

Title: The hybrid conformal bootstrap

Speakers: Ning Su

Series: Quantum Fields and Strings

Date: May 31, 2022 - 3:30 PM

URL: <https://pirsa.org/22050053>

Abstract: Three dimensional conformal field theories(CFTs) describe important critical physics in the real world. In the past few years much progress has been made in 3D CFTs using the conformal bootstrap. In particular, using numerical bootstrap, the critical exponents of the 3D Ising, Super-Ising, O(2), O(3) CFTs have been determined precisely with rigorous error bars, while using analytic bootstrap, the information of the leading twist operators at large spins can be computed analytically. The two bootstrap approaches are sensitive to different regions of the spectrum and complement each other. In this talk, I will first give a general review of the numerical bootstrap and analytic bootstrap. Then I will discuss how to combine the numerical bootstrap with the analytic lightcone bootstrap, i.e. a hybrid bootstrap method. As a result, the bootstrap prediction for the actual CFT can be significantly improved.

Zoom Link: <https://pitp.zoom.us/j/94186668943?pwd=ejRRb0M4VXNjeVVCGdnTHQyaWRyQT09>

The Hybrid Bootstrap

Ning SU
(University of Pisa)

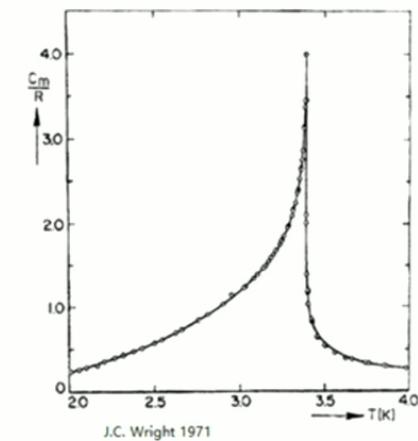
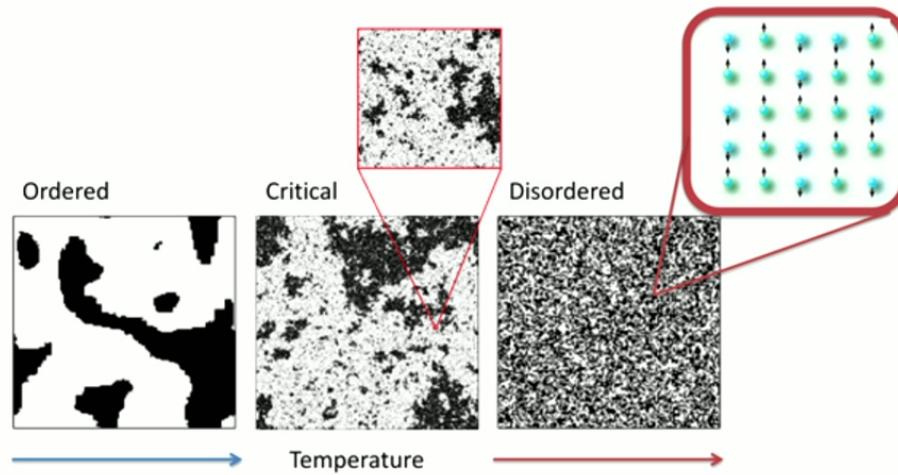
31/05/2022 @ Perimeter

(Based on Ning Su, arXiv:2202.07607

funded by )

Critical phenomena : Ising model

$$\mathcal{H}_{\text{Ising}} = -J \sum_{\langle i j \rangle} \sigma_i \sigma_j$$



$$C_p \simeq (T - T_c)^{-\alpha}$$

critical exponent

$$\alpha = 2 - 3 / (3 - \Delta_\epsilon)$$

$$\eta = 2 \Delta_\sigma - 1$$

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Ising model : studied for 100 years!

1d



Ernst Ising
1925

2d



Lars Onsager
1944

3d



Holy Grail !

The slide is titled "Ising model : studied for 100 years!" and features three sections labeled 1d, 2d, and 3d. Each section contains a black and white photograph of a scientist: Ernst Ising in 1925, Lars Onsager in 1944, and a golden chalice representing the 3D Ising model. Below each photograph is the scientist's name and the year of their work. The slide is presented in a dark-themed Beamer-like interface.

Ising model : studied for 100 years!

1d

Ernst Ising
1925

2d

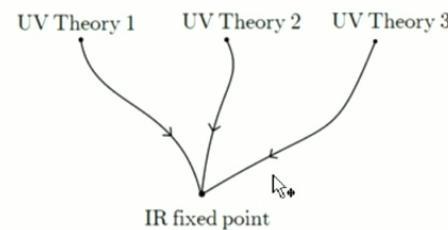
Lars Onsager
1944

3d

Holy Grail !

Traditional methods

Simulate an UV theory



4- ϵ perturbation theory

$$\Delta_\sigma = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} + \frac{109\epsilon^3}{11664} + \epsilon^4 \left(\frac{7217}{1259712} - \frac{2 \operatorname{Zeta}[3]}{243} \right) + \dots$$

diverge



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Better starting point for 3D universality class ?

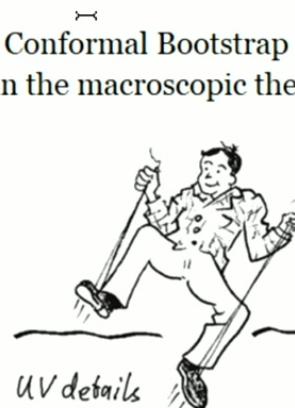
A paradigm shift:

Study the theory **exactly** at the IR fixed point and **non-perturbatively**
(WITHOUT any flow)

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Conformal Bootstrap

Conformal Bootstrap :
use **consistency conditions** to constrain the macroscopic theory **without resorting to the UV details**



Credit: Dennis Boos, Leonard Stefanski

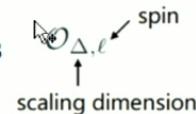
"If the 1971 renormalization group ideas had not been developed, the Migdal-Polyakov bootstrap would have been the most promising framework of its time for trying to further understand critical phenomena. However, the renormalization group methods have proved both easier to use and more versatile, and the bootstrap receives very little attention today." --- K.G. Wilson 1982 Nobel Prize Lecture

Revived in 2008. Many progress in the past decades.
An organic mix of High Energy theory, Condensed matter Physics, Computer Science



Constraints for conformal field theory

1. Spectrum: infinite set of operators

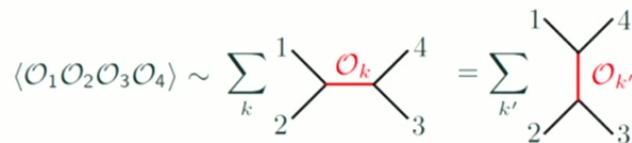


$$\alpha = 2 - 3 / (3 - \Delta_\epsilon)$$

2, Interaction among operators : $\mathcal{O}_i \times \mathcal{O}_j \sim \sum \mathcal{O}_i \text{---} \mathcal{O}_j \sim \sum f_{ijk} \mathcal{O}_k$



3, Crossing symmetry :

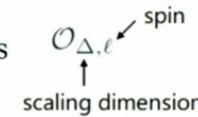


All combined :

$\langle \phi\phi\phi\phi \rangle \implies \sum f_{\phi\phi O}^2 F_{\Delta,r}(z, \bar{z}) = 0$
 What choices of $\{f_{ijk}, \Delta_i\}$ are consistent?

Constraints for conformal field theory

1, Spectrum: infinite set of operators



$$\alpha = 2 - 3 / (3 - \Delta_\epsilon)$$

$$\eta = 2 \Delta_\sigma - 1$$

2, Interaction among operators : $\mathcal{O}_i \times \mathcal{O}_j \sim \sum_k \mathcal{O}_i$

$$\mathcal{O}_j \quad \mathcal{O}_k \sim \sum_k f_{ijk} \mathcal{O}_k$$

Operator Product Expansion (OPE) coefficients

3, Crossing symmetry :

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \sim \sum_k \begin{array}{c} 1 \\[-1ex] 2 \end{array} \mathcal{O}_k \begin{array}{c} 4 \\[-1ex] 3 \end{array} = \sum_{k'} \begin{array}{c} 1 \\[-1ex] 2 \end{array} \mathcal{O}_{k'} \begin{array}{c} 4 \\[-1ex] 3 \end{array}$$

All combined :

$$\langle \phi \phi \phi \phi \rangle \Rightarrow \sum_O f_{\phi \phi O}^2 F_{\Delta, \ell}(z, \bar{z}) = 0$$

What choices of $\{f_{ijk}, \Delta_i\}$ are consistent?



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Example of the bootstrap equation

Consider four-point function $\langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$ in 2D (Dolan and Osborn 2001, 2004) :

$$\sum_{O \in \phi \times \phi} f_{\phi\phi O}^2 F_{\Delta,\ell}(z, \bar{z}) = 0$$

$$F_{\Delta,\ell} = ((1-z)(1-\bar{z}))^{\Delta_\phi} g_{\Delta,\ell}(z, \bar{z}) - (z\bar{z})^{\Delta_\phi} g_{\Delta,\ell}(1-z, 1-\bar{z})$$

$$g_{\Delta,\ell}(z, \bar{z}) = k_{\Delta+l}(z) k_{\Delta-l}(\bar{z}) + (z \leftrightarrow \bar{z})$$

$$k_\beta(z) = z^{\beta/2} {}_2F_1\left(\frac{\beta}{2}, \frac{\beta}{2}, \beta; z\right)$$

$$\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z \bar{z}, \quad \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

$$0 < z < 1, \quad 0 < \bar{z} < 1$$



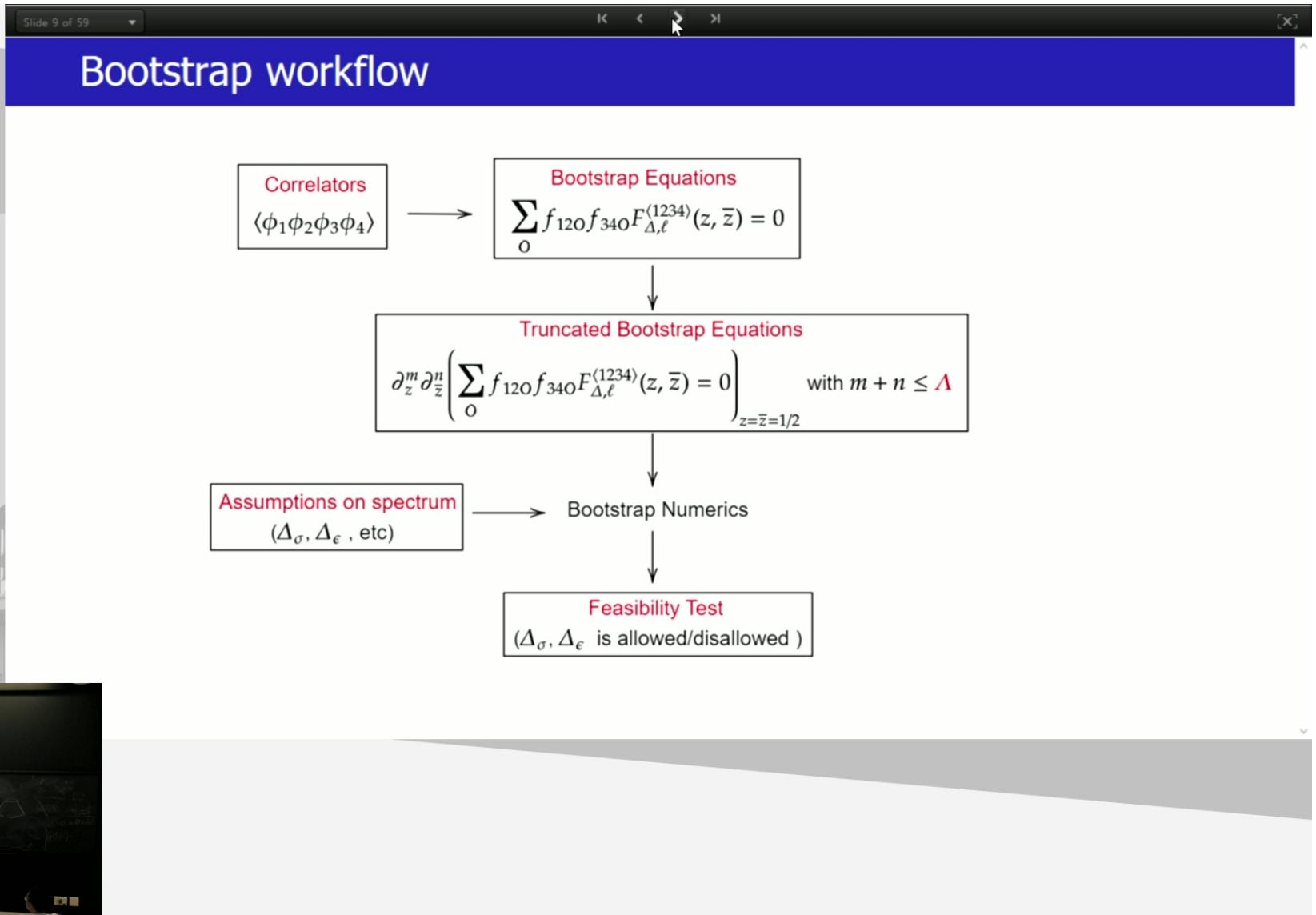
The image is a composite of two views. The top view is a screenshot of a presentation slide titled "Overview". The slide lists five topics under a blue header: "Introduction", "Numerical bootstrap" (highlighted in red), "Analytic bootstrap", "Hybrid bootstrap", and "Outlook". The bottom view is a video feed of a lecture in progress. A male lecturer is standing at a podium in front of a chalkboard filled with mathematical diagrams and equations. To the left of the video, a small window shows the same "Overview" slide from the top view.

