Title: The hybrid conformal bootstrap

Speakers: Ning Su

Series: Quantum Fields and Strings

Date: May 31, 2022 - 3:30 PM

URL: https://pirsa.org/22050053

Abstract: Three dimensional conformal field theories(CFTs) describe important critical physics in the real world. In the past few years much progress has been made in 3D CFTs using the conformal bootstrap. In particular, using numerical bootstrap, the critical exponents of the 3D Ising, Super-Ising, O(2), O(3) CFTs have been determined precisely with rigorous error bars, while using analytic bootstrap, the information of the leading twist operators at large spins can be computed analytically. The two bootstrap approaches are sensitive to different regions of the spectrum and complement each other. In this talk, I will first give a general review of the numerical bootstrap and analytic bootstrap. Then I will discuss how to combine the numerical bootstrap with the analytic lightcone bootstrap, i.e. a hybrid bootstrap method. As a result, the bootstrap prediction for the actual CFT can be significantly improved.

Zoom Link: https://pitp.zoom.us/j/94186668943?pwd=ejRRb0M4VXNjeVVCWGdnTHQyaWRyQT09

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The Hybrid Bootstrap

Ning SU
(University of Pisa)

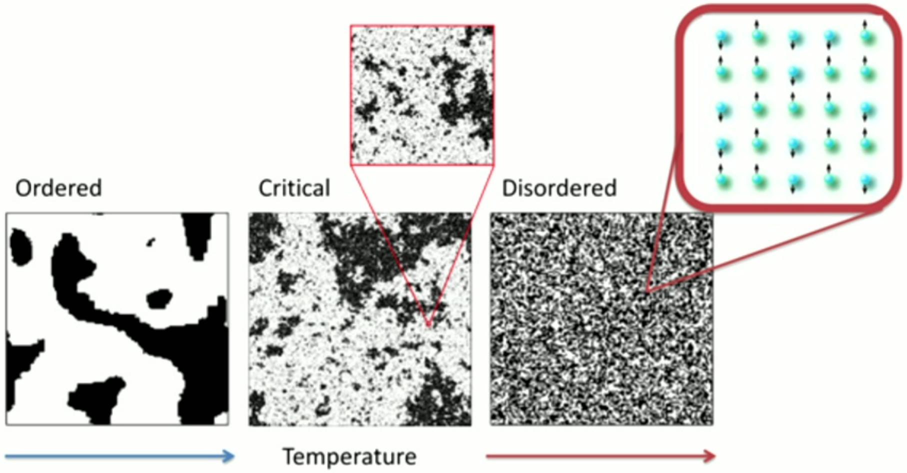
31/05/2022 @ Perimeter

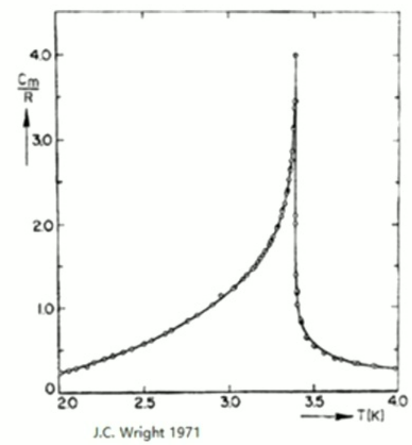
(Based on Ning Su, arXiv:2202.07607

funded by erc)

Critical phenomena : Ising model

$$\mathcal{H}_{\text{Ising}} = -J \sum_{\langle i\, j \rangle} \sigma_i \, \sigma_j$$





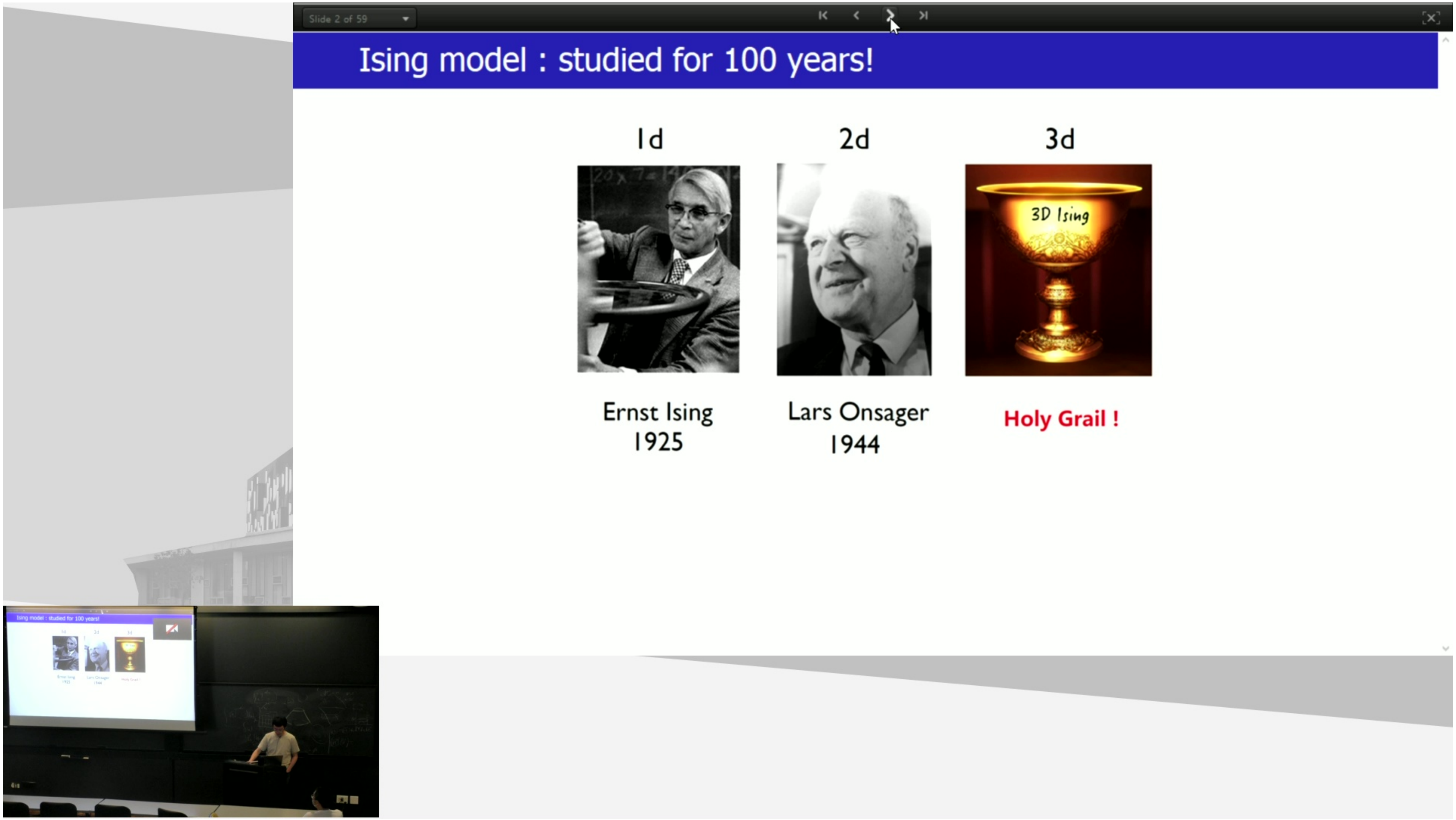
$$C_p \simeq (T - T_c)^{-\alpha}$$

critical exponent

Conformal invariance

$$\alpha = 2 - 3\,/\,(3 - \Delta_\epsilon)$$

$$\eta = 2\,\Delta_\sigma - 1$$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Ising model : studied for 100 years!
1d
2d
3d
3D Ising
Ernst Ising
1925
Lars Onsager
1944
Holy Grail !

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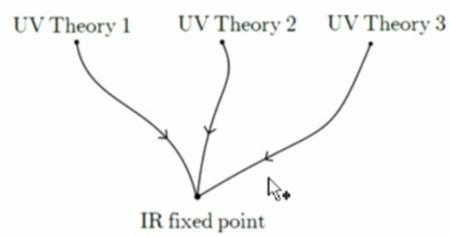
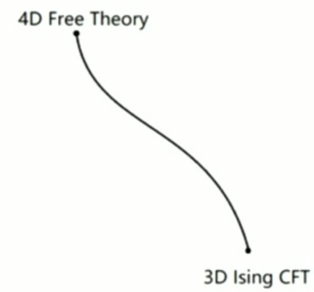
Traditional methods

Simulate an UV theory



4−ϵ perturbation theory



$$\Delta_\sigma = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} + \frac{109\,\epsilon^3}{11\,664} + \epsilon^4\left(\frac{7217}{1\,259\,712} - \frac{2\,\mathrm{Zeta}[3]}{243}\right) + \ldots$$

diverge

Better starting point for 3D universality class ?

A paradigm shift:

Study the theory exactly at the IR fixed point and non-perturbatively
(WITHOUT any flow)

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Conformal Bootstrap

Conformal Bootstrap :

use consistency conditions to constrain the macroscopic theory without resorting to the UV details

"If the 1971 renormalization group ideas had not been developed, the Migdal-Polyakov bootstrap would have been the most promising framework of its time for trying to further understand critical phenomena. However, the renormalization group methods have proved both easier to use and more versatile, and the bootstrap receives very little attention today." --- $K.G.$ Wilson 1982 Nobel Prize Lecture

Revived in 2008. Many progress in the past decades.

An organic mix of High Energy theory, Condensed matter Physics, Computer Science

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Constraints for conformal field theory

1, Spectrum: infinite set of operators $\mathcal{O}_{\Delta,\ell}$ spin scaling dimension

$\alpha = 2 - 3/(3-\Delta_\epsilon)$
$\eta = 2\,\Delta_\sigma - 1$

2, Interaction among operators : $\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k \mathcal{O}_k \sim \sum_k f_{ijk}\mathcal{O}_k$

Operator Product Expansion (OPE) coefficients

3, Crossing symmetry : $\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle \sim \sum_k \mathcal{O}_k = \sum_{k'} \mathcal{O}_{k'}$

All combined : $\boxed{\langle\phi\phi\phi\phi\rangle \Longrightarrow \sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}}\, F_{\Delta,\ell}(z,\bar{z}) = 0}$
What choices of $\{f_{ijk},\ \Delta_i\}$ *are consistent ?*

Constraints for conformal field theory

1, Spectrum: infinite set of operators $\mathcal{O}_{\Delta,\ell}$ (spin: ℓ; scaling dimension: Δ)

$$\alpha = 2 - 3/(3-\Delta_\epsilon)$$
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2, Interaction among operators : $\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k \mathcal{O}_k \sim \sum_k f_{ijk}\mathcal{O}_k$

Operator Product Expansion (OPE) coefficients

3, Crossing symmetry : $\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\mathcal{O}_4\rangle \sim \sum_k (\mathcal{O}_k \text{ exchange, 12} \to 34) = \sum_{k'} (\mathcal{O}_{k'} \text{ exchange, 14} \to 23)$

All combined : $\langle\phi\phi\phi\phi\rangle \Longrightarrow \sum_{\mathcal{O}} f^2_{\phi\phi\mathcal{O}}\, F_{\Delta,\ell}(z,\bar{z}) = 0$

