Title: Deconfined quantum critical points with emergent SO(5) symmetry in fermionic models

Speakers: Hong Yao

Collection: Quantum Criticality: Gauge Fields and Matter

Date: May 20, 2022 - 11:15 AM

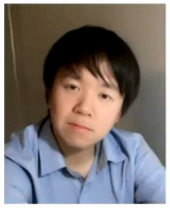
URL: https://pirsa.org/22050050

Deconfined quantum criticality with emergent SO(5) symmetry and superconducting criticality with emergent supersymmetry

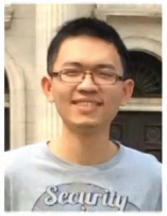
Hong Yao
Institute for Advanced Study, Tsinghua University

Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975
Zi-Xiang Li, A. Vaezi, C. Mendl, and Hong Yao, Science Advances 4, eaau1463 (2018)

Collaborators:

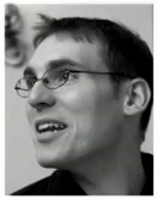


Zi-Xiang Li
(Berkeley → IOP, Beijing)



Shao-Kai Jian
(Brandeis University)

Abolhassan Vaezi
(Sharif University of Technology)



Christian Mendl
(TU Dresden)

Workshop on "Quantum Criticality: Gauge Fields and Matter"
Parameter Institute; May 20, 2022

Outline

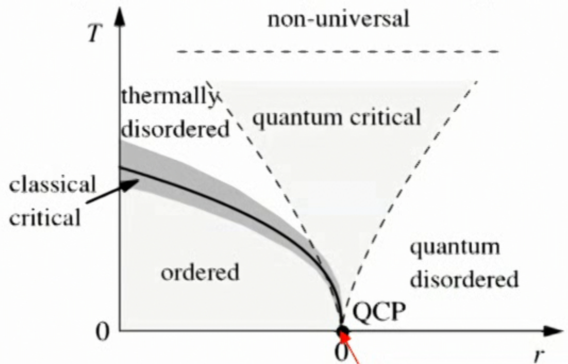
- Introduction
- Deconfined quantum critical point with emergent SO(5) symmetry between antiferromagnetism (AFM) and valence bond solid (VBS) in 2D Dirac semimetals.

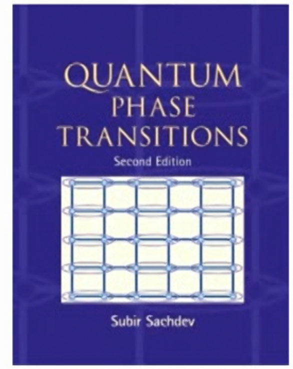
  Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975
- Emergent supersymmetry at superconducting critical point on the surface of 3D topological insulators.

  Zi-Xiang Li, A. Vaezi, C. Mendl, and Hong Yao, Science Advances 2018
- Summary

Quantum phase transitions

- Quantum phase transition (QPT): a phase transition at zero temperature by tuning **one** parameter in the Hamiltonian.





Emergent symmetry

- A quantum phase transition can be first-order or continuous.
- Note that a first-order transition is not forbidden by any principle. **Namely, nothing stops a first-order transition.**

