

Title: Deconfined multi-criticality in quantum spin models and experiments

Speakers: Anders Sandvik

Collection: Quantum Criticality: Gauge Fields and Matter

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Abstract: In the original field theoretical scenario of deconfined quantum criticality, the deconfined quantum-critical point (DQCP) separating antiferromagnetic (AFM) and singlet-solid phases of quantum magnets is generic, i.e., does not require fine-tuning. Recent numerical studies instead point to a fine-tuned multi-critical DQCP [1] that is also the end-point of a gapless spin liquid phase [2]. An example is the Shastry-Sutherland (SS) model, where a narrow spin liquid phase was recently detected [2,3], instead of the previously argued direct transition between plaquette singlet solid (PSS) and AFM phases. The multi-critical DQCP, followed by a direct transition without intervening spin liquid, can be reached when other interactions are included. Very recent NMR experiments on the SS compound $\text{SrCu}_2(\text{BO}_3)_2$ under high pressures and high magnetic fields are consistent with this scenario [4]. Low-temperature (below 0.1 K) direct PSS to XY-AFM transitions were observed that become less strongly first-order at higher pressures. At the highest pressure, quantum-critical scaling of the spin-lattice relaxation was observed, indicating close proximity to a DQCP. This point may be the end-point of a not yet confirmed quantum spin liquid phase existing at slightly higher pressures.

[1] B. Zhao, J. Takahashi, and A. W. Sandvik, PRL 125, 257204 (2020).

[2] J. Yang, A. W. Sandvik, and L. Wang, PRB 105, L060409 (2022).

[3] L. Wang, Y. Zhang, and A. W. Sandvik, arXiv:2205.02476

[4] Y. Cui et al., arXiv:2204.08133.

Quantum Criticality: Gauge Fields and Matter, Perimeter Institute, May 16-20, 2022

Deconfined multi-criticality in models and experiments

Anders W Sandvik, Boston University

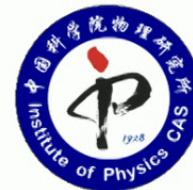
Topic

Ground state phases of the Shastry-Sutherland model
and related 2D $S=1/2$ models

+ new generic insights into deconfined quantum criticality



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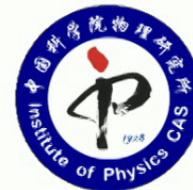
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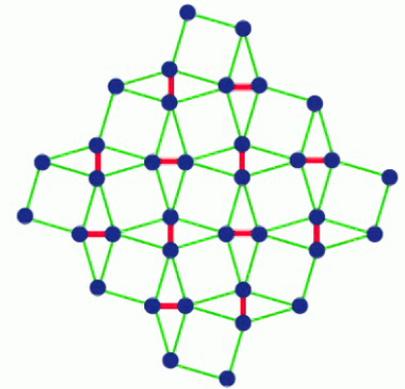
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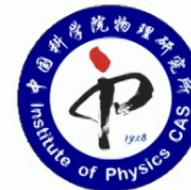
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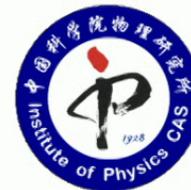
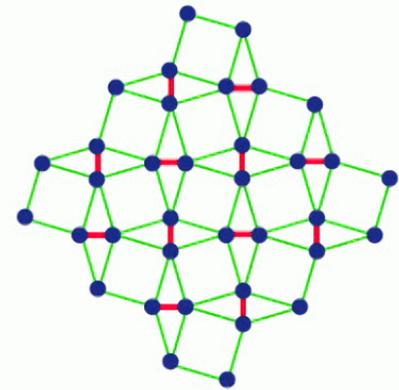
Model studies: Ling Wang (Zhejiang University) + others ...

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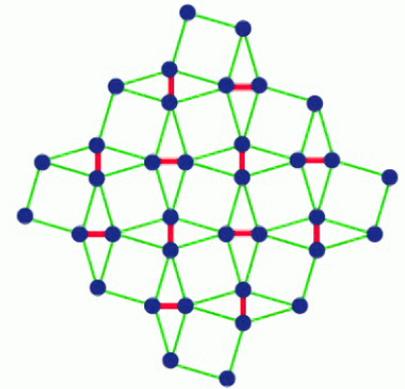
Experiments: Weiqiang Yu, Yi Cui, (Renmin University, Beijing) + many ...

Topic

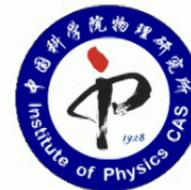
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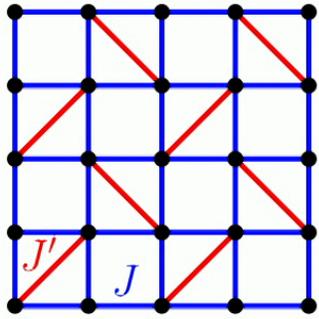


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Shastry-Sutherland model

Shastry & Sutherland, 1982

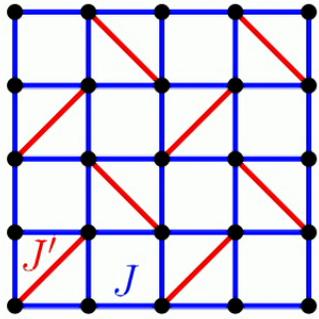


$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J' \sum_{\langle ij \rangle'} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$J, J' > 0 \quad g = J/J'$$

Shastry-Sutherland model

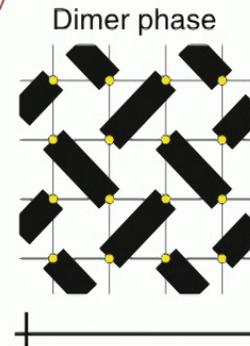
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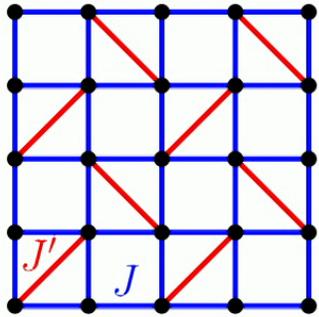
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Exact dimer ground state for $g < g_{c1}$



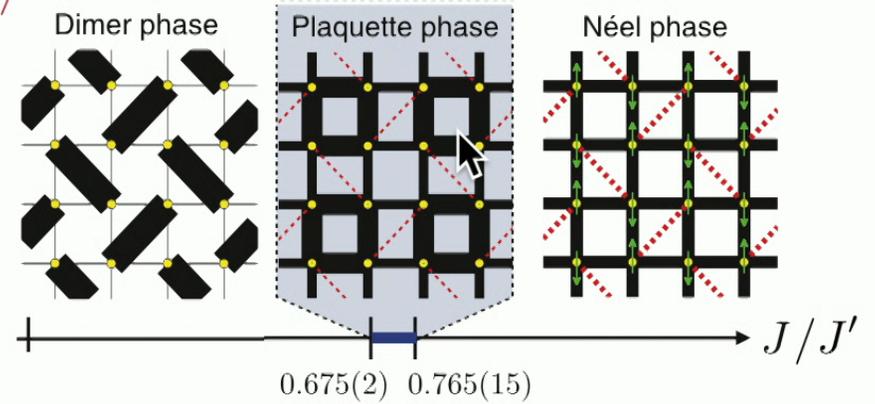
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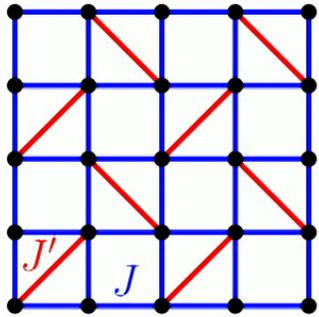
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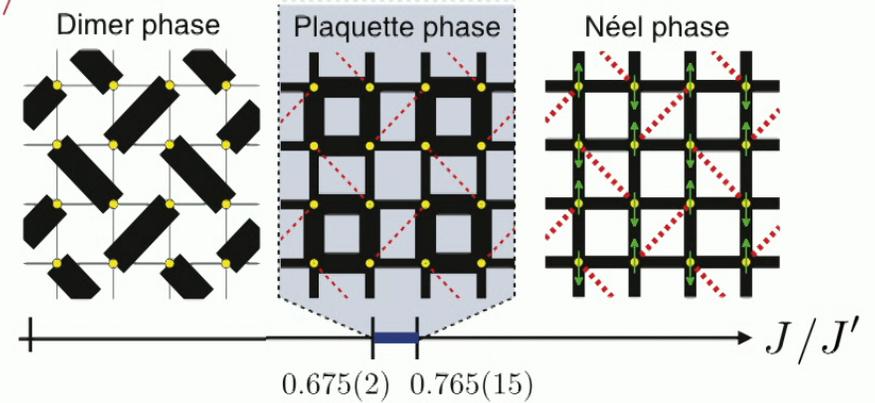


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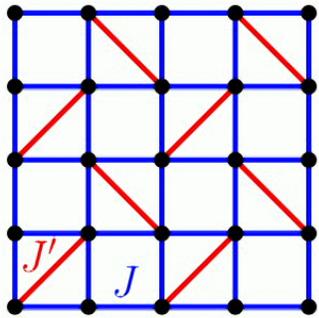
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PEPS (tensor product): [Corbotz & Mila PRB 2013](#)
First-order plaquette - Néel transition



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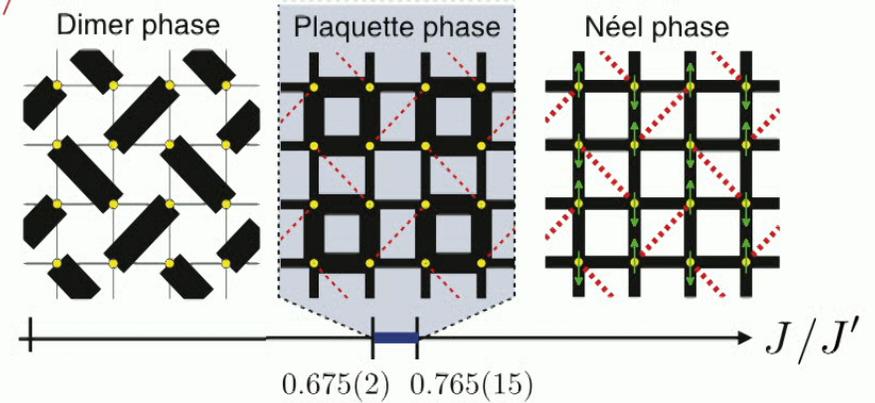


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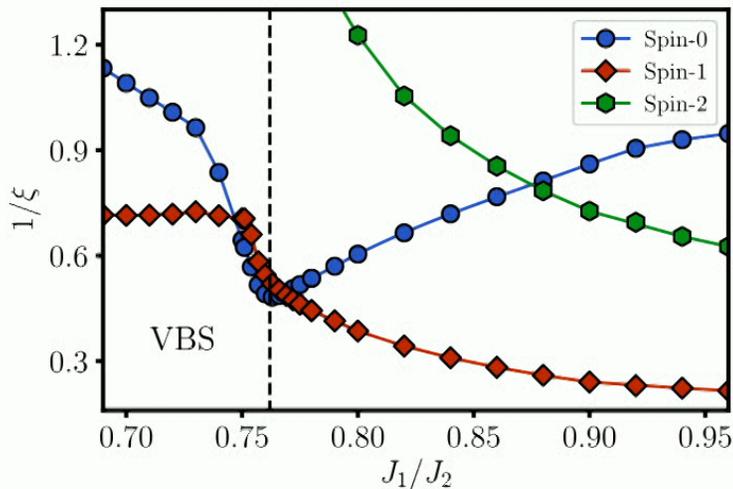
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Shastry-Sutherland Lattice, $L = 10$



[Lee, You, Schdev, Vishwanath, PRX 2019](#)

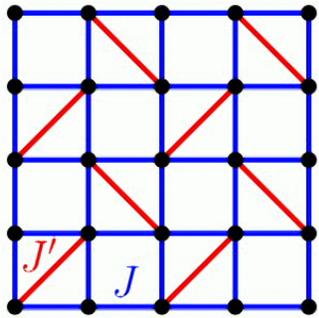
	$L = 6$	$L = 8$	$L = 10$	$L = 12$
$(J_1/J_2)_{c1}$	0.682	0.677	0.675	0.675
$(J_1/J_2)_{c2}$	0.693	0.728	0.762	0.77

DMRG
 long cylinders

Argued continuous deconfined quantum-critical
 plaquette - Neel transition; emergent O(4) symmetry

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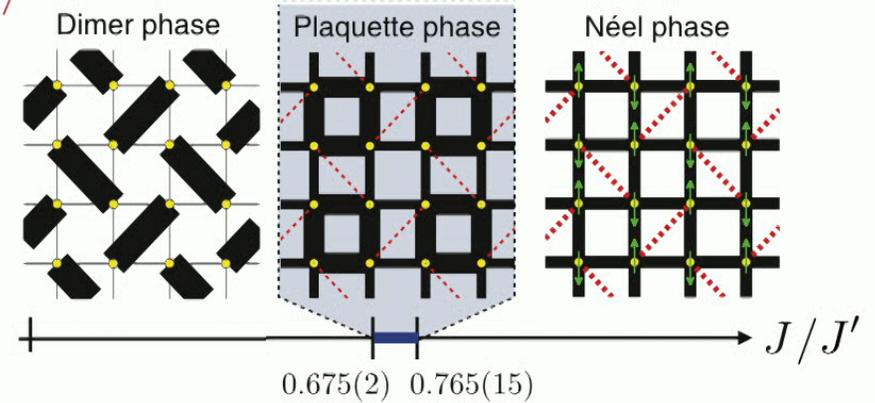


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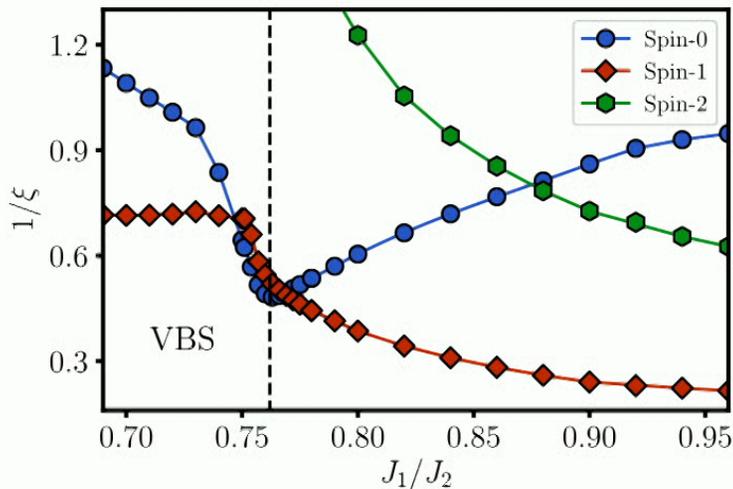
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Here: Level crossing method to detect transitions

[Yang, Wang, Sandvik, PRB 2022 \(DMRG\)](#)

[Wang, Zhang, Sandvik, arXiv:2205.02476 \(exact diag.\)](#)

Level-crossing method

- Example: J_1 - J_2 Heisenberg chain

$$H = J_1 \sum_{i=1}^N \mathbf{S}_i \cdot \mathbf{S}_{i+1} + J_2 \sum_{i=1}^N \mathbf{S}_i \cdot \mathbf{S}_{i+2}$$



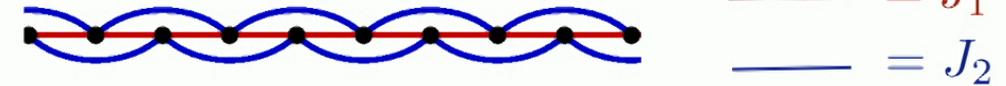
— = J_1
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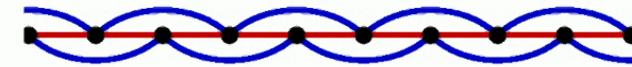
dimerization transition
at $g_c = 0.2411\dots$

- $g < g_c$ - unique singlet ground state, critical state, $1/r$ spin correlations
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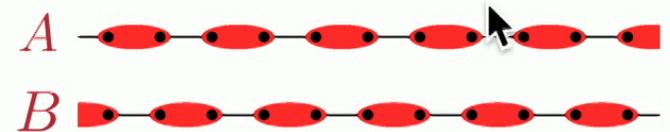


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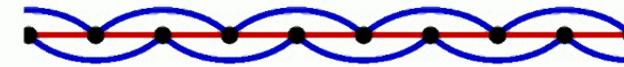
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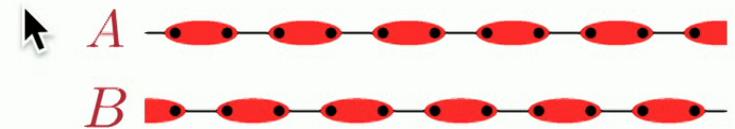
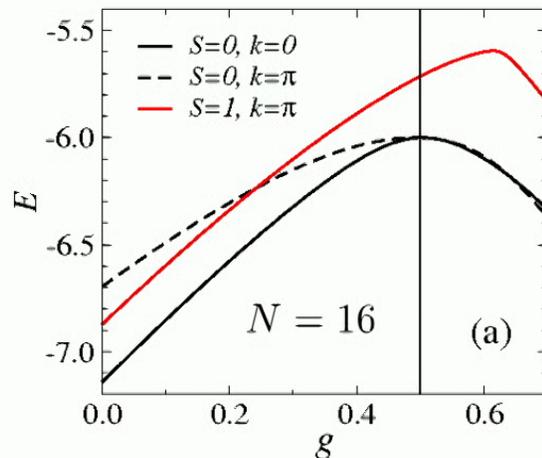
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The two lowest excited states cross at g_c

