

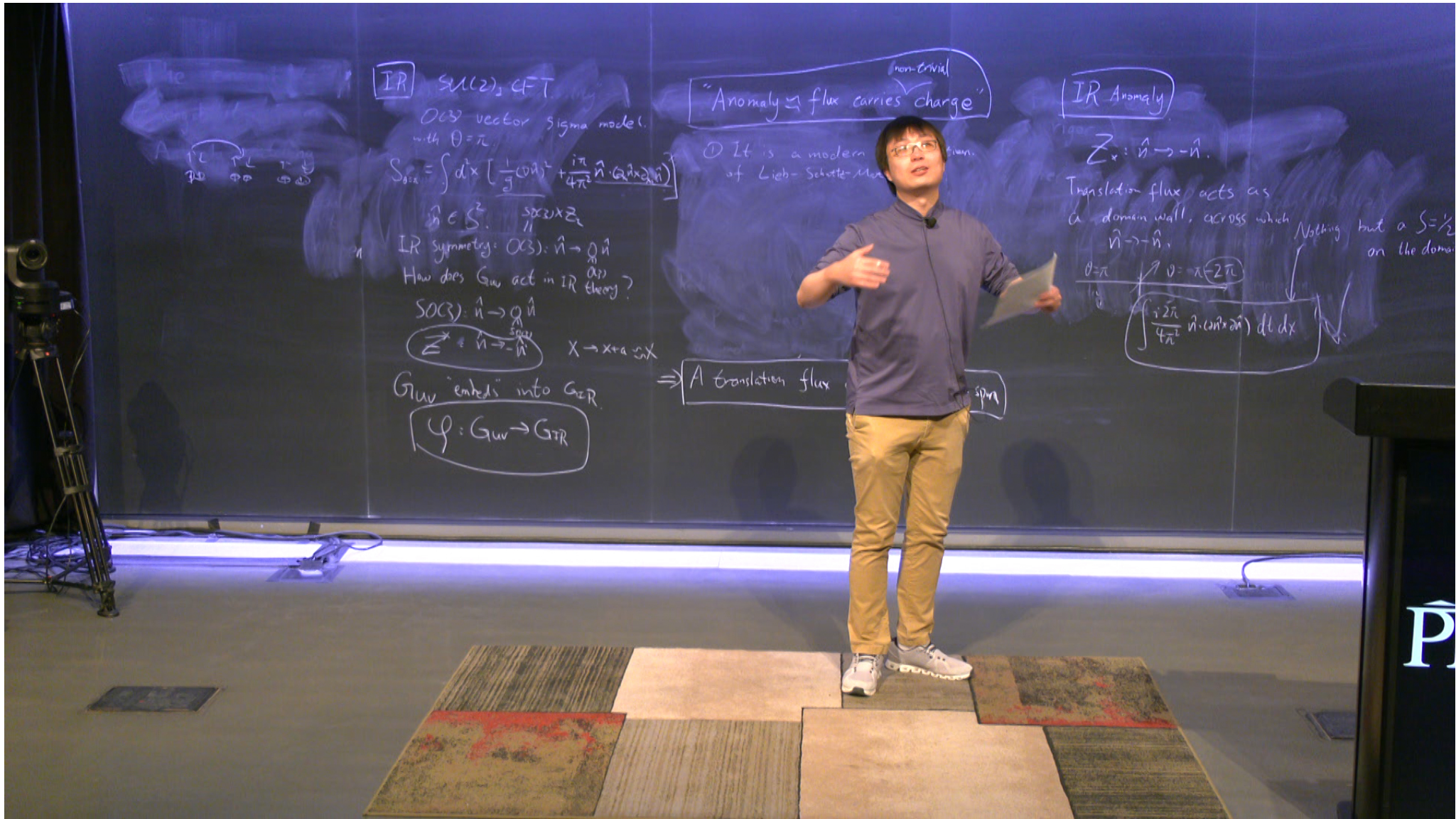
Title: Emergibility: from lattice spins to critical gauge theories and beyond - Talk 2

Speakers: Chong Wang

Collection: Quantum Criticality: Gauge Fields and Matter

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IR $SU(2)$ CFT

O(3) vector sigma model
with $\theta = \pi$

$$S_{\text{O(3)}} = \int d^2x \left[\frac{1}{2} (\partial_\mu \hat{n})^2 + \frac{i\pi}{4\pi} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) \right]$$

$\hat{n} \in S^2$ $S^2 \times \mathbb{R}^2$

IR symmetry: O(3): $\hat{n} \rightarrow Q \hat{n}$

How does G_{UV} act in IR theory?