What choices of $\{f_{ijk}, \Delta_i\}$ *are consistent ?*

Example of the bootstrap equation

Consider four-point function $\langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_4) \rangle$ in 2D (Dolan and Osborn 2001, 2004) :

$$\sum_{O \in \phi \times \phi} f_{\phi\phi O}{}^2\, F_{\Delta,\ell}(z,\, \bar{z}) = 0$$

$$F_{\Delta,\ell} = ((1-z)\,(1-\bar{z}))^{\Delta_\phi}\, g_{\Delta,\ell}(z,\, \bar{z}) - (z\, \bar{z})^{\Delta_\phi}\, g_{\Delta,\ell}(1-z,\, 1-\bar{z})$$

$$g_{\Delta,\ell}(z,\, \bar{z}) = k_{\Delta+l}(z)\, k_{\Delta-l}(\bar{z}) + (z \leftrightarrow \bar{z})$$

$$k_\beta(z) = x^{\beta/2}\, {}_2F_1\left(\frac{\beta}{2},\, \frac{\beta}{2},\, \beta;\, z\right)$$

$$\frac{x_{12}{}^2\, x_{34}{}^2}{x_{13}{}^2\, x_{24}{}^2} = z\, \bar{z}, \quad \frac{x_{14}{}^2\, x_{23}{}^2}{x_{13}{}^2\, x_{24}{}^2} = (1-z)\,(1-\bar{z})$$

$$0 < z < 1,\;\; 0 < \bar{z} < 1$$

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Next Slide

Overview

- Introduction
- Numerical bootstrap
- Analytic bootstrap
- Hybrid bootstrap
- Outlook

Bootstrap workflow

Correlators
$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle$

→

Bootstrap Equations
$$\sum_O f_{12O} f_{34O} F^{\langle 1234 \rangle}_{\Delta,\ell}(z,\bar{z}) = 0$$

↓

Truncated Bootstrap Equations
$$\partial_z^m \partial_{\bar{z}}^n \left(\sum_O f_{12O} f_{34O} F^{\langle 1234 \rangle}_{\Delta,\ell}(z,\bar{z}) = 0 \right)_{z=\bar{z}=1/2} \quad \text{with } m+n \leq \Lambda$$

↓

Assumptions on spectrum
$(\Delta_\sigma, \Delta_\epsilon$, etc)

→ Bootstrap Numerics

↓

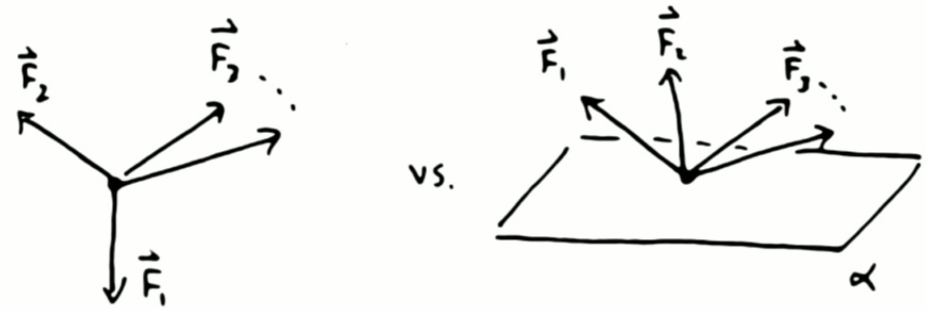
Feasibility Test
$(\Delta_\sigma, \Delta_\epsilon$ is allowed/disallowed)

Numerics

How to solve $\sum_{O\in\phi\times\phi} {f_{\phi\phi O}}^2 \, F_{\Delta,\ell}(z,\bar{z}) = 0$? (bootstrap Equ. from $\langle\phi\phi\phi\phi\rangle$)

Step 1 : $F_{\Delta,\ell}(z,\bar{z}) \simeq \sum_{m+n\leq\Lambda} c_{mn}\, \partial_z^m \partial_{\bar{z}}^n F_{\Delta,\ell}(z,\bar{z})$ $\qquad F_{\Delta,\ell}(z,\bar{z}) \rightarrow$ vector $(c_{01},\, c_{12},\, c_{23}\, \ldots)$

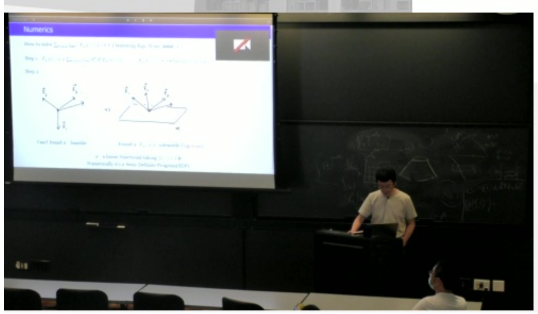
Step 2 :



Can't found α : feasible $\qquad$ Found $\alpha \cdot F_{\Delta,\ell} \geq 0$: infeasible (rigorous)

α : a linear functional taking $f(z,\bar{z}) \rightarrow \mathbb{R}$

Numerically it's a Semi-Definite Program(SDP).



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Bootstrap workflow

Correlators
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↓

Assumptions on spectrum
($\Delta_\sigma, \Delta_\epsilon$, etc)

→ Bootstrap Numerics

↓

Feasibility Test
($\Delta_\sigma, \Delta_\epsilon$ is allowed/disallowed)

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Numerics

How to solve $\sum_{\mathcal{O}\in\phi\times\phi} f_{\phi\phi\mathcal{O}}{}^2\, F_{\Delta,\ell}(z,\bar{z}) = 0$? (bootstrap Equ. from $\langle\phi\phi\phi\phi\rangle$)

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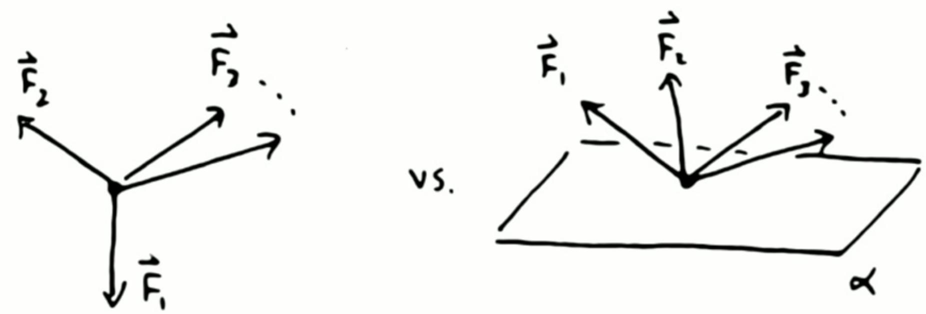

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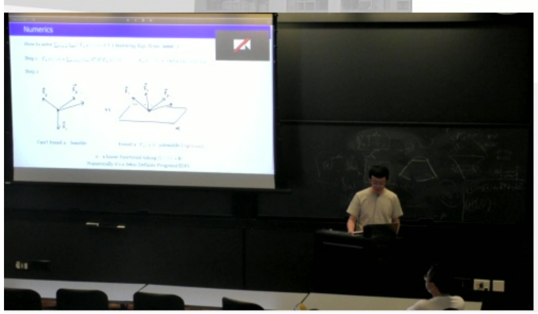

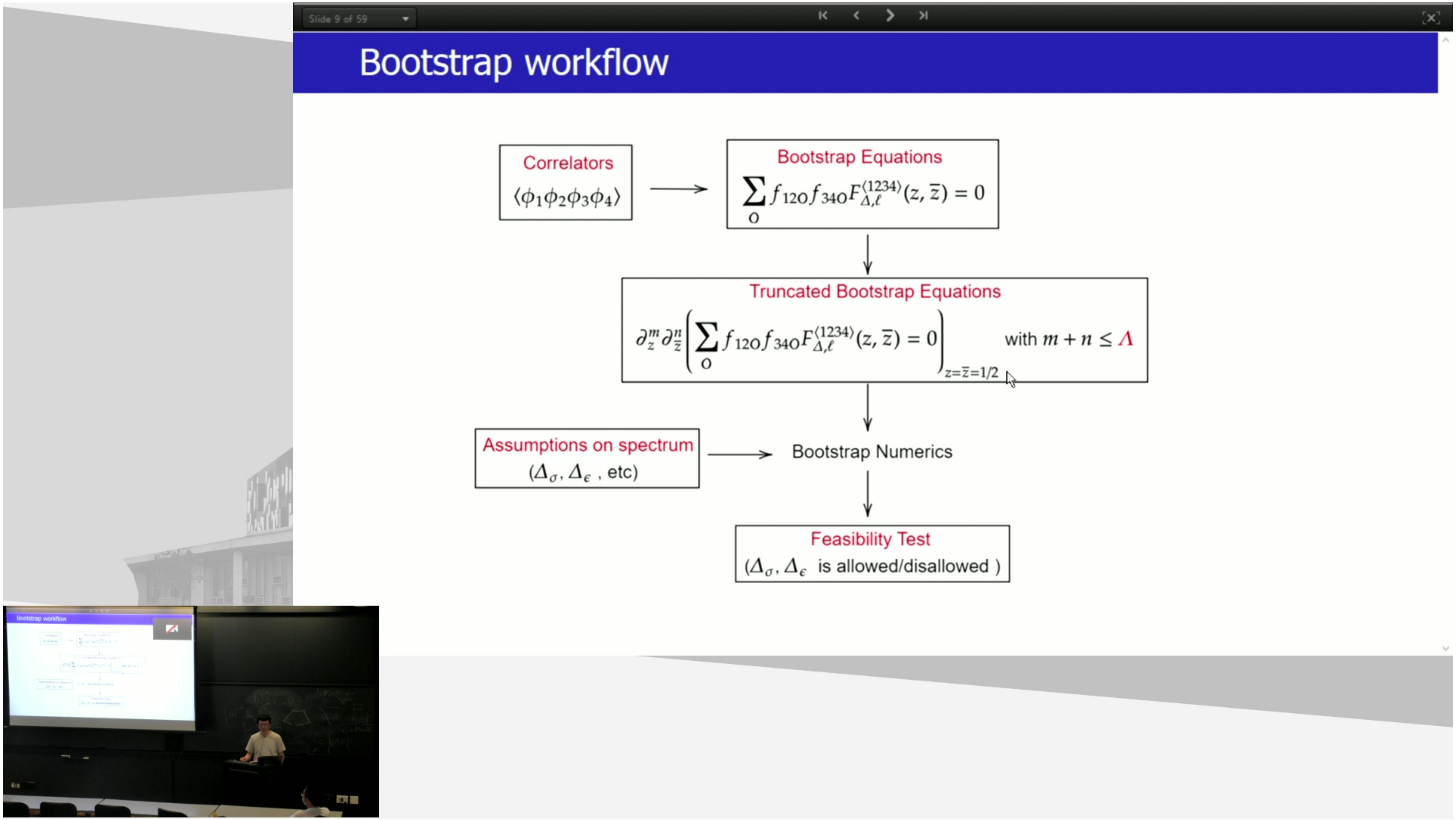
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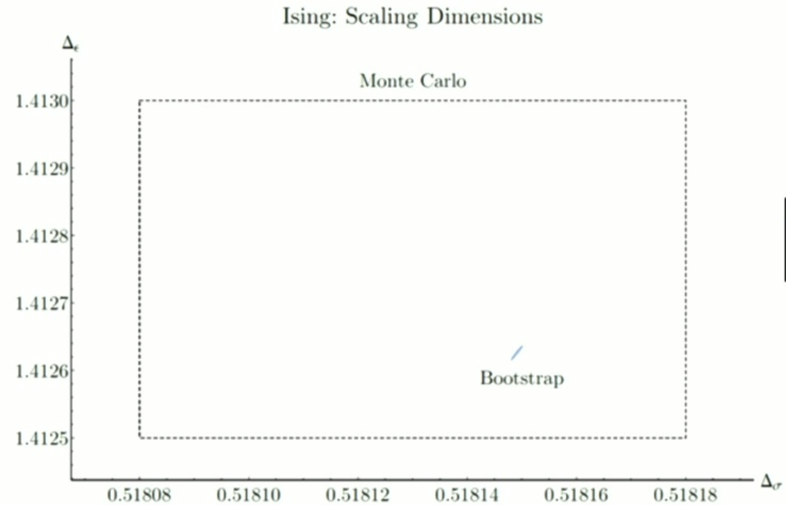
Bootstrap workflow
Correlators
$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle$
Bootstrap Equations
$\sum_{O} f_{12O} f_{34O} F^{\langle 1234 \rangle}_{\Delta,\ell}(z,\bar{z}) = 0$
Truncated Bootstrap Equations
$\partial_z^m \partial_{\bar{z}}^n \left(\sum_{O} f_{12O} f_{34O} F^{\langle 1234 \rangle}_{\Delta,\ell}(z,\bar{z}) = 0 \right)_{z=\bar{z}=1/2}$ with $m+n \leq \Lambda$
Assumptions on spectrum
($\Delta_\sigma, \Delta_\epsilon$, etc)
Bootstrap Numerics
Feasibility Test
($\Delta_\sigma, \Delta_\epsilon$ is allowed/disallowed)

