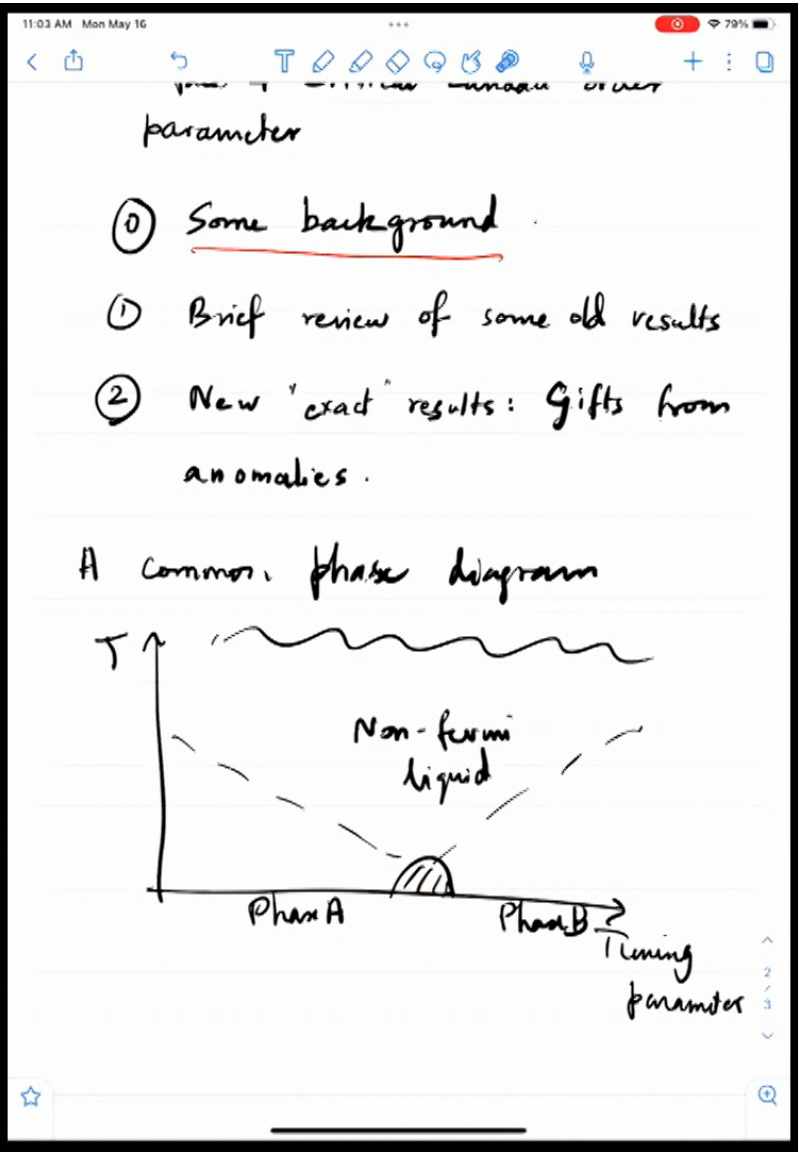
Title: Blackboard Talk 2

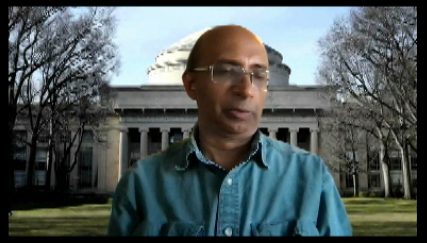
Speakers: Senthil Todadri

Collection: Quantum Criticality: Gauge Fields and Matter

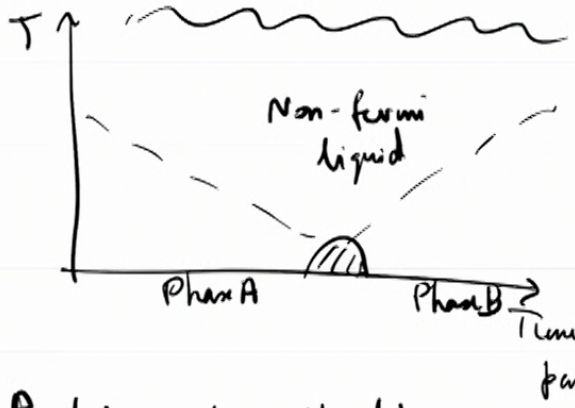
Date: May 16, 2022 - 10:30 AM

URL: https://pirsa.org/22050028


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parameter
0 Some background
1 Brief review of some old results
2 New "exact" results: Gifts from anomalies.
A common phase diagram
T
Non-fermi liquid
Phase A
Phase B
Tuning parameter



