Title: States of Low Energy in Loop Quantum Cosmology

Speakers: Mercedes Martin-Benito

Series: Quantum Gravity

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Abstract: States of Low Energy (SLEs) have been constructed in cosmological spacetimes as the ones that minimize the mode contribution to the regularized energy density. These have been shown to be an adequate choice of vacuum state for primordial perturbations in models that include a period of kinetic domination prior to inflation. This is precisely the case in Loop Quantum Cosmology (LQC). In this talk we will review the construction and properties of SLEs for a general FLRW model and explore their application to LQC.

Zoom Link: https://pitp.zoom.us/j/98112698584?pwd=NkFhMWVNM3gvc1lYQ3cvVDFGL0laUT09

States of Low Energy in Loop Quantum Cosmology

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Perimeter Institute for Theoretical Physics
May 12th 2022

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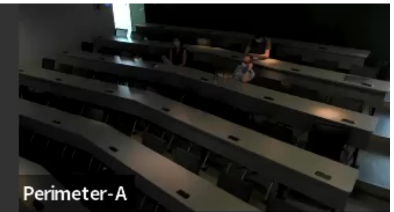
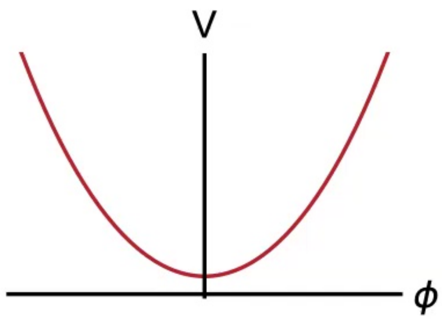
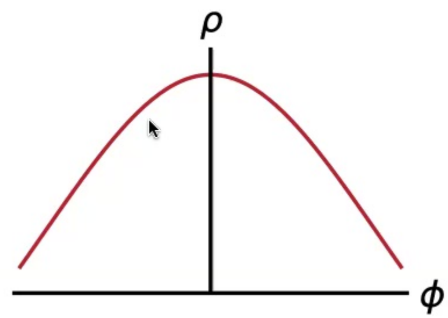


Loop Quantum Cosmology: FLRW

- LQC applies techniques of canonical LQGravity to cosmological models
- Flat FLRW + ϕ (minimally coupled scalar field)
- Most important result: singularity $\rightarrow$ bounce.



Volume has positive minimum.



Energy density has finite maximum ρ_c.

Loop Quantum Cosmology: FLRW

- Effective dynamics: corrected Friedmann equation,

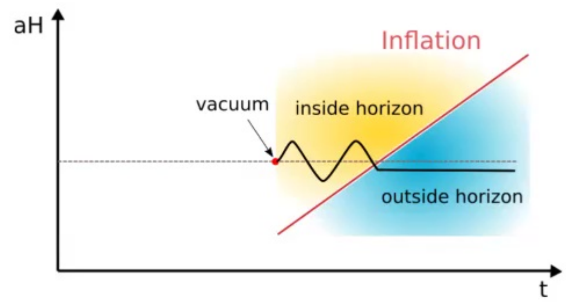
$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right).$$

- $\rho \ll \rho_c \rightarrow$ classical trajectory
- $\rho \sim \rho_c \rightarrow$ quantum effects render gravity repulsive $\rightarrow$ bounce

Cosmological perturbations: motivation

- Connection between theoretical models of the early universe and observations (CMB),
- During inflation modes freeze once they cross out of the horizon
- Cross back in after inflation ends

⇒ What we see in the CMB is generated in the very early universe



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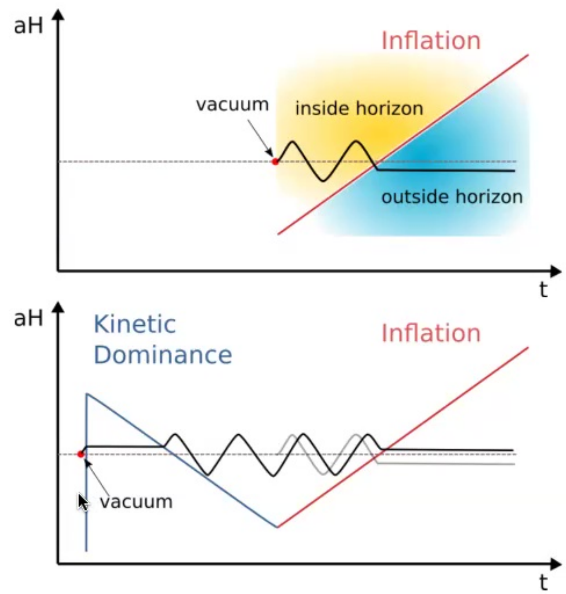

