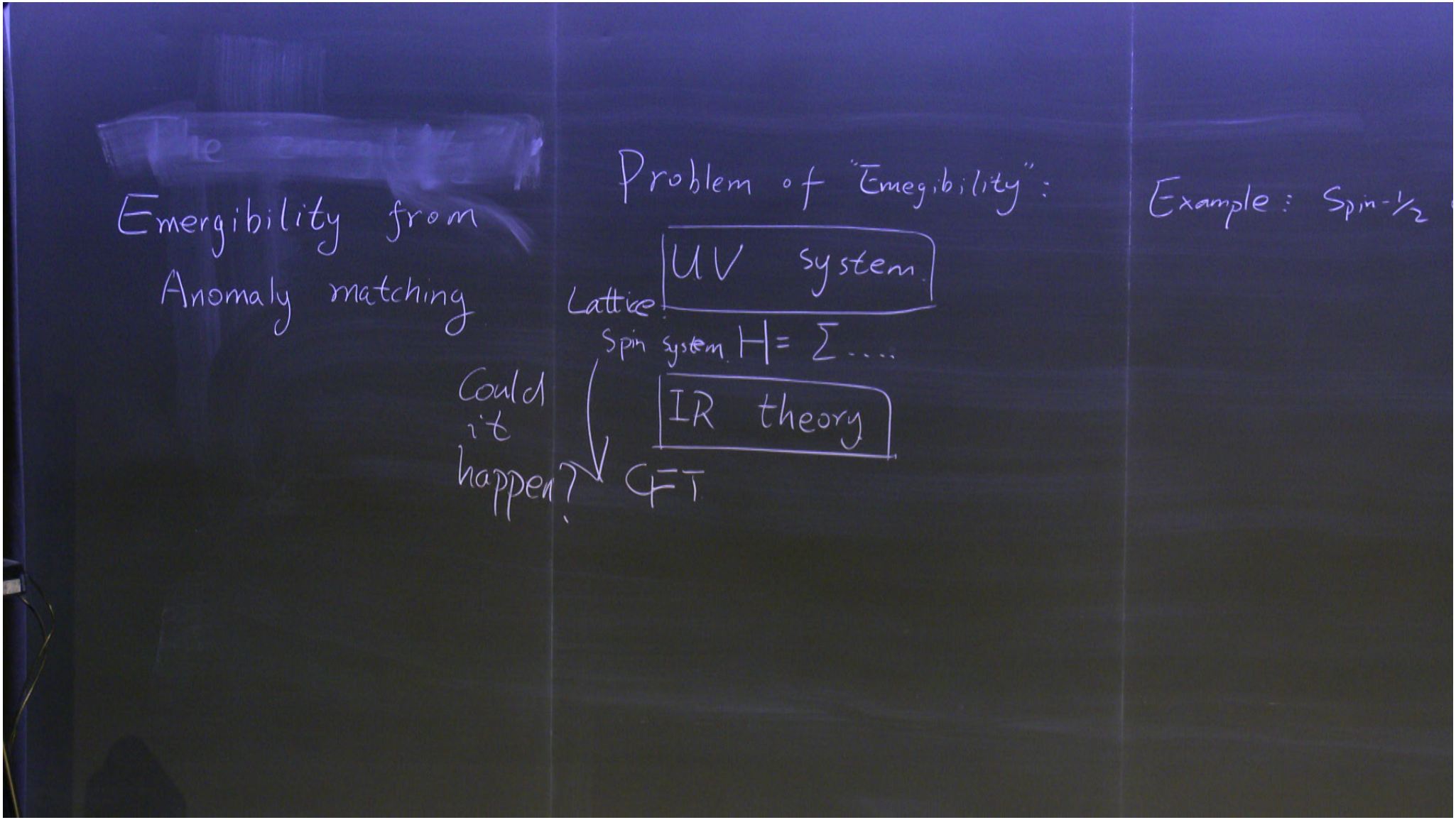
Title: Emergibility: from lattice spins to critical gauge theories and beyond - Talk 1

Speakers: Chong Wang

Collection: Quantum Criticality: Gauge Fields and Matter

Date: May 19, 2022 - 9:00 AM

URL: https://pirsa.org/22050043


Emergibility from
Anomaly matching
Problem of "Emegibility":
UV system
Lattice
Spin system $H = \Sigma \ldots$
IR theory
Could it happen?
QFT
Example: Spin-1/2

...gibility":

Example: Spin-½ chain ← UV.

