

Title: Fractionalized fermionic quantum criticality

Speakers: Lukas Janssen

Collection: Quantum Criticality: Gauge Fields and Matter

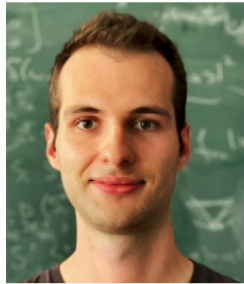
Date: May 19, 2022 - 4:15 PM

URL: <https://pirsa.org/22050047>

Abstract: In frustrated magnets, novel phases characterized by fractionalized excitations and emergent gauge fields can occur. A paradigmatic example is given by the Kitaev model of localized spins  $1/2$  on the honeycomb lattice, which realizes an exactly solvable quantum spin liquid ground state with Majorana fermions as low-energy excitations. I will demonstrate that the Kitaev solution can be generalized to systems with spin and orbital degrees of freedom. The phase diagrams of these Kitaev-Kugel-Khomskii spin-orbital magnets feature a variety of novel phases, including different types of quantum liquids, as well as conventional and unconventional long-range-ordered phases, and interesting phase transitions in between. In particular, I will discuss the example of a continuous quantum phase transition between a Kitaev spin-orbital liquid and a symmetry-broken phase. This transition can be understood as a realization of a fractionalized fermionic quantum critical point.

# Fractionalized fermionic quantum criticality

Lukas Janssen  
TU Dresden



Urban Seifert, Santa Barbara



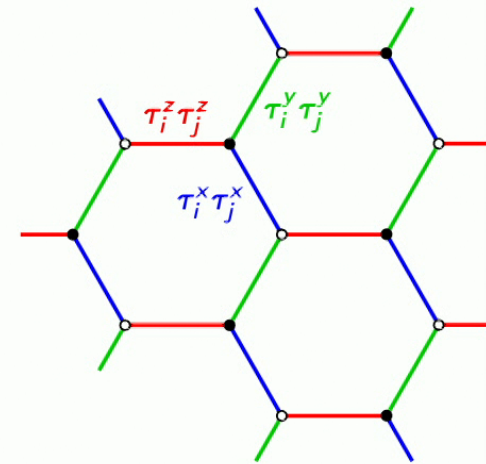
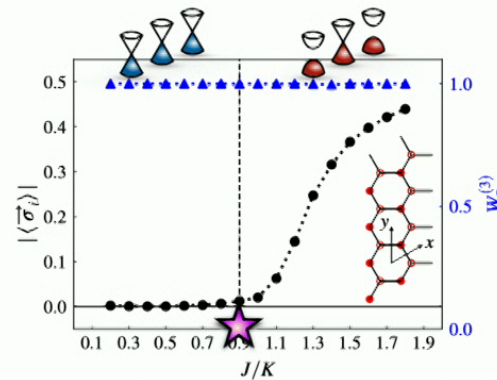
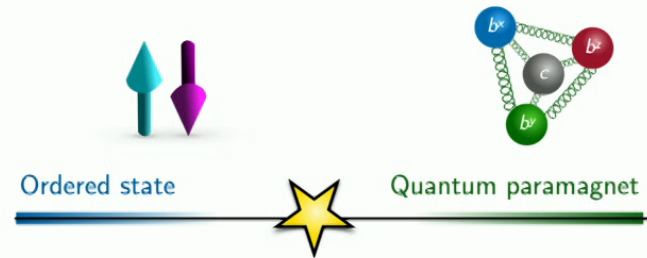
Zihong Liu, Würzburg

Fakher Assaad, Würzburg  
Sreejith Chulliparambil, Dresden  
Xiao-Yu Dong, Ghent  
Hong-Hao Tu, Dresden  
Matthias Vojta, Dresden



# Outline

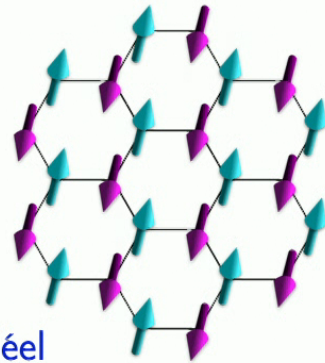
- (1) Fractionalized quantum criticality
- (2) Kitaev spin-orbital models
- (3) Critical fractionalized fermions
- (4) Conclusions



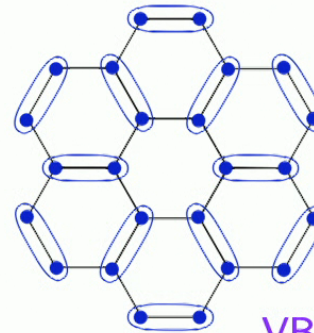
Slides available on <https://tu-dresden.de/physik/qcm/vortraege>

# Deconfined quantum criticality

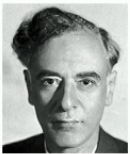
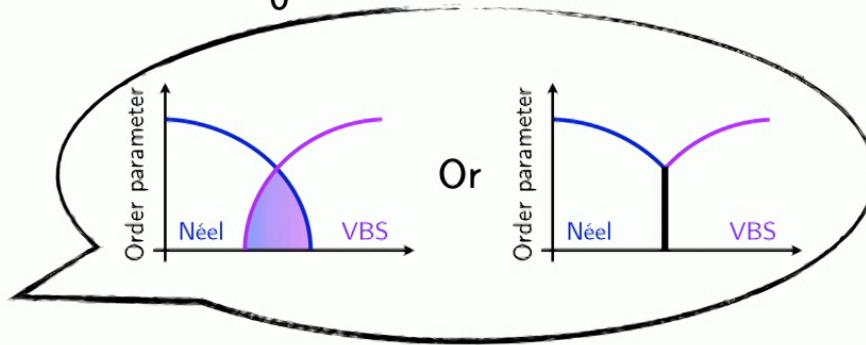
$$\text{⬭} = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$$



