Title: Quantum electrodynamics with massless fermions in three dimensions - Talk 2

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Collection: Quantum Criticality: Gauge Fields and Matter

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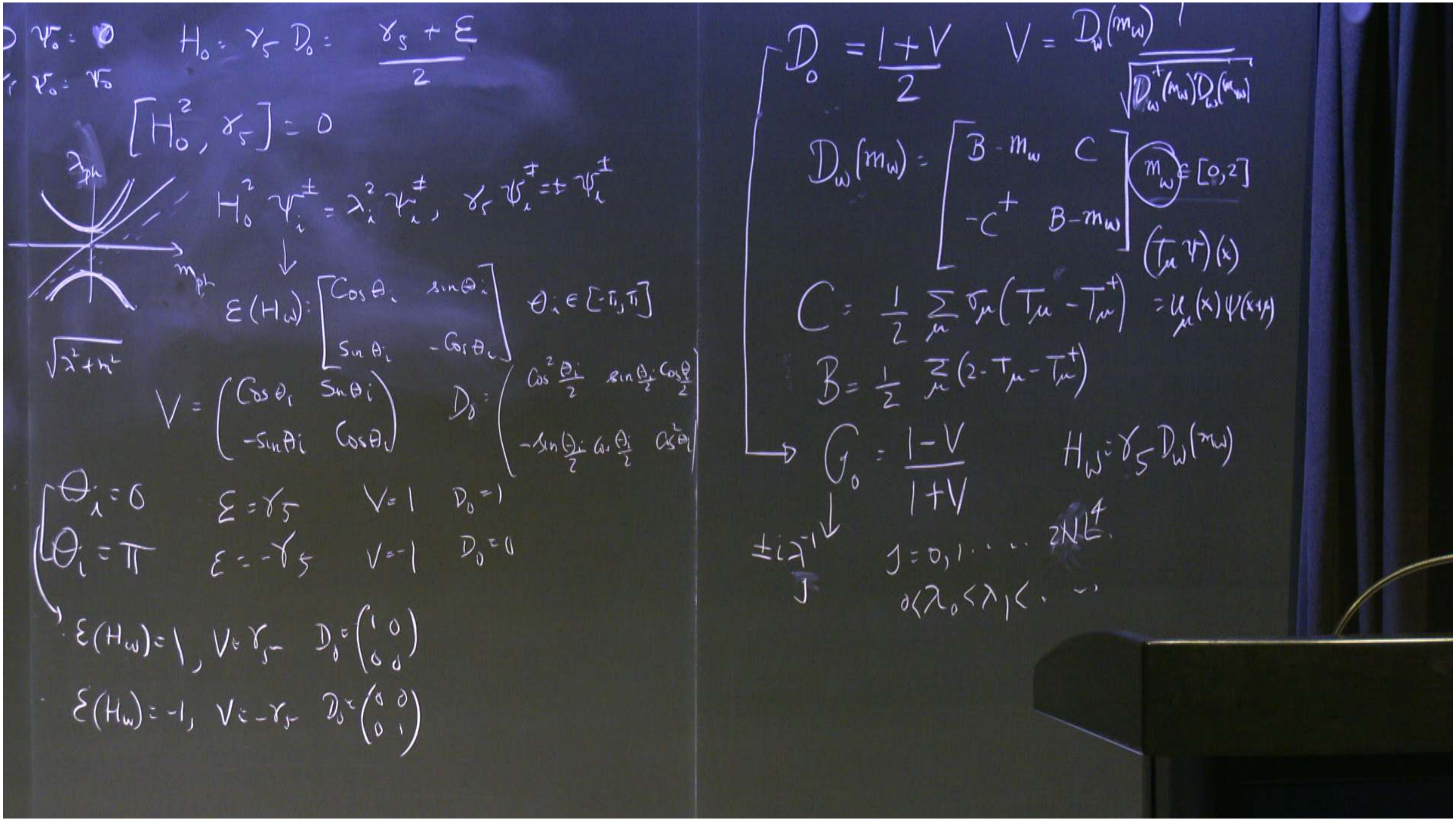
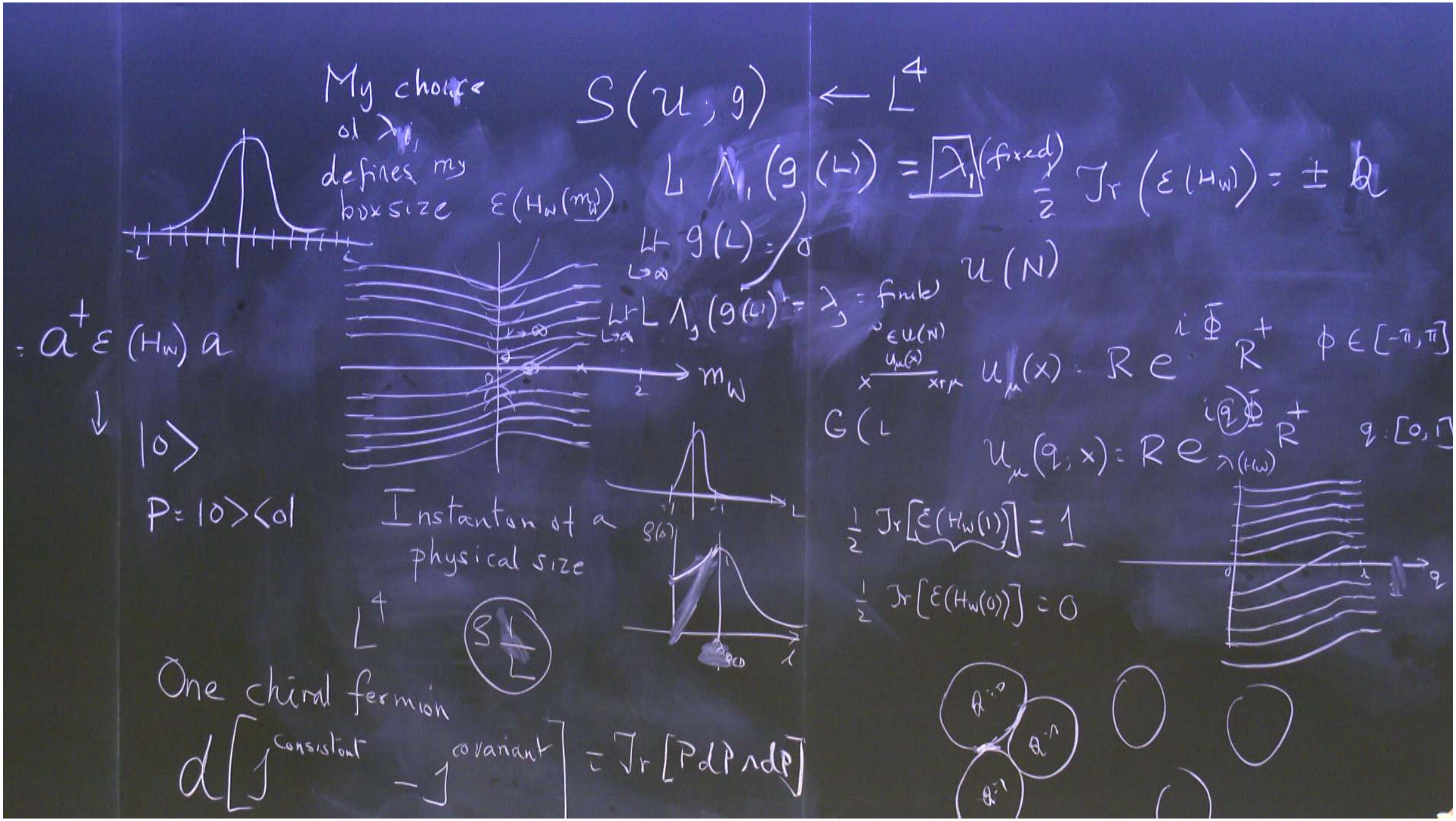

