Title: Dirac criticality from field theory beyond the leading order

Speakers: Michael Scherer

Collection: Quantum Criticality: Gauge Fields and Matter

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Abstract: Two-dimensional gapless Dirac fermions emerge in various condensed-matter settings. In the presence of interactions such Dirac systems feature critical points and the precision determination of their exponents is a prime challenge for quantum many-body methods. In a field-theoretical language, these critical points can be described by Gross-Neveu-Yukawa-type models and in my talk I will show some results on Gross-Neveu critical behavior using field theoretical approaches beyond the leading order. To that end, I will first present higher-loop perturbative RG calculations for generic Gross-Neveu-Yukawa models and compare estimates for the exponents with recent corresponding results from Quantum Monte Carlo simulations and the conformal bootstrap. Then, I will discuss a more exotic variant of Gross-Neveu-Yukawa models which describes the interacting fractionalized excitations of two-dimensional frustrated spin-orbital magnets. Here, we have provided field-theoretical estimates for the critical exponents employing higher-order epsilon expansion, large-N calculations, and functional renormalization group.

Dirac criticality from field theory beyond the leading order

Michael M. Scherer
Ruhr University Bochum

May 18, 2022 @ PI

Outline

RUB

- **Mini-review of Dirac criticality from field theory**
  - Dirac fermions and quantum critical points #Gross-Neveu-models
  - Critical exponents from perturbative RG at 4-loop order #hep
  - What to expect from these results ... and what not?! #many-body-methods

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  - Gross-Neveu-Yukawa* model
  - 3-loop perturbative RG, 2nd-order large-N expansion, functional RG

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- **Collaborators**:
  John Gracey (Liverpool), Igor Herbut (Vancouver), Bernhard Ihrig (Cologne), Lukas Janssen (Dresden), Daniel Kruti (Cologne), Peter Marquard (Berlin), Luminita Mihaila (Heidelberg), Shouryya Ray (Dresden), Nikolai Zerf (Berlin)

Materials and quasi-relativistic fermions

RUB

- emergence of **Dirac, Weyl & Majorana** quasi-particle excitations in materials
- **interacting Dirac fermions** have **quantum critical points**
  - **graphene**: antiferromagnetic/Kekulé-bond/charge order/...
  - ...

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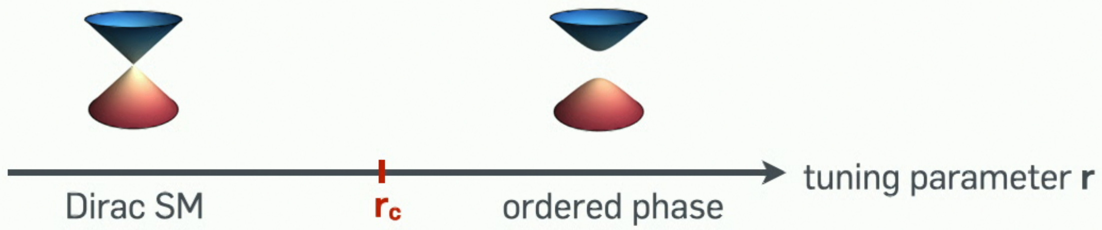

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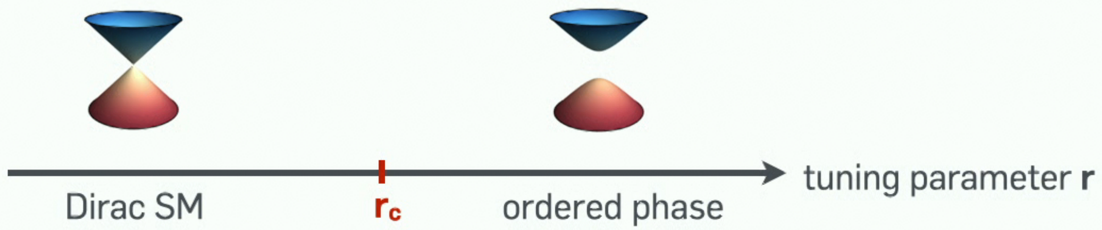
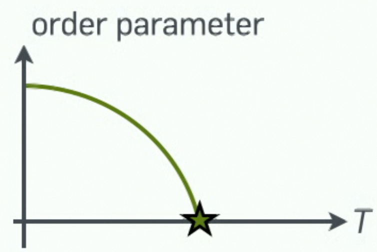


➡ **fermionic universality classes** → **What are their critical exponents?**

Three-dimensional Ising universality class

RUB

- near critical point of continuous phase transition: **universality**
- **correlation length** diverges for $T \to T_c$: $\xi(t) \propto |t|^{-\nu}$
- order parameter **correlation function** at T_c: $G(\vec{r}) \propto |\vec{r}|^{2-D-\eta}$



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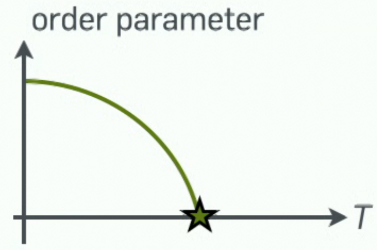