Néel

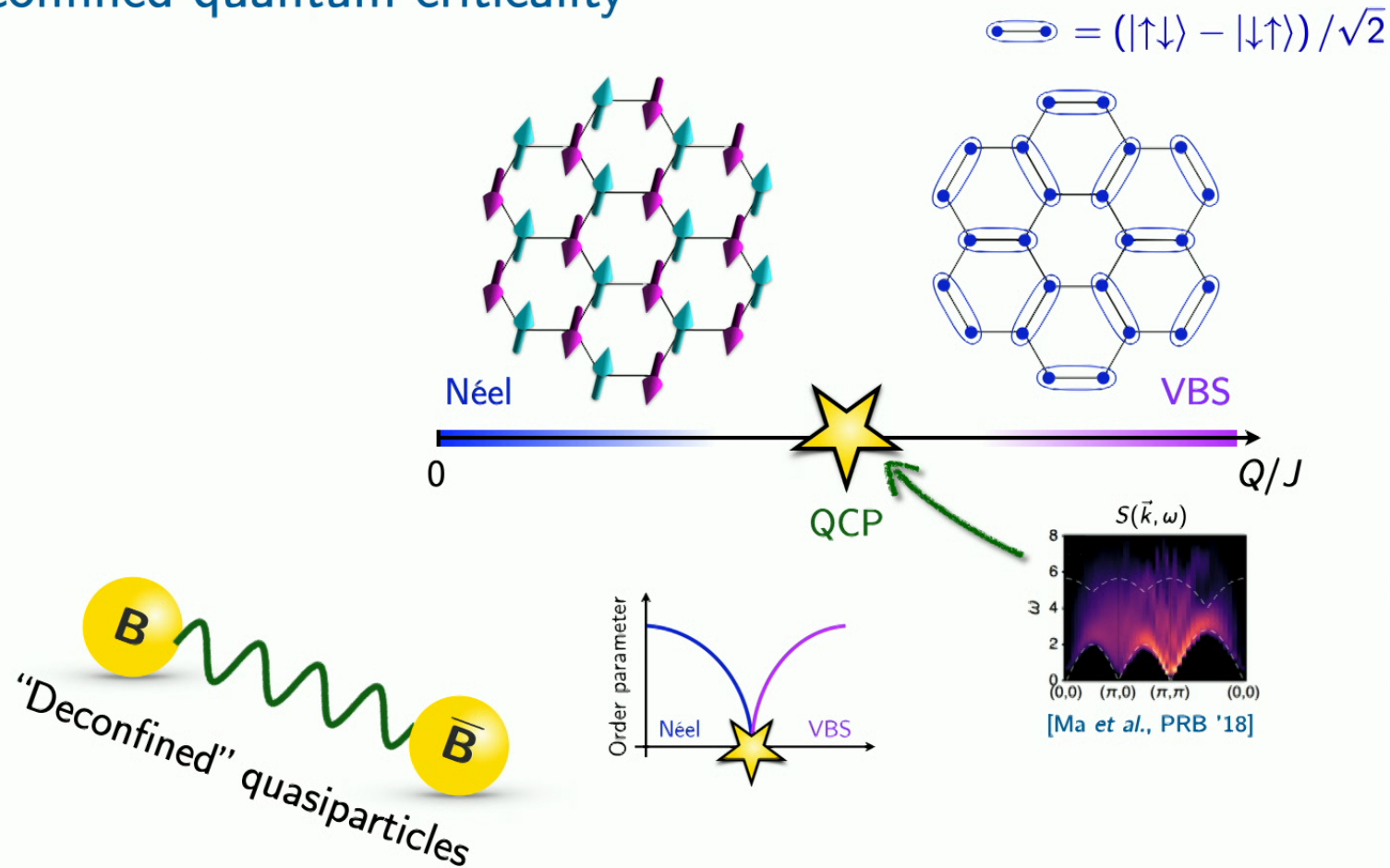


VBS

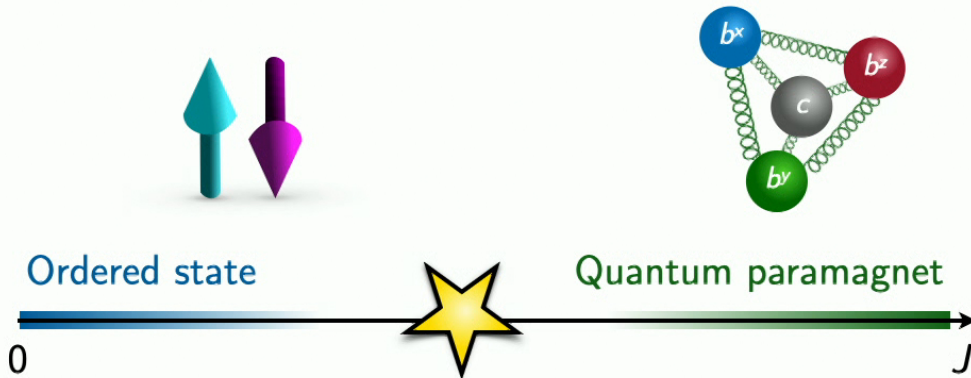


Landau

# Deconfined quantum criticality

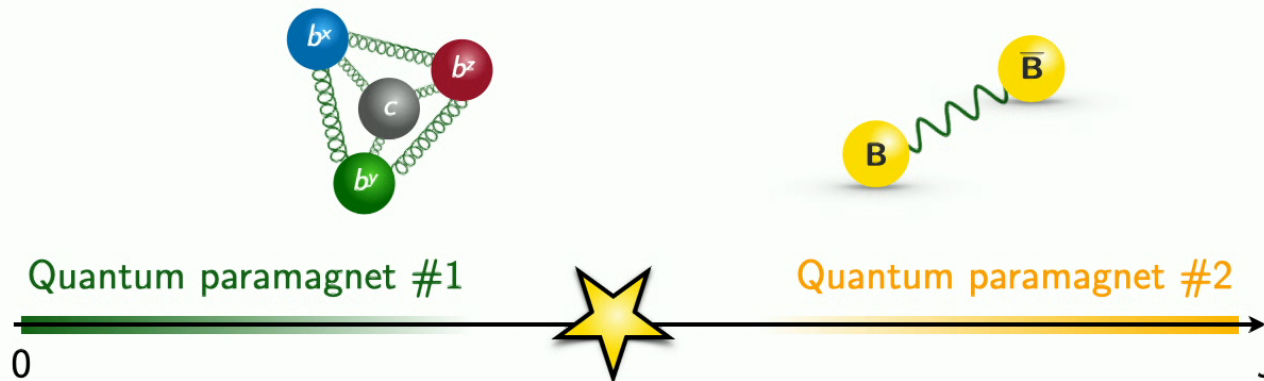


# Spin-liquid transitions



[Assaad & Grover, PRX '16]  
 [LJ, Wang, Scherer, Meng, Xu, PRB '20]  
 ...

→ talk by Z. Y. Meng



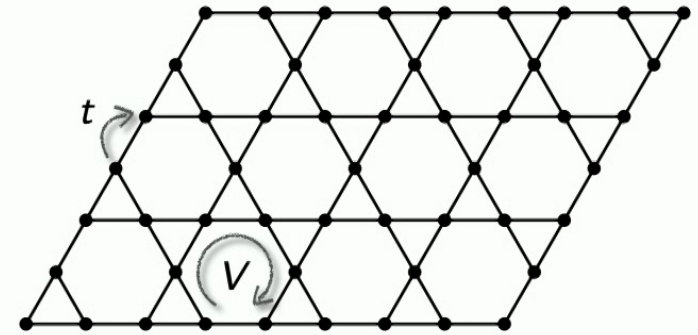
[Metlitski, Mross, Sachdev, Senthil, PRB '15]  
 [LJ & He, PRB '17]  
 [Boyack, Lin, Zerf, Rayyan, Maciejko, PRB '18]  
 ...