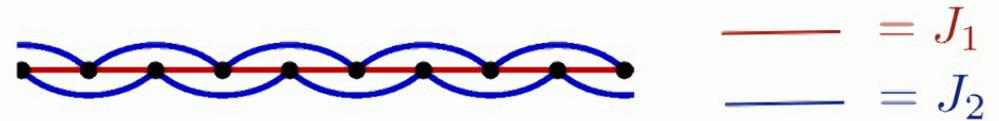


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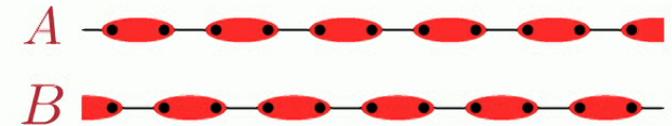
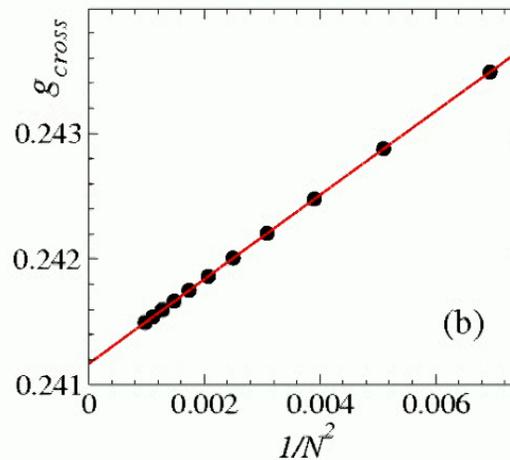
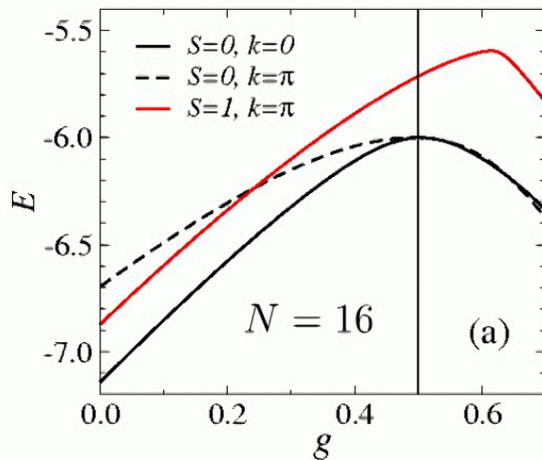
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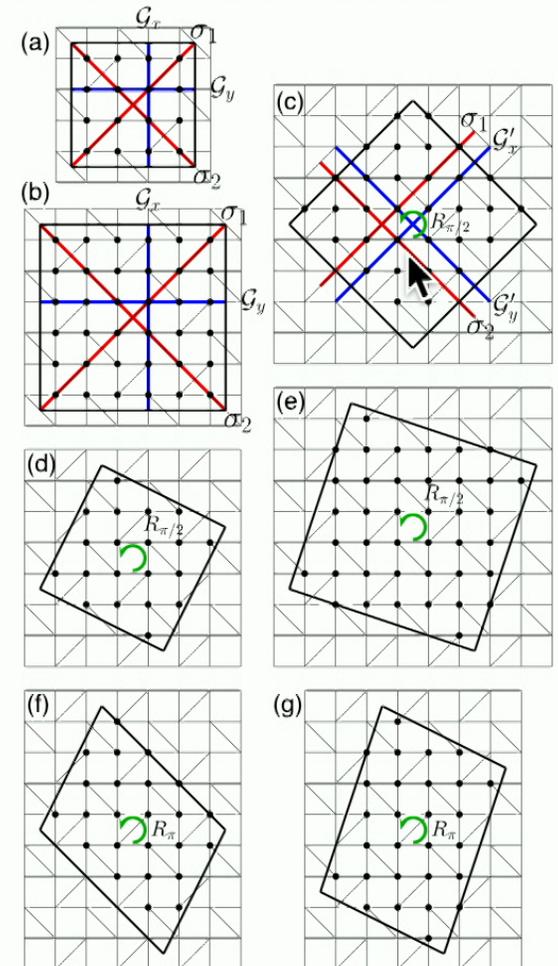
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Nomura & Okamoto (Phys. Lett. 1992)
Eggert (PRB 1996)
Sandvik (AIP Conf. Proc. 2010)

$$g_c = 0.2411674(2)$$

Level crossings in the SS model. 1) Small clusters, exact diag

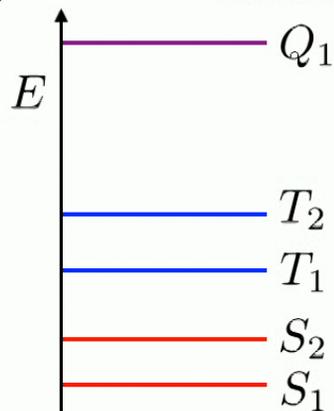
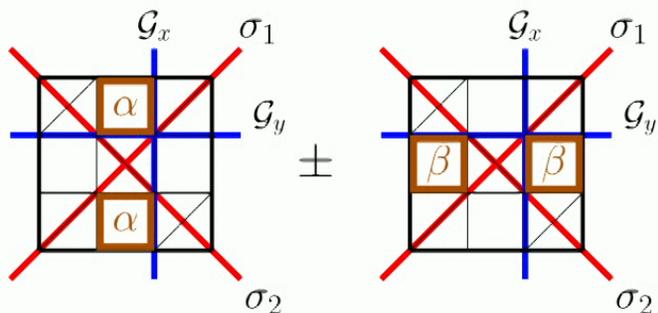
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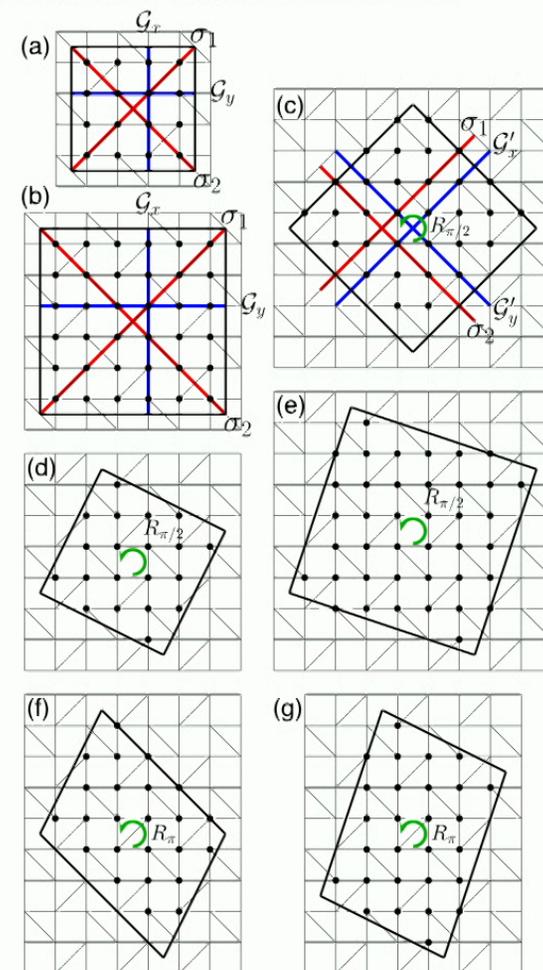
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quasi-degenerate singlet ground state of the plaquette-singlet-solid (PSS)



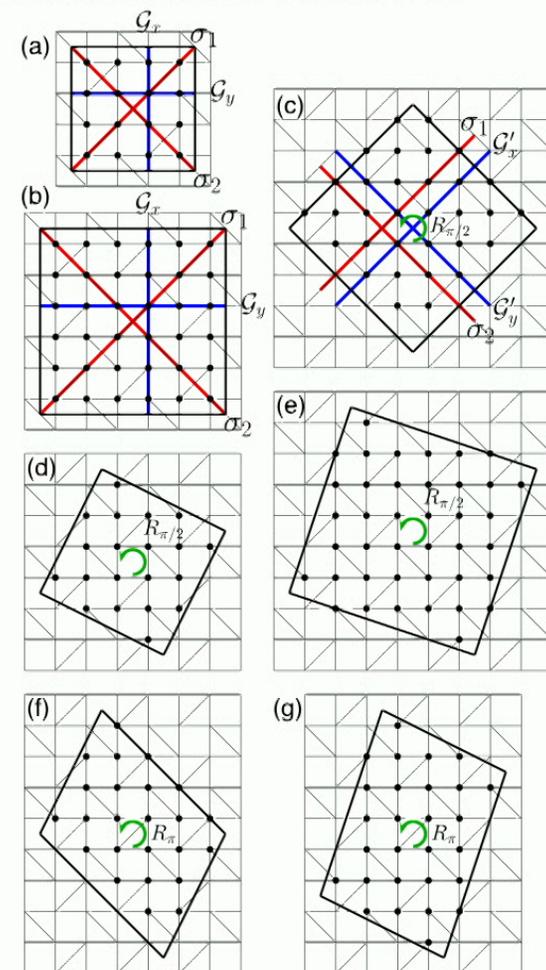
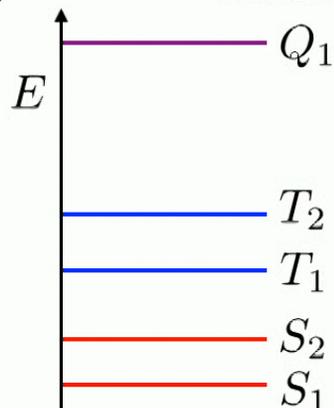
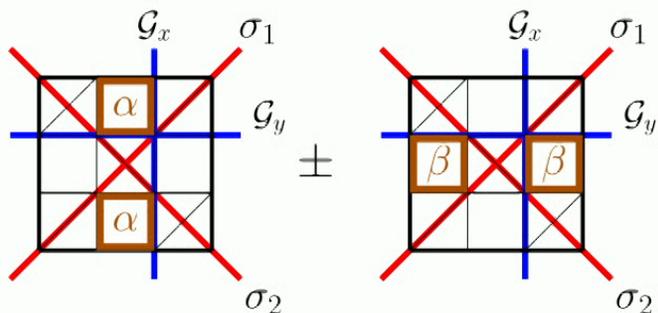
Singlets S_1, S_2 . Triplets T_1, T_2 . Quintuplet Q_1 .



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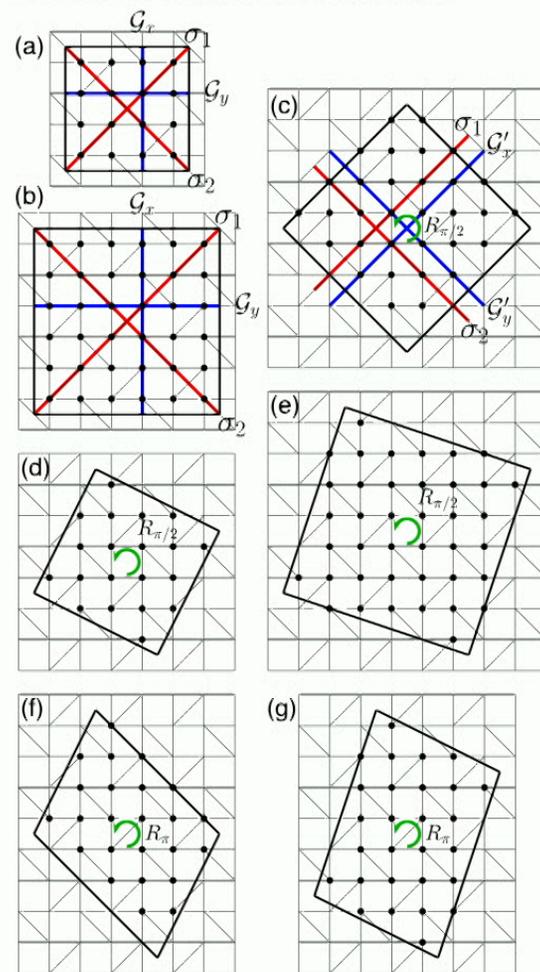
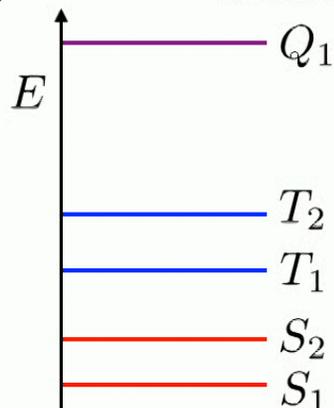
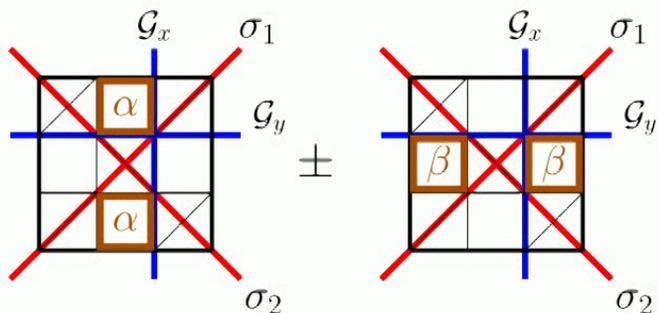
Gaps: $\Delta(S_2) = E(S_2) - E(S_1)$ $\Delta(T_1) = E(T_1) - E(S_1)$

Composite gaps: $\delta_T = E(T_2) - E(S_2)$ $\delta_Q = E(Q_1) - E(T_1)$

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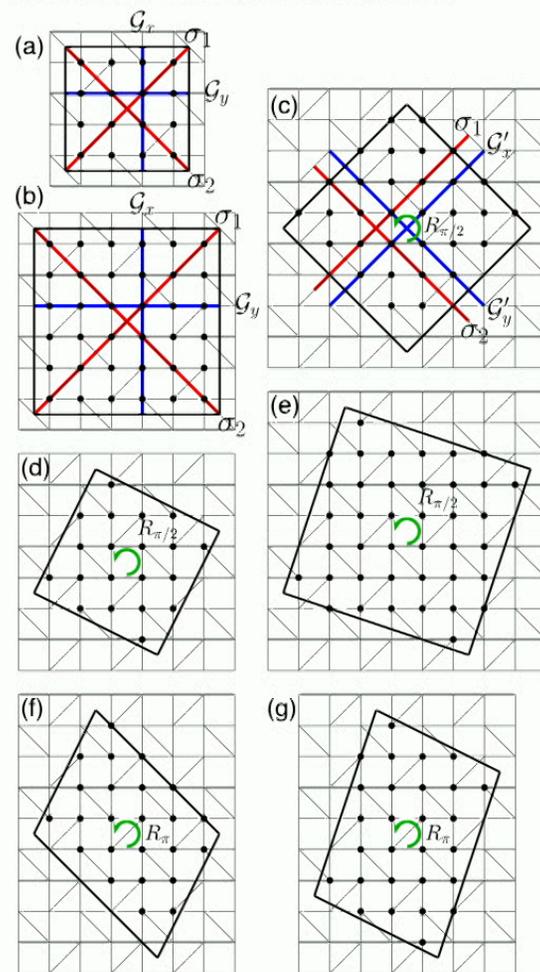
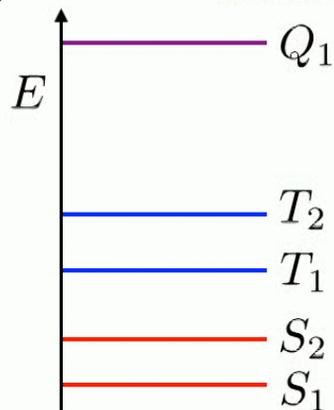
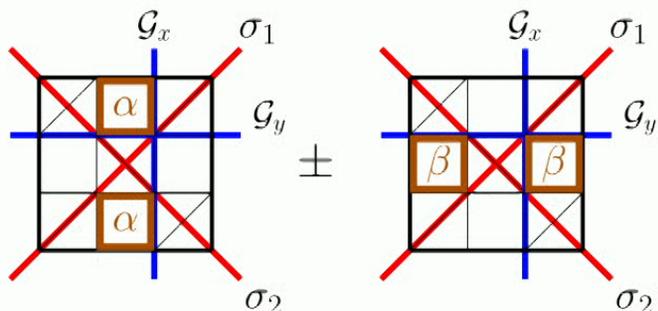
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Gap criteria for PSS and AFM ordered phases

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Gap criteria for PSS and AFM ordered phases

PSS: $\Delta(S_2) < \Delta(T_1)$

AFM: $\Delta(Q_1) < \Delta(S_2), \delta_T < \Delta(S_2)$

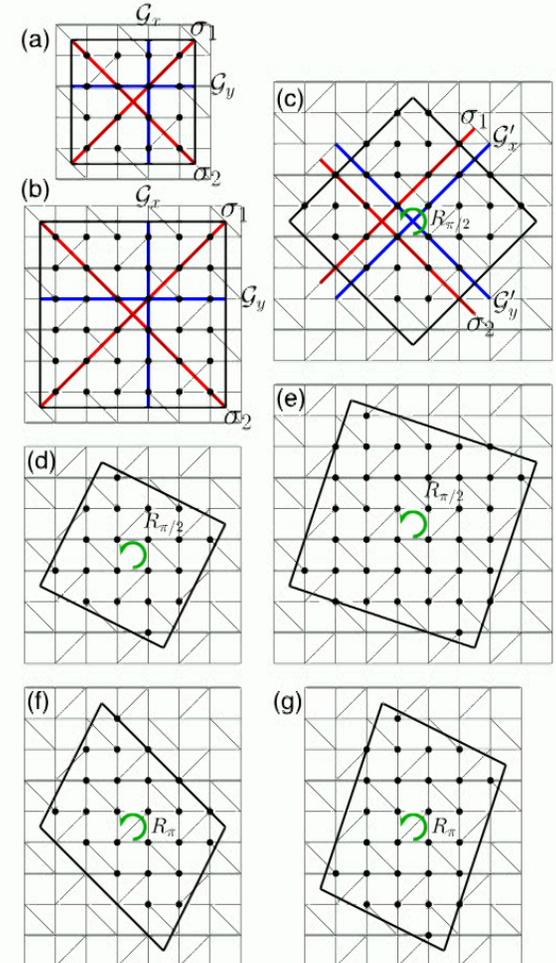
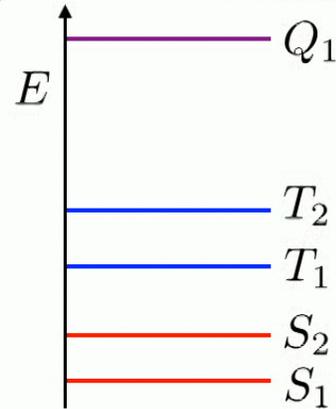
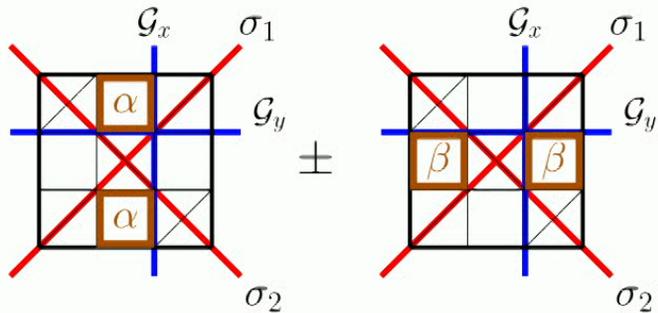
Anderson tower

$$E_S \propto S(S + 1)/N$$

Level crossings in the SS model. 1) Small clusters, exact diag

Wang, Zhang, Sandvik, arXiv:2205.02476

quasi-degenerate singlet ground state of the plaquette-singlet-solid (PSS)



Singlets S_1, S_2 . Triplets T_1, T_2 . Quintuplet Q_1 .

