Title: Quantum gravity here and now, and at the end of the world

Speakers: Joao Magueijo

Series: Cosmology & Gravitation

Date: May 02, 2022 - 12:00 PM

URL: https://pirsa.org/22050024

Abstract: I review a recent approach to connecting quantum gravity and the real world by deconstantizing the constants of nature, and using their conjugate as a time variable. This is nothing but a generalization of unimodular gravity. The wave functions are then packets of plane waves moving in a space that generalizes the Chern-Simons functional. For appropriate states they link up with classical cosmology in the appropriate limit. There are however deviations, namely during the matter to Lambda transition, raising the possibility that quantum gravity could be in action here and now. At the other extreme I show how this approach can be used to resolve the cosmological singularity, and perhaps more.

Zoom Link: https://pitp.zoom.us/j/94010122575?pwd=L291eHNSOG1wZmpCL1lmWHVJaEwwdz09

Quantum gravity: here and now, and at the end of the world

João Magueijo
2022
Imperial College, London

Cosmology and Quantum Gravity

- Frequently not on speaking terms at all.
- When friendly to each other, often talking at cross purposes
- Obviously part of the problem is the so-called problem of time in quantum gravity: if you can't even find the time how can you make contact with the real world?
- The thesis in this talk is that the problem of time is closely related to the problem of the constants of nature.

The thesis in this talk:

- Time is the conjugate of the constants of Nature.
- The fundamental constants will appear side by side with all the other constants of motion in a process I will call "deconstantization"
- The problem, then, is an embarrassment of riches: too many times.
- I will argue that it is not only OK, but it is a feature of the physical world, with different time zones in action and adjustment of clocks across them a fact of life.
- It is possible that phenomenology and testability arises from this feature of the world.

The blueprint for this idea is not that far-fetched: Unimodular gravity

- The original Einstein dilemma (1919! First unification theories): Should the theory be invariant under general diffeomorphisms, or only volume preserving ones?

A. Einstein, *Do gravitational fields play an essential part in the structure of the elementary particles of matter?* Sitzungsberichte der Preussiscehen Akad. D. Wissenschaften, 1919, translated and included in **The Principle of Relativity**, by H. A. Lorentz et al Dover Press, 1923.

W. G. Unruh, Phys. Rev. **D40**, 1048 (1989).

- However, this is equivalent to full diffeormorphism invariance with a Lagrange multiplier term added to the action.

M. Henneaux and C. Teitelboim, Physics Letters **B 222**, 195 (1989).

The covariant reformulation of unimodular gravity:

- Add to the standard action a new term

$$S_0 \to S = S_0 - \int d^4x \Lambda \partial_\mu T^\mu_U = S_0 + \int d^4x (\partial_\mu \Lambda) T^\mu_U$$

where T^μ_U is a vector *density*

- EOMs are the usual equations (inc. E's equations) and:

$$\frac{\delta S}{\delta T^\mu_U} = 0 \implies \partial_\mu \Lambda = -\frac{\delta S_0}{\delta T^\mu_U} = 0,$$

$$\frac{\delta S}{\delta \Lambda} = 0 \implies \partial_\mu T^\mu_U = \frac{\delta S_0}{\delta \Lambda} = -\frac{\sqrt{-g}}{8\pi G_N}.$$

Implications:

- The constancy of Lambda becomes the result of an equation of motion: no longer is its value set in stone…

  (Goes back to Hawking's 3-form: one of the earliest solutions to the CC problem.)

- The (zero mode) of the conjugate variable is a great candidate for a time variable: the 4-Volume elapsed, in this case.
- The quantum mechanics changes dramatically.

Why not do this with every other constant of Nature?

- For any set of constants:

$$S_0 \to S = S_0 - \int d^4x\, \boldsymbol{\alpha} \cdot \partial_\mu \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}$$

- EOM: on-shell constancy and a "time-formula"

$$\frac{\delta S}{\delta \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}} = 0 \implies \partial_\mu \boldsymbol{\alpha} = 0$$
$$\frac{\delta S}{\delta \boldsymbol{\alpha}} = 0 \implies \partial_\mu \boldsymbol{T}^\mu_{\boldsymbol{\alpha}} = \frac{\delta S_0}{\delta \boldsymbol{\alpha}}.$$

- Obviously all equivalent (and equivalent to GR) under a canonical transformation:

$$\boldsymbol{\alpha} \to \beta(\boldsymbol{\alpha}) \qquad \boldsymbol{T}^\mu_\beta = \frac{\delta \boldsymbol{\alpha}}{\delta \beta} \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}$$

- But not quantum mechanically, as we will show.

Side remark: this is not that new.