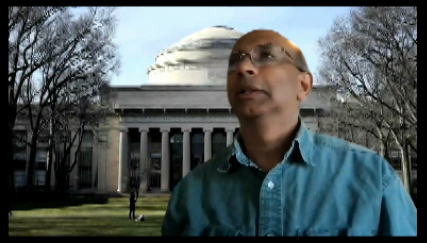
A common phase diagram

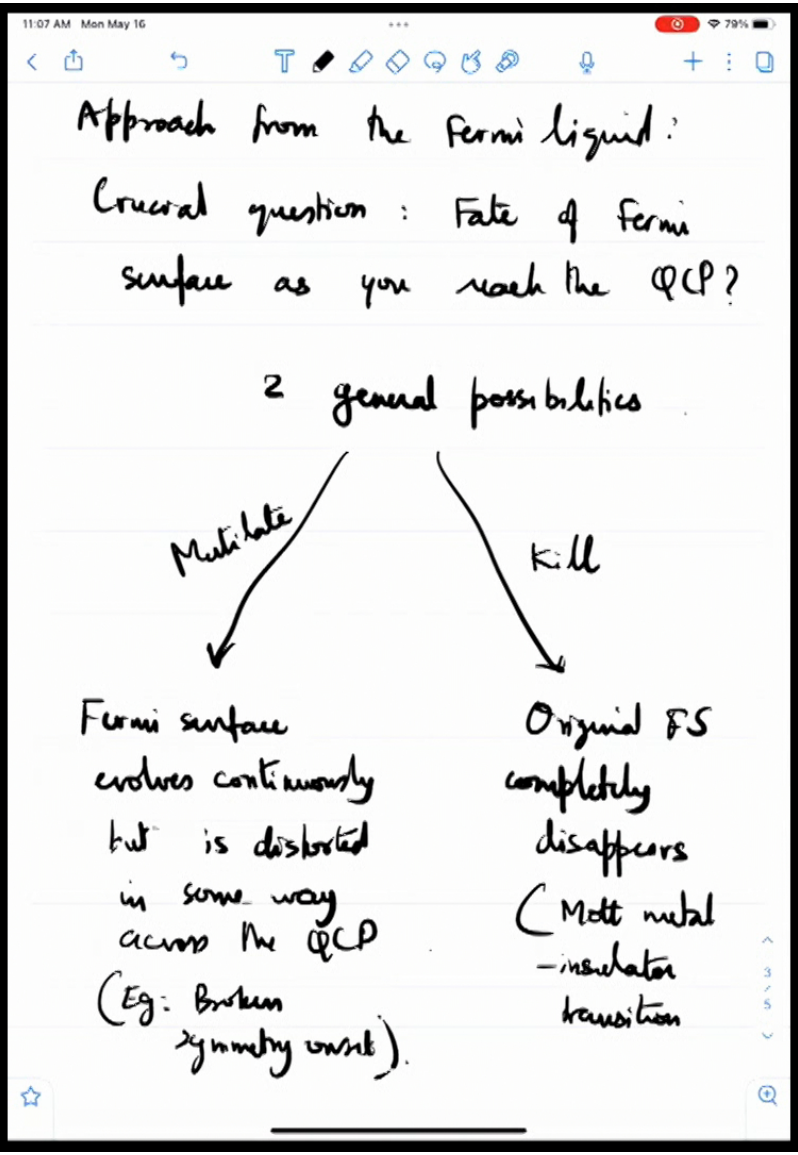


Perhaps qtm criticality is responsible for the non-fermi liquid

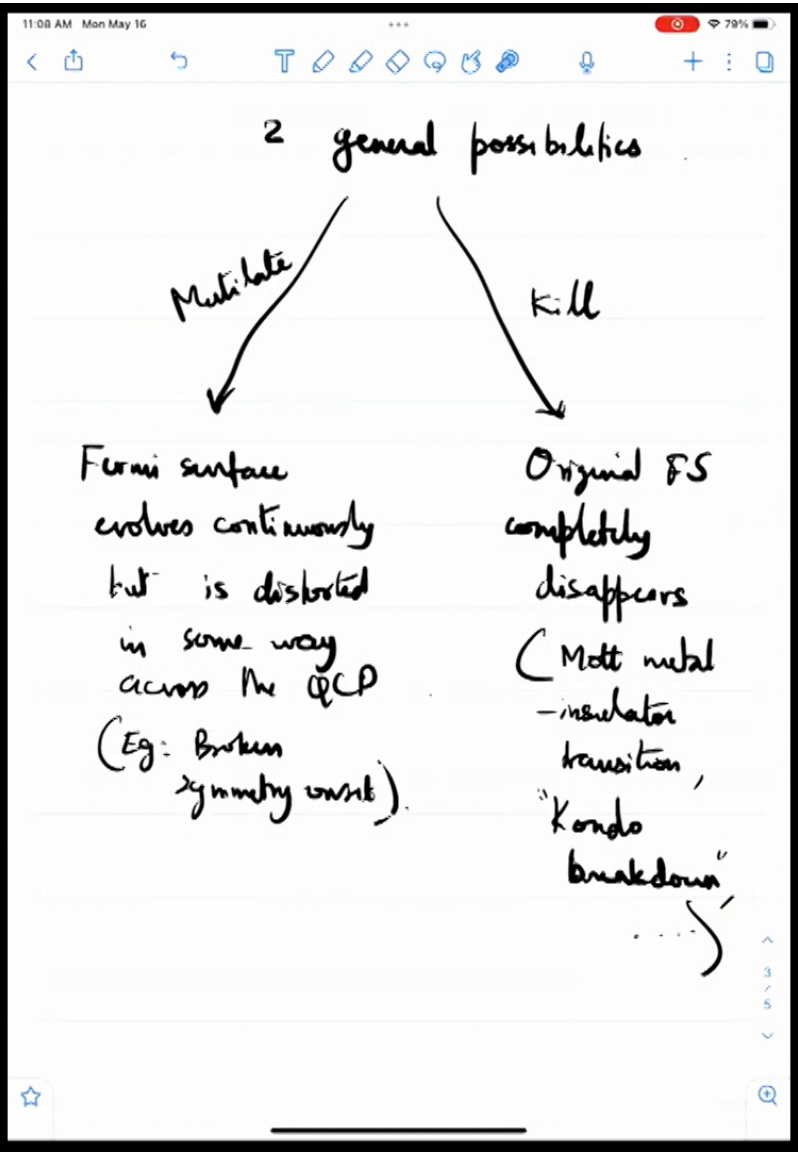
Approach from the fermi liquid:

Crucial question: Fate of fermi surface as you reach the QCP?

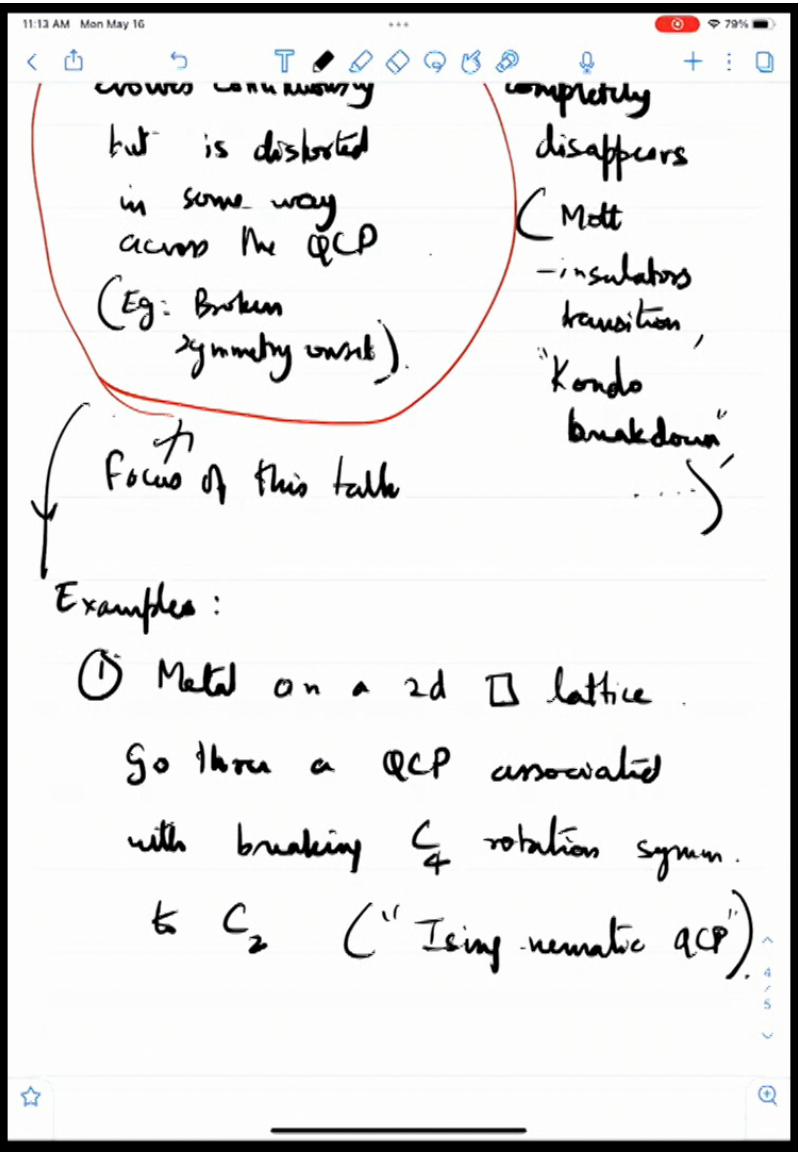



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Approach from the Fermi liquid:
Crucial question: Fate of Fermi surface as you reach the QCP?
2 general possibilities
Multiplate
Kill
Fermi surface evolves continuously but is distorted in some way across the QCP
(Eg: Broken symmetry metal).
Original FS completely disappears
(Mott metal-insulator transition