- 3D **Ising universality class**

Method	ν	η	
conformal bootstrap	0.629971(4)	0.036298(2)	Kos *et al.* (2016)
Monte Carlo	0.63002(10)	0.03627(10)	Hasenbusch (2010)
pRG, 4-ϵ, 6th order	0.6292(5)	0.0362(2)	Panzer & Kompaniets (2017)
functional RGs, DE	0.630(5)	0.034(5)	Litim & Zappala (2010)
experiment (fluid mixture)	0.629(3)	0.032(13)	Sengers & Shanks (2009)

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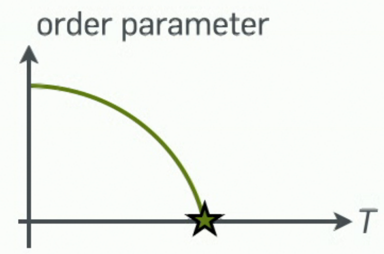

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- textbook example and prime benchmark for **development of many-body methods**
- gapless **Dirac fermions** ***not*** **in Ising/O(N) universality classes**!

Effective theories for phase transitions in Dirac systems

RUB

- critical point described by simple **continuum** field theory in $D = 2+1$ dimensions
  - **Gross-Neveu model:** $\mathscr{L}_{\mathrm{GN}} = \bar{\psi}\gamma_\mu\partial_\mu\psi + \lambda_\psi(\bar{\psi}\psi)^2$

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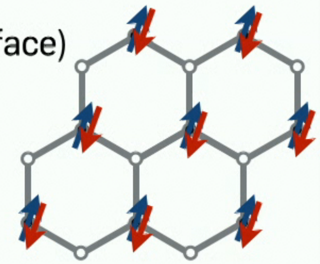
    - example: **CDW** order of electrons in graphene Herbut (2006)
    - **bosonized version** of model...



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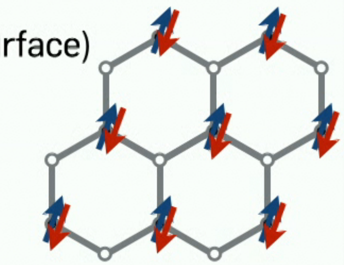

  - **Gross-Neveu-Yukawa model:** $\mathscr{L}_{\mathrm{GNY}} = \bar{\psi}(\gamma_\mu\partial_\mu + g\phi)\psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4$

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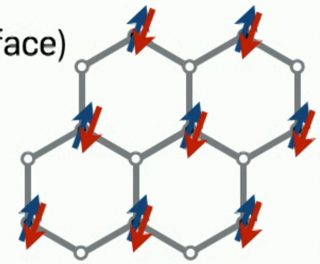

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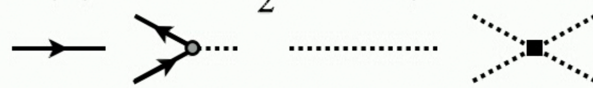

- both models have **critical point** in **2 < D < 4** and lie in **same universality class**

➡ **chiral Ising universality class** (it's the Ising model of critical fermions!)

Fermionic universality classes — developments

RUB

- towards **precision determination** of universality of interacting Dirac fermions
  - **Monte Carlo methods**
    - microscopic lattice models with 2nd order phase transition in GN universality class
    - sign-problem free formulations

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  - **renormalization group**
    - functional RG
    - higher-loop calculations adapted from hep-th…

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GNY model & renormalization group beta functions

RUB

- **renormalized** GNY **Lagrangian:**

$$\mathscr{L} = Z_\psi \bar{\psi}\gamma_\mu \partial_\mu - \frac{1}{2} Z_\phi (\partial_\mu \phi)^2 + Z_{\phi^2}\frac{m^2}{2}\phi^2 + Z_{\phi\bar{\psi}\psi} g \mu^{\epsilon/2} \phi \bar{\psi}\psi + Z_{\phi^4} \lambda \mu^\epsilon \phi^4$$

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  - the Z_i relate bare and renormalized parameters
  - **energy scale** μ parametrizes RG flow
  - RG β functions:

$$\beta_{g^2} = \frac{dg^2}{d\ln\mu}, \quad \beta_\lambda = \frac{d\lambda}{d\ln\mu}$$

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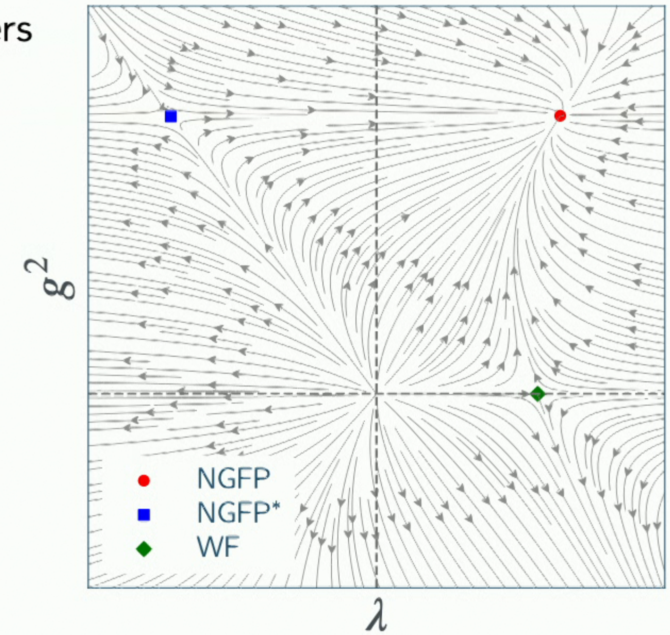

