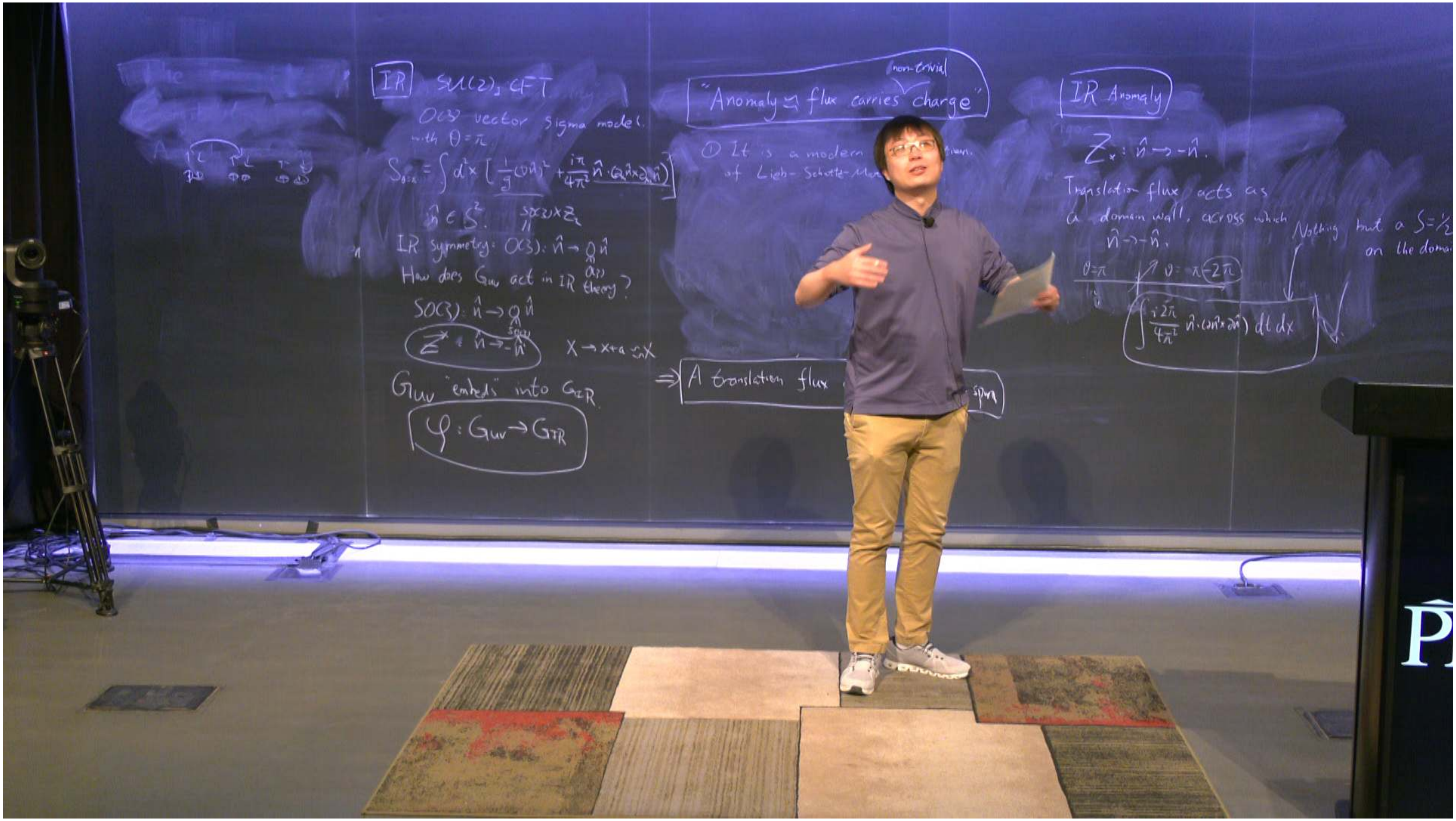
Title: Emergibility: from lattice spins to critical gauge theories and beyond - Talk 2

Speakers: Chong Wang

Collection: Quantum Criticality: Gauge Fields and Matter

Date: May 19, 2022 - 10:15 AM

URL: https://pirsa.org/22050044


IR SU(2)$_1$ CFT
O(3) vector sigma model.
with $\theta=\pi$
IR symmetry: O(3)
How does $G_{UV}$ act in IR theory?
$G_{UV}$ "embeds" into $G_{IR}$
$\varphi: G_{UV} \to G_{IR}$
"Anomaly flux carries non-trivial charge"
① It is a modern
of Lieb-Schultz-Mattis
A translation flux
IR Anomaly
Translation flux acts as a domain wall, across which $\hat{n} \to -\hat{n}$.
Nothing but a S=1/2 on the domain

igma model.

