Title: Superconductivity, charge density wave, and supersolidity in flat bands with tunable quantum metric

Speakers:

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Abstract: Predicting the fate of an interacting system in the limit where the electronic bandwidth is quenched is often highly non-trivial. The complex interplay between interactions and quantum fluctuations driven by the band geometry can drive a competition between various ground states, such as charge density wave order and superconductivity. In this work, we study an electronic model of topologically-trivial flat bands with a continuously tunable Fubini-Study metric in the presence of on-site attraction and nearest-neighbor repulsion, using numerically exact quantum Monte Carlo simulations. By varying the electron filling and the spatial extent of the localized flat-band Wannier wavefunctions, we obtain a number of intertwined orders. These include a phase with coexisting charge density wave order and superconductivity, i.e., a supersolid. In spite of the non-perturbative nature of the problem, we identify an analytically tractable limit associated with a `small' spatial extent of the Wannier functions, and derive a low-energy effective Hamiltonian that can well describe our numerical results.

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Superconductivity, charge density wave and supersolidity in flat bands with tunable quantum metric

ArXiv:2204:02994

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Superconductivity in flat bands

ARTICLE

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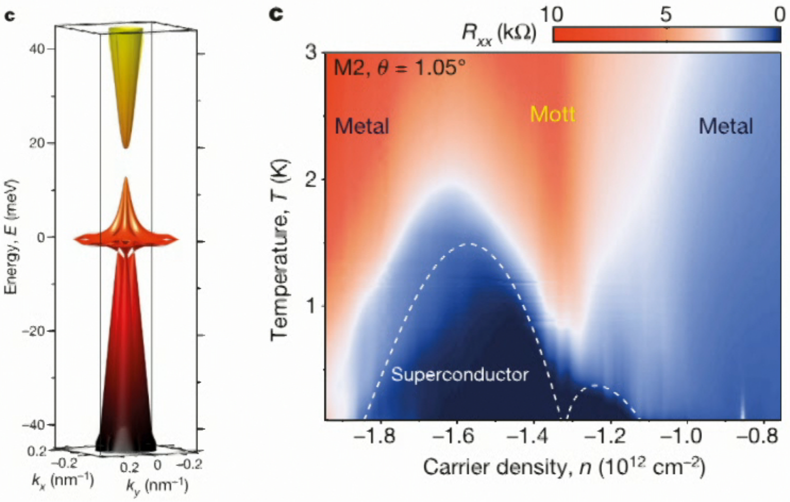
Unconventional superconductivity in magic-angle graphene superlattices

Yuan Cao[1], Valla Fatemi[1], Shiang Fang[2], Kenji Watanabe[3], Takashi Taniguchi[3], Efthimios Kaxiras[2,4] & Pablo Jarillo-Herrero[1]



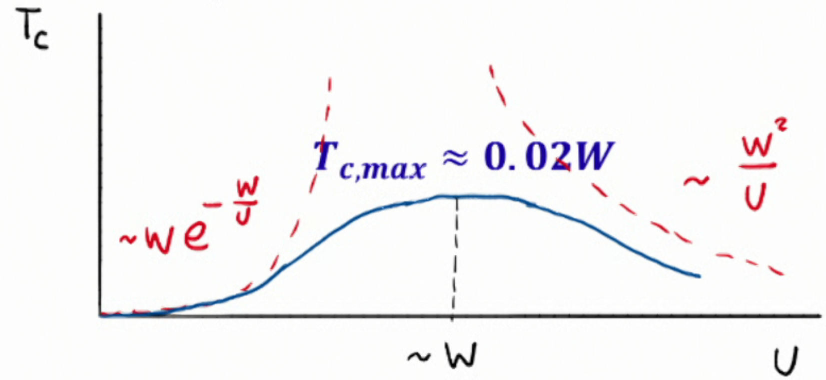
Emery, Kivelson: Nature 1995

Two ingredients:

- pair binding $\sim U$
- phase coherence T_θ

Often: $T_c \to 0$ when $W/U \to 0$ as $T_\theta \to 0$

For example: attractive Hubbard model



Paiva, Scalettar, Randeria and Trivedi: PRL 2010

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Volovik (2011); Peotta, Torma (2015)
Tovmasyan, Huber *et.al.* (2016)
Xie, Bernevig *et.al.* (2019)
Park, Kim, Lee (2020)

Superconductivity in flat bands

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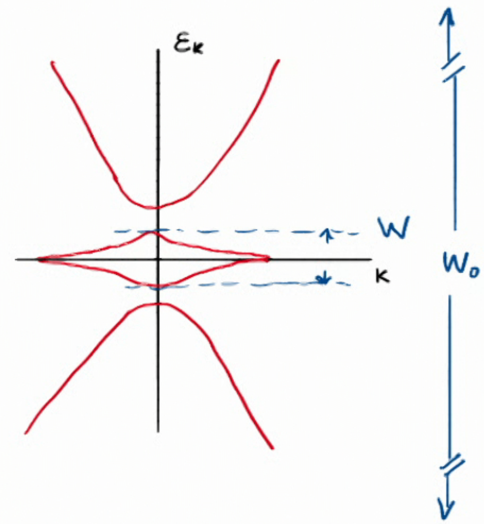
Consider isolated, nearly flat band with W << U << Δ_{gap} :

- **Ground state** as a function of |U|/W $\rightarrow$ ∞ ?
- **Robustness** to infinitesimal perturbations (e.g. nearest-neighbor interactions)?
  - Superconductor, phase separation, CDW, supersolid, …
- What determines **competition** between phases?
  - Mean-field theory is not the way to address these questions