Gaps: $\Delta(S_2) = E(S_2) - E(S_1)$ $\Delta(T_1) = E(T_1) - E(S_1)$

Composite gaps: $\delta_T = E(T_2) - E(S_2)$ $\delta_Q = E(Q_1) - E(T_1)$

Gap criteria for PSS and AFM ordered phases

PSS: $\Delta(S_2) < \Delta(T_1)$

AFM: $\Delta(Q_1) < \Delta(S_2)$, $\delta_T < \Delta(S_2)$

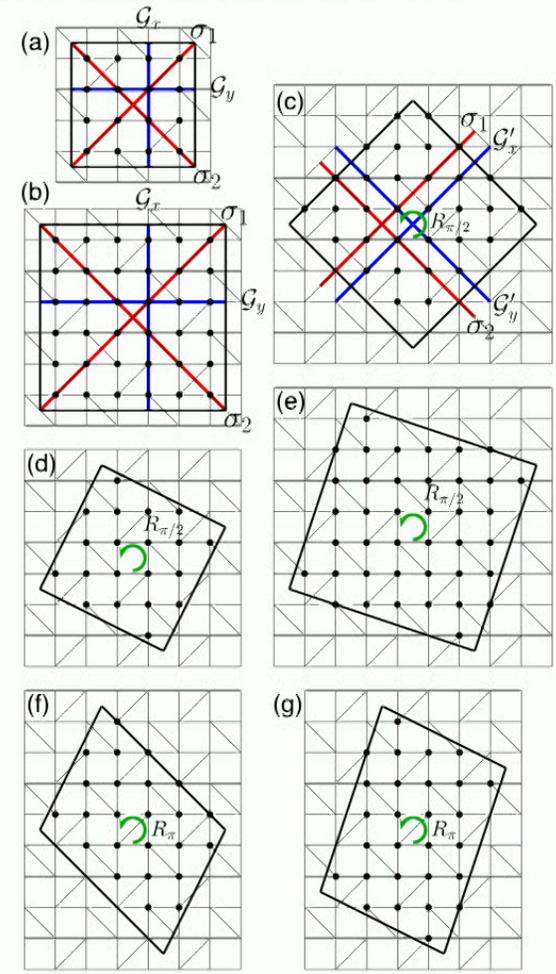
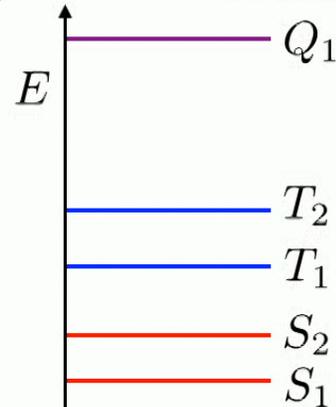
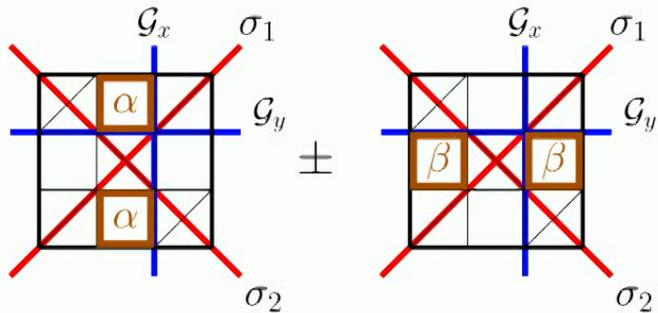
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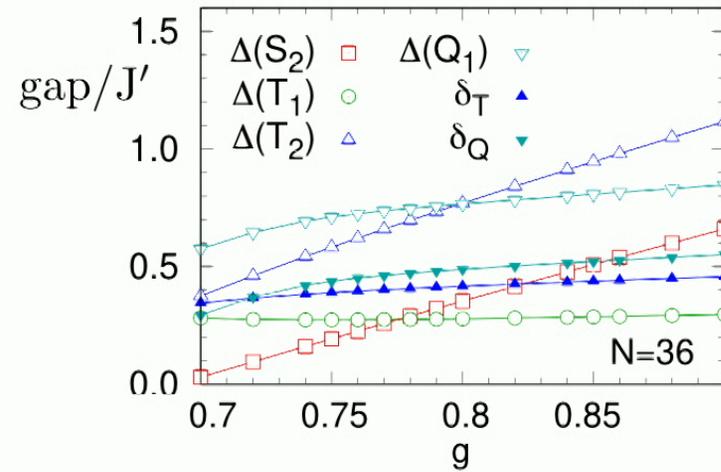
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Gap crossings vs g to detect quantum phase transitions

Example, N=36: gaps versus $g=J/J'$

Size dependence of crossing points

$\Delta(T_1)$ crossing $\Delta(S_2) \rightarrow g_{c1}$ (end of \mathbb{Z}_2 SS phase)

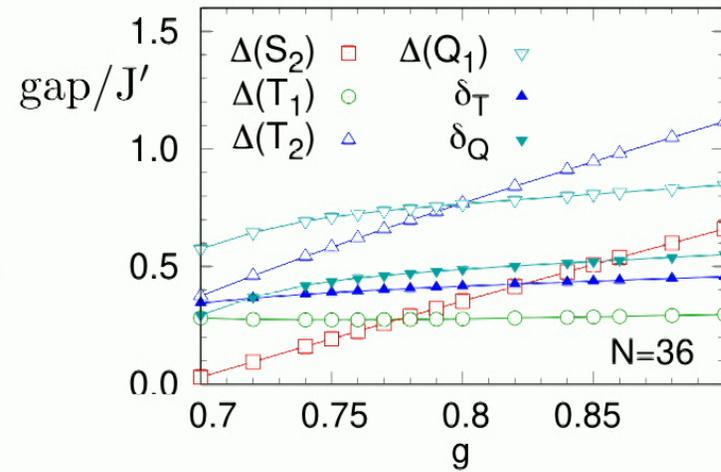


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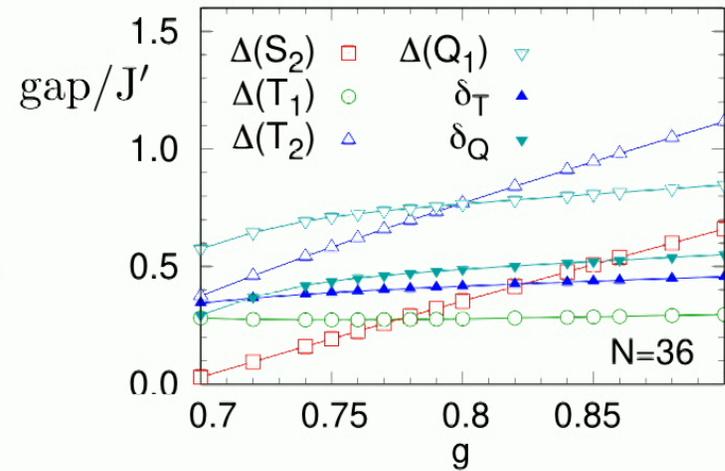
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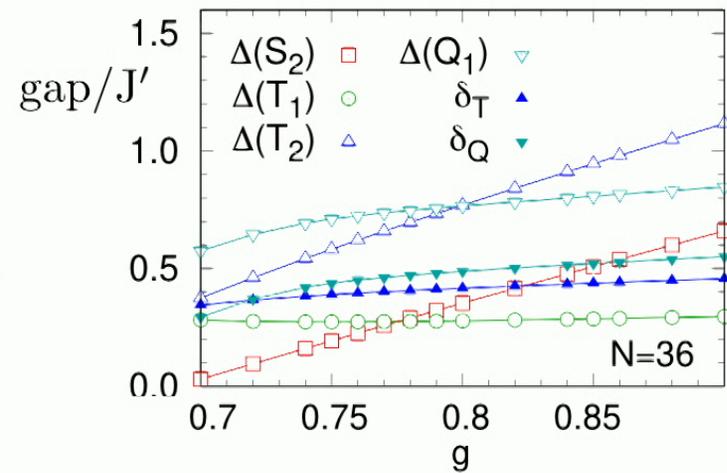
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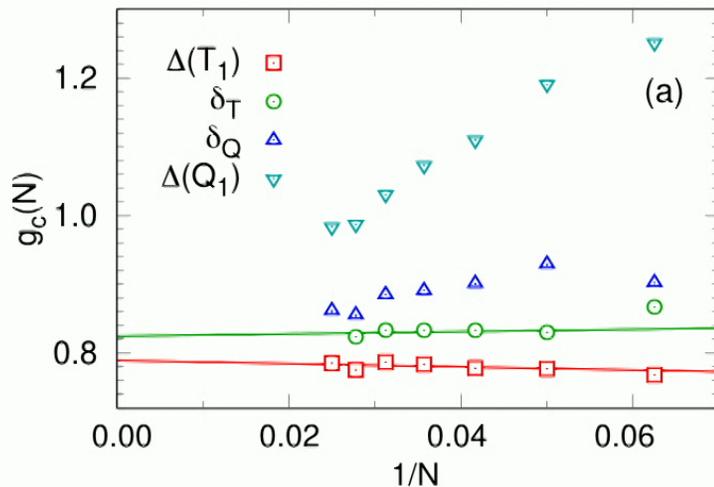
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Crossings points with $\Delta(S_2)$



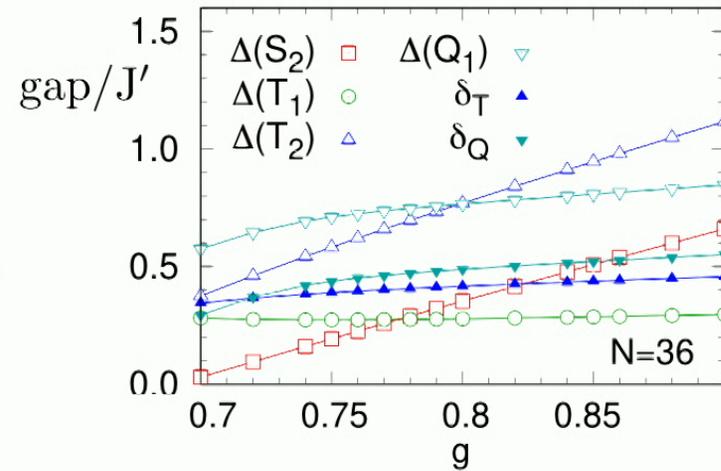
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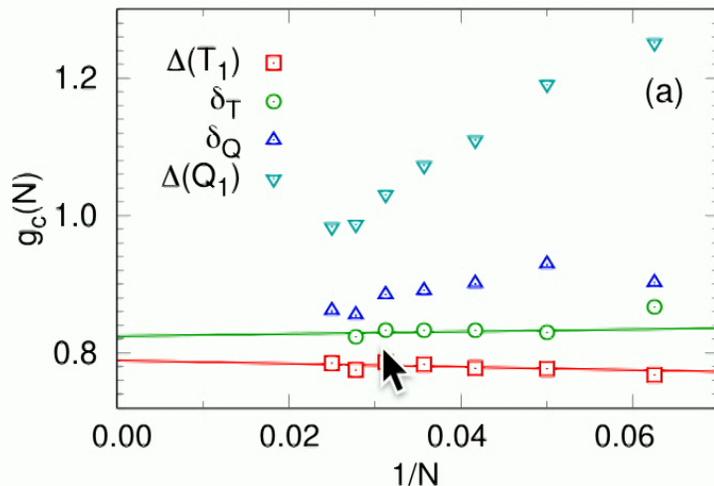
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Crossings points with $\Delta(S_2)$



**crossing points
are correlated**

**shift all points
equally for
given N**

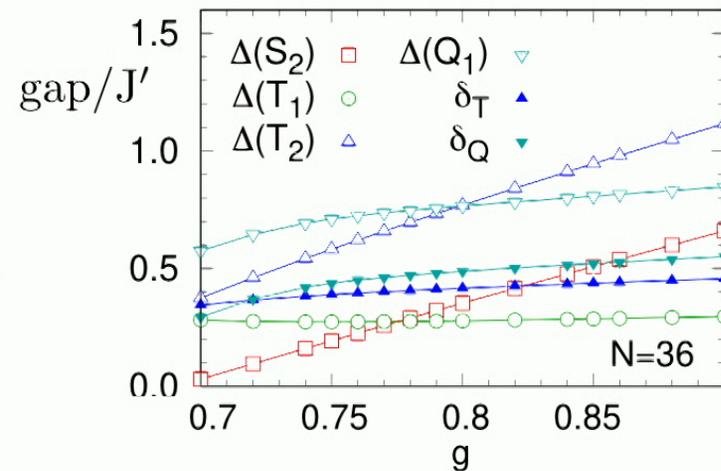
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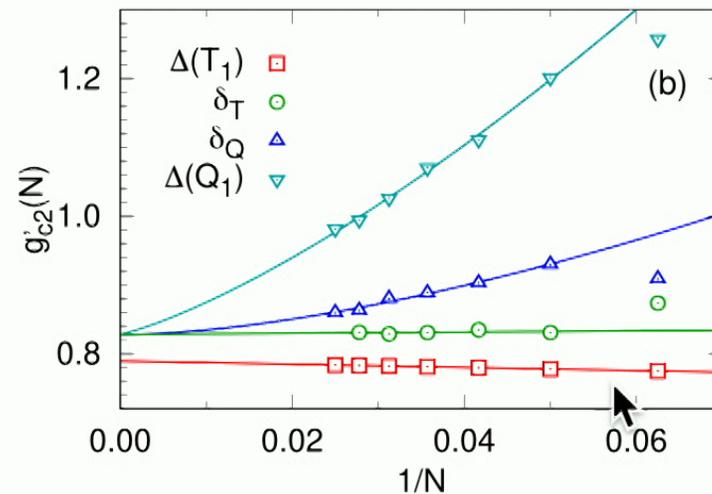
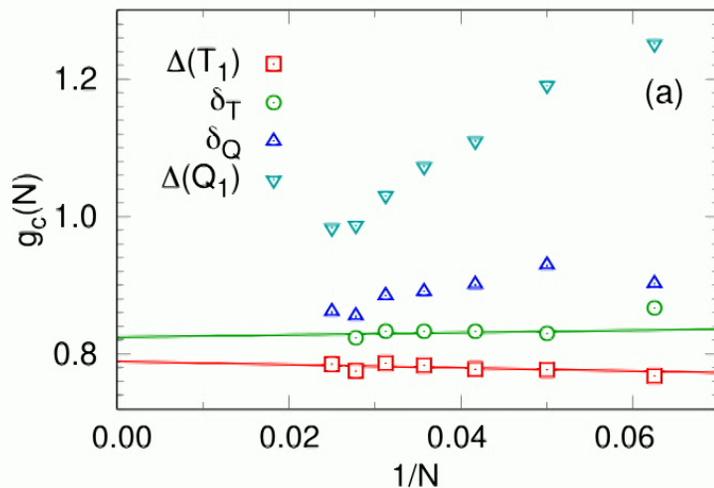
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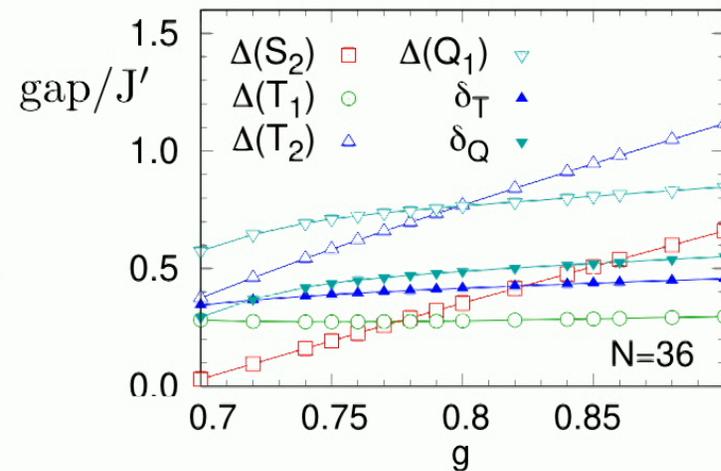
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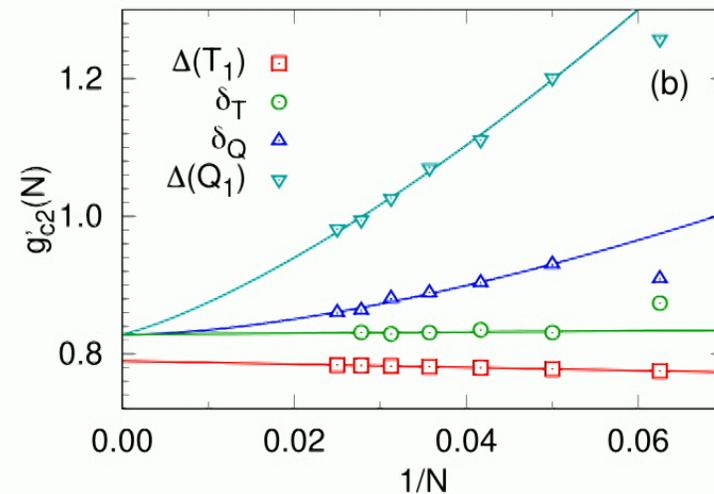
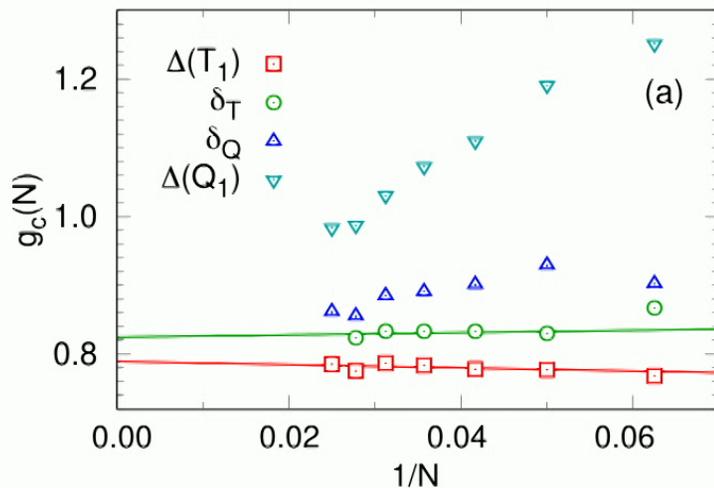
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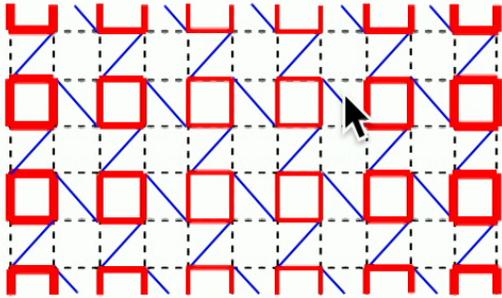
shift all points equally for given N