$SU(2)_1$ CFT.

Many approaches: $+J'\sum \vec{S}_i \cdot \vec{S}_{i+2}$

1. Solve $H = J\sum_i \vec{S}_i \cdot \vec{S}_{i+1}$ with Bethe ansatz.
2. Approximately to some continuum Lagrangian
   e.g. sigma model / parton gauge theory.
3. Anomaly matching "$W_{UV} = W_{IR}$"

From ① → ② → ③.

rigor ↓, generalizability ↑.

$\vec{S}_{i+2}$

Bethe ansatz.

Precisely:

UV $\quad \mathcal{H} = \bigotimes_{i=1}^{L} \mathcal{H}_i, \quad \mathcal{H}_i = \mathrm{span}\{|\uparrow\rangle_i, |\downarrow\rangle_i\}$.

$H = \sum_i$ (local stuff around site i)

Lagrangian

gauge theory

IR

Symmetry: $G_{UV} = SO(3) \times$ translation

$\mathbb{Z}^x = \{T_x^n, n \in \mathbb{Z}\}$

IR $SU(2)_1$ CFT

$O(3)$ vector sigma model with $\Theta = \pi$.

$$S_{\theta=\pi} = \int d^2x \left[\frac{1}{g} (\partial \vec{n})^2 + \frac{i\pi}{4\pi^2} \hat{n} \cdot (\partial_{x_1} \hat{n} \times \partial_{x_2} \hat{n}) \right]$$

$\hat{n} \in S^2$.

IR symmetry: $O(3): \hat{n} \to \underset{O(3)}{Q} \hat{n}$

Example: Spin-$\frac{1}{2}$ $SU(2)_1$

Many approaches

① Solve $H = J \sum_i$

② Approximately

e.g. sigma m

③ Anomaly mat

IR $SU(2)_1$ CFT

$O(3)$ vector sigma model with $\Theta = \pi$.

$$S_{\theta=\pi} = \int d^2x \left[\frac{1}{g} (\partial \hat{n})^2 + \frac{i\pi}{4\pi^2} \hat{n} \cdot (\partial_{x_1} \hat{n} \times \partial_{x_2} \hat{n}) \right]$$

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How does G_{UV} act in IR theory?

Example: Spin-$\frac{1}{2}$ $SU(2)_1$

Many approaches

① Solve $H = J \sum_i$

② Approximately t

e.g. sigma m

③ Anomaly mat

O(3) vector sigma model.

with $\theta = \pi$.

$$S_{\theta=\pi} = \int d^2x \left[\frac{1}{g} (\partial \hat{n})^2 + \frac{i\pi}{4\pi^2} \hat{n} \cdot (\partial_x \hat{n} \times \partial_y \hat{n}) \right]$$

$\hat{n} \in S^2$.

IR symmetry: $O(3): \hat{n} \to \underset{O(3)}{Q} \hat{n}$

How does G_{UV} act in IR theory?

$SO(3): \hat{n} \to \underset{SO(3)}{Q} \hat{n}$

$\mathbb{Z}: \hat{n} \to -\hat{n}$

$X \to X + a \sim X$

$SU(2)_1$

Many approaches

① Solve $H = \sum_i$

② Approximately to

e.g. sigma mo

③ Anomaly matc

IR symmetry: $O(3)$: $\vec{n} \to Q \vec{n}$ ($Q \in O(3)$)

How does G_{UV} act in IR theory?

$SO(3)$: $\vec{n} \to Q \vec{n}$ ($Q \in SO(3)$)

$\mathbb{Z}_2$: $\vec{n} \to -\vec{n}$ $\quad X \to X + a \sim X$

G_{UV} "embeds" into G_{IR}.

$\varphi : G_{UV} \to G_{IR}$

e.g. sigma mo

(3) Anomaly matc

with $\theta = \pi$.

$$S_{\theta=\pi} = \int d^2x \left[\frac{1}{g} (\partial \hat{n})^2 + \frac{i\pi}{4\pi^2} \hat{n} \cdot (\partial_{x_1} \hat{n} \times \partial_{x_2} \hat{n}) \right]$$

$\hat{n} \in S^2$. $SO(3) \times \mathbb{Z}_2$

IR symmetry: $O(3)$: $\hat{n} \to \underset{O(3)}{O} \hat{n}$

How does G_{UV} act in IR theory?