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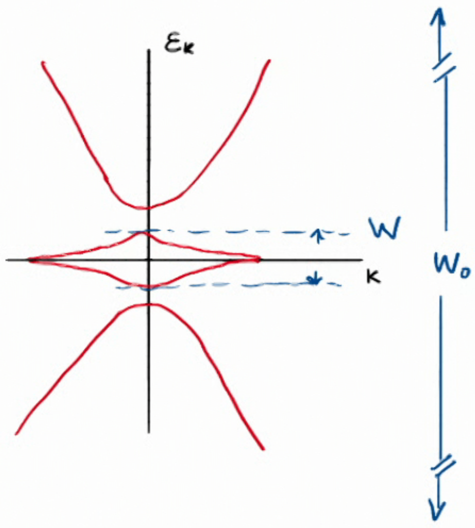

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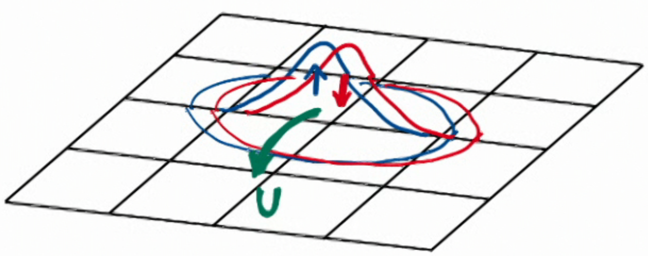
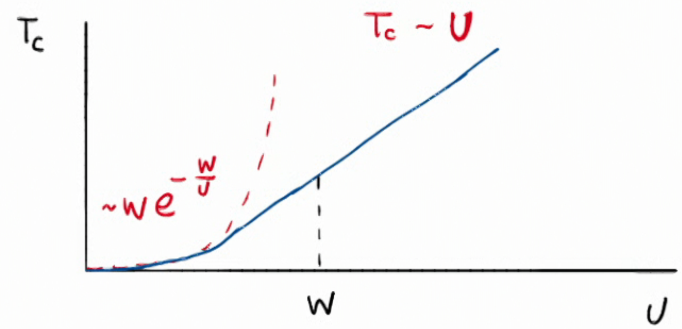
Superconductivity in flat bands

Certain flat bands are “special”:

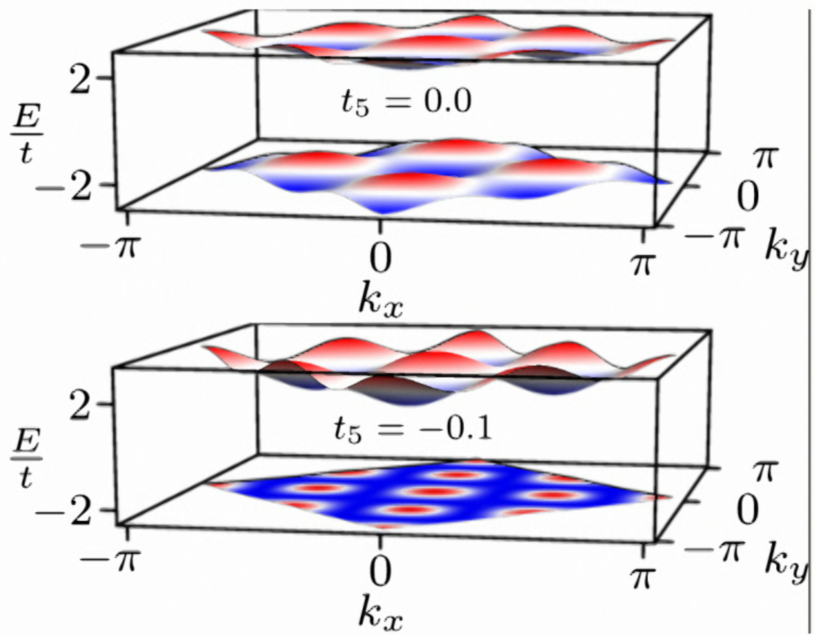
- Possible for local Cooper-pairs to delocalize even in the flat-band limit W $\rightarrow$ 0
  - i.e. $T_c \propto |U|$, instead of the standard $T_c \propto W^2/|U|$