$g_{c1} < g_{c2}$: Other phase between PSS and AFM for $g \in [0.79, 0.82]$

Level crossings in the SS model. 2) DMRG, cylinders

$2L \times L$ spins, open x boundaries, periodic y boundaries

Yang, Wang, Sandvik, PRB 2022

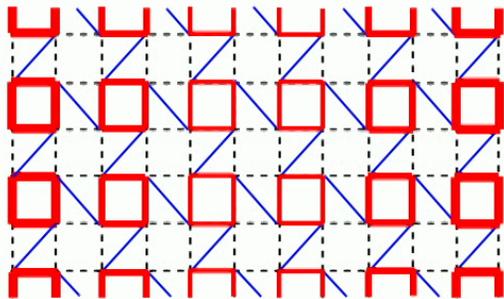


PSS state is unique
- boundaries break symmetry

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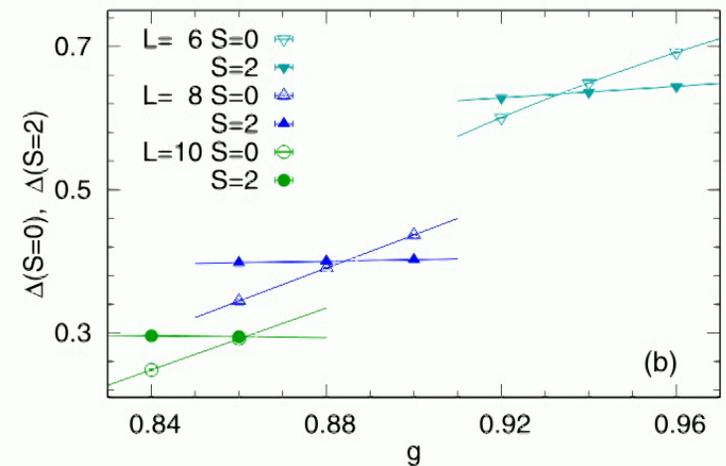
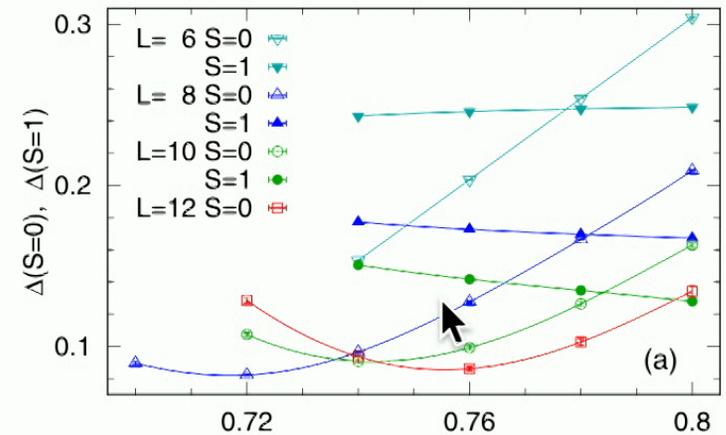
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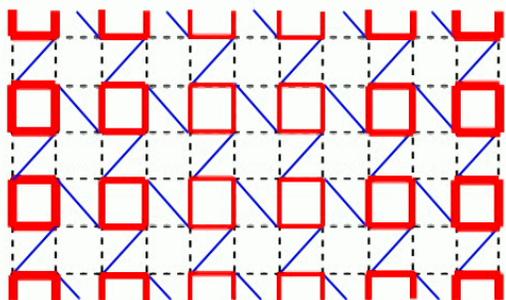
Second singlet is not degenerate ground state
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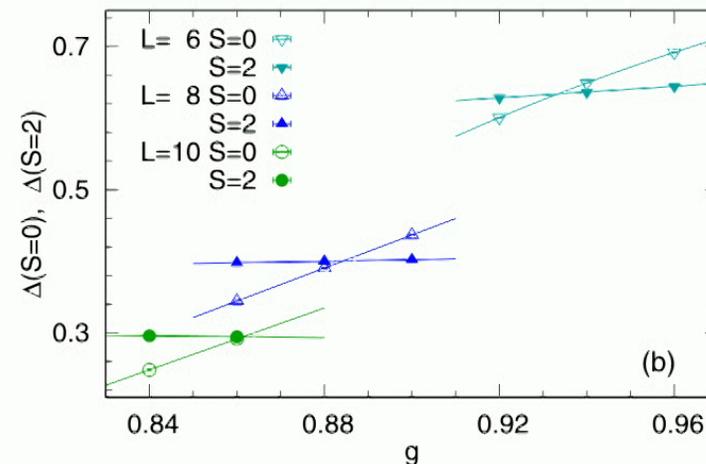
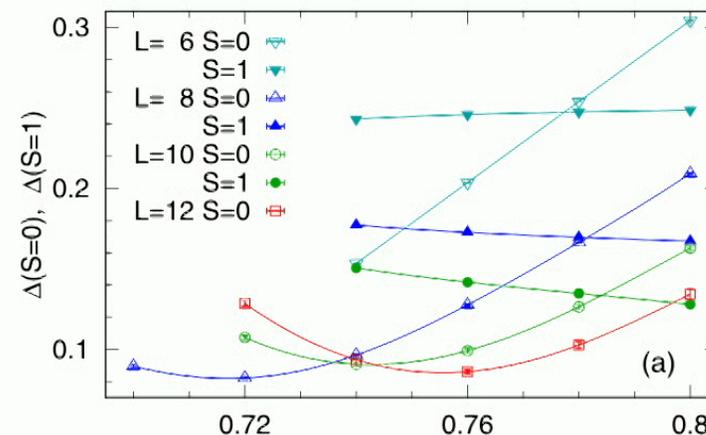
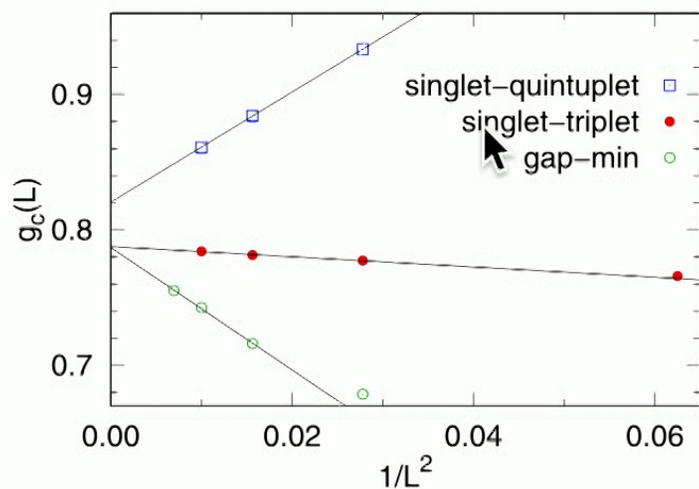
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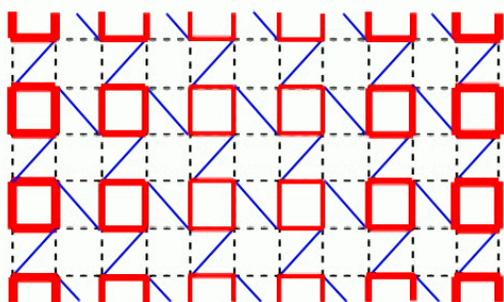
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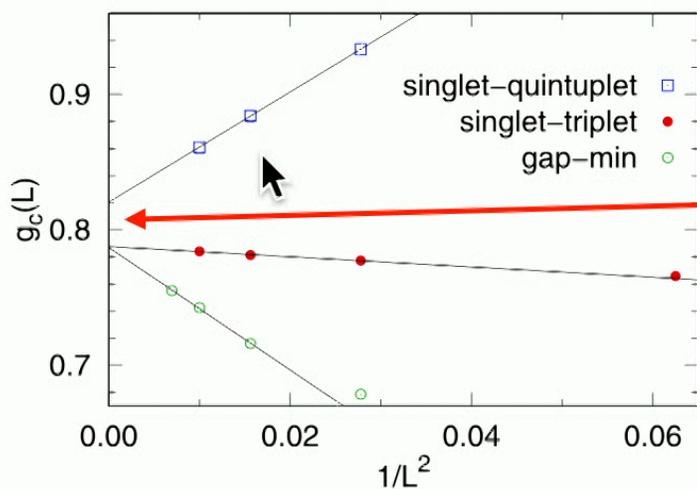
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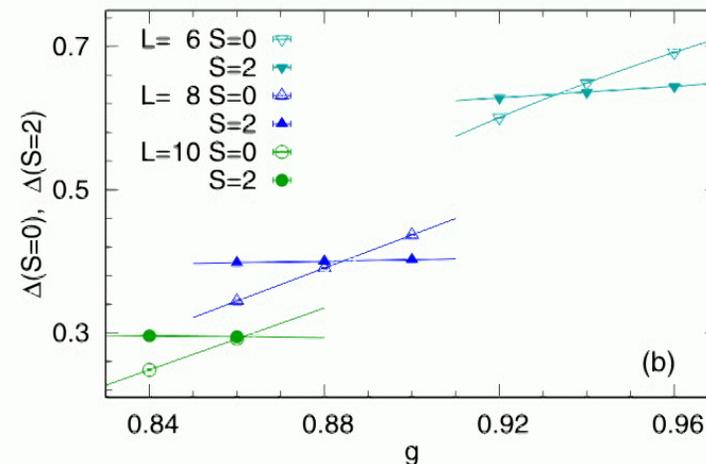
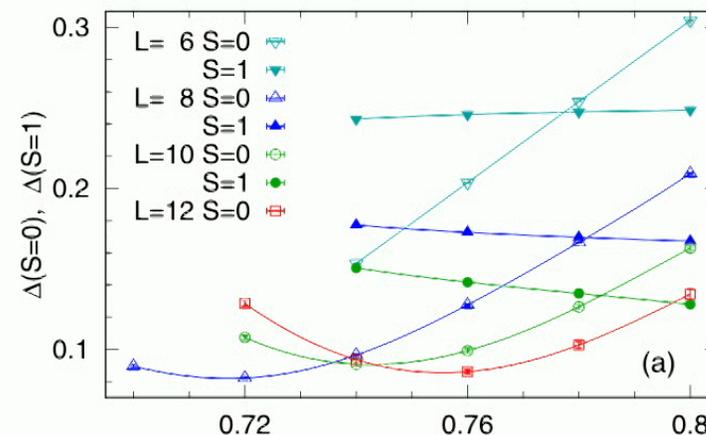


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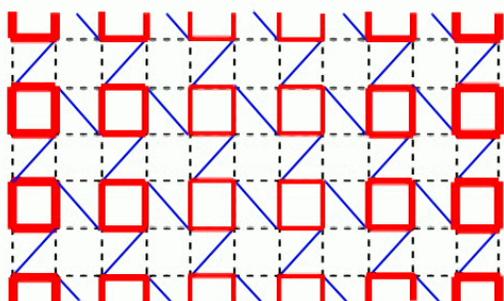
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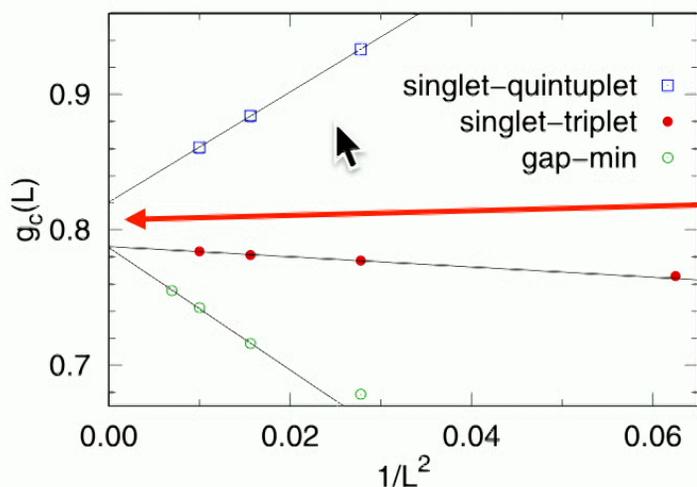
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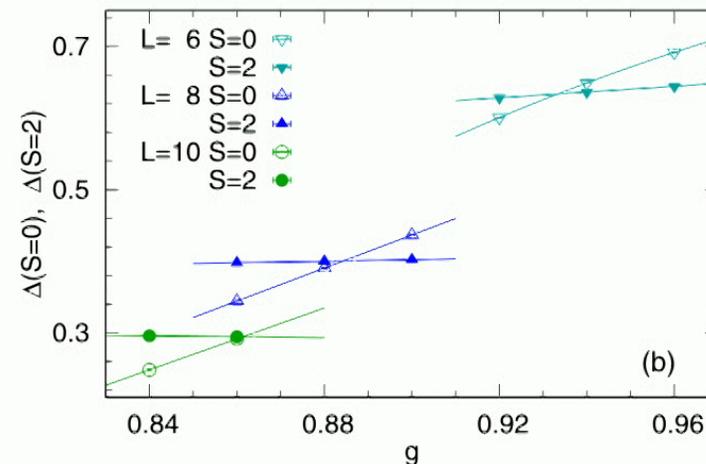
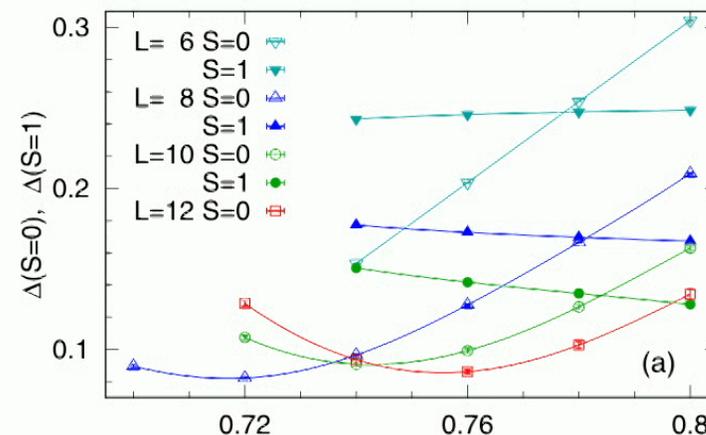
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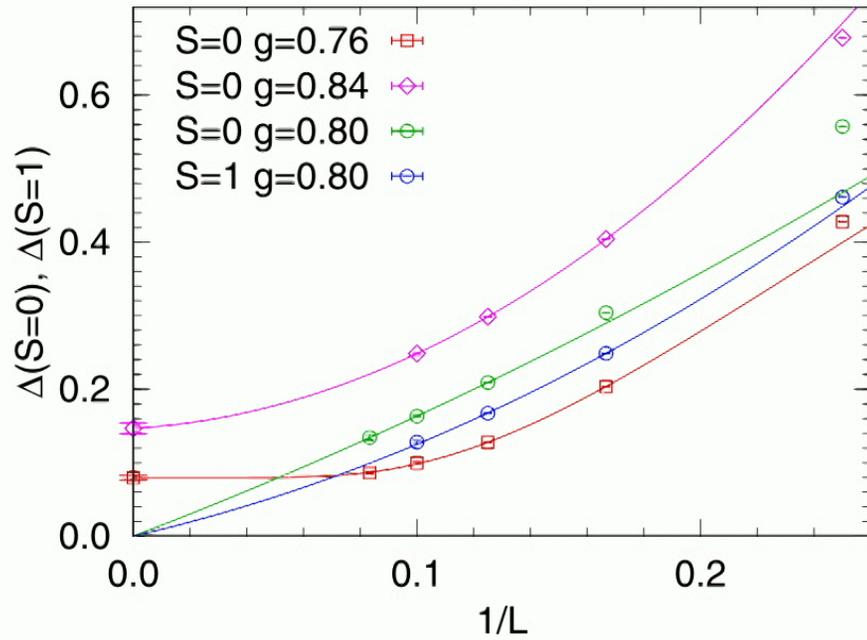