Cosmological perturbations: predictions

- Inflaton field ϕ subject to potential $V(\phi)$,
- Fourier modes of scalar and tensor gauge invariant perturbations, minimally coupled: $\mathcal{Q}$, $\mathcal{T}^I \longrightarrow T$, $\omega^{(s)}$, $\omega^{(t)} \longrightarrow \omega^{(i)}$

$$\ddot{T}_k + 3H\dot{T}_k + \left(\omega_k^{(i)}(t)\right)^2 T_k = 0$$

- Redefinition: $u = aT$, conformal time η

$$u''_{\vec{k}}(\eta) + \left(k^2 + s^{(i)}(\eta)\right) u_{\vec{k}}(\eta) = 0,$$

- $\left(\omega_k^{(i)}(t)\right)^2 = \dfrac{k^2 + s^{(i)}(t)}{a^2} + H^2 + \dfrac{\ddot{a}}{a}$

Cosmological perturbations: predictions

in LQC:

- dressed metric approach
- hybrid LQC:

$$s^{(t)} = -\frac{4\pi G}{3} a^2 (\rho - 3P),$$
$$s^{(s)} = s^{(t)} + \mathcal{U},$$

$$\mathcal{U} = \mathcal{U}\left[V(\phi), V_{,\phi}, V_{,\phi\phi}, \text{bckg}\right]$$

$\hookrightarrow$ No analytical solutions.

Cosmological perturbations: predictions

1. Choose vacuum (initial conditions)
2. Evolve perturbations until all scales have crossed horizon ($\eta_{\rm end}$)
3. Compute power spectra:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z^2}\Big|_{\eta=\eta_{\rm end}} \qquad \mathcal{P}_{\mathcal{T}}(k) = \frac{32k^3}{\pi}\frac{|\mu_k|^2}{a^2}\Big|_{\eta=\eta_{\rm end}},$$

(scalars:u, tensors: μ, $z = a\dot{\phi}/H$)

Cosmological perturbations: vacuum

- Cosmological perturbations in (L)QC: observational window;
- Freedom in choice of vacuum state;
- Natural vacuum?
  - Adiabatic,
  - Diagonalization of Hamiltonian[1]
  - Minimization of renormalized stress-energy tensor[2], uncertainty relations[3],
  - Minimization of smeared quantities[4].

[1] Fahn, et. al, Universe **5**, 170 (2019), Elizaga Navascués, et. al, Class. Quant. Grav. **36**, 185010 (2019)

[2] Agullo, et. al, PRD **91**, 064051 (2015), Handley, et. al, PRD **94**, 024041 (2016)

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[4] de Blas, et. al, JCAP **06**, 029 (2016), Elizaga Navascués, et. al, Class. Quant. Grav. **38**, 035001 (2020), Martín-Benito, RN, Olmedo: Phys. Rev. D **103**, 123524 (2021), Front.Astron.Space Sci. 0 (2021) **133**

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States of Low Energy [Olbermann, Class. Quant. Grav. **24**, 5011 (2007)]

- Fewster 2000 [Class. Quant. Grav. **17**, 1897–1911 (2000)]:
  - Renormalized energy density, smeared along time-like curve, is bounded from below as function of state.
- Olbermann 2007:
  - Particularize for generic FLRW models,
  - Define procedure to find state that minimizes it mode by mode.
- Candidates for vacuum of perturbations in LQC:
  - Minimization of smeared energy density,
  - Exact Hadamard states.

