Title: Probing sign structure using measurement-induced entanglement

Speakers: Timothy Hsieh

Collection: Quantum Criticality: Gauge Fields and Matter

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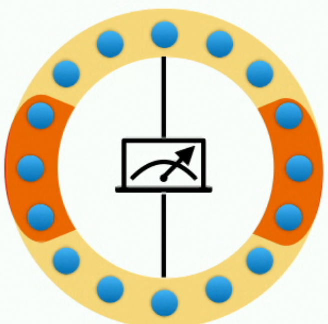
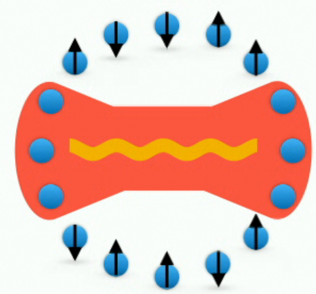
Abstract: The sign structure of quantum states is closely connected to quantum phases of matter, yet detecting such fine-grained properties of amplitudes is subtle. We employ as a diagnostic measurement-induced entanglement (MIE)-- the average entanglement generated between two parties after measuring the rest of the system. We propose that for a sign-free state, the MIE upon measuring in the sign-free basis decays no slower than correlations in the state before measurement. Concretely, we prove that MIE is upper bounded by mutual information for sign-free stabilizer states (essentially CSS codes), which establishes a bound between scaling dimensions of conformal field theories describing measurement-induced critical points in stabilizer systems. We also show that for sign-free qubit wavefunctions, MIE between two qubits is upper bounded by a simple two-point correlation function, and we verify our proposal in several critical ground states of one-dimensional systems, including the transverse field and tri-critical Ising models. In contrast, for states with sign structure, such bounds can be violated, as we illustrate in critical hybrid circuits involving both Haar or Clifford random unitaries and measurements, and gapless symmetry-protected topological states.

Probing Sign Structure with Measurement Induced Entanglement

Tim Hsieh

PERIMETER INSTITUTE





Quantum Criticality Workshop, Perimeter Institute
5/18/22

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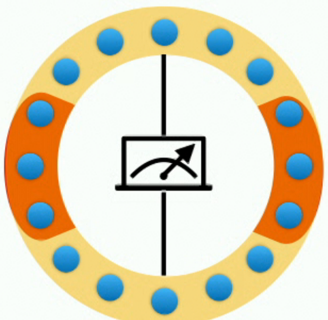

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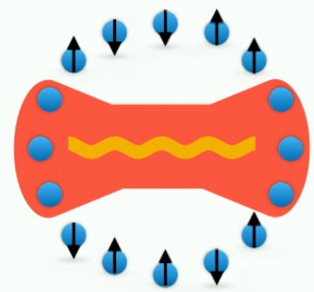

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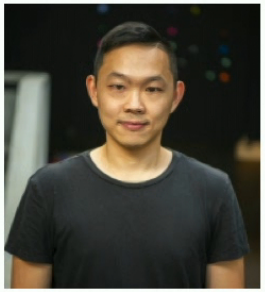
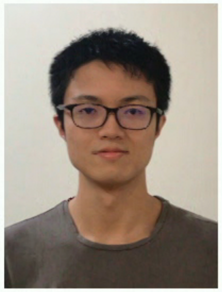
Collaborators



Cheng-Ju (Jacob) Lin
(Perimeter)

Weicheng Ye
(Perimeter)

Yijian Zou
(Stanford)

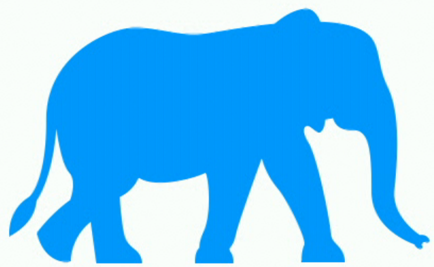


Shengqi Sang
(Perimeter)

arXiv: 2205.05692

Motivation

Sign structure: computational barrier for Monte Carlo approaches



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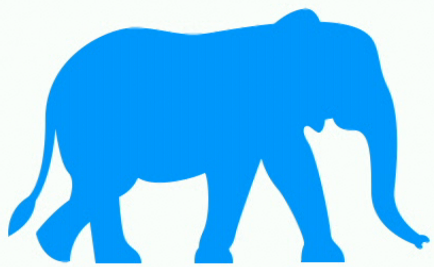

How physical is sign structure?

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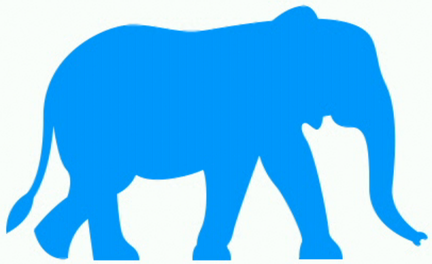
Sign structure: computational barrier for Monte Carlo approaches



How physical is sign structure?

How to probe sign structure? What are observable manifestations?

Sign Structure

Sign Structure

Monte Carlo needs local basis in which
Hamiltonian off-diagonal matrix elements are real and non-positive

$\langle x | H | y \rangle \leq 0$ $\qquad$ $\langle x | e^{-\beta H} | y \rangle \geq 0$

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Quantum phases of matter with “intrinsic” sign problem:

Every commuting projector Hamiltonian w/ double semion topological order has sign problem

Hastings (2015)

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Sign structure at level of ground state implies sign problem for parent Hamiltonian

How to detect sign structure?

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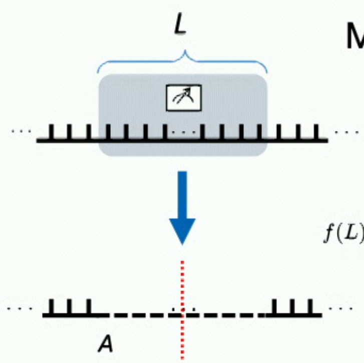
1d non-negative state with zero correlation length:



Measure in reference basis

$$S(\rho_A) \leq f(L)$$

$f(L)$ decays rapidly to zero

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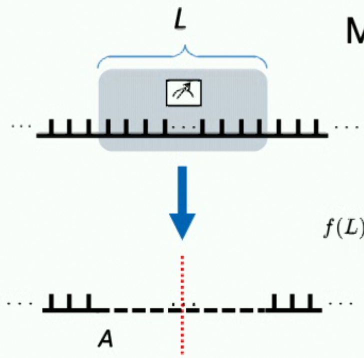

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Rough picture:

$$|\psi\rangle \approx \sum_s e^{-H_{cl}(s)} |s\rangle$$

where H_{cl}
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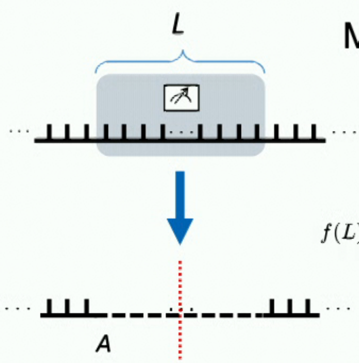

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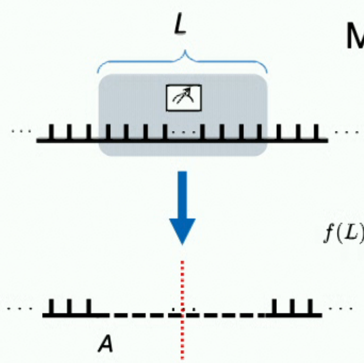

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"measurement-induced entanglement" (MIE)

See also "localizable entanglement"

Verstraete, Popp, Cirac (2004)

This talk

How to detect sign structure more generally?