$+\frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_{x_1}\hat{n}\times\partial_{x_2}\hat{n})$

$\times \mathbb{Z}_2$

$\hat{n}\to O\hat{n}$, $O \in O(3)$

IR theory?

$X \to X+a$

$\boxed{\text{Anomaly} \simeq \text{flux carries charge}}$

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

$\Rightarrow$ $\boxed{\text{A translation flux carries half-integer spin}}$

gma model.

$+\frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_{x_1}\hat{n}\times\partial_{x_2}\hat{n})$

$\times\mathbb{Z}_2$

$\hat{n}\to O\hat{n}$, $O\in O(3)$

IR theory?

$X\to X+a\simeq X$

Anomaly $\simeq$ flux carries charge

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

② Can also put in an SO(3) flux (a.k.a. a large gauge transform)
→ changes lattice momentum by $\pi$!

$\Rightarrow$ $\boxed{\text{A translation flux carries half-integer spin}}$

gma model.

$+\frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_x\hat{n}\times\partial_{x}\hat{n})$

$\times \mathbb{Z}_2$

$\hat{n} \to \underset{O(3)}{\underset{\pi}{O}}\,\hat{n}$

IR theory?

$X \to X + a \cdot \sum_n X$

# Anomaly ⇝ flux carries charge

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

② Can also put in an SO(3) flux (a.k.a. a large gauge transform)

→ changes lattice momentum by $\pi$.

$e^{iS_z \cdot \theta(x)} \qquad \theta = \frac{2\pi}{L}\cdot x$

⇒ $\boxed{\text{A translation flux carries half-integer spin}}$

Anomaly ⇔ flux carries charge

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

② Can also put in an $SO(3)$ flux (a.k.a. a large gauge transform)
→ changes lattice momentum by $\pi$!

$$\prod_x e^{i S_z^{(x)} \cdot \theta(x)} \qquad \theta = \frac{2\pi}{L} \cdot x$$

⇒ A translation flux carries half-integer spin

...igma model.

$+\frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_{x_1}\hat{n}\times\partial_{x_2}\hat{n})$

$\hat{n}$

R...ory?