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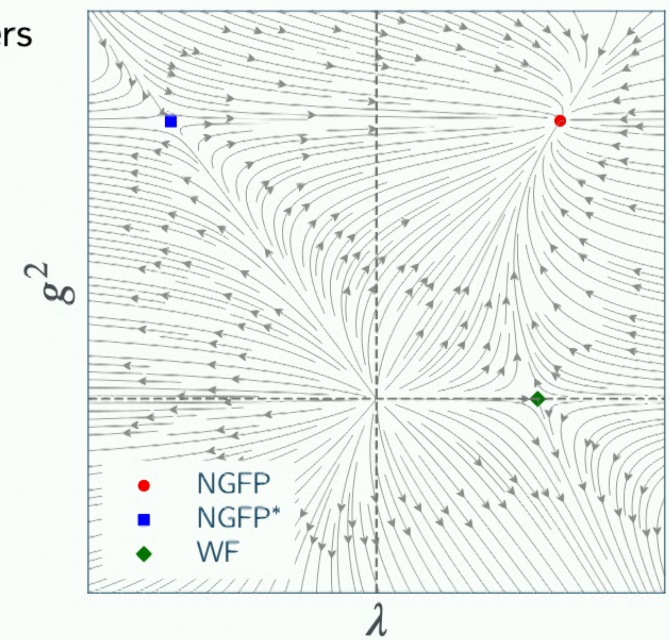
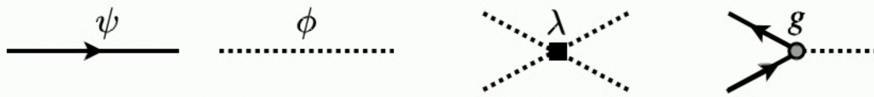
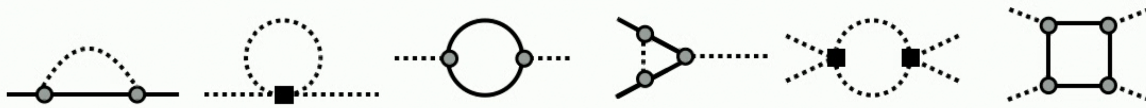
- **continuous** phase transition:
  - **IR attractive fixed point**

➡ **critical exponents!**



Renormalization group constants — tool chain

- evaluate renormalization group constants Z_i up to 4-loop order
- tree level: 

- 1 loop: 

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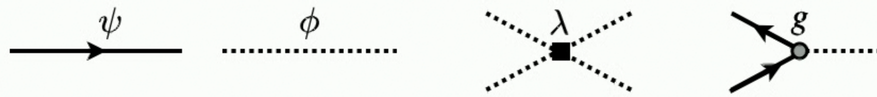

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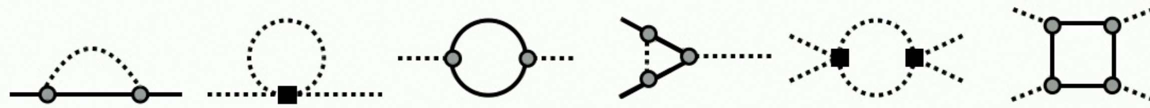
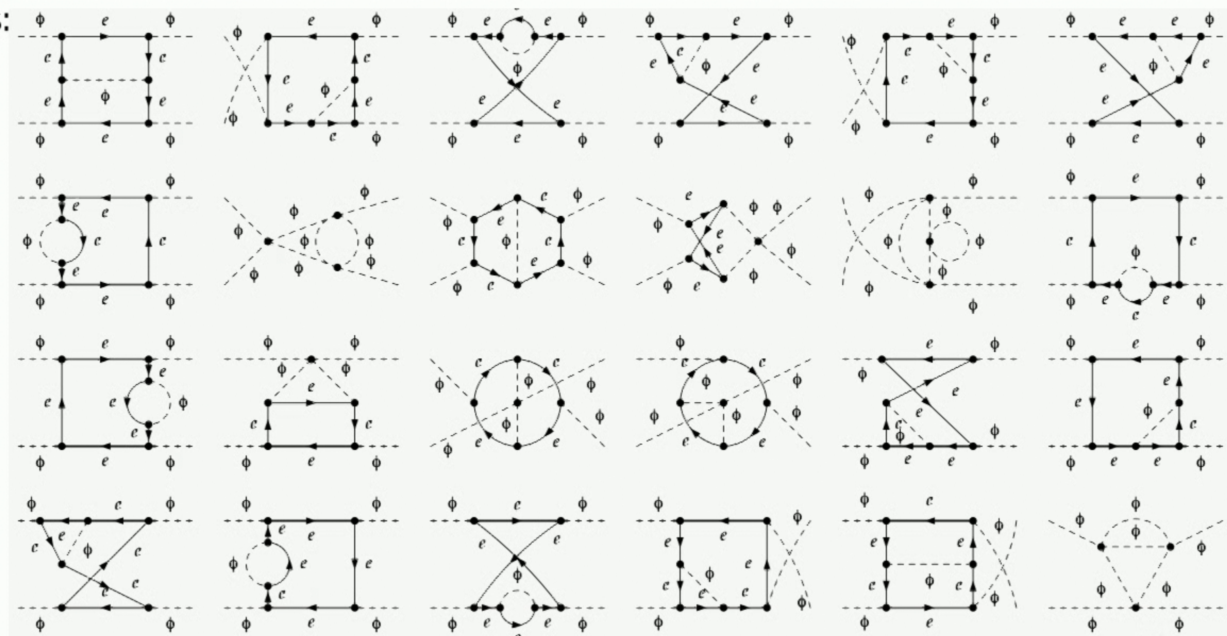