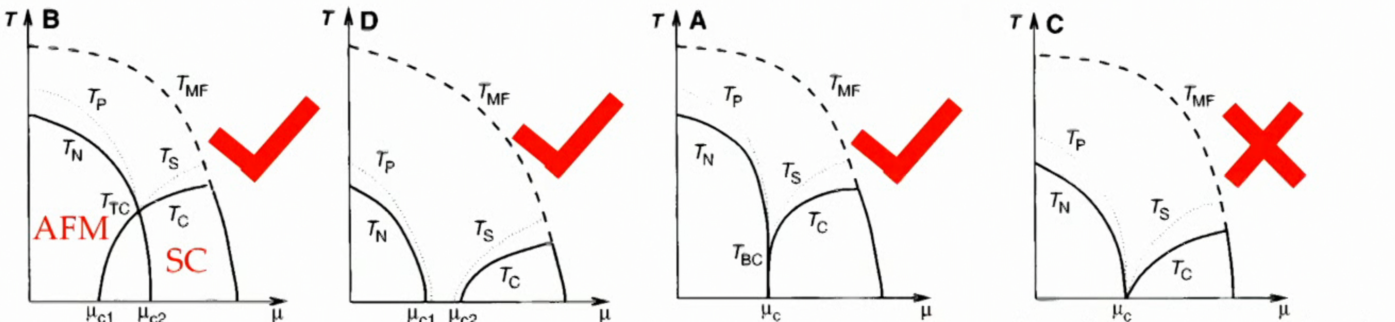
$F = F_0 + r\phi^2 + u\phi^4 + w\phi^6$, ($u < 0$ and $w > 0$: first order transition)

- Quantum critical point (QCP): a continuous QPT.
- Emergent symmetries, such as SO(5) symmetry and supersymmetry, can occur at QCP.

Direct transitions between two broken-symmetry phases (two competing orders): first-order or second order?

- Landau theory states that a continuous transition is forbidden if one broken symmetry is not the subgroup of the other.

- For example, according to the Landau theory, a direct continuous transition between SC and AFM is not allowed.



Shou-Chang Zhang, Science 1997

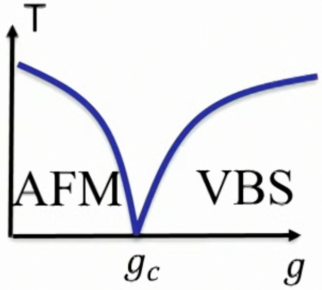
First-order transition is allowed in principle.

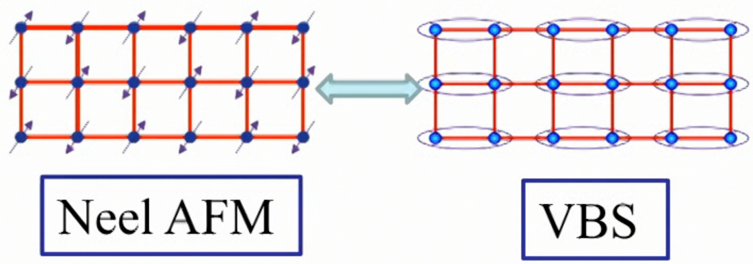
Continuous transition is forbidden by Landau.

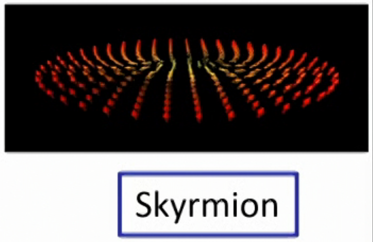
- Questions: Can Landau-forbidden continuous transition occur?

Proposal of deconfined QCP between AFM and VBS: a non-Landau QCP

- Proposal: Deconfined quantum critical point (DQCP) in quantum magnets.

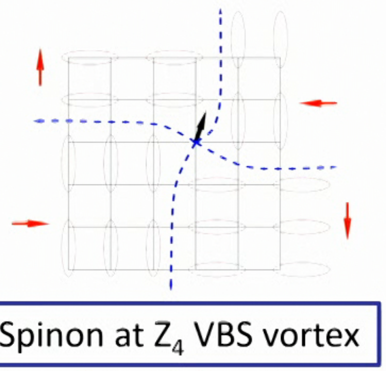






Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004

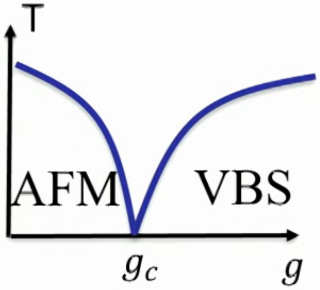
- AFM and VBS are two competing orders.
- Skyrmion carries VBS quantum number.
- VBS vortex carries spinon quantum number.
- Spinons are deconfined at the QCP.