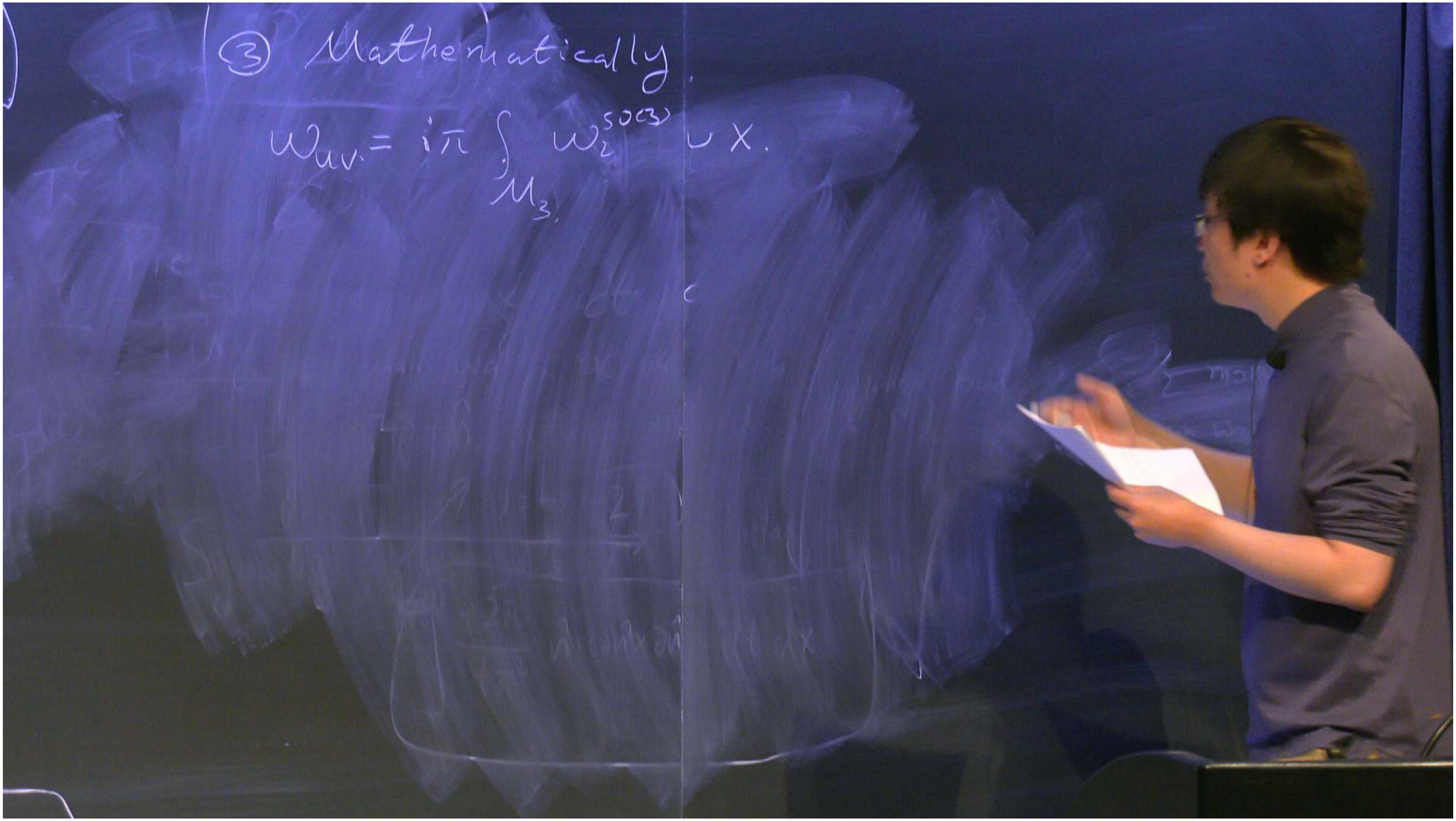
$\boxed{\text{Anomaly} \rightleftharpoons \text{flux carries charge}}$

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

② Can also put in an SO(3) flux (a.k.a. a large gauge transform)
→ changes lattice momentum by $\pi$!

$$\prod_x e^{i\frac{\sigma_z(x)}{2}\theta(x)} \qquad \theta = \frac{2\pi}{L}\cdot x$$

$\Rightarrow$ $\boxed{\text{A translation flux carries half-integer spin}}$


③ Mathematically.
$\omega_{UV} = i\pi \int_{M_3} w_2^{SO(3)} \cup X.$

③ Mathematically.

$$\omega_{UV} = i\pi \int_{M_3} w_2^{SO(3)} \cup x, \quad \omega_{IR} = i\pi \int_{M_3} w_3^{O(3)}.$$

③ Mathematically.

$$\omega_{UV} = i\pi \int_{\mathcal{M}_3} w_2^{SO(3)} \cup x \,, \qquad \omega_{IR} = i\pi \int_{\mathcal{M}_3} w_3^{O(3)}.$$

Symmetry embedding $\varphi: G_{UV} \to G_{IR}$.

Anomaly matching means $\varphi^*(\omega_{IR}) = \omega_{UV}$.

③ Mathematically.

$$\omega_{UV} = i\pi \int_{\mathcal{M}_3} w_2^{SO(3)} \cup x, \quad \omega_{IR} = i\pi \int_{\mathcal{M}_3} w_3^{O(3)}.$$

Symmetry embedding $\varphi : G_{UV} \to G_{IR}$.

Anomaly matching means $\varphi^*(\omega_{IR}) = \omega_{UV}$. (*)

For given $\omega_{UV}$ and $\omega_{IR}$, (*) becomes an algebraic equation on $\varphi$.