Bootstrap 3D Ising CFT

Correlators : $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\epsilon\epsilon\epsilon\epsilon\rangle$, $\langle\epsilon\epsilon\sigma\sigma\rangle$

Derivative truncation : $\Lambda = 43$

Assumptions : σ, ϵ are the only two relevant scalars.



$\alpha = 2 - 3/(3 - \Delta_\epsilon)$
$\eta = 2\,\Delta_\sigma - 1$

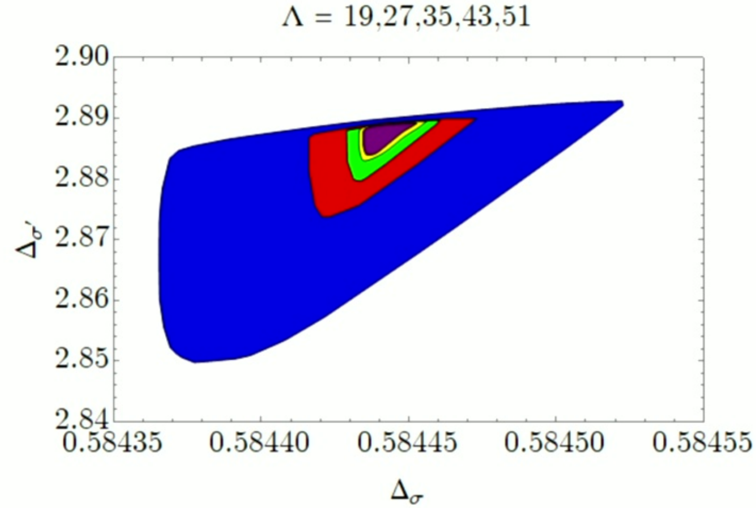
$\eta = 0.036298\,(\mathbf{2}),\ \ \alpha = 0.11008\,(\mathbf{1})$

(Kos, Poland, Simmons-Duffin, Vichi 2016)

Bootstrap 3D super-Ising CFT

$$\mathcal{L}_{\text{SuperIsing}} = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}\,\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{\lambda}{2}\,\sigma\,\bar{\psi}\,\partial\!\!\!/\,\psi + \frac{\lambda^2}{8}\,\sigma^4$$

Bootstrapping $\langle\sigma\sigma\sigma\sigma\rangle$ (Rong, NS 2018)



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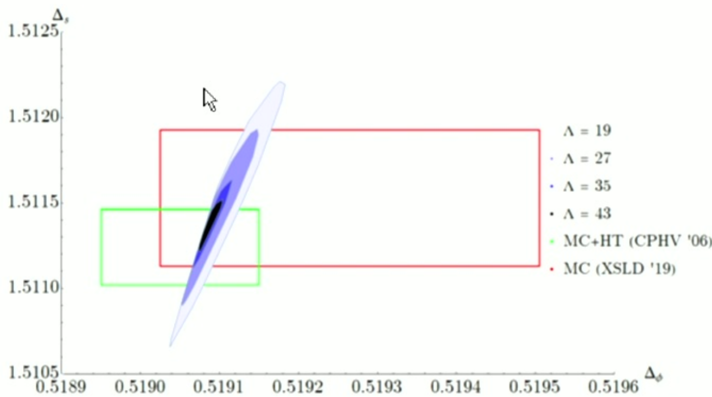
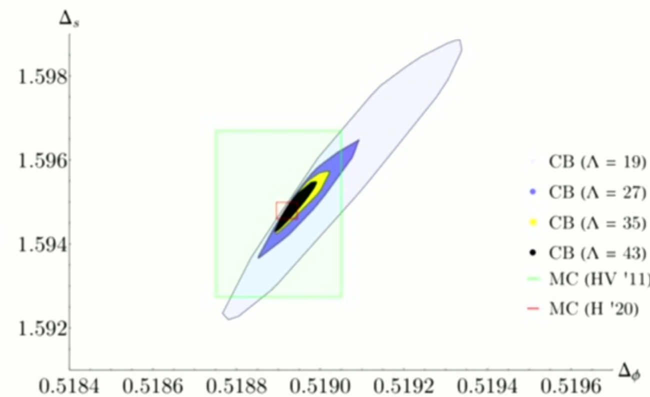
Bootstrap 3D O(N) CFTs

Lagrangian : $\mathcal{L} = \partial\phi_i\,\partial\phi_i + m^2\,(\phi_i\,\phi_i) + \lambda\,(\phi_i\,\phi_i)^2$

Bootstrapping all 4pt involves $\{v = \phi_i,\ s = \phi^2,\ t = \phi_i\,\phi_j\}$:





(Chester, Landry, Liu, Poland, Simmons-Duffin, SN, Vichi 2019, 2020)

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Difficulties in numerical bootstrap

- Need to scan a parameter space with a large dimensional
- Slow convergence at large Λ

Bootstrap 3D O(N) CFTs

Lagrangian : $\mathcal{L} = \partial\phi_i\,\partial\phi_i + m^2\,(\phi_i\,\phi_i) + \lambda\,(\phi_i\,\phi_i)^2$

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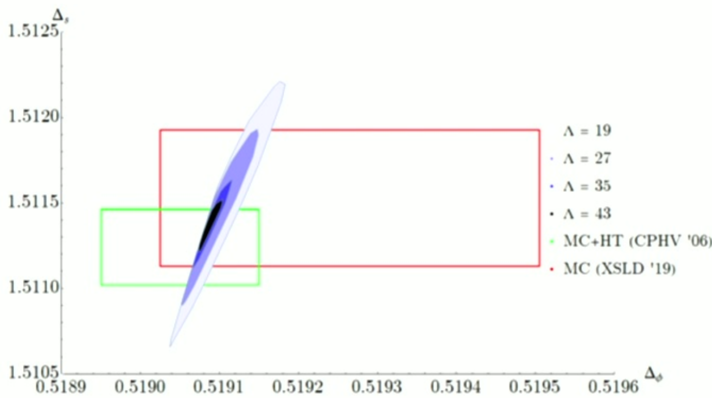
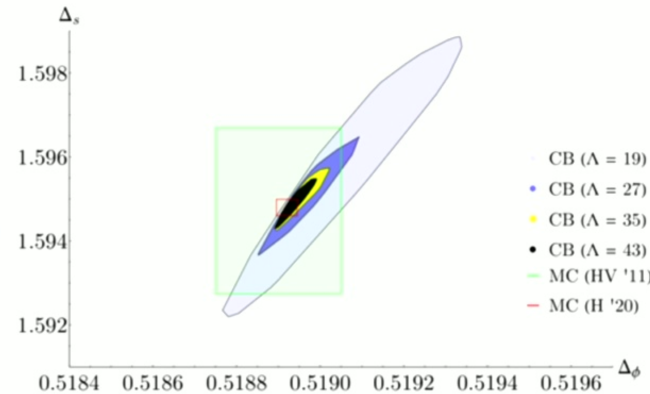
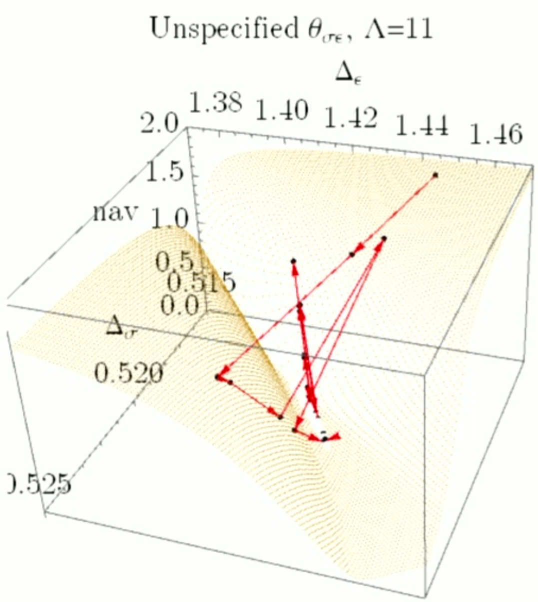


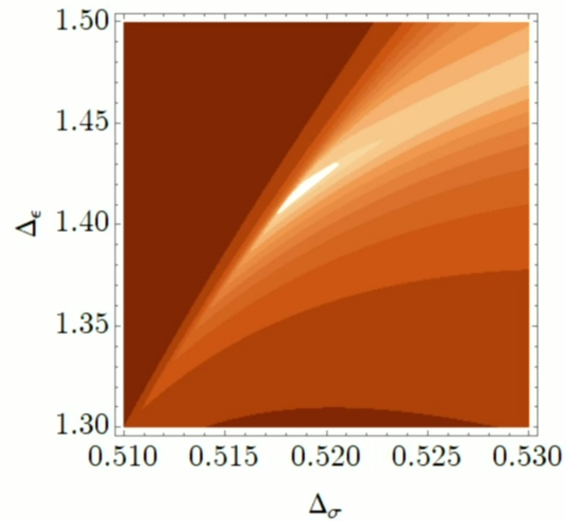


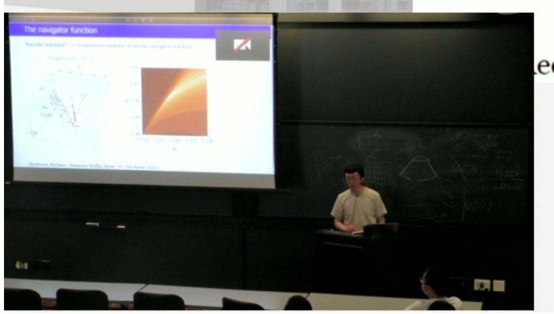
(Chester, Landry, Liu, Poland, Simmons-Duffin, SN, Vichi 2019, 2020)

