Title: Quantum electrodynamics with massless fermions in three dimensions - Talk 1

Speakers: Rajamani Narayanan

Collection: Quantum Criticality: Gauge Fields and Matter

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Overlap-Dirac operator

$$D_0 = \frac{1+V}{2} \qquad V = D_w(m_w)\frac{1}{\sqrt{D_w^{\dagger}(m_w)D_w(m_w)}}$$

$$D_w(m_w) = \begin{bmatrix} B - m_w & C \\ -C^{\dagger} & B - m_w \end{bmatrix} \qquad m_w \in [0,2]$$

$$(T_\mu \psi)(x) = U_\mu(x)\,\psi(x+\mu)$$

$$C = \frac{1}{2}\sum_\mu \sigma_\mu \left(T_\mu - T_\mu^{\dagger}\right)$$

$$B = \frac{1}{2}\sum_\mu \left(2 - T_\mu - T_\mu^{\dagger}\right)$$

$$G_0 = \frac{1-V}{1+V}$$

$$V = \gamma_5 \, \varepsilon(H_W)$$

$$H_0 = \gamma_5 D_0 = \frac{\gamma_5 + \varepsilon}{2}$$

$$[H_0^2, \gamma_5] = 0$$

$$H_0^2 \psi_i^\pm = \lambda_i^2 \psi_i^\pm, \quad \gamma_5 \psi_i^\pm = \pm \psi_i^\pm$$

$$\downarrow$$

$$\varepsilon(H_W) = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ \sin\theta_i & -\cos\theta_i \end{bmatrix} \qquad \theta_i \in [-\pi, \pi]$$

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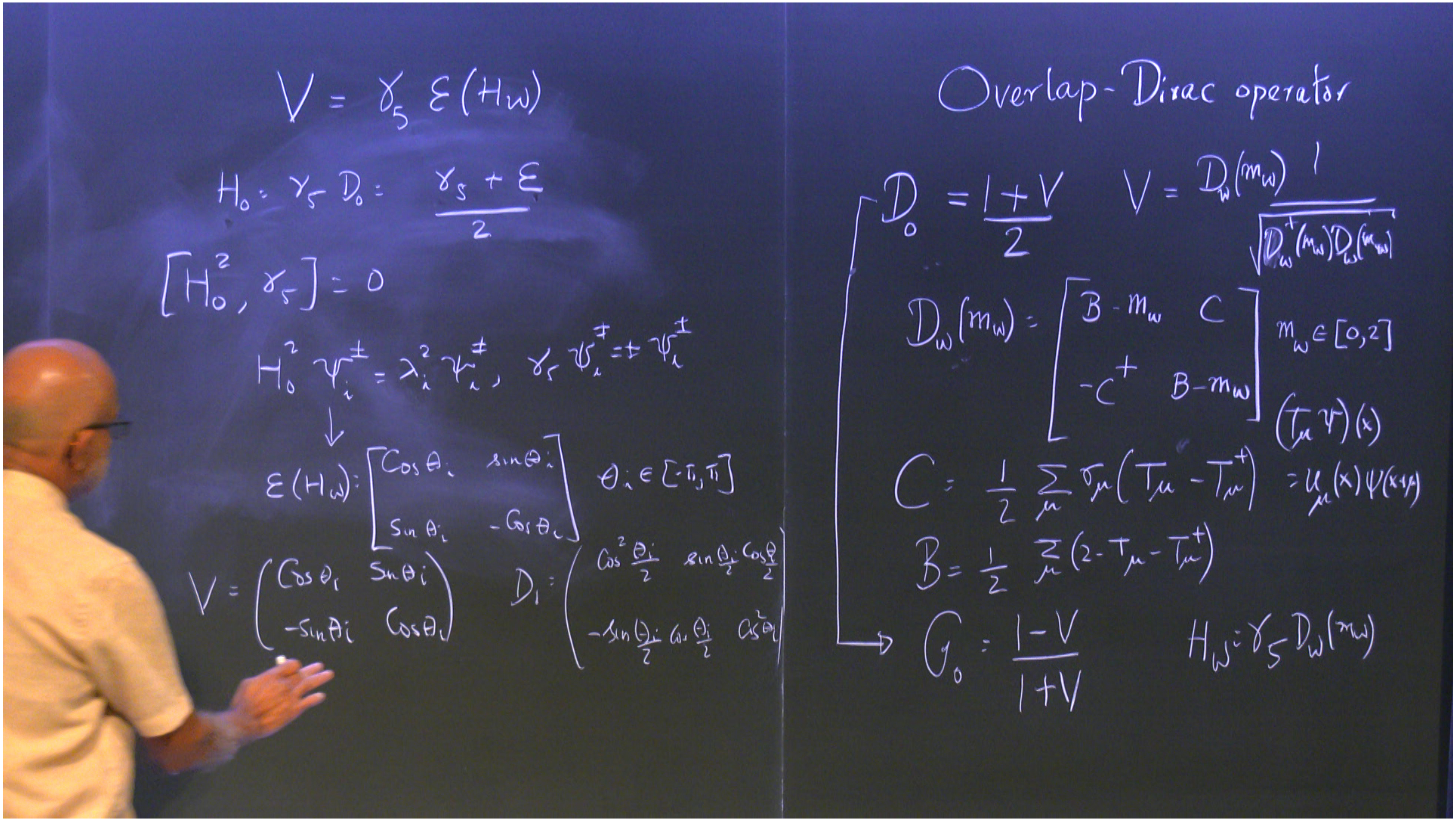
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$\theta_i = 0 \quad \varepsilon = \gamma_5 \quad V = 1 \quad D_0 = 1$