States of Low Energy

Modes e.o.m.:

$$\ddot{T}_k + 3H\dot{T}_k + \omega_k^2(t)T_k = 0,$$

Minimize mode contribution to smeared energy density:

$$E(T_k) = \frac{1}{2}\int dt\, f^2(t)\left(\frac{|\pi_{T_k}|^2}{a^6} + \omega_k^2|T_k|^2\right), \quad \pi_{T_k} = a^3\dot{T}_k$$

- $f(t)$ smearing function (on the worldline of an isotropic observer).

Procedure

- Arbitrary solution S_k,
- Bogoliubov transformation:

$$T_k = \lambda(k) S_k + \mu(k) \bar{S}_k,$$

with $|\lambda(k)|^2 - |\mu(k)|^2 = 1$, fixing $\mu(k) \in \mathbb{R}^+$,

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- Defining:

$$c_1(k) := \frac{1}{2} \int dt\, f^2(t) \left(|\dot{S}_k|^2 + \omega_k^2 |S_k|^2 \right) = E(S_k),$$
$$c_2(k) := \frac{1}{2} \int dt\, f^2(t) \left(\dot{S}_k^2 + \omega_k^2 S_k^2 \right).$$

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Procedure

$$E(T_k) = \left(2\mu^2(k) + 1\right)\, c_1(k) + \underbrace{2\,\mu(k)\,\mathrm{Re}[\lambda(k)\,c_2(k)]}_{2\mu|\lambda||c_2|\cos[\mathrm{Arg}(\lambda)+\mathrm{Arg}(c_2)]}\ .$$

Minimize $E(T_k) \Rightarrow \mathrm{Arg}[\lambda(k)] = \pi - \mathrm{Arg}[c_2(k)]$:

$$E(T_k) = (2\mu^2(k) + 1)c_1(k) - 2\mu(k)\sqrt{\mu^2(k) + 1}|c_2(k)|.$$

Minimizing $E(T_k)$ with respect to μ:

$$\mu(k) = \sqrt{\frac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} - \frac{1}{2}},$$

$$\lambda(k) = -e^{-i\mathrm{Arg}[c_2(k)]}\sqrt{\frac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} + \frac{1}{2}}\ .$$

Procedure

State of Low Energy associated with $f(t)$

$$T_k = \lambda(k) S_k + \mu(k) \bar{S}_k,$$

$$\begin{cases} \mu(k) &= \sqrt{\dfrac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} - \dfrac{1}{2}}, \\ \lambda(k) &= -e^{-i\mathrm{Arg}[c_2(k)]}\sqrt{\dfrac{c_1(k)}{2\sqrt{c_1^2(k) - |c_2(k)|^2}} + \dfrac{1}{2}}\,. \end{cases}$$

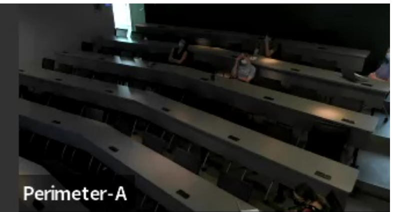
↪ Dependence on smearing function (through c_1 and c_2).

State of Minimal Energy

The state T_k that is of low energy for *any* $f(t)$.
Only exists in ultrastatic spacetimes.

PROPERTIES [BANERJEE, NIEDERMAIER, J. MATH. PHYS. **61**, 103511 (2020)]

- In maximally symmetric spacetimes:
  - Minkowski: SLE trivially identical to Minkowski vacuum;
  - de Sitter: singles out Bunch-Davies;
- Independence of fiducial solution S_k;
- T_k admits UV and IR expansions;
- UV aymptotic behavior of $|T_k|^2$ is independent of f (Minkowski-like);
- IR asymptotic behavior is the same for all f (up to a constant);
- As vacuum states of perturbations: agreement with observations for cosmological models with period of kinetic dominance prior to inflation.