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Next Slide

Overview

- Introduction
- Numerical bootstrap
- Analytic bootstrap
- Hybrid bootstrap
- Outlook

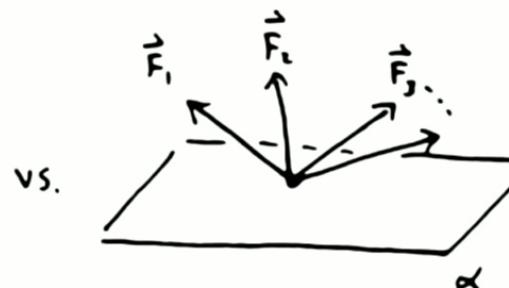
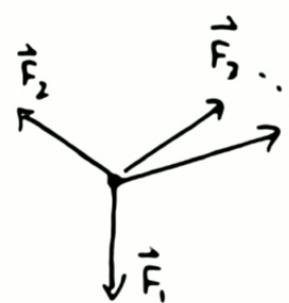


Numerics

How to solve $\sum_{O \in \phi \times \phi} f_{\phi\phi O}^2 F_{\Delta,\ell}(z, \bar{z}) = 0$? (bootstrap Equ. from $\langle \phi\phi\phi\phi \rangle$)

Step 1 : $F_{\Delta,\ell}(z, \bar{z}) \simeq \sum_{m+n \leq \Lambda} c_{mn} \partial_z^m \partial_{\bar{z}}^n F_{\Delta,\ell}(z, \bar{z})$ $F_{\Delta,\ell}(z, \bar{z}) \rightarrow$ vector $(c_{01}, c_{12}, c_{23} \dots)$

Step 2 :

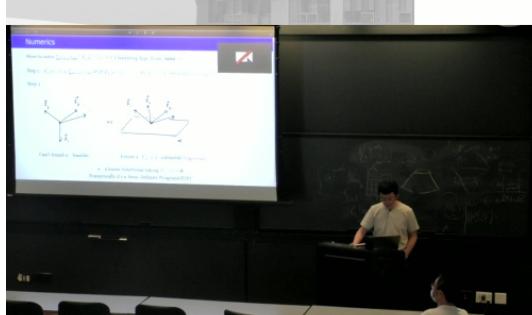


Can't find α : feasible

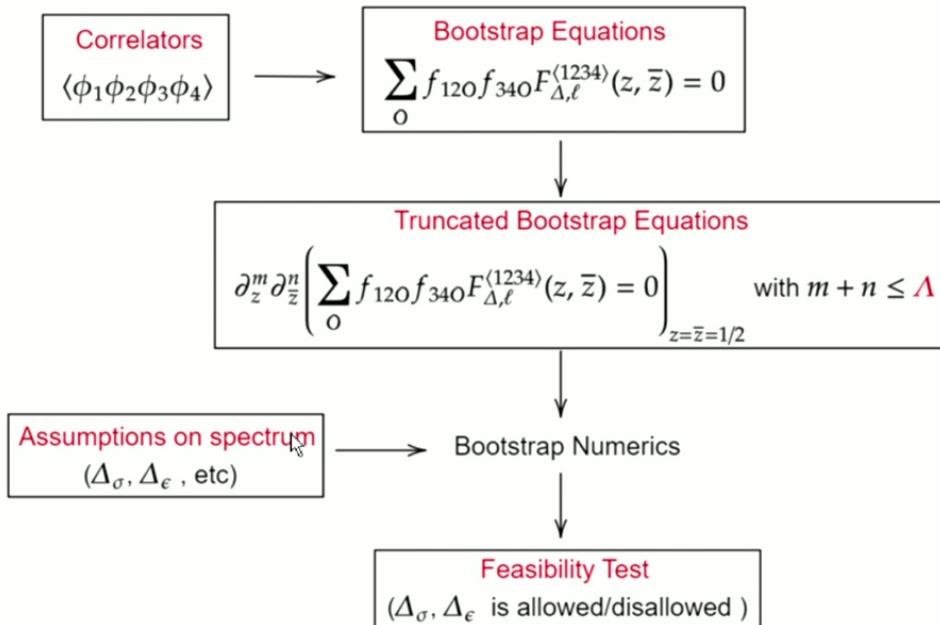
Found $\alpha \cdot F_{\Delta,\ell} \geq 0$: infeasible (**rigorous**)

α : a linear functional taking $f(z, \bar{z}) \rightarrow \mathbb{R}$

Numerically it's a Semi-Definite Program(SDP).



Bootstrap workflow

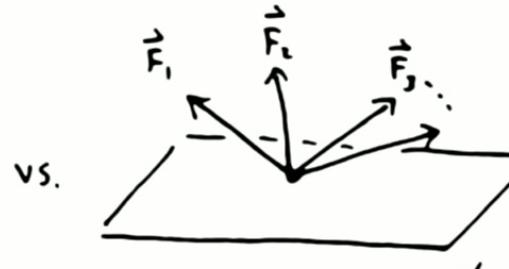
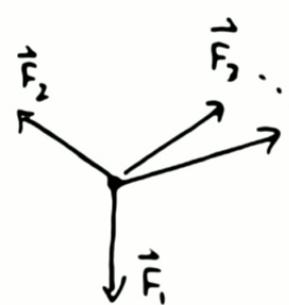


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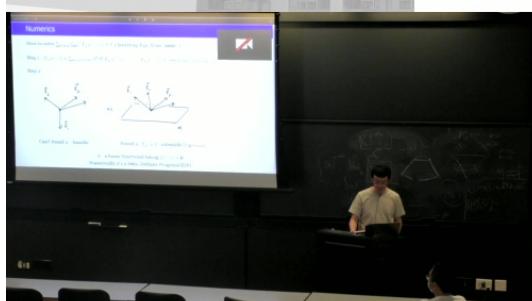


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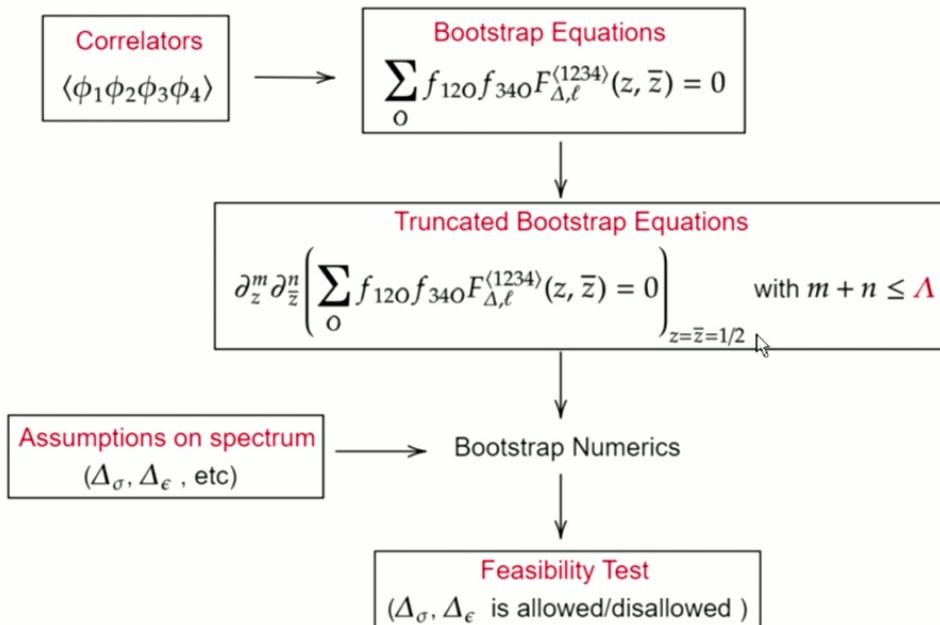
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Bootstrap workflow

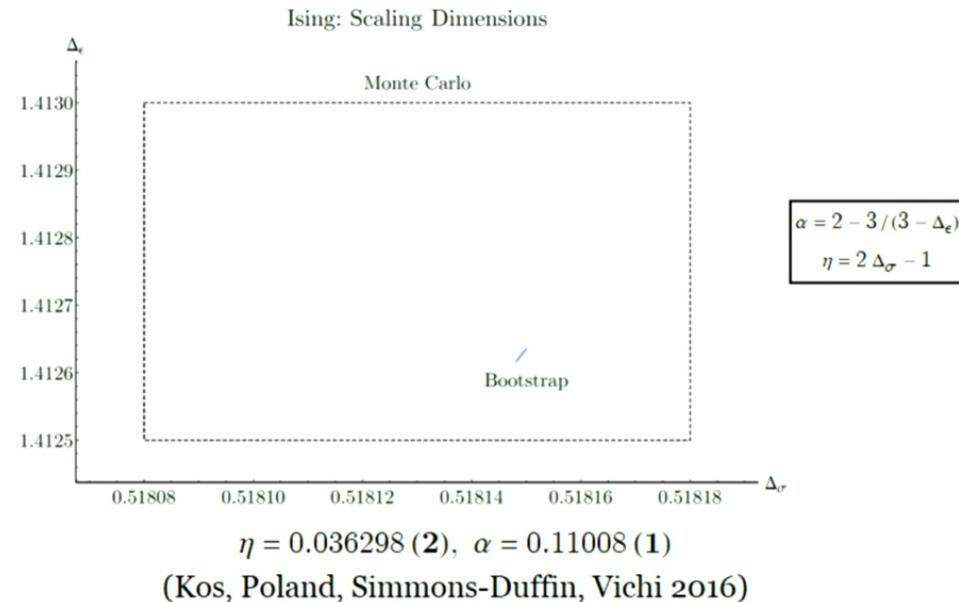


Bootstrap 3D Ising CFT

Correlators : $\langle \sigma\sigma\sigma\sigma \rangle, \langle \epsilon\epsilon\epsilon\epsilon \rangle, \langle \epsilon\epsilon\sigma\sigma \rangle$

Derivative truncation : $\Lambda = 43$

Assumptions : σ, ϵ are the only two relevant scalars.