- Realized by one version of the "sequester":

$$\alpha = \left(\frac{1}{16\pi G_N}, \rho_0 = \frac{\Lambda}{8\pi G_N}\right) \qquad \partial_\mu T^\mu_{\boldsymbol{\alpha}} = \sqrt{-g}\,(-R, 1) \qquad \Lambda_{obs} = \frac{1}{4}\langle R\rangle = -\frac{\Delta T_1}{4\Delta T_2}$$

N. Kaloper, A. Padilla, D. Stefanyszyn, and G. Zahariade, "Manifestly Local Theory of Vacuum Energy Sequestering," *Phys. Rev. Lett.* **116** (2016), 051302, arXiv:1505.01492.

- It puts the constants and their conjugate times on the same footing as other the conserved momenta of scalar fields used as relational clocks.
- Or as perfect fluids according to some Lagrangian formulations, e.g:

J. D. Brown, "Action functionals for relativistic perfect fluids," *Class. Quant. Grav.* **10** (1993), 1579–1606, gr-qc/9304026.

$$S_{\rm fl} = \int \mathrm{d}^4x \left[-\sqrt{-g}\,\rho\left(\frac{|J|}{\sqrt{-g}}\right) + J^\mu\left(\partial_\mu\varphi + \beta_A\partial_\mu\alpha^A\right)\right]$$

Why not do this with every other constant of Nature?

- For any set of constants:

$$S_0 \to S = S_0 - \int d^4x \, \boldsymbol{\alpha} \cdot \partial_\mu \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}$$

- EOM: on-shell constancy and a "time-formula"

$$\frac{\delta S}{\delta \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}} = 0 \implies \partial_\mu \boldsymbol{\alpha} = 0$$
$$\frac{\delta S}{\delta \boldsymbol{\alpha}} = 0 \implies \partial_\mu \boldsymbol{T}^\mu_{\boldsymbol{\alpha}} = \frac{\delta S_0}{\delta \boldsymbol{\alpha}}.$$

- Obviously all equivalent (and equivalent to GR) under a canonical transformation:

$$\boldsymbol{\alpha} \to \boldsymbol{\beta}(\boldsymbol{\alpha}) \qquad \boldsymbol{T}^\mu_{\boldsymbol{\beta}} = \frac{\delta \boldsymbol{\alpha}}{\delta \boldsymbol{\beta}} \boldsymbol{T}^\mu_{\boldsymbol{\alpha}}$$

- But not quantum mechanically, as we will show.

Side remark: this is not that new.

- Realized by one version of the "sequester":

$$\alpha = \left(\frac{1}{16\pi G_N}, \rho_0 = \frac{\Lambda}{8\pi G_N}\right) \qquad \partial_\mu T^\mu_{\boldsymbol{\alpha}} = \sqrt{-g}\,(-R, 1) \qquad \Lambda_{obs} = \frac{1}{4}\langle R\rangle = -\frac{\Delta T_1}{4\Delta T_2}$$

N. Kaloper, A. Padilla, D. Stefanyszyn, and G. Zahariade, "Manifestly Local Theory of Vacuum Energy Sequestering," *Phys. Rev. Lett.* **116** (2016), 051302, arXiv:1505.01492.

- It puts the constants and their conjugate times on the same footing as other the conserved momenta of scalar fields used as relational clocks.
- Or as perfect fluids according to some Lagrangian formulations, e.g:

J. D. Brown, "Action functionals for relativistic perfect fluids," *Class. Quant. Grav.* **10** (1993), 1579–1606, gr-qc/9304026.

$$S_{\rm fl} = \int \mathrm{d}^4x \left[-\sqrt{-g}\,\rho\left(\frac{|J|}{\sqrt{-g}}\right) + J^\mu\left(\partial_\mu\varphi + \beta_A\partial_\mu\alpha^A\right)\right]$$

The quantum mechanics is totally different…

- Not that surprising. Quantum mechanics sniffs the off-shell regions of phase space.
- Even abstracting from path integral formulation it is obvious that constants and their times will become canonical conjugates.
- Can expect a time to appear in the usually timeless WdW equation.
- Can expect superpositions with different values of the constants to be possible.
- Can expect a better grasp of normalizability and inner product (usual solutions will be like monochromatic plane waves).