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Central proposal: for a sign-free state, entanglement induced by measuring in sign-free basis must decay at least as fast as correlations before measurement

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Emphasis on quantum critical states— sign structure can constrain critical data

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For sign-free stabilizer state, $\mathrm{MIE}(A{:}B) \leq \mathrm{MI}(A{:}B)$

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For sign-free stabilizer state, MIE(A:B) $\leq$ MI(A:B)

Constrains measurement-driven criticality: $h^{(1)}_{f|f} \geq h^{(1)}_{a|a}$

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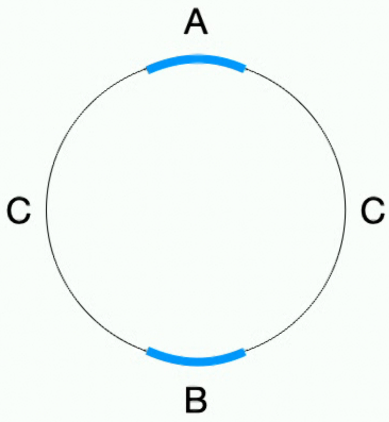
Constrains measurement-driven criticality: $h^{(1)}_{f|f} \geq h^{(1)}_{a|a}$

For sign-free spin state, MIE(A:B) $\leq \langle X_A X_B \rangle$

Measurement-induced Entanglement (MIE)



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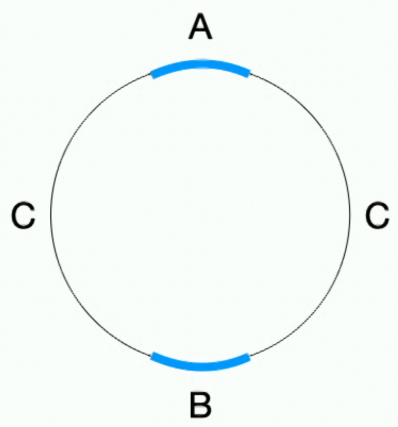
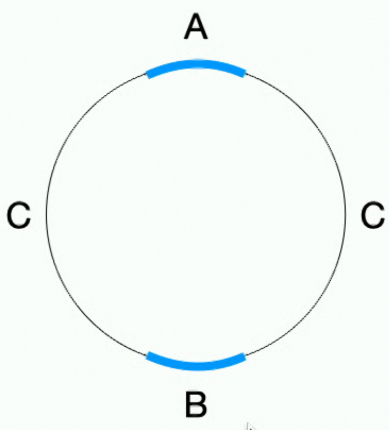


$$\text{MIE(A:B)} = \sum_c p_c S_A(\psi_{AB,c})$$

Probability of outcome c when measuring C

Resulting state on AB after getting outcome c

Measurement-induced Entanglement (MIE)



von Neumann entanglement entropy between A and B

$$\text{MIE(A:B)} = \sum_c p_c S_A(\psi_{AB,c})$$

Probability of outcome c when measuring C

Resulting state on AB after getting outcome c

Will compare with correlation functions or mutual information MI(A:B) = $S_A + S_B - S_{AB} \geq \frac{\mathcal{C}(M_A, M_B)^2}{2||M_A||^2||M_B||^2}$

Wolf, Hastings, Verstraete, Cirac (2008)

Stabilizer States

Pauli strings: $\mathcal{P} = \cdots \otimes I \otimes X \otimes X \otimes I \otimes Z \otimes \cdots$

Stabilizer states: Eigenstates of commuting Pauli strings

Example: Toric code ground states (stabilizers are star and plaquette operators)

Sign-Free Stabilizer States

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Property 1: Can choose all stabilizers to consist of only X operators or Z operators ("CSS code")

e.g. toric code

Property 2: MIE(A:B) $\leq$ MI(A:B)

(measuring C in Z or X basis)

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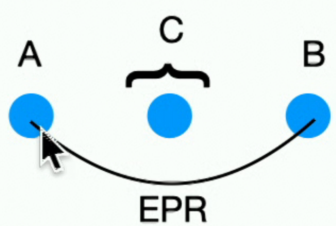

MI = 2
MIE = 1

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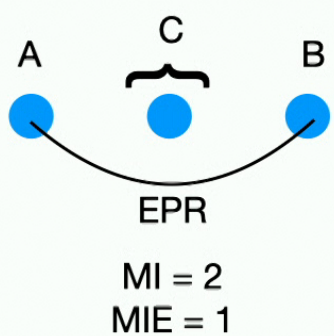
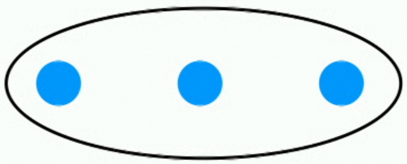


GHZ: stabilized by $Z_A Z_C, Z_C Z_B, X_A X_C X_B$



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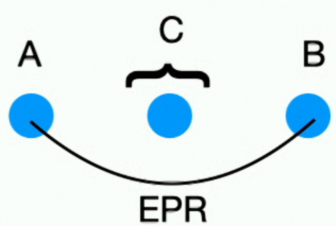

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MI = 1

After measurement: $Z_A Z_B, X_A X_B$

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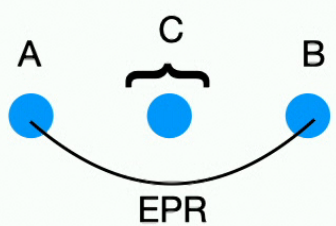

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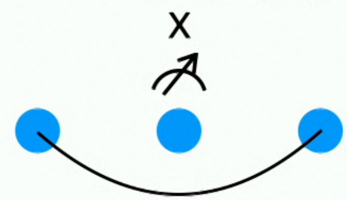
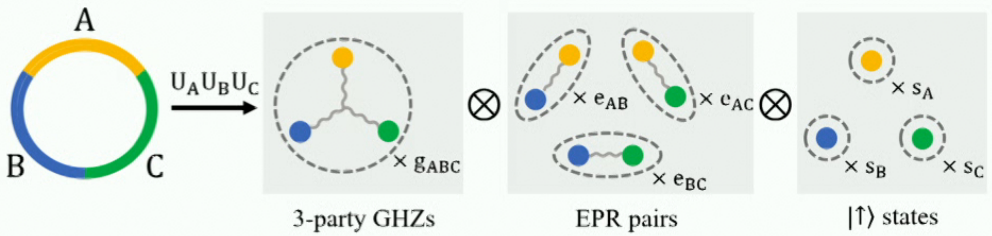