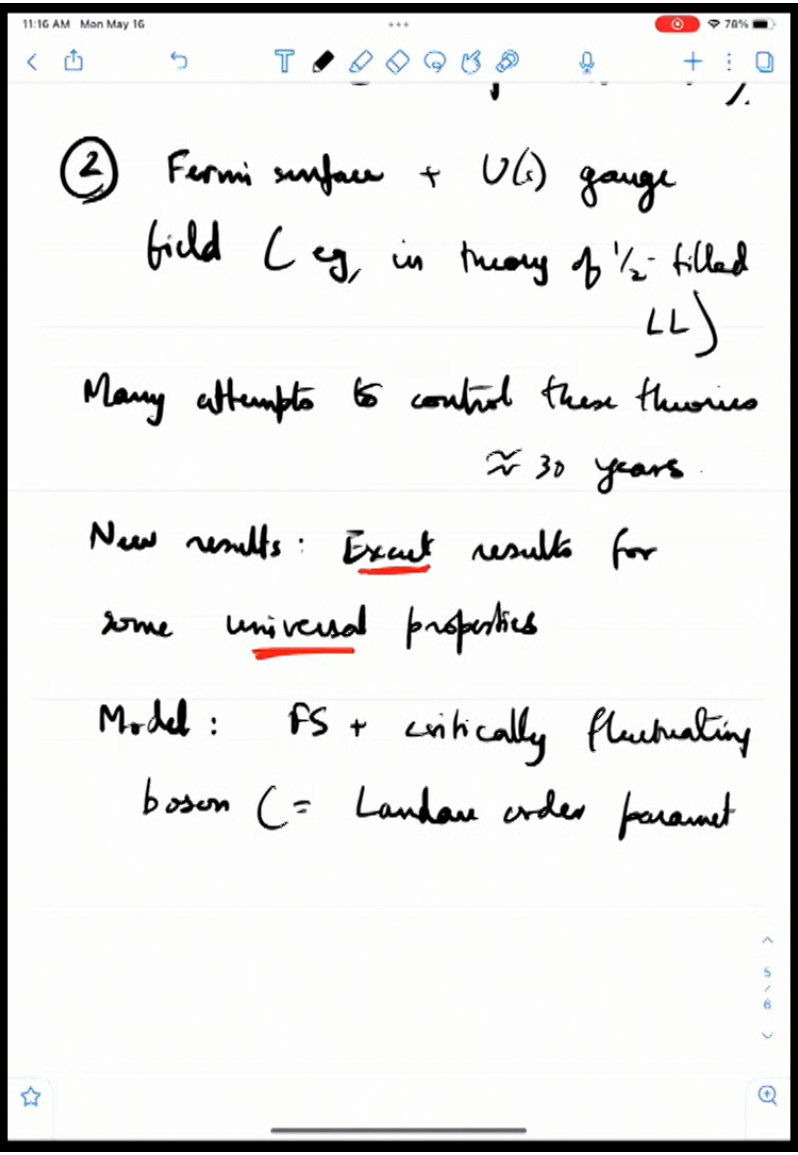



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2 general possibilities
Mutilate
Kill
Fermi surface evolves continuously but is distorted in some way across the QCP (Eg: Broken symmetry onset).
Original FS completely disappears (Mott metal-insulator transition, "Kondo breakdown" ...)
3 / 5




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but is distorted
in some way
across the QCP
(Eg: Broken
symmetry ones).
completely
disappears
(Mott
-insulators
transition,
"Kondo
breakdown"
...)
Focus of this talk
Examples:
1. Metal on a 2d □ lattice.
Go thru a QCP associated
with breaking C_4 rotation symm.
to C_2 ("Ising-nematic QCP").




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(2) Fermi surface + U(1) gauge field (eg, in theory of 1/2-filled LL)
Many attempts to control these theories ≈ 30 years.
New results: Exact results for some universal properties
Model: FS + critically fluctuating boson (= Landau order parameter
5/6



≈ 30 years.

New results: Exact results for some universal properties

Model: FS + critically fluctuating boson (= Landau order parameter)

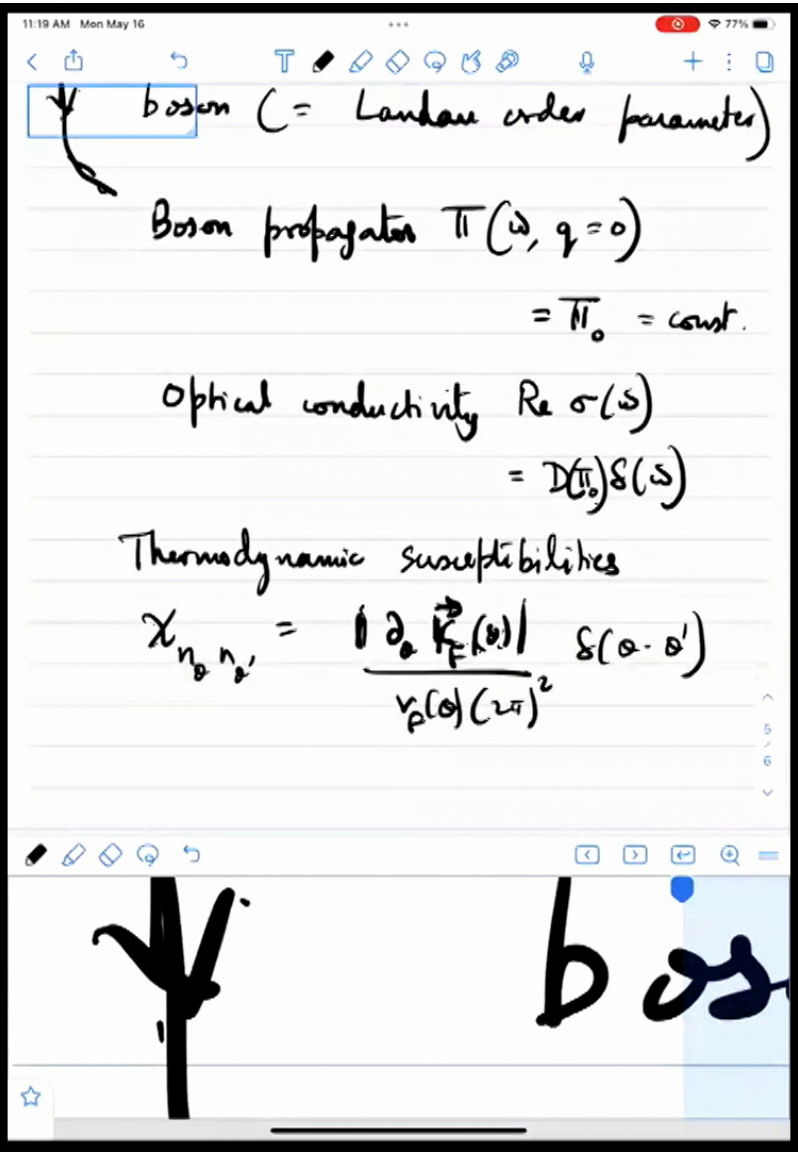
Boson propagator $\Pi(\omega, q=0)$

$= \Pi_0 = \text{const.}$

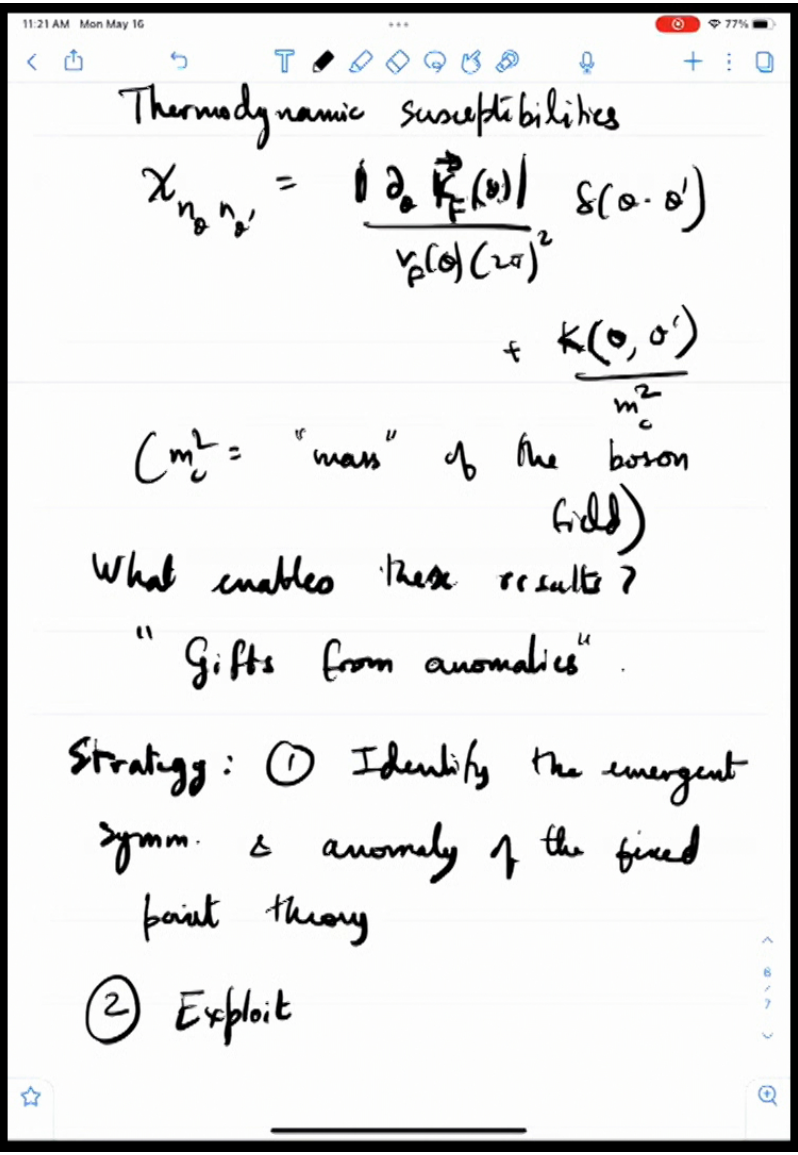
Optical conductivity $\text{Re}\,\sigma(\omega)$