Topological flat bands

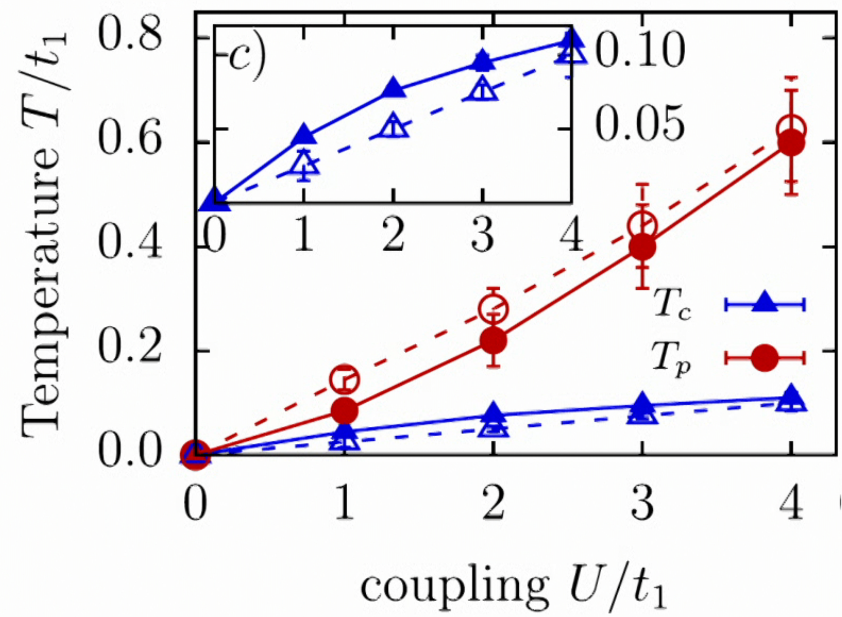
$C = \pm 1$ Chern bands



Flatness ratio: $\mathcal{F} = \dfrac{W}{\Delta_{\text{gap}}} (= 0.2, 0.01)$

$$H = \sum_{i,j,\sigma} t^{\sigma}_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} - U \sum_i n_{i\uparrow} n_{i\downarrow} \; [\text{TRS} : (t^{\uparrow}_{ij})^* = t^{\downarrow}_{ij}]$$

Non-perturbative DQMC results:



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Hofmann, Berg, Chowdhury (2020)

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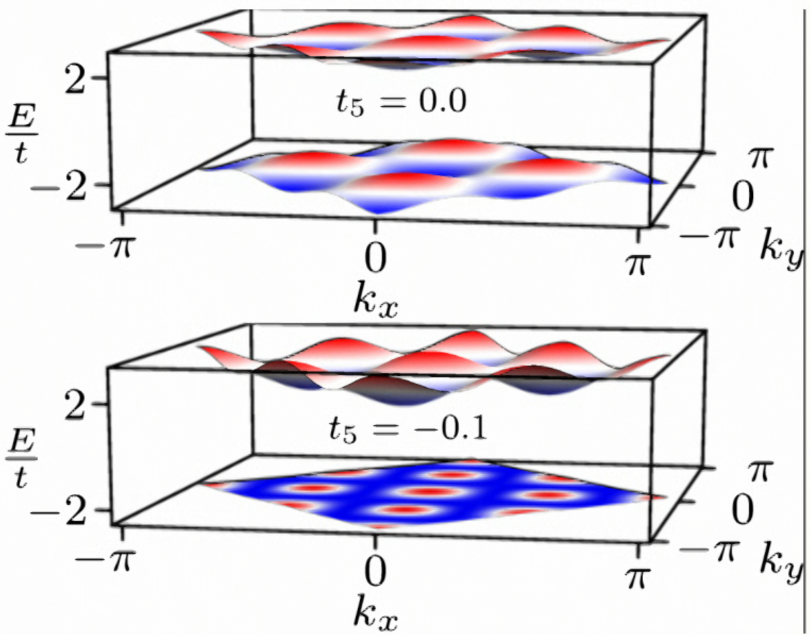

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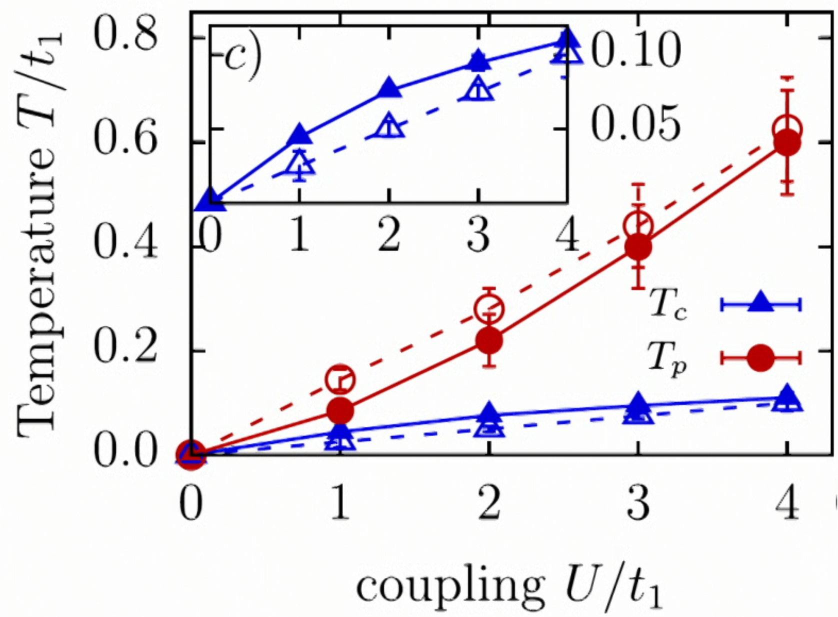
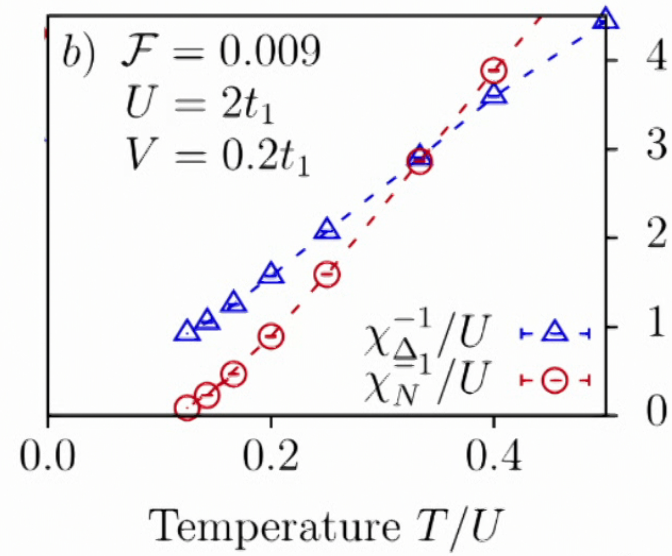


