Title: Bootstrapping critical gauge theories

Speakers: Yin-Chen He

Collection: Quantum Criticality: Gauge Fields and Matter

Date: May 19, 2022 - 3:15 PM

URL: https://pirsa.org/22050046

Abstract: I will talk about our recent progress on bootstrapping critical gauge theories. In specific, I will first introduce the current understanding that why bootstrap works, for example, why a CFT can sit at a kink of bootstrap bounds and why CFT can be isolated as an island. Then, I will apply these idea to a prototypical critical gauge theory--the scalar QED (i.e. SU(N) deconfined phase transition), and demonstrate it can be isolated in a bootstrap island when the matter flavour is large.

Bootstrapping critical gauge theories

Yin-Chen He
(何寅琛)
Perimeter Institute

May 2022

arXiv:2005.04250, 2101.07262, 2107.14637

Junchen Rong
IHES, Paris

Ning Su
Pisa U., Italy

Critical gauge theories

Critical bosonic/fermionic matter fields interact with gauge fields.

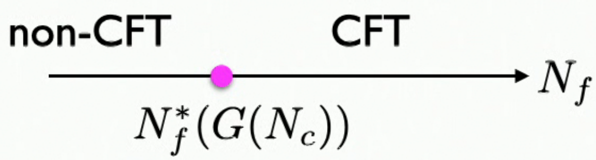
Eg.

$$\mathcal{L} = \sum_{I=1}^{N_f} \bar{\psi}_I (i\partial\!\!\!/ + a\!\!\!/) \psi_I + \frac{1}{4e^2} f_{\mu\nu}^2$$

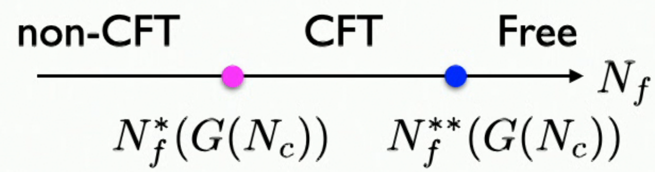
$$\mathcal{L} = \sum_{I=1}^{N_f} |(\partial_\mu - i a_\mu)\phi_I|^2 - u(\sum_{I=1}^{N_f} |\phi_I|^2)^2 + \frac{1}{4e^2} f_{\mu\nu}^2$$

Gauge field a_μ: $U(N_c)$, $SU(N_c)$, $SO(N_c)$, $USp(2N_c)$, $\cdots$

3D



4D



Open questions:

1. What are the boundaries of the conformal window?
2. What are the conformal data of CFTs in the conformal window?

Correlation functions of a CFT

2pt, 3pt corre. functions are fixed by the conformal symmetry.

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{|x_{12}|^{2\Delta}}, \quad x_{12} = x_1 - x_2$$

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{\lambda_{ijk}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_3}|x_{13}|^{\Delta_1+\Delta_3-\Delta_2}|x_{23}|^{\Delta_2+\Delta_3-\Delta_1}}$$

Operator product expansion (OPE):

$$\mathcal{O}_i(x)\mathcal{O}_j(y) = \sum_k \lambda_{ijk} C_k(x-y, \partial_y)\mathcal{O}_k(y)$$

Primary operators: $\{\mathcal{O}_{\Delta,l}\}$

OPE coefficients: $\{\lambda_{ijk}\}$

⇒ All properties of a CFT on R^D

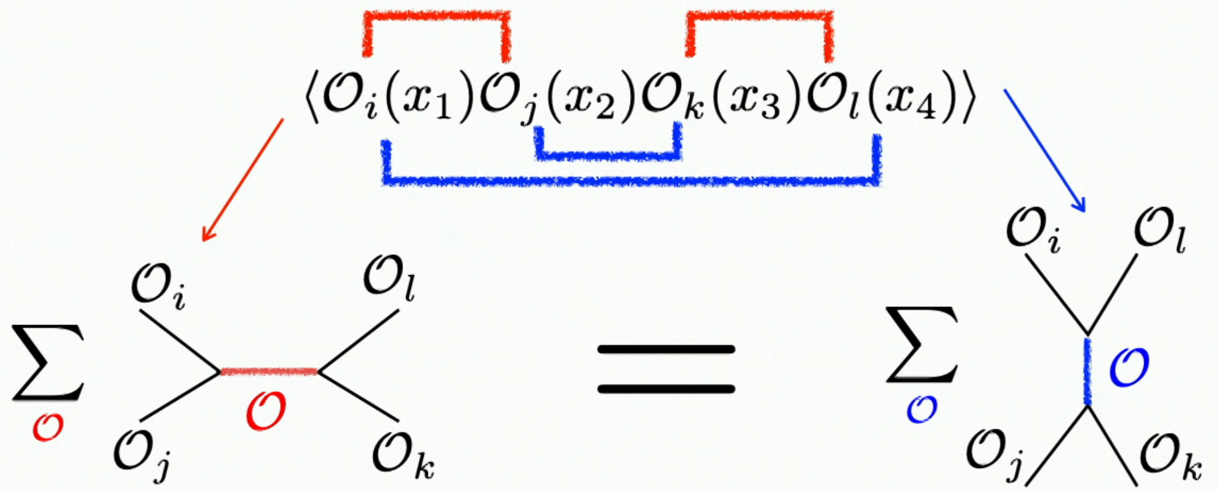
Four point correlation function

NOT completely fixed by conformal symmetry

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\mathcal{O}_l(x_4)\rangle = \frac{1}{|x_{12}|^{2\Delta}|x_{34}|^{2\Delta}} G[u,v]$$

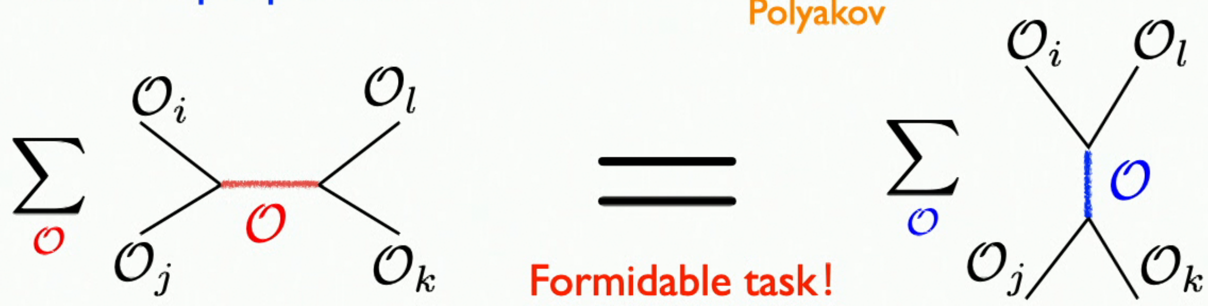
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Crossing symmetry: bootstrap equation



Conformal bootstrap

Conformal bootstrap: find a set of CFT data that solves all the bootstrap equations.