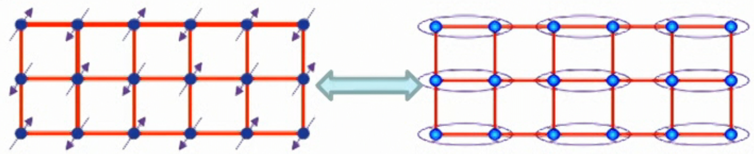


Levin and Senthil, PRB 2004

Proposal of deconfined QCP between AFM and VBS: a non-Landau QCP

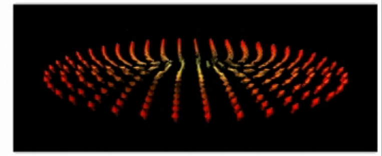
- Proposal: Deconfined quantum critical point (DQCP) in quantum magnets.





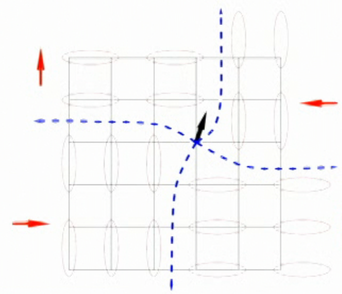
Neel AFM

VBS



Skyrmion

Senthil, Vishwanath, Balents, Sachdev, Fisher, Science 2004



Spinon at Z_4 VBS vortex

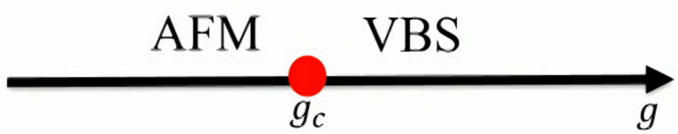
Levin and Senthil, PRB 2004

- AFM and VBS are two competing orders.
- Skyrmion carries VBS quantum number.
- VBS vortex carries spinon quantum number.
- Spinons are deconfined at the QCP.

Conditions of DQCP with emergent SO(5) symmetry: bounds from conformal bootstrap

- Assuming emergent SO(5) symmetry at the DQCP, conformal bootstrap analysis shows that the critical exponents have to satisfy the following bounds: $\eta > 0.52$.

Poland, Rychkov, and Vichi, Review of Modern Physics 2019



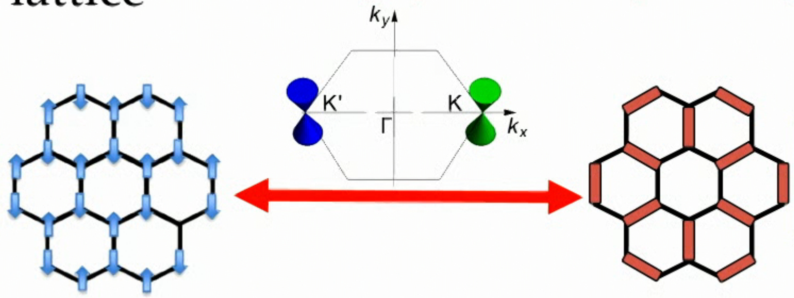
- Previous QMC simulations of spin models obtained $\eta \approx 0.2 \sim 0.4$, which violates the conformal bounds.
- This implies those spin models cannot support genuine DQCP with emergent SO(5) symmetry. Instead, the AFM-VBS transition in those models are weakly first-order with long correlation length.
- Note that the results from those models do not rule out DQCP.
- Question: Is it possible to have DQCP with emergent SO(5) symmetry in other 2D models?