Lessons

- An incoherent metallic normal state **can** give rise to a superconducting ground state.
  (Unlike any "mean-field BCS" transition)
  - Strongly fluctuating density and pairing correlations
  - Approximate SU(2) invariant limit can be a useful starting point
    Tovmasyan, Huber *et.al.* (2016)
- **BUT:** weak nearest neighbor attraction can tip the balance in favor of phase separation!

$$H_V = -V \sum_{\langle i,j \rangle} n_i n_j$$



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Key questions

- What is the nature of the *competing phases* and the associated quantum phase-transitions?
- How does varying the *spatial extent of the localized Wannier functions* tune the system between different orders?
- How do the *interactions induce a finite dispersion* for the excitations?
- Is there a *theoretical limit* in which this band competition can be explored in a controlled fashion, without resorting to BCS mean-field theory?

Hofmann, Berg, Chowdhury; ArXiv:2204:02994

Superconductivity in flat bands with tunable metric

$$\hat{H} = \hat{H}_{kin}(\zeta) + \hat{H}_{int}$$

- $\hat{H}_{kin}(\zeta)$ engineered to yield isolated, flat bands with vanishing Berry curvature
  - ζ controls the spatial extent of the Wannier function
  - $\zeta = 0$ corresponds to a decoupled "atomic limit"
- $$\hat{H}_{int} = -\frac{U}{2}\sum_{i,l}\left(\sum_{\sigma}\hat{n}_{i,l,\sigma} - \frac{1}{2}\right)^2 + V\sum_{\langle ij\rangle,l}\left(\sum_{\sigma}\hat{n}_{i,l,\sigma} - \frac{1}{2}\right)\left(\sum_{\sigma}\hat{n}_{j,l,\sigma} - \frac{1}{2}\right)$$
- $$\hat{H}_{kin}(\zeta) = t\sum_{\mathbf{k},\sigma}\hat{c}^{\dagger}_{\mathbf{k},\sigma}\left(\sin\alpha_{\mathbf{k}}\tau_x + (-1)^{\sigma}\cos\alpha_{\mathbf{k}}\tau_y - \mu\tau_0\right)\hat{c}_{\mathbf{k},\sigma}$$
  $$\alpha_{\mathbf{k}} = \zeta(\cos k_x + \cos k_y)$$

Hofmann, Berg, Chowdhury; ArXiv:2204:02994

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Effective theory

- For small ζ, we can derive the following effective model:

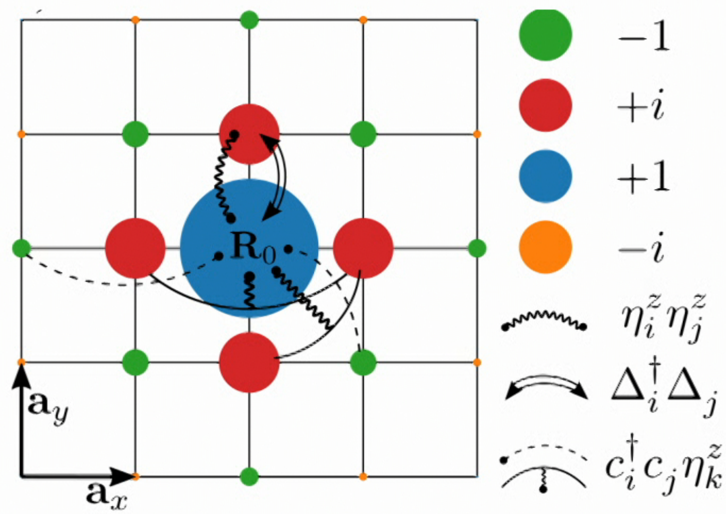
$$\hat{H}_{\rm int} = -\frac{U_{\rm eff}}{2}\sum_{\mathbf{r}}(2\hat{\eta}^z_{\mathbf{r}})^2 - \sum_{\langle \mathbf{r},\mathbf{r}'\rangle}[J_\perp(\hat{\eta}^x_{\mathbf{r}}\hat{\eta}^x_{\mathbf{r}'} + \hat{\eta}^y_{\mathbf{r}}\hat{\eta}^y_{\mathbf{r}'}) + J_z\hat{\eta}^z_{\mathbf{r}}\hat{\eta}^z_{\mathbf{r}'}] + \hat{H}_{\rm hop}$$

$$\hat{\eta}^\alpha_{\mathbf{r}} \equiv (\Psi^\dagger_{\mathbf{r}}\eta^\alpha\Psi_{\mathbf{r}})/2 \qquad \Psi^\dagger_{\mathbf{r}} = (\hat{d}^\dagger_{\mathbf{r},\uparrow}, \hat{d}_{\mathbf{r},\downarrow})$$

$$J_\perp = \zeta^2 U/4 + \ldots, \quad J_z = \zeta^2 U/4 - 2V + \ldots$$

$$U_{\rm eff} = U(2-\zeta^2)/4$$

$$\tilde{H}_{\rm hop} \propto U\zeta^2 t_{ij}(k) c^\dagger_i c_j \hat{\eta}^z_k \equiv$$ Density-assisted hopping