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1/L² drifts of crossing points

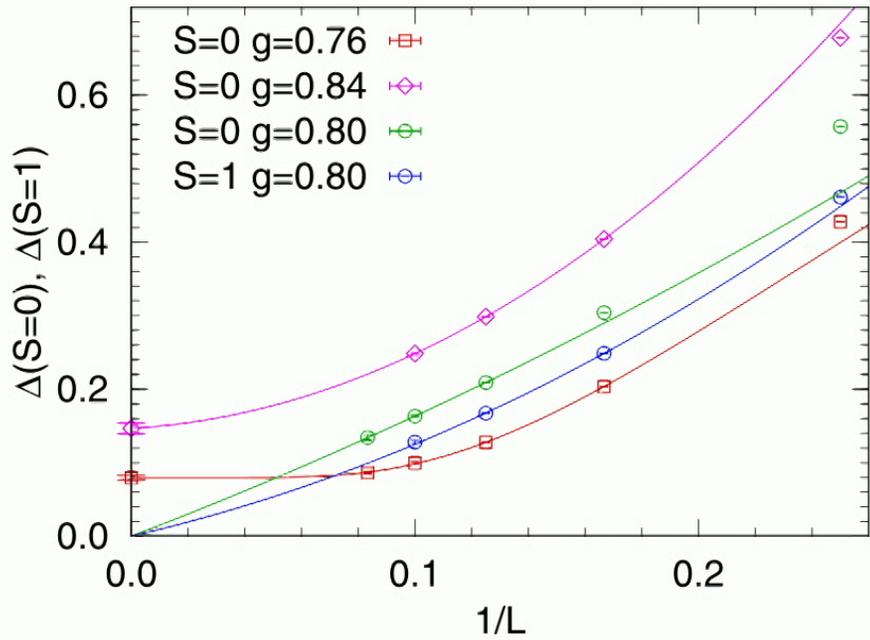


SS model DMRG, spin liquid properties



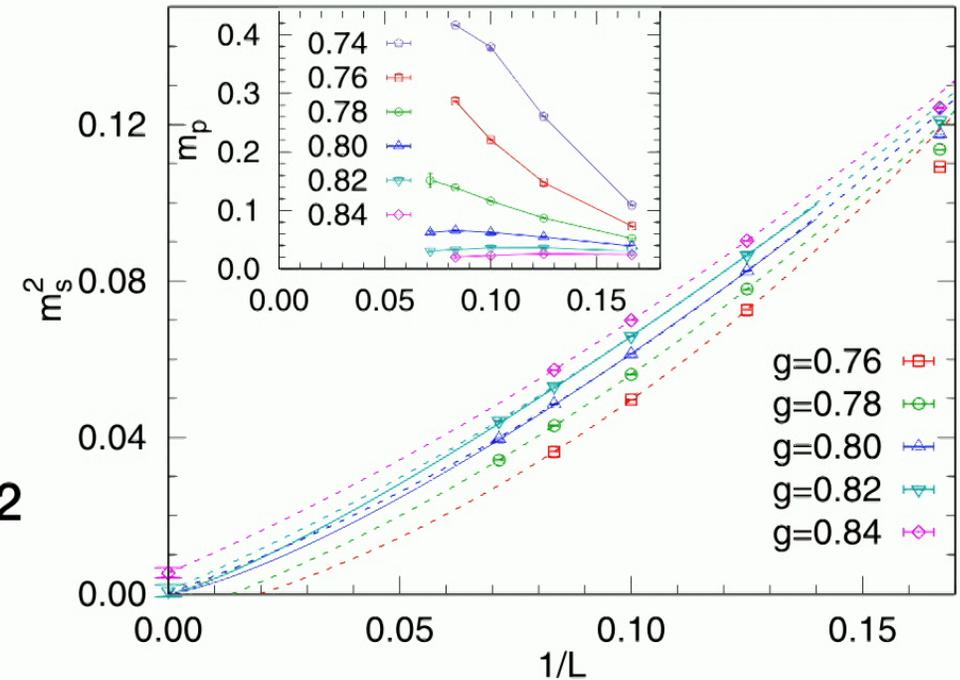
Gaps in the spin liquid phase decay as $1/L$ ($z=1$)

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Spin correlations decay as $1/r^{1+\eta}$, $\eta \approx 0.2$

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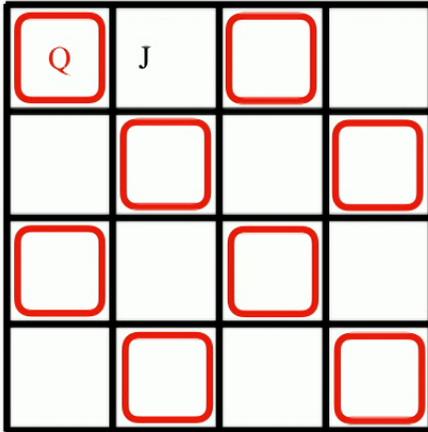


Checker-board J-Q (CBJQ) model, QMC

Zhao, Weinberg, Sandvik, Nat. Phys. 2019

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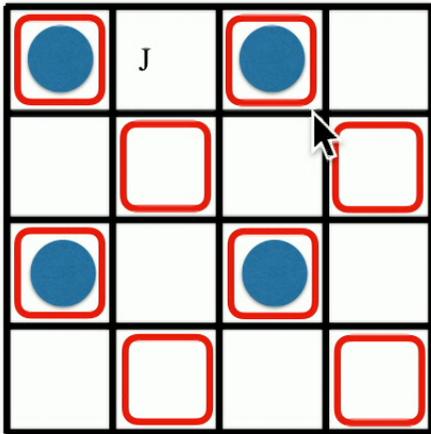


$$\mathcal{H} = -J \sum_{\langle ij \rangle} P_{ij} - Q \sum_{ijkl \in \square'} (P_{ij} P_{kl} + P_{ik} P_{jl})$$

$$P_{ij} = \frac{1}{4} - \mathbf{S}_i \cdot \mathbf{S}_j$$

Checker-board J-Q (CBJQ) model, QMC

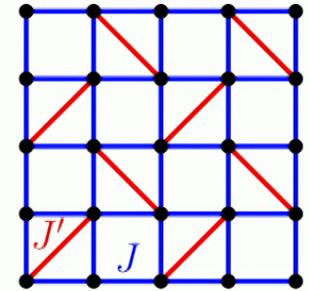
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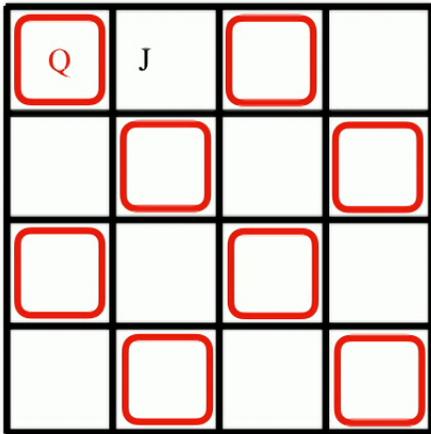
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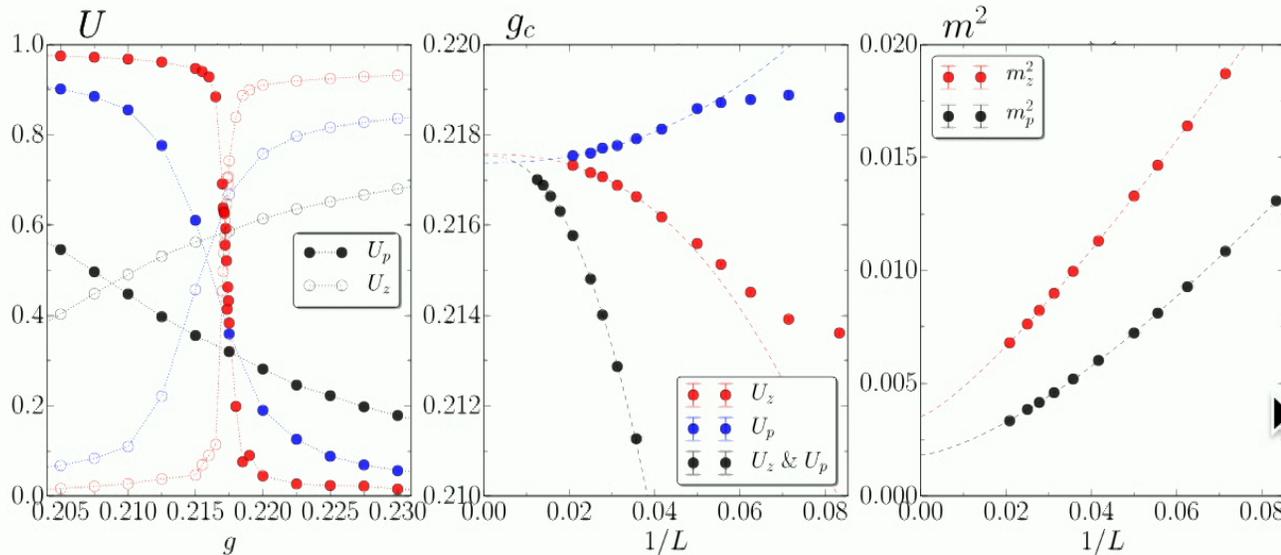
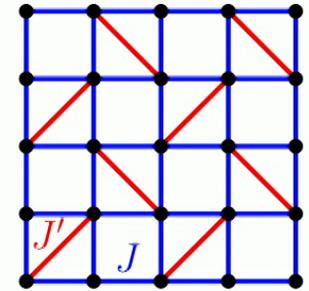
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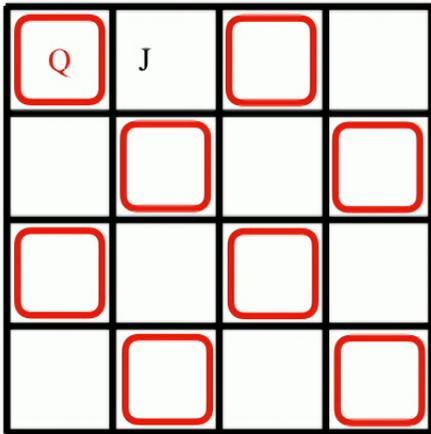
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QMC simulations
- periodic $L \times L$ lattices
- periodic boundaries

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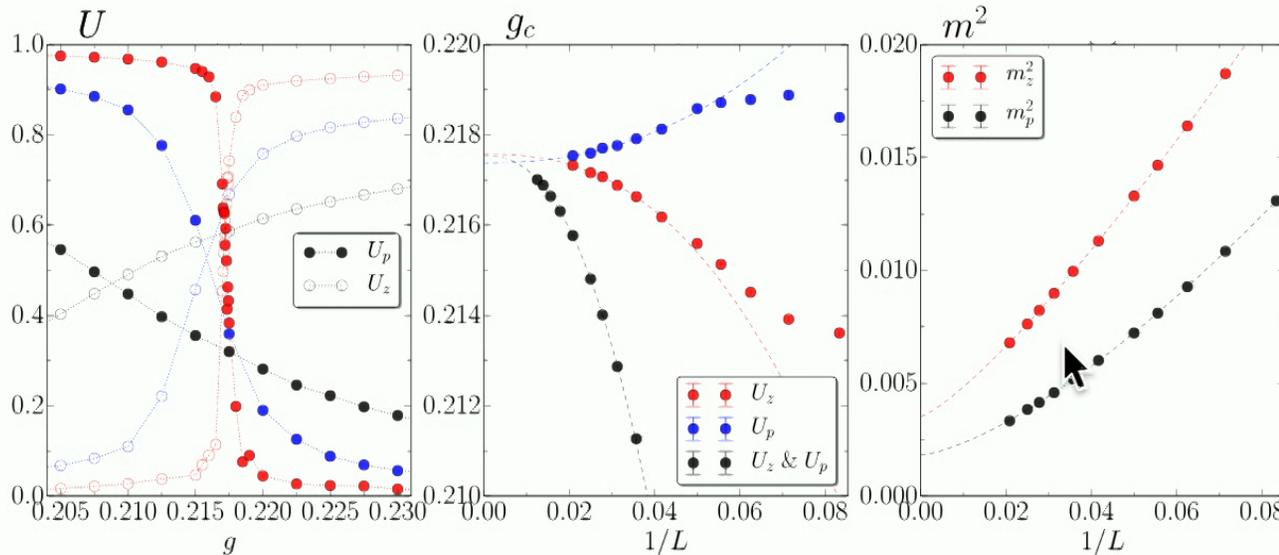
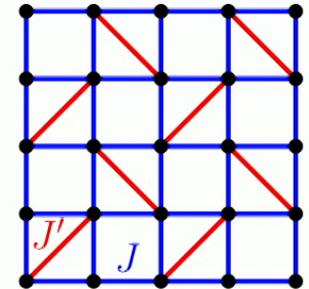
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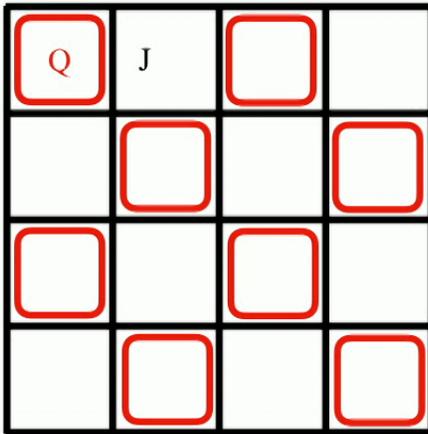
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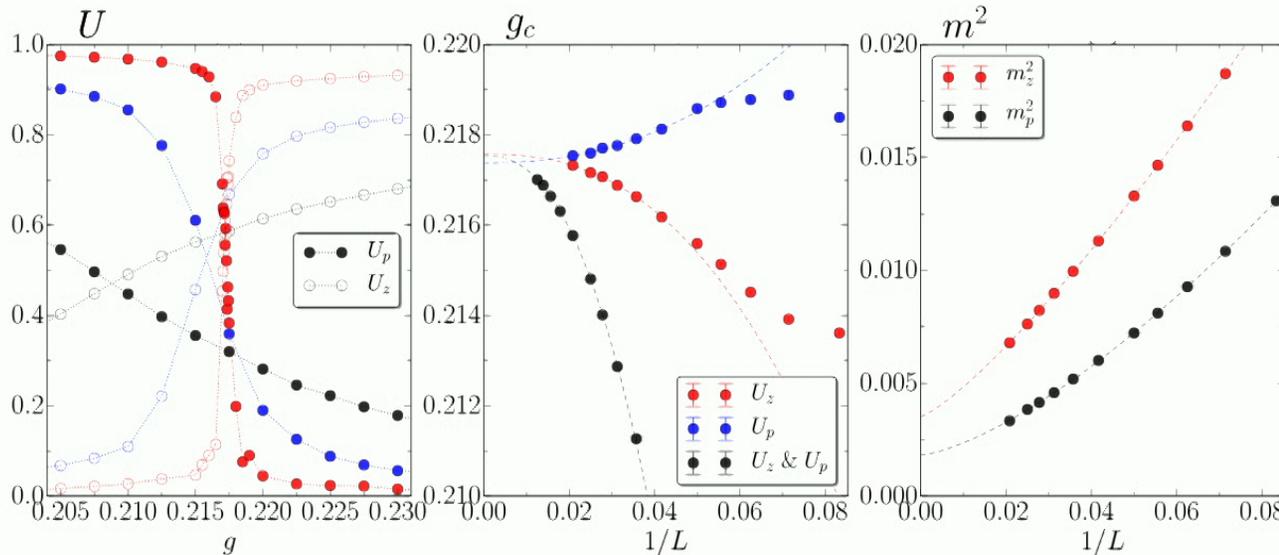
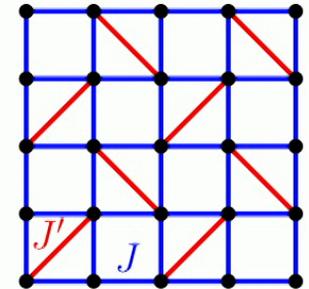
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First-order direct
PSS - AFM transition

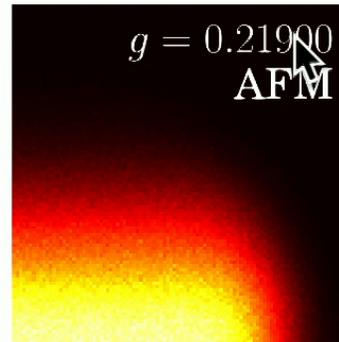
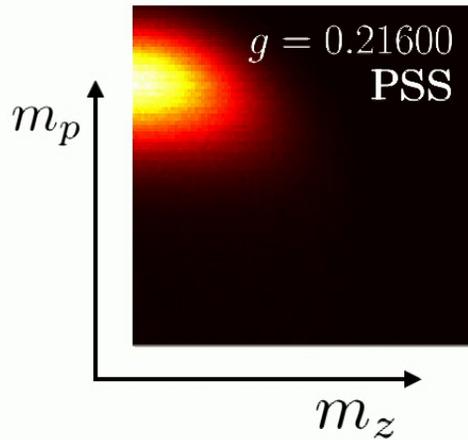
Emergent O(4) symmetry of the coexistence state