SLEs in LQC [Phys. Rev. D **103**, 123524 (2021)]

1. Obtain S_k numerically
   - give initial conditions (irrelevant)
     ↪ 0th order adiabatic at bounce:

$$u_k(0) = \frac{1}{\sqrt{2k}},\ u'_k(0) = -i\sqrt{\frac{k}{2}}$$

   - $V(\phi) = m^2\phi^2/2$
   - $m = 1.2 \times 10^{-6}\, m_{\mathrm{Pl}}$
   - $\phi_B = 1.225\, m_{\mathrm{Pl}}$

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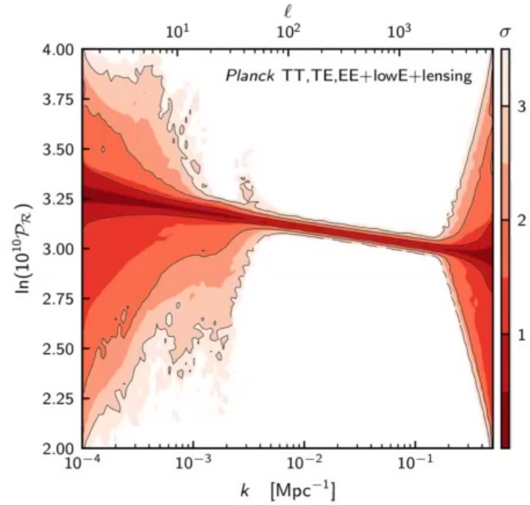
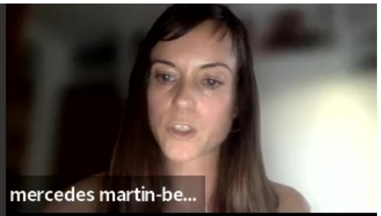


Planck 2018: predictive posterior plot of (Bayesian reconstruction of) primordial power spectrum.



SLEs in LQC [Phys. Rev. D **103**, 123524 (2021)]

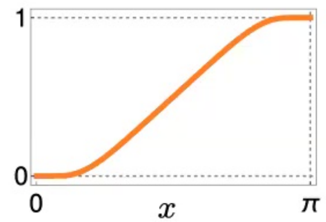
❷ choose $f^2(t)$:

- usually in LQC, 2 strategies for choosing initial conditions of perturbations
  - at bounce
  - at asymptotic time well before bounce
- here: no need for initial conditions, but need to choose f^2:
  - support on "whole" evolution (since well before to well after bounce):
    $\rightarrow$results insensitive to shape and support of f^2 as long as it is wide enough
  - support on expanding branch only (f^2 is sharp (but smooth) step function starting at bounce):
    $\rightarrow$results insensitive to support as long as it is wide enough

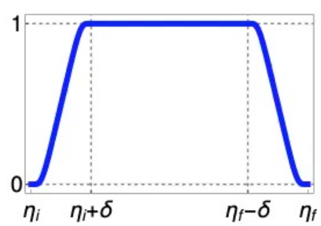
SLEs in LQC [Phys. Rev. D **103**, 123524 (2021)]

❷ choose $f^2(t)$:

- support on "whole" evolution vs expanding branch only
- smooth step function:



$$S(x) = \frac{1 - \tanh\left[\cot(x)\right]}{2},$$



$$f^2(\eta) = \begin{cases} S\left(\frac{\eta - \eta_i}{\delta}\pi\right) & \eta_i \leq \eta < \eta_i + \delta, \\ 1 & \eta_i + \delta \leq \eta \leq \eta_f - \delta, \\ S\left(\frac{\eta_f - \eta}{\delta}\pi\right) & \eta_f - \delta < \eta \leq \eta_f. \end{cases}$$

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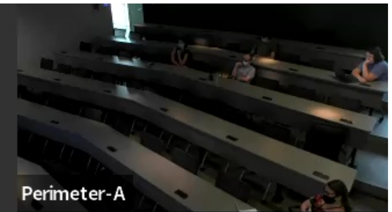

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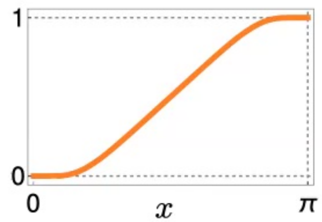

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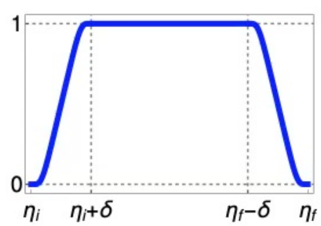