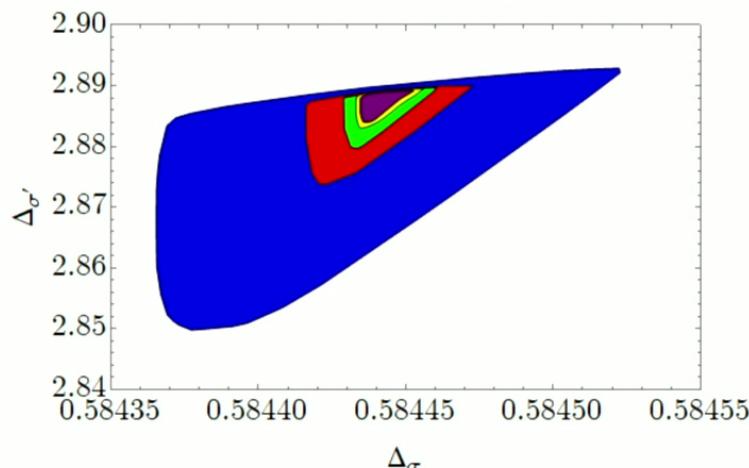


Bootstrap 3D super-Ising CFT

$$\mathcal{L}_{\text{SuperIsing}} = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \bar{\psi} \not{\partial} \psi + \frac{\lambda}{2} \sigma \bar{\psi} \not{\partial} \psi + \frac{\lambda^2}{8} \sigma^4$$

Bootstrapping $\langle \sigma \sigma \sigma \sigma \rangle$ (Rong, NS 2018)

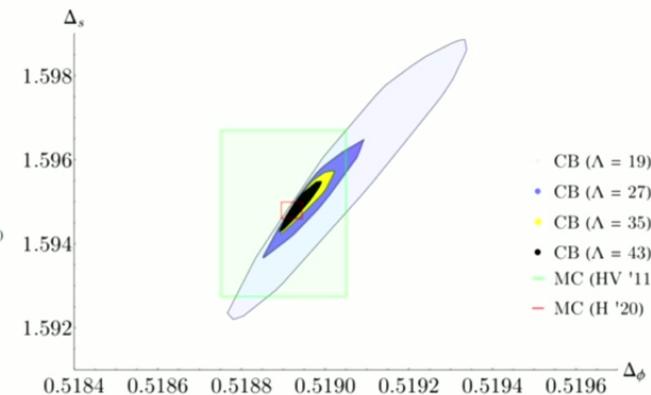
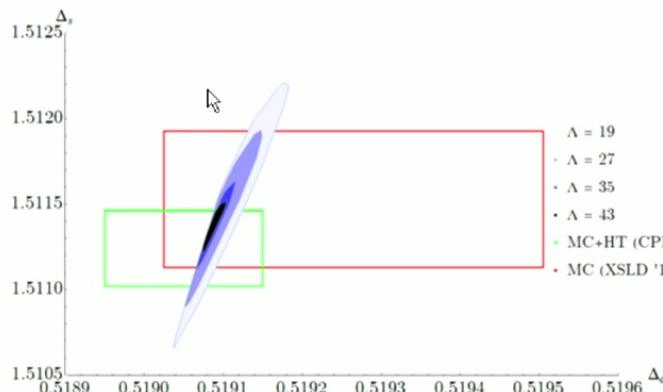
$$\Lambda = 19, 27, 35, 43, 51$$



Bootstrap 3D O(N) CFTs

Lagrangian : $\mathcal{L} = \partial\phi_i \partial\phi_i + m^2 (\phi_i \phi_i) + \lambda (\phi_i \phi_i)^2$

Bootstrapping all 4pt involves $\{v = \phi_i, s = \phi^2, t = \phi_i \phi_j\}$:



(Chester, Landry, Liu, Poland, Simmons-Duffin, **SN**, Vichi 2019, 2020)



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Difficulties in numerical bootstrap

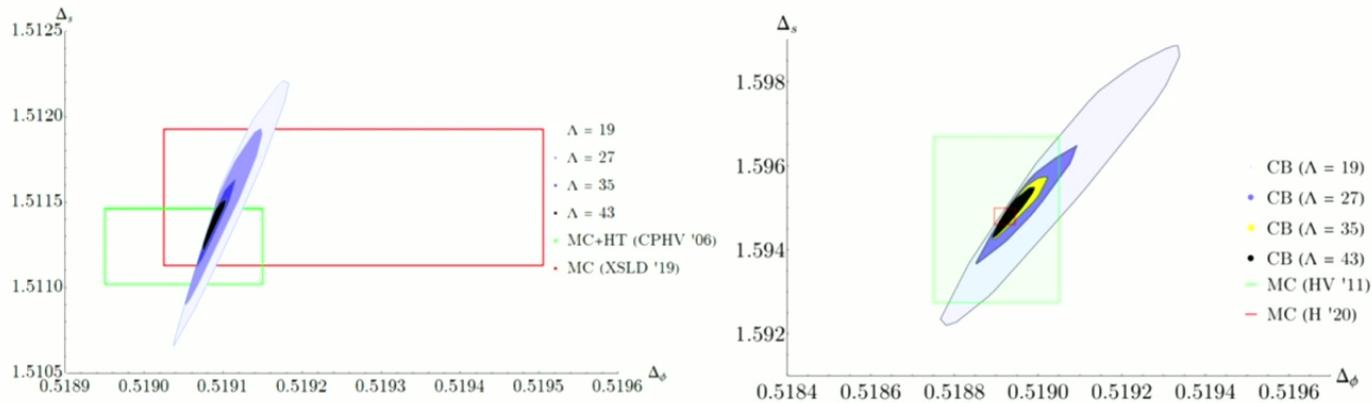
- Need to scan a parameter space with a large dimensional
- slow convergence at large Λ

The image shows a presentation slide titled "Difficulties in numerical bootstrap". The slide contains two bullet points: "Need to scan a parameter space with a large dimensional" and "slow convergence at large Λ ". The background of the slide is blue. Below the slide, there is a small image of a person standing at a chalkboard in a lecture hall. The chalkboard has some mathematical equations written on it.

Bootstrap 3D O(N) CFTs

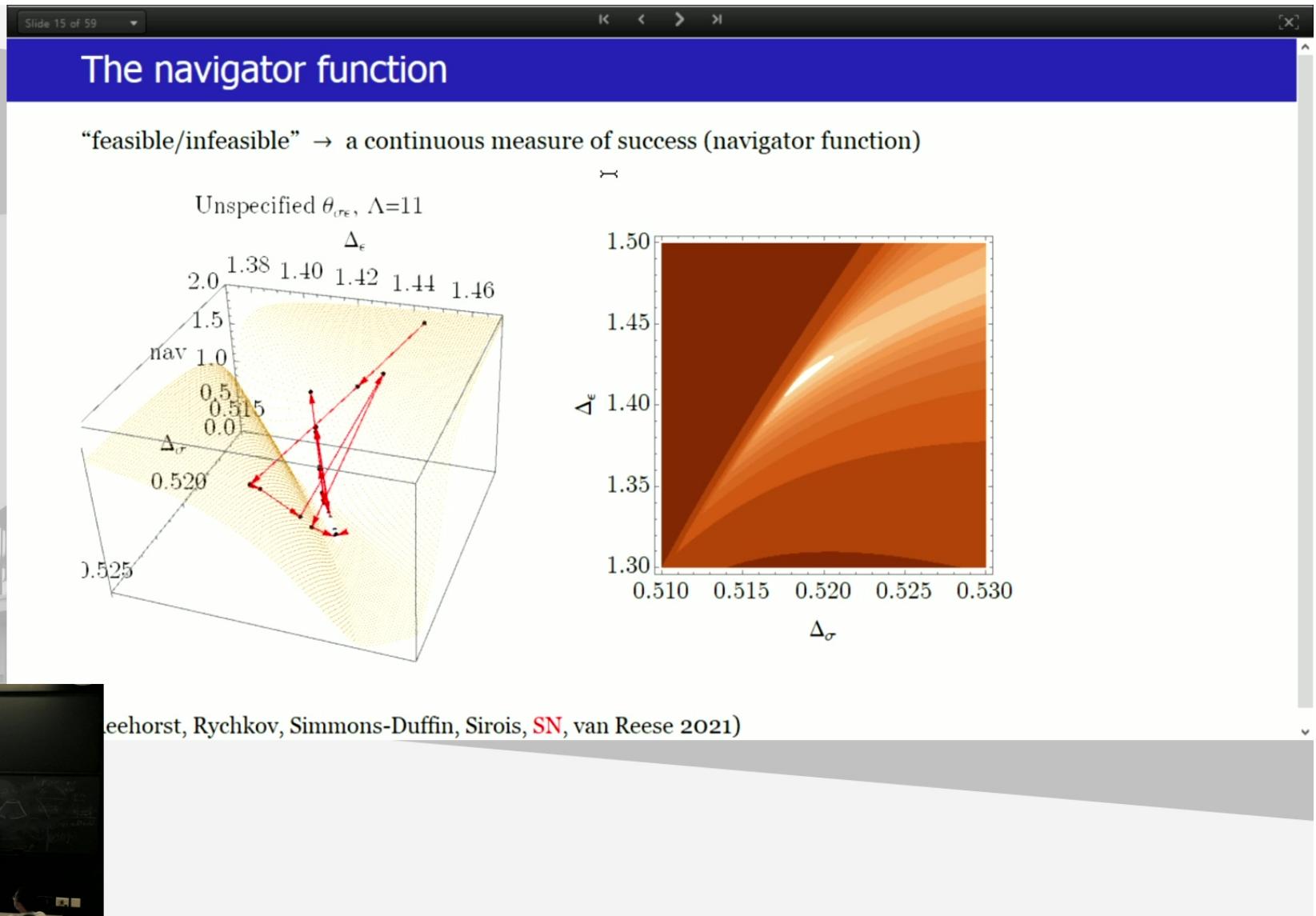
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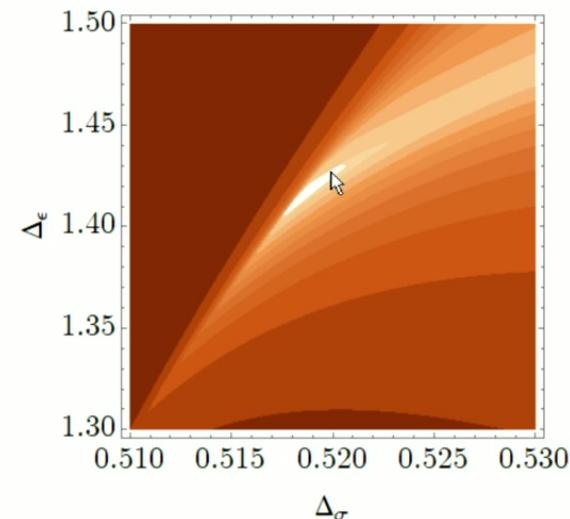
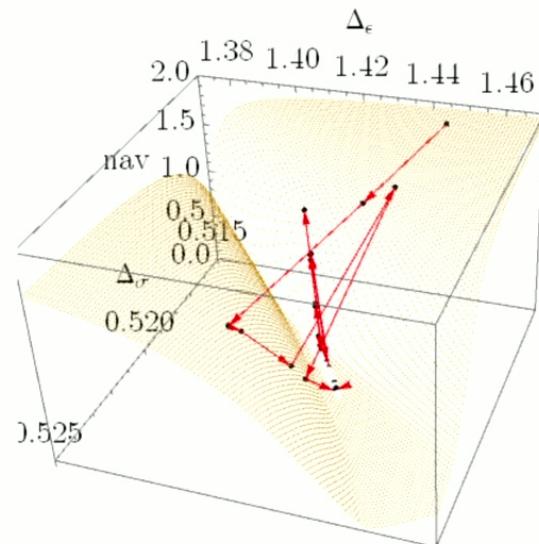




The navigator function

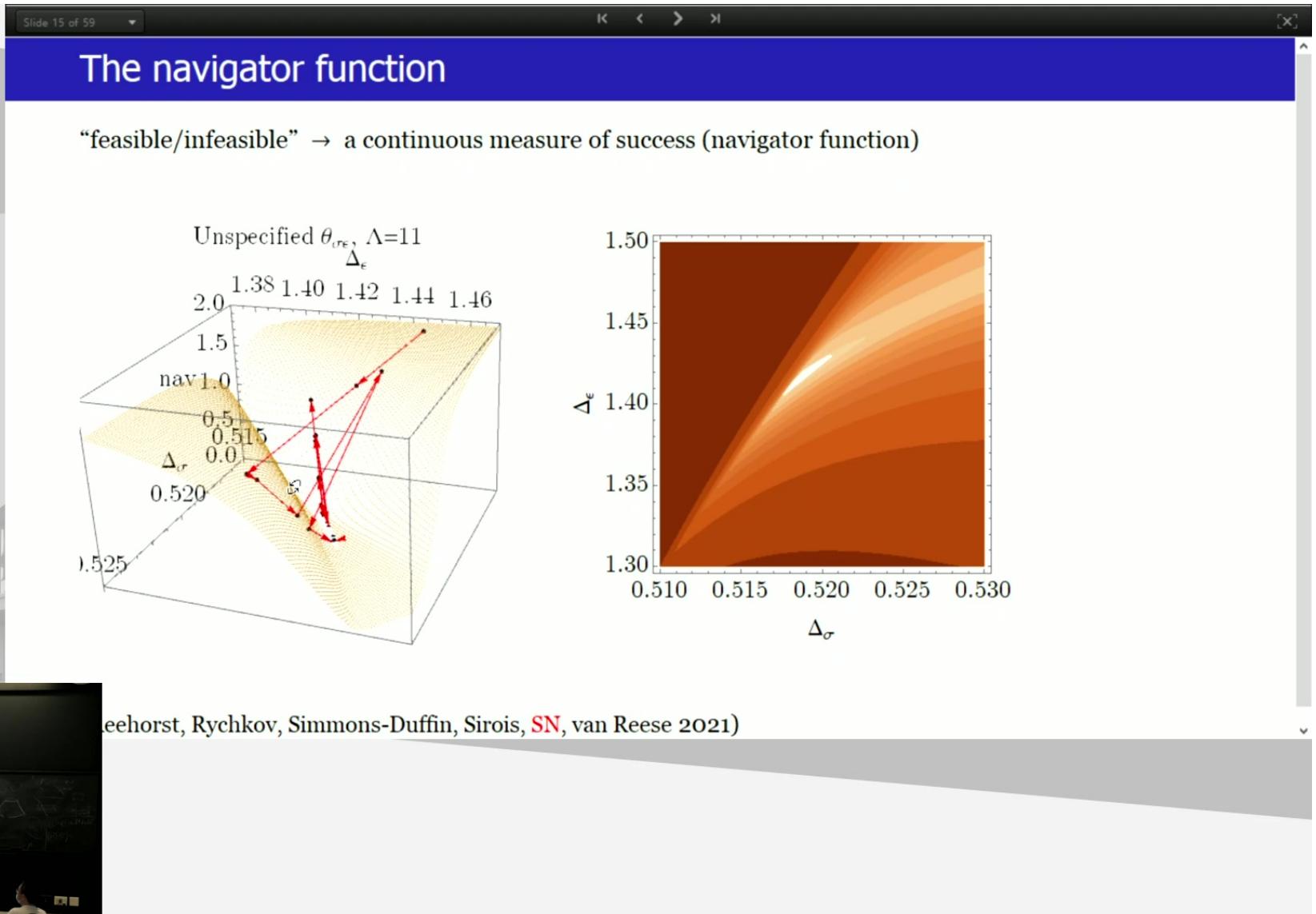
"feasible/infeasible" → a continuous measure of success (navigator function)