$\theta_i = \pi \quad \varepsilon = -\gamma_5 \quad V = -1 \quad D_0 = 0$

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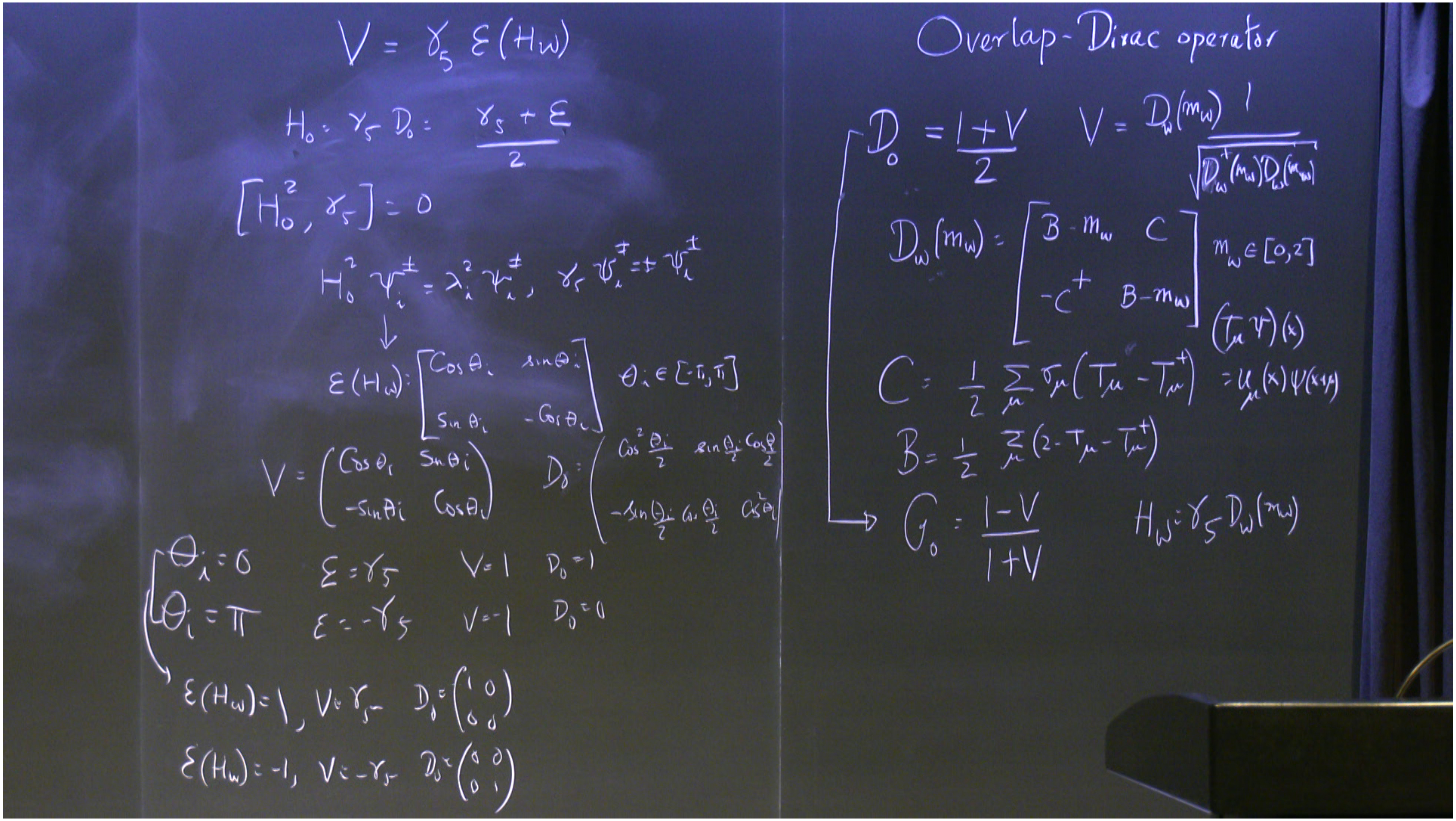
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$$\frac{1}{2} \mathrm{Tr}\left(\mathcal{E}(H_W)\right) = \pm 1$$