## Example: Kagome-lattice Bose-Hubbard model

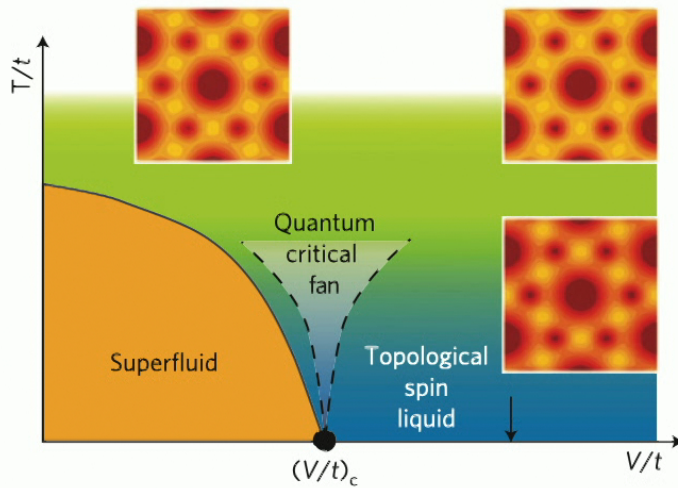
Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left[ b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\hexagon} (n_{\hexagon})^2$$

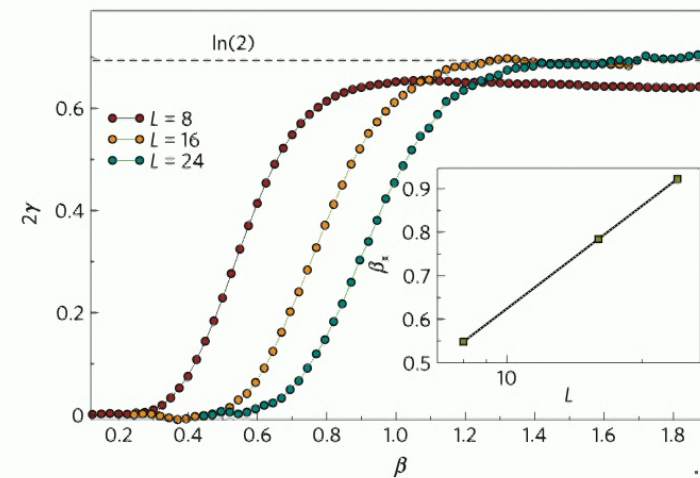
...  $b_i$  hard-core bosons



Phase diagram:



Entanglement entropy:  $S_n(A) = a\ell - \gamma + \dots$

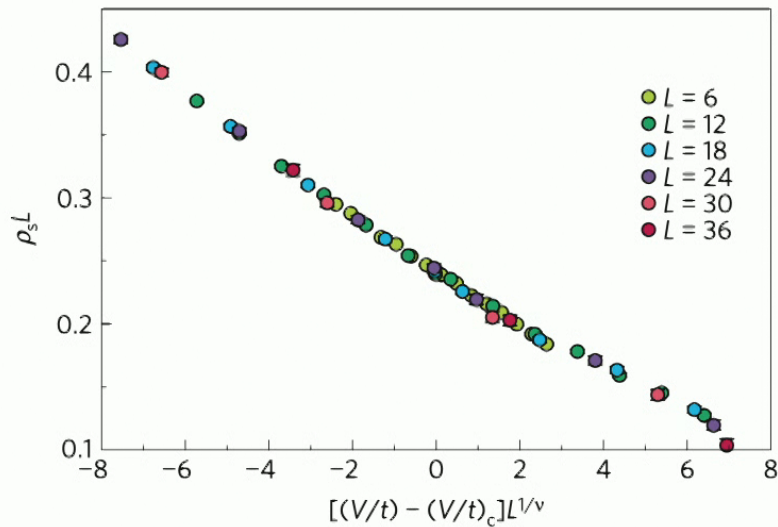


... in spin liquid phase

[Isakov, Hastings, Melko, Nat. Phys. '11]

# Quantum critical scaling: XY\*

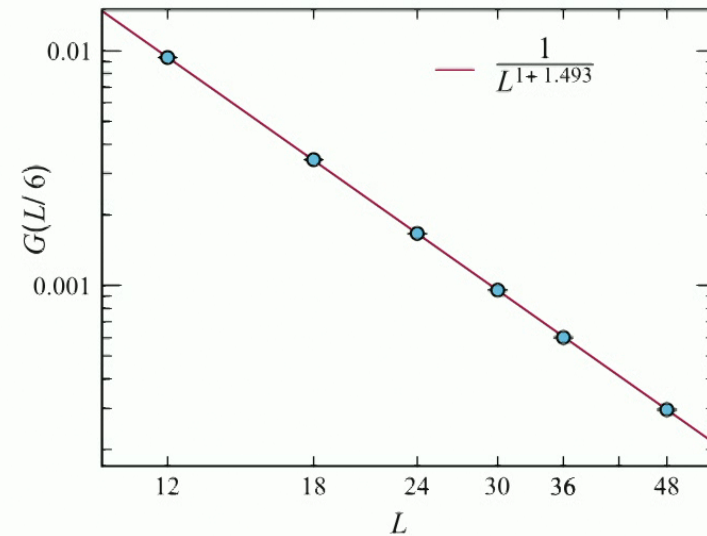
Superfluid density:



[Isakov, Hastings, Melko, Nat. Phys. '11]

$$\nu \approx 0.67 = \nu_{XY}$$

Two-point superfluid correlator:



[Isakov, Melko, Hastings, Science '12]

$$\eta \approx 1.49 \neq \eta_{XY} \approx 0.038$$

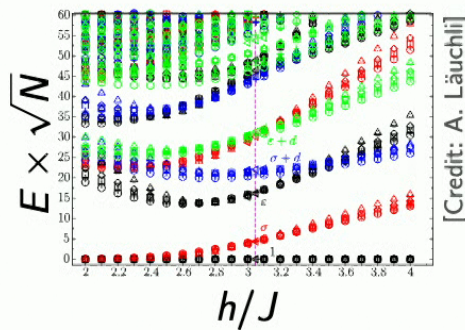
Order parameter *composite* of fractionalized particles!

... cf.  $\eta_T \approx 1.54$  from field theory  
[Chubukov, Sachdev, Senthil, NPB '94]

# Finite-size spectroscopy: Ising vs Ising\*

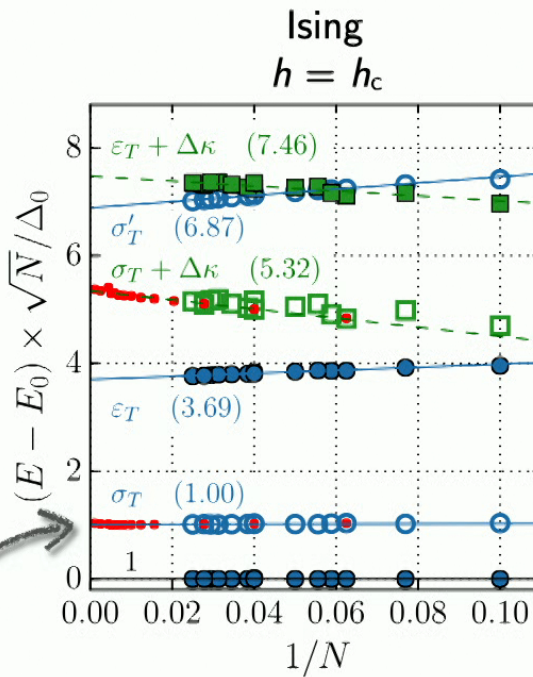
Transverse-field Ising:

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$



[Credit: A. Läuchli]

missing in Ising\*

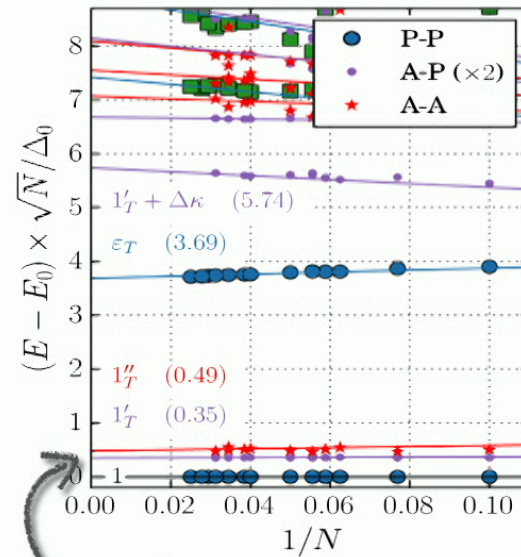


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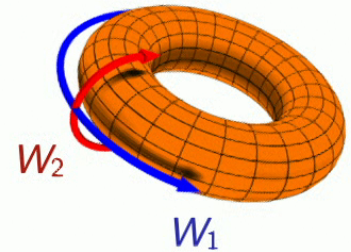
Transverse-field toric code:

$$\mathcal{H} = -J \sum_s \prod_{i \in s} \sigma_i^x - J \sum_p \prod_{i \in p} \sigma_i^z - h \sum_i \sigma_i^x$$

Ising\*  
 $h = h_c$



topological "copies"

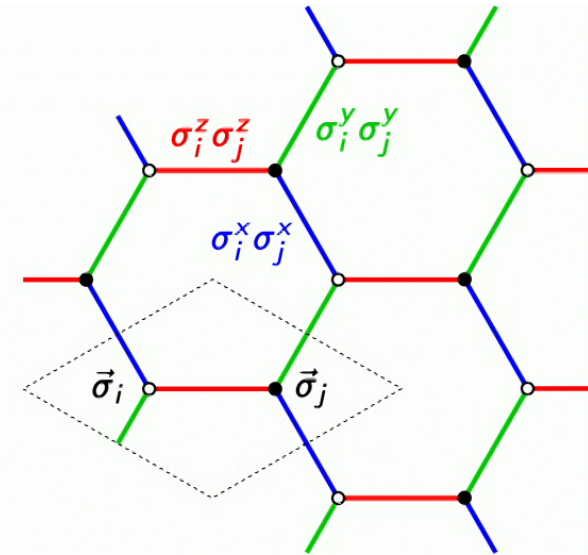


[Schuler, Whitsitt, Henry, Sachdev, Läuchli, PRL '16]

# Kitaev spin-1/2 model

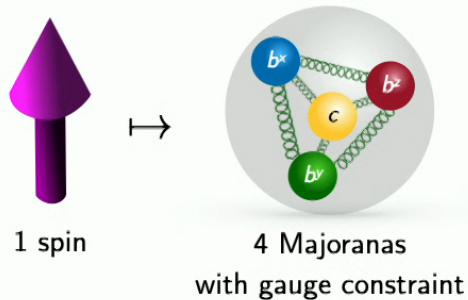
Hamiltonian:

$$\mathcal{H} = K \sum_{\text{blue links}} \sigma_i^x \sigma_j^x + K \sum_{\text{green links}} \sigma_i^y \sigma_j^y + K \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$



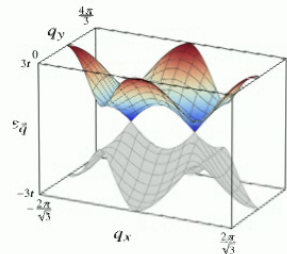
Majorana representation:

$$\begin{aligned} \sigma^x &\mapsto \tilde{\sigma}^x = ib^x c \\ \sigma^y &\mapsto \tilde{\sigma}^y = ib^y c \\ \sigma^z &\mapsto \tilde{\sigma}^z = ib^z c \end{aligned}$$



Fractionalization:

$$\mathcal{H} \mapsto \tilde{\mathcal{H}} = iK \sum_{\langle ij \rangle_\gamma} \underbrace{(ib_i^\gamma b_j^\gamma)}_{\equiv \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j$$



with  $[\hat{u}_{ij}, \tilde{\mathcal{H}}] = 0 \Rightarrow$  static  $\mathbb{Z}_2$  gauge field!

Ground-state flux pattern:  $u = 1$   
[Lieb, PRL '94]

→ talk by N. Trivedi

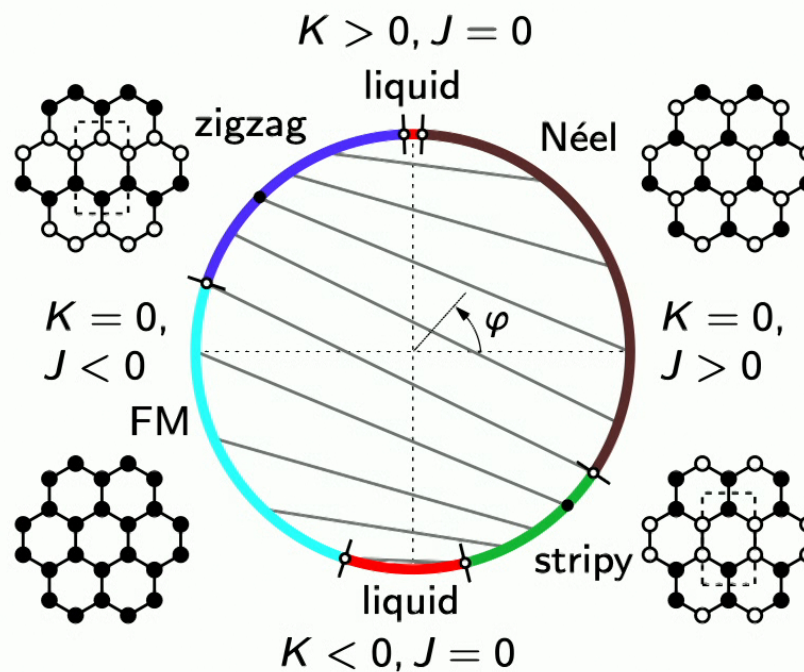
[Kitaev, Ann. Phys. '06]

# Kitaev-Heisenberg spin-1/2 model

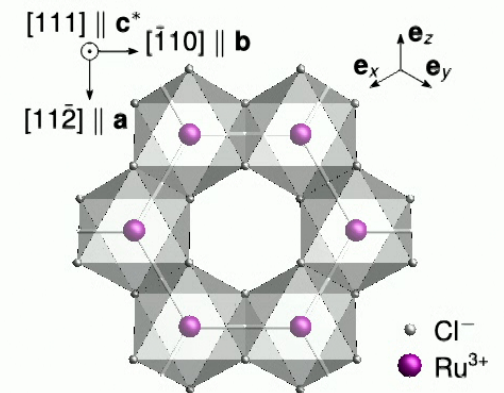
Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \sigma_i^\gamma \sigma_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Phase diagram:



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... possible relevance to  $\alpha\text{-RuCl}_3$ ,  $\text{Na}_2\text{IrO}_3$ ,  $\text{Na}_2\text{Co}_2\text{TeO}_6$ , ...

$$J = A \cos \varphi$$

$$K = 2A \sin \varphi$$

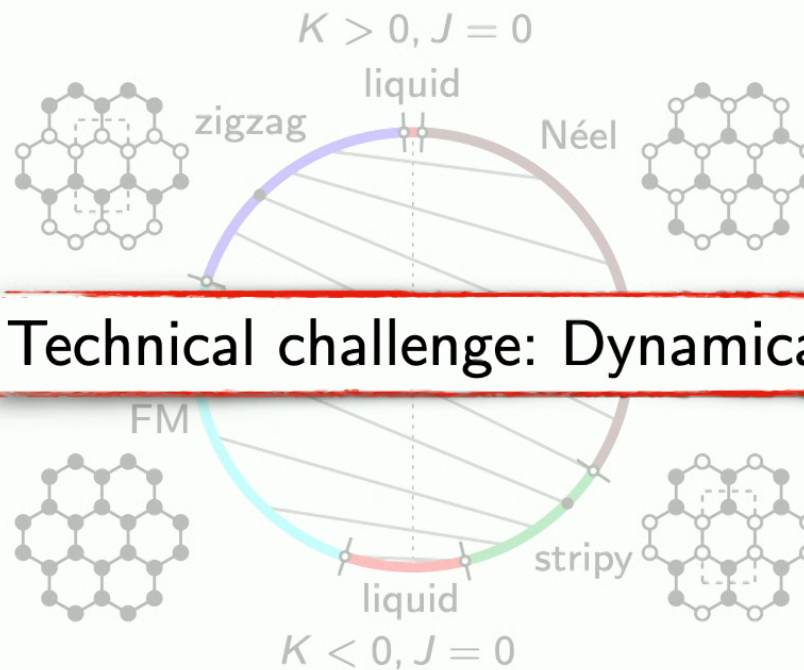
... from 24-site ED: [Chaloupka, Jackeli, Khaliullin, PRL '13]

# Kitaev-Heisenberg spin-1/2 model

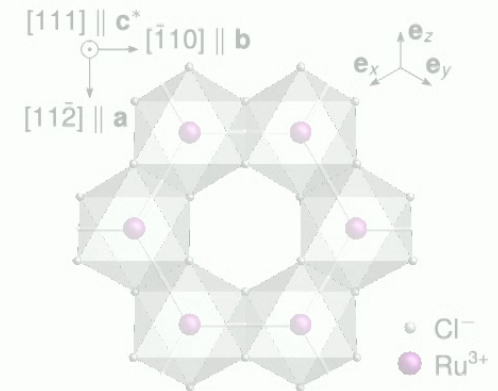
Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \sigma_i^\gamma \sigma_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

Phase diagram:



**Technical challenge: Dynamical  $\mathbb{Z}_2$  gauge field!**



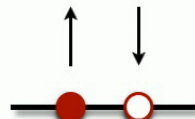
... possible relevance to  $\alpha\text{-RuCl}_3$ ,  $\text{Na}_2\text{IrO}_3$ ,  $\text{Na}_2\text{Co}_2\text{TeO}_6$ , ...

... no sign-problem-free QMC available: [Sato & Assaad, PRB '21]

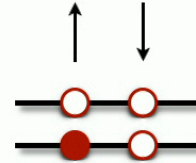
... from 24-site ED: [Chaloupka, Jackeli, Khaliullin, PRL '13]

# Kitaev spin-orbital models

Spin-orbital generalization:

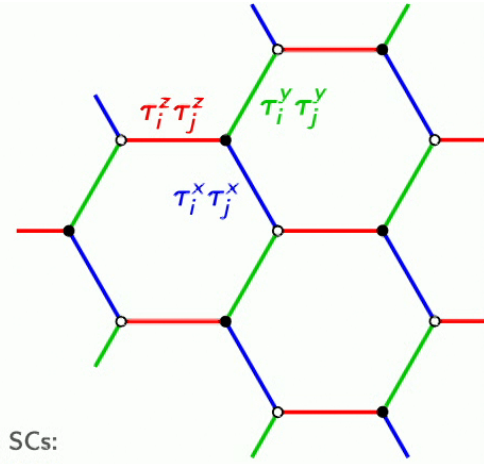


$$\sigma^\alpha \quad 2 \times 2$$



$$\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$$

... can realize all 16 topological SCs:  
[Chulliparambil, ..., LJ, Tu, PRB '20]



Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma$$

Heisenberg spin

Kitaev orbital

Majorana representation:

$$\sigma^y \otimes \tau^x = ib^1 c^x$$

$$\sigma^y \otimes \tau^y = ib^2 c^x$$

$$\sigma^y \otimes \tau^z = ib^3 c^x$$

$$\sigma^x \otimes \mathbb{1} = ic^y c^x$$

$$\sigma^z \otimes \mathbb{1} = ic^z c^x$$

Fractionalization:

$$\mathcal{H} \mapsto \tilde{\mathcal{H}} = iK \sum_{\langle ij \rangle_\gamma} \hat{u}_{ij} c_i^\top c_j$$

$$\text{with } [\hat{u}_{ij}, \tilde{\mathcal{H}}] = 0$$

... cf. also [Yao & Lee, PRL '11]

$$c \equiv \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$$

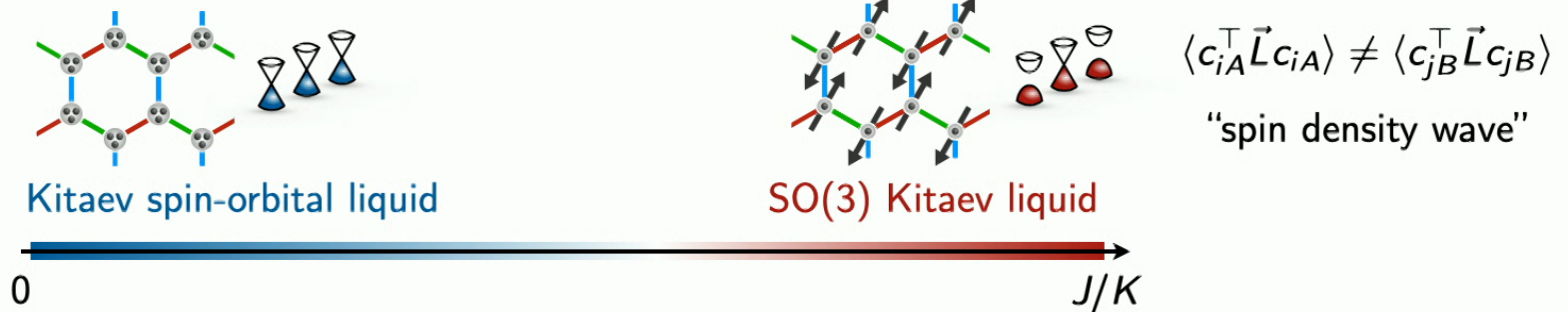
# Kitaev-Heisenberg spin-orbital model

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j}_{\mapsto \frac{1}{4} (c_i^\top \vec{L} c_i) \cdot (c_j \vec{L} c_j)} \quad \begin{array}{c} \swarrow \\ \text{spin-1 matrices} \end{array}$$