$SO(3)$: $\hat{n} \to \underset{SO(3)}{O} \hat{n}$

$\mathbb{Z}$: $\hat{n} \to -\hat{n}$

$X \to X + a \simeq X$

G_{UV} "embeds" into G_{IR}

$SU(2)_1$

Many approaches

1. Solve $H = \bar{J} \sum_i$

2. Approximately to
   e.g. sigma mo

3. Anomaly matc

$\hat{n} \in S^2$ $\quad SO(3) \times \mathbb{Z}_2$

IR symmetry: $O(3)$: $\hat{n} \to \underset{\uparrow}{O}\,\hat{n}$ $\quad O(3)$

How does G_{UV} act in IR theory?

$SO(3)$: $\hat{n} \to \underset{\uparrow}{O}\,\hat{n}$ $\quad SO(3)$

$\mathbb{Z}^X$: $\hat{n} \to -\hat{n}$ $\qquad X \to X + a \in \mathbb{Z}X$

G_{UV} "embeds" into G_{IR}.

$\varphi : G_{UV} \to G_{IR}$

(2) Approximately to
e.g. sigma mo

(3) Anomaly matc

"Anomaly ⇝ flux carries charge"

From

sigma model.

$\frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_x\hat{n}\times\partial_x\hat{n})$

UV Anomaly

$\mathcal{H}(L)$ vs. $\mathcal{H}(L+1)$



Going around the space (A full cycle) involves T_x^L

Recall: a g-flux

$\hat{n} \to \hat{n}$

$O(3)$

IR theory?

$X \to X+a \sim X$

$\mathcal{H}(L)$ vs. $\mathcal{H}(L+1)$



Going around the space (A full cycle) involves T_x^L

Recall: a g-flux means picking up a g-action when going around the flux.

"Anomaly ⇝ flux carries charge"

UV Anomaly

$\mathcal{H}(L)$ vs. $\mathcal{H}(L+1)$



Going around the space. (A full cycle) involves T_x^L

Recall: a g-flux means picking up a g-action when going around the flux.

⇒ A translation flux carries half-integer spin

From ① →

rigor

Precisely:

UV $\mathcal{H}$

$H = \sum$

Symmetry:

or Sigma model.

$(\partial\hat{n})^2 + \frac{i\pi}{4\pi^2}\hat{n}\cdot(\partial_x\hat{n}\times\partial_x\hat{n})$

$SO(3)\times\mathbb{Z}_2$

$O(3)$: $\hat{n}\to\hat{n}$

$O(3)$

ict in IR theory?

$X\to x+a\sim X$

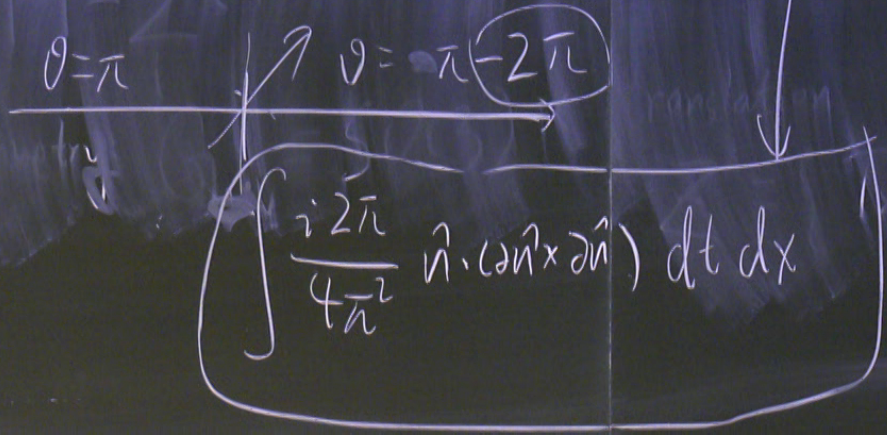
into G_{IR}

IR Anomaly

$Z_x : \hat{n} \to -\hat{n}$.

Translation flux acts as a domain wall, across which $\hat{n} \to -\hat{n}$.

Nothing but a $S = 1/2$ moment on the domain wall.



$$\int \frac{i\,2\pi}{4\pi^2}\, \hat{n}\cdot(\partial\hat{n}\times\partial\hat{n})\, dt\, dx$$