$$V = \gamma_5 \, \mathcal{E}(H_W)$$

$$H_0 = \gamma_5 D_0 = \frac{\gamma_5 + \mathcal{E}}{2}$$

$$[H_0^2, \gamma_5] = 0$$

$$H_0^2 \psi_i^\pm = \lambda_i^2 \psi_i^\pm, \quad \gamma_5 \psi_i^\pm = \pm \psi_i^\pm$$

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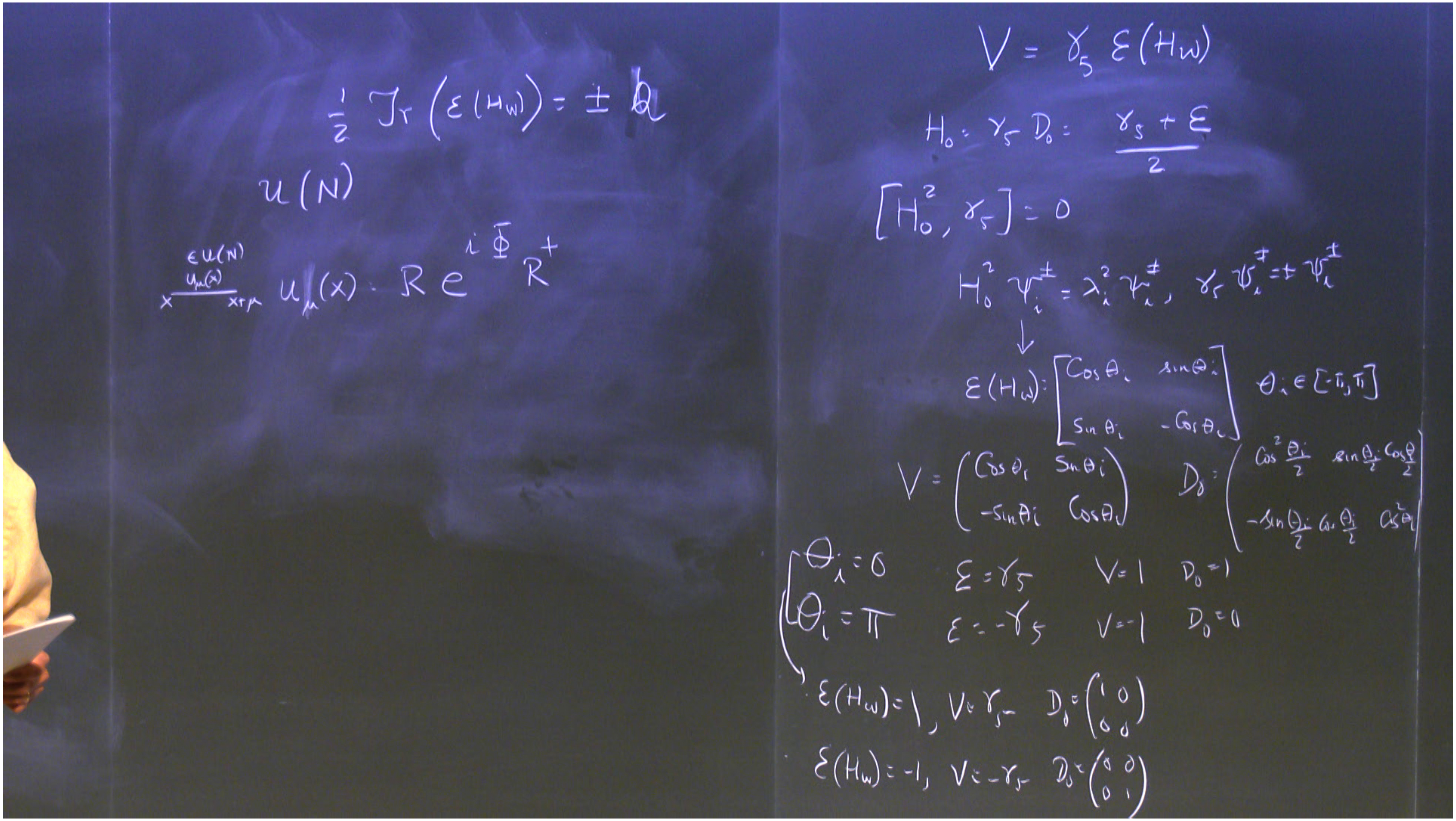
$\frac{1}{2}\mathrm{Tr}\left(\varepsilon(H_W)\right) = \pm Q$
$\mathcal{U}(N)$
$\in \mathcal{U}(N)$
$x \xrightarrow{U_\mu(x)} x+\mu$
$U_\mu(x) = R\, e^{i\Phi} R^\dagger$
$V = \gamma_5\, \varepsilon(H_W)$
$H_0 = \gamma_5 D_0 = \frac{\gamma_5 + \varepsilon}{2}$
$[H_0^2, \gamma_5] = 0$
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$$\frac{1}{2}\,\mathrm{Tr}\left(\varepsilon(H_W)\right) = \pm Q$$