H_0 = γ_5 D_0 = (γ_5 + ε)/2
[H_0^2, γ_5] = 0
H_0^2 ψ_i^± = λ_i^2 ψ_i^±, γ_5 ψ_i^± = ± ψ_i^±
ε(H_w) = [cos θ_i sin θ_i ; sin θ_i −cos θ_i]
θ_i ∈ [−π, π]
V = (cos θ_i sin θ_i ; −sin θ_i cos θ_i)
θ_i = 0   ε = γ_5   V = 1   D_0 = 1
θ_i = π   ε = −γ_5   V = −1   D_0 = 0
ε(H_w) = 1, V = γ_5   D_0 = (1 0 ; 0 0)
ε(H_w) = −1, V = −γ_5   D_0 = (0 0 ; 0 1)
D_0 = (1+V)/2   V = D_w(m_w) / √(D_w^†(m_w) D_w(m_w))
D_w(m_w) = [B − m_w C ; −C^† B − m_w]
m_w ∈ [0, 2]
(T_μ ψ)(x) = U_μ(x) ψ(x+μ)
C = 1/2 Σ_μ σ_μ (T_μ − T_μ^†)
B = 1/2 Σ_μ (2 − T_μ − T_μ^†)
G_0 = (1−V)/(1+V)
H_w = γ_5 D_w(m_w)
±iλ_j^{-1}
j = 0, 1, … 2NL^4
0 < λ_0 < λ_1 < …