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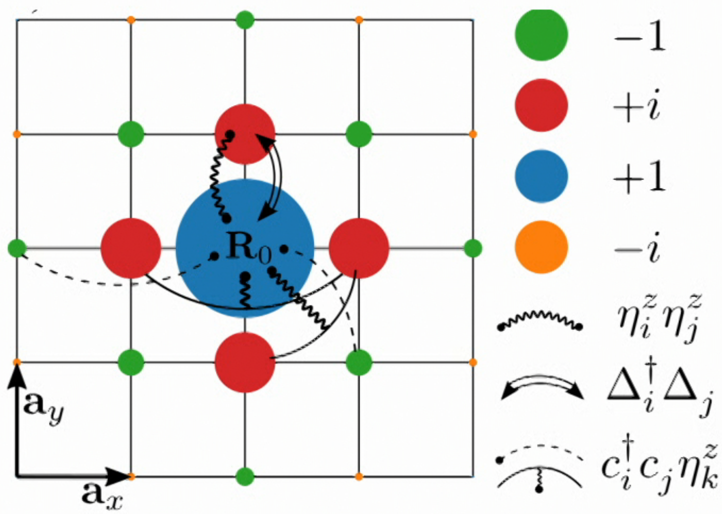

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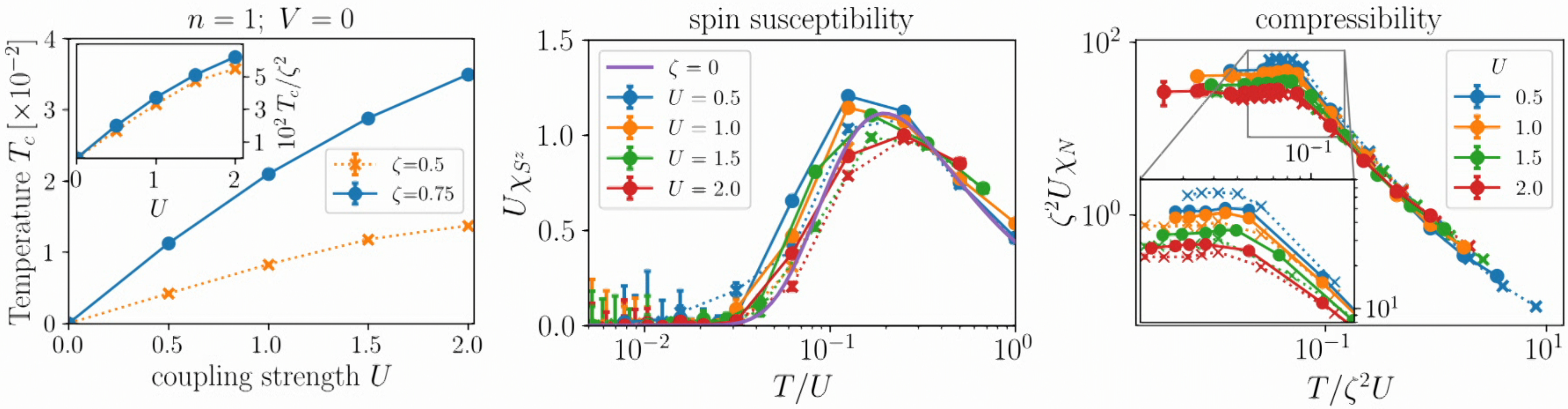

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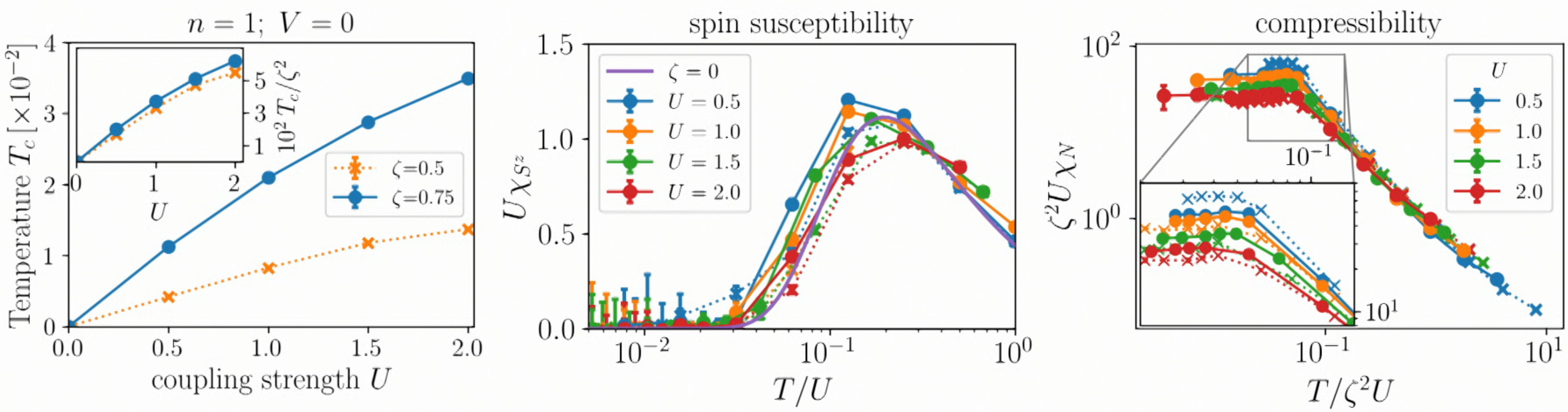