- 2 loops:



+ 96 more

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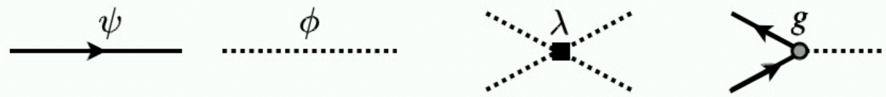
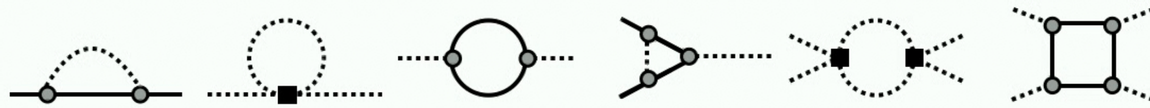
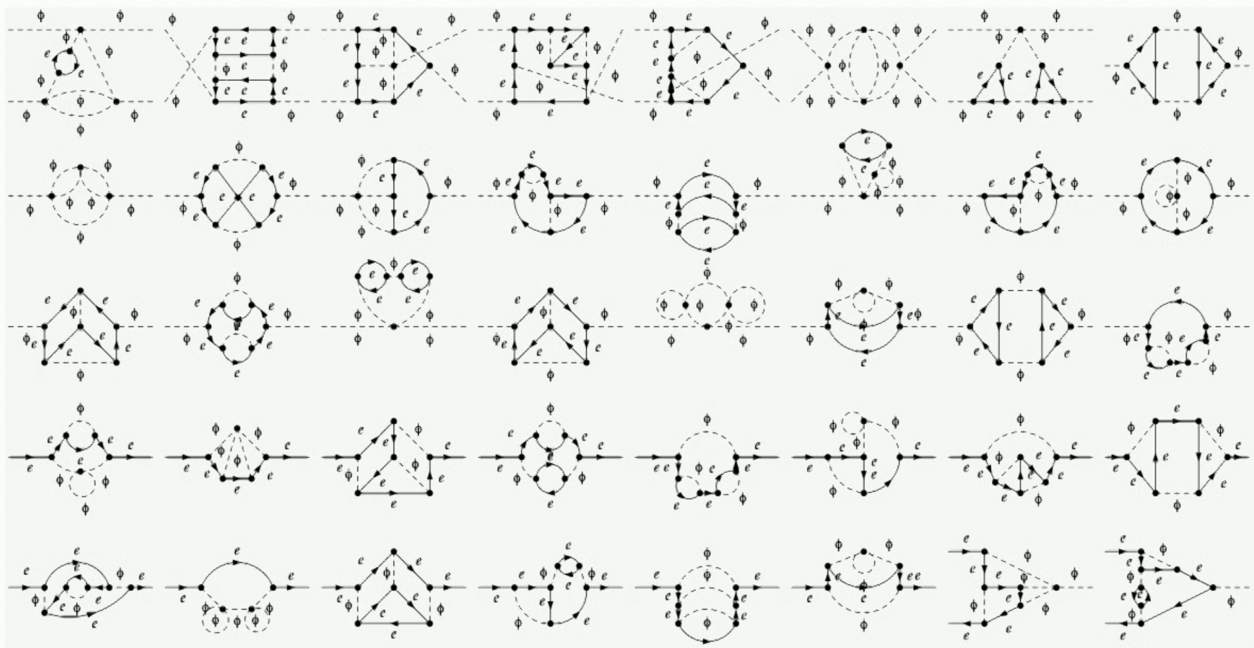
- tree level: 

- 1 loop: 
- 3 loops: 

\+ 1697 more

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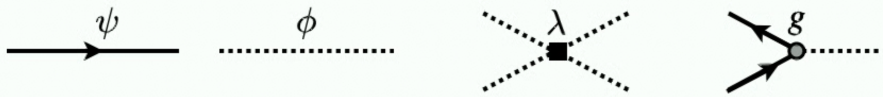
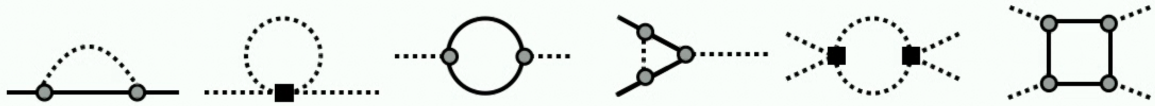
- tree level: 

- 1 loop: 

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- 4 loops: in total **31,671 diagrams!**

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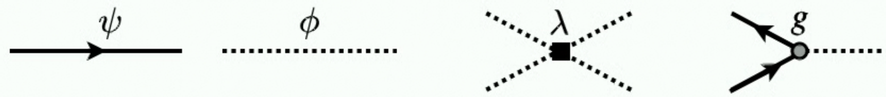

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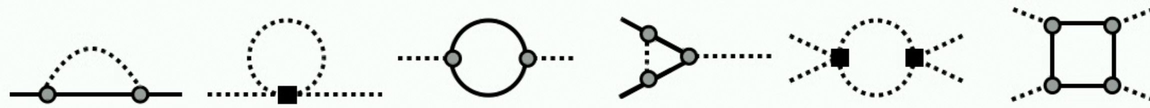

  ...