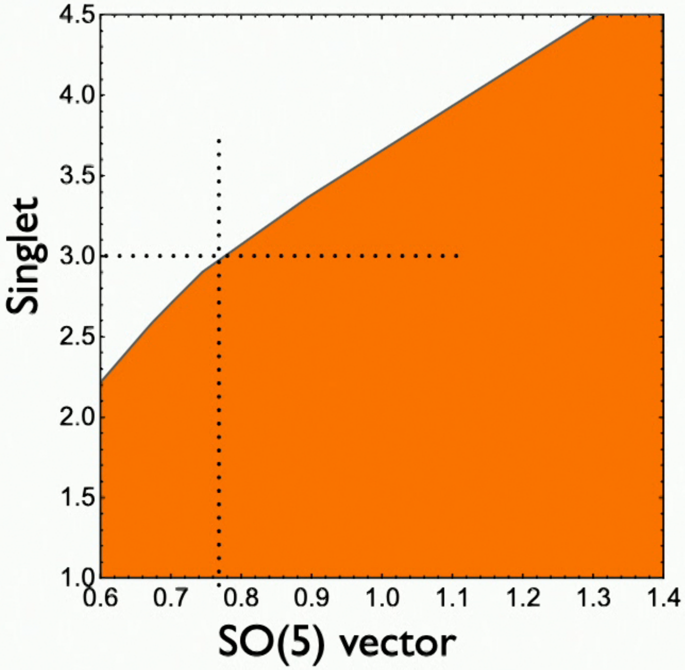
Rattazzi, Rychkov, Tonni, Vichi, 2008

Numerical bootstrap

- Rather than solving it, let us try to get some constraint from bootstrap equations.

Bootstrap bounds for CFTs

Deconfined transition

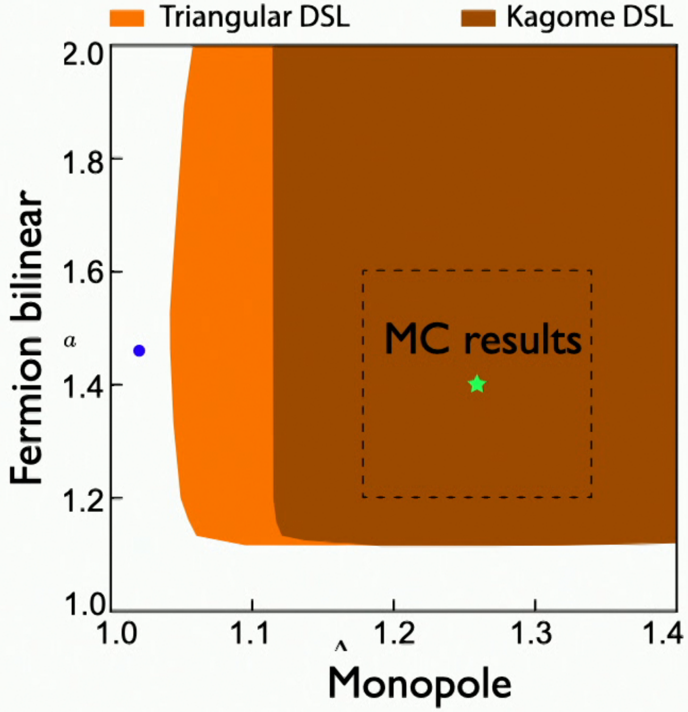


$\Delta_\phi > 0.775$ or $\eta > 0.55$

But numerically $\eta \approx 0.2 \sim 0.3$

Sandvik, Nahum, Kaul, Melko, Assaad,…

Dirac spin liquid (Nf=4 QED3)



YCH, Rong, Su, arXiv:2107.14637

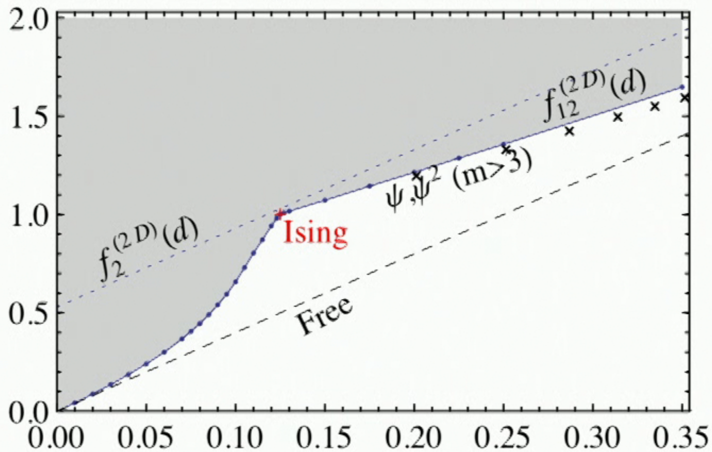
MC results: Karthik & Narayanan

Ising CFT

$\langle \sigma\sigma\sigma\sigma \rangle \qquad\qquad \sigma \times \sigma \sim \epsilon + \cdots$

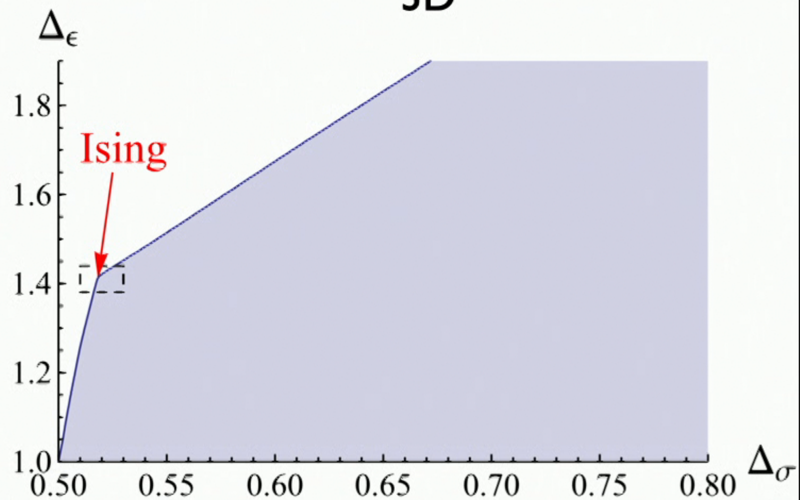
Ising CFT sits at a kink!