Combined PS,AFM order parameter: (m_p, m_x, m_y, m_z)

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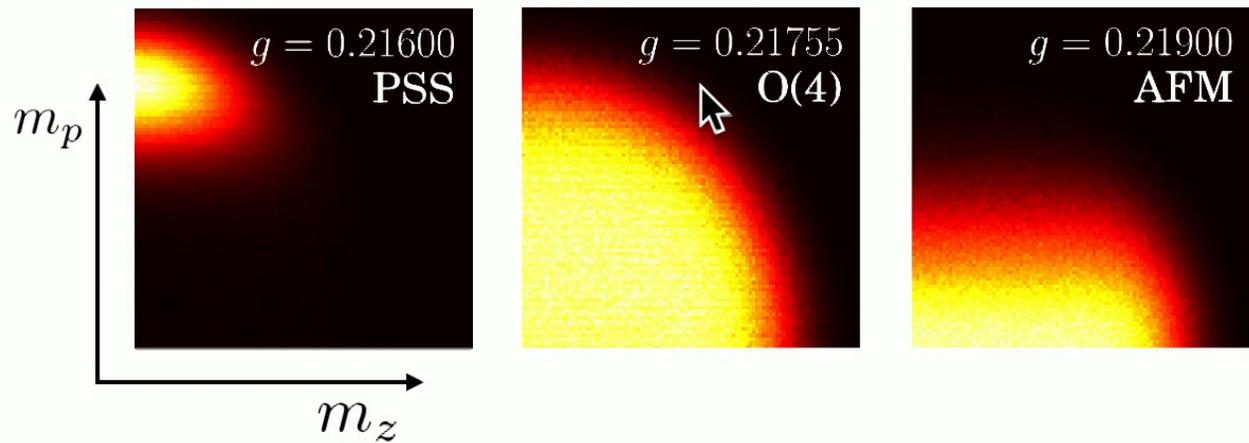
Distribution $P(m_p, m_z)$ (PS, AFM z-component), $L=96$



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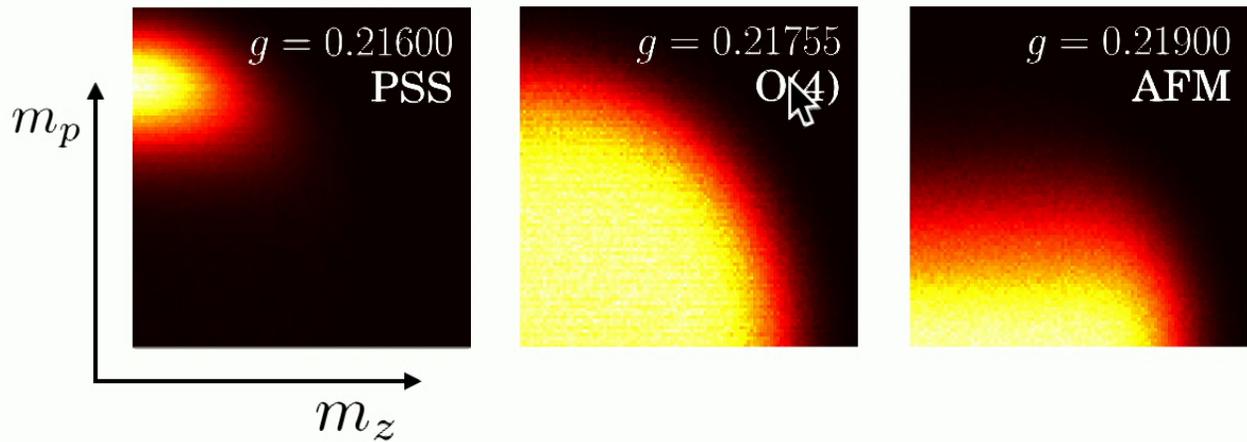
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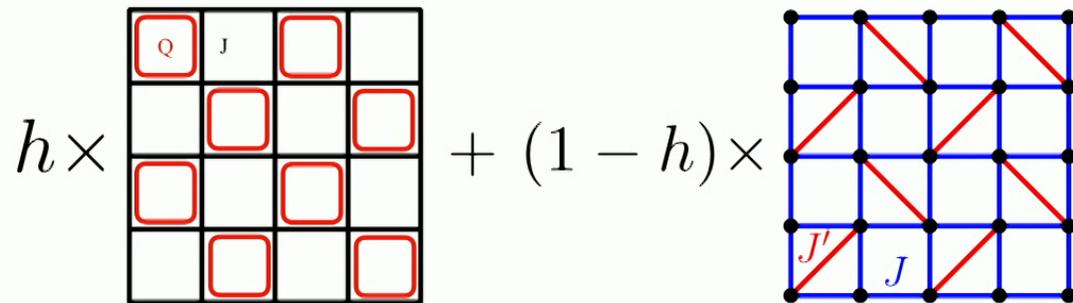
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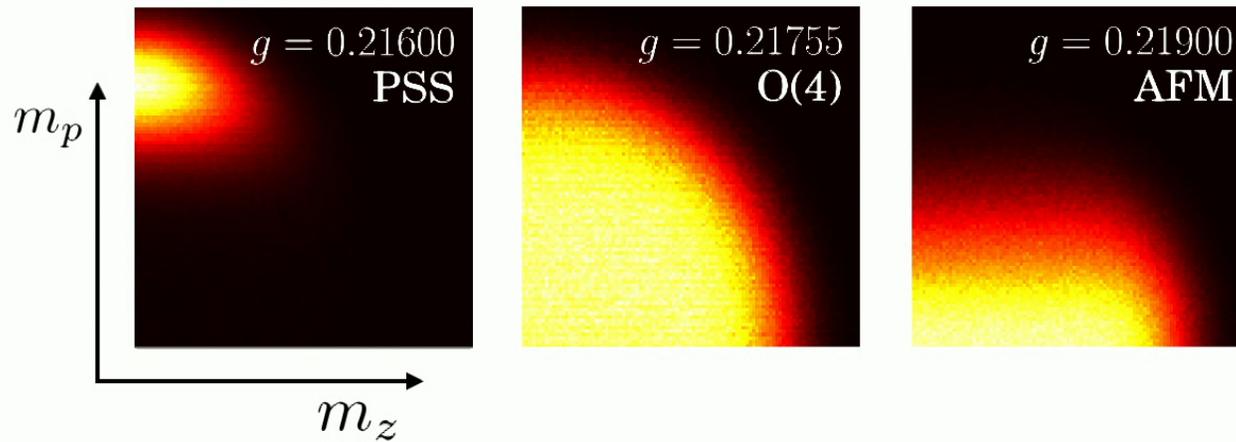
Combine CBJQ and SS models



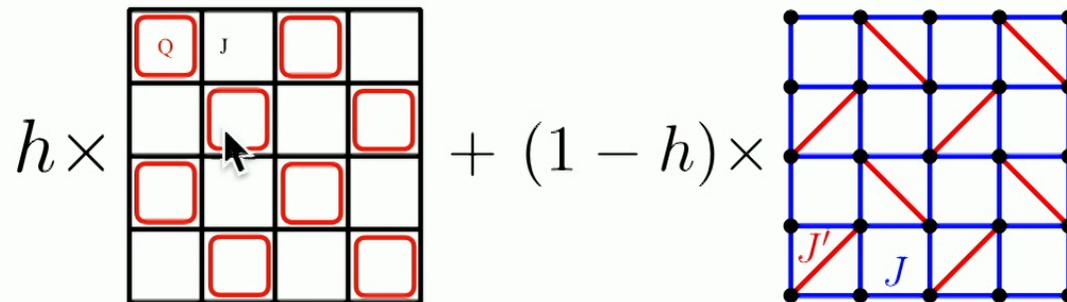
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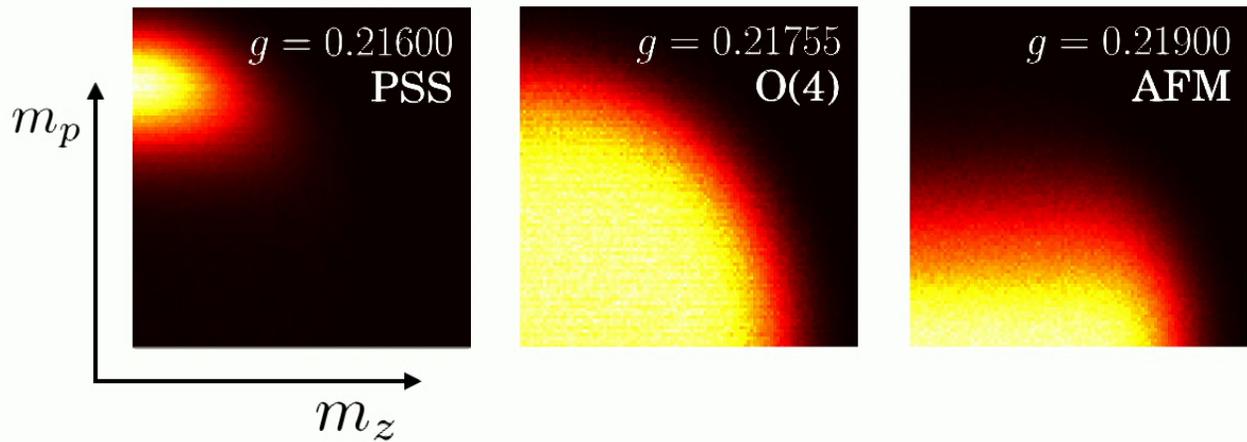
Combine CBJQ and SS models



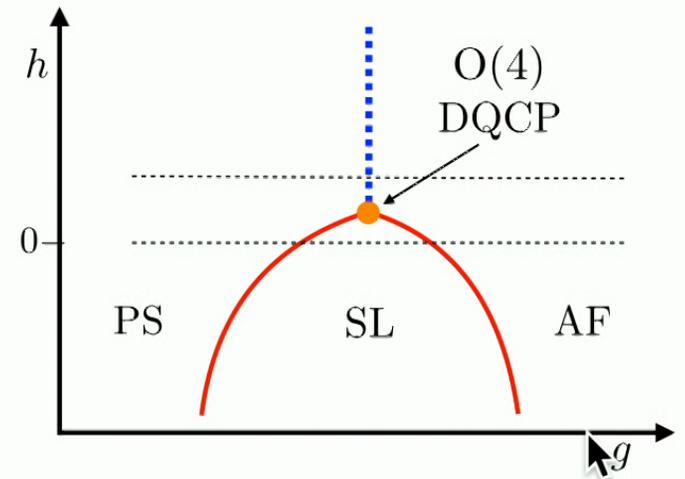
Emergent O(4) symmetry of the coexistence state

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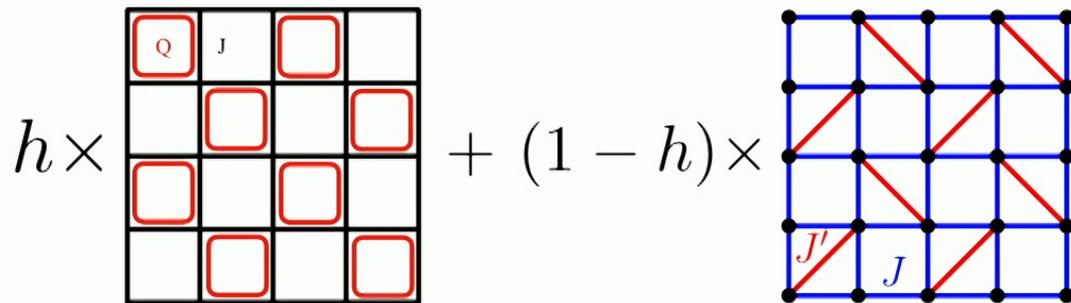
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Multi-criticality scenario



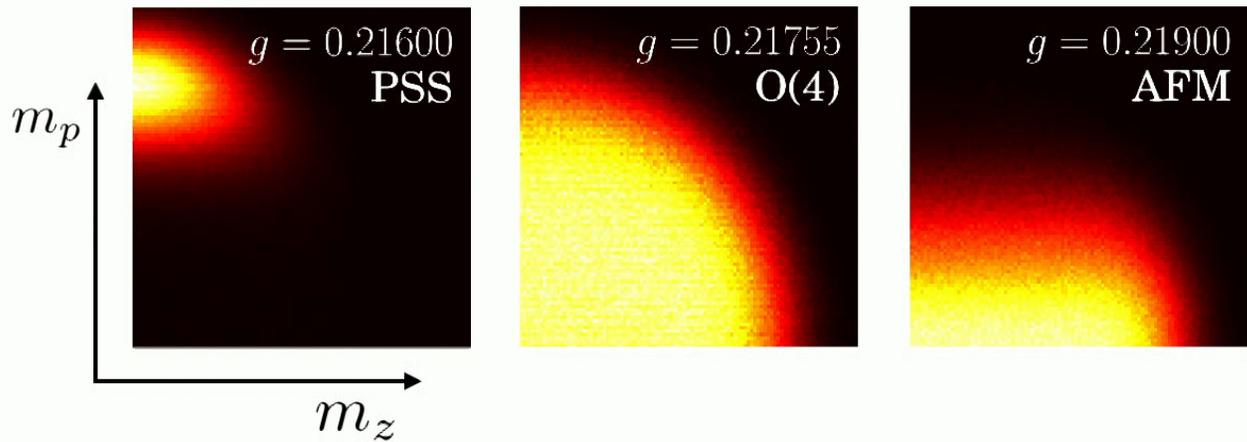
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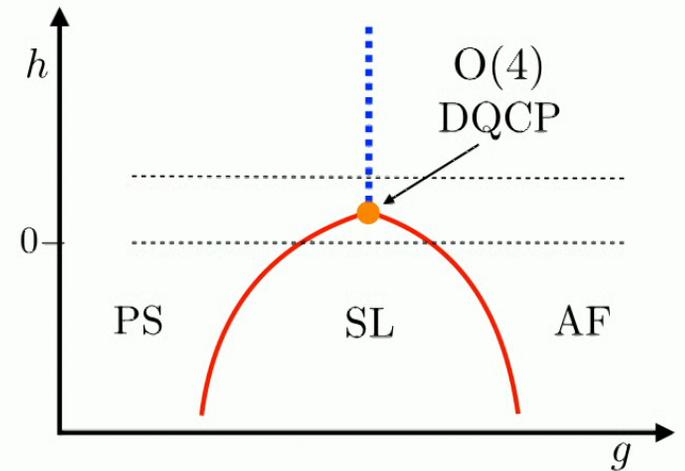
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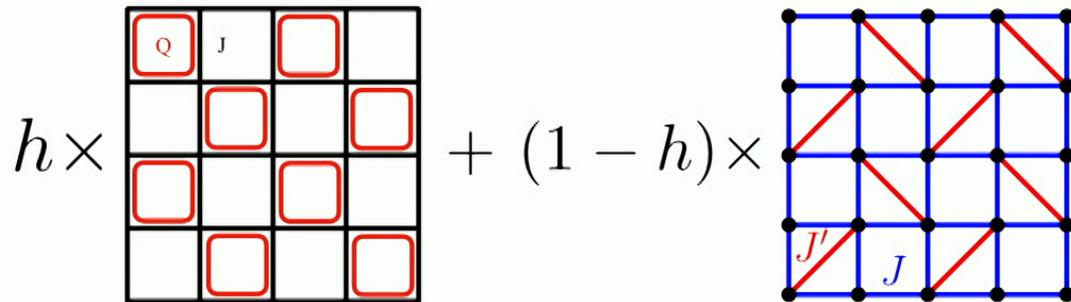
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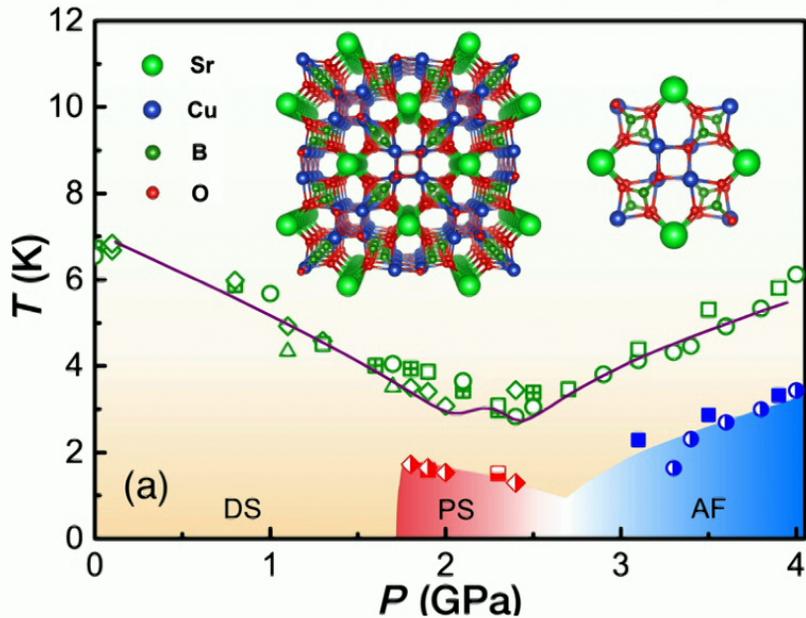
Also discussed in the context of Z_4 breaking VBS; $SO(5)$ DQCP - identified new relevant operator
 Zhao, Takahashi, Sandvik (PRL 2020)

Experiments: Shastry-Sutherland material $\text{SrCu}_2(\text{BO}_3)_2$

PHYSICAL REVIEW LETTERS **124**, 206602 (2020)

Quantum Phases of $\text{SrCu}_2(\text{BO}_3)_2$ from High-Pressure Thermodynamics