My choice of λ defines my box size
S(U; g) ← L^4
L Λ(g(L)) = λ (fixed)
1/2 Tr(ε(H_W)) = ± Q
ε(H_W(m_W))
U(N)
U_μ(x) = R e^{iΦ} R^+
φ ∈ [-π, π]
U_μ(q; x) = R e^{iqΦ} R^+
q : [0, 1]
a^+ ε(H_W) a
|0>
P = |0><0|
m_W
Instanton of a physical size
1/2 Tr[ε(H_W(1))] = 1
1/2 Tr[ε(H_W(0))] = 0
One chiral fermion
d[J^consistent − J^covariant] = Tr[PdP∧dP]

My choice of λ_1 defines my box size

$S(\mathcal{U};\underline{g}) \leftarrow \underline{L}^4$

$\mathcal{E}(H_W(m_W))$

$L\,\Lambda_1\left(g(L)\right) = \boxed{\lambda_1}$ (fixed)

$\lim_{L\to\infty} g(L) = 0$

$\frac{1}{2}\,\mathrm{Tr}\left(\mathcal{E}(H_W\right.$

$\mathcal{U}(N)$

$\lim_{L\to\infty} L\,\Lambda_j\, g(\ell L) = \lambda_j(\ell)$

$\lim_{L\to\infty} L\,\Lambda_j(g(L)) = \lambda_j$ = finite

$\in \mathcal{U}(N)$

dimensionless box size

$x \xrightarrow{U_\mu(x)} x+\mu$

$U_\mu(x) = R e^{i\Phi}$

$\lim_{L\to\infty} G_S(x, L, \ell)$

$\downarrow$

$\lim_{\ell\to\infty} G_S(x, \ell)$

$\lim_{\kappa\to\infty} G_S(x)$

$G(L$

If $\ell \to 0$ $\quad \lambda_j(\ell) \sim \frac{1}{\ell}$

$U_\mu(q;x) = R e^{i q}$

What happens when $\ell \to \infty$

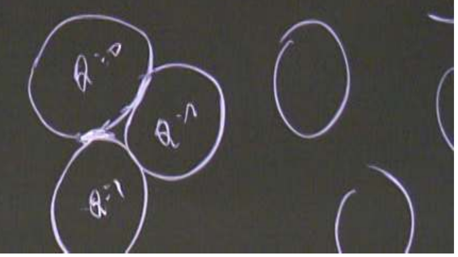
$\frac{1}{2}\,\mathrm{Tr}\left[\underbrace{\mathcal{E}(H_W(1))}\right] = 1$