2D



Rychkov, Vichi (2009)

3D



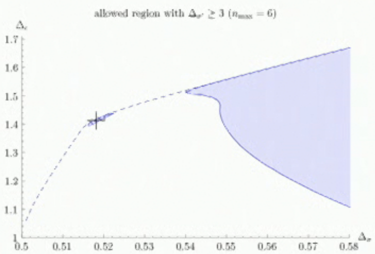
Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi (2012)

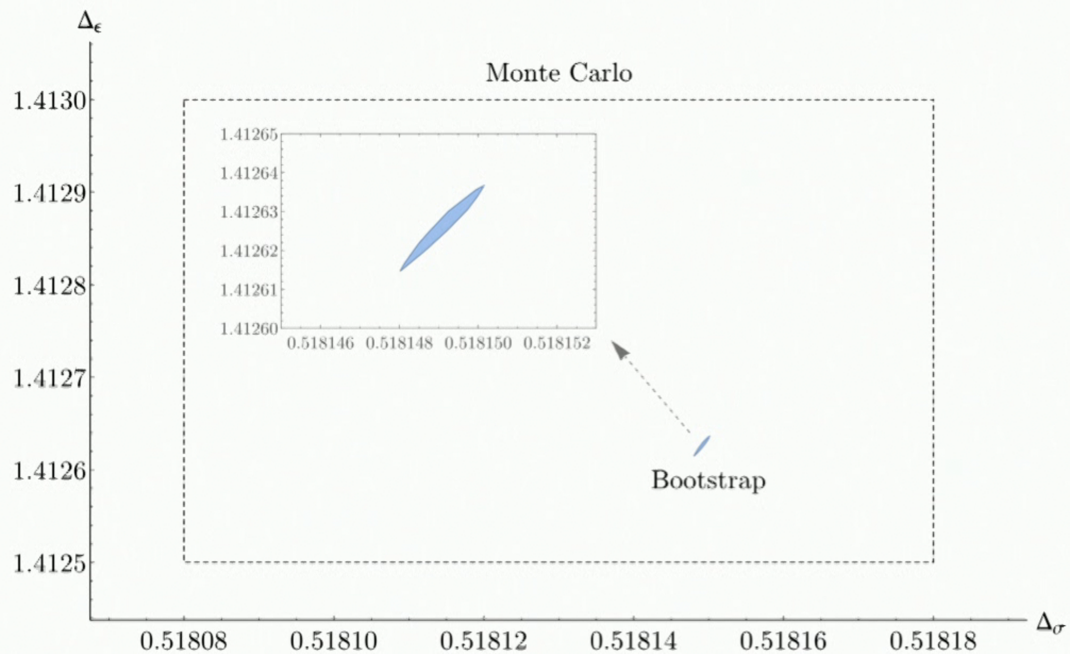
CFT Island

$$\langle \sigma\sigma\sigma\sigma \rangle, \quad \langle \sigma\sigma\epsilon\epsilon \rangle, \quad \langle \epsilon\epsilon\epsilon\epsilon \rangle$$

Most precise critical exponents of 3D Ising!

Kos, Poland, Simmons-Duffin, Vichi, 2016





Bootstrapping critical gauge theories

Challenge: it's hard to target a CFT of interest because we don't use a Lagrangian/Hamiltonian.

A common setup/input of the conformal bootstrap:

	Global symmetry	Operators being bootstrapped
Wilson-Fisher CFT	$O(N)$	$O(N)$ vector, ϕ_i
Bosonic U(Nc) gauge theory	$\frac{SU(N_f)}{Z_{N_f}} \times U(1)$	Fermion bilinear
Fermionic U(Nc) gauge theory	$\frac{SU(N_f)}{Z_{N_f}} \times U(1)$	Monopole

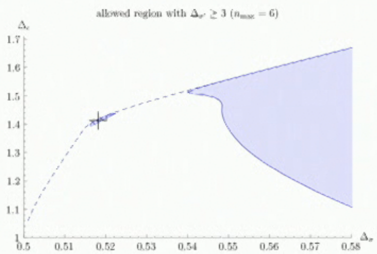
Gauge fields don't appear in the bootstrap equations!