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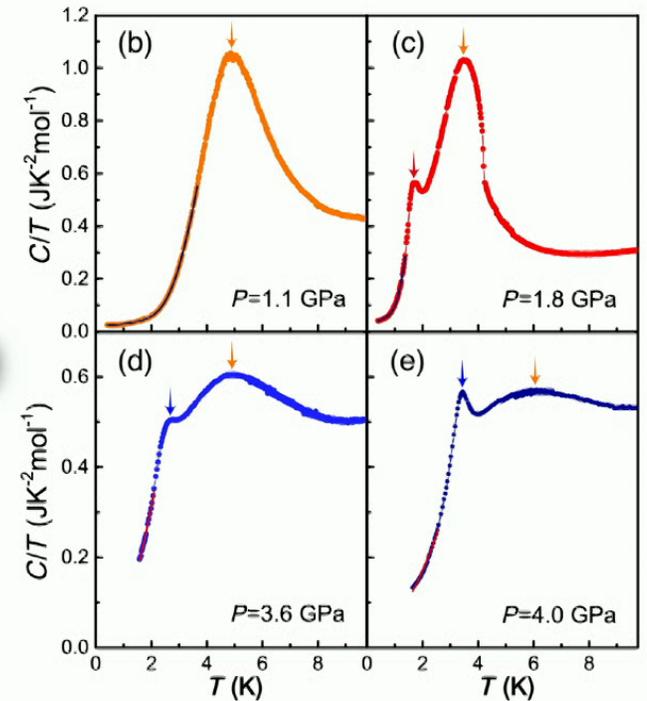
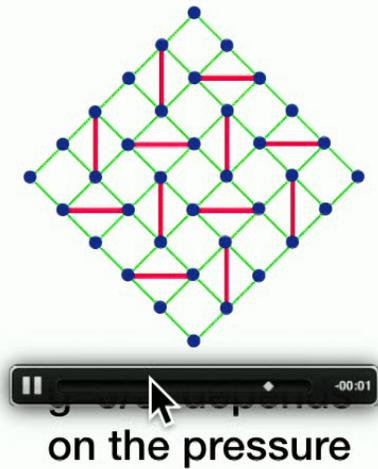
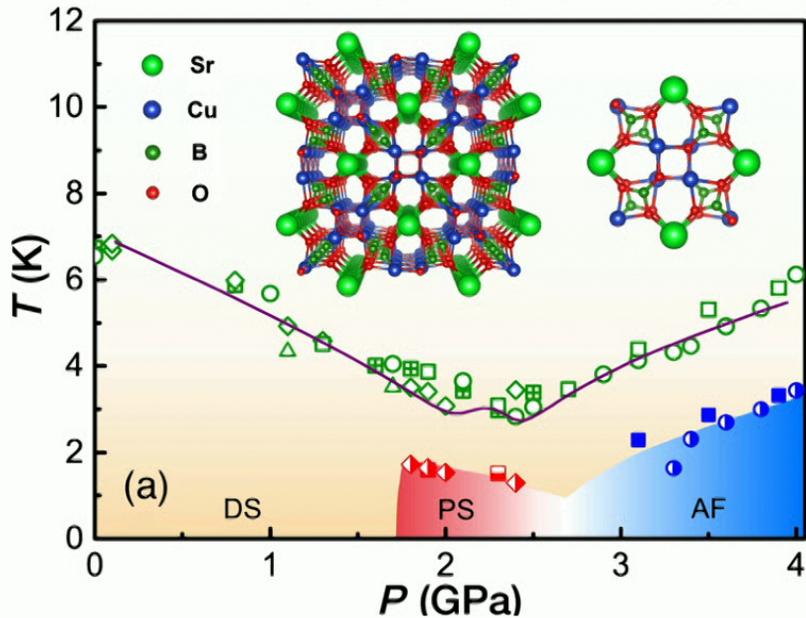


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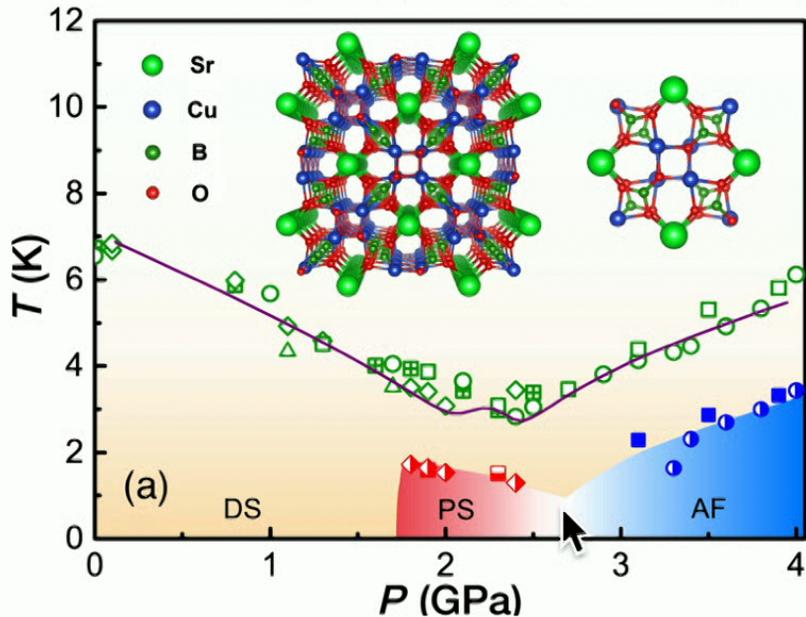
First experimental phase diagram
- consistent with the SS model

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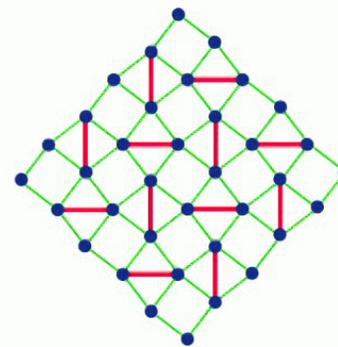
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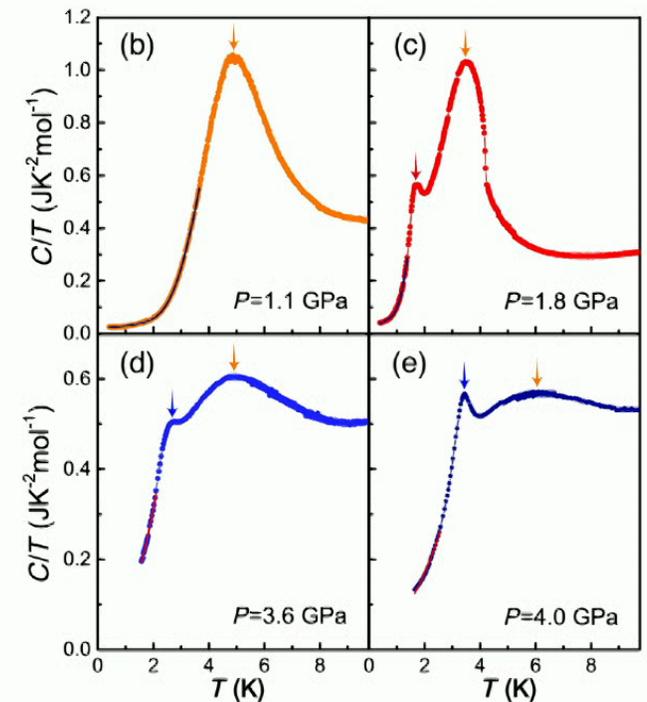


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$g=J/J'$ depends
on the pressure

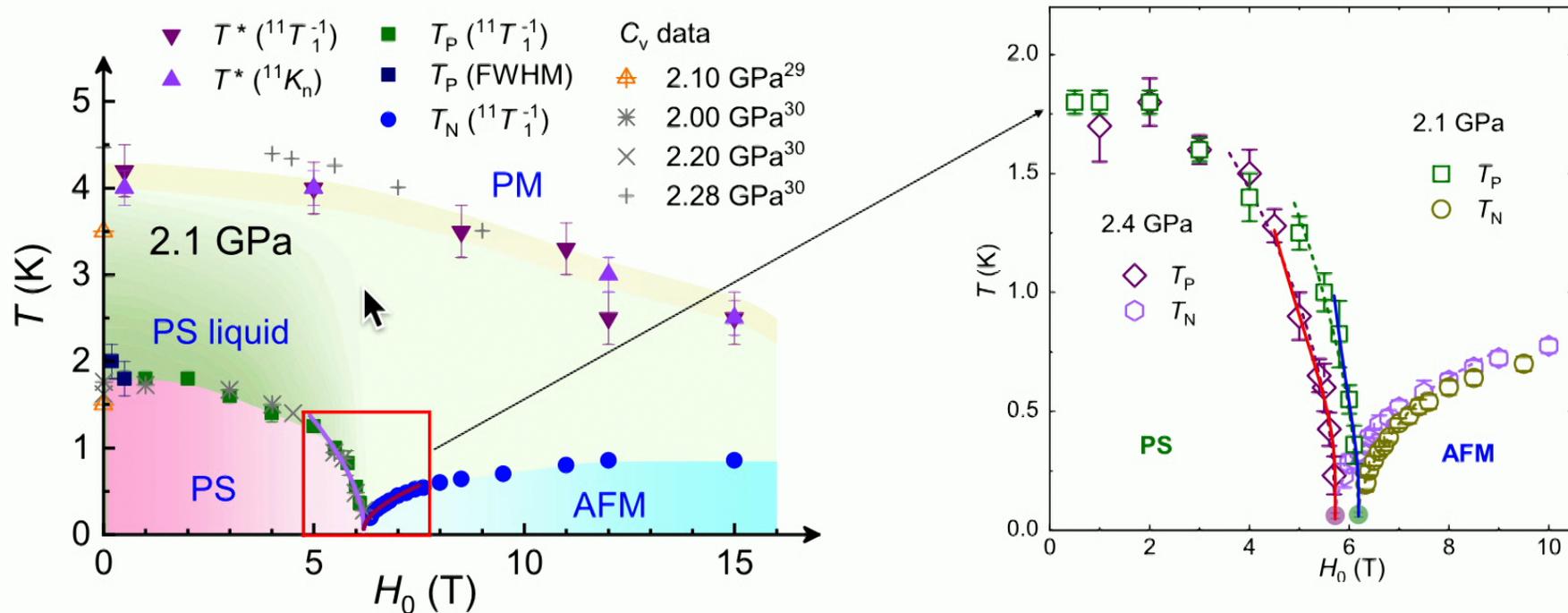
Spin liquid
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Deconfined quantum criticality and emergent symmetry in $\text{SrCu}_2(\text{BO}_3)_2$

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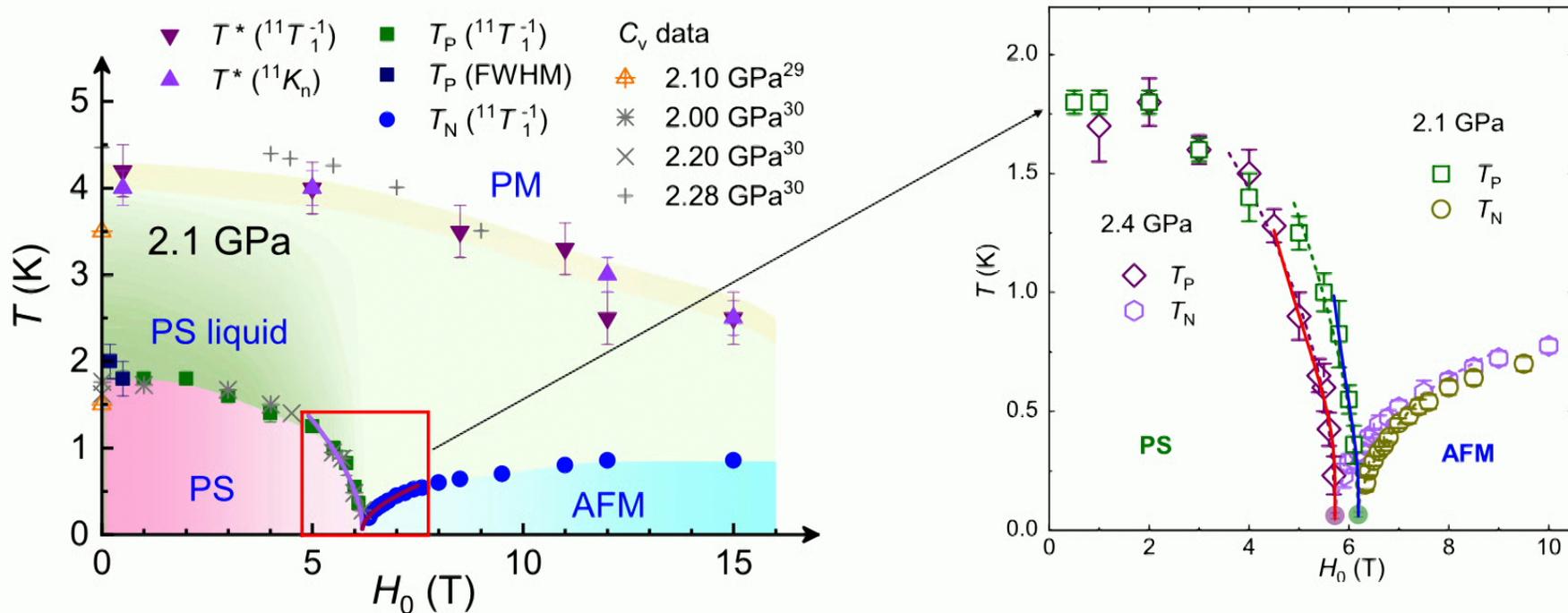
Suppression of PS phase by magnetic field, NMR (^{11}B) measurements



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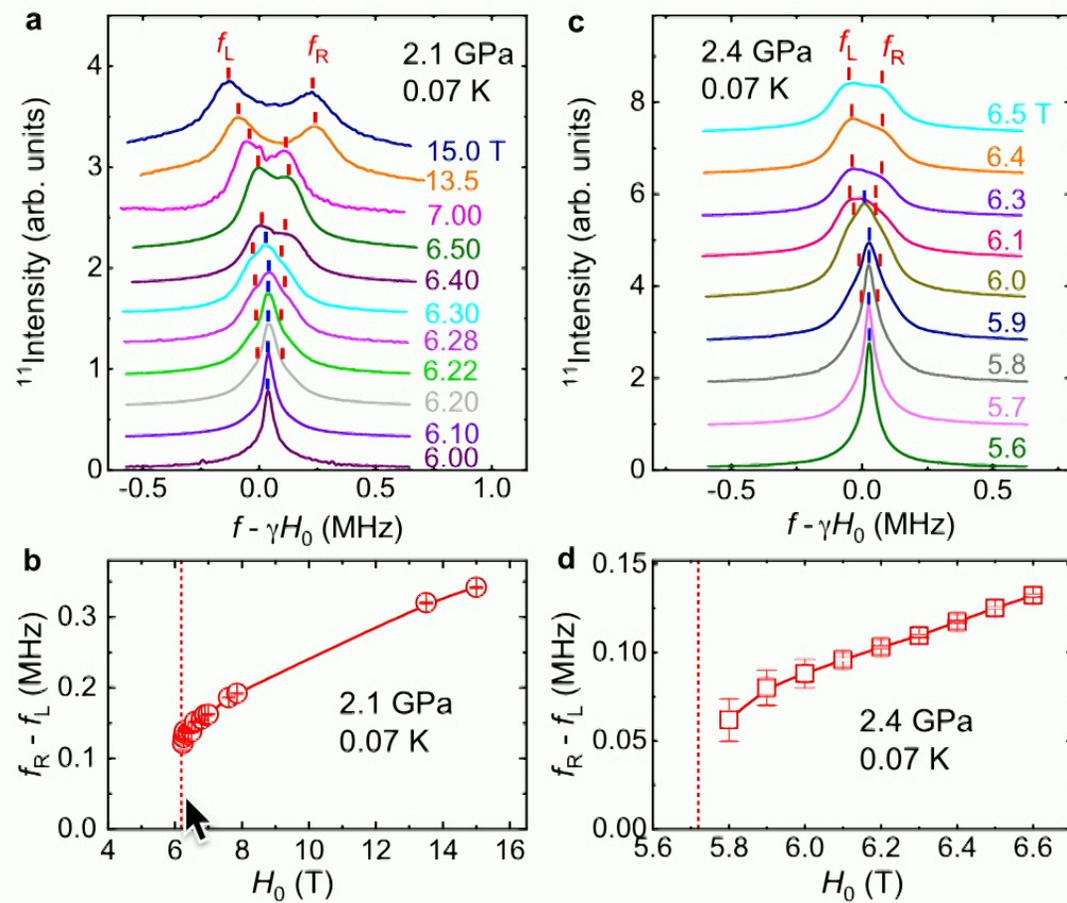
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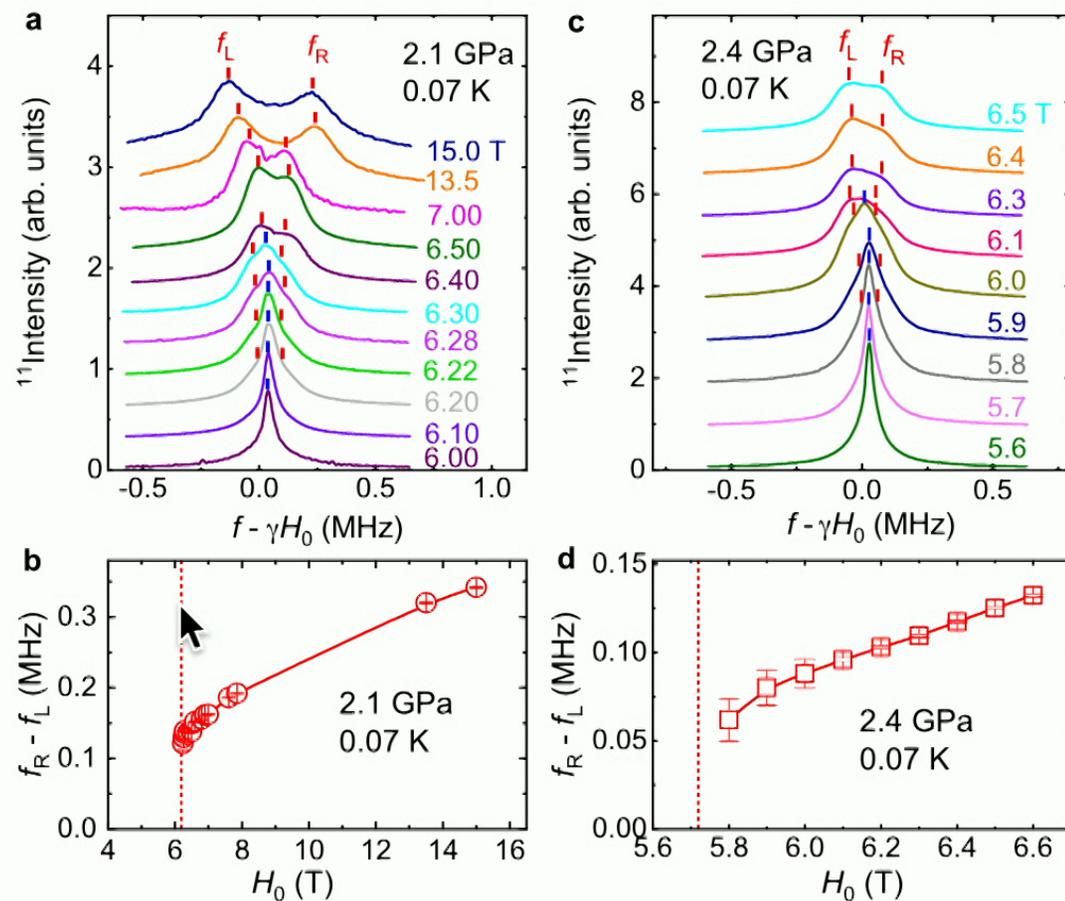