Our study: looking for DQCP and SO(5) symmetry in Dirac semimetals on honeycomb lattice

- Consider AFM ↔ VBS transition in Dirac semimetals on honeycomb lattice

Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975

AFM breaks spin SO(3) symmetry

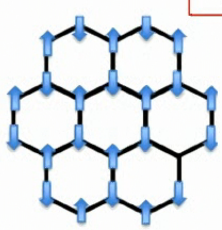


Kekule VBS breaks translation symmetry

- A unified view from Dirac fermions
  - Dirac fermions ψ_{+K} at valley +K and ψ_{-K} at valley –K
  - AFM + VBS = five mass terms of the Dirac fermions

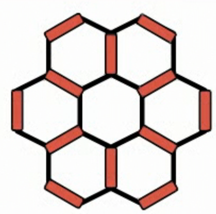
$$\phi_\mu = (N_1, N_2, N_3, \Phi_1, \Phi_2)$$

five-dimensional "superspin"



= sublattice AFM

$$\vec{N} = \psi^\dagger_{\pm K}\sigma^z \vec{s}\psi_{\pm K}$$



= intervalley scattering

$$\Phi = \left(\psi^\dagger_K \sigma^x \psi_{-K}, \psi^\dagger_{-K}\sigma^x \psi_K\right)$$

WZW terms from integrating out Dirac fermions

- Integrating out Dirac fermions can lead to topological WZW terms of five-component superspin.

$$\phi_\mu = (N_1, N_2, N_3, \Phi_1, \Phi_2),\ \phi^2 = 1$$

$$S = \log \int D\bar{\psi} D\psi \exp\left[-\int \bar{\psi} \left(\mathcal{D} + \vec{N} \cdot \vec{s} + \Phi_1 \gamma_3 + \Phi_2 \gamma_5 \right) \psi \right]$$

$$S = \int d^3x \frac{1}{2g} (\partial\phi)^2 + \frac{2\pi i \epsilon^{abcde}}{Area(S^4)} \int d\xi d^3x \phi_a \partial\phi_b \partial\phi_c \partial\phi_d \partial\phi_e + \cdots$$

WZW term

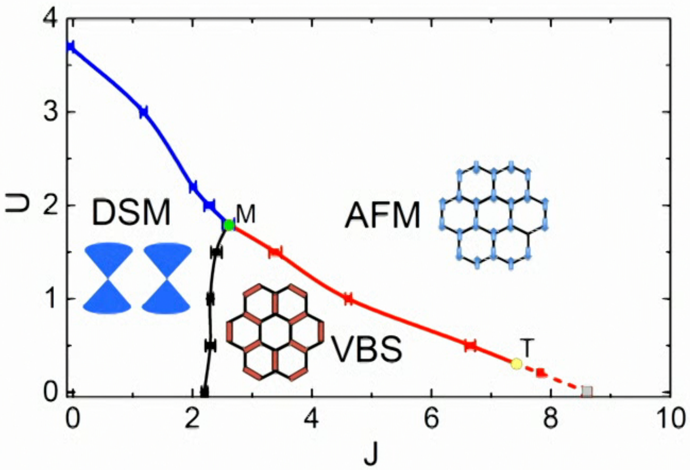
- The appearance of topological WZW term is the hallmark of Landau-forbidden transitions and is conjectured to play an essential role in driving a deconfined QCP.
- The bare value of the coupling g: $g \sim 1/m_f$ (m_f is the fermion mass at the transition between AFM and VBS). Small fermon mass means strong coupling g.

QMC of the extended Hubbard model on honeycomb lattice: Direct transition between AFM and VBS

- The extended Hubbard model on the honeycomb lattice:

$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + H.c.) + U \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) - \frac{J}{4} \sum_{\langle ij \rangle} \Big(\sum_\sigma c_{i\sigma}^\dagger c_{j\sigma} + H.c. \Big)^2$$

- The ground-state phase diagram is obtained by sign-problem-free QMC simulations.