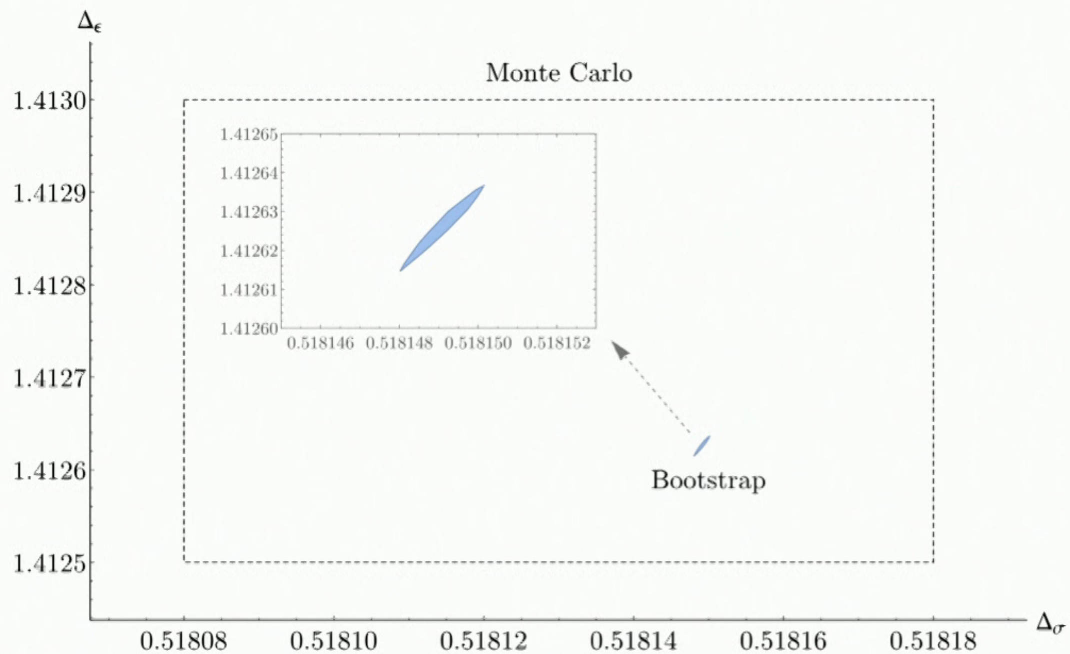
CFT Island

$$\langle \sigma\sigma\sigma\sigma \rangle, \quad \langle \sigma\sigma\epsilon\epsilon \rangle, \quad \langle \epsilon\epsilon\epsilon\epsilon \rangle$$

Most precise critical exponents of 3D Ising!

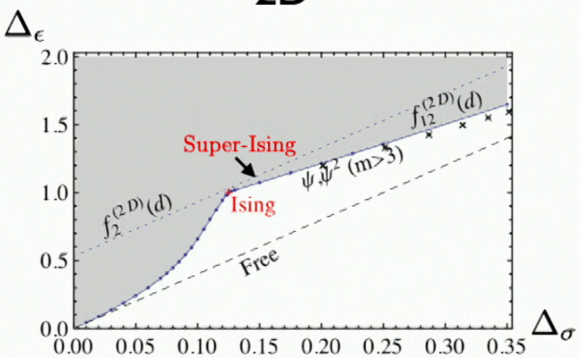
Kos, Poland, Simmons-Duffin, Vichi, 2016

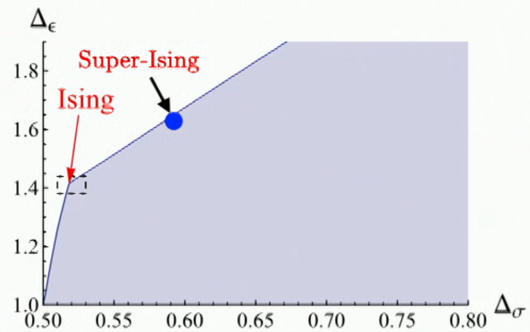




Why kinks: Ising CFT

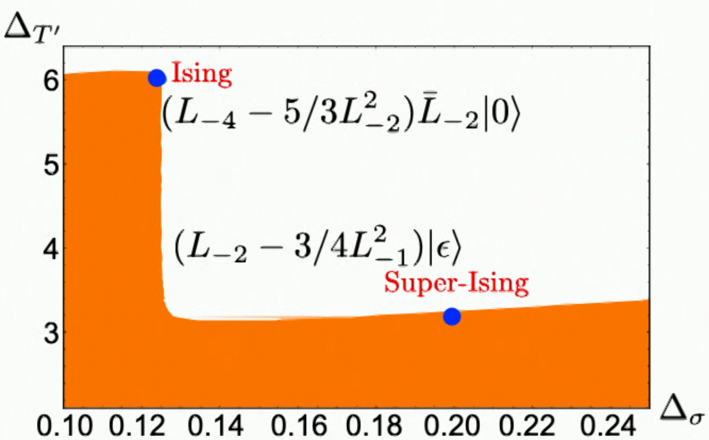
2D $\langle \sigma\sigma\sigma\sigma \rangle$ 3D

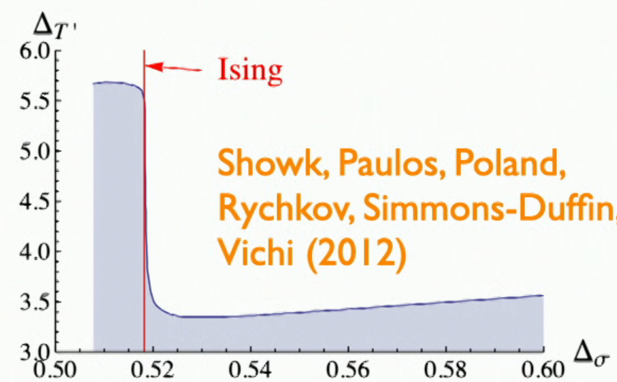




The kink is due to the Virasoro null operator in the spin-2, Z2 even channel.

Similar operator decoupling phenomenon has been observed.

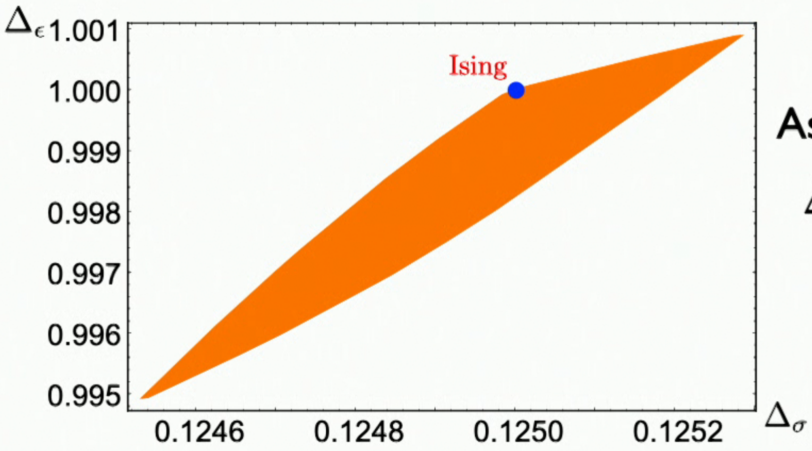




Why islands?