The navigator function

"feasible/infeasible" $\rightarrow$ a continuous measure of success (navigator function)







eehorst, Rychkov, Simmons-Duffin, Sirois, SN, van Reese 2021)

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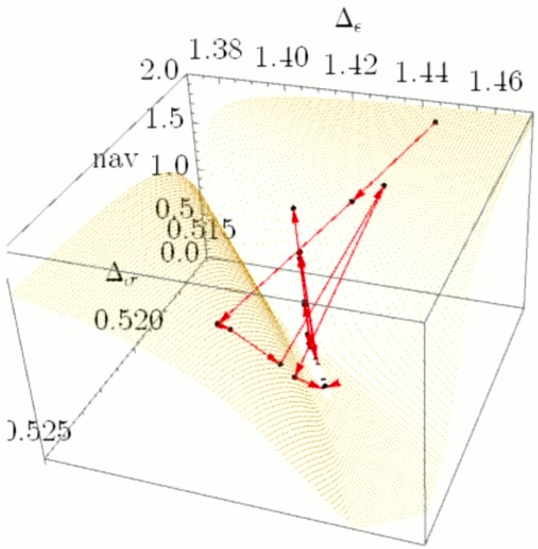

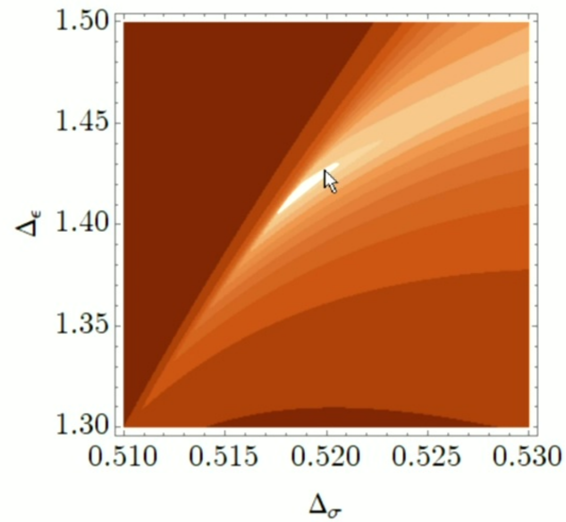

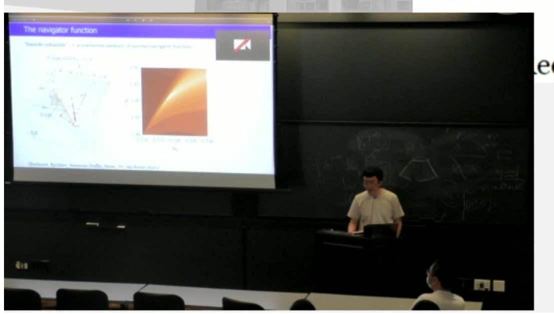

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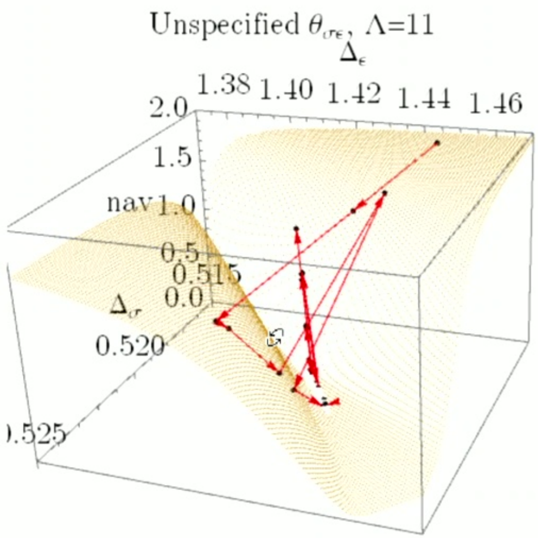

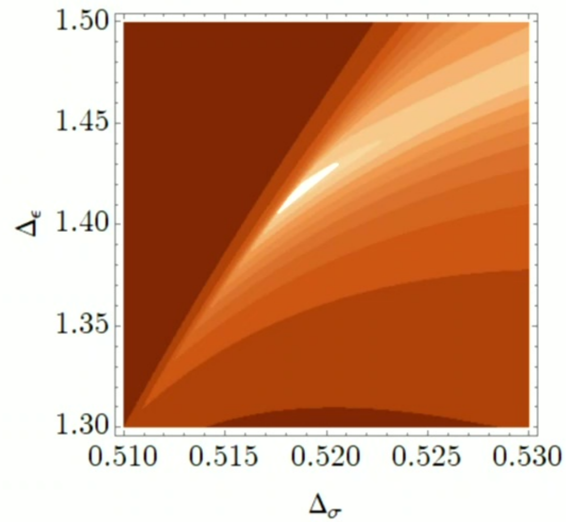

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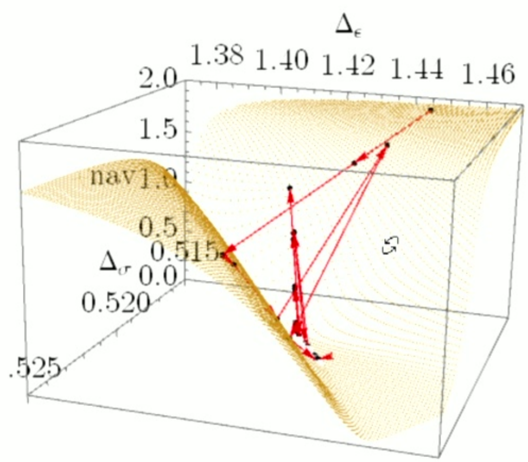
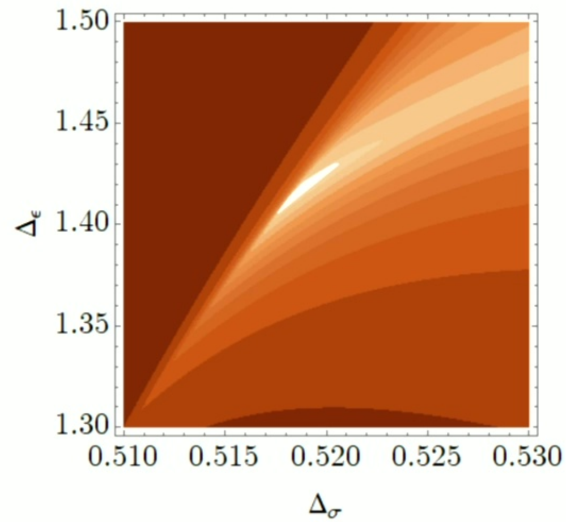
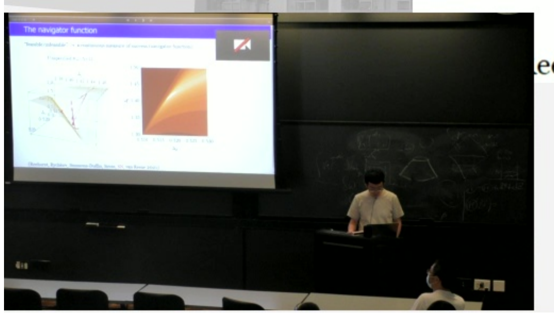
The navigator function

"feasible/infeasible" → a continuous measure of success (navigator function)

Unspecified $\theta_{\sigma\epsilon}$, Λ=11

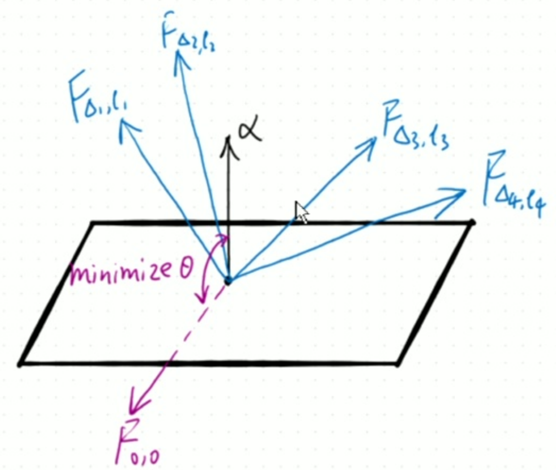






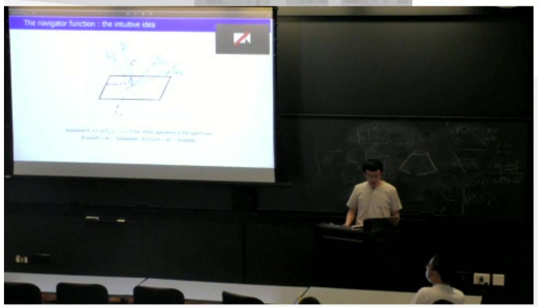
eehorst, Rychkov, Simmons-Duffin, Sirois, SN, van Reese 2021)

The navigator function : the intuitive idea



minimize θ , s.t. $\alpha(F_{\Delta,\ell}(z,\bar{z})) \geq 0$ for other operators in the spectrum

If $\min(\theta) \geq 90^\circ$: Infeasible; If $\min(\theta) < 90^\circ$: Feasible



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Overview

- *Introduction*
- *Numerical bootstrap*
- *Analytic bootstrap*
- *Hybrid bootstrap*
- *Outlook*

The navigator function

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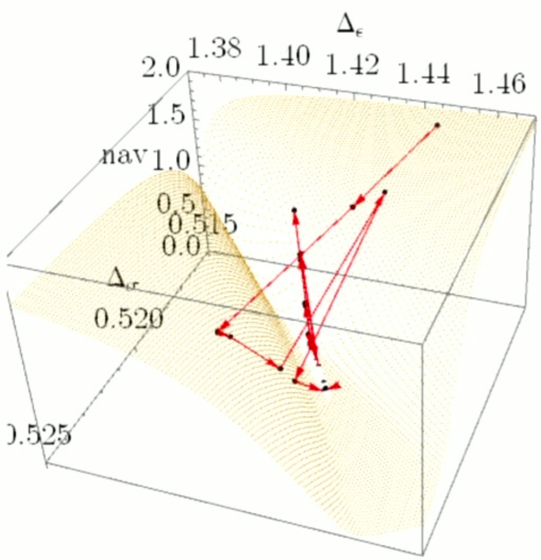

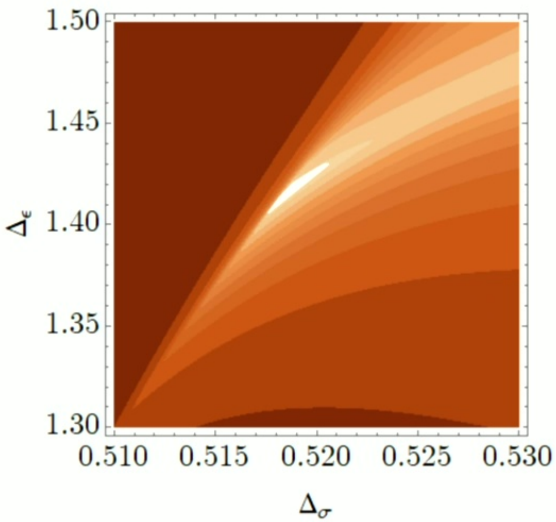

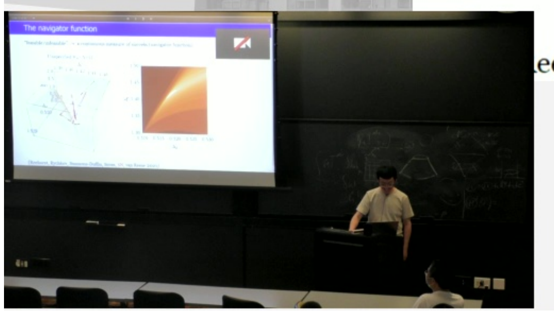

(…eehorst, Rychkov, Simmons-Duffin, Sirois, SN, van Reese 2021)