③ Mathematically.

$$\omega_{UV} = i\pi \int_{\mathcal{M}_3} w_2^{SO(3)} \cup x, \quad \omega_{IR} = i\pi \int_{\mathcal{M}_3} w_3^{O(3)}.$$

Symmetry embedding $\varphi: G_{UV} \to G_{IR}$.

Anomaly matching means $\varphi^*(\omega_{IR}) = \omega_{UV}$. (*)

For given $\omega_{UV}$ and $\omega_{IR}$, (*) becomes an algebraic equation on $\varphi$, i.e. on possible realizations of UV symmetry in IR.

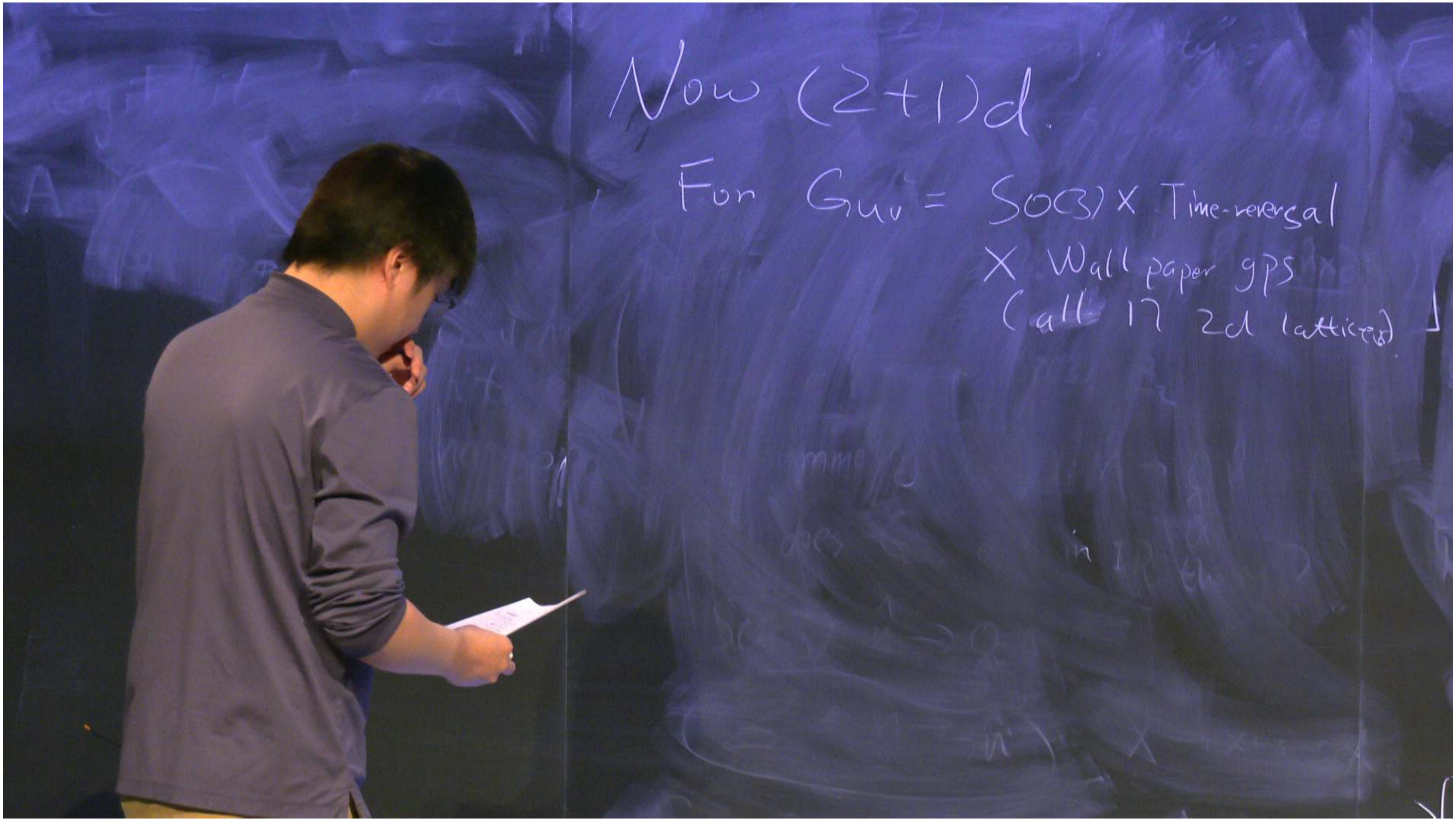
③ Mathematically.

$$\omega_{UV} = i\pi \int_{M_3} w_2^{SO(3)} \cup x \,, \quad \omega_{IR} = i\pi \int_{M_3} w_3^{O(3)}.$$

Symmetry embedding $\varphi : G_{UV} \to G_{IR}$.