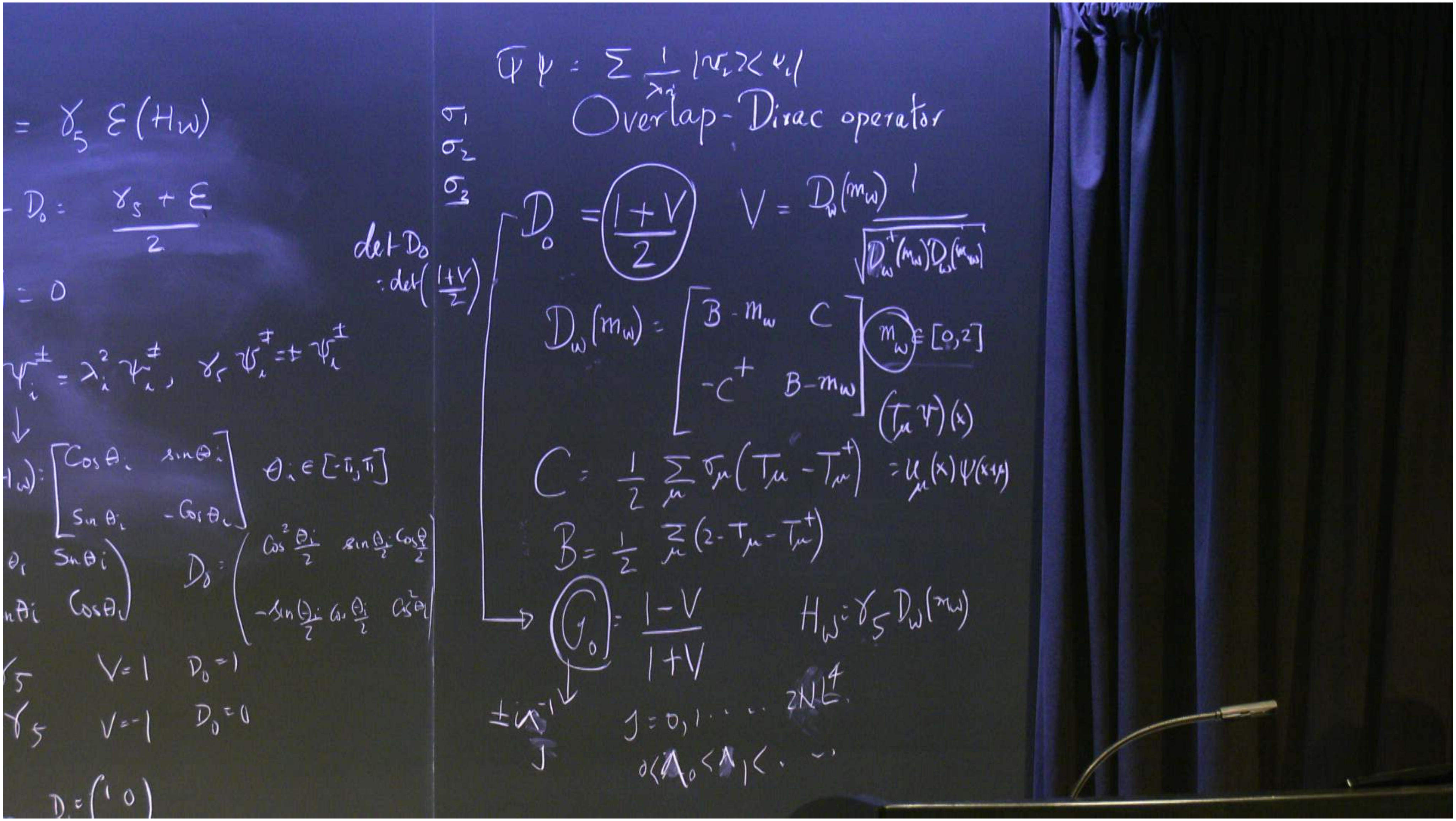
$\lambda_j(\ell) \propto \frac{1}{\ell^{1+\boxed{\gamma_m}}}$

$\frac{1}{2}\,\mathrm{Tr}\left[\mathcal{E}(H_W(0))\right] = 0$

If $0 < \gamma_m < 3 \leftarrow$ scale invariant

$\gamma_m = 3$ = Condensate




Overlap-Dirac operator
$\bar{\psi}\psi = \sum \frac{1}{\lambda_i} |v_i\rangle\langle v_i|$
$D_o = \frac{1+V}{2}$
$V = D_w(m_w) \frac{1}{\sqrt{D_w^\dagger(m_w) D_w(m_w)}}$
$D_w(m_w) = \begin{bmatrix} B - m_w & C \\ -C^\dagger & B - m_w \end{bmatrix}$
$m_w \in [0,2]$
$(T_\mu \psi)(x) = U_\mu(x)\psi(x+\mu)$
$C = \frac{1}{2}\sum_\mu \sigma_\mu (T_\mu - T_\mu^\dagger)$
$B = \frac{1}{2}\sum_\mu (2 - T_\mu - T_\mu^\dagger)$
$G_0 = \frac{1-V}{1+V}$
$H_w = \gamma_5 D_w(m_w)$
$\pm i\lambda_j^{-1}$
$j = 0, 1, \ldots, 2NL^4$
$0 < \lambda_0 < \lambda_1 < \ldots$
σ_1
σ_2
σ_3
$\det D_0 = \det\left(\frac{1+V}{2}\right)$
$= \gamma_5 \varepsilon(H_w)$
$D_0 = \frac{\gamma_5 + \varepsilon}{2}$
$= 0$
$\psi_i^\pm = \lambda_i^2 \psi_i^\pm, \quad \gamma_5 \psi_i^\mp = \pm \psi_i^\pm$
$\theta_i \in [-\pi, \pi]$
$V = 1 \quad D_0 = 1$
$V = -1 \quad D_0 = 0$

