Title: Anomalies and symmetric mass generation for staggered fermions â€“ Talk 1

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Collection: Quantum Criticality: Gauge Fields and Matter

Date: May 20, 2022 - 9:00 AM

URL: https://pirsa.org/22050048

Anomalies for staggered fermions

Plan

* Staggered = (discrete) Kähler-Dirac.
  Kähler-Dirac ≟ Dirac
* Γ, $[\Gamma, K]_+ = 0 \rightarrow U(1)$ symmetry
  ↑ generalizes to curved space
* Naturalization discretization ⇒ staggered fermions
* Anomaly $U(1) \rightarrow \mathbb{Z}_4$ (even dim)
* mixed anomalies
* Pati-Salam GUT

$(k-m)\underline{\Phi} = 0$

$K = d - d^\dagger \qquad K^2 = dd^\dagger + d^\dagger d = \Box$

$\underline{\Phi} = (\phi, \phi_\mu, \phi_{\mu\nu} \cdots)$

Connection to Dirac?

$\underline{\Psi} = \sum_{p=0}^{D} \phi_{\mu_1 \cdots \mu_p} \gamma^{\mu_1} \cdots \gamma^{\mu_p}$

flat space $\qquad (\gamma^\mu \partial_\mu - m)\underline{\Psi} = 0$

$2^{D/2}$ degenerate Dirac fermions

$=$ (discrete) Kähler-Dirac.

$\equiv$ Dirac

$k]_t = 0 \longrightarrow U(1)$ symmetry

generalizes to curved space

discretization $\Rightarrow$ staggered fermions

$U(1) \to \mathbb{Z}_4$ (chirality)

GUT

$D=2 \qquad \underline{\Phi} = (\eta, \psi_\mu, \chi_{\mu\nu})$

$\partial_\mu \psi_\mu$ 0-form.

$\partial_\mu \eta + \partial_\nu \chi_{\nu\mu}$ 1-form.

$\partial_\mu \psi_\nu - \partial_\nu \psi_\mu$ 2-form.

$\gamma_\mu \partial_\mu \Psi = \gamma_\mu \partial_\mu \left(\eta \underline{I} + \psi_\nu \gamma_\nu + \chi_{\mu\nu} \gamma_\mu \gamma_\nu \right)$

$m) \underline{\Phi} = 0$

$d - d^\dagger \qquad \Box = dd^\dagger + d^\dagger d = \Box$

$= (\phi, \phi_\mu, \phi_{\mu\nu} \ldots)$

to Dirac?

$= \sum_{p=0}^{D} \phi_{\eta_1 \ldots \eta_p} \gamma^{\eta_1} \ldots \gamma^{\eta_p}$

$(\gamma^\mu \partial_\mu - m) \underline{\Psi} = 0$

$d - d^\dagger \qquad \Delta = dd^\dagger + d^\dagger d = \Box$

$\Psi = (\phi, \phi_\mu, \phi_{\mu\nu} \cdots)$

ion to Dirac?

$$= \sum_{p=0}^{D} \phi_{\eta_1 \cdots \eta_p} \gamma^{\eta_1} \cdots \gamma^{\eta_p}$$

$$(\gamma^\eta \partial_\mu - m)\Psi = 0$$

degenerate Dirac fermions

$\partial_\mu \psi_\mu$ 0-form.

$\partial_\mu \eta + \partial_\nu \chi_{\nu\mu}$ 1-form.

$\partial_\mu \psi_\nu - \partial_\nu \psi_\mu$ 2-form.

$$\gamma_\mu \partial_\mu \Psi = \gamma_\mu \partial_\mu \left(\eta \mathbb{I} + \psi_\nu \gamma_\nu + \chi_{\mu\nu} \gamma_\mu \gamma_\nu \right)$$

$$S = \int d^D x \, \mathrm{Tr}\left(\bar{\Psi} (\not{\partial} - m) \Psi \right)$$

Lorentz × flavor sym.

$SU(2^{D/2})$

$D=2 \quad \underline{\Phi} = (\eta, \psi_\mu, \chi_{\mu\nu})$

$\partial_\mu \psi_\mu$ 0-form

$\partial_\mu \eta + \partial_\nu \chi_{\nu\mu}$ 1-form

$\partial_1 \psi_2 - \partial_2 \psi_1$ 2-form

$\gamma_\mu \partial_\mu \Psi = \gamma_\mu \partial_\mu \left(\eta \mathbb{I} + \psi_\nu \gamma_\nu + \chi_{\mu\nu} \gamma_\mu \gamma_\nu \right)$

$S = \int d^D x \, \mathrm{Tr} \left(\bar{\Psi} (\not{\partial} - u) \Psi \right)$

Lorentz $\times$ flavor sym. $\uparrow L \Psi \Gamma^\dagger$

$SU(2^{D/2})$

KD field transforms

Lorentz

$L \Psi L^\dagger$

Γ

$\phi_{\mu_1 \cdots \mu_p} \to (-1)^p \phi_{\mu_1 \cdots \mu_p}$

$[\Gamma, K]_+ = 0$

$\underline{\Phi} \to e^{i\alpha\Gamma} \underline{\Phi} \quad \bar{\varphi} \to \bar{\varphi} e^{i\alpha\Gamma}$

$\Psi \to \gamma_5 \Psi \gamma_5$ twist $\times$ sym

Anomalies for staggered fermions

$$\bar{\psi}_\pm = \frac{1}{2}(1 \pm \Gamma)\bar{\psi}$$

$$S = \int \bar{\psi}_+ K \psi_- + \bar{\psi}_- K \psi_+$$

reduced KD field (massless fields)