Anomaly matching means $\boxed{\varphi^*(\omega_{IR}) = \omega_{UV} \quad (*)}$

For given $\omega_{UV}$ and $\omega_{IR}$, (*) becomes an algebraic equation on $\varphi$, i.e. on possible realizations of UV symmetry in IR.


Now (2+1)d.
For G_UV = SO(3) × Time-reversal
× Wall paper gps
(all 17 2d lattices)

Now (2+1)d.

For $G_{uv} = SO(3) \times$ Time-reversal

$\times$ Wall paper gps

(all 17 2d lattices)

$W_{uv}$ listed in 2111.12097

Now (2+1)d.

For $G_{uv}$ = $SO(3)$ × Time-reversal

× Wall paper gps

(all 17 2d lattices)

$W_{uv}$ listed in 2111.12097

Some results:

- The "standard" DQCP (square & honeycomb) and $U(1)$ Dirac spin liquids (triangular & Kagome)
  ($N_f = 4$ $QED_3$)

$\boxed{\text{Anomaly} \Leftrightarrow \text{flux carries charge}}$

① A "non-parton" U(1) Dirac spin liquid.

$$\mathcal{L} = \sum_{i=1}^{4} \bar{\psi}_i \, i \not{D}_a \psi_i + \frac{1}{4e^2} f_{\mu\nu}^2$$

al

lattices)

honeycomb)

2 Kagome)

ates.