My choice of λ_{g_j} defines my box size

$S(u;g) \leftarrow$

$m\int \bar\psi \psi \, d^d x$

$\varepsilon(H_w(m_w))$

$L\,\Lambda_1(g(L))$

$F_{\mu\nu}(x) = [A_\nu(x+\mu) - A_\nu(x)] - [A_\mu(x+\nu) - A_\mu(x)]$

$-S_g F = \frac{1}{g^2} F^2$

e

$f(u = e^{iA_\mu})$

$A_\mu \to A_\mu + 2\pi n_\mu$

$F \to F + N$

$\vec{\nabla} \cdot \vec{N} = 0$

$L\,\Lambda_g \quad g(\ell = L) = \lambda_g(\ell)$

$\lim_{L\to\infty} g(L) = g$

$= \lim_{L\to\infty} L\,\Lambda_g(g(L)) = \lambda_g$

dimensionless box size

If $\ell \to 0 \quad \lambda_g(\ell) \sim \frac{1}{\ell}$

What happens when $\ell \to \infty$

$\frac{1}{x^{2[1-\gamma_m]}}$

$\lambda_g(\ell) \propto \frac{1}{\ell^{1+\boxed{\gamma_m}}} \propto \frac{1}{V}$

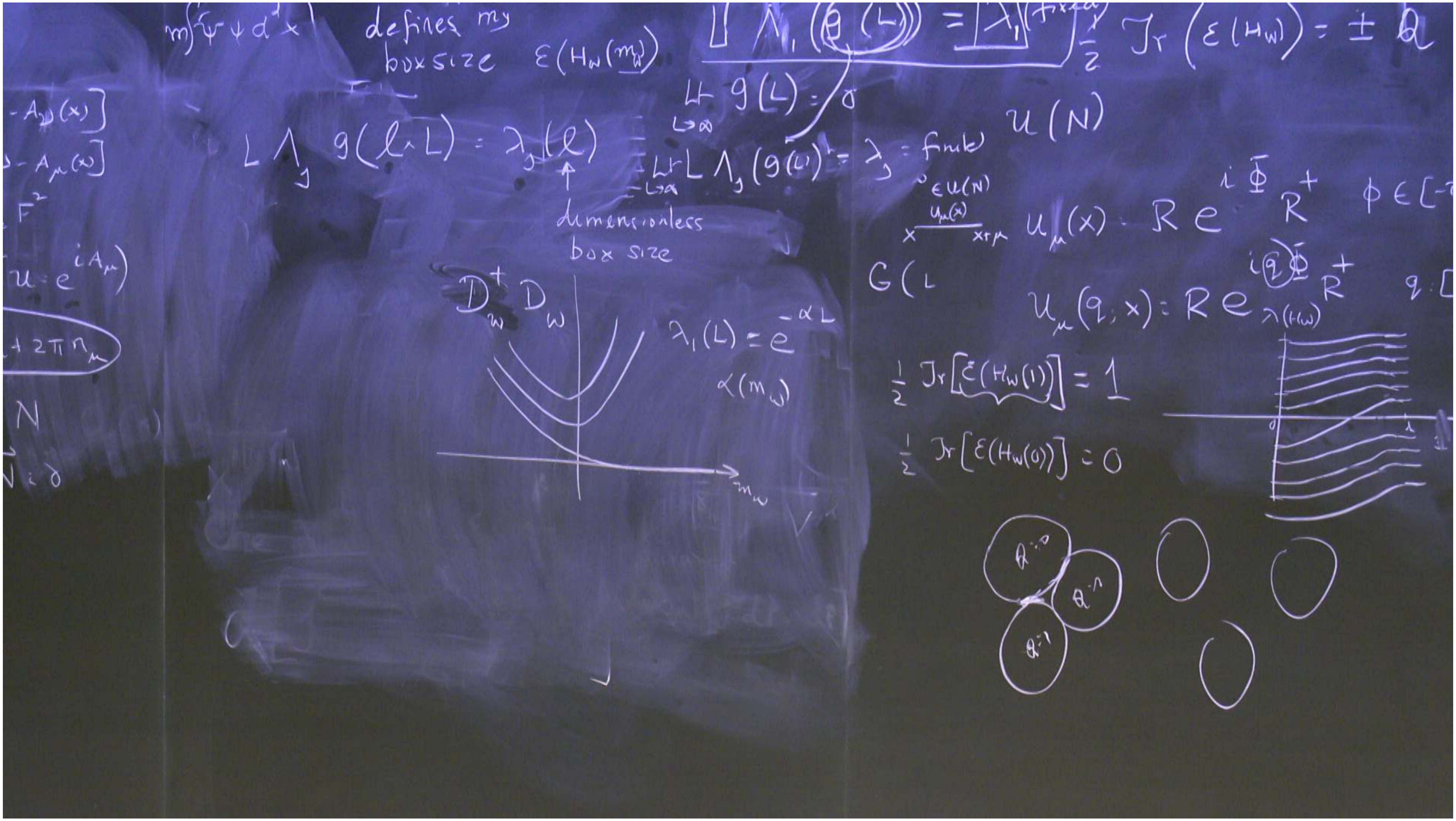
If $0 < \gamma_m < 3 \leftarrow$ Scale invariant