$$\mathcal{U}(N)$$

$$x \xrightarrow[\in\, \mathcal{U}(N)]{U_\mu(x)} x+\mu \qquad U_\mu(x) = R\, e^{i\bar{\Phi}} R^{+} \qquad \phi \in [-\pi, \pi]$$

$$U_\mu(q; x) = R\, e^{iq\bar{\Phi}} R^{+} \qquad q : [0, 1]$$

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$$U(N)$$

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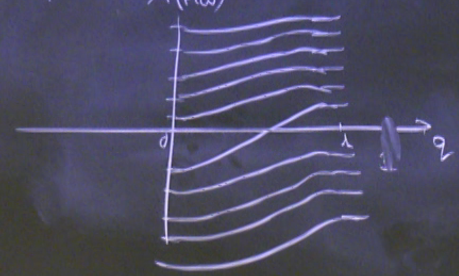
$\lambda(H_W)$



$$\frac{1}{2}\mathrm{Tr}\left[\varepsilon(H_W(1))\right] = 1$$

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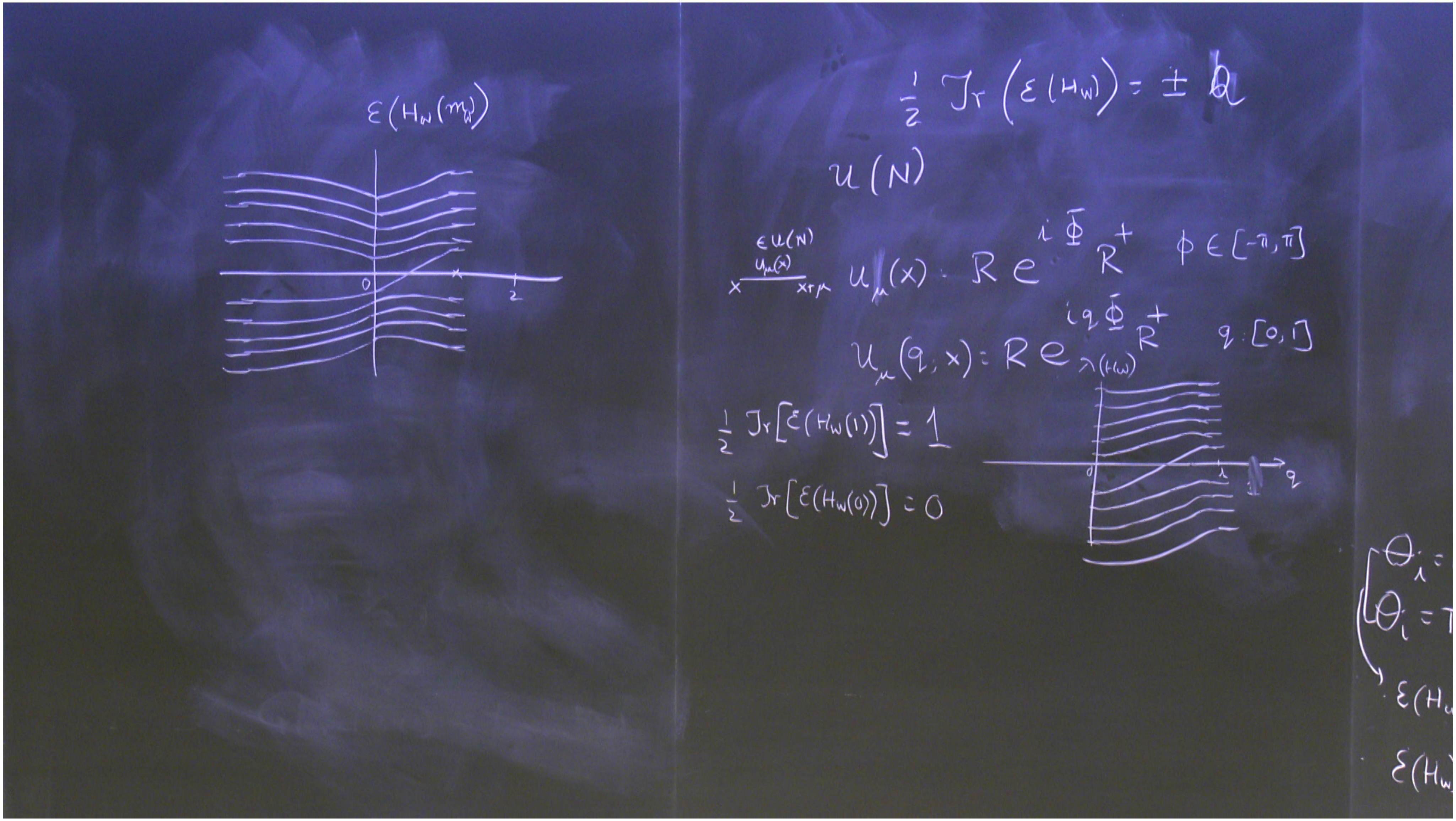
$\mathcal{E}(H_W(m_W))$
$\frac{1}{2}\,\mathrm{Tr}\left(\mathcal{E}(H_W)\right) = \pm Q$
$U(N)$
$\in U(N)$
$U_\mu(x)$
x
$x+\mu$
$U_\mu(x) = R\, e^{i\Phi} R^+$
$\phi \in [-\pi, \pi]$
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$\lambda(H_W)$
$\frac{1}{2}\,\mathrm{Tr}\left[\mathcal{E}(H_W(1))\right] = 1$
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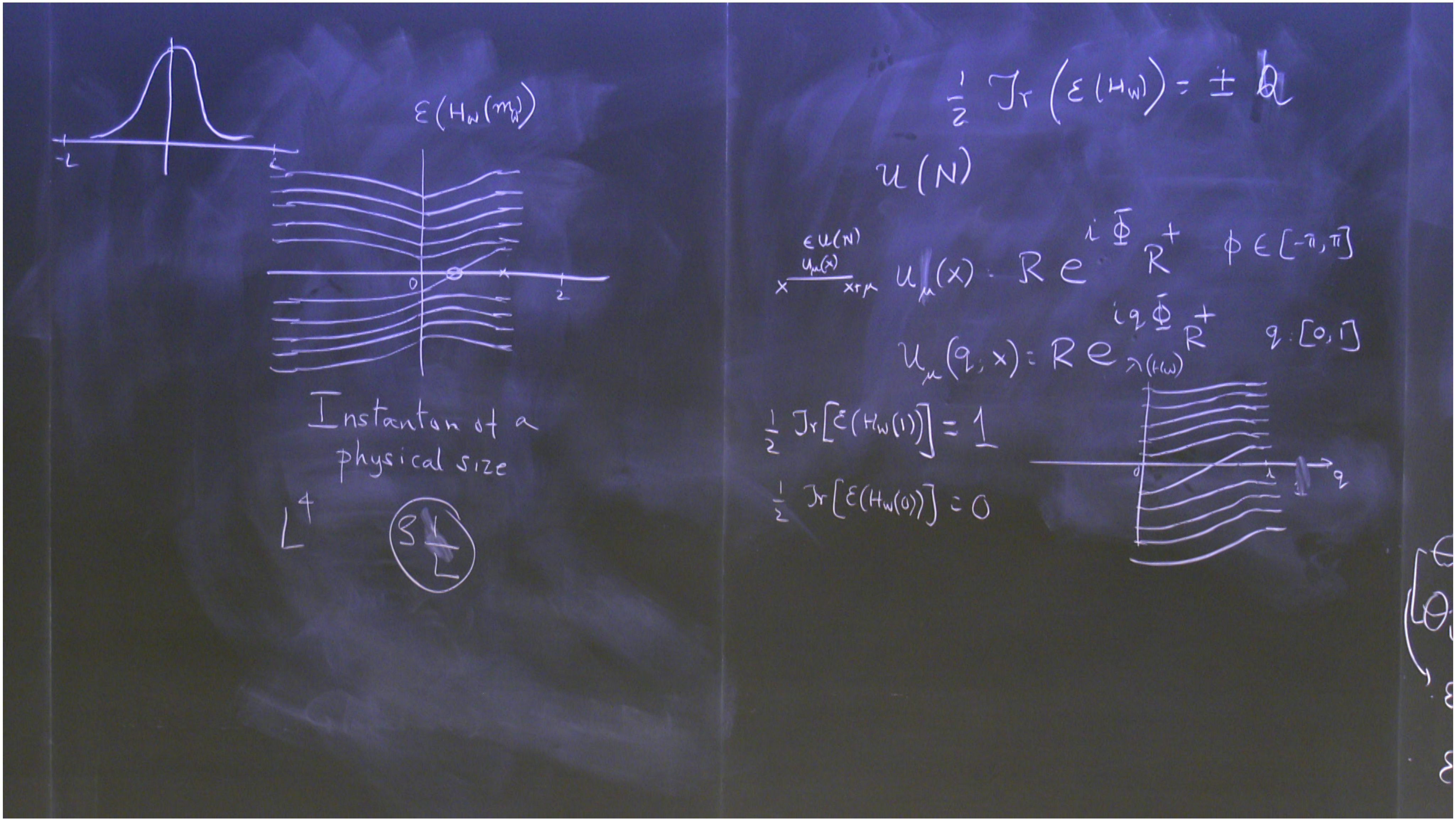
0
1
2
q