- 4 loops: in total **31,671 diagrams!**
  - **use tool chain developed for relativistic high-energy physics:**

QGRAF
- insert Lagrangian - generates complete set of diagrams with symmetry factors

Q2E
- declare mass scales and Feyman rules

EXP
- reduce to one-scale integrals by asymptotic expansion

FORM
- reduce integrals to 19 known master-integrals using **Crusher**

epsilon expansion for chiral Ising model

RUB

- **CDW transition in graphene**: $N_f = 2$ flavors of 4-component Dirac fermions
  - critical exponents in $D = 4 - \epsilon$

$$\frac{1}{\nu} = 2 - \frac{20\epsilon}{21} + \frac{325\epsilon^2}{44982} - \frac{(271572144\zeta_3 + 36133009)\epsilon^3}{3821940612} + \frac{\left(73192843310400\zeta_3 + 179520471709200\zeta_5 - 2472257012904\pi^4 - 86141171013035\right)\epsilon^4}{4175164363361040} + \mathcal{O}(\epsilon^5)\,,$$

$$\eta_\phi = \frac{4\epsilon}{7} + \frac{109\epsilon^2}{882} + \left(\frac{1170245}{26449416} - \frac{144\zeta_3}{2401}\right)\epsilon^3 + \frac{\left(162669869280\zeta_3 + 171915696000\zeta_5 - 1203409872\pi^4 + 102456536695\right)\epsilon^4}{2407822585560} + \mathcal{O}(\epsilon^5)\,,$$

$$\eta_\psi = \frac{\epsilon}{14} - \frac{71\epsilon^2}{10584} - \left(\frac{18\zeta_3}{2401} + \frac{2432695}{158696496}\right)\epsilon^3 + \frac{\left(1155813964920\zeta_3 + 515747088000\zeta_5 - 3610229616\pi^4 - 556332486445\right)\epsilon^4}{57787742053440} + \mathcal{O}(\epsilon^5)\,,$$

Zerf, Mihaila, Marquard, Herbut, MMS (2017)

Quantum critical behavior of N_f=2 chiral Ising model

- GNY exponents from 4-loop RG in (4-ϵ):
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$$\nu^{-1} \approx 2 - 0.95\epsilon + 0.0072\epsilon^2 - 0.095\epsilon^3 - 0.013\epsilon^4$$
$$\eta_B \approx 0.57\epsilon + 0.12\epsilon^2 - 0.028\epsilon^3 + 0.15\epsilon^4$$
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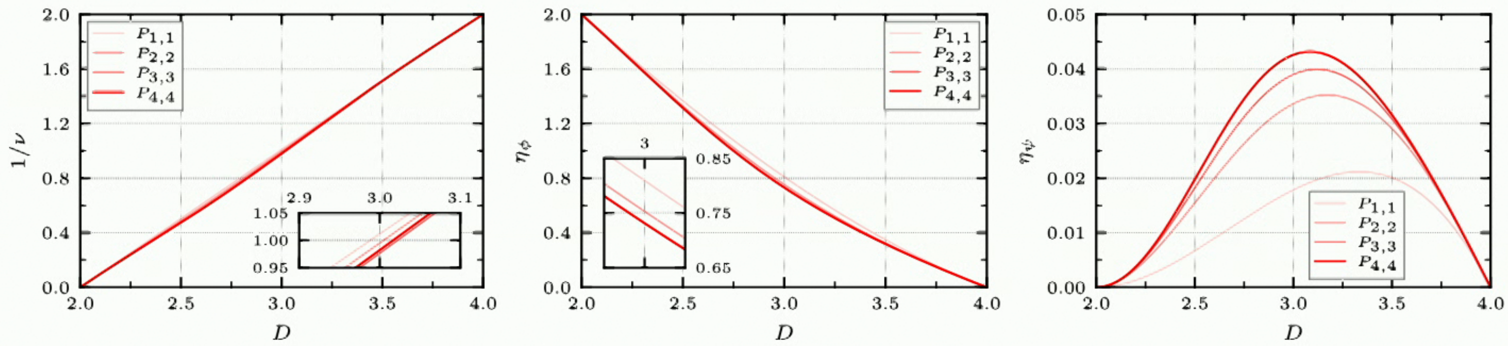