$(\not{k} - m)\bar{q} = 0$

no spin structure to gravity

Lattice



cochain ⟶ triangulation

p-form ⟶ field on p-simplex

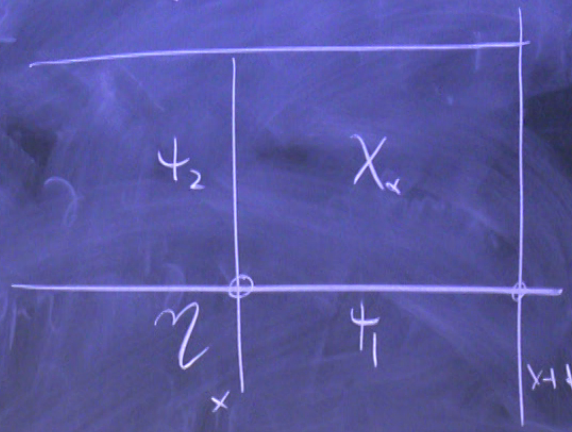
$d, d^\dagger \longrightarrow \delta, \bar{\delta}$

$(\delta - \bar{\delta} - m)\phi_i = 0$

Example

Special case - stagger fermions

2d



$$\delta\psi_1 = \eta(x+1) - \eta(x)$$

$$\bar{\delta}\psi_1 = \chi_{12}(x) - \chi_{12}(x-1)$$

$\bar{q} = 0$

we to gravity



angulation

eld on p simplex Γ ✓

$\delta, \bar{\delta}$

$-m)\Phi = 0$

kD field th

Lorentz

L4

Γ Φ_{μ_1}

$[\Gamma, K]_1 = 0$

$\bar{\Phi} \to$

$\Psi \to \gamma_5$

$\bar{q} = 0$

ve to gravity

angulation

eld an n-p simplex Γ ✓

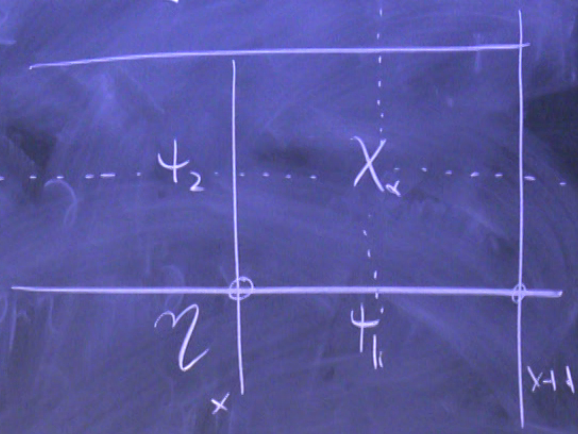
$\delta, \bar{\delta}$

$-m)\varphi = 0$

Example

Special case - staggered fermions

2d



$$\delta\psi_1 = \eta(x+1) - \eta(x)$$

$$\bar{\delta}\psi_1 = \chi_{12}(x) - \chi_{12}(x-1)$$

$$\xi(y) = \eta \quad \xi(y+1) = \psi_1 \quad \xi(y+2) = \psi_2$$

$$\xi(y+1+2) = \chi_{12}$$

kD fuld m

Lorentz

$L\psi$

Γ φ_{μ}

$[\Gamma, K]_1 = 0$

$\bar{\varphi} \to e$

$\psi \to \gamma_5$

Anomalies for staggered fermions

$$\Psi = \xi(x)\mathbb{1} + \xi(x+\hat{1})\gamma_1 + \xi(x+\hat{2})\gamma_2 + \xi(x+\hat{1}+\hat{2})\gamma_1\gamma_2$$

$$S = \int \mathrm{Tr}(\bar{\psi}\not{\partial}\psi)$$

$$\hookrightarrow \sum \eta_\mu(x)\bar{\xi}(x)\Delta_\mu\xi(x)$$

curved space

$$(\not{k} - m)\tilde{\psi} = 0$$

no spin structure to gravity

Lattice — oriented

cont. ⟶ triangulation

p-form ⟶ field on p-simplex

$d, d^\dagger \longrightarrow \delta, \bar{\delta}$

$$(\delta - \bar{\delta} - m)\varphi = 0$$

$$\Psi = \xi(x)\mathbb{I} + \xi(x+n)\gamma_1 + \xi(x+2)\gamma_2 + \xi(x+1+2)\gamma_1\gamma_2$$

$$S = \int \Pi(\bar{\psi}\not{\partial}\psi)$$

$$\hookrightarrow \sum \eta_p(x)\bar{\xi}(x)\Delta_p\xi(x)$$

$$\Gamma(k) \to (-1)^{\sum x_i}$$

$$(k-m)\bar{q} = 0$$

no spin structure to gravity

Lattice — oriented

cont^n ⟶ triangulation

p-form ⟶ field on p-simplex

$d, d^\dagger \longrightarrow \delta, \bar{\delta}$

$$(\delta - \bar{\delta} - m)\varphi_c = 0$$

So far

* KD fermions naturally discretized on any curved space preserving Γ_i zero modes...

* Symmetries of free Dirac at distance of Dirac flat space weakly coupled $\{ L\psi(t) \}$

* Finite lattice spacing lose degeneracy

2d

ψ_2

η

$\delta\psi_1 = \eta(x+1)$

$\xi\psi_1 = \chi_{12}(x$

$\xi(y) = \eta$ $\xi(y+$

$\xi($