(If measure in Z basis, MIE = 0)

MI = 1
MIE = 1

After measurement: $Z_A Z_B, X_A X_B$

Sketch of Proof



Structure theorem for stabilizers   Bravyi, Fattal, Gottesman (2005)

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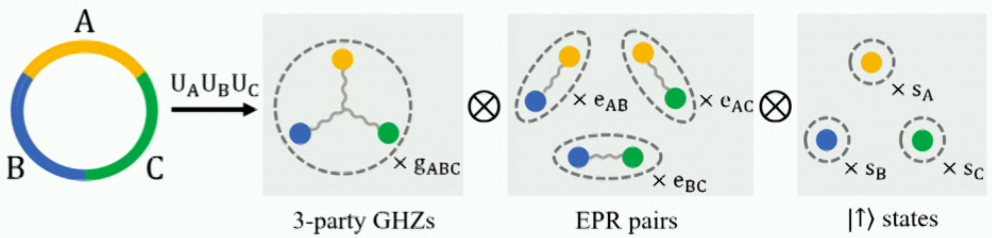

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For a sign-free stabilizer state, local unitaries can be chosen to be sign-free preserving, thus commuting with projective Z-measurements on C

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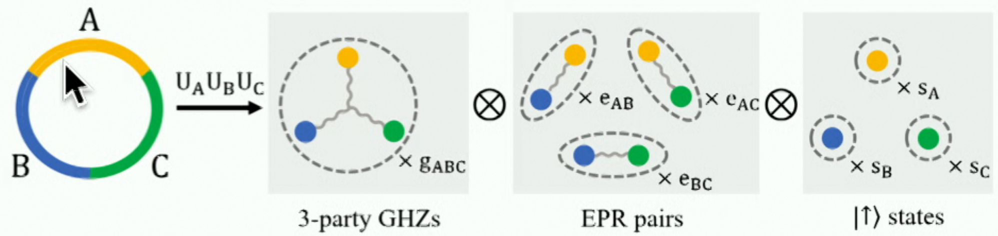

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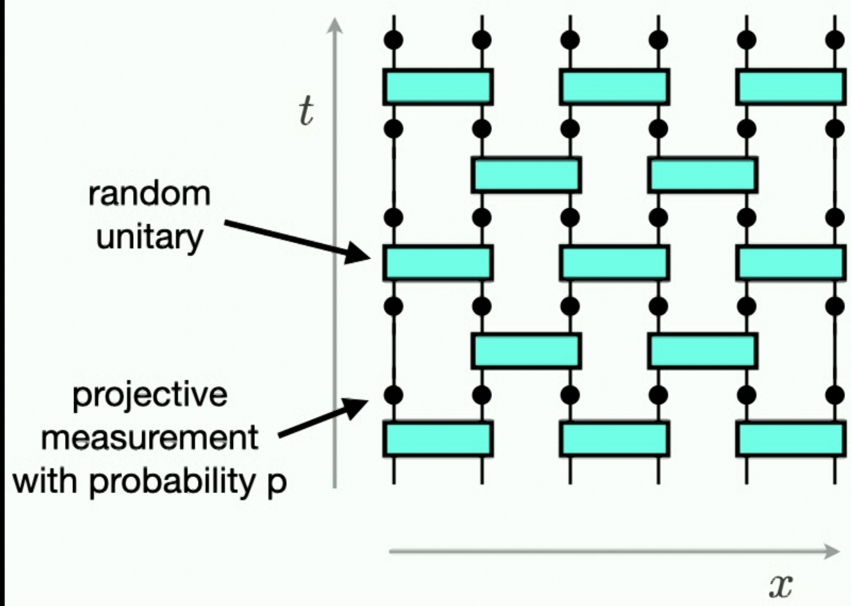
Evaluation of MIE and MI on GHZ, EPR reveals MIE(A:B) $\leq$ MI(A:B)

Hybrid Quantum Circuits



Li, Chen, Fisher (2018)
Skinner, Ruhman, Nahum (2019)
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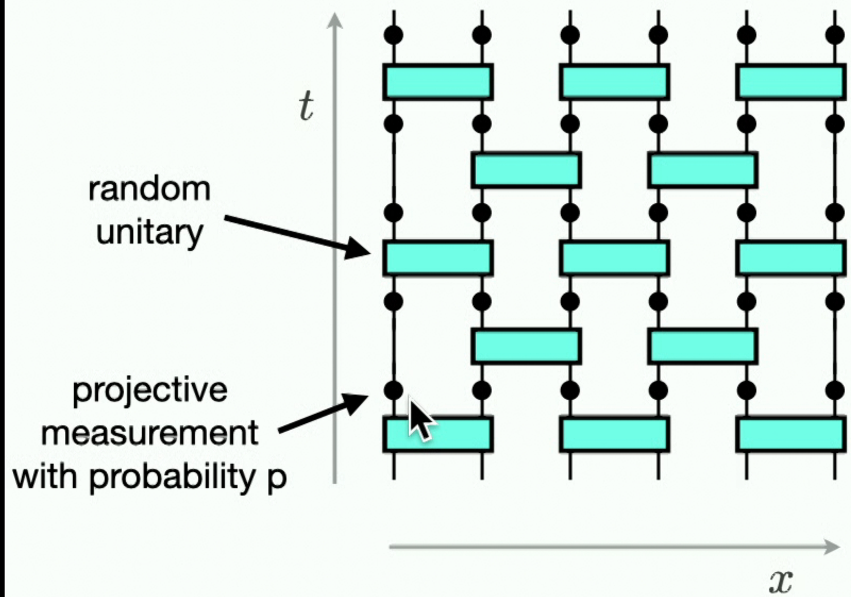
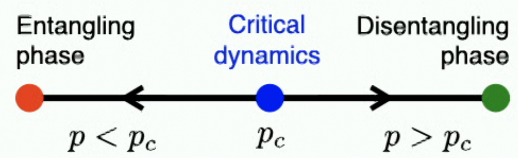
Hybrid Quantum Circuits





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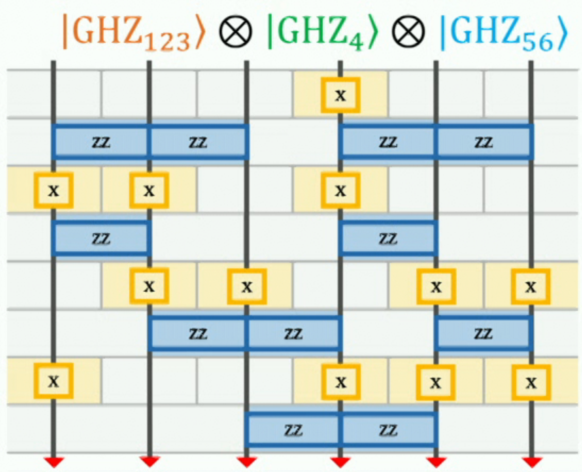
Measurement-only critical point



