

Title: Anomalies and symmetric mass generation for staggered fermions “ Talk 2

Speakers: Simon Catterall

Collection: Quantum Criticality: Gauge Fields and Matter

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Anomaly

lattice

$$S_{KD}(\phi, \bar{\phi}) \quad m=0$$

$$\phi \rightarrow e^{i\alpha} \phi \quad \bar{\phi} \rightarrow e^{-i\alpha} \bar{\phi}$$

classical symmetry

$$Z = \int D\phi D\bar{\phi} e^{-S_{KD}}$$

$$\prod_P D\phi_P D\bar{\phi}_P$$

$$\prod_{\text{sites}} D\phi_i^0 = e^{i2\alpha N_0}$$

0 perms

$$\prod_{\text{links}} D\phi_i^1 = e^{-i2\alpha N_1}$$

$$D\phi D\bar{\phi} \rightarrow D\phi D\bar{\phi} e^{i2\alpha N_0} e^{-i2\alpha N_1} e^{i2\alpha N_2}$$

$$e^{i2\alpha (N_0 - N_1 + N_2 - \dots)}$$

only

lattice

$S_{KD}(\phi, \bar{\phi})$

$m=0$

$$\phi \rightarrow e^{i\alpha} \phi \quad \bar{\phi} \rightarrow e^{-i\alpha} \bar{\phi}$$

(classical symmetry)

$$\int D\phi D\bar{\phi} e^{-S_{KD}}$$

$$\prod_P D\phi_P D\bar{\phi}_P$$

$$\prod_{\text{sites}} D\bar{\phi}^0 \prod_{\text{links}} D\phi_i^0 = e^{i2\alpha N_0}$$

$$\prod_{\text{links}} D\phi_i^1 = e^{-i2\alpha N_1}$$

$$D\phi D\bar{\phi} \rightarrow D\phi D\bar{\phi} e^{2i\alpha N_0 - 2i\alpha N_1 + 2i\alpha N_2}$$

$$e^{2i\alpha (N_0 - N_1 + N_2 - \dots)}$$

reduced KP theory cannot be coupled to gravity

$$\prod_{\text{of rms}} D\bar{\phi}^0 \prod D\phi_i^0 = e^{i2\alpha N_0}$$

$$\prod_{\text{1 rms}} D\phi_i^1 = e^{-i2\alpha N_1}$$

$$D\phi D\bar{\phi} \rightarrow D\phi D\bar{\phi} e^{i2\alpha N_0} e^{-i2\alpha N_1} e^{i2\alpha N_2} \dots$$

$$e^{i2\alpha (N_0 - N_1 + N_2 - \dots)}$$

reduced KP theorem cannot be applied
to gravity

$$\mathcal{D}^N \rightarrow \mathcal{S}^N$$

$$\chi = z$$

$$U(1) \rightarrow \mathbb{Z}_4$$

KD $\psi \rightarrow U \psi U^\dagger$

$$d^D x \text{Tr}(\bar{\psi} \gamma^\mu \gamma^\nu D_\mu \psi)$$

$$D_\mu \psi = \partial_\mu \psi + [W_\mu, \psi]$$

$e^a_\nu = S_\mu$ $d=2$

$$X = \frac{1}{4\pi} \int R^{ab}_{\mu\nu} \epsilon^{\mu\nu} \epsilon_{ab}$$

curvature ω

Ex: global Z_2 symmetry

$d=2$

$(T, \bar{T})$
 $L \sim e^{i\alpha\sigma_3} (0, \pi)$

Act on Wilson lines

$W = e^{\oint W dx}$

$W \rightarrow -W \quad (e^{i\pi} W)$

non-contractible loop

KD

$$\psi \rightarrow U \psi U^\dagger$$

$$\int d^D x \text{Tr}(\bar{\psi} e^{\gamma_a} \gamma^a D_\mu \psi)$$

$$D_\mu \psi = \partial_\mu \psi + [W_\mu, \psi]$$

$$e^a_\mu e^b_\nu = \delta_{\mu\nu}$$

$$d=2$$

$$X = \frac{1}{4\pi} \int R^{ab} \epsilon^{12} \epsilon_{ab}$$

↑
curvature ω

global Z_2 symmetry

$$d=2$$

$$(1, -1)$$

$$L \sim e^{i\alpha\sigma_3} (0, \pi)$$

Act on Wilson lines

$$W = e^{\oint W dx}$$

non-contractible loop

$$W \rightarrow -W \quad (e^{i\pi} W) \quad \underline{\underline{1\text{-form symmetry}}}$$

Z_4, Z_2^c second symmetries of them.

$$d^D x \text{Tr}(\bar{\psi} e^{\gamma_\mu} \gamma^\mu D_\mu \psi)$$

$$D_\mu \psi = \partial_\mu \psi + [W_\mu, \psi]$$

$$e^a_\mu = g_{\mu a}$$

$$d=2$$

$$X = \frac{1}{4\pi} \int R^{\alpha\beta}_{\mu\nu} \epsilon^{\mu\nu} \epsilon_{\alpha\beta}$$

curvature ϵ

$$d=2$$

$$L \sim e^{i\alpha\sigma_3} \begin{pmatrix} 1 & -1 \\ 0 & \pi \end{pmatrix}$$

Act on Wilson lines

$$W = e^{\oint W dx}$$

non-contractible loop

$$W \rightarrow -W \quad (e^{i\pi} W) \quad \underline{\text{1-form symmetry}}$$

$$\underline{Z_1, Z_2 \text{ second symmetric of them.}}$$

↑ marginal gauge Z_2

(t Hooft)