$= D(\Pi_0)\delta(\omega)$




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boson (= Landau order parameter)
Boson propagator $\Pi(\omega, q=0)$
$= \Pi_0 = \text{const.}$
Optical conductivity $\text{Re}\,\sigma(\omega)$
$= D(\Pi_0)\delta(\omega)$
Thermodynamic susceptibilities
$\chi_{n_\sigma n_{\sigma'}} = \frac{|\partial_\sigma \vec{k}_F(\sigma)|}{v_F(\sigma)(2\pi)^2}\,\delta(\sigma-\sigma')$




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Thermodynamic susceptibilities
$\chi_{n_\theta n_{\theta'}} = \frac{|\partial_\theta \vec{k}_F(\theta)|}{v_F(\theta)(2\pi)^2}\,\delta(\theta-\theta') + \frac{K(\theta,\theta')}{m_c^2}$
(m_c^2 = "mass" of the boson field)
What enables these results?
"Gifts from anomalies".
Strategy: (1) Identify the emergent symm. & anomaly of the fixed point theory
(2) Exploit

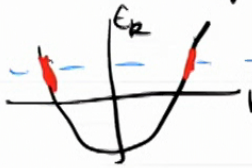


Strategy: (1) Identify the emergent symm. & anomaly of the fixed point theory

(2) Exploit these to obtain constraints on the IR properties

Illustrate:

(1) Return to simple 1d interacting fluid of fermions



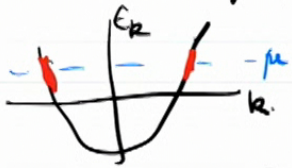
IR theory: massless Dirac fermion with $G_{IR} = U(1)$



constraints on the IR properties

Illustrate :

① Return to simple 1d interacting fluid of fermions



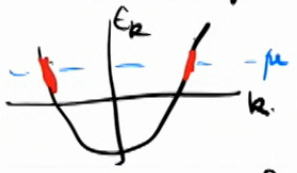
IR theory : massless Dirac fermion with $G_{I/R} = U(1) \times U(1)$

Anomaly eqns : In an E-field

$$\partial_\mu j^\mu_R = E/2\pi \; ; \; \partial_\mu j$$

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fluid of fermions



IR theory:

massless Dirac fermion with $G_{IR} = U(1) \times U(1)$

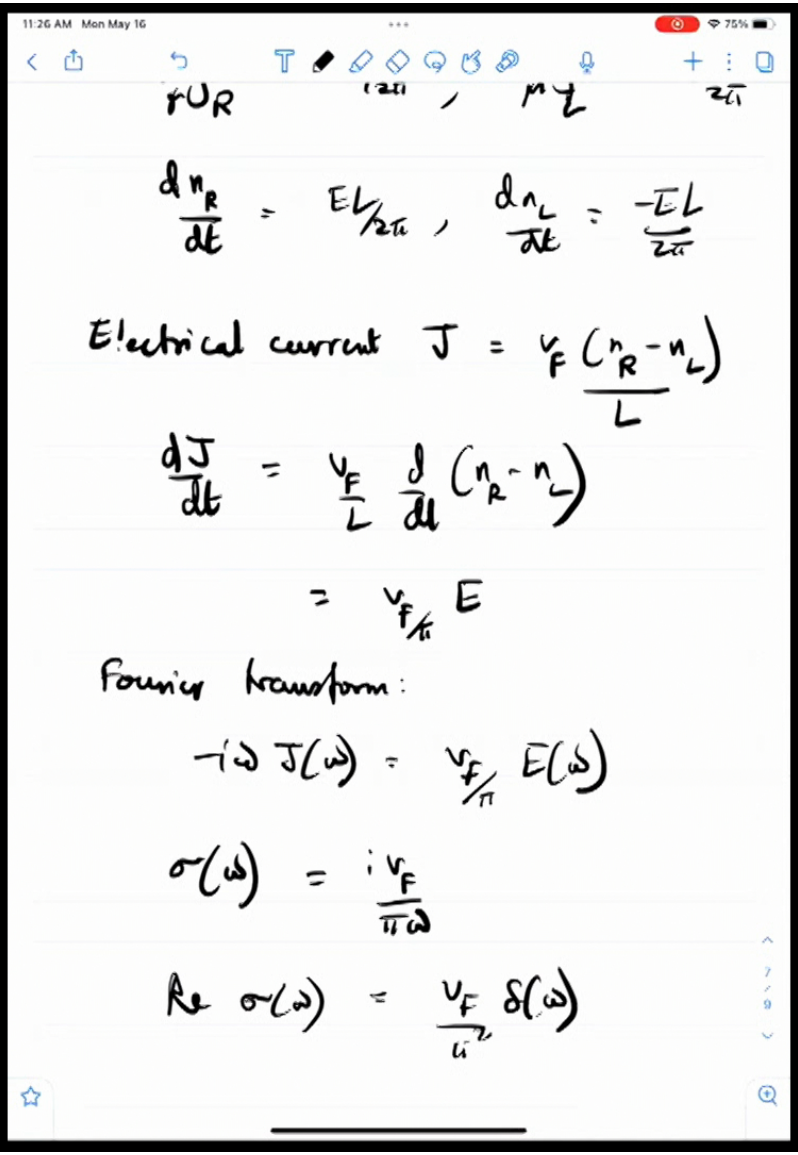
Anomaly eqns: In an E-field

$$\partial_\mu J^\mu_R = E/2\pi \;, \quad \partial_\mu J^\mu_L = -E/2\pi$$

$$\frac{dn_R}{dt} = EL/2\pi \;, \quad \frac{dn_L}{dt} = \frac{-EL}{2\pi}$$

Electrical current $J = v_F \frac{(n_R - n_L)}{L}$




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$v_F n_R$ $\frac{dn_R}{dt} = \frac{EL}{2\pi}$, $\frac{dn_L}{dt} = \frac{-EL}{2\pi}$
Electrical current $J = v_F \frac{(n_R - n_L)}{L}$
$\frac{dJ}{dt} = \frac{v_F}{L} \frac{d}{dt}(n_R - n_L)$
$= \frac{v_F}{\pi} E$
Fourier transform:
$-i\omega J(\omega) = \frac{v_F}{\pi} E(\omega)$
$\sigma(\omega) = \frac{i v_F}{\pi \omega}$
$\mathrm{Re}\ \sigma(\omega) = \frac{v_F}{\omega^2} \delta(\omega)$



$$\sigma(\omega) = \frac{i v_F}{\pi \omega}$$

$$\mathrm{Re}\ \sigma(\omega) = \frac{v_F}{\omega^2}\,\delta(\omega)$$

(2) Schwinger model in 1+1-D

$$S = \int \bar{\psi}\,(i\partial\!\!\!/ + A\!\!\!/)\,\psi - \frac{1}{2g^2} f^2$$

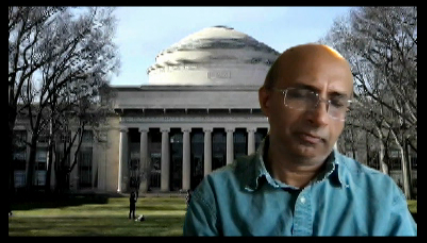
Anomaly: $\partial_\mu J^\mu_L = \frac{1}{2\pi} f$