SO(3): $\hat{n} \rightarrow Q \hat{n}$

$\hat{z} \in \hat{n} \rightarrow -\hat{n}$ $X \rightarrow X + a - \pi X$

G_{UV} embeds into G_{IR}

$\mathcal{Y}: G_{UV} \rightarrow G_{IR}$

^{non-trivial}
"Anomaly \Rightarrow flux carries charge"

① It is a modern version of Lieb-Schultz-Mattis

IR Anomaly

$$Z_x: \hat{n} \rightarrow -\hat{n}$$

Translation flux acts as a domain wall, across which $\hat{n} \rightarrow -\hat{n}$. Nothing but a $S = \frac{1}{2}$ on the domain.

$$\theta = \pi \rightarrow \theta = \pi - 2\pi$$

$$\int \frac{i2\pi}{4\pi} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) dt dx$$

\Rightarrow A translation flux spin

Anomaly \Rightarrow flux carries charge

gma model.

$$+ \frac{i\pi}{4\pi^2} \hat{n} \cdot (\partial_x \hat{n} \times \partial_x \hat{n})$$

① It is a modern manifestation of Lieb-Schultz-Mattis Thm.

$\times \mathbb{Z}_2$

$$\hat{n} \rightarrow \hat{n}$$

IR theory?

$$X \rightarrow X + a \sim X$$

\Rightarrow A translation flux carries half-integer spin

Anomaly \Leftarrow flux carries charge

gma model.

$$+ \frac{i\pi}{4\pi^2} \hat{n} \cdot (\hat{q}_x \times \hat{q}_x \hat{n})$$

$\times \hat{z}$

$$\hat{n} \rightarrow \hat{0} \hat{n}$$

IR theory?

$$X \rightarrow X + a \sim X$$

① It is a modern manifestation of Lieb-Schultz-Mattis Thm

② Can also put in an SO(3) flux (a.k.a. a large gauge transform) \rightarrow changes lattice momentum by π .

\Rightarrow A translation flux carries half-integer spin

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$$iS_z \cdot \hat{q}(x) \quad \theta = \frac{2\pi}{L} \cdot x$$

$$X \rightarrow X + a \cdot \hat{n} \cdot X$$

\Rightarrow A translation flux carries half-integer spin

Anomaly \Leftarrow flux carries charge

gma model.

$$+ \frac{i\pi}{4\pi^2} \hat{n} \cdot (\hat{a}_x \times \hat{a}_y \cdot \hat{n})$$

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$$\frac{\pi e}{x} \quad iS_z^a \cdot O(x) \quad 0 = \frac{2\pi}{c} \cdot x$$

\hat{n}
 $\alpha(x)$
theory?

$$x \rightarrow x+a \sim x$$

\Rightarrow A translation flux carries half-integer spin

gma model.

$$+ \frac{i\pi}{4\pi^2} \hat{n} \cdot (\hat{a}_x \times \hat{a}_y \cdot \hat{n})$$

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- ① It is a modern manifestation of Lieb-Schultz-Mattis Thm
- ② Can also put in an SO(3) flux (a.k.a. a large gauge transform) \rightarrow changes lattice momentum by π .

$$\frac{\pi \rho}{x} \frac{i\delta_z^0 \cdot 0(x)}{z} \quad 0 = \frac{2\pi}{c} \cdot x$$

\Rightarrow A translation flux carries half-integer spin

③ Mathematically

$$\omega_{uv} = i\pi \int_{M_3} \omega_2^{SO(3)} \cup X.$$

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$$, \quad \omega_{IR} = i\pi \int_{M_3} \omega_3^{SO(3)}$$

③ Mathematically

$$\omega_{uv} = i\pi \int_{\mathcal{M}_3} \omega_2^{SO(3)} \wedge X, \quad \omega_{IR} = i\pi \int_{\mathcal{M}_3} \omega_3^{O(3)}.$$

Symmetry embedding $\mathcal{G} = G_{uv} \rightarrow G_{IR}$.