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❸ compute $c_1, c_2 \longrightarrow \mu, \lambda \longrightarrow T_k$

SLEs in LQC - Results [Phys. Rev. D **103**, 123524 (2021)]

- Power spectra ($z = a\dot{\phi}/H$):

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|u_k|^2}{z^2}\Big|_{\eta=\eta_{\rm end}},$$

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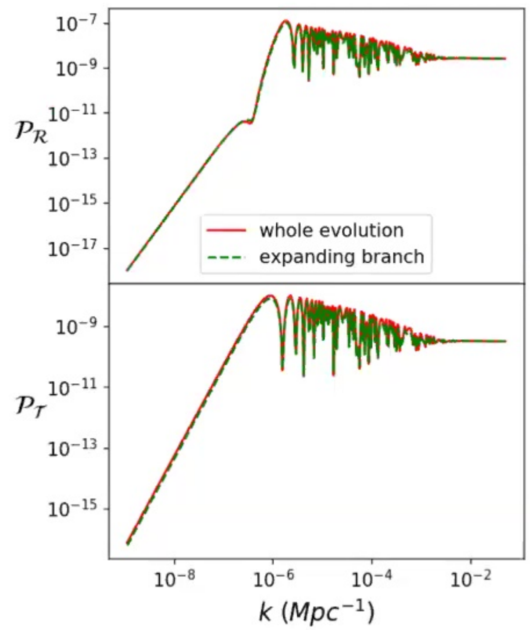

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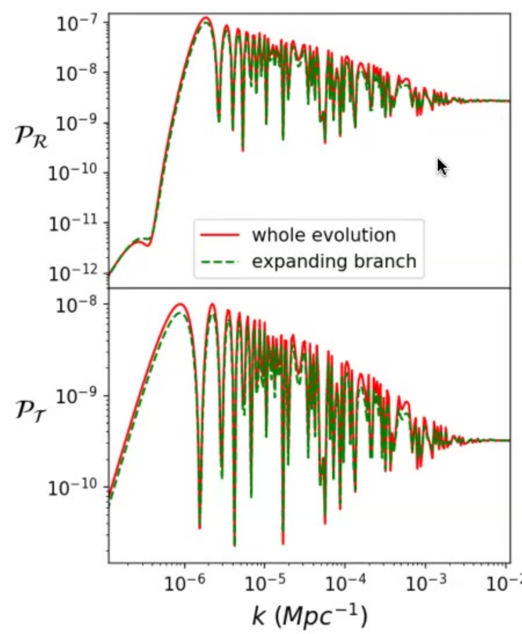

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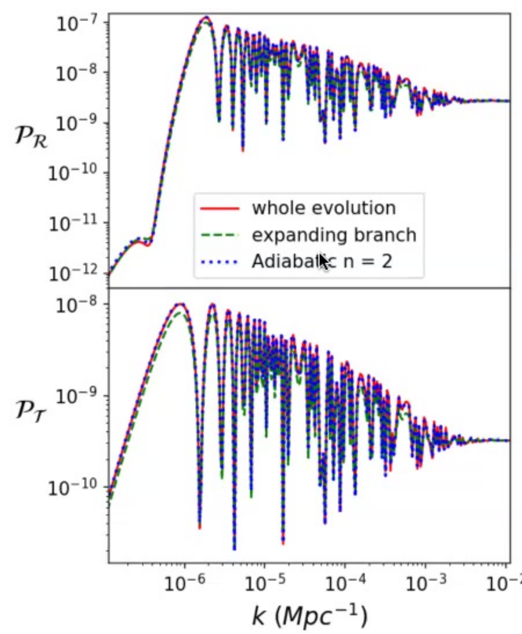
- Observationally: close to 2nd order Adiabatic;



