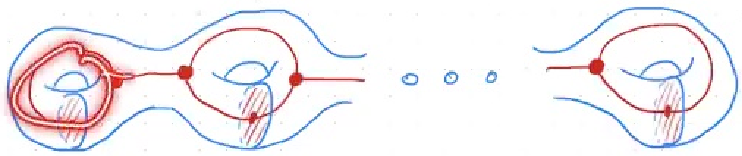
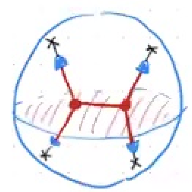
CHECKS

- Partition function on handle bodies

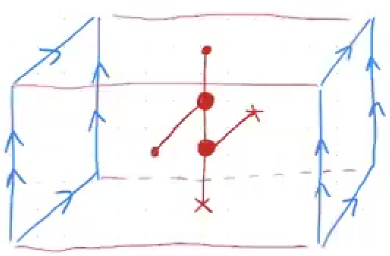


- Correlation functions



} Network of A $\Updownarrow$ expansion in conformal blocks

- Factorization on $T^2 \times I$



$\longrightarrow$ check with explicit formula in terms of R, F for $SU(2)_k$ [Fuchs, Runkel, Schweigert]

$\longrightarrow$ general proof by cutting and simplification of the network

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FUTURE DIRECTIONS

- Spin Chern-Simons Theories
- Orbifold branch of Compact Scalar
- Non-compact case $\longleftrightarrow$ 3d gravity ?
- Higher dimensional examples with TQFT in the bulk ?

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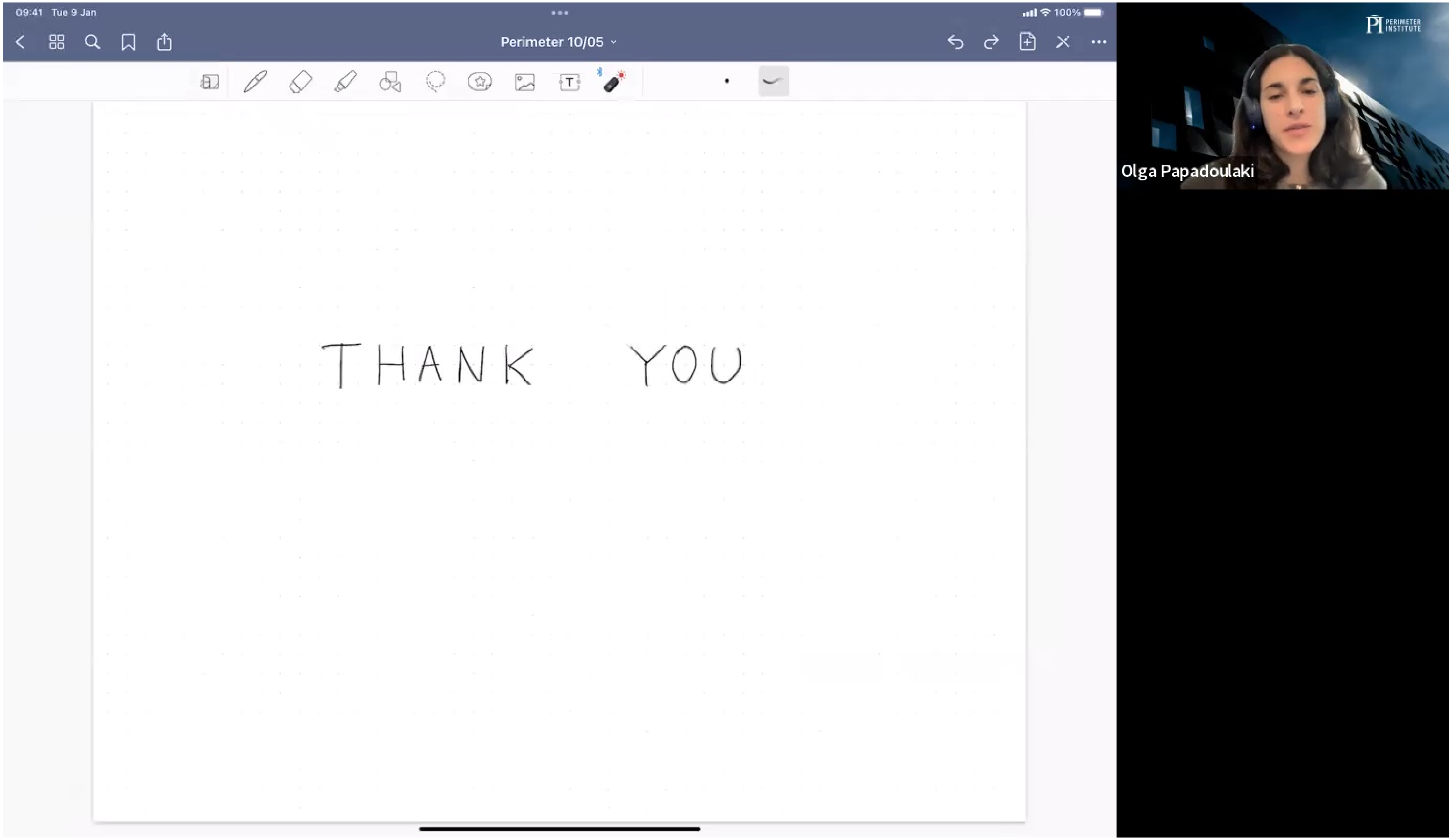
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