Congjun Wu and Shou-Cheng Zhang, PRB 2005

Huffman and Chandrasekharan, PRB 2014

Zi-Xiang Li, Yi-Fan Jiang, and **HY**, PRB 2015

- We find a direct continuous transition between VBS and AFM.
- What is the nature of the direct transition: first-order or second-order?
- Does SO(5) symmetry emerge at the transition?

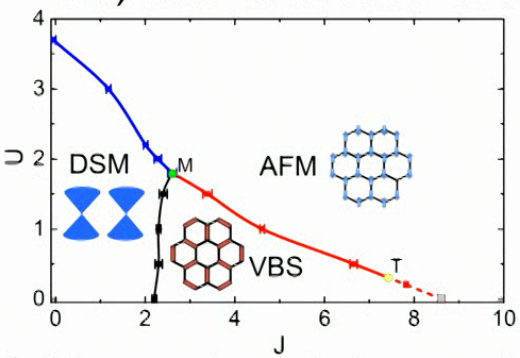
Zi-Xiang Li, Shao-Kai Jian, and **HY,** arXiv:1904.10975

Evidences of a second-order transitions between VBS and AFM: a deconfined quantum critical point

- The extended Hubbard model on the honeycomb lattice:

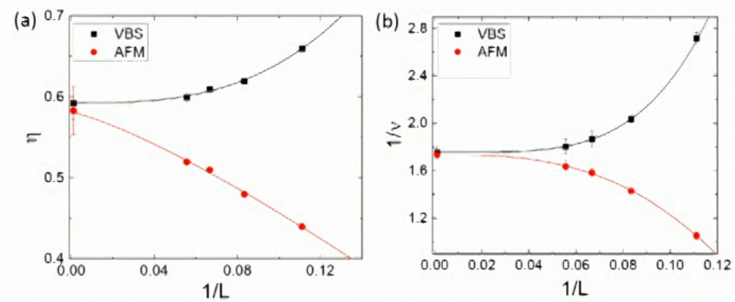
$$H = -t \sum_{\langle ij \rangle \sigma} (c^\dagger_{i\sigma} c_{j\sigma} + H.c.) + U \sum_i (n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) - \frac{J}{4} \sum_{\langle ij \rangle} \Big(\sum_\sigma c^\dagger_{i\sigma} c_{j\sigma} + H.c. \Big)^2$$

- QMC evidences of second-order VBS-AFM transition for U=0.5, 1.0, 1.5. It is first-order transition for U=0 and U=0.2.



For U=1.0:

- $J_c \approx 4.6$
- $\eta \approx 0.59$
- $\frac{1}{\nu} \approx 1.74$



Zi-Xiang Li, Shao-Kai Jian, and **HY,** arXiv:1904.10975

◆ Rigorous bounds from conformal field theory calculations assuming SO(5) symmetry: $\eta > 0.52$ and $\frac{1}{\nu} < 1.957$.

◆ **Our results are consistent with the conformal bounds: implying that it is possible to have DQCP with emergent SO(5) symmetry at the VBS-AFM transition in our model.**

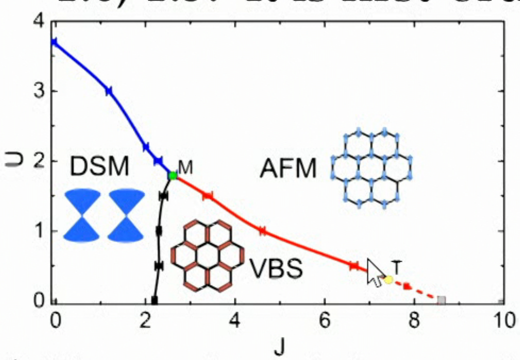
➢ Very recently, $\eta \approx 0.58$ was obtained for a similar VBS-AFM transition in a square lattice π-flux model. Liao et al, arXiv:2204.04884; Zhu et al, arXiv:2204.12147

Evidences of a second-order transitions between VBS and AFM: a deconfined quantum critical point