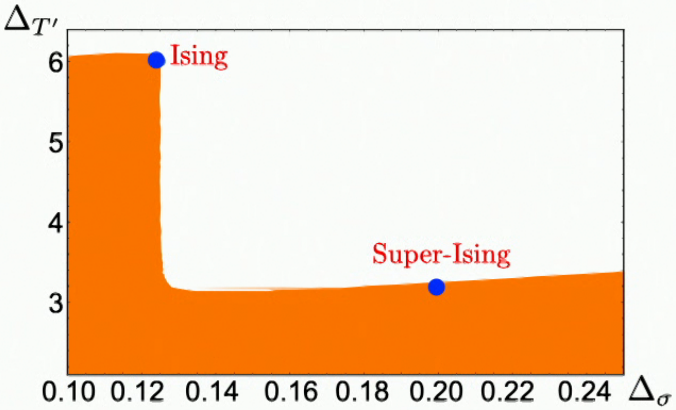
We can impose a few assumptions to exclude spaces outside the Ising CFT.

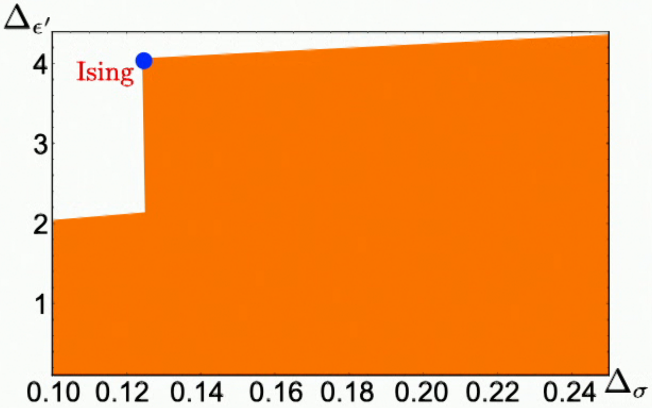
Example: 2D



Assumptions imposed

$\Delta_{T'} > 4, \Delta_{\epsilon'} > 2$





Recipe of bootstrapping a CFT of interest

Make use of the information of operator decoupling of the target CFT.

Types of operator decoupling (we know so far):

- Null operator of 2D CFTs.
- Decoupling operator in gauge theories.
- Decoupling (missing) operator due to equation of motion.

2D WZW CFT

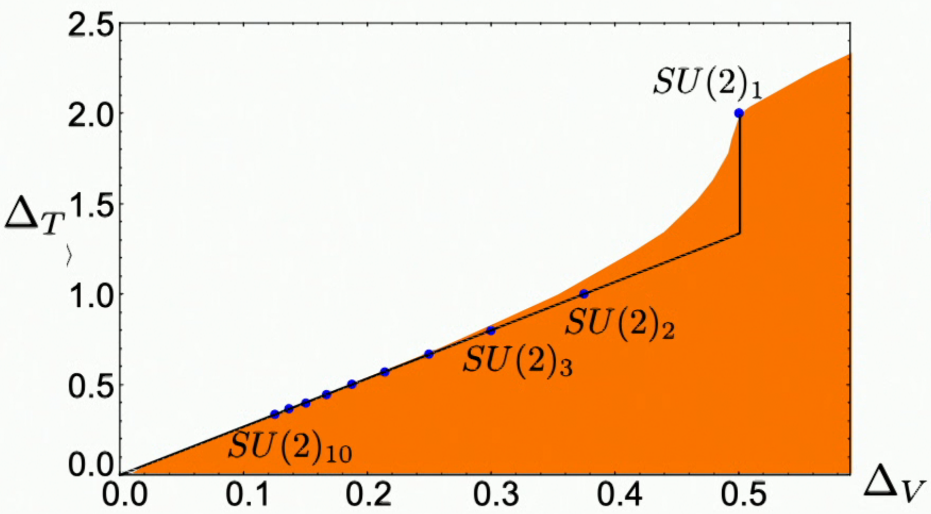
$SU(2)_k$ WZW

Global symmetry: $(SU(2)_L \times SU(2)_R)/Z_2 \cong SO(4)$.

Operator spectrum (lowest weight in each channel):

1. $SO(4)$ V: $\Delta = \frac{3}{2(k+2)}$ (Kac-Moody primary)
2. $SO(4)$ T: $\Delta = \frac{4}{(k+2)}$ for $k > 1$ (Kac-Moody primary) Null at k=1.

   $\Delta = 2$ for $k = 1$ $(J^L_{-1} J^R_{-1}|0\rangle)$



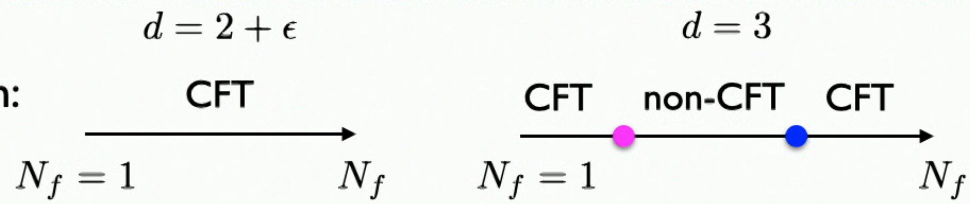
Null operator → Kink

YCH, Rong, Su (2020)

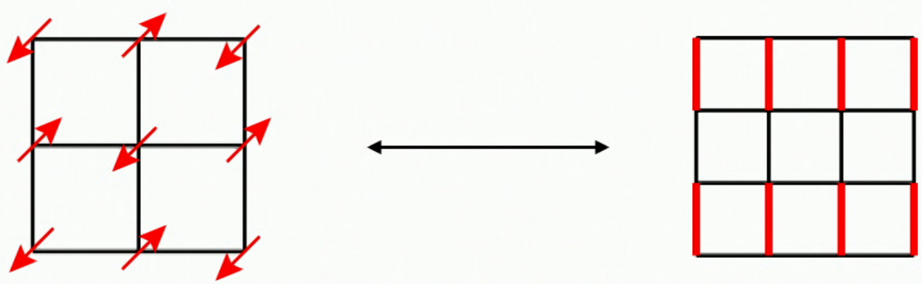
Bootstrap scalar QED

$$\mathcal{L} = \sum_{i=1}^{N_f} |(\partial_\mu - iA_\mu)\phi_i|^2 + \frac{g}{4}|\phi|^4 + \frac{1}{4e^2}F_{\mu\nu}^2.$$

Schematic phase diagram:



- Nf=1: particle-vortex duality. Peskin 1978; Dasgupta & Halperin 1981
- Nf=2: deconfined phase transition. Senthil, et al. 2004
- General Nf: SU(N) deconfined phase transition. Kaul & Sandvik 2012



Decoupling operators of gauge theories

N_f flavors of critical bosons coupled to a $U(N_c)$ gauge field in 3d.