$\gamma_m = 3 =$ Condensate

$G($

$\frac{1}{2}$

$\frac{1}{2}$


defines m_W
box size
$\mathcal{E}(H_W(m_W))$
$\frac{1}{2}\mathrm{Tr}(\mathcal{E}(H_W)) = \pm Q$
$\lim_{L\to\infty} g(L) = 0$
$L\Lambda_J\, g(\ell, L) = \lambda_J(\ell)$
dimensionless
box size
$U(N)$
$U_\mu(x) = R e^{i\Phi} R^\dagger$
$U_\mu(q;x) = R e^{iq\Phi}_{\lambda(H_W)} R^\dagger$
$u = e^{iA_\mu}$
$+2\pi n_\mu$
$D_W^\dagger D_W$
$\lambda_1(L) = e^{-\alpha L}$
$\alpha(m_W)$
m_W
$\frac{1}{2}\mathrm{Tr}[\mathcal{E}(H_W(1))] = 1$
$\frac{1}{2}\mathrm{Tr}[\mathcal{E}(H_W(0))] = 0$

$$\boxed{\lambda_1} \text{ (fixed)}$$

$\mathcal{U}(N)$

$= \lambda_1 = \text{finite}$

$\in \mathcal{U}(N)$

$U_\mu(x) \quad x \xrightarrow{U_\mu(x)} x+\mu$

$G(L$

$U_\mu(q; x)$

$$\frac{1}{2}\,\mathrm{Tr}\left[\underbrace{\varepsilon(H_w(1))}\right] = 1$$

$$\frac{1}{2}\,\mathrm{Tr}\left[\varepsilon(H_w(0))\right] = 0$$

1 2-component theory

$g^2(L) = \frac{\ell}{L}$

Charge $q = 2$

$$F_{\mu\nu}(x) = [A_\nu(x+\mu) - A_\nu(x)] - [A_\mu(x+\nu) - A_\mu(x)]$$

$$S_g = \frac{1}{g^2} F^2$$

$e \qquad f(u = e^{iA_\mu})$

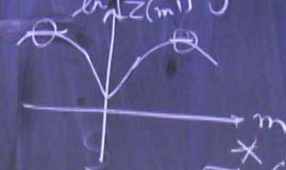
$$A_\mu \to A_\mu + 2\pi n_\mu$$

$F \to F + N$

$\vec{\nabla}\cdot\vec{N} \neq 0$

$$Z(m) = \int [dA_\mu]\, e^{-\frac{L}{\ell}F^2} \det\left[\frac{1+m+(1-m)V_{2A}}{2}\, V_A^+ V_A^+\right]$$