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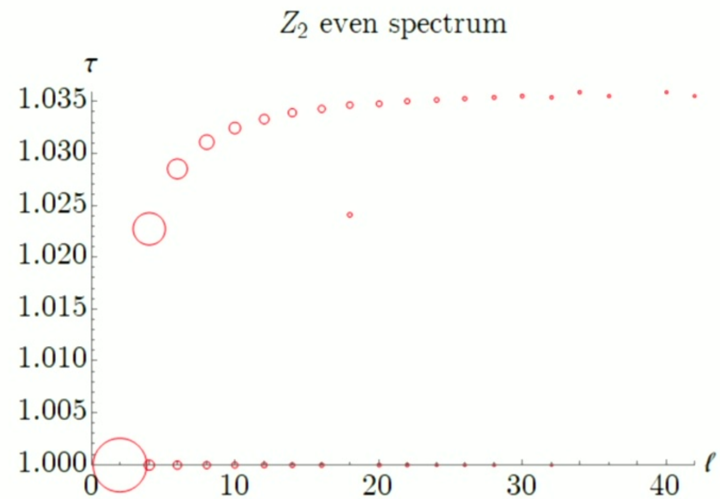
Analytic lightcone bootstrap



Leading twist operator in $\sigma \times \sigma$: $\Delta_{[\sigma\sigma]_{0,\ell}} \approx 2\,\Delta_\sigma + \ell + \delta$

Inversion formula at the lightcone limit $z << 1 - \bar{z} << 1$:
Information of low lying operators $\longrightarrow$ Information of the leading twist operators at large spin
pproximately).

Solving bootstrap equation at the lightcone limit

Prediction from the Inversion formula (Caron-Huot 2017; Liu, Meltzer, Poland, Simmons-Duffin 2020)

Generating function : $C(z, \bar{h}) = f_{\sigma\sigma|\sigma\sigma|_0}(\bar{h})\, z^{2\, h_{|\sigma\sigma|_0}(\bar{h})} + \ldots$

$C_O(z, \bar{h})_{\text{small } z} \approx f_{\sigma\sigma O}^2\, 2 \sin(\pi\, (h_O - 2\, h_\sigma))^2 \sum_{p=0}^{\infty} \sum_{q=-p}^{p} \mathcal{A}_{p,q}(h_O, \bar{h}_O)\, \frac{z^{2\, h_\sigma}\, k_{\bar{h}_O+q}(1-z)}{(1-z)^{2\, h_\sigma}}\, \kappa_{2\, \bar{h}}\, \Omega_{\bar{h}, h_{O+p}, 2\, h_\sigma}^{h_\sigma}$

$C(z, \bar{h}) = \sum_{O \in \sigma\times\sigma} C_O(z, \bar{h})$

At large $\bar{h}$, the main contribution to $C(z, \bar{h})$ at small z come from $C_O(z, \bar{h})$ with small h_O .

Information of small twist operators → Information about $[\sigma\sigma]_{0,\ell}$ at large ℓ .

Analytic lightcone bootstrap : Example

Information of low lying operator : $\{\epsilon,\ T\}$

Input : $\Delta_\sigma,\ \Delta_\epsilon,\ f_{\sigma\sigma\epsilon},\ f_{\sigma\sigma \mathrm{T}}$

Output :

$$\Delta_{[\sigma\sigma]_{0,\ell}} \approx 2\,\Delta_\sigma + \ell + \left(2\left(-\frac{128\, f_{\sigma\sigma\mathrm{T}}^2\,\Gamma(\bar h)^2\,\Gamma\left(\bar h+\Delta_\sigma-\frac{3}{2}\right)}{3\,\pi\,\Gamma(2\,\bar h-1)\,\Gamma\left(\Delta_\sigma-\frac{1}{2}\right)^2\,\Gamma\left(\bar h-\Delta_\sigma+\frac{3}{2}\right)} - \frac{f_{\sigma\sigma\epsilon}^2\,\Gamma(\bar h)^2\,\Gamma(\Delta_\epsilon)\,\Gamma\left(\bar h-\frac{\Delta_\epsilon}{2}+\Delta_\sigma-1\right)}{\Gamma(2\,\bar h-1)\,\Gamma\left(\frac{\Delta_\epsilon}{2}\right)^2\,\Gamma\left(\Delta_\sigma-\frac{\Delta_\epsilon}{2}\right)^2\,\Gamma\left(\bar h+\frac{\Delta_\epsilon}{2}-\Delta_\sigma+1\right)}\right)\right) \Big/$$

$$\left(-\frac{128\,\left(2\,\gamma+2\,\psi^{(0)}\left(\frac{5}{2}\right)\right)\, f_{\sigma\sigma\mathrm{T}}^2\,\Gamma(\bar h)^2\,\Gamma\left(\bar h+\Delta_\sigma-\frac{3}{2}\right)}{3\,\pi\,\Gamma(2\,\bar h-1)\,\Gamma\left(\Delta_\sigma-\frac{1}{2}\right)^2\,\Gamma\left(\bar h-\Delta_\sigma+\frac{3}{2}\right)} - \right.$$

$$\left.\frac{f_{\sigma\sigma\epsilon}^2\,\Gamma(\bar h)^2\,\Gamma(\Delta_\epsilon)\,\left(2\,\psi^{(0)}\left(\frac{\Delta_\epsilon}{2}\right)+2\,\gamma\right)\,\Gamma\left(\bar h-\frac{\Delta_\epsilon}{2}+\Delta_\sigma-1\right)}{\Gamma(2\,\bar h-1)\,\Gamma\left(\frac{\Delta_\epsilon}{2}\right)^2\,\Gamma\left(\Delta_\sigma-\frac{\Delta_\epsilon}{2}\right)^2\,\Gamma\left(\bar h+\frac{\Delta_\epsilon}{2}-\Delta_\sigma+1\right)} + \frac{\Gamma(\bar h)^2\,\Gamma\left(\bar h+\Delta_\sigma-1\right)}{\Gamma(2\,\bar h-1)\,\Gamma(\Delta_\sigma)^2\,\Gamma\left(\bar h-\Delta_\sigma+1\right)}\right)$$

$$\bar h = \left(\Delta_{[\sigma\sigma]_{0,\ell}} + \ell\right)/2$$

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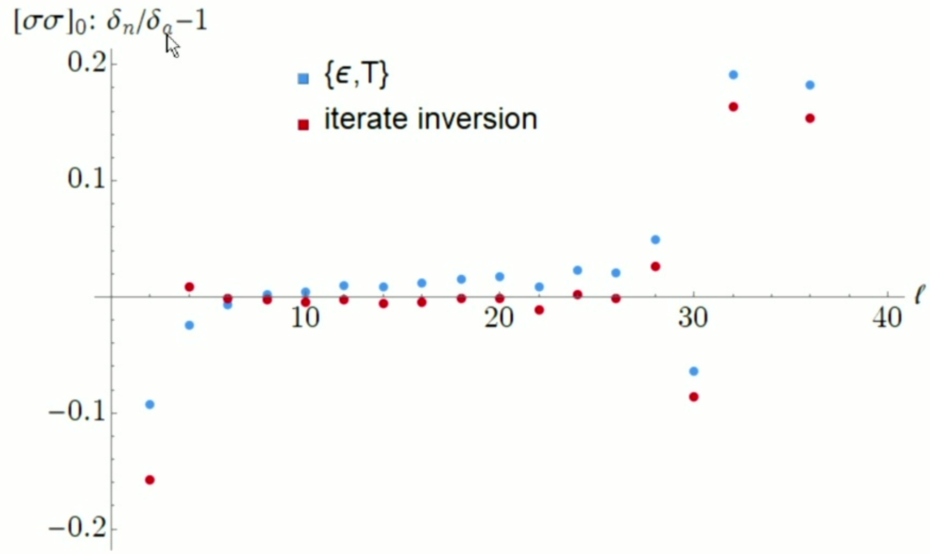
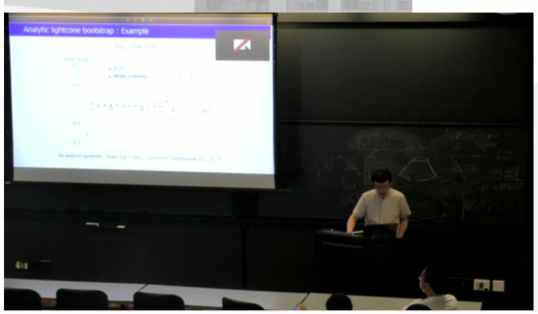
Analytic lightcone bootstrap : Example

$$\Delta_{[\sigma\sigma]_{0,\ell}} = 2\,\Delta_\sigma + \ell + \delta$$



An analytic question : exact 4 pt $= \lim_{n\to\infty}$ inversion^n [information of ϵ, T] ?



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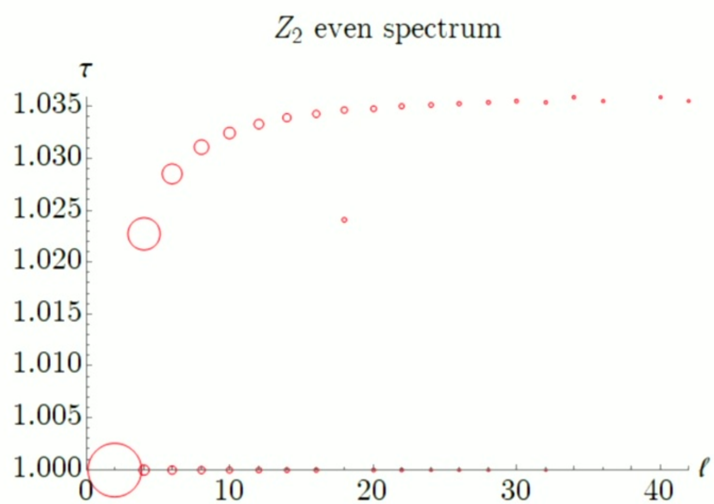
Analytic lightcone bootstrap



Leading twist operator in $\sigma \times \sigma$: $\Delta_{[\sigma\sigma]_{0,\ell}} \approx 2\,\Delta_\sigma + \ell + \delta$

Inversion formula at the lightcone limit $z << 1 - \bar{z} << 1$:
Information of low lying operators $\longrightarrow$ Information of the leading twist operators at large spin (approximately).