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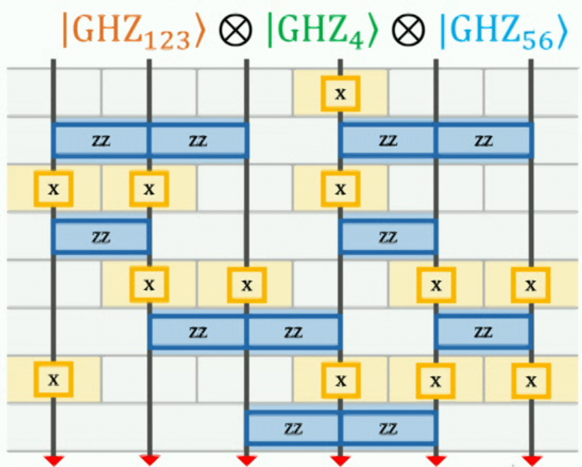

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Sign-free and direct product of GHZ states at all times

MI(A:B) nonzero if A,B belong to same GHZ cluster

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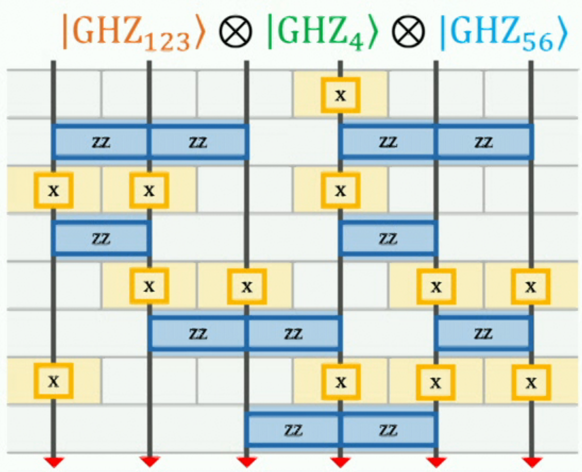

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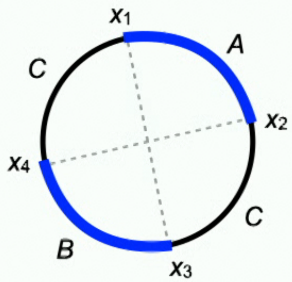
Sign-free and direct product of GHZ states at all times

MI(A:B) nonzero if A,B belong to same GHZ cluster

Also the condition for $MIE_X(A:B)$ to be nonzero

ZZ vs. X measurement-only critical point



Li, Chen, Fisher (2019)

Cross ratio $\eta = \dfrac{x_{12}x_{34}}{x_{13}x_{24}}$

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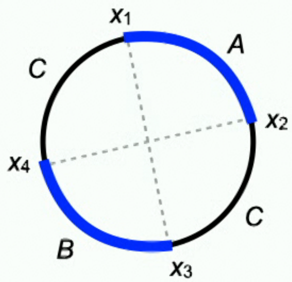

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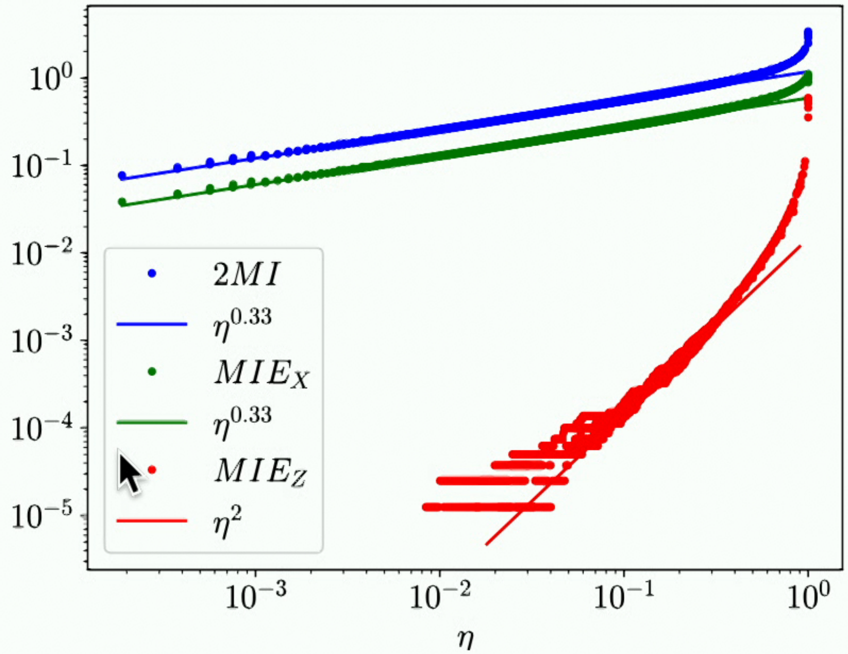

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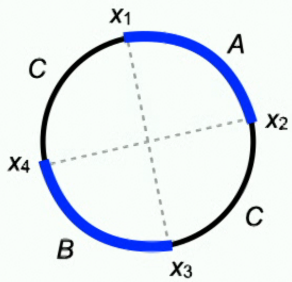

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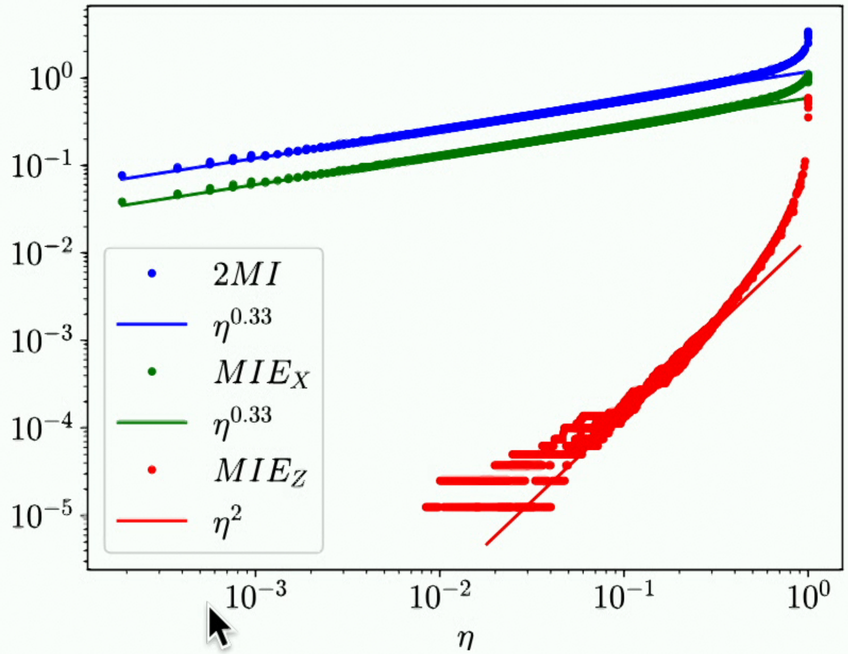