- The extended Hubbard model on the honeycomb lattice:

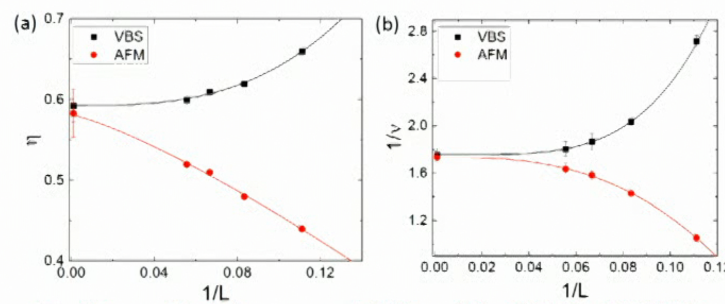
$$H = -t\sum_{\langle ij\rangle\sigma}(c_{i\sigma}^{\dagger}c_{j\sigma} + H.c.) + U\sum_{i}(n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2}) - \frac{J}{4}\sum_{\langle ij\rangle}\Big(\sum_{\sigma} c_{i\sigma}^{\dagger}c_{j\sigma} + H.c.\Big)^2$$

- QMC evidences of second-order VBS-AFM transition for U=0.5, 1.0, 1.5. It is first-order transition for U=0 and U=0.2.



For U=1.0:

- ➢ $J_c \approx 4.6$
- ➢ $\eta \approx 0.59$
- ➢ $\frac{1}{\nu} \approx 1.74$



Zi-Xiang Li, Shao-Kai Jian, and **HY,** arXiv:1904.10975

- ◆ Rigorous bounds from conformal field theory calculations assuming SO(5) symmetry: $\eta > 0.52$ and $\frac{1}{\nu} < 1.957$.
- ◆ **Our results are consistent with the conformal bounds: implying that it is possible to have DQCP with emergent SO(5) symmetry at the VBS-AFM transition in our model.**

- ➢ Very recently, $\eta \approx 0.58$ was obtained for a similar VBS-AFM transition in a square lattice π-flux model. Liao et al, arXiv:2204.04884; Zhu et al, arXiv:2204.12147

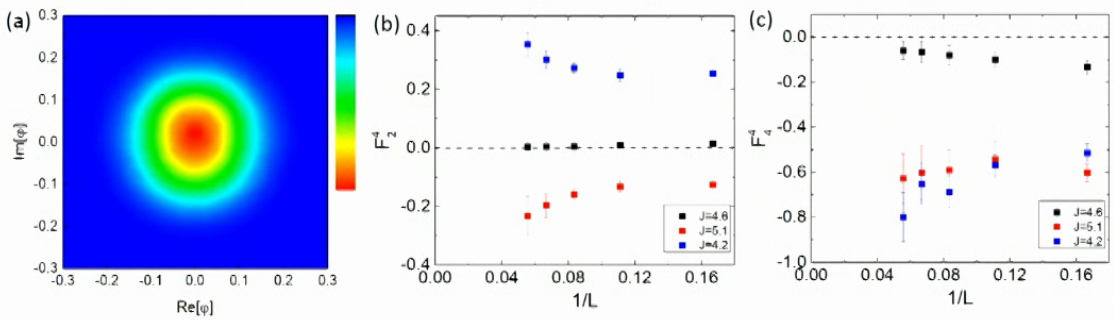
Direct evidences of emergent SO(5) symmetry

- QMC evidences of emergent SO(5) symmetry at the DQCP
- Five-component order-parameter or superspin:

$$\phi_\mu = (N_1, N_2, N_3, \Phi_1, \Phi_2),\ \phi^2 = 1$$

- Emergent SO(5) symmetry requires the following:
  - Rotational symmetry between Φ_1 and Φ_2
  - Rotational symmetry between N_1 and Φ_1

$$F_2^4 = \langle N_1^4 - \Phi_1^4 \rangle = 0 \text{ and } F_4^4 = \langle N_1^4 + \Phi_1^4 - 6N_1^2\Phi_1^2 \rangle = 0$$