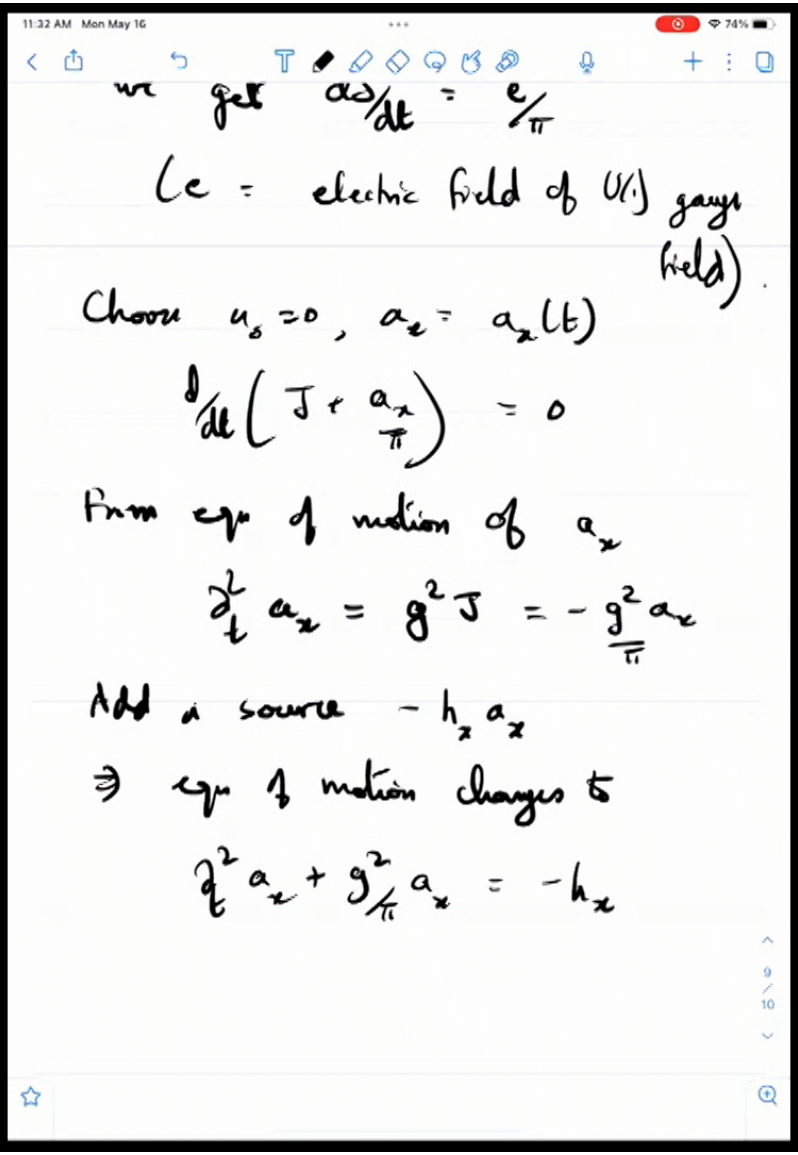
$$\partial_\mu J^\mu_R = -\frac{1}{2\pi} f$$