MIE(A:B) $\leq$ MI(A:B)

Sign Structure Constrains Scaling Dimensions

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MIE power law given by $h^{(1)}_{f|f}$

MI power law given by $h^{(1)}_{a|a}$

Li, Chen, Ludwig, Fisher (2020)

Sign Structure Constrains Scaling Dimensions

MIE power law given by $h^{(1)}_{f|f}$

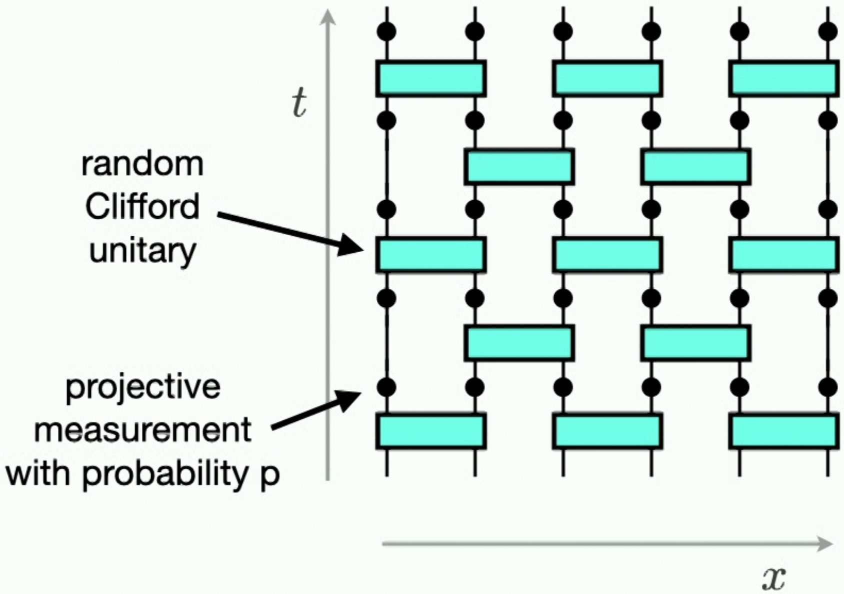
MI power law given by $h^{(1)}_{a|a}$

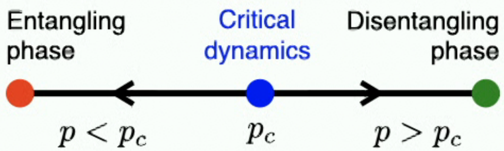
Lowest scaling dimensions of certain boundary condition changing operators

Li, Chen, Ludwig, Fisher (2020)

MIE(A:B) $\leq$ MI(A:B) implies $h^{(1)}_{f|f} \geq h^{(1)}_{a|a}$ for sign-free stabilizer critical point

Hybrid Quantum Circuits





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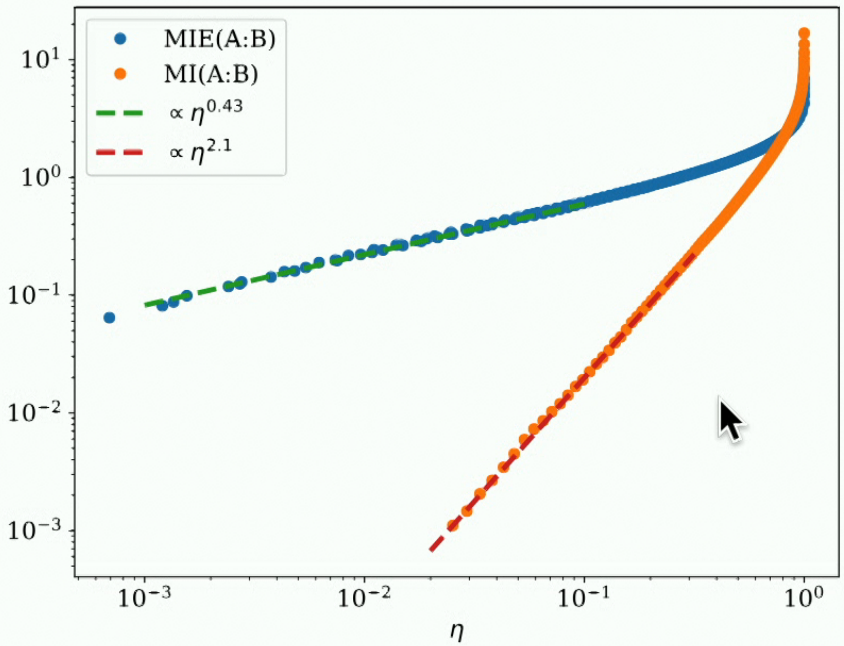
Clifford hybrid circuit transition



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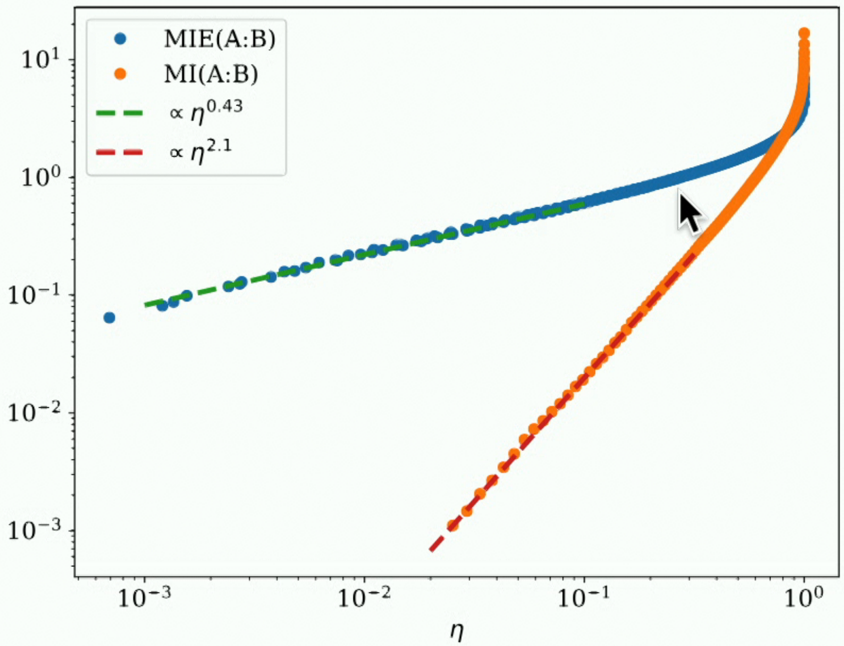