Common PS and AFM transition at $T \sim 0.07$ K, $H_0 \sim 6$ T

Proxy AFM order parameter: NMR line splitting

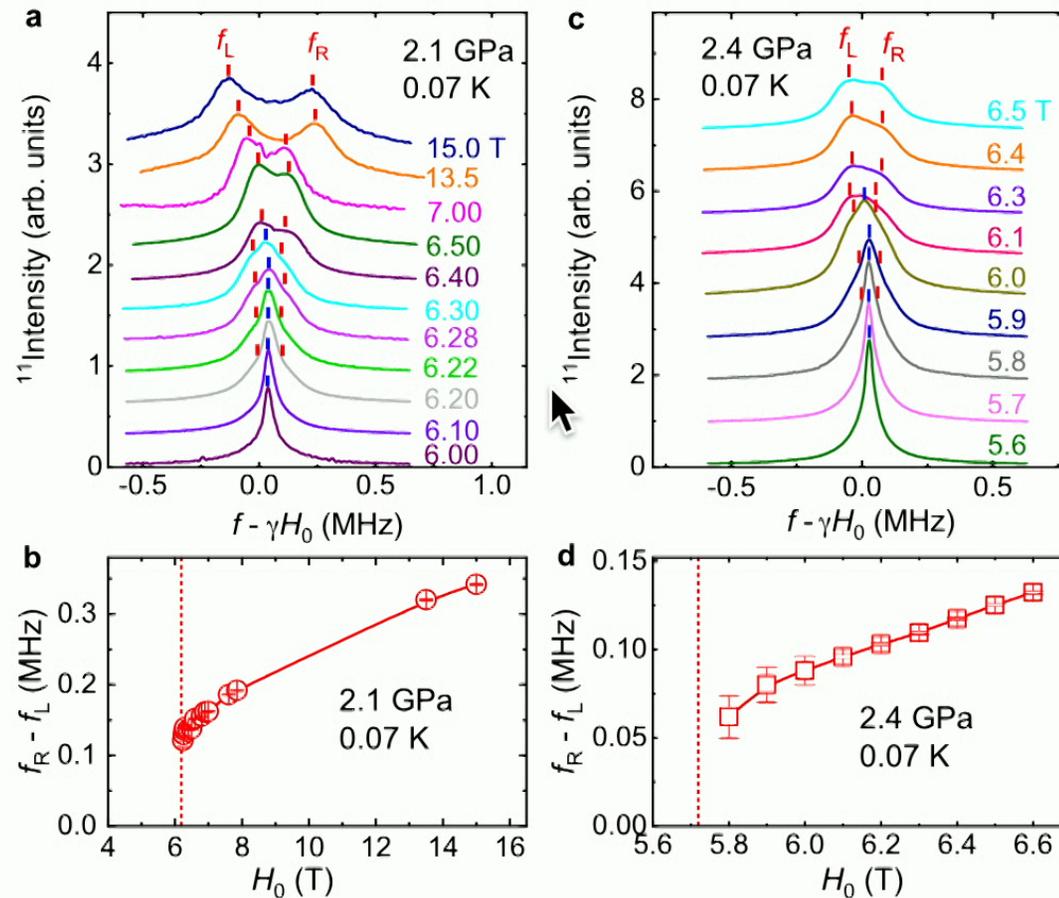


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Transition point (dashed lines) determined more precisely from spin-lattice relaxation rate $1/T_1$

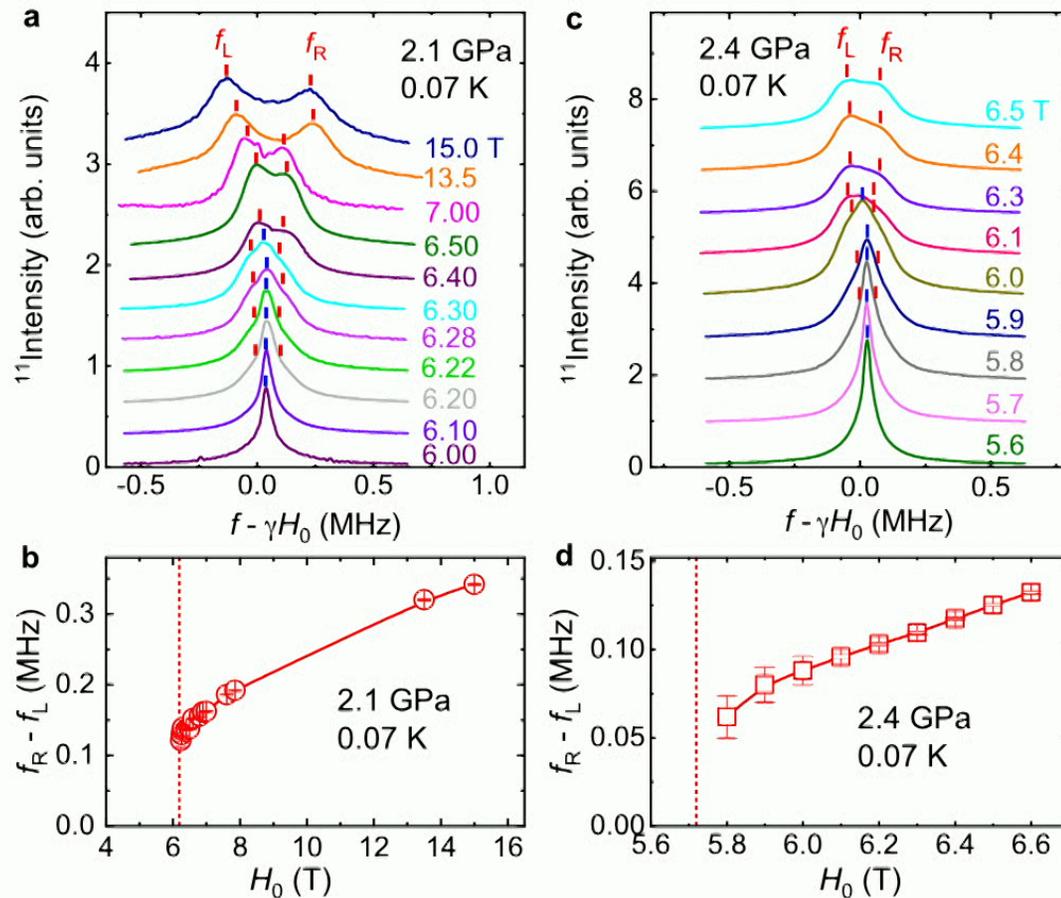
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- Discontinuity smaller at higher pressure
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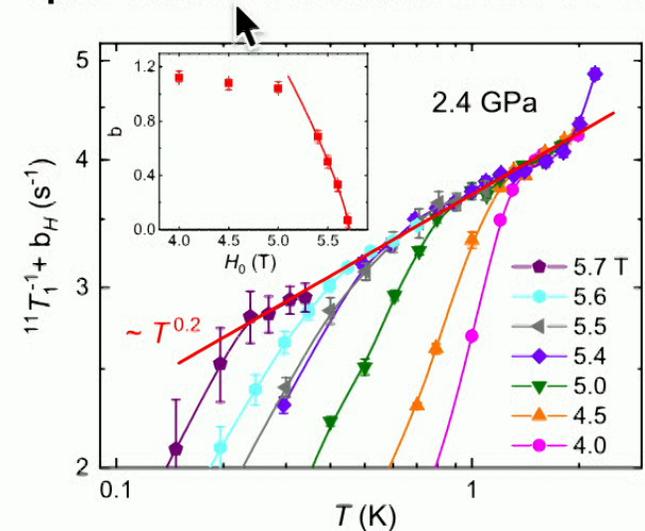
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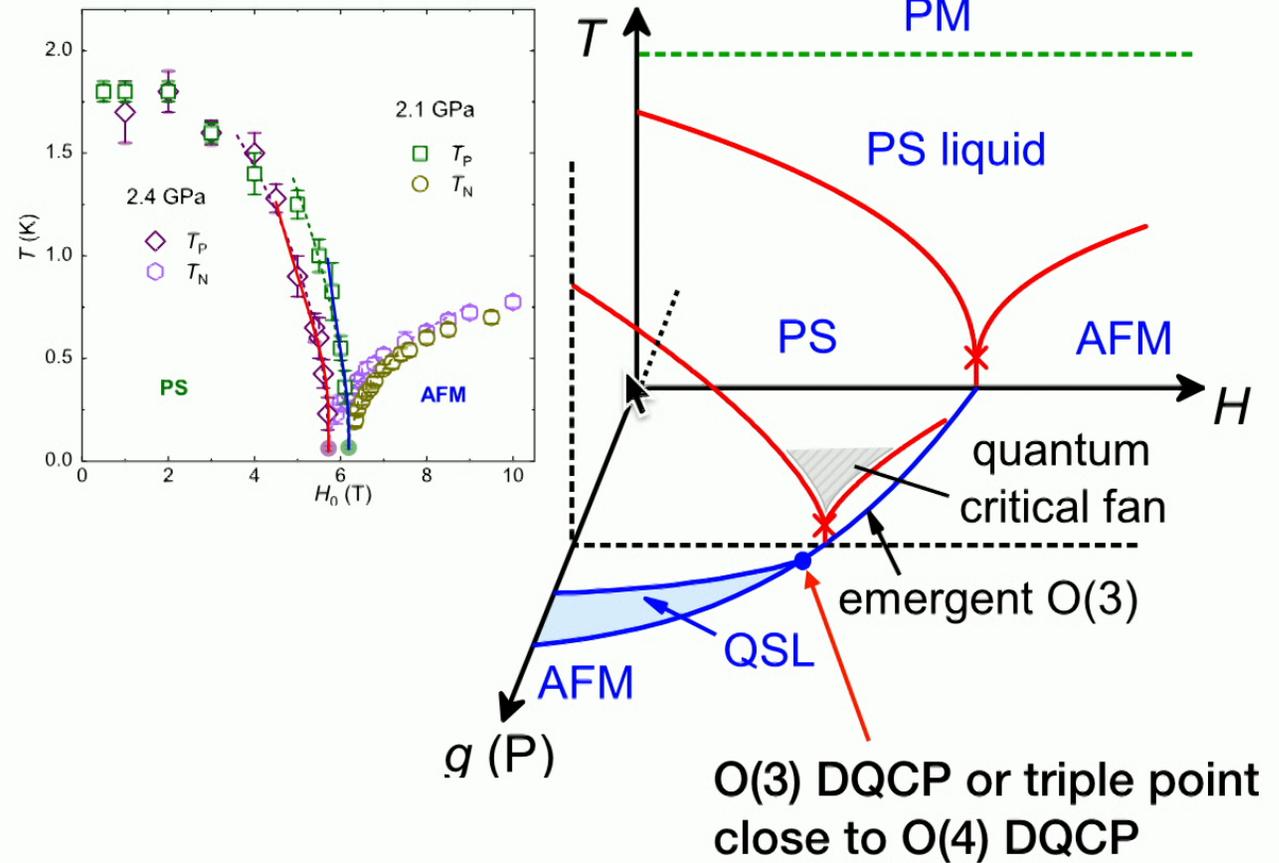
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Theoretical scenario

Picture emerging from models and experiments

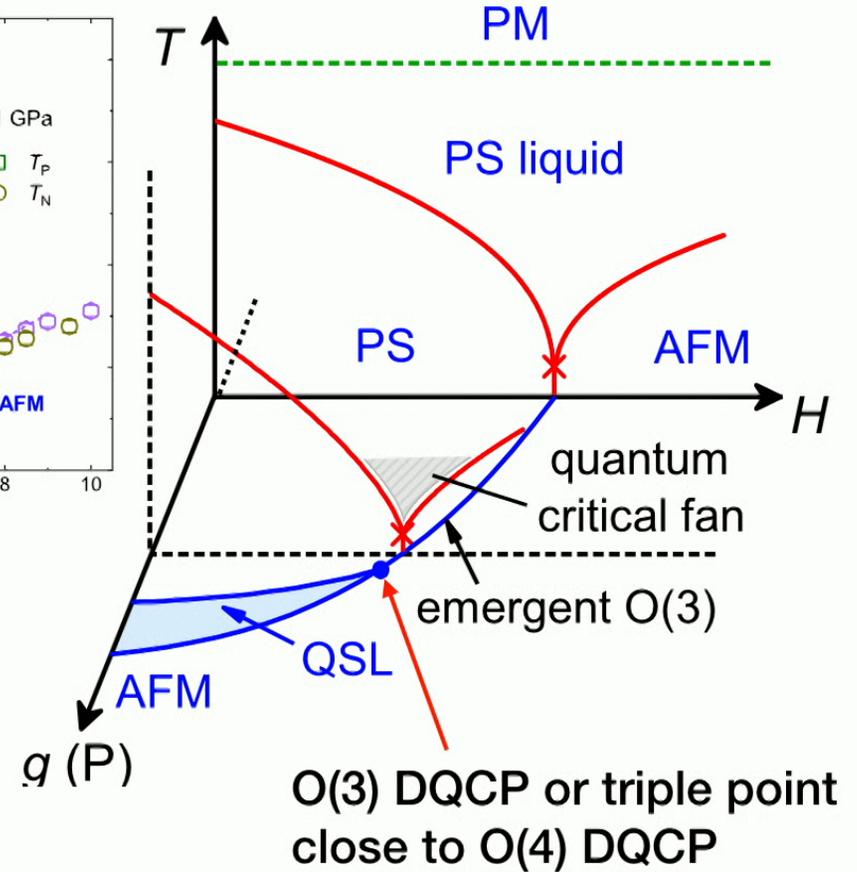
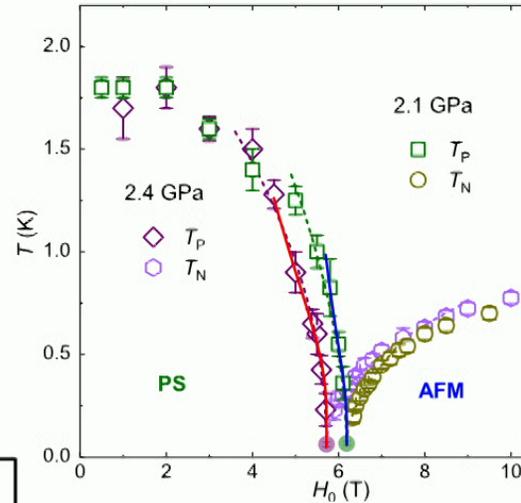
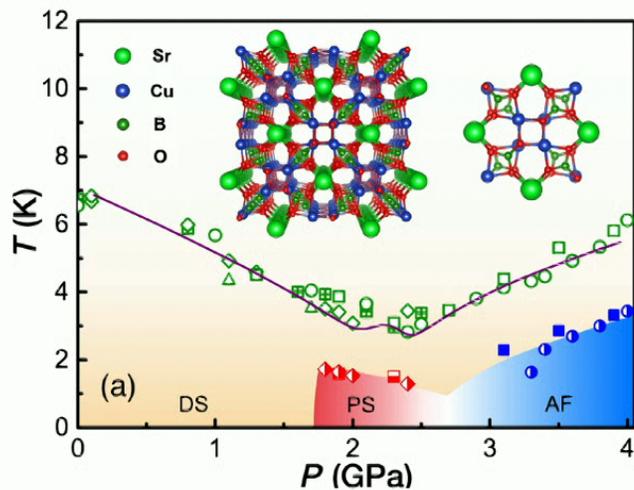
- Emergent O(3) symmetry (large length scale) on 1st-order PS-AFM line
- Only weak 3D effects



Theoretical scenario

Picture emerging from models and experiments

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- Only weak 3D effects
- At highest P, 2.4 GPa, influence of DQCP



Summary, conclusions

Shastry-Sutherland model hosts a narrow gapless spin liquid phase

Surprising? Yes: previous numerical studies missed it, no theory predictions

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NMR experiments match near-DQCP scenario

Is there a spin liquid?

- Not explicitly detected yet but there as a promising unexplored “whire region”
- SCBO single crystals are very clean, may be best spin liquid candidate