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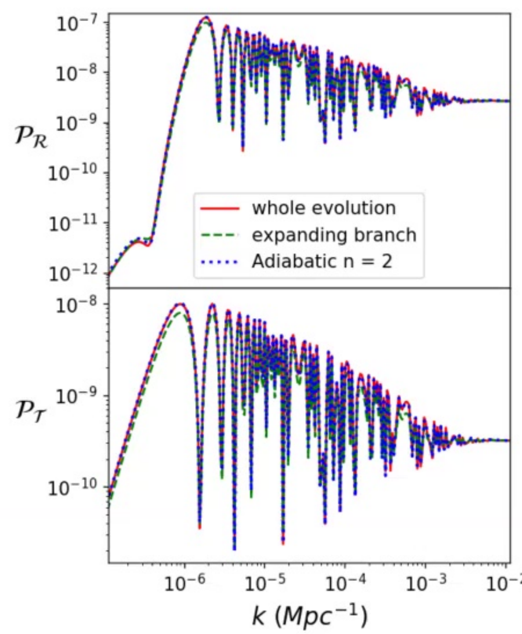
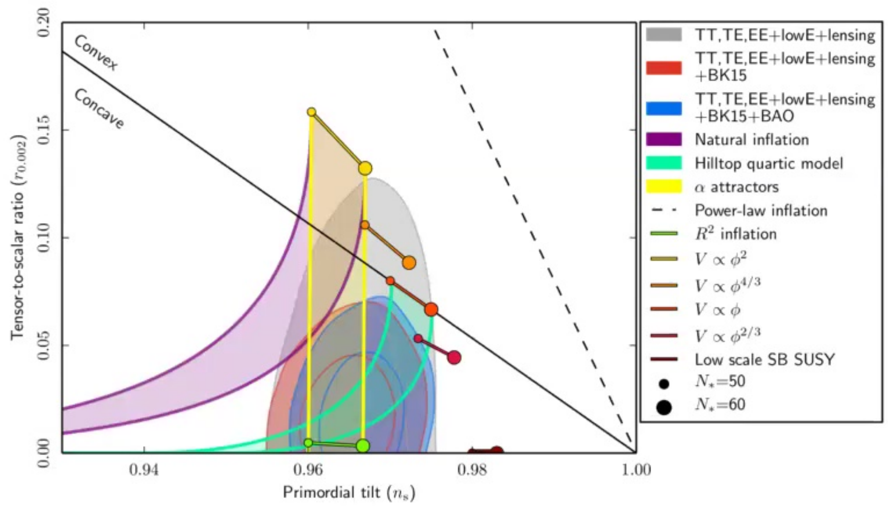
- Observationally: close to 2nd order Adiabatic;
- Fundamentally different, SLEs:
  - minimize smeared energy density,
  - are exact Hadamard states



SLEs in LQC - Results

- Tensor-to-scalar ratio: $r_{0.002} \simeq 0.117$
- Spectral index: $n_s \simeq 0.969$



Planck 2018: Y. Akrami et al.(Planck), Astron. Astrophys. **641**, A10 (2020)

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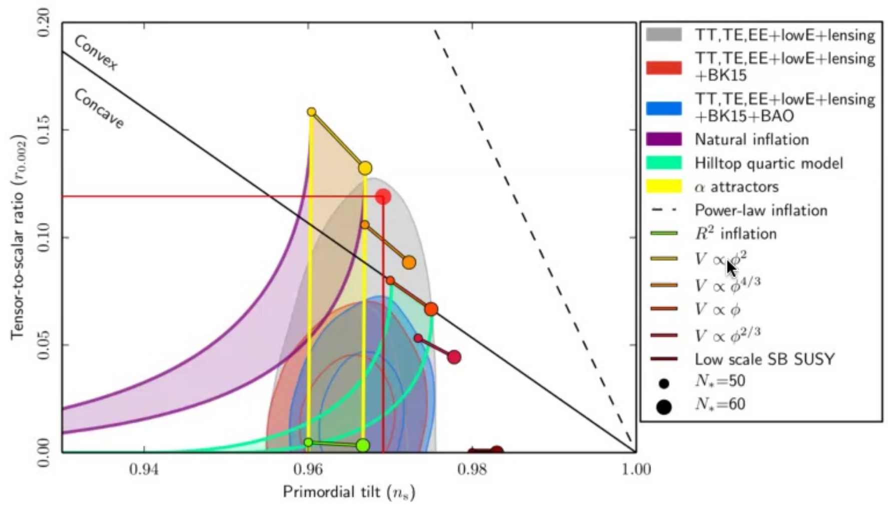