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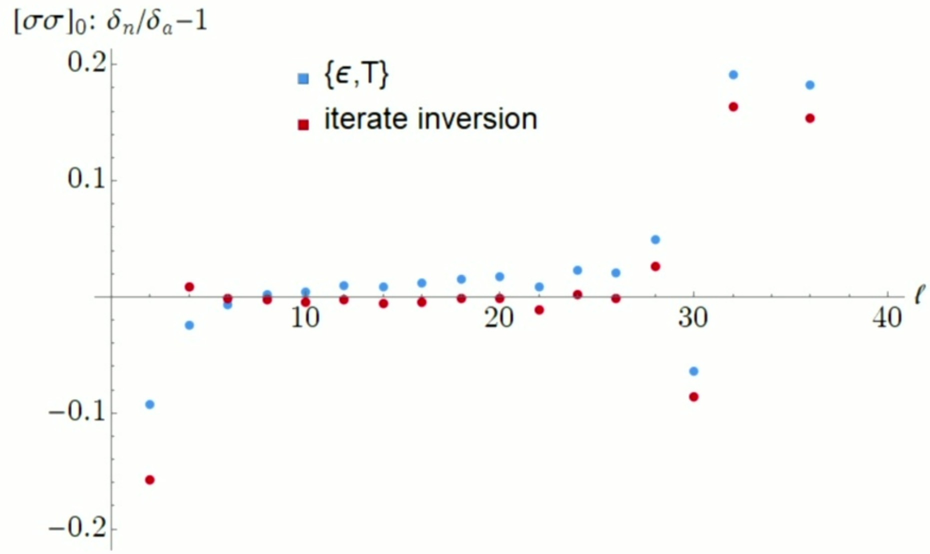
Analytic lightcone bootstrap : Example

$$\Delta_{[\sigma\sigma]_{0,\ell}} = 2\,\Delta_\sigma + \ell + \delta$$



An analytic question : exact 4 pt = $\lim_{n\to\infty}$ inversion^n [information of ϵ, T] ?

Overview

- Introduction
- Numerical bootstrap
- Analytic bootstrap
- Hybrid bootstrap
- Outlook

Why hybrid?

Does bootstrap equations from $\langle\sigma\sigma\sigma\sigma\rangle$, $\langle\epsilon\epsilon\epsilon\epsilon\rangle$, $\langle\sigma\sigma\epsilon\epsilon\rangle$ uniquely determine 3D Ising Δ_σ, Δ_ϵ ?
Numerically it means the island should shrink to a point.

So far the shrinking is very slow at high Λ, but we know a reason :

Traditional numerics : derivative expansion at $z = \bar{z} = 1/2$
Conformal block at $z << 1 - \bar{z} << 1$: $G \sim \log(z)$

Derivative expansion is exponentially inefficient for $z \to 0$

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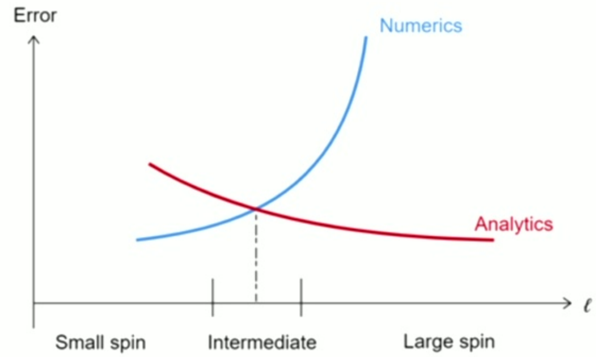
Why hybrid?

Numerics : around $z = \bar{z} = 1/2$: sensitive to small Δ, small ℓ.

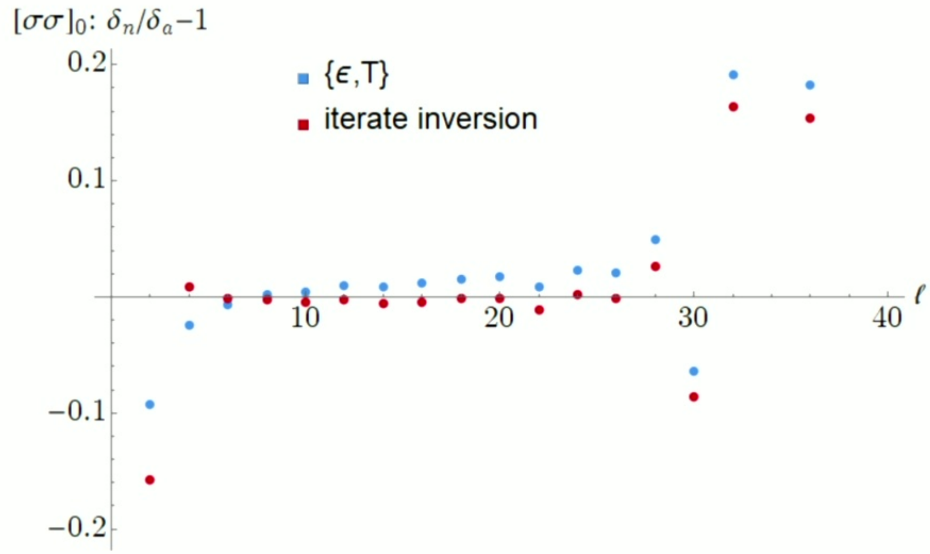
Analytics : around $z << 1 - \bar{z} << 1$: $\Delta_{[\sigma\sigma]_{0,\ell}} \approx f(\Delta_\sigma, \Delta_\epsilon, f_{\sigma\sigma\epsilon}, f_{\sigma\sigma T})$ accurate for leading twist at large ℓ.



We should combine the numerics & analytics :
two difficulties in the past (before the navigator was invented)...

Analytic lightcone bootstrap : Example

$$\Delta_{[\sigma\sigma]_{0,\ell}} = 2\,\Delta_\sigma + \ell + \delta$$



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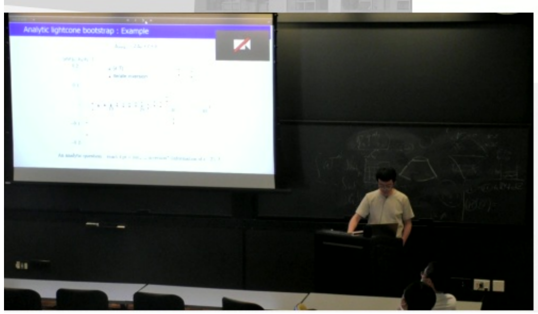
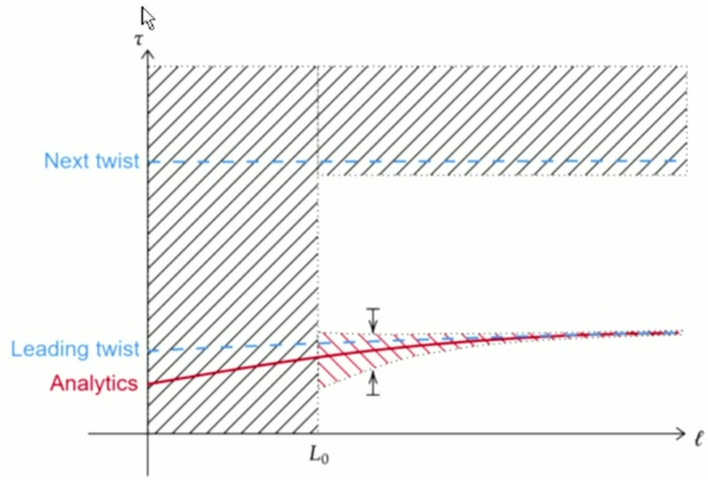


The hybrid bootstrap



1, Choose (Δ_σ, Δ_ϵ, $f_{\epsilon\epsilon\epsilon}$, $f_{\sigma\sigma\epsilon}$, $f_{\sigma\sigma T}$, *Area of red*)

2, Compute analytic bootstrap

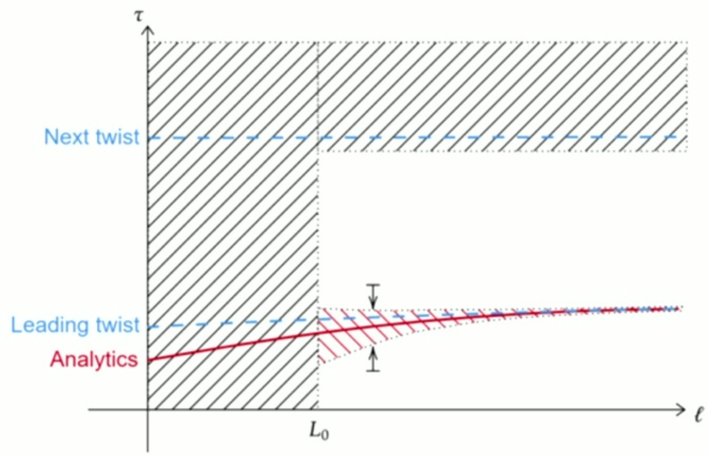
3, Run the numerics (navigator),

demanding operators exist in the shaded region

4, Repeat and minimize the *Area*

while navigator function ≤ 0

The hybrid bootstrap



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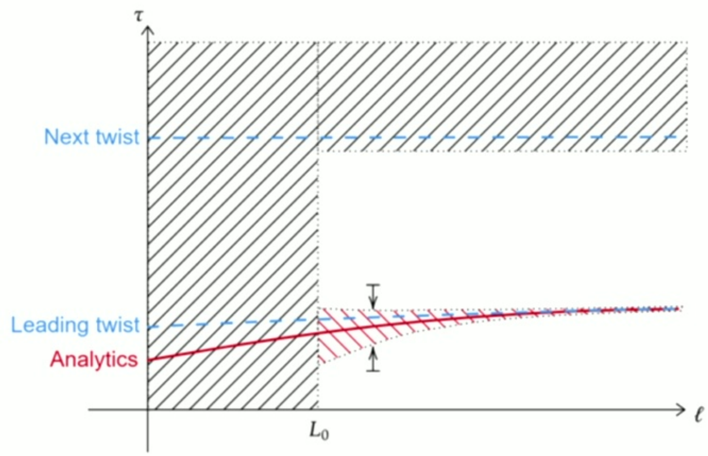