➢ **It provides the first convincing example of DQCP with emergent SO(5) symmetry.**

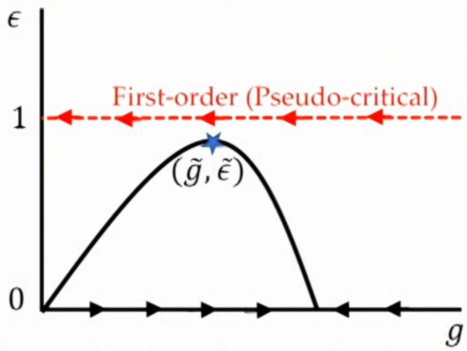
Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975

Genuine DQCP or pseudo-criticality?

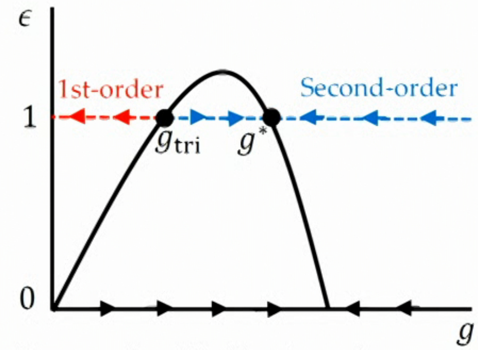
- Previous numerical results of η being not consistent with conformal bootstrap bounds inspired the proposal of pseudo-criticality for the VBS-AFM transition.

$$S = \int d^3x \frac{1}{2g}(\partial\phi)^2 + \frac{2\pi i \epsilon^{abcde}}{Area(S^4)} \int d\xi d^3x \phi_a \partial\phi_b \partial\phi_c \partial\phi_d \partial\phi_e$$

$\epsilon = d - 2$ ($d = 3$ for the physical systems)



Scenario #1: always first-order for all the possible bare value of g. Pseudo-criticality occurs if $g \geq \tilde{g}$.
Ruochen Ma and Chong Wang, PRB 2020
Nahum, PRB 2020



Scenario #2: first-order or second-order depending on the value of g. There is a tricritical point g_{tri} separating the first- and second-order transitions.

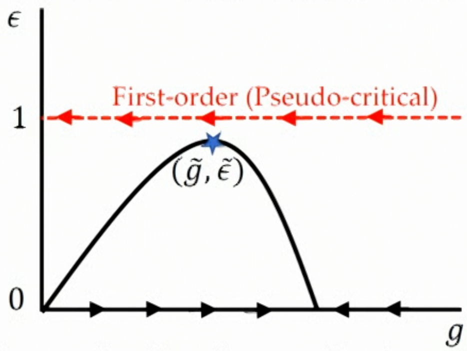
- Our model (arXiv:1904.10975) supports the second scenario. Dirac fermions play an important role for setting the bare value of g.

Genuine DQCP or pseudo-criticality?

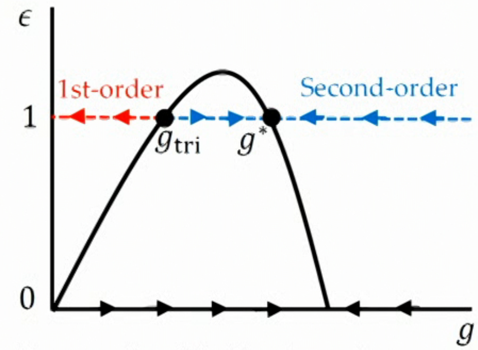
- Previous numerical results of η being not consistent with conformal bootstrap bounds inspired the proposal of pseudo-criticality for the VBS-AFM transition.