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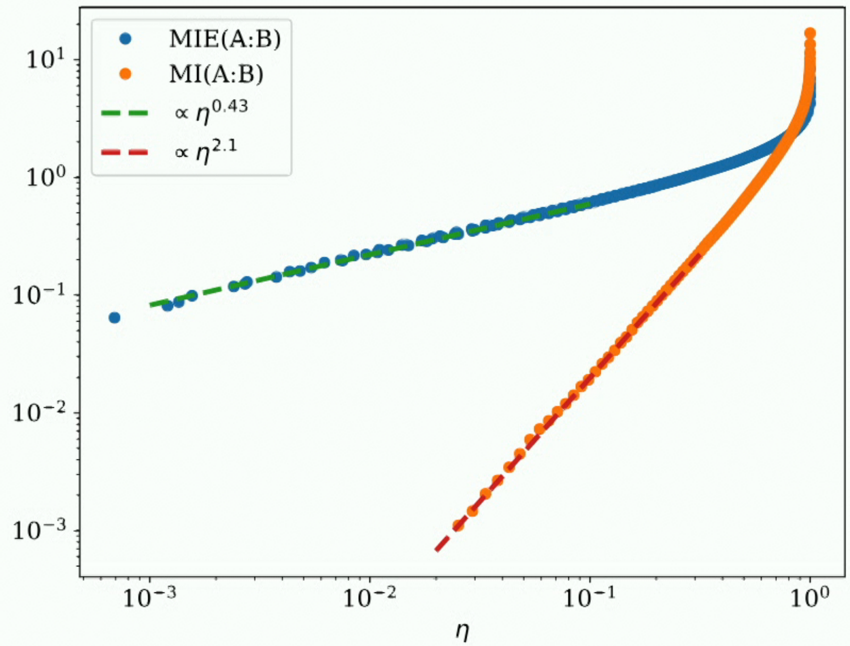

Intrinsic sign problem?

$h^{(1)}_{f|f} \geq h^{(1)}_{a|a}$ (for sign-free) is violated here

Spin (Qubit) Systems

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Let $|\psi\rangle$ be a non-negative wavefunction in the computational basis, and let A and B be single qubits. Then the measurement-induced entanglement, upon measuring C in computational basis, is upper bounded by the correlation function $\langle\psi|X_A X_B|\psi\rangle$.

Spin (Qubit) Systems

Let $|\psi\rangle$ be a non-negative wavefunction in the computational basis, and let A and B be single qubits. Then the measurement-induced entanglement, upon measuring C in computational basis, is upper bounded by the correlation function $\langle\psi|X_A X_B|\psi\rangle$.

(Inspired by an observation in Popp, Verstraete, Cirac (2004))

Sketch of Proof

$$|\psi\rangle = \sum_{abc} \psi_{abc} |abc\rangle \qquad \psi_{abc} \geq 0$$

Measurement-induced concurrence is

$$MIC = \sum_{c} p_c |\langle \psi^*_{AB,c} | Y_A Y_B | \psi_{AB,c}\rangle |$$

$$= \sum_{c} | \sum_{ab,a'b'} \psi_{abc}\psi_{a'b'c} \langle a'b' | Y_A Y_B | ab\rangle |$$

$$\leq \sum_{c} \sum_{ab,a'b'} \psi_{abc}\psi_{a'b'c} \langle a'b' | X_A X_B | ab\rangle$$

$$= \langle \psi | X_A X_B | \psi\rangle$$

Given Ising symmetry,
strict equality
(observed in Popp, Verstraete, Cirac (2004))

$$MIE \leq MIC \leq \langle \psi | X_A X_B | \psi\rangle$$

MIE of Critical 1d Transverse Field Ising Model

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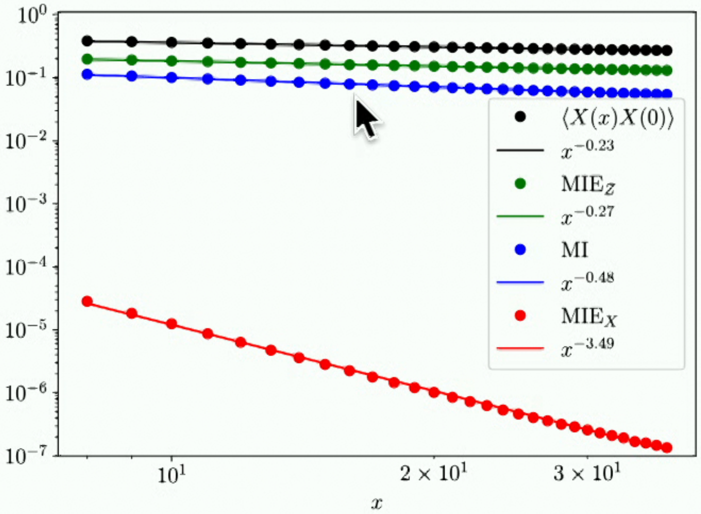
$$H = -\sum XX - \sum Z$$



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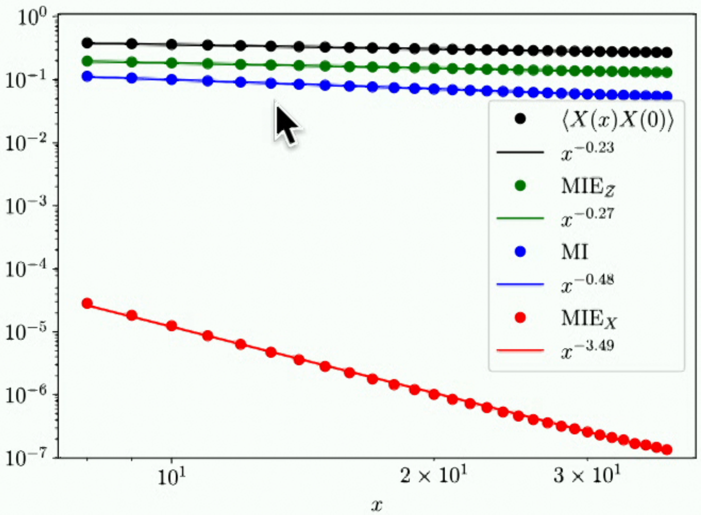

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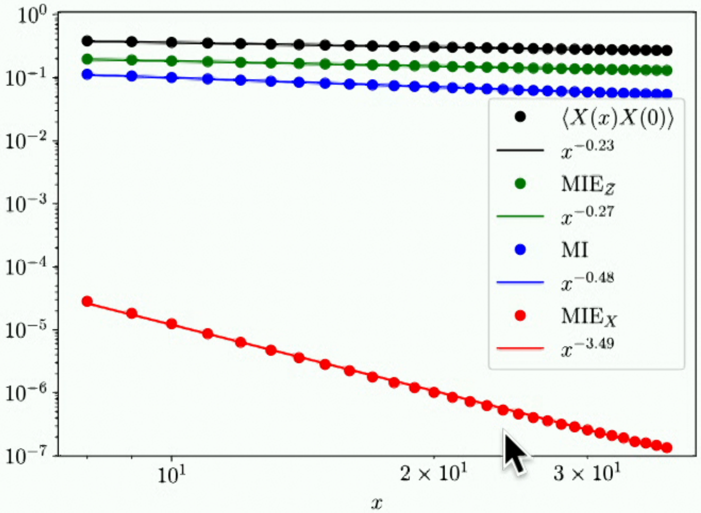