Unspecified θ_{re} , $\Lambda=11$



(deehorst, Rychkov, Simmons-Duffin, Sirois, **SN**, van Reese 2021)

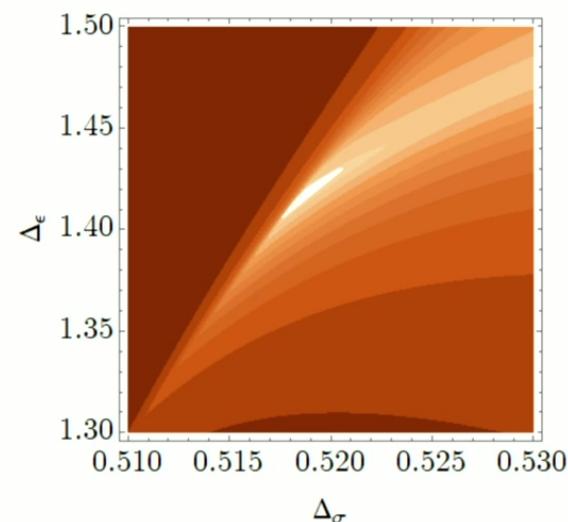
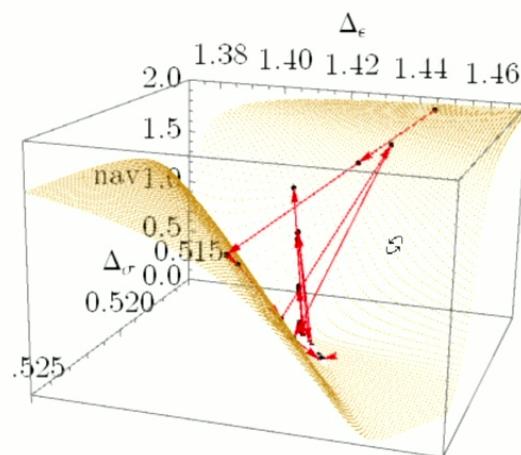




The navigator function

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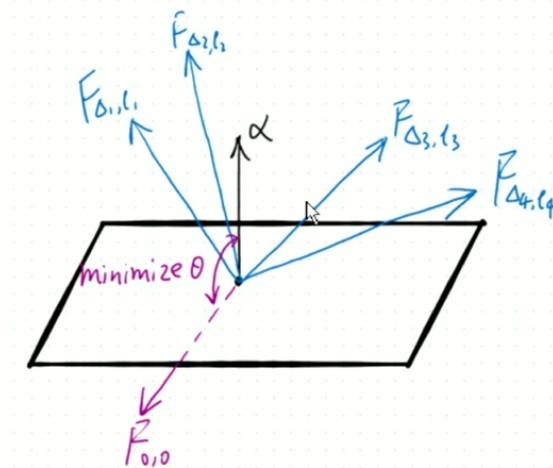
Unspecified θ_{re} , $\Lambda=11$



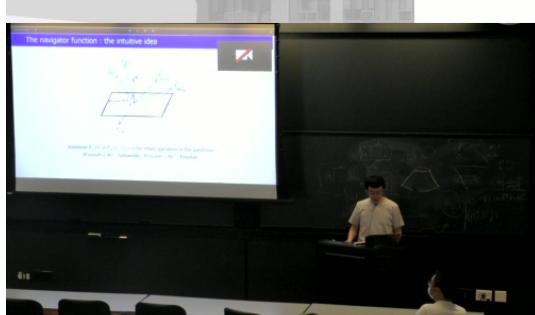
(dehorst, Rychkov, Simmons-Duffin, Sirois, SN, van Reese 2021)



The navigator function : the intuitive idea



minimize θ , s.t. $\alpha(F_{\Delta,\ell}(z, \bar{z})) \geq 0$ for other operators in the spectrum
 If $\min(\theta) \geq 90^\circ$: Infeasible; If $\min(\theta) < 90^\circ$: Feasible



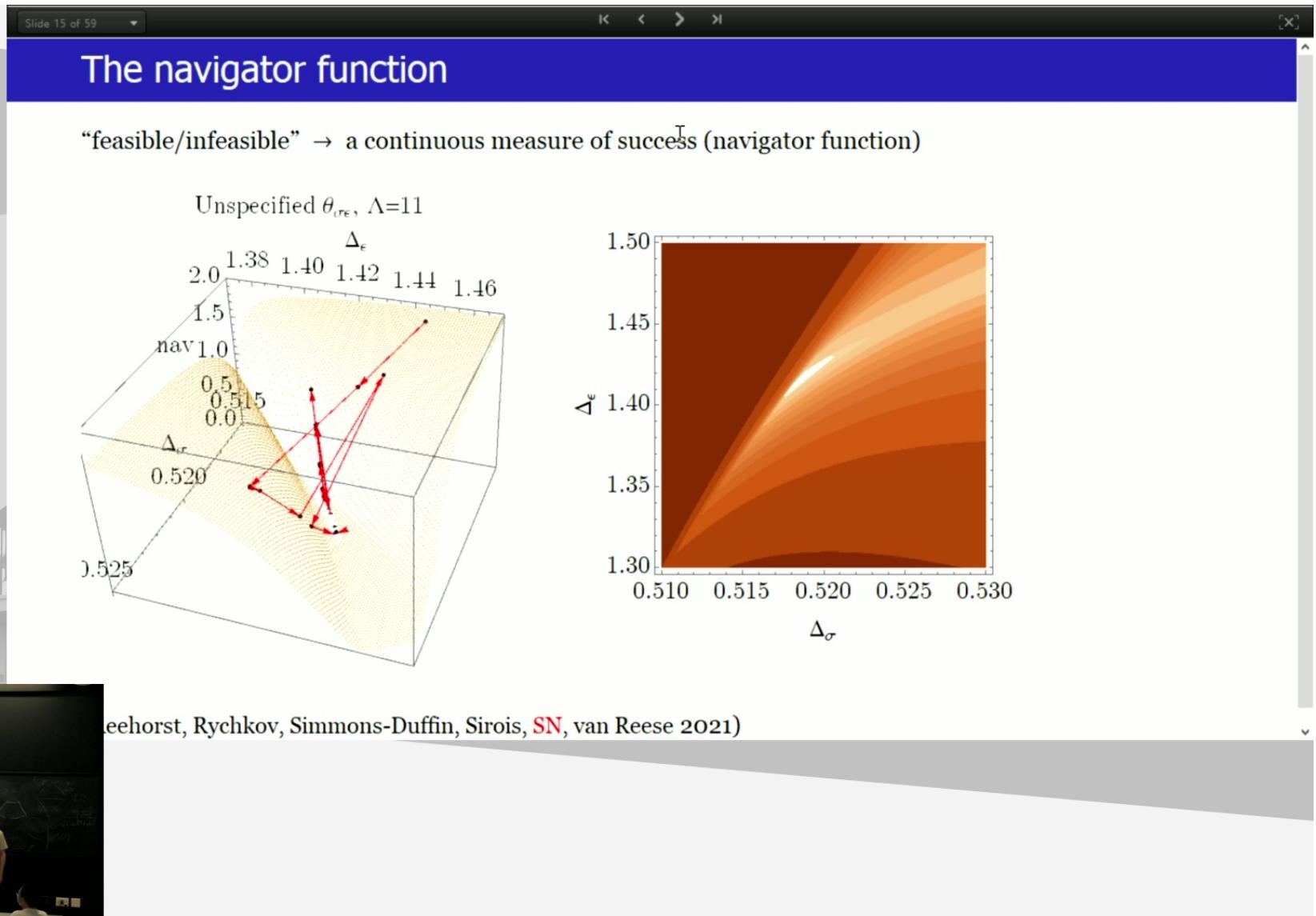
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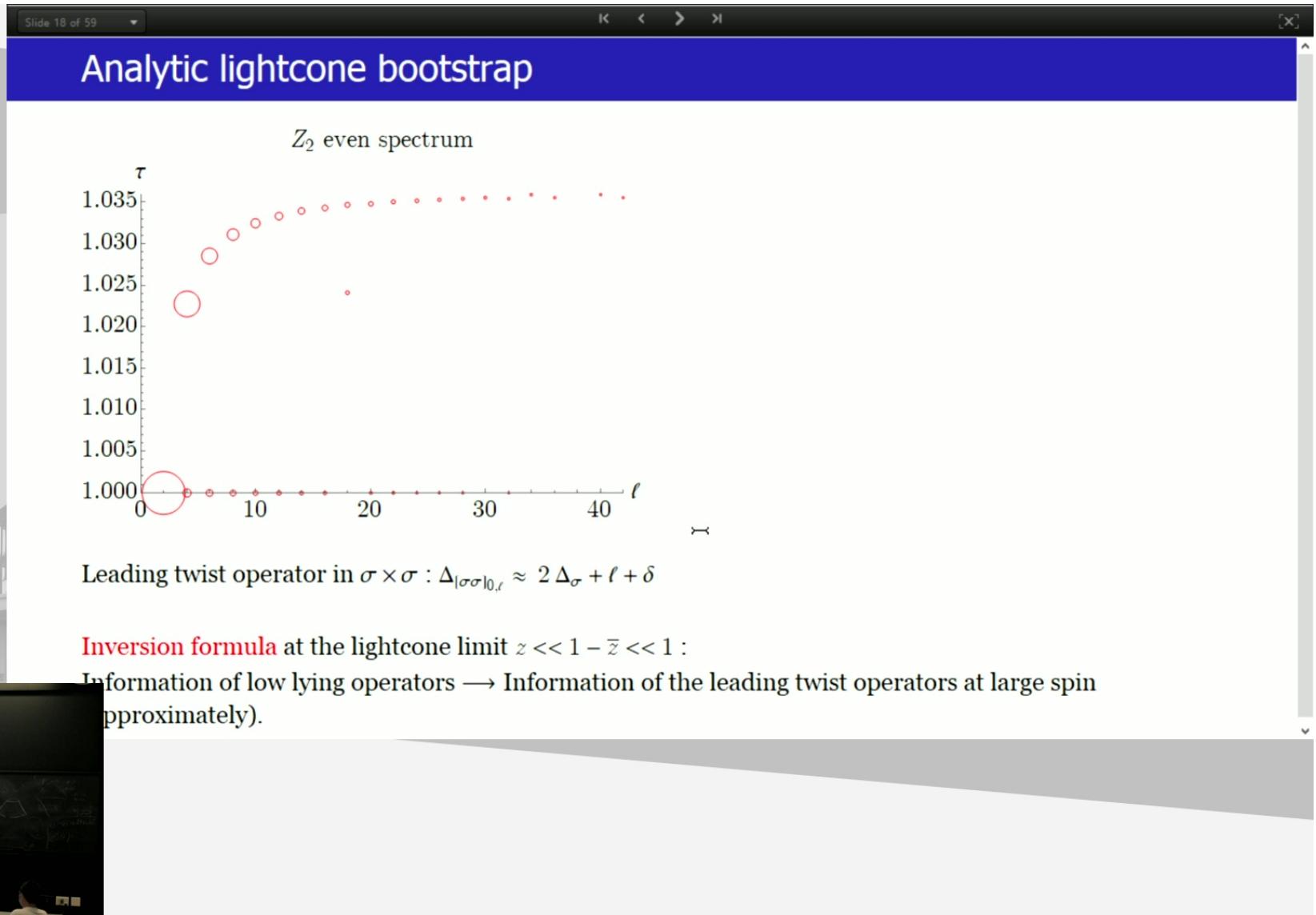
Overview