Anomaly matching means $\mathcal{G}^*(\omega_{IR}) = \omega_{uv}$.

③ Mathematically

$$\omega_{uv} = i\pi \int_{\mathcal{M}_3} \omega_2^{SO(3)} \text{tr} X, \quad \omega_{IR} = i\pi \int_{\mathcal{M}_3} \omega_3^{SO(3)}.$$

Symmetry embedding $\varphi: G_{uv} \rightarrow G_{IR}$.

Anomaly matching means $\varphi^*(\omega_{IR}) = \omega_{uv} \quad (*)$

For given ω_{uv} and ω_{IR} , $(*)$ becomes an algebraic equation on φ .

③ Mathematically

$$\omega_{UV} = i\pi \int_{\mathcal{M}_3} \omega_2^{SO(3)} \cup X, \quad \omega_{IR} = i\pi \int_{\mathcal{M}_3} \omega_3^{O(3)}$$

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on φ , i.e. on possible realizations of UV symmetry in IR.

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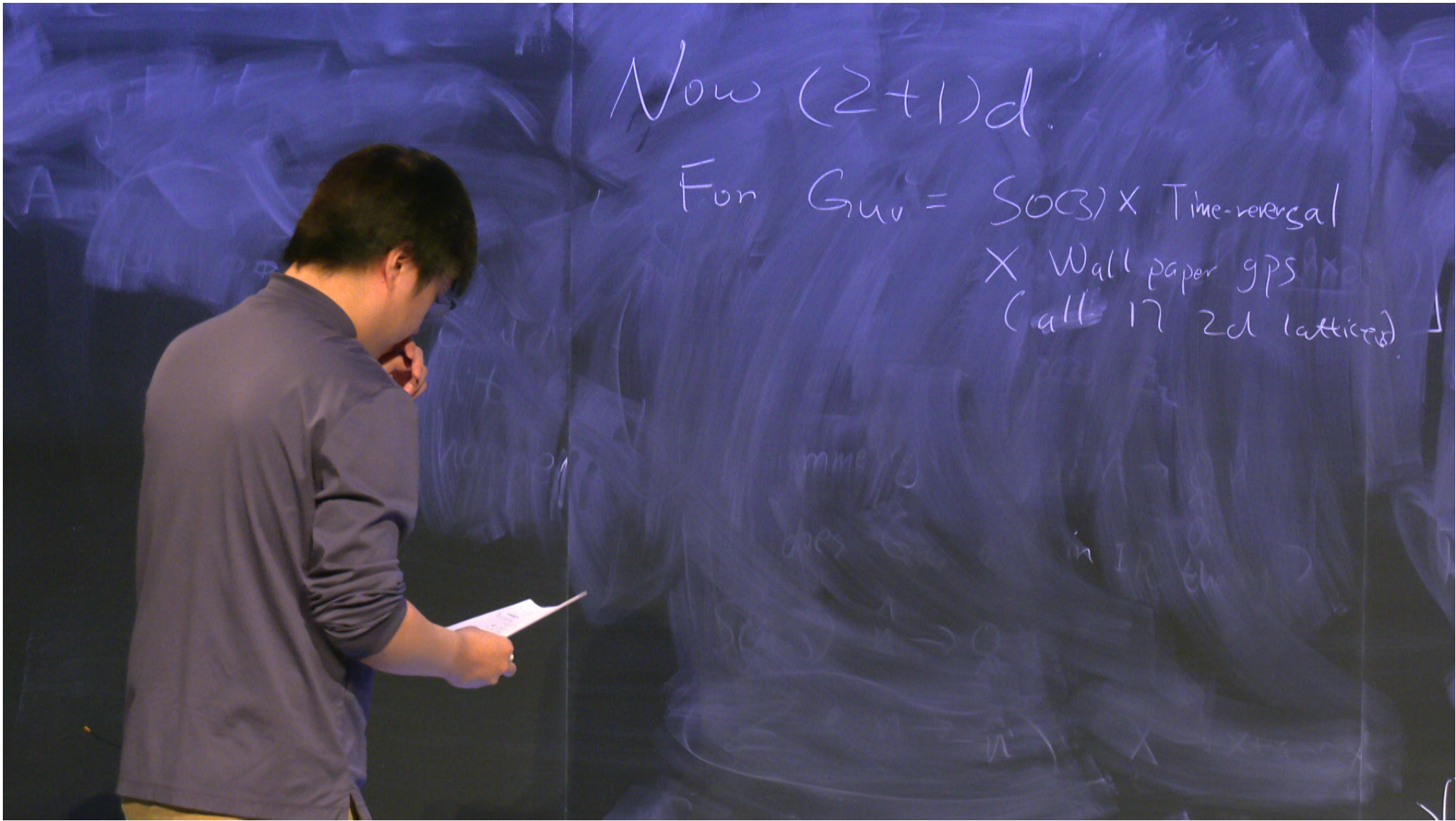
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on φ , i.e. on possible realizations of UV symmetry in IR.