$\ln[Z(m)]$



$z(\pm 1) > 0$

$$Z^*(m) = Z(m)$$

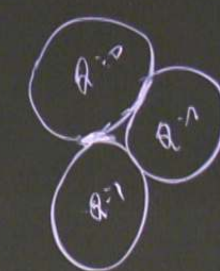
$$Z(m) = Z(-m)$$

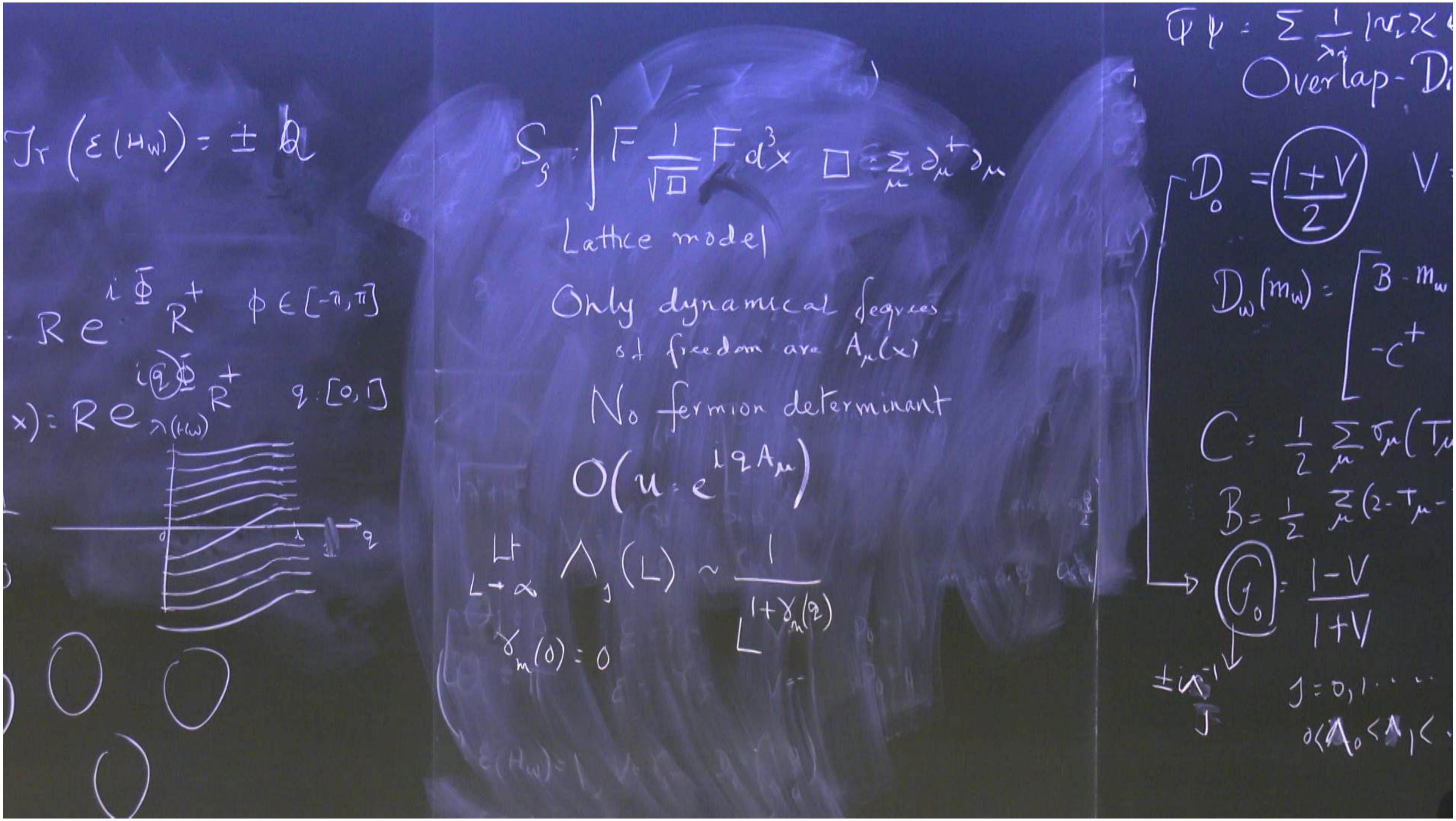
$$m = 0$$

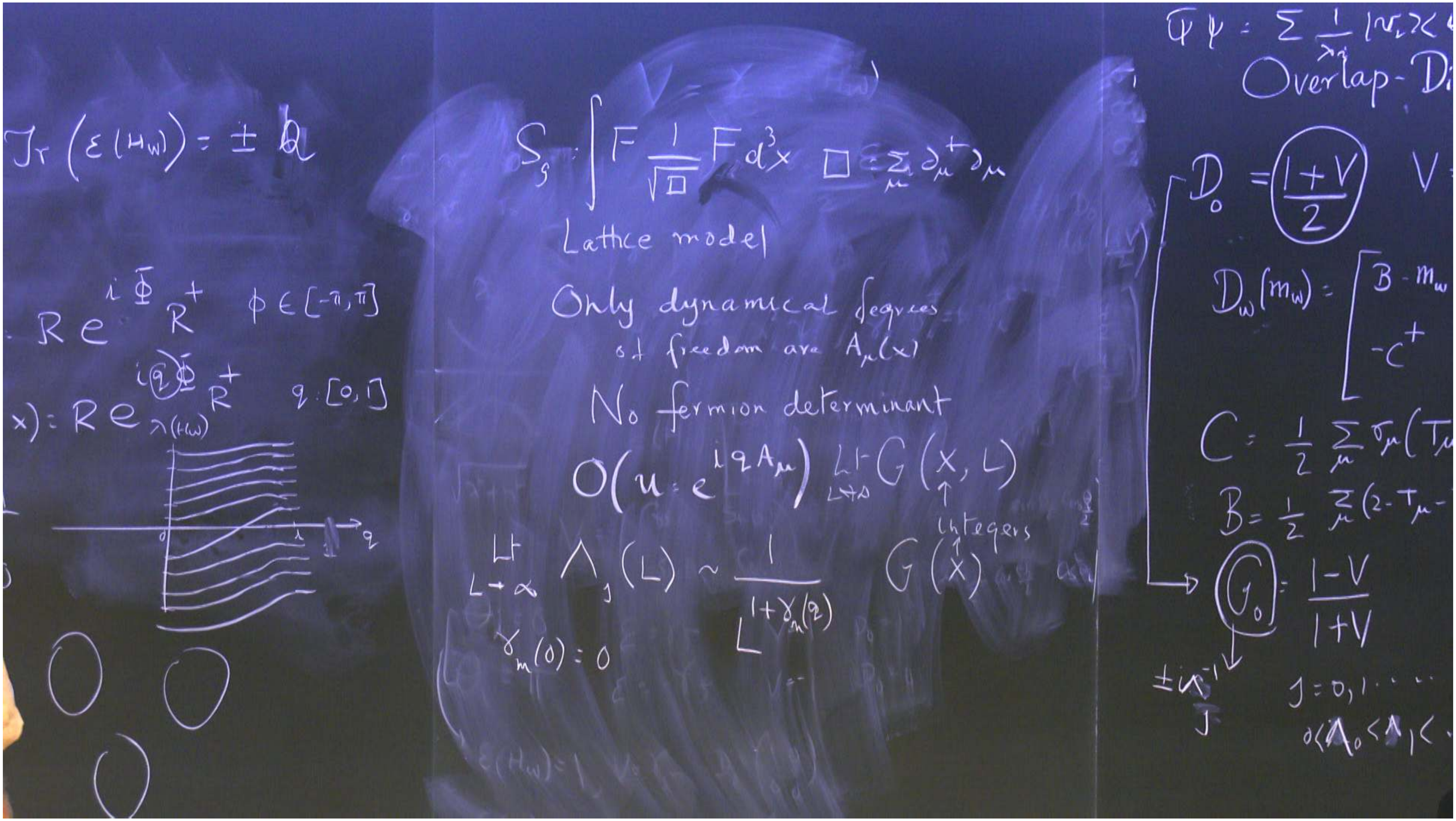
One two component Charge 2 fermion

$$V_{2A}\,(V_A^+)^4 \neq 1$$

$$e^{\frac{i}{\pi}AdA} \quad \left(e^{-\frac{i}{4\pi}AdA}\right)^4$$




Tr (ε(H_W)) = ± Q
Re^{iΦ} R^+   φ ∈ [-π, π]
x) = Re^{iqΦ}_{λ(H_W)} R^+   q : [0, 1]
S_g = ∫ F (1/√□) F d^3x   □ = Σ_μ ∂_μ^+ ∂_μ
Lattice model
Only dynamical degrees of freedom are A_μ(x)
No fermion determinant
O(u = e^{iqA_μ})
Lt_{L→∞} Λ_j(L) ~ 1 / L^{1+γ_m(q)}
γ_m(0) = 0
Overlap-D