$$S = \int d^3x \frac{1}{2g}(\partial\phi)^2 + \frac{2\pi i \epsilon^{abcde}}{Area(S^4)} \int d\xi d^3x \phi_a \partial\phi_b \partial\phi_c \partial\phi_d \partial\phi_e$$

$\epsilon = d - 2$ ($d = 3$ for the physical systems)



Scenario #1: always first-order for all the possible bare value of g. Pseudo-criticality occurs if $g \geq \tilde{g}$.
Ruochen Ma and Chong Wang, PRB 2020
Nahum, PRB 2020



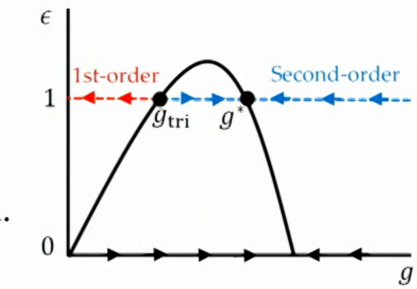
Scenario #2: first-order or second-order depending on the value of g. There is a tricritical point g_{tri} separating the first- and second-order transitions.

- Our model (arXiv:1904.10975) supports the second scenario. Dirac fermions play an important role for setting the bare value of g.

DQCP in fermionic models

- Our model features massless Dirac fermions, which is different from previous pure spin models.

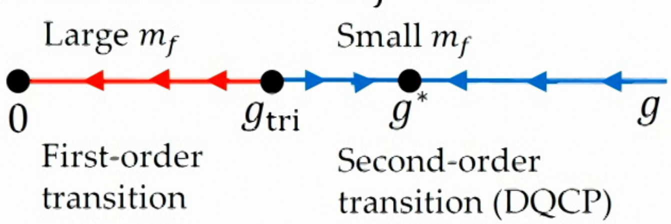
- The coupling g of the NLSM with WZW terms depends on the mass of Dirac fermions at the VBS-AFM transition.

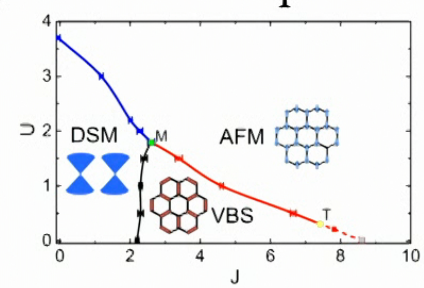


$$S = \int d^3x \frac{1}{2g}(\partial\phi)^2 + \frac{2\pi i \epsilon^{abcde}}{Area(S^4)} \int d\xi d^3x \phi_a \partial\phi_b \partial\phi_c \partial\phi_d \partial\phi_e$$

$g \sim 1/m_f$, m_f is the mass of Dirac fermions at the transition.

- Our fermionic model has a multi-critical point **M**, where three phases meet and where $m_f = 0$.





Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975

Outline

- Introduction
- Deconfined quantum critical point with emergent SO(5) symmetry between antiferromagnetism (AFM) and valence bond solid (VBS) in 2D Dirac semimetals.
  Zi-Xiang Li, Shao-Kai Jian, and Hong Yao, arXiv:1904.10975
- Emergent supersymmetry at superconducting critical point on the surface of 3D topological insulators.
  Zi-Xiang Li, A. Vaezi, C. Mendl, and Hong Yao, Science Advances 2018
- Summary

Conclusions

- DQCP with emergent SO(5) symmetry is shown to occur at the AFM-VBS quantum phase transition in Dirac fermion models. It is the first convincing example of DQCP with emergent SO(5) symmetry and with critical exponents consistent with conformal bootstrap bounds.

  Zi-Xiang Li, Shao-Kai Jian, and **Hong Yao**, arXiv:1904.10975

- Supersymmetry is shown to emerge at the superconducting transition in a single-Dirac cone model (like the surface of 3D topological insulators).

  Zi-Xiang Li, A. Vaezi, C. Mendl, and **Hong Yao**, Science Advances 2018