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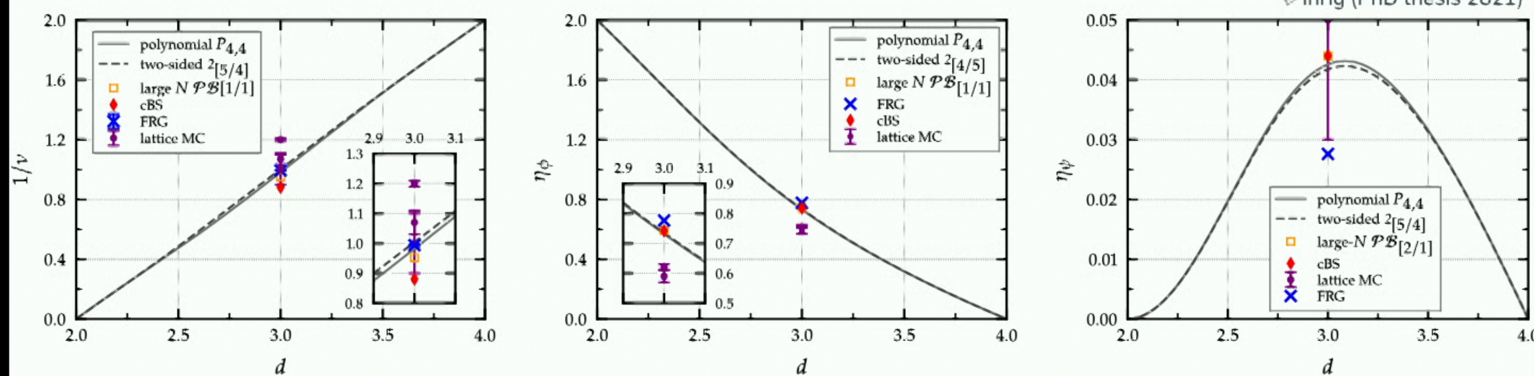
Ihrig (PhD thesis 2021)



Method	$1/\nu$	η_B	η_F	
4-loop pert. RG, ϵ^4	0.99	0.731	0.043	Ihrig, Mihaila, MMS (2018)
large-N, $1/N^{2,3}$	0.95	0.743	0.044	Gracey (2018)
conformal bootstrap	0.88	0.742	0.044	Iliesiu et al. (2017)
functional RG	0.994	0.7765	0.0274	Knorr (2016)
Monte Carlo	1.20(1)	0.62(1)	0.38(1)	Chandrasekharan & Li (2013)
Monte Carlo	1.1(1)	0.60(2)	0.05(2)	Liu, Sun, Meng (2020)

Perspective for N_f=2 chiral Ising criticality

- **anomalous dimensions** η_ϕ, η_ψ:
  - very good agreement of perturbative approaches and conformal bootstrap
  - discrepancies with Monte Carlo simulations
- **correlation length exponent** ν:
  - ~10% discrepancies across the methods
  - update on conformal bootstrap ... Erramilli, Iliesiu, Kravchuk, Liu, Poland, Simmons-Duffin (in progress)
- **model with best chance to settle critical behavior across methods, soon #textbook**

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Related models

- **different choices for** $N_f = N/4$
  - results from $4-\epsilon$ expansion at 4-loop order with Padé approximants

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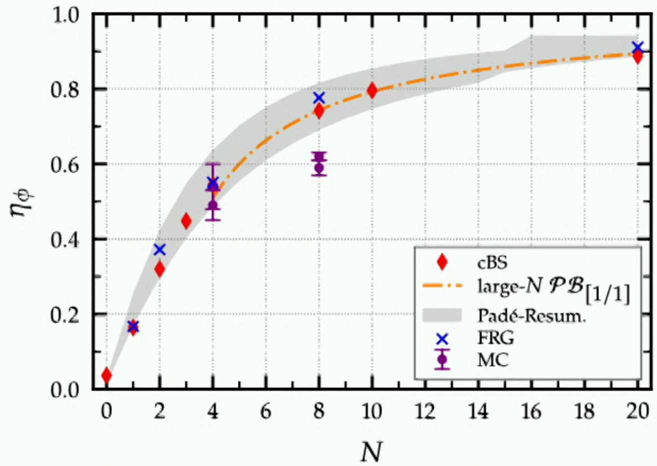