As before $J = J^0_R - J^0_L$, and

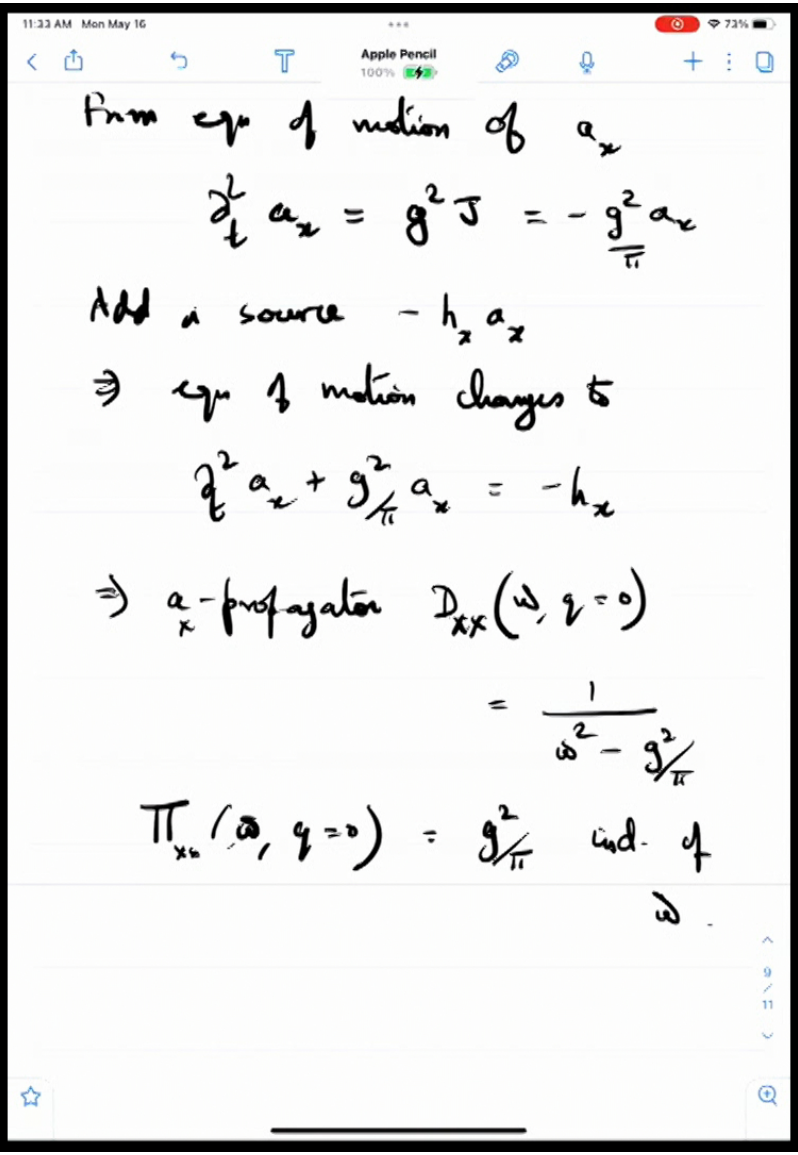
we get $dJ/dt = e/\pi$

f_e = electric field of U(1) gauge

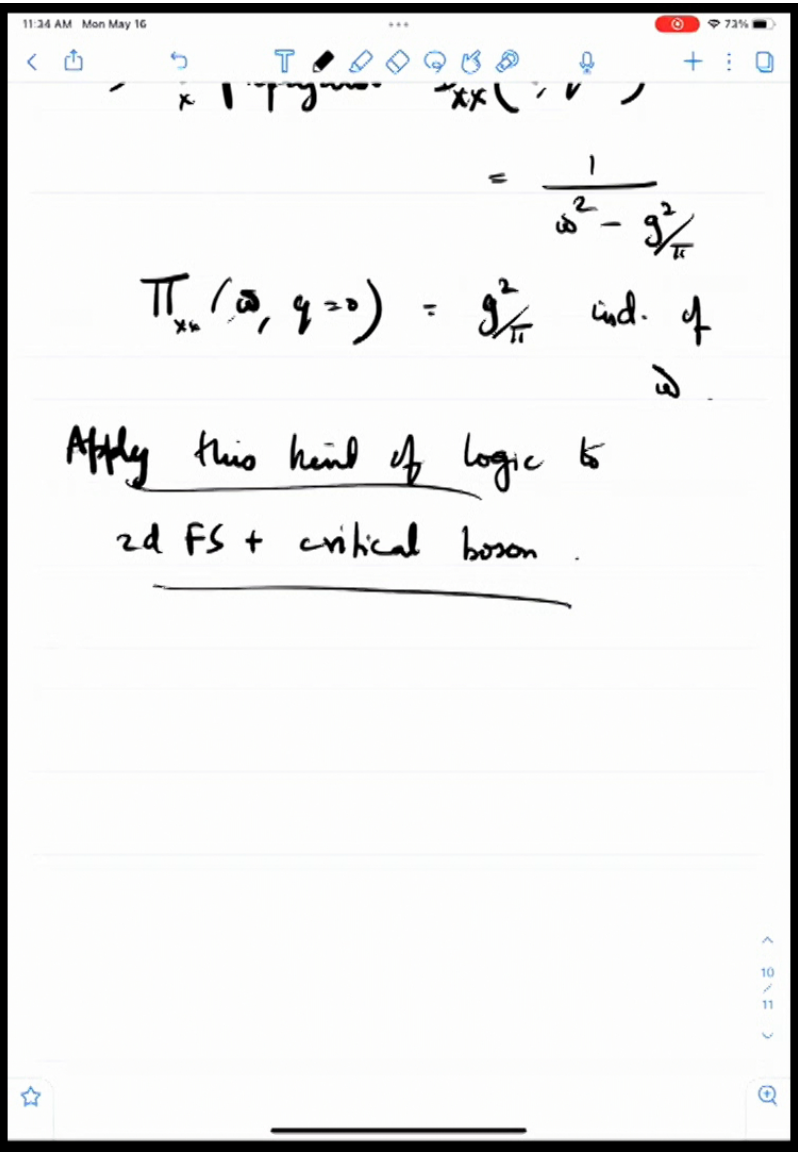



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we get $d\omega/dt = e/\pi$
E = electric field of U(1) gauge field)
Choose $a_0 = 0$, $a_x = a_x(t)$
$\frac{d}{dt}\left(J + \frac{a_x}{\pi}\right) = 0$
From eqn of motion of a_x
$\partial_t^2 a_x = g^2 J = -\frac{g^2}{\pi} a_x$
Add a source $-h_x a_x$
$\Rightarrow$ eqn of motion changes to
$\partial_t^2 a_x + \frac{g^2}{\pi} a_x = -h_x$
9/10




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73%
Apple Pencil
100%
From eqn of motion of a_x
$\partial_t^2 a_x = g^2 J = -\frac{g^2}{\pi} a_x$
Add a source $-h_x a_x$
$\Rightarrow$ eqn of motion changes to
$\partial_t^2 a_x + \frac{g^2}{\pi} a_x = -h_x$
$\Rightarrow$ a_x-propagator $D_{xx}(\omega, q=0)$
$= \frac{1}{\omega^2 - \frac{g^2}{\pi}}$
$\Pi_{xx}(\omega, q=0) = \frac{g^2}{\pi}$ ind. of ω.
9/11




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73%
$= \frac{1}{\omega^2 - g^2/\pi}$
$\Pi_{xx}(\omega, q=0) = g^2/\pi$ ind. of ω
Apply this kind of logic to
2d FS + critical boson.
10 / 11



11:42 AM Mon May 16

71%

Some dispersion, coupled by Yukawa coupling to a boson that is tuned to be massless.

Preliminary look:

Boson self energy 

$\sim \frac{|\omega|}{|q|}$

(C for Ising Reg.)



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71%

Apply this kind of logic to

2d FS + critical boson.

Microscopic model: Fermions with some dispersion, coupled by Yukawa coupling to a boson that is tuned to be massless.

Preliminary look:

Boson self energy 

$\sim \frac{|\omega|}{|q|}$

(C for Landau damping)



11:42 AM Mon May 16 71%

Some dispersion, coupled by Yukawa coupling to a boson that is tuned to be massless.

Preliminary look:

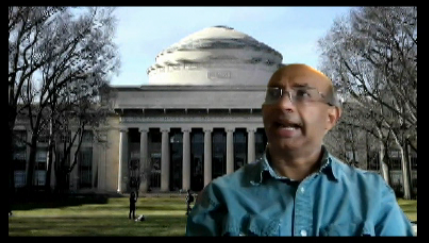
Boson self energy 

$\sim \frac{|\omega|}{|q|}$ (for irrep. freq.)

Fermion self energy 

$\sim -i|\omega|^{2/3}\,\mathrm{sgn}(\omega)$

10 / 12



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71%

Boson self energy 

$\sim \frac{|\omega|}{|q|}$

(C for (mag. freq.))

Fermion self energy 

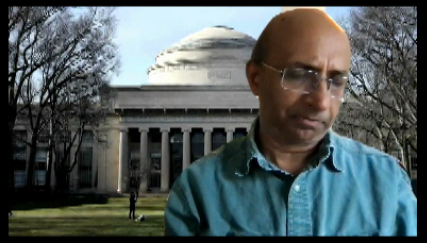
$\sim -i|\omega|^{2/3}\,\mathrm{sgn}(\omega)$

Fermi liquid is destroyed!

Long history of attempts to control

IR physics; b

11 / 12



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Long history of attempts to control IR physics; but nothing fully satisfactory as of today.

Physics input: Critical boson can only scatter fermions by a small angle.

Divide FS into small patches & couple each pair



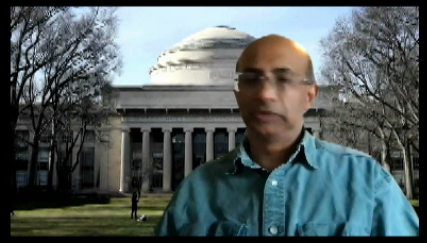
can only scatter fermions by a small angle.

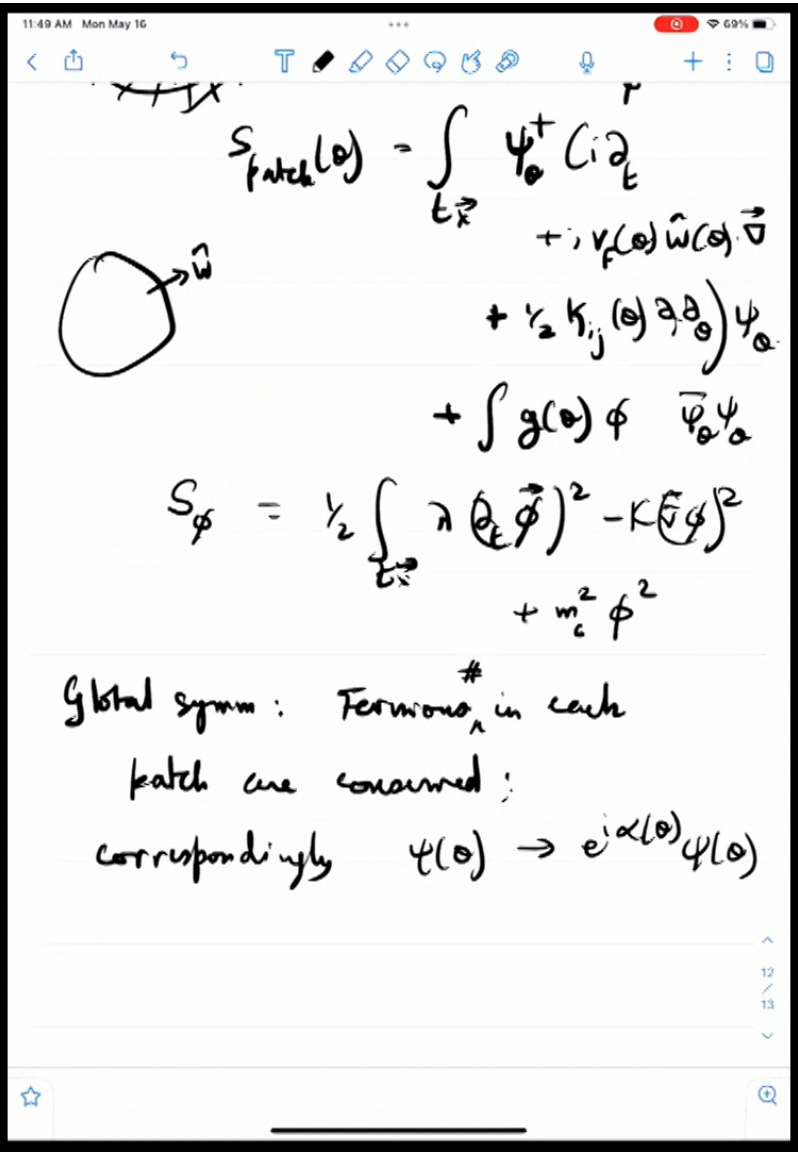
Divide FS into small patches & couple each patch to boson.