YCH, Rong, Su (2021)

$SU(N_f)$ rep		Operator	Scaling dimension
Adjoint		$\bar{\phi}^{f_1}_{c_1}\phi^{c_1}_{f_2}$	$\Delta = 1 + O(1/N_f)$
$A^{[f_1,f_2]}_{[f_3,f_4]}$	$N_c > 1$	$\bar{\phi}^{[f_1}_{[c_1}\bar{\phi}^{f_2]}_{c_2]}\phi^{[c_1}_{[f_3}\phi^{c_2]}_{f_4]}$	$\Delta = 2 + O(1/N_f)$
	$N_c = 1$	$\bar{\phi}^{[f_1}\partial\bar{\phi}^{f_2]}\phi_{[f_3}\partial\phi_{f_4]}$	$\Delta = 4 + O(1/N_f)$

The lowest operator of $U(N_c > 1)$ gauge theories is decoupled at $N_c = 1$.

The lowest operator in the anti-symmetric representation $A^{[i_1,\cdots,i_m]}_{[j_1,\cdots,j_m]}$ of $N_c > m-1$ is decoupled at $N_c \leq m-1$.

Also see Reehorst, Rifinetti & Vichi (2020); Manenti & Vichi (2021)

Bounding the decoupling operator

$SU(N_f)$ adjoint

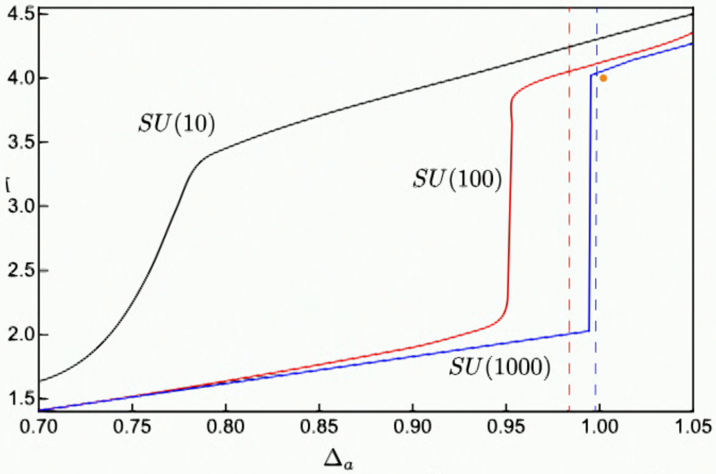
$a = \bar{\phi}^i \phi_j - \delta^i_j / N_f |\phi|^2$

Bootstrap correlator: $\langle a(x_1) a(x_2) a(x_3) a(x_4) \rangle$

The bootstrap bound can capture the essential physics of the operator decoupling, but the scalar QED does not precisely sit at the kink.

YCH, Rong, Su (2021)

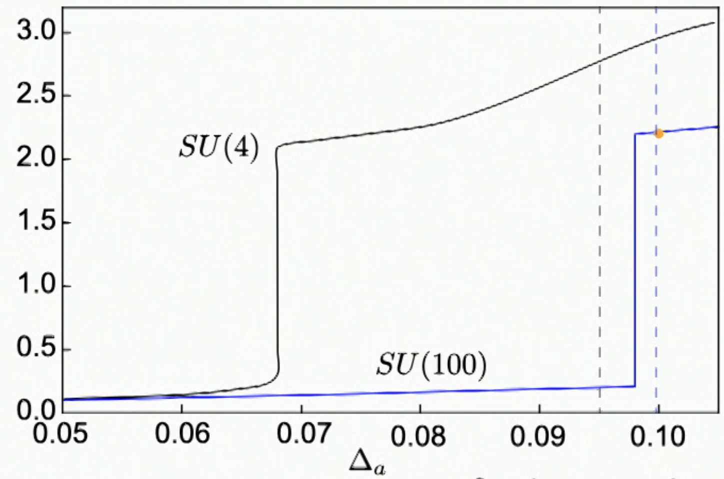
$d = 3$



Large-N_f: $\Delta_a = 1 - \frac{48}{3\pi^2 N_f} + O(1/N_f^2)$

Kaul & Sachdev (2008);

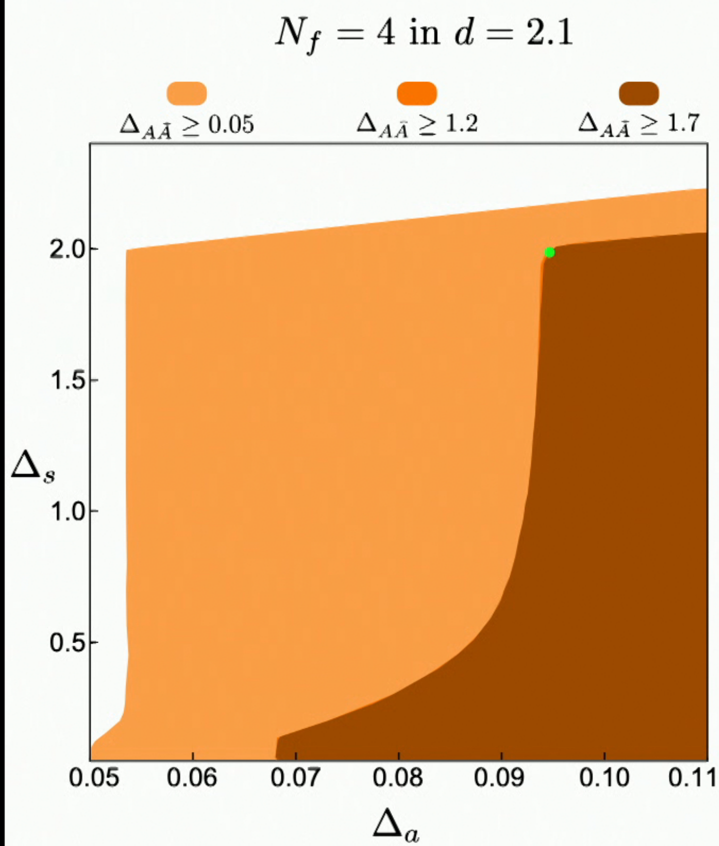
$d = 2.1$



$2 + \epsilon$: $\Delta_a = \epsilon - \frac{2}{N_f}\epsilon^2 + O(\epsilon^3)$

Hikami, 1979, 1981

Scalar QED kink



YCH, Rong, Su (2021)

The scalar QED appears as a kink once a mild gap is added for the decoupling operator, and it is stable against the change of the gap.