- *Introduction*
- *Numerical bootstrap*
- *Analytic bootstrap*
- *Hybrid bootstrap*
- *Outlook*

Overview

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Solving bootstrap equation at the lightcone limit

Prediction from the Inversion formula (Caron-Huot 2017; Liu, Meltzer, Poland, Simmons-Duffin 2020)

Generating function : $C(z, \bar{h}) = f_{\sigma\sigma|\sigma\sigma|_0}(\bar{h}) z^{2 h_{[\sigma\sigma]_0}(\bar{h})} + \dots$

$$C_O(z, \bar{h})_{\text{small } z} \approx f_{\sigma\sigma O}^2 2 \sin(\pi (h_O - 2 h_\sigma))^2 \sum_{p=0}^{\infty} \sum_{q=-p}^p \mathcal{A}_{p,q}(h_O, \bar{h}_O) \frac{z^{2 h_\sigma k_{\bar{h}_O+q}(1-z)}}{(1-z)^{2 h_\sigma}} \kappa_{2 \bar{h}} \Omega_{\bar{h}, h_{O+p}, 2 h_\sigma}^{h_\sigma}$$

$$C(z, \bar{h}) = \sum_{O \in \sigma \times \sigma} C_O(z, \bar{h})$$

At large \bar{h} , the main contribution to $C(z, \bar{h})$ at small z come from $C_O(z, \bar{h})$ with small h_O .

Information of small twist operators \rightarrow Information about $[\sigma\sigma]_{0,\ell}$ at large ℓ .



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Analytic lightcone bootstrap : Example

Information of low lying operator : $\{\epsilon, T\}$

Input : $\Delta_\sigma, \Delta_\epsilon, f_{\sigma\sigma\epsilon}, f_{\sigma\sigma T}$

Output :

$$\Delta_{|\sigma\sigma|_{0,\ell}} \approx 2 \Delta_\sigma + \ell +$$

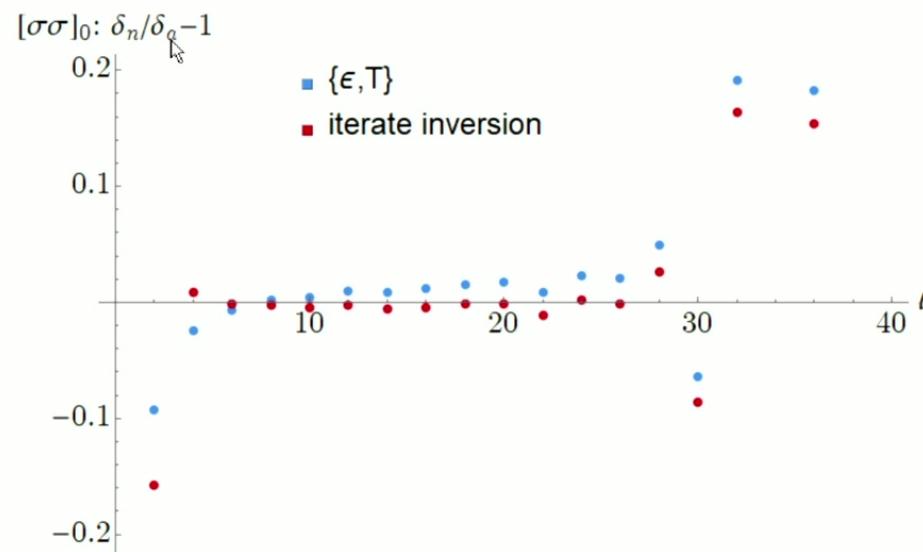
$$\left(2 \left(- \frac{128 f_{\sigma\sigma T}^2 \Gamma(\bar{h})^2 \Gamma(\bar{h} + \Delta_\sigma - \frac{3}{2})}{3 \pi \Gamma(2 \bar{h} - 1) \Gamma(\Delta_\sigma - \frac{1}{2})^2 \Gamma(\bar{h} - \Delta_\sigma + \frac{3}{2})} - \frac{f_{\sigma\sigma\epsilon}^2 \Gamma(\bar{h})^2 \Gamma(\Delta_\epsilon) \Gamma(\bar{h} - \frac{\Delta_\epsilon}{2} + \Delta_\sigma - 1)}{\Gamma(2 \bar{h} - 1) \Gamma(\frac{\Delta_\epsilon}{2})^2 \Gamma(\Delta_\sigma - \frac{\Delta_\epsilon}{2})^2 \Gamma(\bar{h} + \frac{\Delta_\epsilon}{2} - \Delta_\sigma + 1)} \right) \right) /$$

$$\left(- \frac{128 (2 \gamma + 2 \psi^{(0)}(\frac{5}{2})) f_{\sigma\sigma T}^2 \Gamma(\bar{h})^2 \Gamma(\bar{h} + \Delta_\sigma - \frac{3}{2})}{3 \pi \Gamma(2 \bar{h} - 1) \Gamma(\Delta_\sigma - \frac{1}{2})^2 \Gamma(\bar{h} - \Delta_\sigma + \frac{3}{2})} - \frac{f_{\sigma\sigma\epsilon}^2 \Gamma(\bar{h})^2 \Gamma(\Delta_\epsilon) (2 \psi^{(0)}(\frac{\Delta_\epsilon}{2}) + 2 \gamma) \Gamma(\bar{h} - \frac{\Delta_\epsilon}{2} + \Delta_\sigma - 1)}{\Gamma(2 \bar{h} - 1) \Gamma(\frac{\Delta_\epsilon}{2})^2 \Gamma(\Delta_\sigma - \frac{\Delta_\epsilon}{2})^2 \Gamma(\bar{h} + \frac{\Delta_\epsilon}{2} - \Delta_\sigma + 1)} + \frac{\Gamma(\bar{h})^2 \Gamma(\bar{h} + \Delta_\sigma - 1)}{\Gamma(2 \bar{h} - 1) \Gamma(\Delta_\sigma)^2 \Gamma(\bar{h} - \Delta_\sigma + 1)} \right)$$

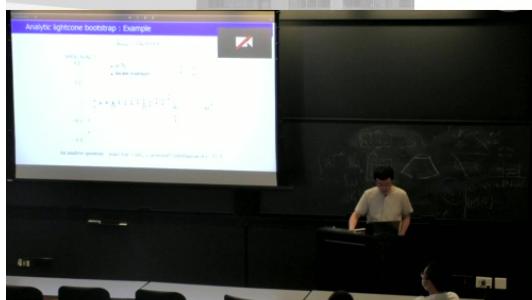
$$\bar{h} = (\Delta_{|\sigma\sigma|_{0,\ell}} + \ell) / 2$$

Analytic lightcone bootstrap : Example

$$\Delta_{[\sigma\sigma]_0, \ell} = 2 \Delta_\sigma + \ell + \delta$$



An analytic question : exact 4 pt = $\lim_{n \rightarrow \infty}$ inversionⁿ [information of ϵ, T] ?



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Analytic lightcone bootstrap

Z_2 even spectrum

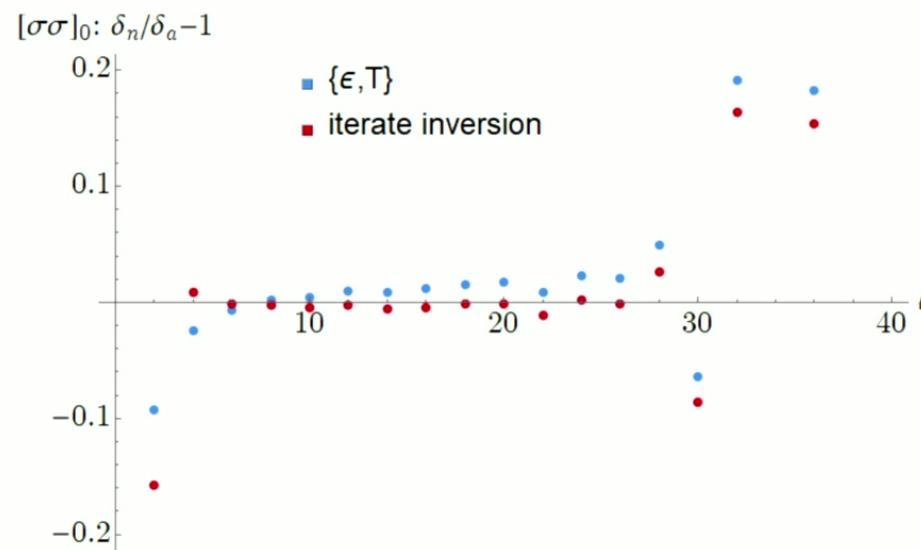
Leading twist operator in $\sigma \times \sigma$: $\Delta_{|\sigma\sigma|0,\ell} \approx 2\Delta_\sigma + \ell + \delta$

Inversion formula at the lightcone limit $z \ll 1 - \bar{z} \ll 1$:
Information of low lying operators → Information of the leading twist operators at large spin (approximately).

Small inset image shows a person giving a presentation in a lecture hall.

Analytic lightcone bootstrap : Example

$$\Delta_{[\sigma\sigma]_0,\ell} = 2 \Delta_\sigma + \ell + \delta$$



An analytic question : exact 4 pt = $\lim_{n \rightarrow \infty} \text{inversion}^n$ [information of ϵ, T] ?



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Overview

- *Introduction*
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- *Analytic bootstrap*
- *Hybrid bootstrap*
- *Outlook*

Overview

- Introduction
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- Analytic bootstrap
- Hybrid bootstrap
- Outlook

A video feed shows a lecture hall. A professor stands at a podium in front of a chalkboard filled with mathematical equations. A large projection screen behind the professor displays the same list of topics as the slide.

Why hybrid?

Does bootstrap equations from $\langle \sigma\sigma\sigma\sigma \rangle$, $\langle \epsilon\epsilon\epsilon\epsilon \rangle$, $\langle \sigma\sigma\epsilon\epsilon \rangle$ uniquely determine 3D Ising Δ_σ , Δ_ϵ ?
Numerically it means the island should shrink to a point.

So far the shrinking is very slow at high Λ , but we know a reason :

Traditional numerics : derivative expansion at $z = \bar{z} = 1/2$
Conformal block at $z \ll 1 - \bar{z} \ll 1 : G \sim \log(z)$

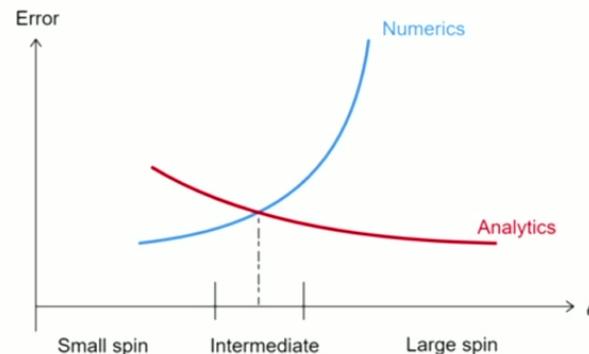
Derivative expansion is exponentially inefficient for $z \rightarrow 0$



Why hybrid?

Numerics : around $z = \bar{z} = 1/2$: sensitive to small Δ , **small ℓ** .

Analytics : around $z \ll 1 - \bar{z} \ll 1$: $\Delta_{|\sigma\sigma|0,\ell} \approx f(\Delta_\sigma, \Delta_\epsilon, f_{\sigma\sigma\epsilon}, f_{\sigma\sigma T})$ accurate for leading twist at **large ℓ** .