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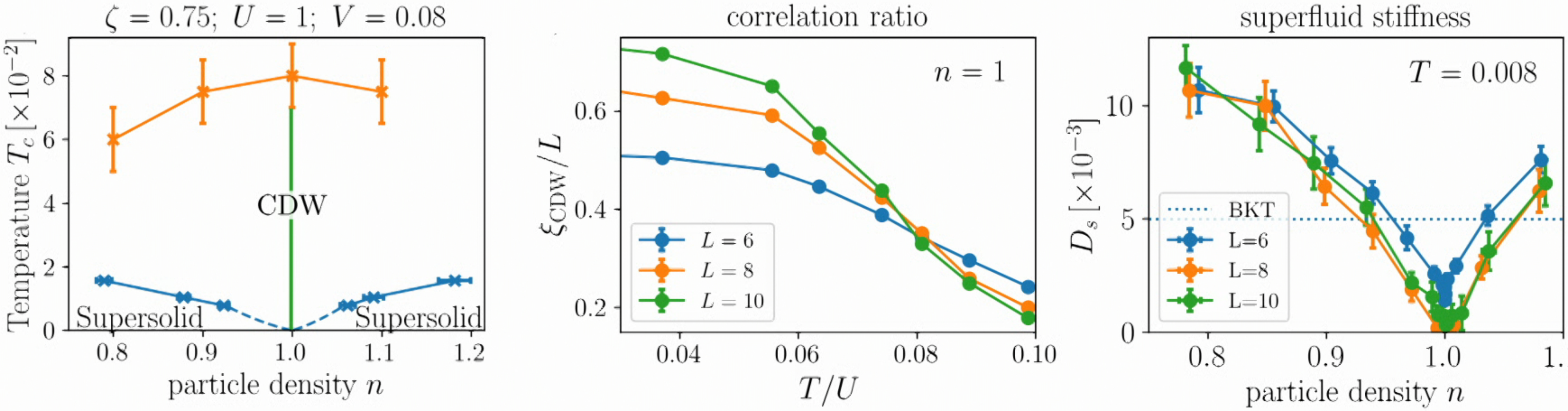
Supersolid at incommensurate filling

Non-perturbative DQMC results (V=0.08):



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Excitation spectra & supersolid at quarter filling

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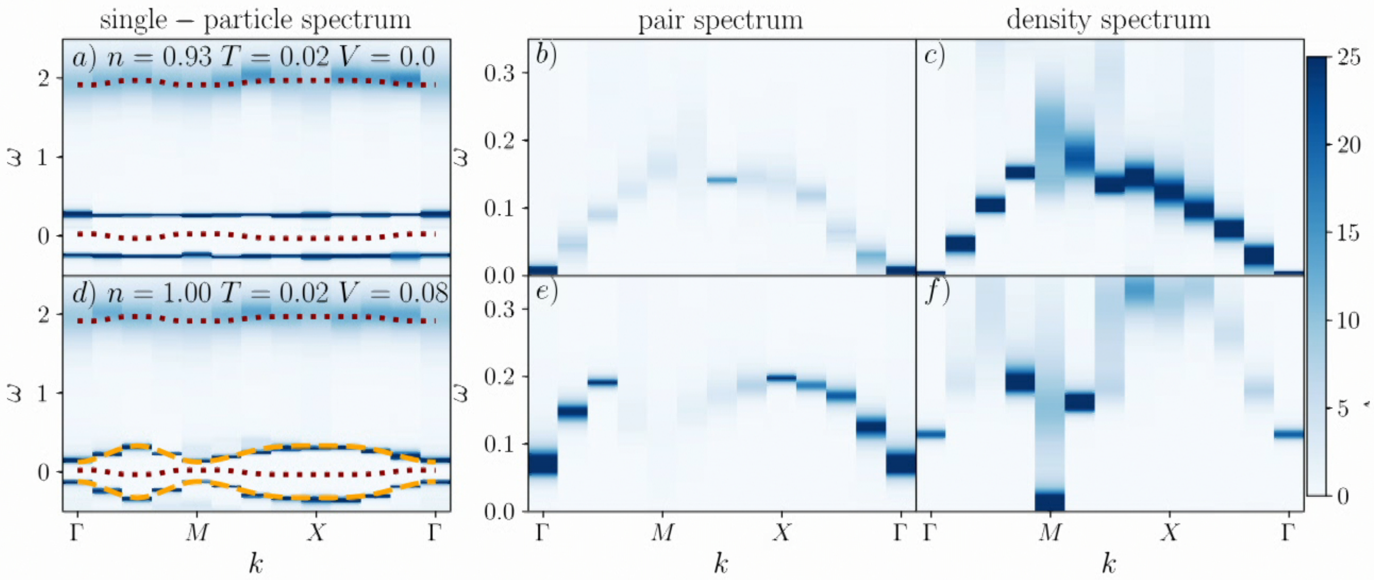