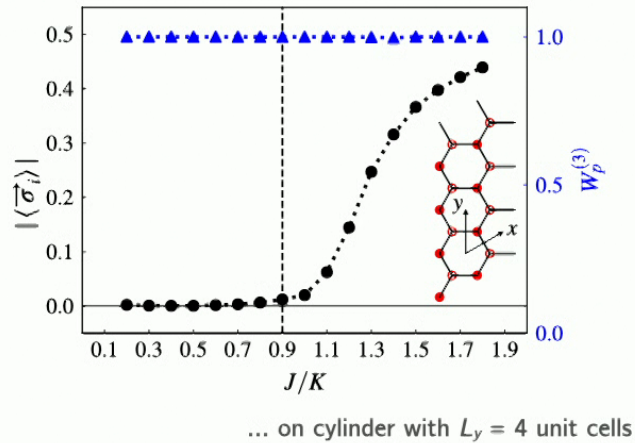
with  $[\hat{u}_{ij}, \mathcal{H}] = 0$  still static!

Phase diagram:

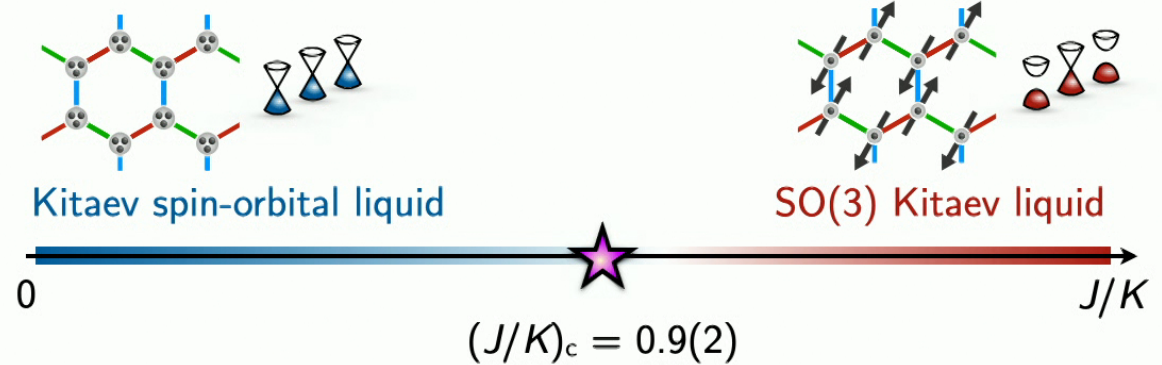


# Gross-Neveu-SO(3)\* transition

iDMRG:



Phase diagram:



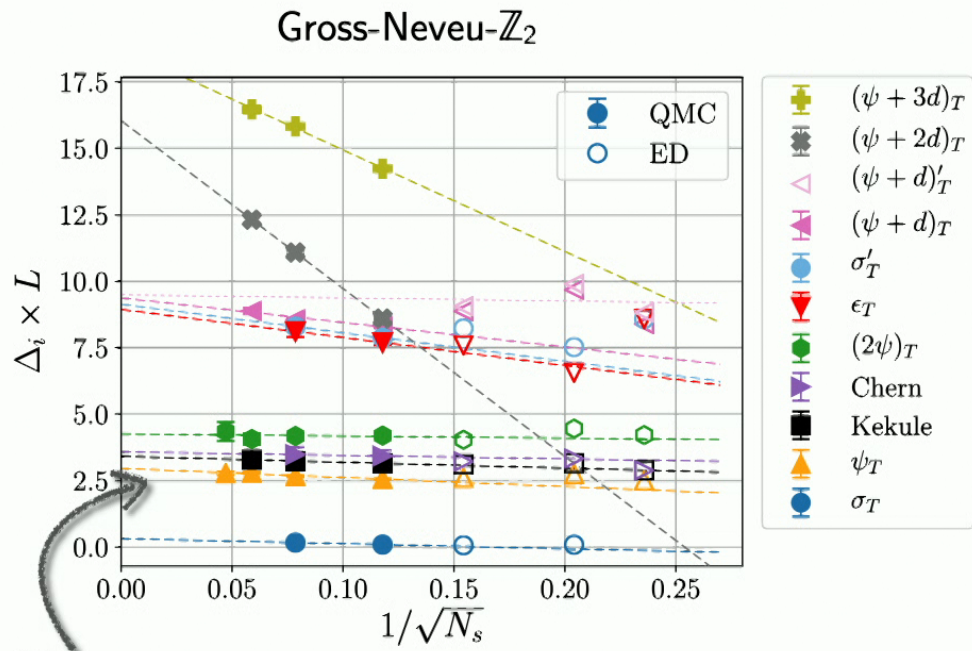
[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Effective field theory: 
$$\mathcal{S} = \int d^2\vec{x} d\tau \left[ \bar{\psi} \gamma^\mu \partial_\mu \psi + g \vec{\varphi} \cdot \bar{\psi} (\mathbb{1}_2 \otimes \vec{L}) \psi \right] \quad \text{“Gross-Neveu-SO(3)”}$$

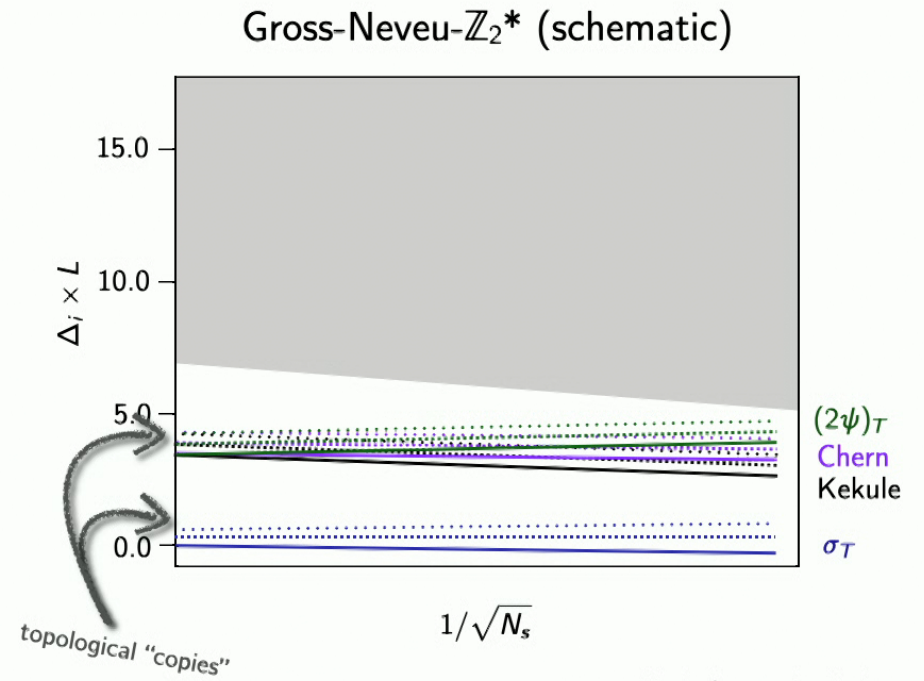
→ talk by M. Scherer

[Ray, Ihrig, Kruti, Gracey, Scherer, LJ, PRB '21]