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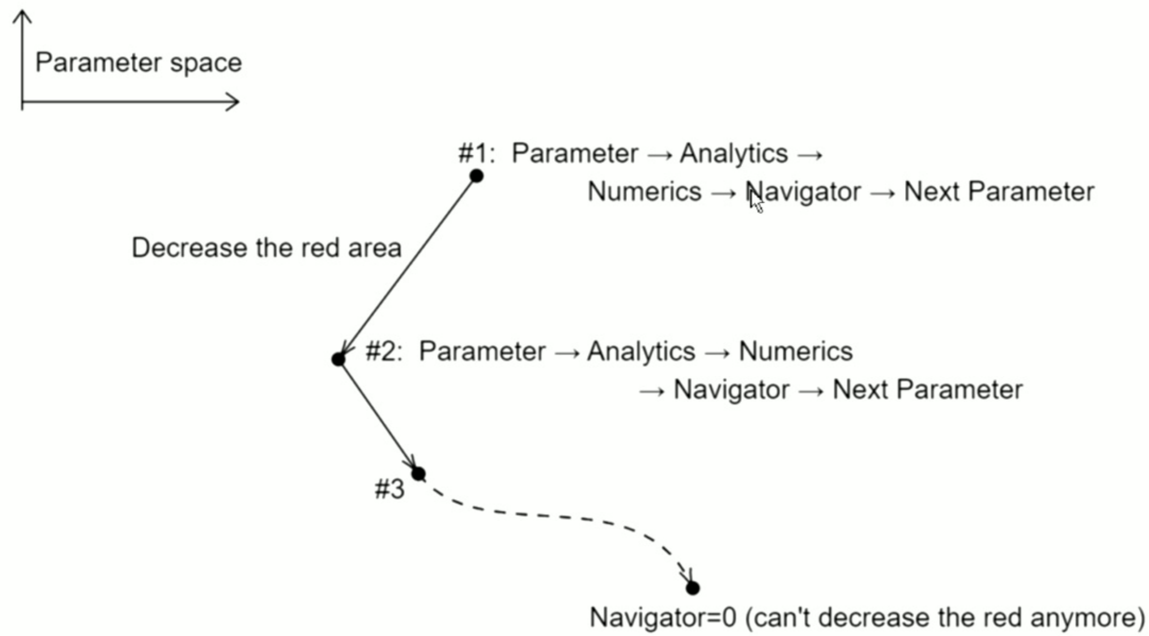
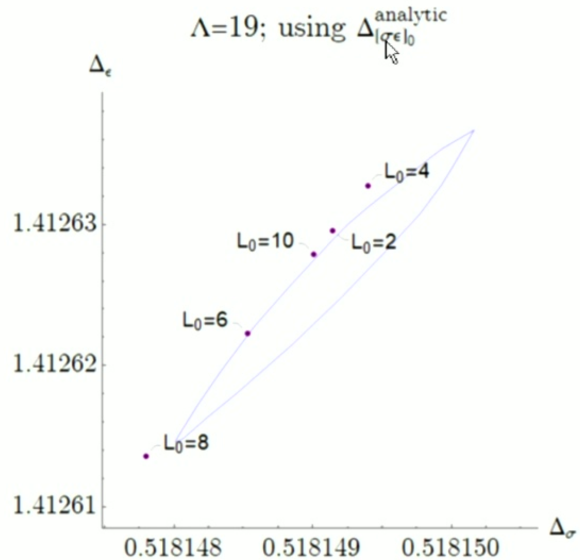
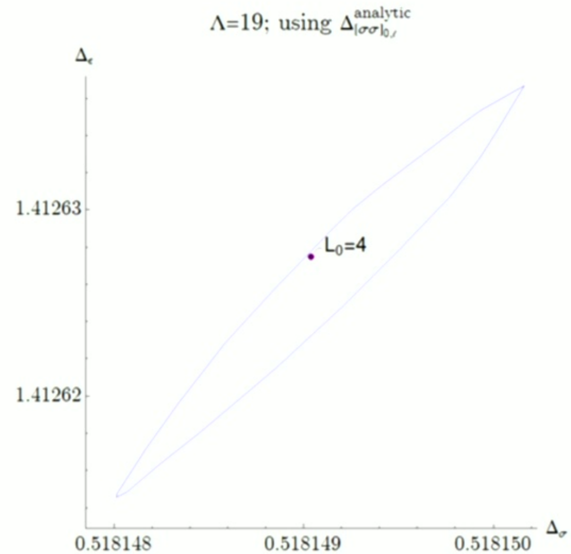
The hybrid bootstrap



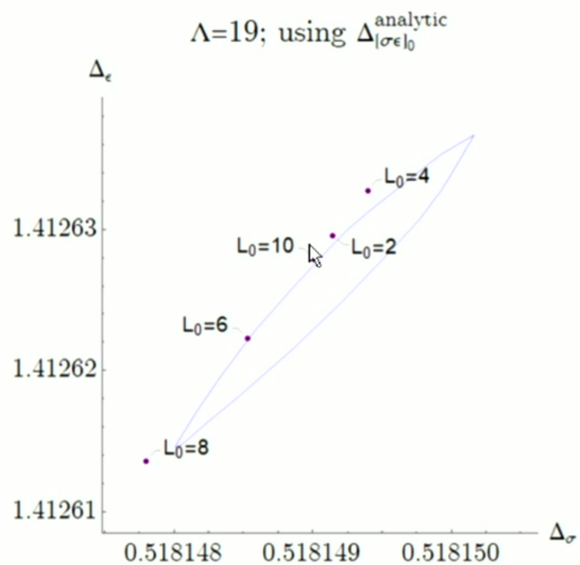
The hybrid bootstrap

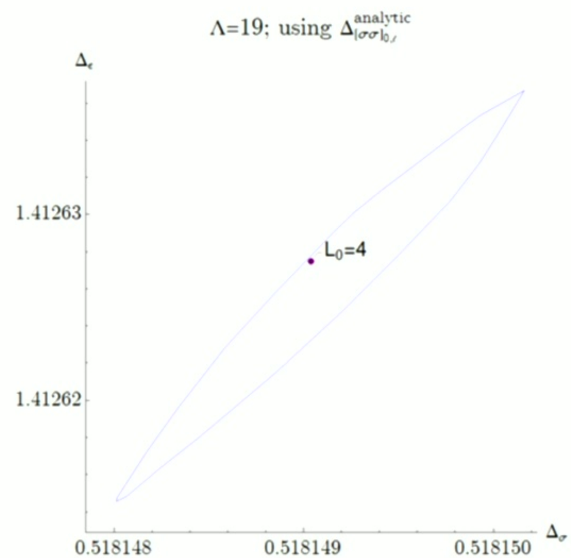




hybrid bootstrap at $\Lambda = 19$ predicts very precise values that are within the previous $\Lambda = 43$ rigorous error bars!

The hybrid bootstrap





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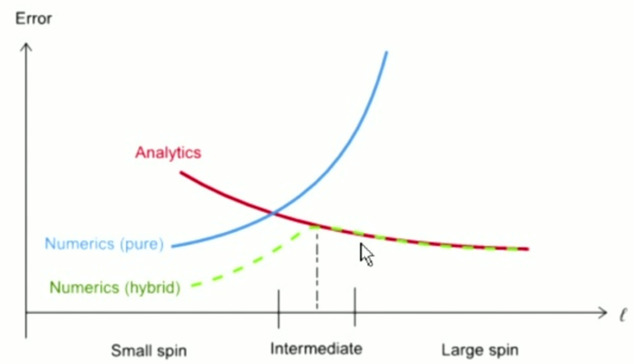
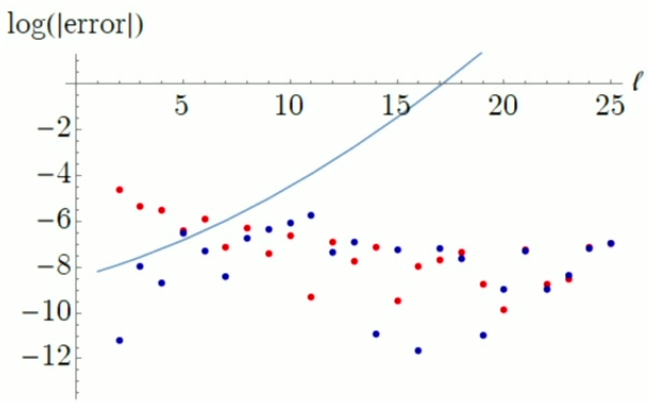




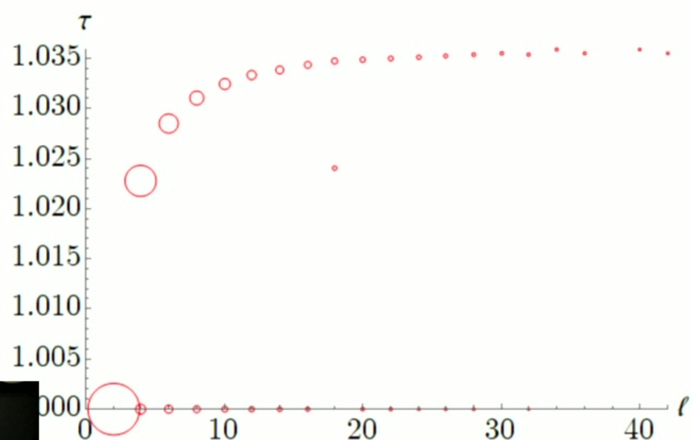
An lesson for numerical bootstrapper

A lesson for bootstrap numerics : the leading twist is important

In pure numerics, we demand $\Delta \geq d - 2 - \ell$ (unitarity bound)
In hybrid numerics, we demand $\Delta \geq \Delta_{\text{leading twist}} - \epsilon$

Λ=19 pure numerics: 2D Ising : $\delta(\Delta_\sigma) \sim 10^{-6}$; 3D Ising : $\delta(\Delta_\sigma) \sim 10^{-5}$; 3D Super-Ising : $\delta(\Delta_\sigma) \sim 10^{-4}$

Z_2 even spectrum



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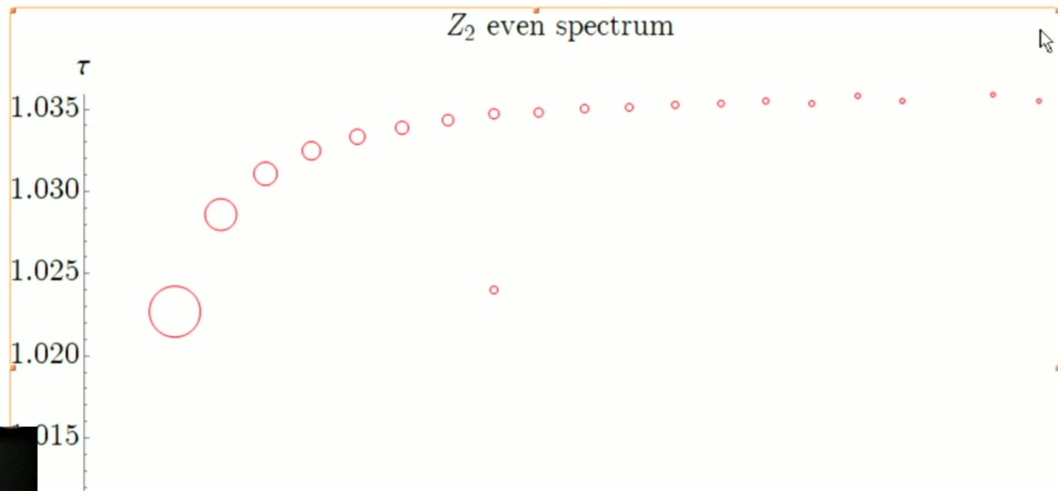

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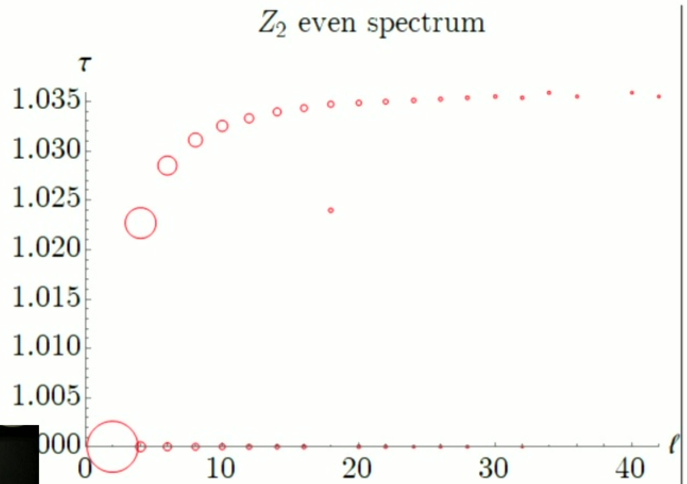