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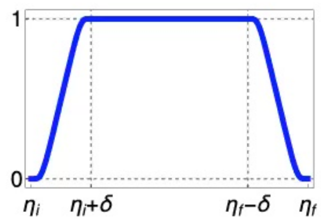
SLEs away from LQC [Front.Astron.Space Sci. 0 (2021) **133**]

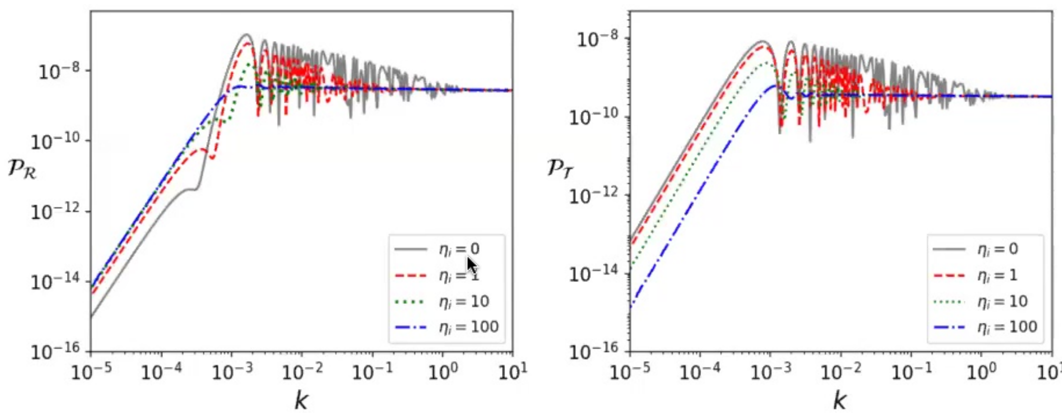
Disentangle effects of LQC from kinetic dominance [not necessarily quantum]

- choose $f^2(t)$:
  - support on expanding branch including vs excluding bounce
  - smooth step function:



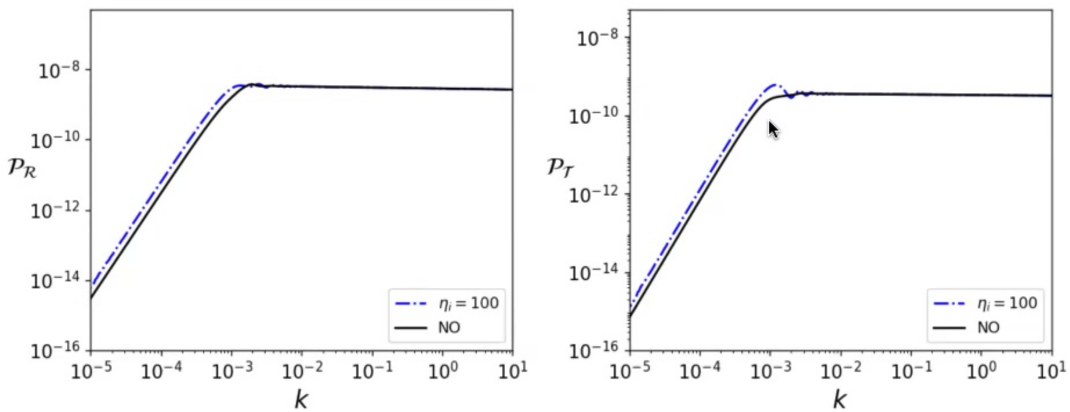
$\longrightarrow \eta_i \geq \eta_{\text{Bounce}} = 0$

SLEs away from LQC - Results [Front.Astron.Space Sci. 0 (2021) **133**]

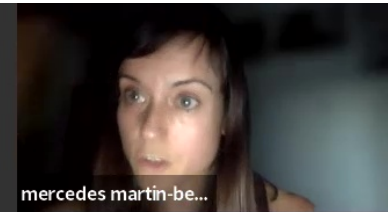


SLEs vs NO vacuum [Front.Astron.Space Sci. 0 (2021) **133**]