③ Mathem…

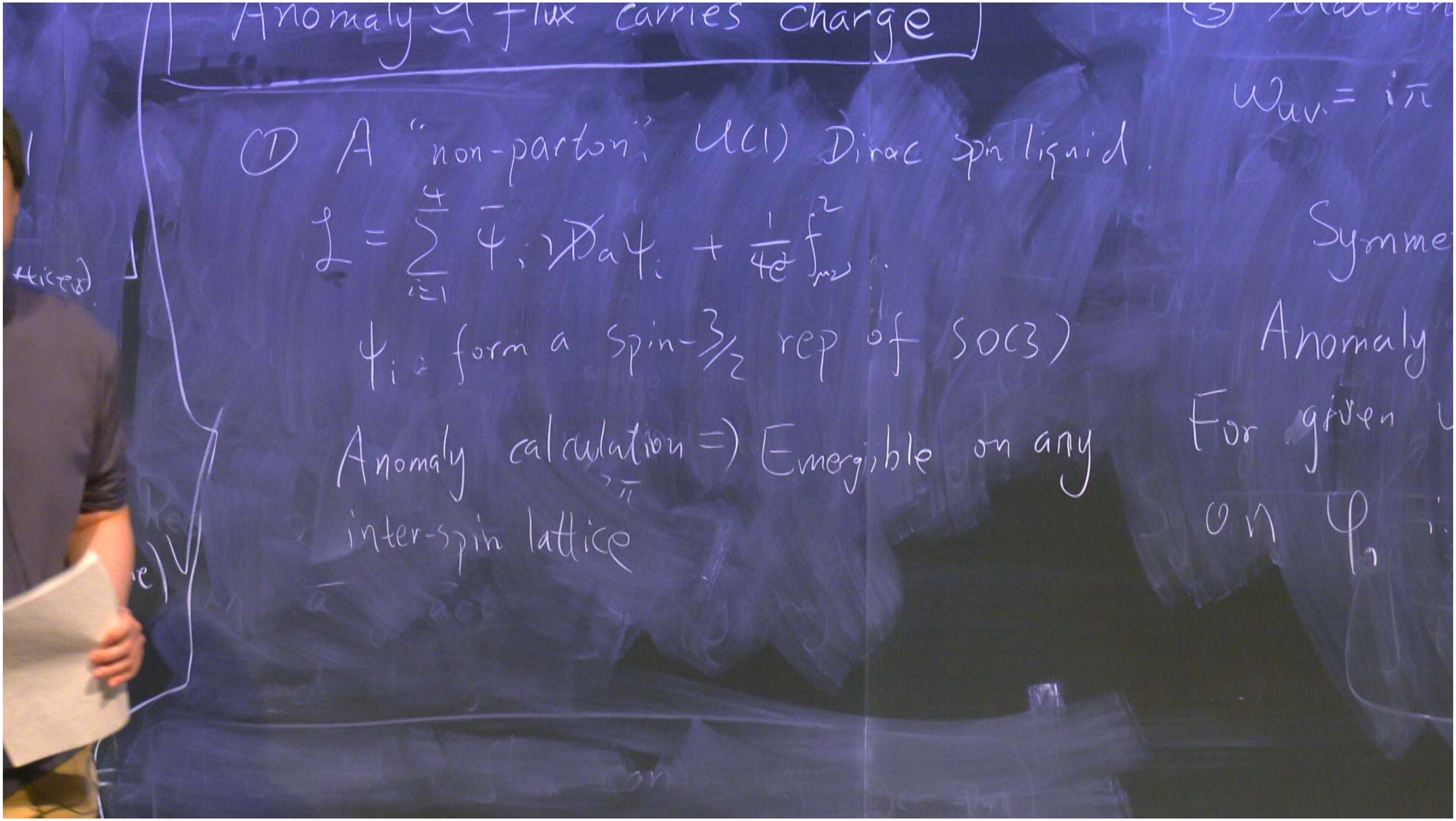
$\omega_{UV} = i\pi$

Symme…

Anomaly

For given U…

on $\varphi_1$ …


Anomaly ⇔ flux carries charge
(1) A "non-parton" U(1) Dirac spin liquid.
$\mathcal{L} = \sum_{i=1}^{4} \bar{\psi}_i \, i \not{D}_a \psi_i + \frac{1}{4e^2} f_{\mu\nu}^2$
$\psi_{i}$ form a spin-3/2 rep of SO(3)
Anomaly calculation ⇒ Emergible on any inter-spin lattice
$\omega_{UV} = i\pi$
Anomaly

$\boxed{\text{Anomaly} \Leftrightarrow \text{flux carries charge}}$

(1) A "non-parton" $U(1)$ Dirac spin liquid.

$$\mathcal{L} = \sum_{i=1}^{4} \bar{\psi}_i \, i \not{D}_a \psi_i + \frac{1}{4e^2} f_{\mu\nu}^2$$

$\psi_i$ form a spin-$3/2$ rep of $SO(3)$

Anomaly calculation $\Rightarrow$ Emergible on any inter-spin lattice ($S=\frac{1}{2}$ on honeycomb also O.K.)

(3) Mathem

$\omega_{UV} = i\pi$

Symme

Anomaly

For given U

on $\varphi$, i

Anomaly $\Leftrightarrow$ flux carries charge

① A "non-parton" $U(1)$ Dirac spin liquid.

$$\mathcal{L} = \sum_{i=1}^{4} \bar{\psi}_i \, i \not{D}_a \psi_i + \frac{1}{4e^2} f_{\mu\nu}^2$$

$\psi_i$: form a spin-$3/2$ rep of $SO(3)$

Anomaly calculation $\Rightarrow$ Emergible on any inter-spin lattice ($S=\frac{1}{2}$ on honeycomb also o.k.)

The most relevant spin excitations carry spin-2.

# The meaning of “constructing” a phase on lattice

- Classic:

$$J\sum \mathbf{S}_i \cdot \mathbf{S}_j \to J\sum \langle \mathbf{S}_i \rangle \cdot \mathbf{S}_j$$

- Modern:

$$\begin{aligned} \mathbf{S} &= \frac{1}{2} f^\dagger_\alpha \sigma_{\alpha\beta} f_\beta \\ H_{MF} &= \sum_{ij} t_{ij} f^\dagger_i f_j \end{aligned}$$

- Postmodern?

$$\varphi : G_{UV} \to G_{IR}$$

$$w[G_{UV}] = \varphi^* w[G_{IR}]$$