MIE decays with same power as slowest correlator $\langle XX\rangle$

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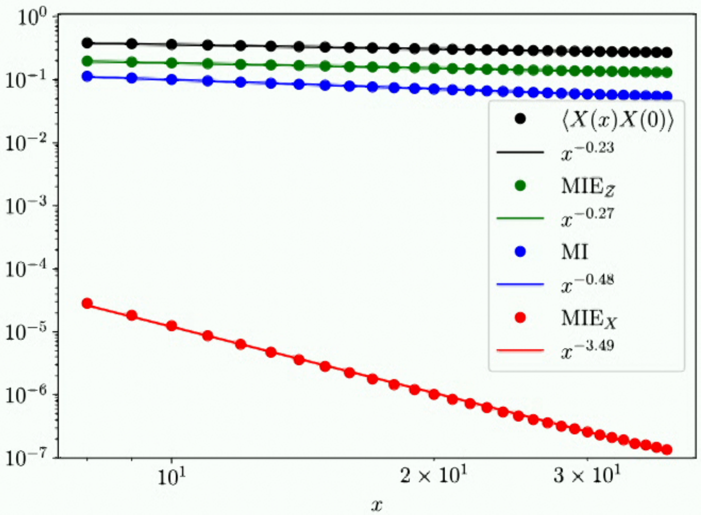

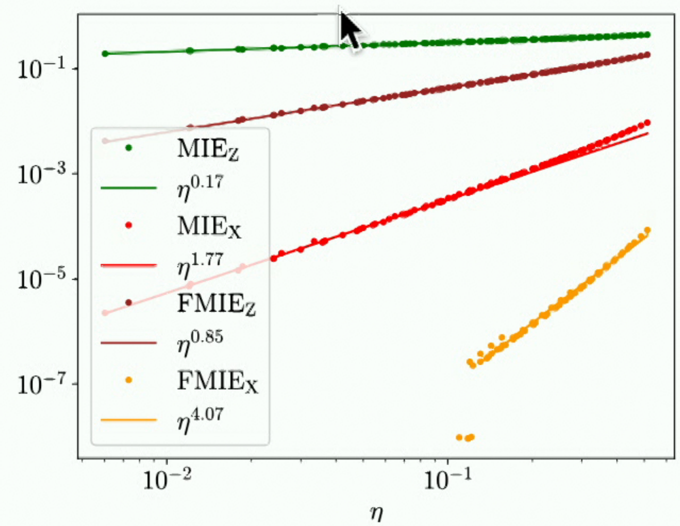

(cross ratio)

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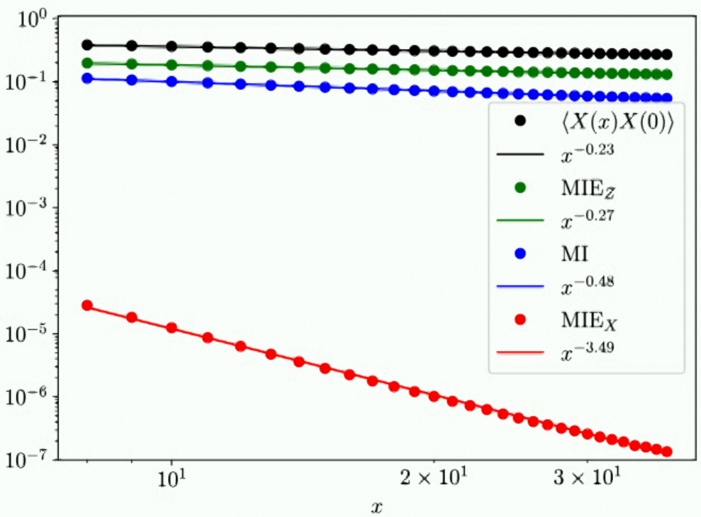

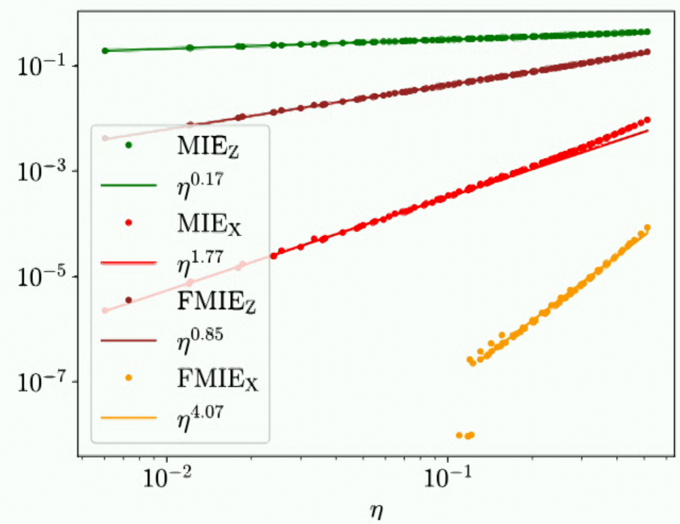

(cross ratio)

MIE decays with same power as slowest correlator $\langle XX \rangle$

MIE a function of cross-ratio only; which conformal boundary condition?

MIE of Critical Ground State with Sign Structure

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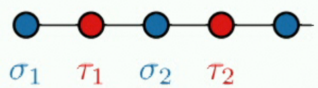
"Gapless symmetry-protected topological (SPT) state"

Scaffidi, Parker, Vasseur (2018)
Verresen, Thorngren, Jones, Pollmann (2019)

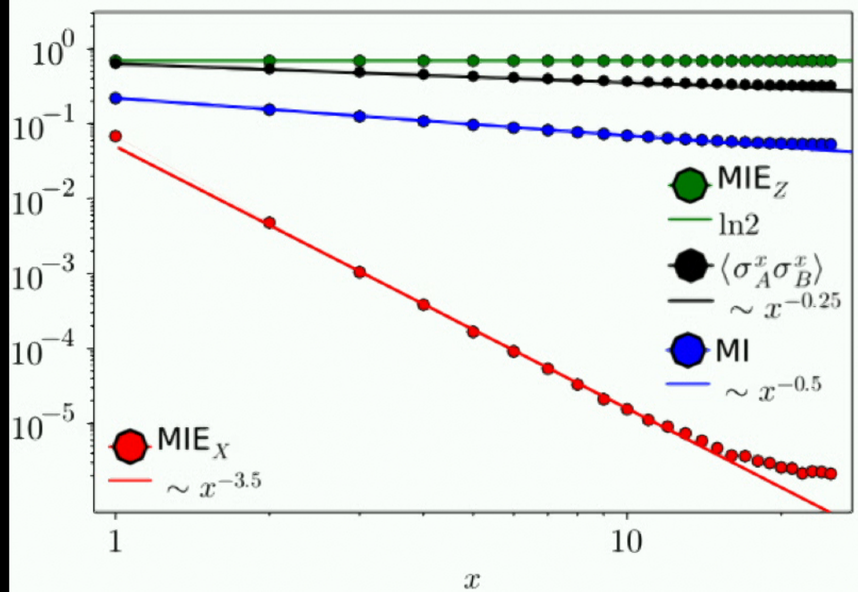
Transition between cluster state SPT (protected by $Z_2 \times Z_2$ symmetry) and ferromagnet:

$$H_{gSPT} = -\sum_j (\tau^x_{j-\frac{1}{2}} \sigma^z_j \tau^x_{j+\frac{1}{2}} + \sigma^x_{j-1} \tau^z_{j-\frac{1}{2}} \sigma^x_j) - \sum_j \sigma^x_j \sigma^x_{j-1}$$