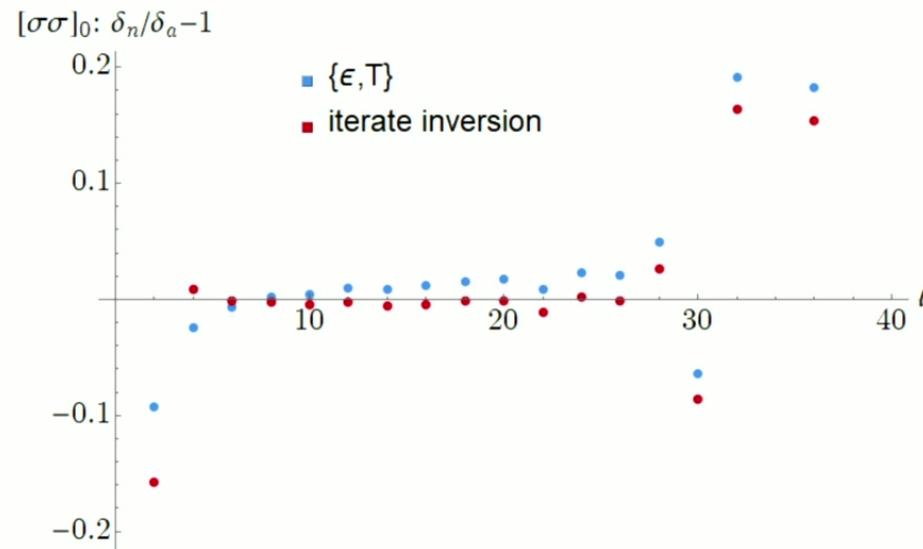
We should combine the numerics & analytics :
two difficulties in the past (before the navigator was invented)...



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Analytic lightcone bootstrap : Example

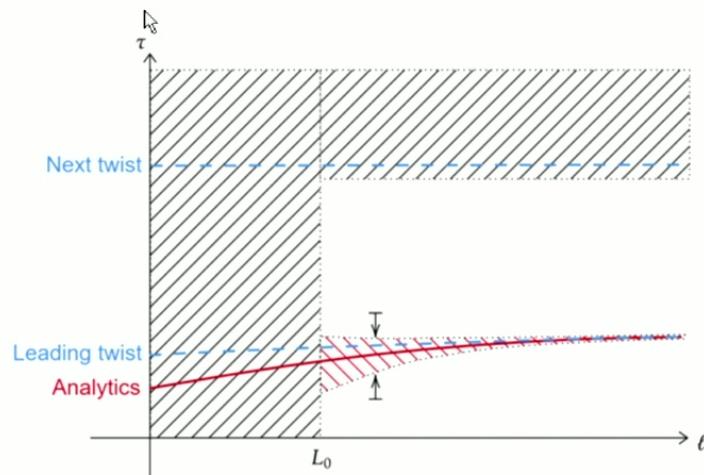
$$\Delta_{[\sigma\sigma]_0,\ell} = 2 \Delta_\sigma + \ell + \delta$$



An analytic question : exact 4 pt = $\lim_{n \rightarrow \infty} \text{inversion}^n$ [information of ϵ, T] ?



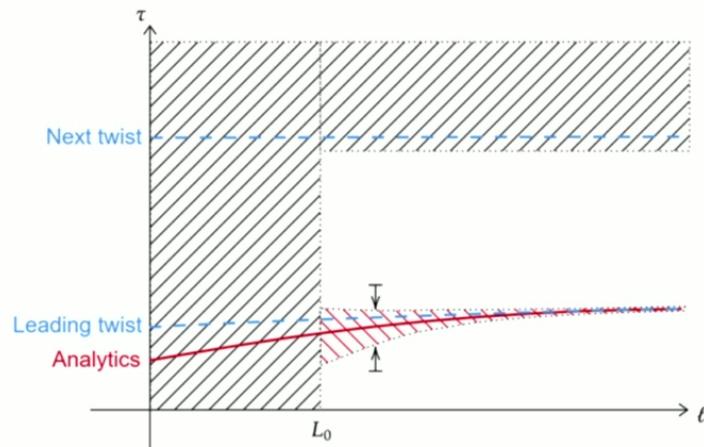
The hybrid bootstrap



- 1, Choose (Δ_σ , Δ_ϵ , $f_{\epsilon\epsilon}$, $f_{\sigma\sigma\epsilon}$, $f_{\sigma\sigma T}$, *Area of red*)
- 2, Compute analytic bootstrap
- 3, Run the numerics (navigator),
demanding operators exist in the shaded region
- 4, Repeat and minimize the *Area*
while navigator function ≤ 0



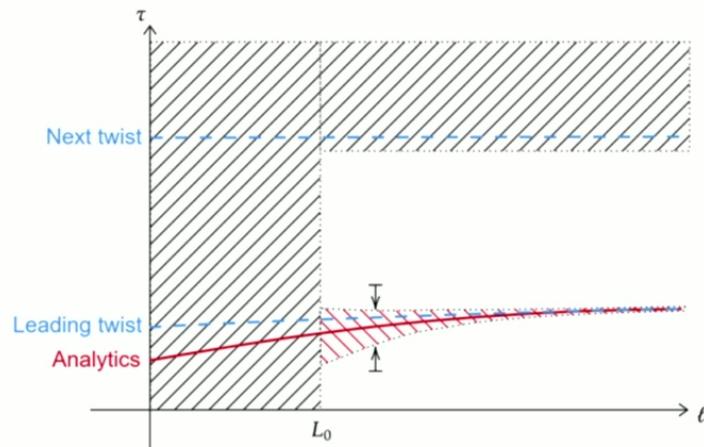
The hybrid bootstrap



- 1, Choose (Δ_σ , Δ_ϵ , $f_{\epsilon\epsilon}$, $f_{\sigma\sigma\epsilon}$, $f_{\sigma\sigma T}$, **Area of red**)
- 2, Compute analytic bootstrap
- 3, Run the numerics (navigator),
demanding operators exist in the shaded region
- 4, Repeat and minimize the *Area*
while navigator function ≤ 0



The hybrid bootstrap



- 1, Choose (Δ_σ , Δ_ϵ , $f_{\epsilon\epsilon}$, $f_{\sigma\sigma\epsilon}$, $f_{\sigma\sigma T}$, *Area of red*)
- 2, Compute analytic bootstrap
- 3, Run the numerics (navigator),
demanding operators exist in the shaded region
- 4, Repeat and minimize the *Area*
while navigator function ≤ 0 ∇



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The hybrid bootstrap

Parameter space

#1: Parameter → Analytics → Numerics → Navigator → Next Parameter

Decrease the red area

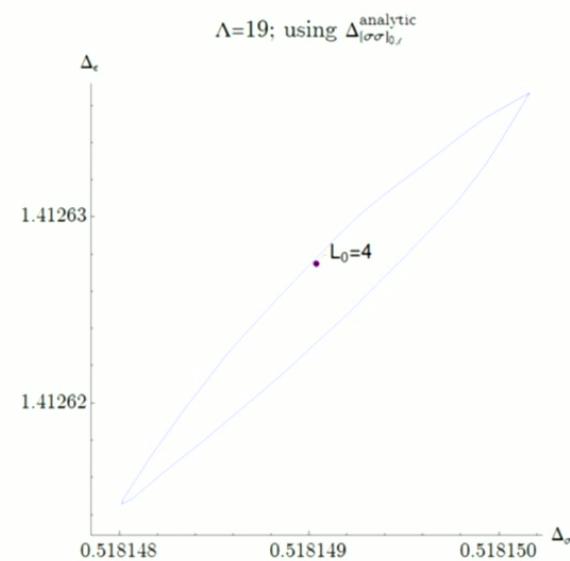
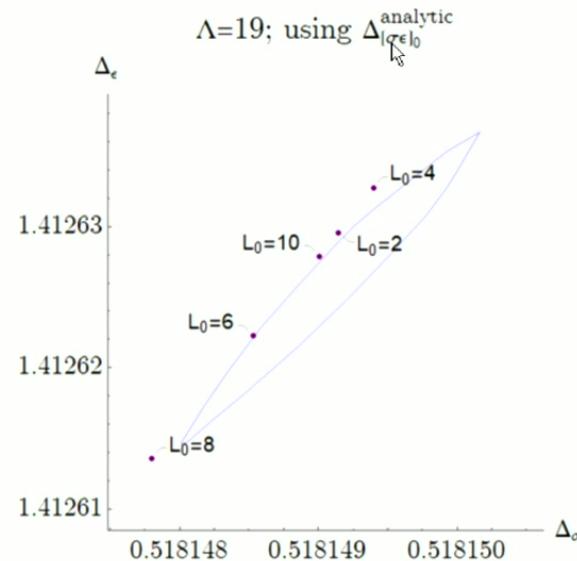
#2: Parameter → Analytics → Numerics → Navigator → Next Parameter

#3

Navigator=0 (can't decrease the red anymore)

The diagram illustrates the hybrid bootstrap process. It starts with a 'Parameter space' represented by a horizontal and vertical axis. Three points represent the flow of the process: point #1 is at the top right; point #2 is below it; and point #3 is further down and to the right. A solid arrow points from #1 to #2, labeled '#1: Parameter → Analytics → Numerics → Navigator → Next Parameter'. Another solid arrow points from #2 to #3, labeled '#2: Parameter → Analytics → Numerics → Navigator → Next Parameter'. A dashed arrow points from #3 downwards and to the right, labeled 'Navigator=0 (can't decrease the red anymore)'. A text box on the left says 'Decrease the red area' with an arrow pointing towards the dashed arrow. The slide has a blue header bar.

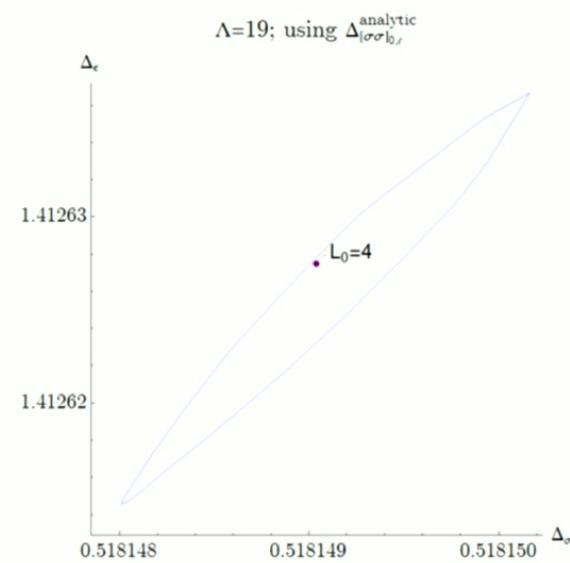
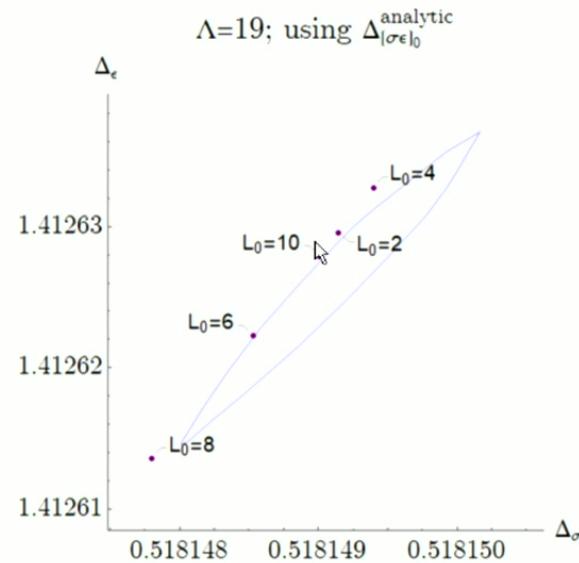
The hybrid bootstrap



hybrid bootstrap at $\Lambda = 19$ predicts very precise values that are within the previous $\Lambda = 43$ rigorous error bars!