# Gross-Neveu vs Gross-Neveu\*



[Schuler, Hesselmann, Whitsitt, Lang, Wessel, Läuchli, PRB '21]

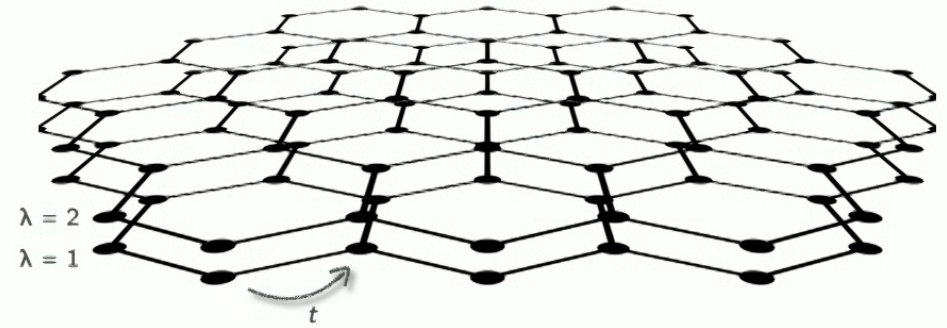


... testable in future simulations

# Sign-problem-free bilayer model

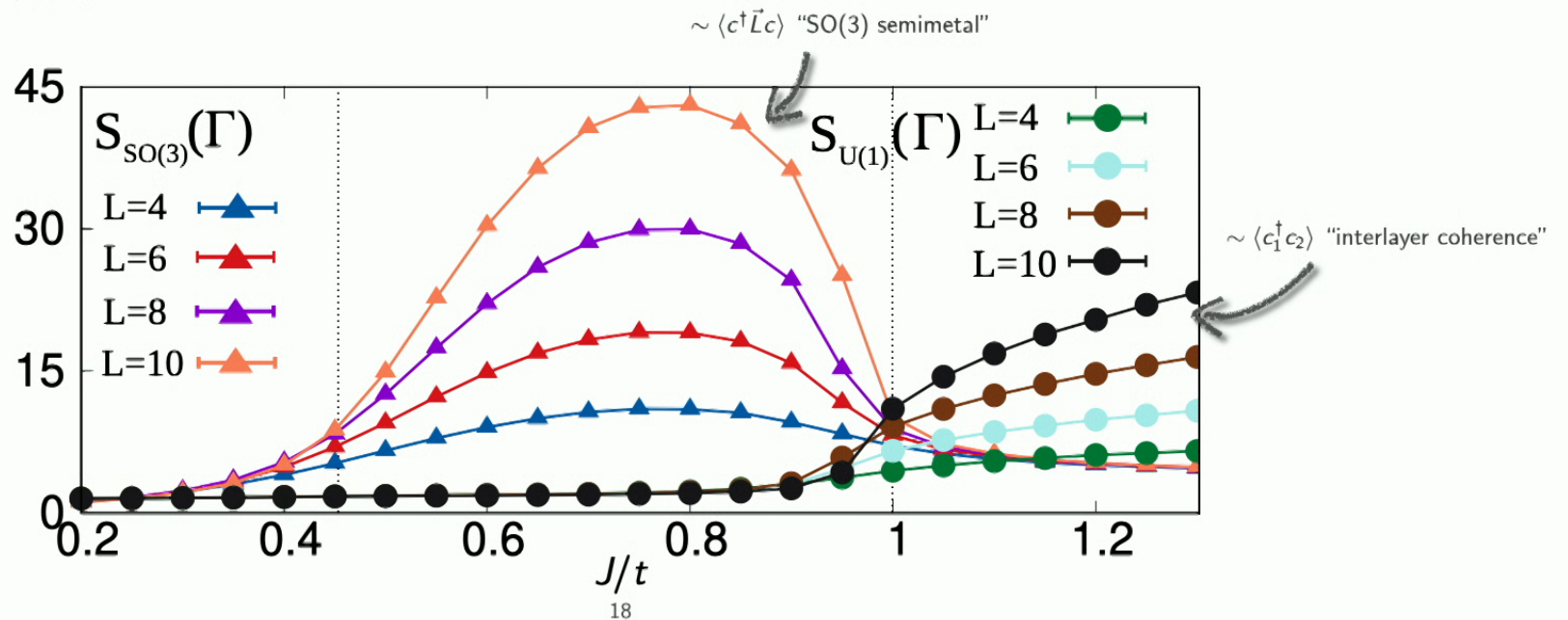
Hamiltonian:

$$\mathcal{H} = -t \sum_{\langle ij \rangle} c_{i\lambda}^\dagger c_{j\lambda} - J \sum_i \left( c_{i\lambda}^\dagger \vec{L} \tau_{\lambda\lambda'}^z c_{i\lambda'} \right)^2$$



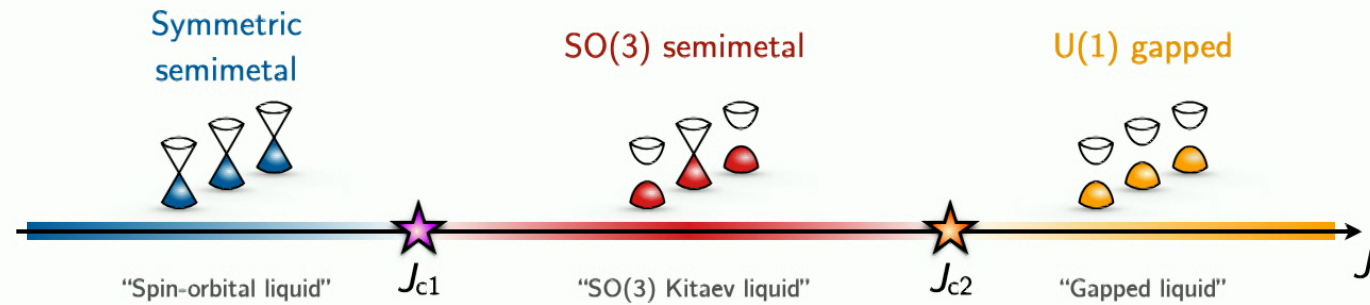
... with  $SO(3) \times U_\lambda(1) \times U_c \times \mathbb{Z}_2$  symmetry

QMC structure factors:

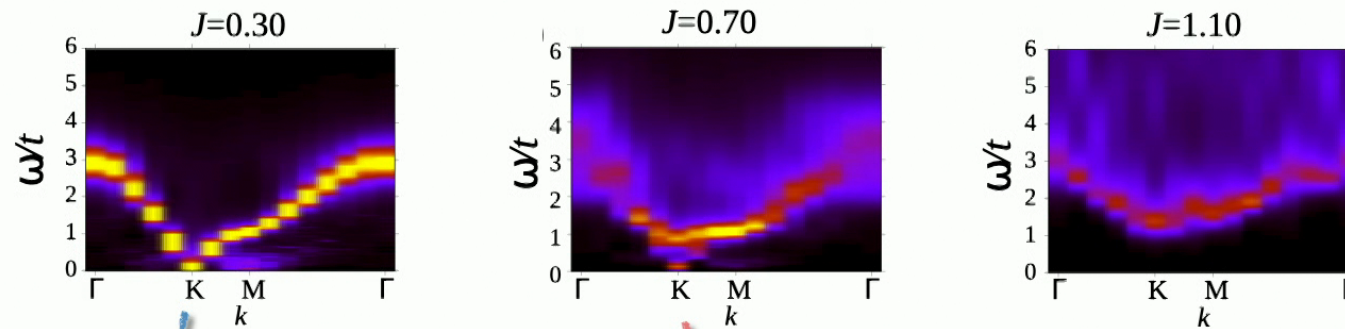


# Sign-problem-free bilayer model

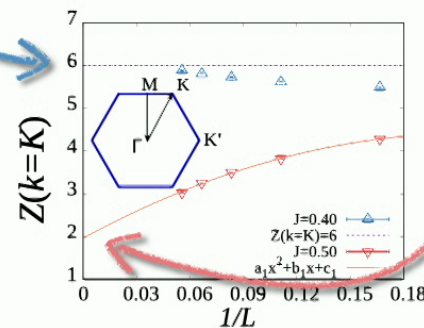
Phase diagram:



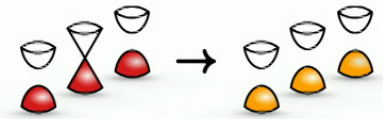
Fermion spectral function:



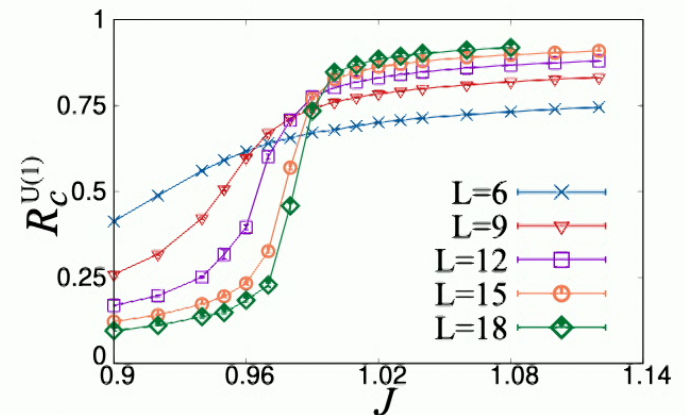
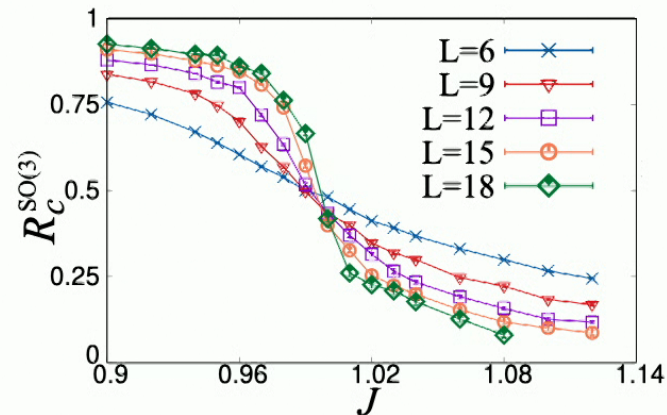
Quasiparticle weight:



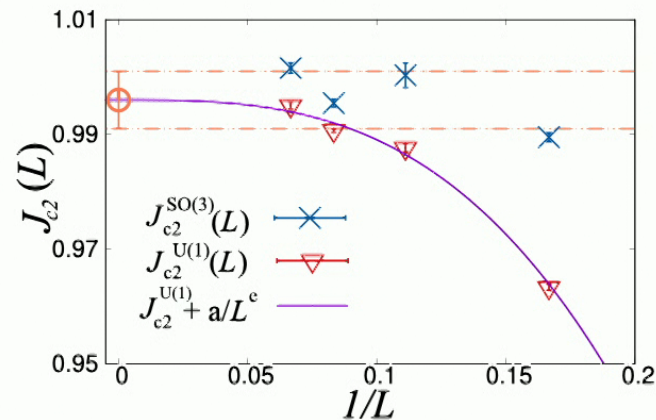
# SO(3)-U(1) transition at $J_{c2}$



Correlation ratios:



Critical couplings:



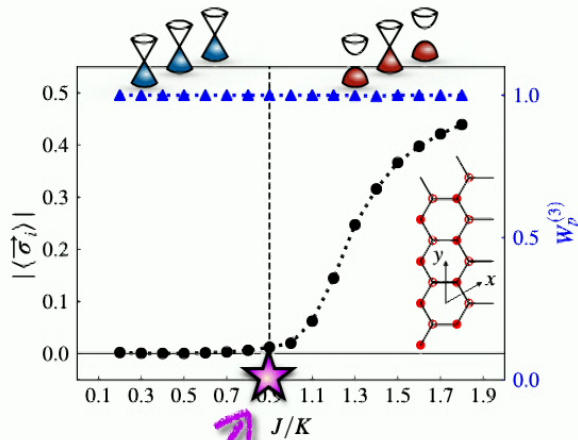
$$\Rightarrow J_{c2}^{SO(3)} = J_{c2}^{U(1)} \text{ unique!}$$

**Metallic deconfined QCP?**

[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]

# Conclusions

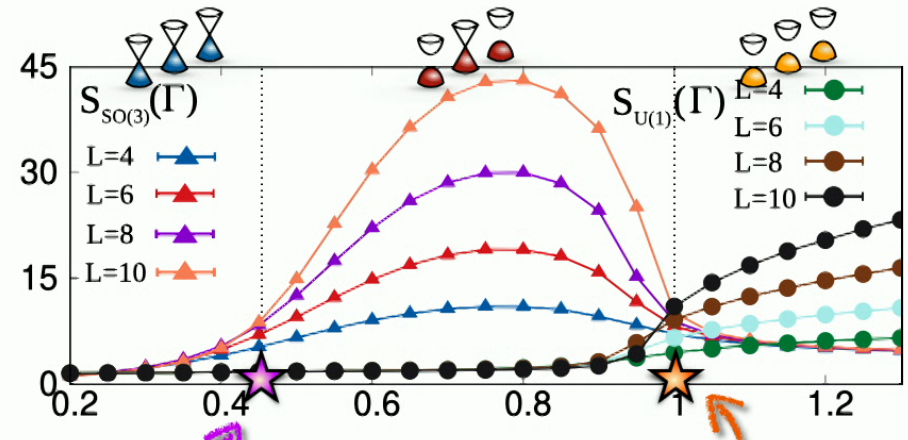
Kitaev-Heisenberg spin-orbital model:



Gross-Neveu-SO(3)\*

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

Effective bilayer honeycomb model:



Gross-Neveu-SO(3)

Metallic  
deconfined QCP?

[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]