MIE of Gapless SPT
(with sign structure)

For A,B on σ sublattice:



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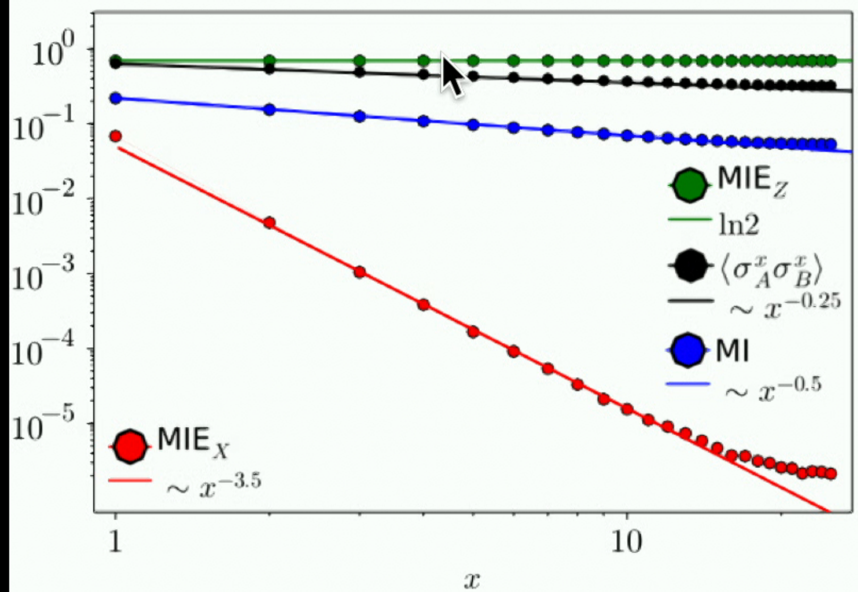

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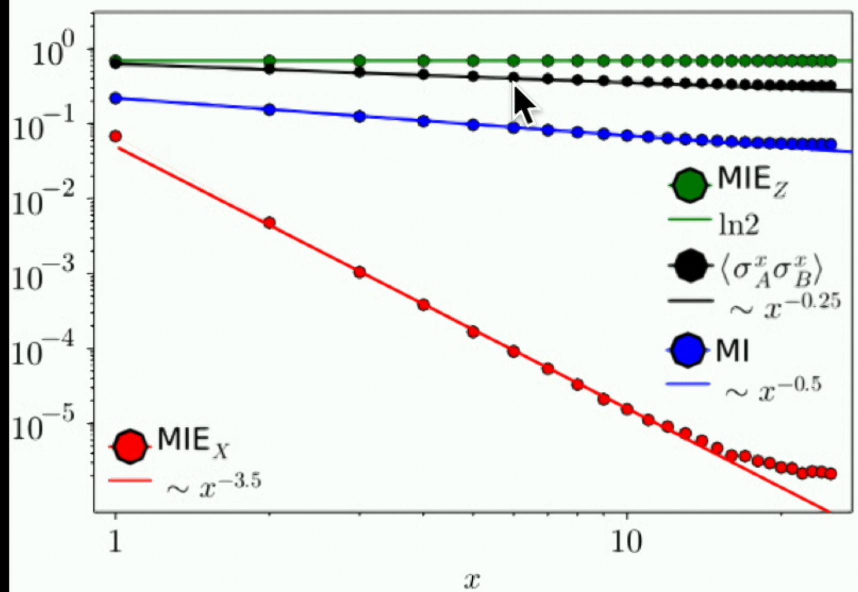

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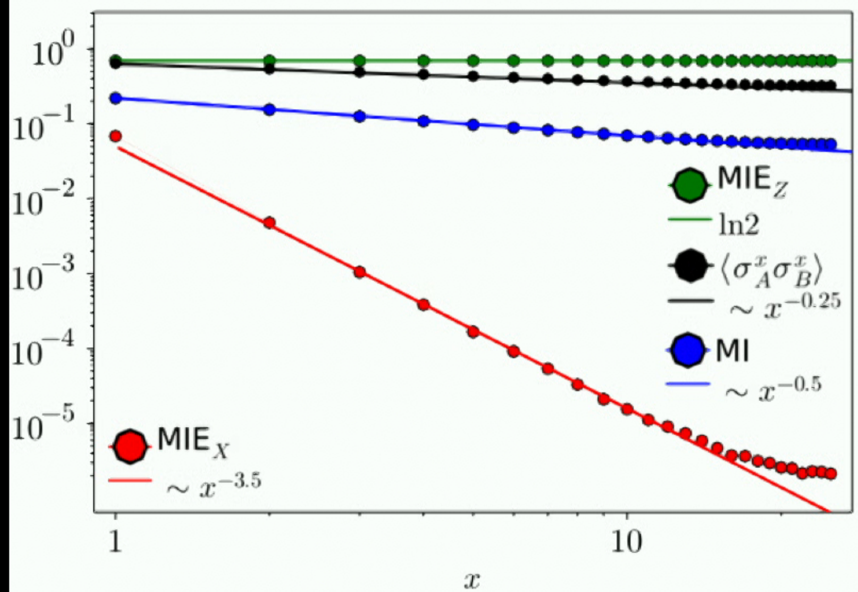

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Gapless SPT likely has symmetry-protected sign problem

Open Questions

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Prove hybrid Clifford circuit transition has intrinsic sign problem?

Find critical ground state with intrinsic sign problem?

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How are MIE / sign problem encoded in IR / CFT data / conformal boundary conditions?

Overview

For a sign-free state, entanglement induced by measuring in sign-free basis decays at least as fast as correlations before measurement

For sign-free stabilizer state, MIE(A:B) $\leq$ MI(A:B)

Constrains measurement-driven criticality: $h^{(1)}_{f|f} \geq h^{(1)}_{a|a}$

For sign-free spin state, MIE(A:B) upper bounded by $\langle X_A X_B \rangle$

Violations in hybrid circuit criticality, gapless SPT indicate sign structure

C-J. Lin, W. Ye, Y. Zou, S. Sang, and TH

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