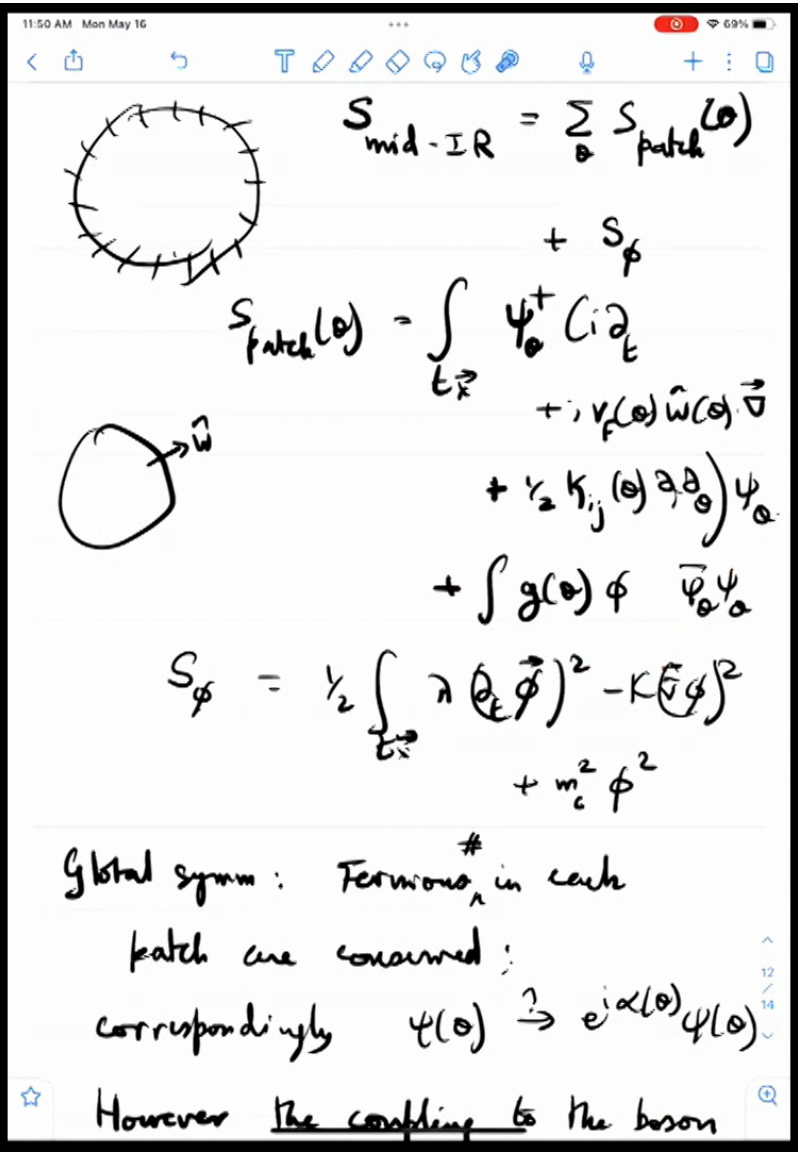
$$S_{\text{mid-IR}} = \sum_{\theta} S_{\text{patch}}(\theta) + S_{\phi}$$

$$S_{\text{patch}}(\theta) = \int_{t,\vec{x}} \psi_{\theta}^{+} (i\partial_t + i v_F(\theta)\, \hat{w}(\theta)\cdot\vec{\nabla})$$

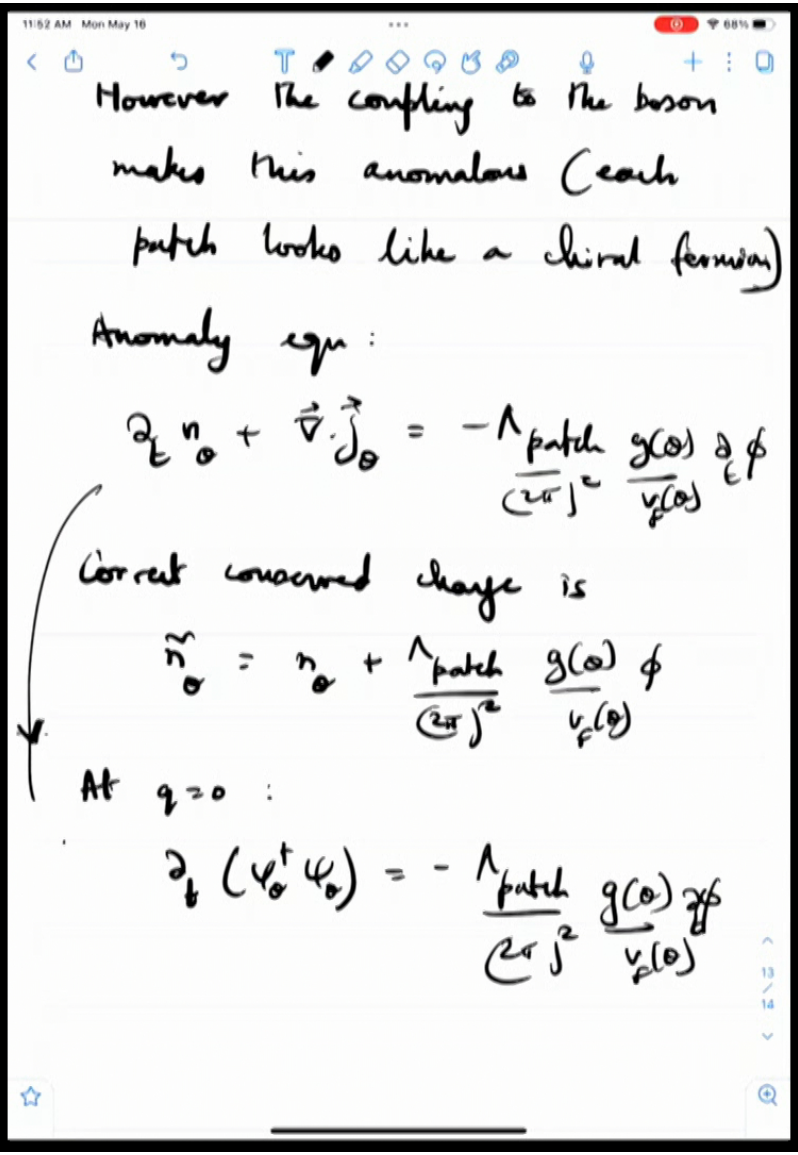


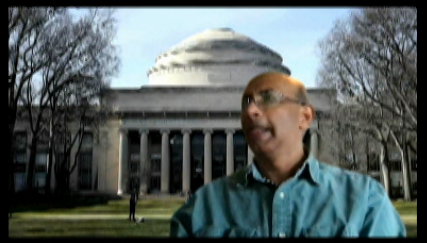

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$S_{patch}(\theta) = \int_{t,\vec{x}} \psi_\theta^\dagger (i\partial_t$
$+ i v_F(\theta)\hat{w}(\theta)\cdot\vec{\nabla}$
$+ \tfrac{1}{2} k_{ij}(\theta)\partial_i\partial_j)\psi_\theta$
$+ \int g(\theta)\phi\ \bar{\psi}_\theta\psi_\theta$
$\hat{w}$
$S_\phi = \tfrac{1}{2}\int_{t,\vec{x}} \pi(\partial_t\phi)^2 - K(\vec{\nabla}\phi)^2$
$+ m_c^2\phi^2$
Global symm: Fermions in each patch are conserved;
correspondingly $\psi(\theta) \to e^{i\alpha(\theta)}\psi(\theta)$
12
13




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$S_{mid-IR} = \sum_\theta S_{patch}(\theta)$
$+ S_\phi$
$S_{patch}(\theta) = \int_{t\vec{x}} \psi_\theta^+ (i\partial_t$
$+ v_F(\theta)\hat{w}(\theta)\cdot\vec{v}$
$+ \tfrac{1}{2} k_{ij}(\theta) q_i q_j)\psi_\theta$
$+ \int g(\theta)\phi\ \bar{\psi}_\theta \psi_\theta$
$\hat{w}$
$S_\phi = \tfrac{1}{2}\int_{t\vec{x}} \lambda(\partial_t\vec{\phi})^2 - K(\vec{\nabla}\phi)^2$
$+ m_c^2\phi^2$
Global symm: Fermions in each patch are conserved;
correspondingly $\psi(\theta) \to e^{i\alpha(\theta)}\psi(\theta)$
However the coupling to the boson