-L
L
ε(H_W(m_W))
0
2
Instanton of a
physical size
L^4
ε U(N)
U_μ(x)
x
x+μ
1/2 Tr (ε(H_W)) = ± Q
U(N)
U_μ(x) = R e^{iΦ} R^+
φ ∈ [-π, π]
U_μ(q; x) = R e^{iqΦ}_{λ(H_W)} R^+
q : [0, 1]
1/2 Tr[ε(H_W(1))] = 1
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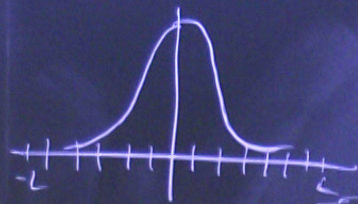
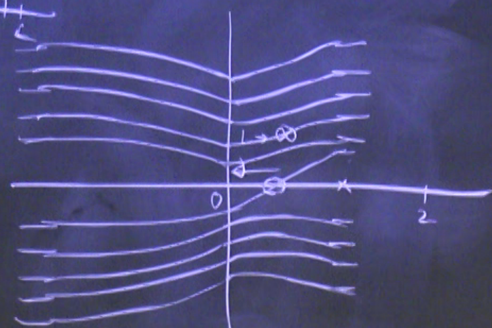
0
1
q



$\mathcal{E}(H_W(m_W))$



Instanton of a physical size

L^4 $\quad \frac{\delta}{L}$

$$\frac{1}{2}\mathrm{Tr}\left(\mathcal{E}(H_W)\right) = \pm Q$$

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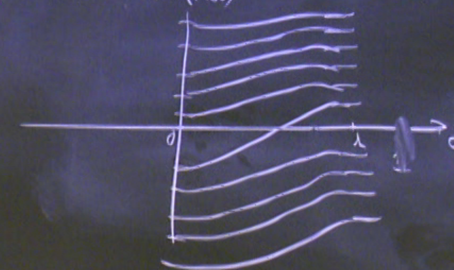