D_o = (1+V)/2
D_w(m_w) =
B - m_w
-C^+
C = 1/2 Σ_μ σ_μ (T_μ
B = 1/2 Σ_μ (2 - T_μ -
G_o = (1-V)/(1+V)
j = 0, 1, ...
0 < Λ_0 < Λ_1 <


Tr (ε(H_W)) = ± Q
R e^{iΦ} R^+   ϕ ∈ [−π, π]
x) = R e^{iqΦ}_{λ(H_W)} R^+   q : [0, 1]
S_g : ∫ F (1/√□) F d³x   □ = Σ_μ ∂_μ^+ ∂_μ
Lattice model
Only dynamical degrees of freedom are A_μ(x)
No fermion determinant
O(u = e^{iqA_μ})   Lt_{L→∞} G(X, L)
integers
Lt_{L→∞} Λ_j(L) ~ 1 / L^{1+γ_m(q)}   G(X)
γ_m(0) = 0
Φ ψ = Σ 1/λ_i |v_i⟩⟨
Overlap-Di
D_0 = (1+V)/2   V
D_W(m_W) = [B − m_W ; −C^+]
C = 1/2 Σ_μ σ_μ (T_μ
B = 1/2 Σ_μ (2 − T_μ −
G_0 = (1−V)/(1+V)
±iλ_j^{-1}
j = 0, 1 ...
0 < λ_0 < λ_1 <