The stability of bootstrap bounds against spectrum assumptions is a necessary requirement for reliable bootstrap results.

Towards a bootstrap island

Goal: exclude all other crossing solutions of the bootstrap equation.

The bootstrap equation we use here comes from the 4pt of SU(N) adjoint.

$$\langle a(x_1)a(x_2)a(x_3)a(x_4)\rangle \quad \Delta_a = d - 2 + O(1/N_f)$$

Theories that are consistent with this bootstrap equation:

1. Scalar QED.
2. U(Nc) gauge theories.
3. SU(Nc) gauge theories.
4. Tri-critical QED.
5. Chern-Simons matter theories in 3d.
6. O(2Nf)*.
7. Generalized free theory.

Scalar QED v.s. O(2Nf)*

$$\mathcal{L} = \sum_{i=1}^{N_f} |(\partial_\mu - iA_\mu)\phi_i|^2 + \frac{g}{4}|\phi|^4 + \frac{1}{4e^2}F_{\mu\nu}^2.$$

	Scalar QED	$O(2N_f)^*$
A_μ	$U(1)$ gauge field	Z_N gauge field
Δ_a	$1 + O(1/N_f)$	$1 + O(1/N_f)$
Δ_s	$2 + O(1/N_f)$	$2 + O(1/N_f)$
$\Delta_{S\bar{S}}$	$2 + O(1/N_f)$	$2 + O(1/N_f)$
$\Delta_{A\bar{A}}$	$4 + O(1/N_f)$	$4 + O(1/N_f)$

OPE: $a \times a = S^+ + Adj^\pm + A\bar{A}^+ + S\bar{S}^+ + S\bar{A}^- + A\bar{S}^-.$

$A\bar{A}$: $T^{[f_3,f_4]}_{[f_1,f_2]}$ $S\bar{S}$: $T^{(f_3,f_4)}_{(f_1,f_2)}$ $S\bar{A}$: $T^{(f_3,f_4)}_{[f_1,f_2]}$ $A\bar{S}$: $T^{[f_3,f_4]}_{(f_1,f_2)}$

Decoupling operator from equation of motion

Large-N_f Lagrangian $\quad \mathcal{L} = \sum_{i=1}^{N_f} |(\partial_\mu - iA_\mu)\phi_i|^2 + \sigma|\phi|^2,$

Equation of motion:

$$\bar{\phi}^k \phi_k = |\phi|^2 = 0 \qquad \bar{\phi}^k D_\mu \phi_k = 0 \text{ (unique for QED)}$$

$SU(N_f)$ adjoint, $l = 1$ operators:

$O(2N_f)^*$: $J_\mu = \bar{\phi}^i D_\mu \phi_j,$

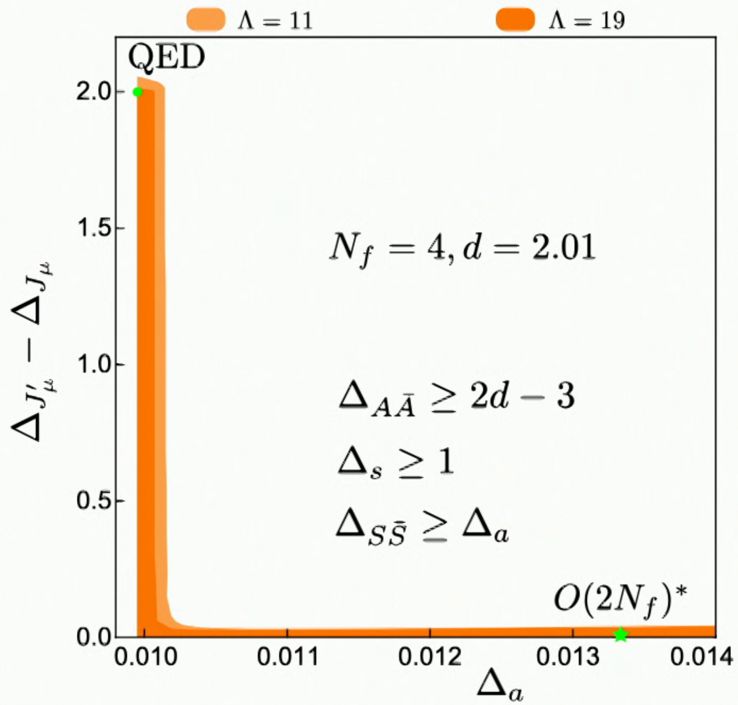
$$J'_\mu = (\bar{\phi}^i \phi_j)\bar{\phi}^k D_\mu \phi_k$$