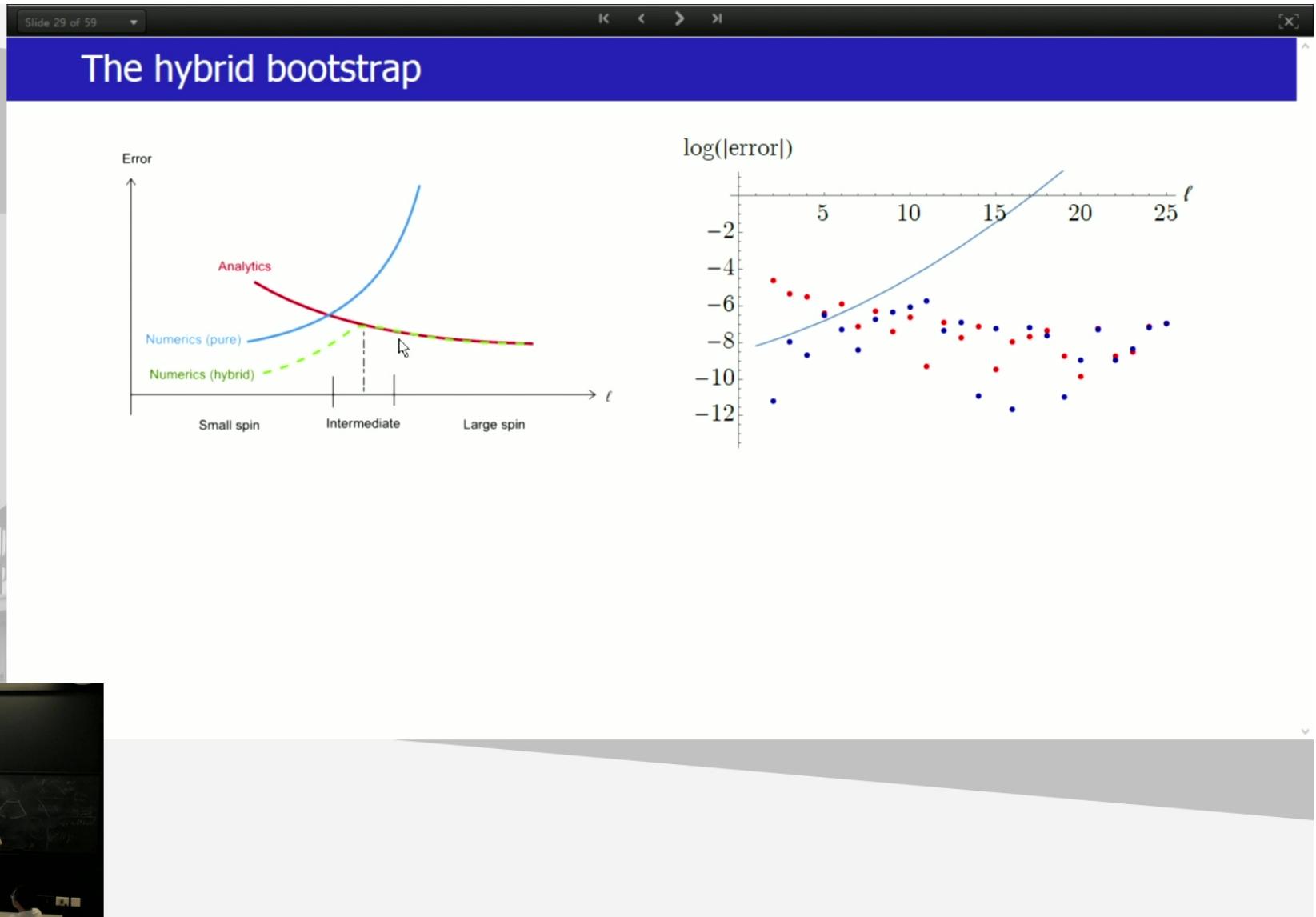


The hybrid bootstrap



hybrid bootstrap at $\Lambda = 19$ predicts very precise values that are within the previous $\Lambda = 43$ rigorous error bars!





An lesson for numerical bootstrapper

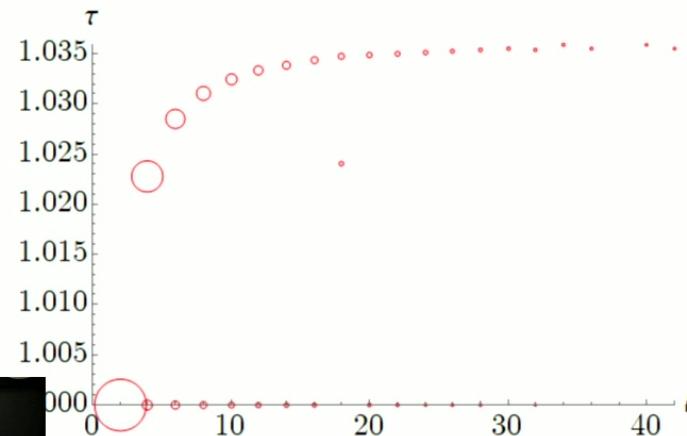
A lesson for bootstrap numerics : the leading twist is important

In pure numerics, we demand $\Delta \geq d - 2 - \ell$ (unitarity bound)

In hybrid numerics, we demand $\Delta \geq \Delta_{\text{leading twist}} - \epsilon$

$\Lambda=19$ pure numerics: 2D Ising : $\delta(\Delta_\sigma) \sim 10^{-6}$; 3D Ising : $\delta(\Delta_\sigma) \sim 10^{-5}$; 3D Super-Ising : $\delta(\Delta_\sigma) \sim 10^{-4}$

Z_2 even spectrum



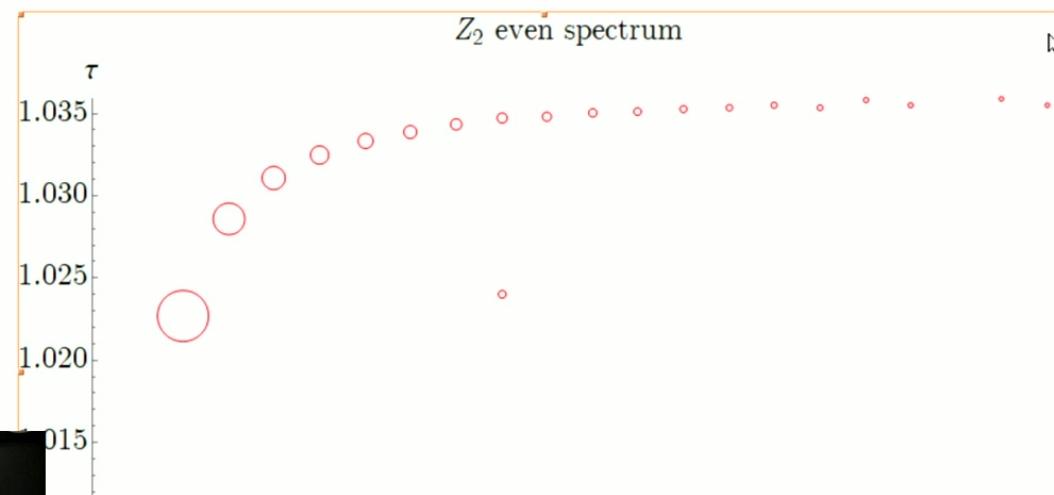
An lesson for numerical bootstrapper

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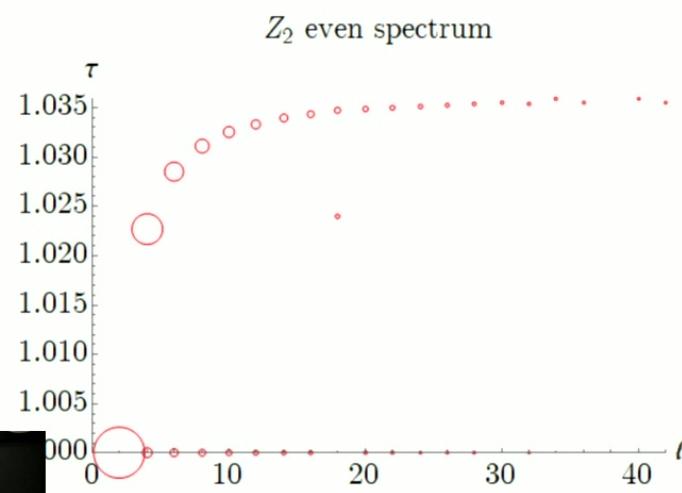
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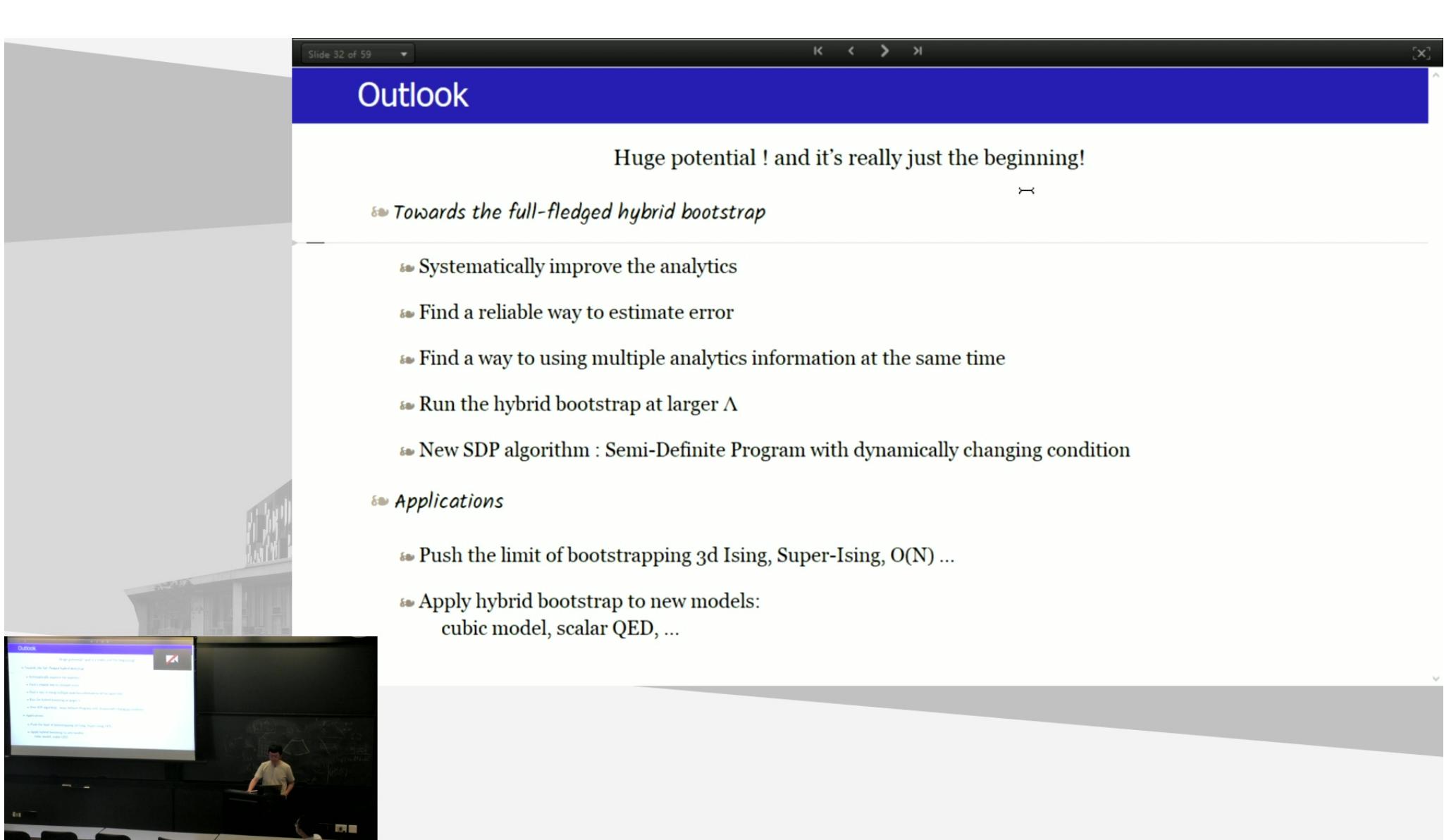
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Overview

- *Introduction*
- *Numerical bootstrap*
- *Analytic bootstrap*
- *Hybrid bootstrap*
- *Outlook*

Next Slide

The image shows a presentation slide titled "Overview" on a computer screen. The slide contains a list of five items, each preceded by a small orange icon. The items are: "Introduction", "Numerical bootstrap", "Analytic bootstrap", "Hybrid bootstrap", and "Outlook". The "Outlook" item is highlighted in red. At the top of the slide, there are navigation icons for back, forward, and search, along with a "Next Slide" button. In the bottom left corner of the slide area, there is a small video window showing a person standing at a chalkboard in a lecture hall. The chalkboard has some mathematical equations written on it.