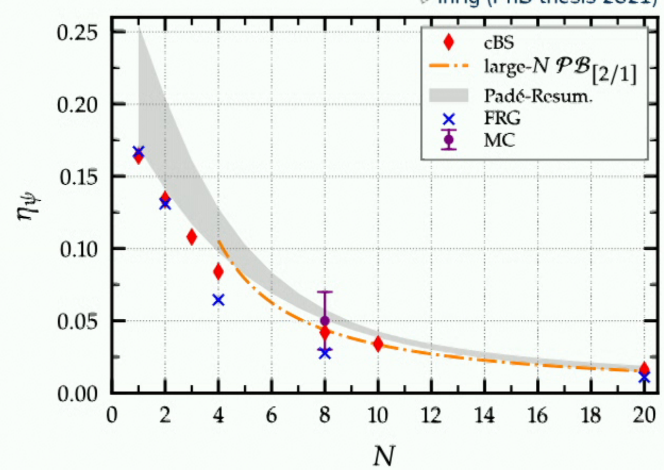

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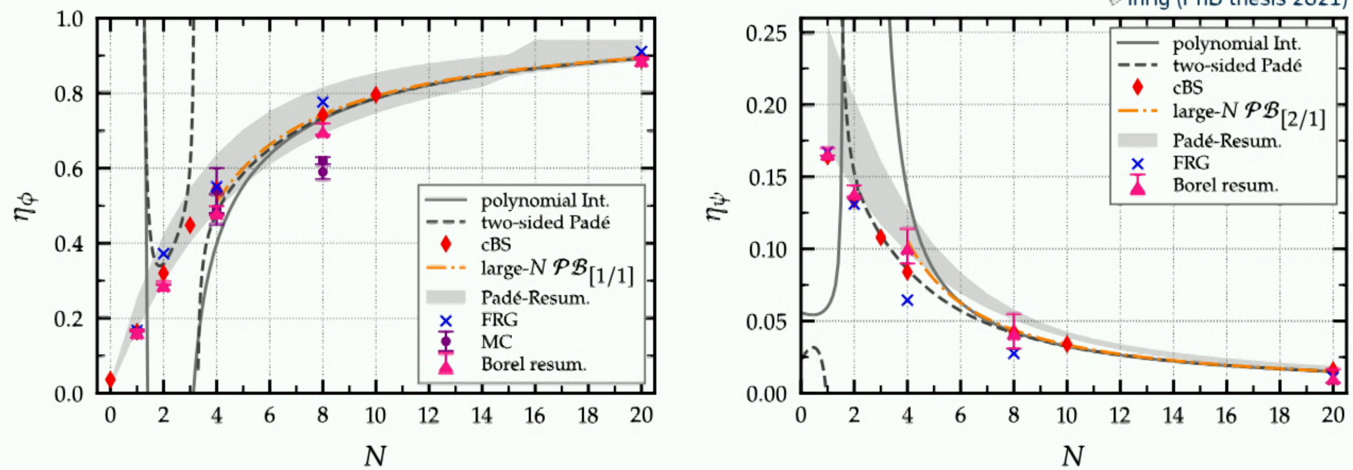

  - critical exponents from $2+\epsilon$ expansion exhibit pole at $N=2$ $(N_f = 1/2)$
  - impact of pole already shows at $N=4$ $(N_f = 1)$
  - results from 2-sided interpolations not as good at $N_f\!=\!1$ as they are at $N_f\!=\!2$!
  - **chiral Ising model at $N_f = 2$ is a sweet spot for comparisons of different methods!**

Related models II

- **antiferromagnetic transition in graphene** $(N_f = 2)$
  - e.g., Hubbard model on the honeycomb lattice
  - **chiral Heisenberg model**

$$\mathscr{L}_{\mathrm{GNY}} = \bar{\psi}\gamma_\mu\partial_\mu\psi + g\vec{\phi}\cdot\bar{\psi}\left(\vec{\sigma}\otimes\mathbb{1}_{2N_f}\right)\psi + \frac{1}{2}\vec{\phi}(m^2-\partial_\mu^2)\vec{\phi} + \lambda\left(\vec{\phi}\cdot\vec{\phi}\right)^2$$

Zerf, Mihaila, Marquard, Herbut, MMS (2017)

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  - expansion in $D = 4 - \epsilon$ at 4-loop order

$$\frac{1}{\nu} \approx 2 - 1.527\epsilon + 0.4076\epsilon^2 - 0.8144\epsilon^3 + 2.001\epsilon^4,$$
$$\eta_\phi \approx 0.8\epsilon + 0.1593\epsilon^2 + 0.02381\epsilon^3 + 0.2103\epsilon^4\ ,$$
$$\eta_\psi \approx 0.3\epsilon - 0.05760\epsilon^2 - 0.1184\epsilon^3 + 0.04388\epsilon^4,$$

Zerf, Mihaila, Marquard, Herbut, MMS (2017)

Related models III

- 2-dimensional frustrated magnets with spin and orbital degrees of freedom
  - low-energy Dirac fermion excitations from fractionalization
- microscopic model leads to effective **Gross-Neveu-SO(3) model** (→ talk by L. Janssen)

$$\mathscr{L} = \bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}\phi_a(-\partial_\mu^2 + m^2)\phi_a + \lambda(\phi_a\phi_a)^2 - g\phi_a\bar{\psi}(\mathbb{1}_{2N/3} \otimes L_a)\psi$$

Seifert, Dong, Chulliparam

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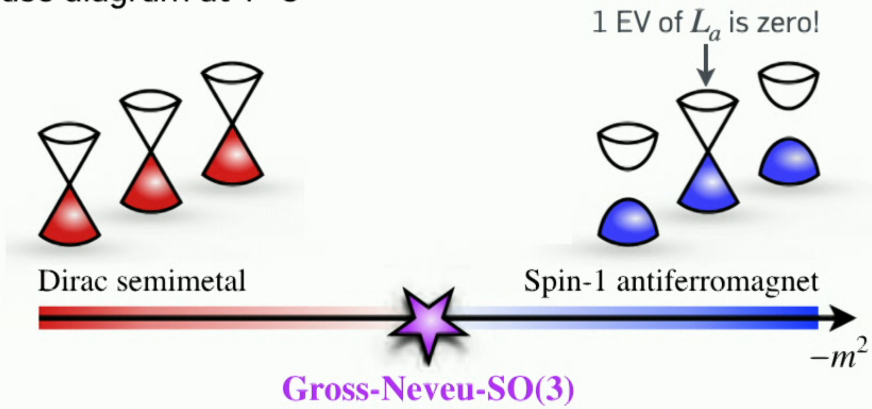
  - mean-field phase diagram at T=0



  - **only 2N/3 of the Dirac cones acquire mass gap** and 1N/3 remain massless

Fractionalized quantum criticality in spin-orbital liquids

- critical exponents from field-theory beyond the leading order:

1. expansion in $D = 4 - \epsilon$ at 3-loop order

$$\frac{1}{\nu} \approx 2 - 0.917\epsilon - 0.0340\epsilon^2 - 0.0735\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\eta_\phi \approx 0.333\epsilon + 0.0922\epsilon^2 - 0.0338\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$\eta_\psi \approx 0.167\epsilon + 0.0360\epsilon^2 - 0.0303\epsilon^3 + \mathcal{O}(\epsilon^4)$$

2. large-N expansion at order $1/N^2$ for ν, η_ϕ and at order $1/N^3$ for η_ψ

$$\frac{1}{\nu} \approx 1 - \frac{1.621}{N} + \frac{19.922}{N^2} + \mathcal{O}(1/N^3)$$

$$\eta_\phi \approx 1 - \frac{2.026}{N} + \frac{1.564}{N^2} + \mathcal{O}(1/N^3)$$

$$\eta_\psi \approx \frac{0.4053}{N} + \frac{1.0403}{N^2} - \frac{0.7972}{N^3} + \mathcal{O}(1/N^4)$$

3. functional renormalization group...

Ray, Ihrig, Kruti, Gracey, MMS, Janssen (2021)

Conclusions

- even at high loop order: no guarantee to get quantitative results!
- sometimes it seems to work, though!
  - **chiral Ising model at $N_f = 2$ is a sweet spot for comparisons of different methods!**
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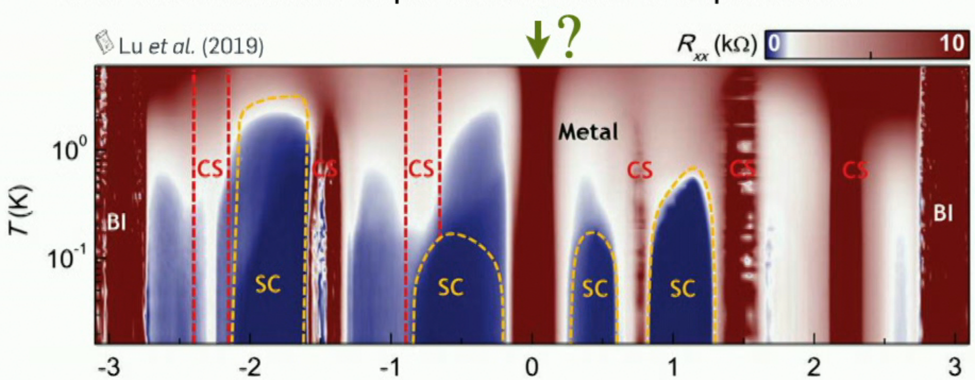

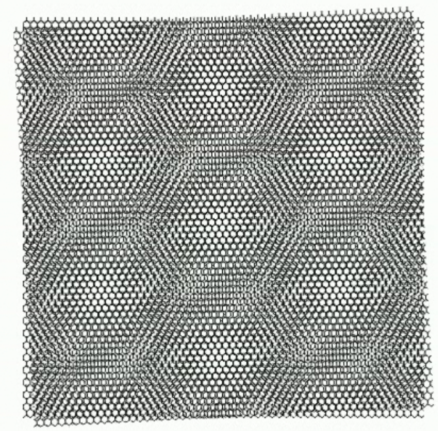

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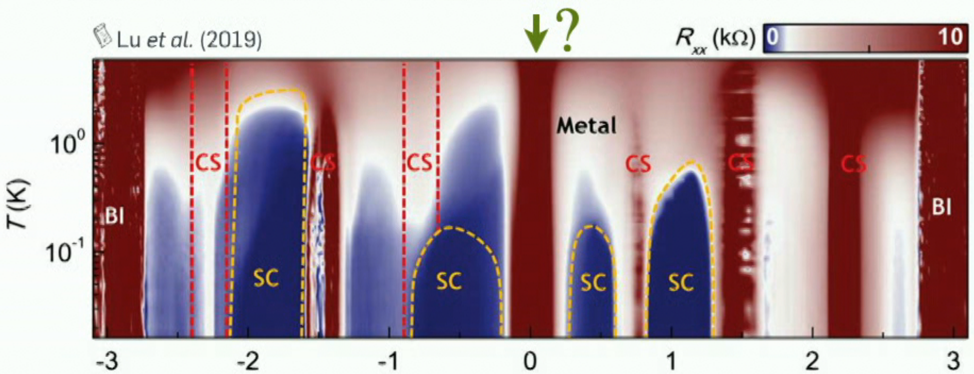

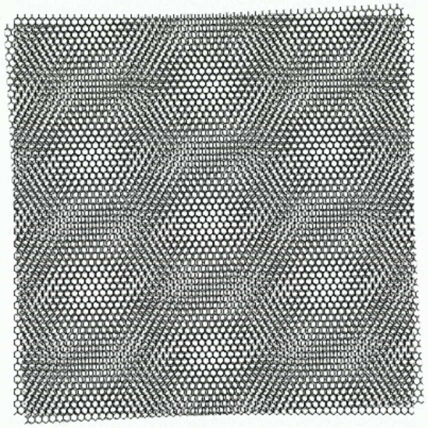

- chiral Heisenberg model at 4 loops less clear (also no conformal bootstrap data)
- abelian Higgs model $n_c \approx 183(1 - 1.75\epsilon + 0.80\epsilon^2 + 0.36\epsilon^3) \quad \rightarrow \quad n_c(d=3) \approx 12 \pm 4$

Ihrig, Zerf, Marquard, Herbut, MMS (2019)