However the coupling to the boson makes this anomalous (each patch looks like a chiral fermion)
Anomaly eqn :
$\partial_t n_\sigma + \vec{\nabla}\cdot\vec{j}_\sigma = -\frac{\Lambda_{patch}}{(2\pi)^2}\frac{g(\sigma)}{v_F(\sigma)}\partial_t\phi$
Correct conserved charge is
$\tilde{n}_\sigma = n_\sigma + \frac{\Lambda_{patch}}{(2\pi)^2}\frac{g(\sigma)}{v_F(\sigma)}\phi$
At $q=0$:
$\partial_t(\psi_\sigma^\dagger\psi_\sigma) = -\frac{\Lambda_{patch}}{(2\pi)^2}\frac{g(\sigma)}{v_F(\sigma)}\partial_t\phi$



Correct conserved charge is

$$\tilde{n}_\sigma = n_\sigma + \frac{\Lambda_{patch}}{(2\pi)^2} \frac{g(\sigma)}{v_F(\sigma)} \phi$$

At $q = 0$:

$$\partial_t (\psi_\sigma^\dagger \psi_\sigma) = - \frac{\Lambda_{patch}}{(2\pi)^2} \frac{g(\sigma)}{v_F(\sigma)} \chi\phi$$

Eqn of motion for ϕ :

$$(\lambda \partial_t^2 + m_c^2)\phi = \sum_\sigma g(\sigma) \psi_\sigma^\dagger \psi_\sigma + h(t)$$

source

Fourier and look at $q = 0$:

$$(-\lambda \omega^2 + m_c^2$$



$$\partial_t (\psi_\sigma^\dagger \psi_\sigma) = - \frac{\Lambda_{patch}}{(2\pi)^2} \frac{g(\sigma)}{v_F(\sigma)} \chi\phi$$

Eqn of motion for ϕ :

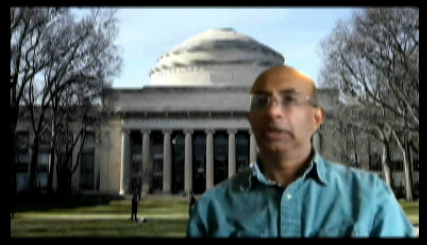
$$\left(\lambda \partial_t^2 + m_c^2\right)\phi = \sum_\sigma g(\sigma) \psi_\sigma^\dagger \psi_\sigma + h(t)$$

source

Fourier and look at $q = 0$:

$$\left(-\lambda \omega^2 + m_c^2 + \Pi\right) \phi(q=0, \omega) = h(\omega)$$

with $\Pi = \sum_\sigma \frac{\Lambda_{patch}}{(2\pi)^2} \frac{(g(\sigma))^2}{v_F(\sigma)}$



$$\left(\lambda \partial_t^2 + m_c^2\right)\phi = \sum_\theta g(\theta)\,\psi_\theta^\dagger \psi_\theta + h(t)$$

source

Fourier and look at $q = 0$:

$$\left(-\lambda\omega^2 + m_c^2 + \Pi\right)\phi(q=0,\omega) = h(\omega)$$

with $\Pi = \sum_\theta \frac{\Lambda_{patch}}{(2\pi)^2} \frac{(g(\theta))^2}{v_F(\theta)}$

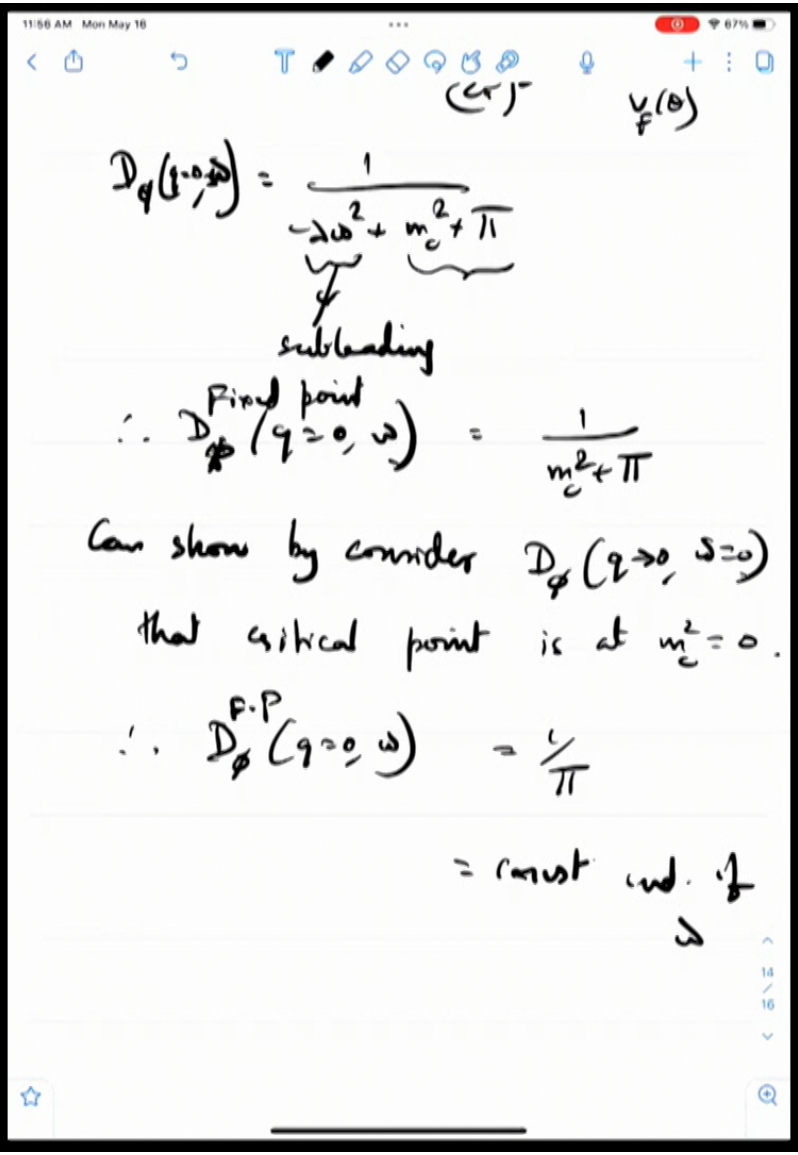
$$D_\phi(q=0,\omega) = \frac{1}{-\lambda\omega^2 + m_c^2 + \Pi}$$

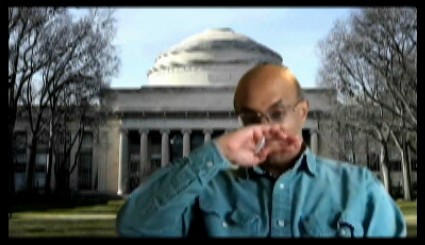
$-\lambda\omega^2$: subleading

Pinned point

$$\therefore D_\phi(q=0,\omega) = \frac{1}{m_c^2 + \Pi}$$




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$(\zeta\tau)^-$
$V_f(\theta)$
$D_\phi(q=0,\omega) = \frac{1}{-\omega^2 + m_c^2 + \Pi}$
subleading
$\therefore D_\phi^{\text{Fixed point}}(q=0,\omega) = \frac{1}{m_c^2+\Pi}$
Can show by consider $D_\phi(q\to 0, \omega=0)$
that critical point is at $m_c^2 = 0$.
$\therefore D_\phi^{F.P}(q=0,\omega) = \frac{1}{\Pi}$
= const ind. of ω
14/16



$\therefore D_{\phi}^{\text{Fixed point}}(q=0,\omega) = \frac{1}{m_c^2 + \Pi}$

Can show by consider $D_{\phi}(q\to 0, \omega=0)$

that critical point is at $m_c^2 = 0$.

$\therefore D_{\phi}^{F.P}(q=0,\omega) = \frac{1}{\Pi}$

= const ind. of ω

Central question:

$\mathrm{Re}\,\sigma(\omega) = D\,\delta(\omega) + \sigma_{inc}$

Schematic $D = \sum_{0}$ (conserved) $\frac{\chi_{J0}^2}{\chi_{00}}$