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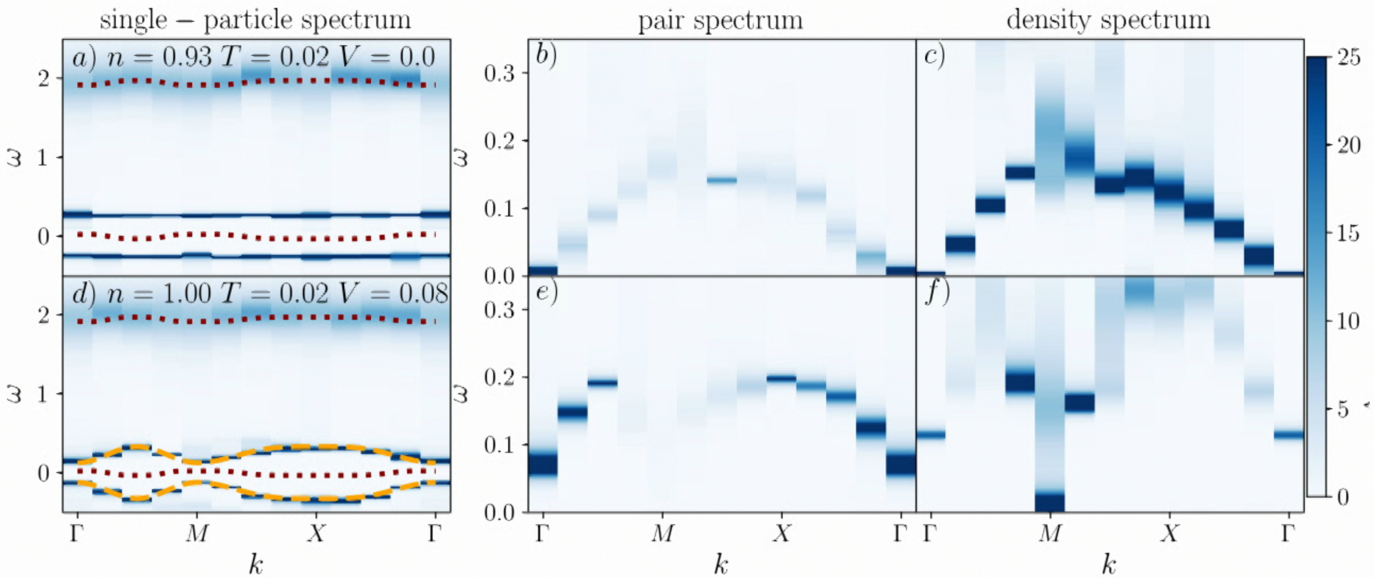

$$\tilde{\Delta} - \frac{U\zeta^2\Delta_{\mathrm{CDW}}}{4}\left(2\sum_{a=\pm}\cos k_a + \sum_{a=x,y}\cos 2k_a\right)$$

Hofmann, Berg, Chowdhury; ArXiv:2204:02994

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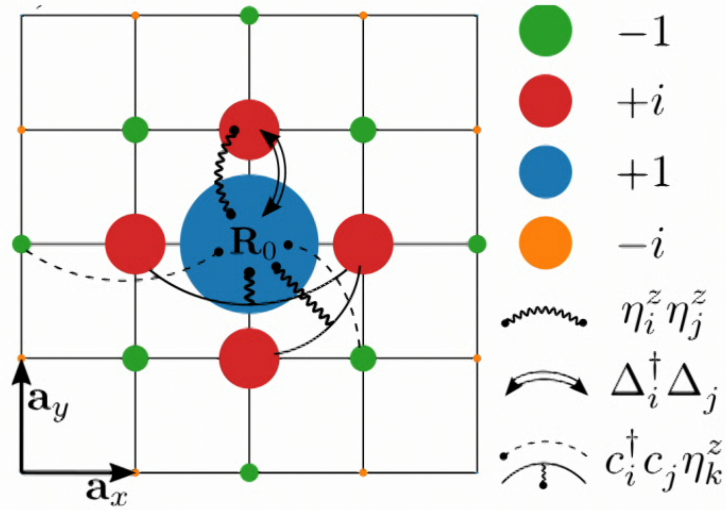

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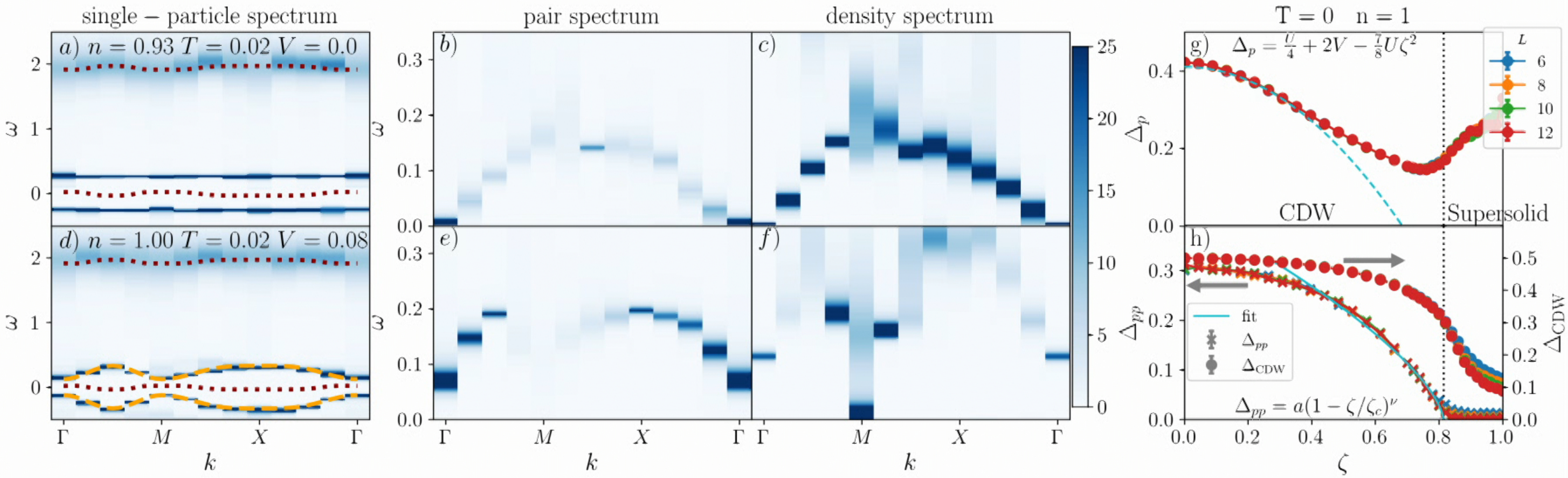