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Next Slide

Overview

- Introduction
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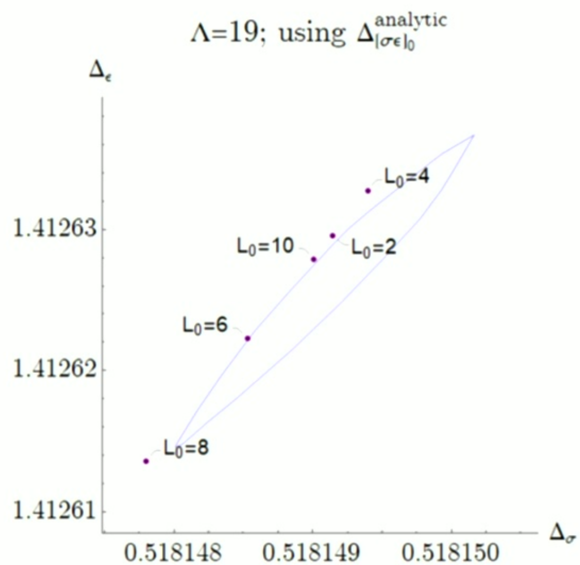
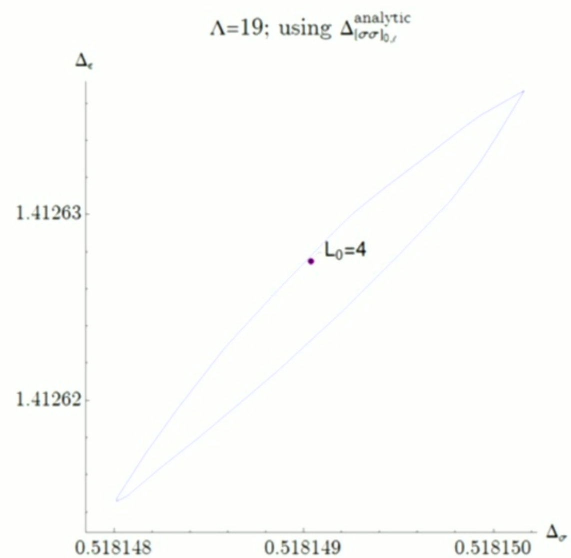
Outlook

Huge potential ! and it's really just the beginning!

- *Towards the full-fledged hybrid bootstrap*
  - Systematically improve the analytics
  - Find a reliable way to estimate error
  - Find a way to using multiple analytics information at the same time
  - Run the hybrid bootstrap at larger Λ
  - New SDP algorithm : Semi-Definite Program with dynamically changing condition
- *Applications*
  - Push the limit of bootstrapping 3d Ising, Super-Ising, O(N) ...
  - Apply hybrid bootstrap to new models:
    cubic model, scalar QED, ...

The hybrid bootstrap





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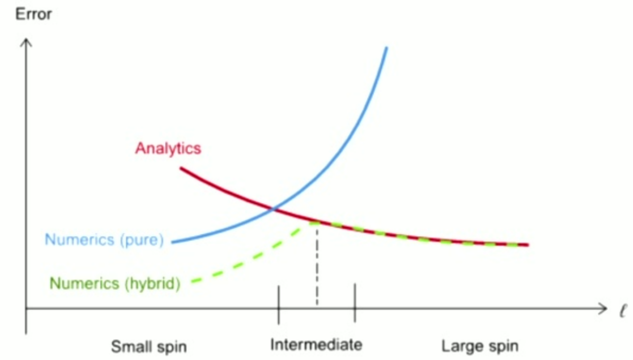

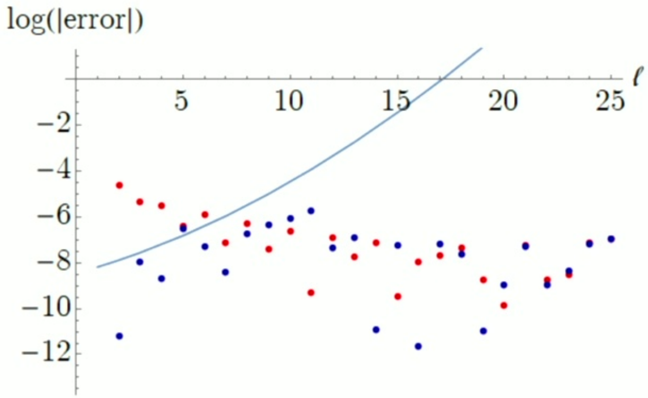

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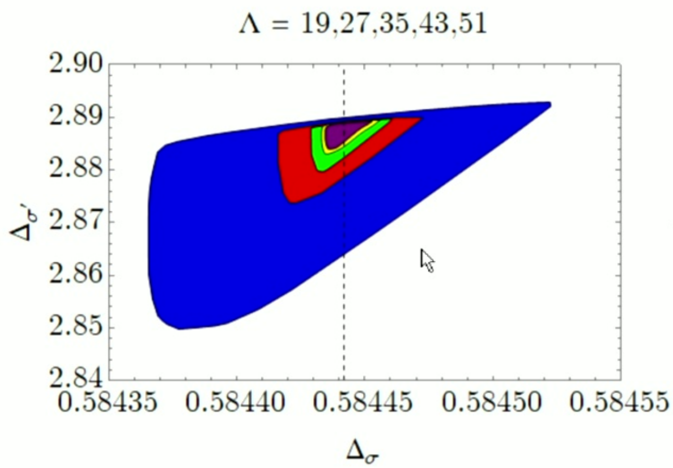
A concrete goal

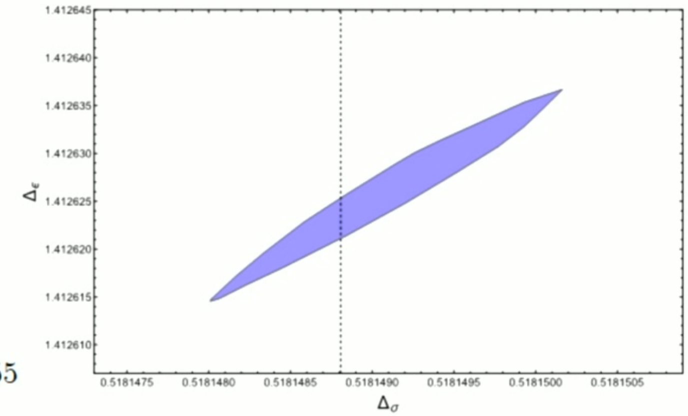
In 2D : $\Delta_{\sigma,\text{Ising}} = \frac{1}{8}$ $\Delta_{\sigma,\text{SuperIsing}} = \frac{1}{5}$ (related by Kac table)

In 3D : $\Delta_{\sigma,\text{Ising}} = 0.518148$ $\Delta_{\sigma,\text{SuperIsing}} = 0.58444$ (related by ?)

My guess : $\Delta_{\sigma,\text{Ising}} \stackrel{?}{=} \frac{\Gamma\left(\frac{1}{6}\right)+4}{\Gamma\left(\frac{1}{24}\right)-5} = 0.518148808$ $\Delta_{\sigma,\text{superIsing}} \stackrel{?}{=} \frac{\Gamma\left(\frac{5}{24}\right)-4}{\Gamma\left(\frac{1}{3}\right)-2} = 0.58444185$

A concrete goal





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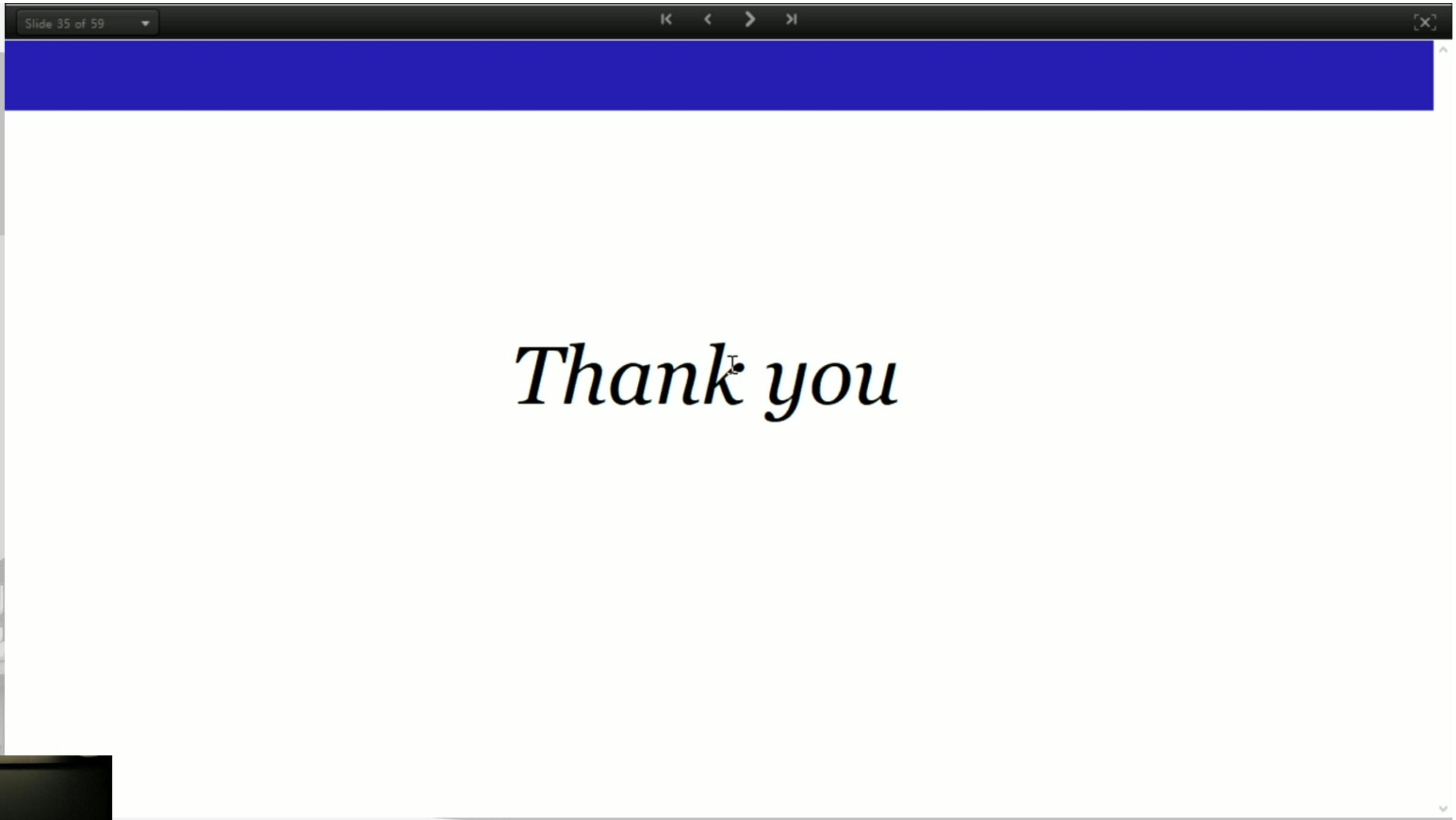
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Thank you


Thank you