$$\Delta_{J'_\mu} - \Delta_{J_\mu} \approx d - 2$$

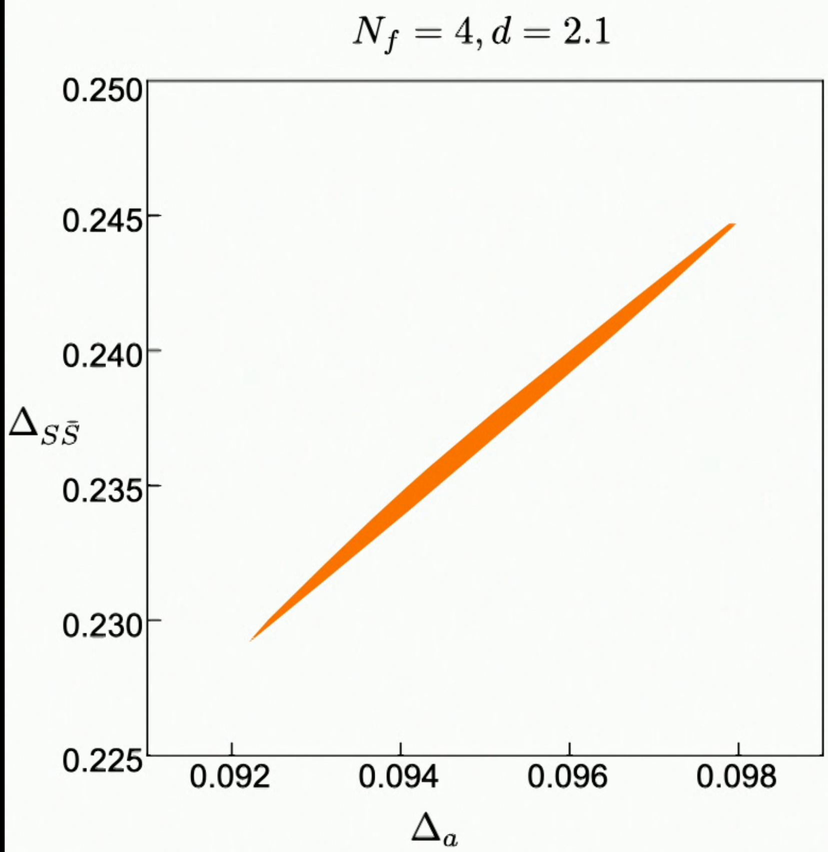
QED: $J_\mu = \bar{\phi}^i D_\mu \phi_j,$

$$J'_\mu = \sigma\bar{\phi}^i D_\mu \phi_j, \cdots$$

$$\Delta_{J'_\mu} - \Delta_{J_\mu} \approx 2$$



Scalar QED island



Single correlator

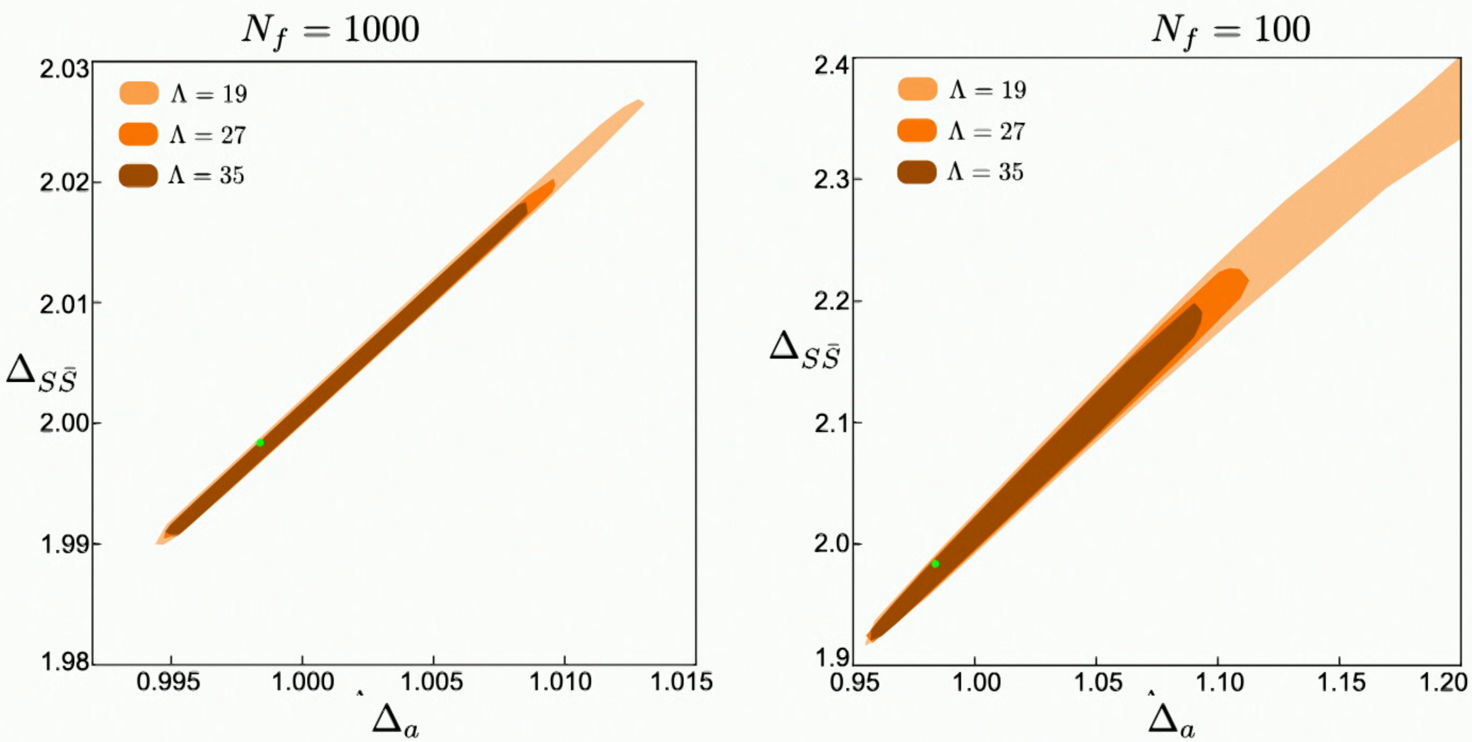
	gap imposed	physical gap
$\Delta_{A\bar{A}}$	$2d-3$	$2d-2$
Δ_s	1	2
$\Delta_{J'_\mu}$	d	$d+1$
$\Delta_{S\bar{S}'}$	$2d-3$	$2d-2$

Scalar QED islands in 3d

YCH, Rong, Su (2021)

	gap imposed	physical gap
$\Delta_{A\bar{A}}$	3	4
$\Delta_{J'_\mu}$	3.1	4
$\Delta_{S\bar{S}'}$	3	4

Single correlator



Summary and outlook

- I introduced conformal bootstrap, and talked about the relation between bootstrap kinks/islands and CFT null/decoupling operators.
- We proposed a recipe to bootstrap critical gauge theories and benchmarked it with scalar QED by obtaining its bootstrap kink and islands.
- Future: We are working on the mixed correlator and spinning correlator to numerically solve scalar QED, QED.

Thanks!

arXiv:2005.04250
arXiv: 2101.07262
arXiv:2107.14637