Now $(2+1)d$.

For $G_{uv} = SO(3) \times \text{Time-reversal}$

\times Wall paper gps

(all 17 2d lattices)

Wuv listed in 211-12097

Now (2+1)d.

For $G_{UV} = \text{SO}(3) \times \text{Time-reversal}$
 $\times \text{Wall paper gps}$
(all 17 2d lattices)
Wuv listed in 2111-12097

Some results:

- The "standard" DQCP (square & honeycomb)
and U(1) Dirac spin liquids (triangular & Kagome)
($N_f = 4$ QED₃)

Anomaly \Rightarrow flux carries charge

① A "non-parton" U(1) Dirac spin liquid

$$\mathcal{L} = \sum_{i=1}^4 \bar{\psi}_i i \not{D}_a \psi_i + \frac{1}{4e} f_{\mu\nu}^2$$

$$W_{uv} = i\pi$$

Symme

Anomaly

For given

on ψ_i

Anomaly \Rightarrow flux carries charge

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ψ_i form a spin- $3/2$ rep of SO(3)

Anomaly calculation \Rightarrow Emergible on any
inter-spin lattice

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$$\mathcal{L} = \sum_{i=1}^4 \bar{\psi}_i \not{D}_a \psi_i + \frac{1}{4e} f_{\mu\nu}^2$$

$\psi_{i=2}$ form a spin- $3/2$ rep of SO(3)

Anomaly calculation \Rightarrow Emergible on any

inter-spin lattice ($S=1/2$ on honeycomb also o.k.)

$$W_{uv} = i\pi$$

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The most relevant spin excitations
carry spin-2.

$$W_{uv} = i\pi$$

Symme

Anomaly

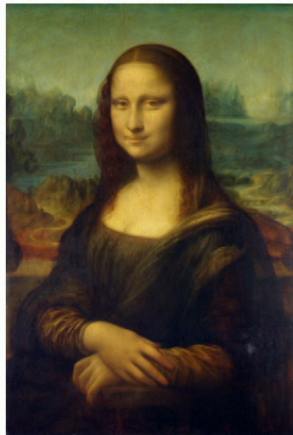
For given

on ψ_i

The meaning of “constructing” a phase on lattice

- Classic:

$$J \sum \mathbf{s}_i \cdot \mathbf{s}_j \rightarrow J \sum \langle \mathbf{s}_i \rangle \cdot \mathbf{s}_j$$



- Modern:

$$\mathbf{S} = \frac{1}{2} f_{\alpha}^{\dagger} \sigma_{\alpha\beta} f_{\beta}$$
$$H_{MF} = \sum_{ij} t_{ij} f_i^{\dagger} f_j$$



- Postmodern?

$$\varphi : G_{UV} \rightarrow G_{IR}$$
$$w[G_{UV}] = \varphi^* w[G_{IR}]$$