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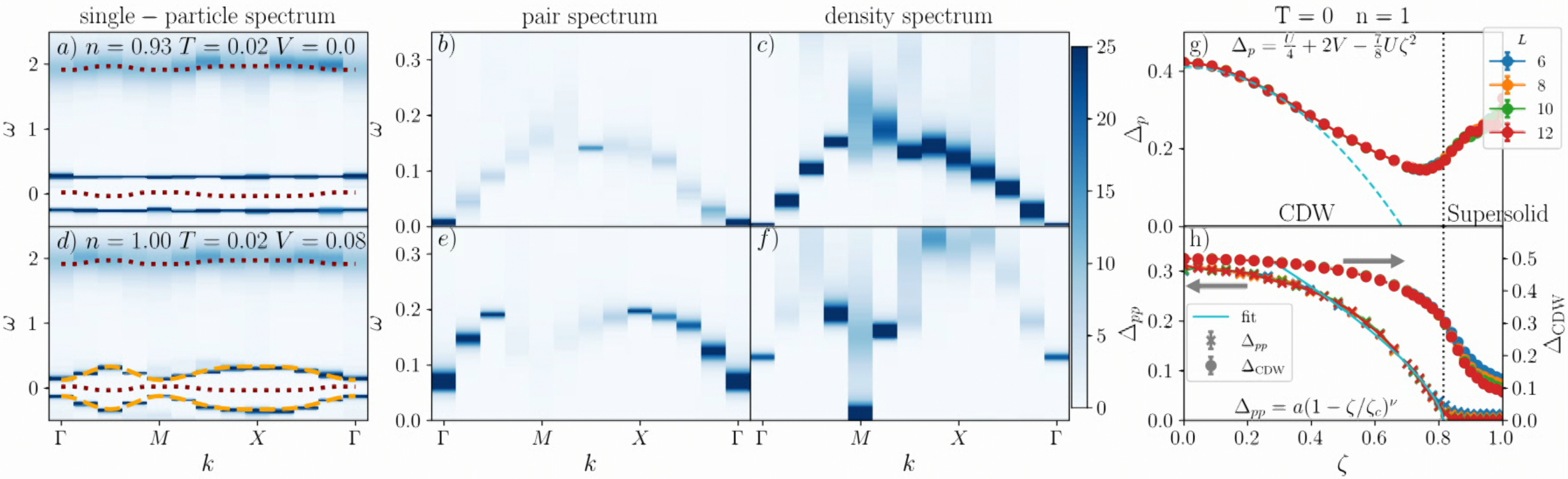

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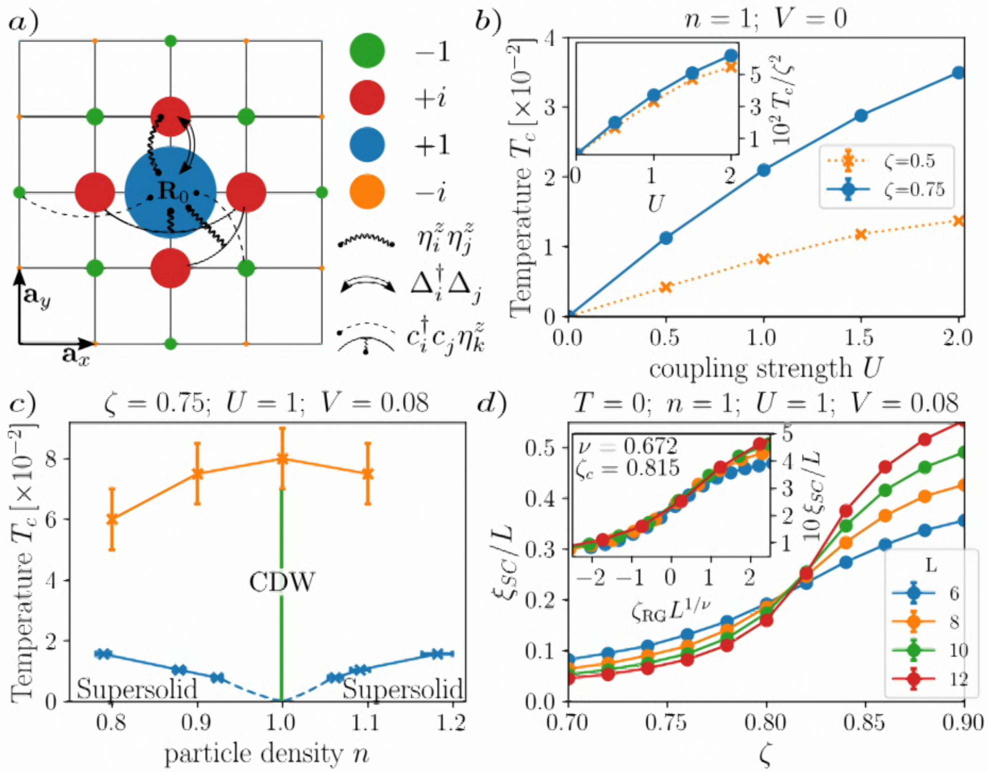
Conclusion

Extended Wannier orbitals → XXZ model + assisted hopping

Charge density wave; supersolid ground state at finite doping



Superconductivity

Density-assisted hopping → Supersolid at quarter filling

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