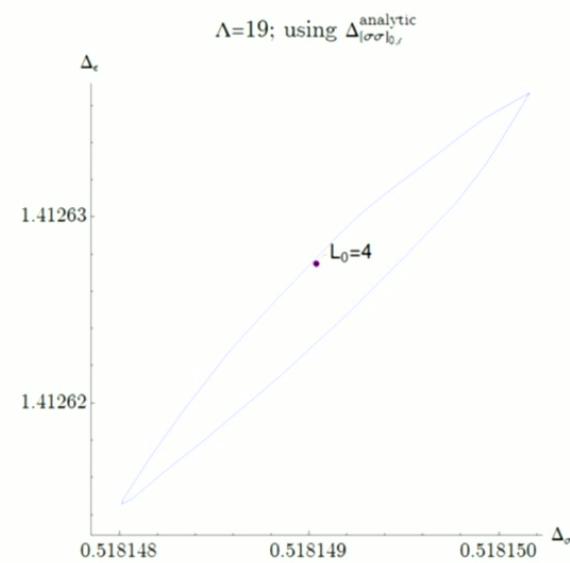
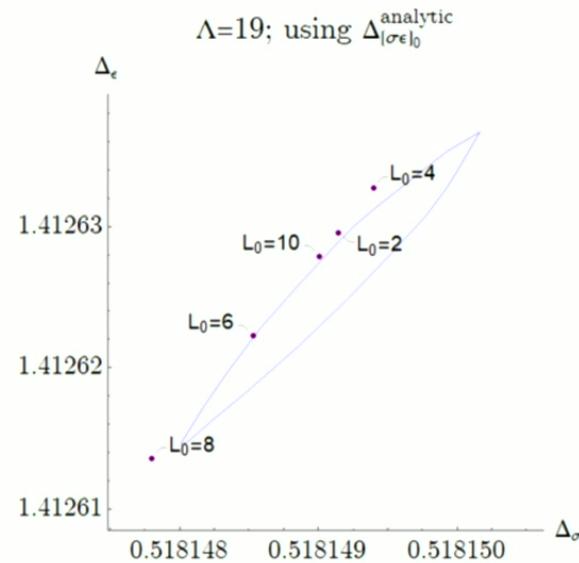
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Outlook

Huge potential ! and it's really just the beginning!

- ❖ Towards the full-fledged hybrid bootstrap
 - ❖ Systematically improve the analytics
 - ❖ Find a reliable way to estimate error
 - ❖ Find a way to using multiple analytics information at the same time
 - ❖ Run the hybrid bootstrap at larger Λ
 - ❖ New SDP algorithm : Semi-Definite Program with dynamically changing condition
- ❖ Applications
 - ❖ Push the limit of bootstrapping 3d Ising, Super-Ising, O(N) ...
 - ❖ Apply hybrid bootstrap to new models:
cubic model, scalar QED, ...

The hybrid bootstrap

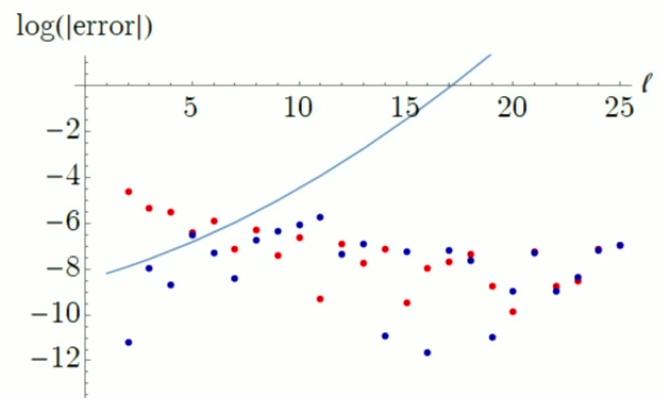
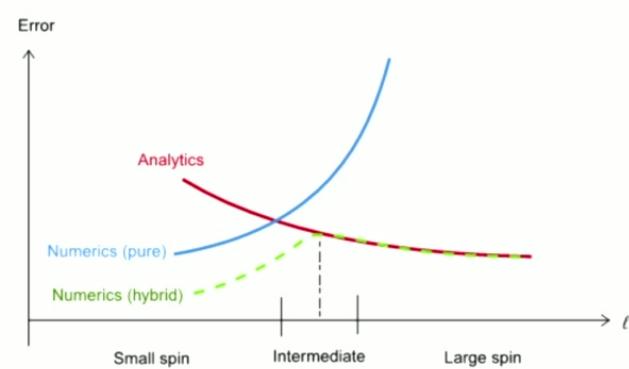


hybrid bootstrap at $\Lambda = 19$ predicts very precise values that are within the previous $\Lambda = 43$ rigorous error bars!



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The hybrid bootstrap



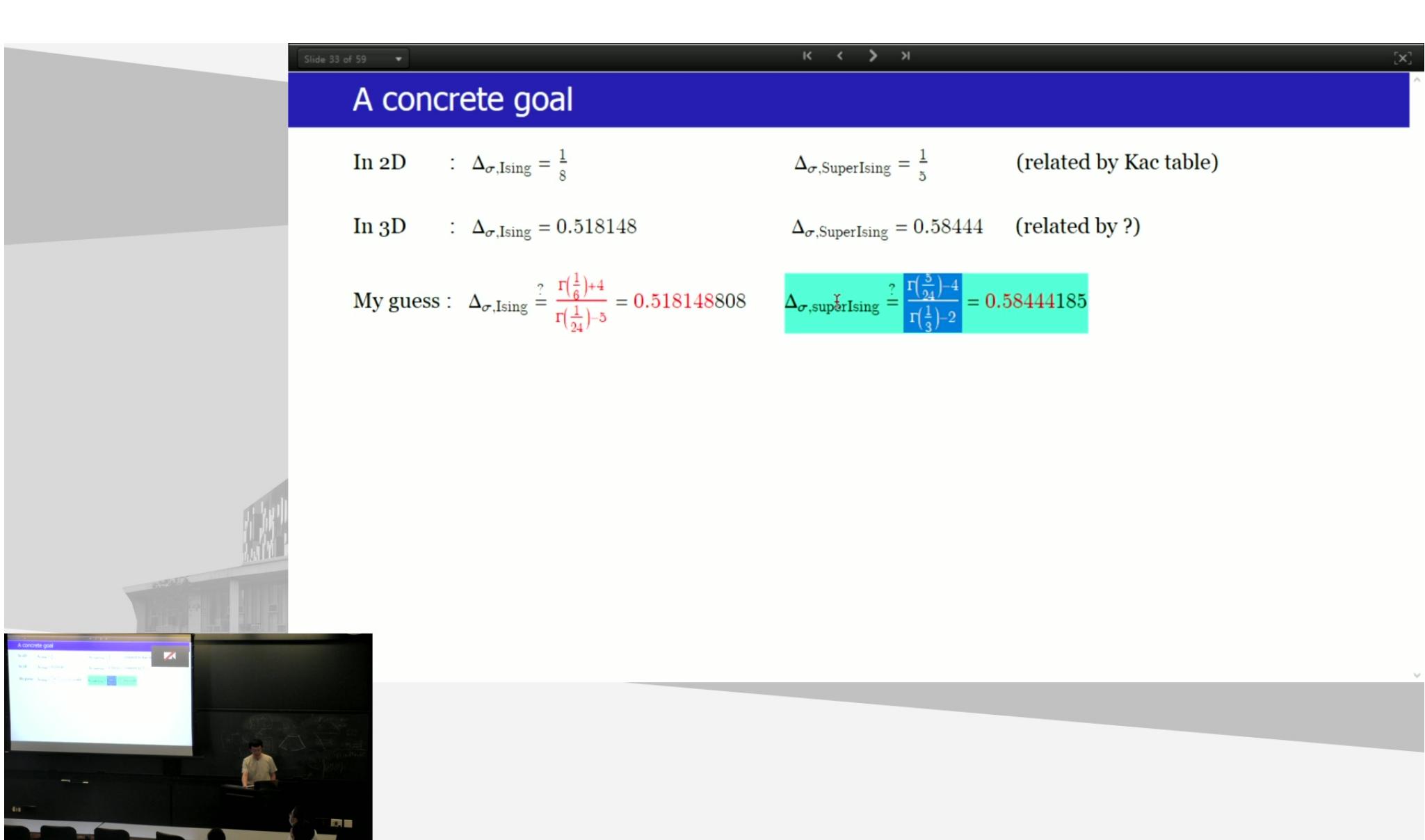
Slide 33 of 59

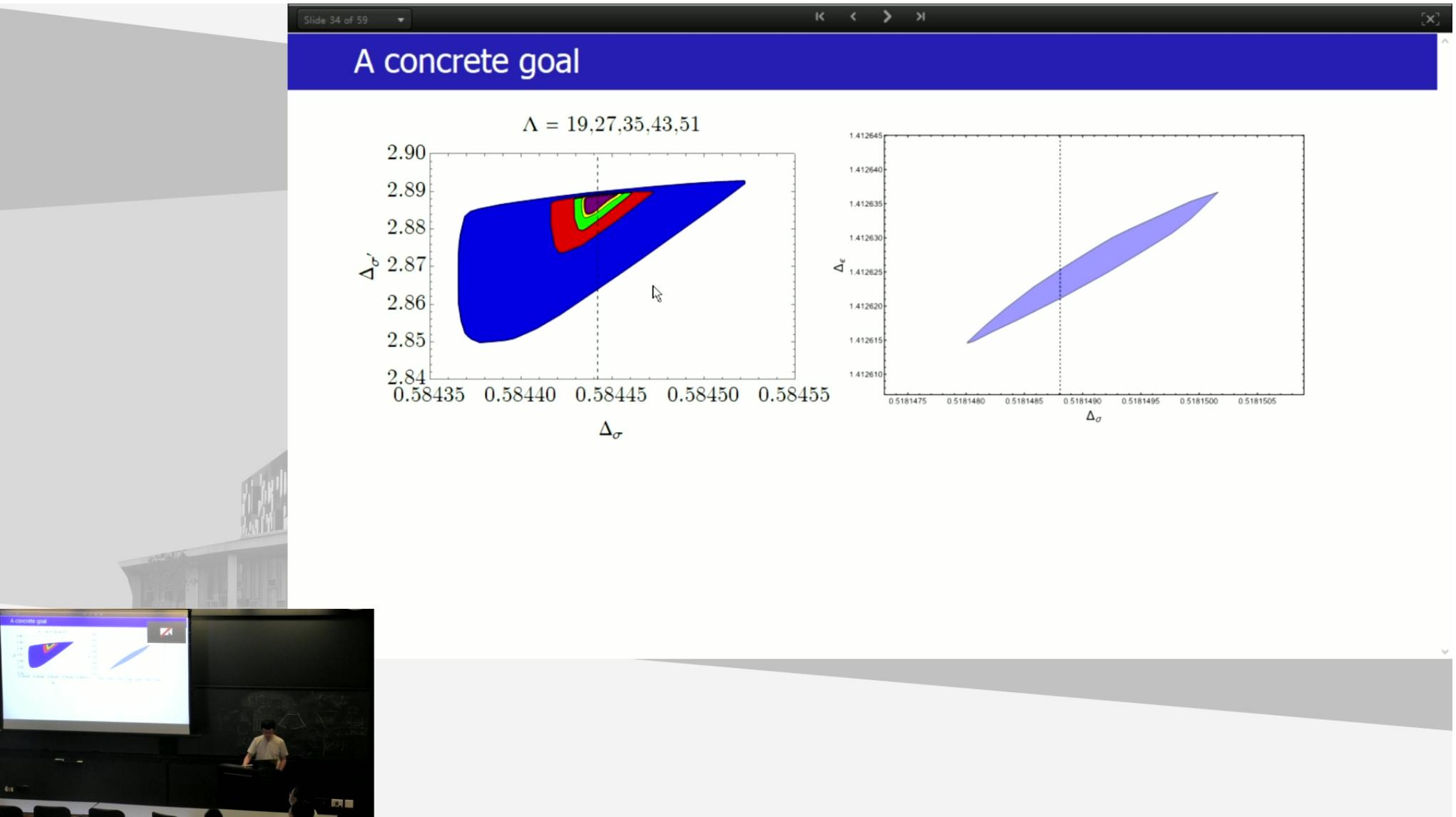
A concrete goal

In 2D : $\Delta_{\sigma,\text{Ising}} = \frac{1}{8}$ $\Delta_{\sigma,\text{SuperIsing}} = \frac{1}{5}$ (related by Kac table)

In 3D : $\Delta_{\sigma,\text{Ising}} = 0.518148$ $\Delta_{\sigma,\text{SuperIsing}} = 0.58444$ (related by ?)

My guess : $\Delta_{\sigma,\text{Ising}} = \frac{\Gamma(\frac{1}{6})+4}{\Gamma(\frac{1}{24})-5} = 0.518148808$ $\Delta_{\sigma,\text{SuperIsing}} = \frac{\Gamma(\frac{5}{24})-4}{\Gamma(\frac{1}{3})-2} = 0.58444185$





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A concrete goal

In 2D : $\Delta_{\sigma,\text{Ising}} = \frac{1}{8}$ $\Delta_{\sigma,\text{SuperIsing}} = \frac{1}{5}$ (related by Kac table)

In 3D : $\Delta_{\sigma,\text{Ising}} = 0.518148$ $\Delta_{\sigma,\text{SuperIsing}} = 0.58444$ (related by ?)

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Thank you