$$d^D x \text{Tr}(\bar{\psi} e^{\gamma_5} \gamma^\mu D_\mu \psi)$$

$$D_\mu \psi = \partial_\mu \psi + [w_\mu, \psi]$$

$$e^a_\mu = S_\mu$$

$$d=2$$

$$X = \frac{1}{4\pi} \int R^{ab}_{\mu\nu} \epsilon^{\mu\nu} \epsilon_{ab}$$

↑
curvature ω

$$d=2$$

$$L \sim e^{i\alpha\sigma_3} (0, \pi)$$

Act on Wilson lines

$$W = e^{\oint w dx}$$

non-contractible
loop

$$W \rightarrow -W \quad (e^{i\pi} W) \quad \underline{\text{1-form symmetry}}$$

$$\underline{Z_1, Z_2 \text{ second symmetry of them.}}$$

↑ marginal gauge Z_2
() if Higgs anomaly

$$D_1 \psi$$

$$+ [W_1 \psi]$$

$$Z_2$$

$$\frac{1}{4\pi} \int R^{ab} \epsilon^{cd} G_{ab} G_{cd}$$

curvature G

$$\alpha = 2\pi$$

$$L \sim e^{i\alpha\sigma_3} (0, \pi)$$

Act on Wilson lines

$$W = e^{\oint W dx}$$

non-contractible loop

$$W \rightarrow -W \quad (e^{i\pi} W) \quad \underline{\text{1-form symmetry}}$$

$$\underline{Z_4, Z_2 \text{ symmetries of them.}}$$

↑ Imaginary gauge Z_2

() 't Hooft anomaly

2-form
Sym (Rel)

2d fermions

$B_{\text{pr}} \leftarrow$ background $(4\text{-fermi} \rightarrow \mathbb{Z}_4)$

non-zero flux

$$\int B = \pi$$

$\hookrightarrow$ fractionalize top charges

$B_{pr} \leftarrow$ background

non-zero flux

$$\int B = \pi$$

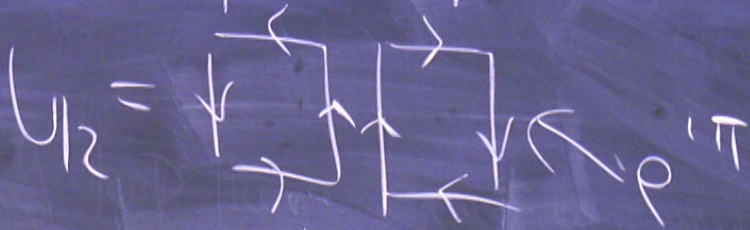
$\hookrightarrow$ fractionalize top charges $\rightarrow$ break Z_4

lattice

$$U_p \rightarrow Z U_p \quad Z \in \pm 1$$

latta

$U_p \rightarrow Z U_p \quad Z \subset \pm 1$ B-plagut



$\phi_4 \phi_4 e^{i\pi X}$ under Z_2

$\phi_2 e^{i\pi X} e^{i\pi}$

(4-fermi
→ Z_4)

$= \pi$

$g \rightarrow$ break Z_4

$\pm$ B-plaquette

Example

global Z_2 symmetry

dz_2

$(1, -1)$
 Z_2

$L \sim e^{i\alpha_3}$ $(0, \pi)$

Act on Wilson lines

$\oint w dx$

$w = e$

non-contractible
loop

$w \rightarrow -w$ $(e^{i\pi} w)$ 1-form symmetry

Z_4, Z_2 symmetries of them. Φ

cancel anomaly of
with 2 copies of KD field

$$d=2$$

$$1 \text{ KD} = 2 \text{ Dirac} \times 2 \\ = 8 \text{ Majorana}$$

$$d=4$$

$$1 \text{ KD} = 4 \text{ Dirac} \times 2 \\ = 16 \text{ Majorana}$$

$B_{\text{pr}} \leftarrow$ background

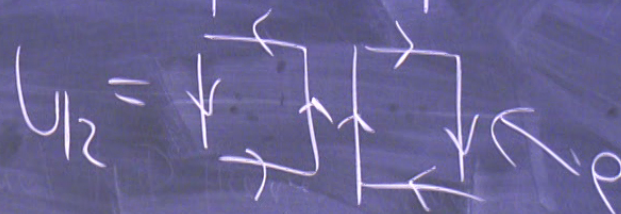
non-zero flux

$$\int B =$$

$\hookrightarrow$ fractionalize top charges

lattice

$$U_{\mu} \rightarrow Z U_{\mu} \quad Z = \pm 1$$



$$4 \text{ Dirac} \quad \text{under } \mathbb{Z}_2$$

Cancel anomaly of
with 2 copies of KD field

$$d=2$$

$$1 \text{ KD} = 2 \text{ Dirac} \times 2 \\ = 8 \text{ Majorana}$$

$$d=4$$

$$1 \text{ KD} = 4 \text{ Dirac} \times 2 \\ = 16 \text{ Majorana}$$

$B_{\text{pr}} \leftarrow$ background

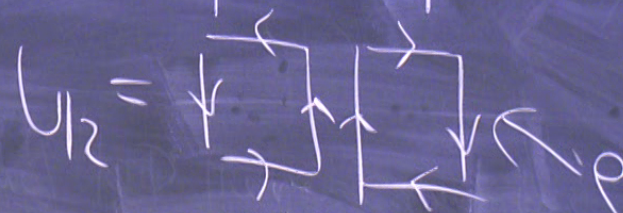
non-zero flux

$$\int B =$$

$\hookrightarrow$ fractionalize top charges

lattice

$$U_{\mu} \rightarrow Z U_{\mu} \quad Z \in \pm 1$$



$$4 \text{ Dirac} \quad \text{under } \mathbb{Z}_2$$