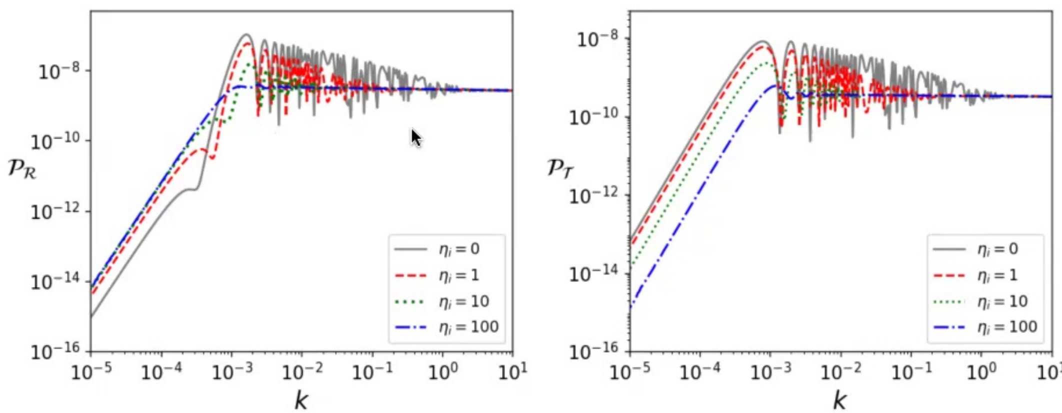
- Non-Oscillatory (NO) vacuum [de Blas, Olmedo, JCAP **06**, 029 (2016)]:
  minimize mode by mode amplitude of oscillations in given *time* interval (includes bounce) $\longrightarrow$ minimizes oscillations *in k*.



$\longrightarrow$ Comparing: SLEs are Hadamard $\Rightarrow$ NO is Hadamard



SLEs away from LQC - Results [Front.Astron.Space Sci. 0 (2021) **133**]



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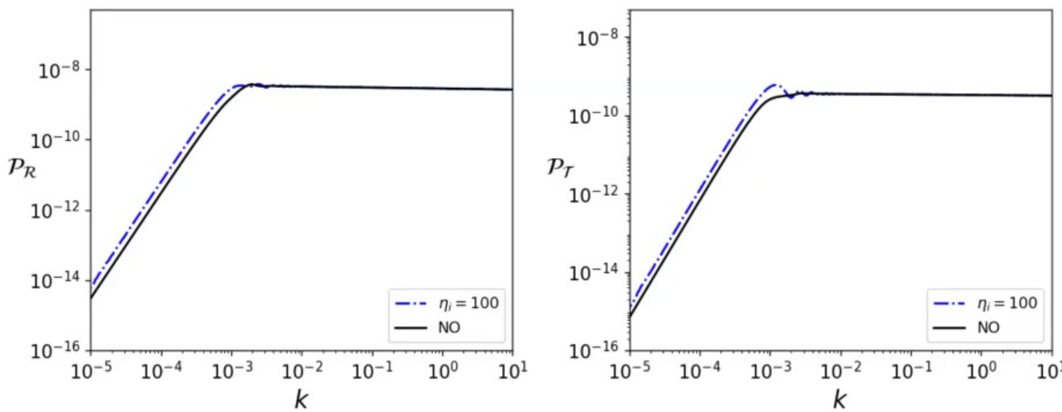

$\longrightarrow$ Comparing: SLEs are Hadamard $\Rightarrow$ NO is Hadamard

Summary and outlook

- SLEs are good candidates for vacua:
  - Hadamard states,
  - Minimize smeared energy density,
  - IR and UV asymptotic behaviors independent of test function;
- IR and UV agreement with observations when KD precedes inflation;
- In LQC:
  - qualitative agreement with observations,
  - insensitive to choice of test function;

Summary and outlook

- Not including bounce:
  - effectively compare LQC with (classical) model with kinetic dominance before inflation
  - effects from the bounce visible in primordial power spectra
  - reproduces power spectra of NO vacuum of LQC
- Rigorous statistical Bayesian analysis:
  - parameters of the background;
  - realistic $V(\phi)$;