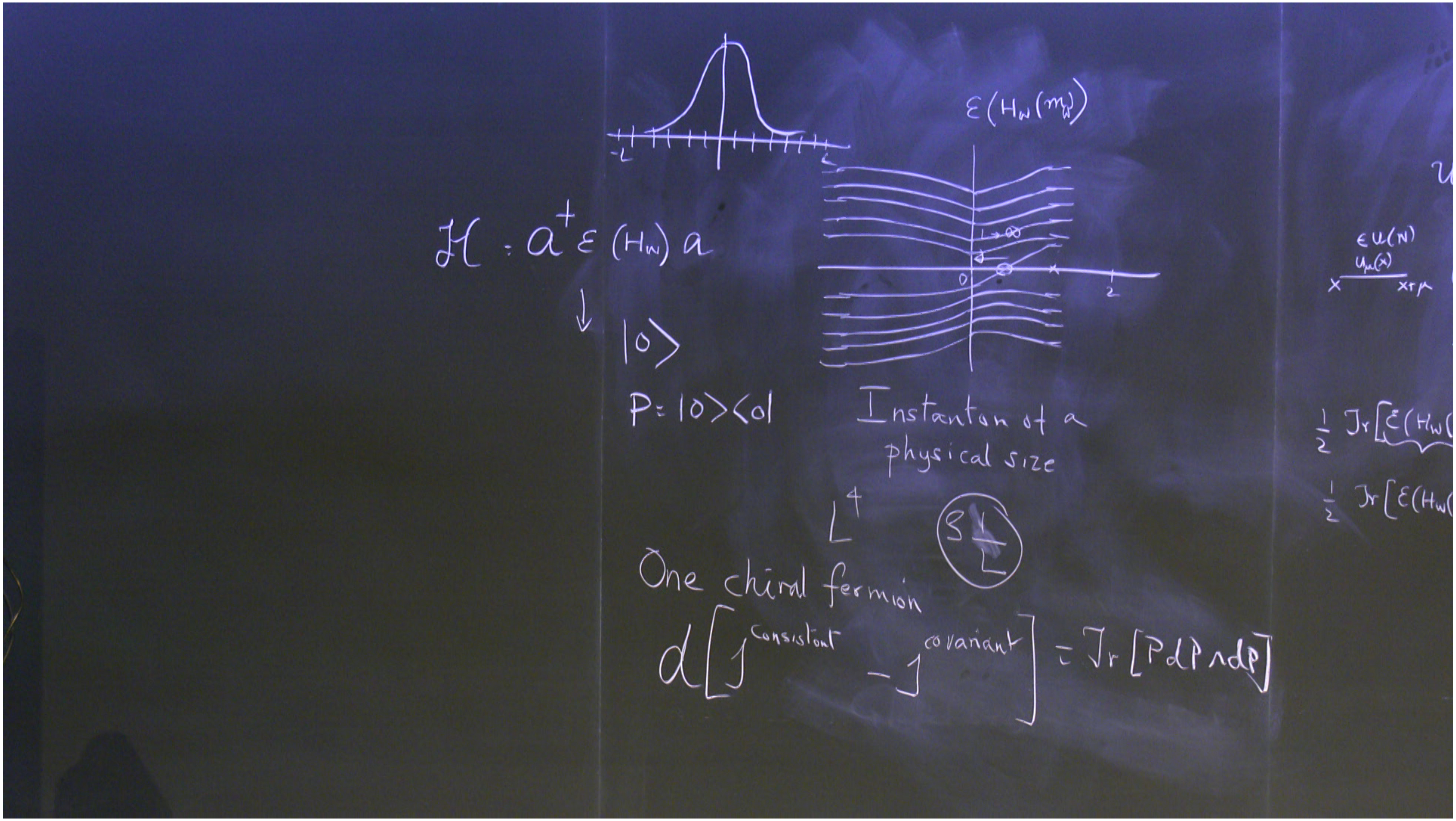
ℋ = a† ε(H_W) a
|0⟩
P = |0⟩⟨0|
ε(H_W(m_W))
Instanton of a physical size
L⁴
One chiral fermion
d[J^consistent − J^covariant] = Tr[P dP ∧ dP]
∈ U(N)
U_μ(x)
x
x+μ
½ Tr[ε(H_W(
½ Tr[ε(H_W(