BEFORE DECONSTANTIZATION:

- Action in MSS starts as:

$$S_0 = 6\kappa V_c \int dt \Big(a^2 \dot{b} - N a \Big[-(b^2 + kc^2)$$

$$\kappa = 1/(16\pi G_N),\ k = 0, \pm 1$$

$$V_c = \int d^3x$$

- The metric and connection appear as conjugates. On-shell we have: $b = \dot{a}$

(b is like the inverse comoving Hubble volume, in cosmology speak)

BEFORE DECONSTANTIZATION: (./cont)

- The example of Lambda. The WdW equation for GR (in the connection representation):

$$H = 6\kappa V_c N a\left(-(b^2+k)+\frac{\Lambda}{3}a^2\right) \qquad \left[\hat{b},\hat{a^2}\right] = i\frac{l_P^2}{3V_c} = i\mathfrak{h}, \qquad \left[-(b^2+k) - i\frac{\Lambda l_P^2}{9V_c}\frac{\partial}{\partial b}\right]\psi = 0$$

or:

$$\mathcal{H}_0 = \frac{1}{b^2+k}a^2 - \frac{3}{\Lambda} = 0 \qquad h_\alpha(b) = \frac{1}{b^2+k}, \quad \alpha = \phi = \frac{3}{\Lambda}. \qquad \left[-i\frac{l_P^2}{3V_c}h_\alpha(b)\frac{\partial}{\partial b} - \frac{3}{\Lambda}\right]\psi_s = 0$$

with solution:

$$\psi_{CS} = \mathcal{N}\exp\left[i\frac{9V_c}{\Lambda l_P^2}\left(\frac{b^3}{3}+bk\right).\right], \qquad \psi_s(b;\phi) = \mathcal{N}\exp\left[i\frac{3V_c}{l_P^2}\phi X_\phi(b)\right] \qquad X_\phi(b) = \int\frac{db}{h_\alpha(b)} = \frac{b^3}{3}+kb.$$

- I.e. the real Chern-Simons state reduced to MSS:

$$\psi_K(A) = \mathcal{N}\exp\left(\frac{3}{l_P^2\Lambda}Y_{CS}\right) \qquad Y_{CS} \to i\Im(Y_{CS})$$

AFTER DECONSTANTIZATION:

- Your WdW equation becomes a Schrodinger equation:

$$[\alpha, T_\alpha] = i\mathfrak{h} \qquad \left[-i\mathfrak{h} h_\alpha(b)\frac{\partial}{\partial b} - i\mathfrak{h}\frac{\partial}{\partial T_\alpha}\right]\psi(b, T_\alpha) = 0$$

- You gain a time factor in your monochromatic solutions (with the spatial factor satisfying the original WdW eqn)

$$\psi = \psi_s(b;\phi)\exp\left[-i\frac{3V_c}{l_P^2}\phi T_\phi\right] \qquad \psi(b, T_\phi) = \mathcal{N}\exp\left[i\frac{3V_c}{l_P^2}\phi(T_\phi - X_\phi(b))\right]$$

- You get to superpose plane waves, obtaining normalizable physical states:

$$\psi(b) = \int \frac{d\phi}{\sqrt{2\pi\mathfrak{h}}}\mathcal{A}(\phi)\exp\left[\frac{i}{\mathfrak{h}}\phi(X_\phi(b) - T_\phi)\right]$$

AFTER DECONSTANTIZATION (./cont):

- You acquire a natural conserved inner product:

$$\langle \psi_1 | \psi_2 \rangle = \int d\phi \mathcal{A}_1^\star(\phi) \mathcal{A}_2(\phi)$$

- Which for (and only for) pure Lambda states translates to:

$$\langle \psi_1 | \psi_2 \rangle = \int dX_\phi \psi_1^\star(b, T_\phi) \psi_2(b, T_\phi)$$

- (Note that this is not invariant under canonical transformations: the choice of wave number 3/Lambda can be important.)

Lee was almost there before (in his 2009 paper on unimodular gravity)

If we define that constant value $\lambda(x) = \Lambda$ we find that the wavefunction evolves as

$$\Psi(A,\tau) = \int d\Lambda \Psi(A,\Lambda) e^{\imath\tau\Lambda} \tag{70}$$

This is now an ordinary integral over Λ.

It is amusing to note that the Kodama state[12] $\Psi_k(A,\lambda) = e^{\frac{3}{\lambda}\int Y_{CS}(A)}$ is still a solution to (65), with the state considered a function of variables A and λ. With the standard point split regularizations, it solves the ordering

$$\epsilon^{ijk} \hat{\hat{E}}^a_i \hat{\hat{E}}^b_j \left(\hat{F}_{abk} - \frac{\lambda}{3} \epsilon_{abc} \hat{\hat{E}}^c_k \right) \Psi(A,\lambda) = 0 \tag{71}$$

Whether this offers any improvement of the interpretational issues facing the Kodama state is unclear.

The classical limit is obtained (don't laugh!)

- Take a Gaussian packet: $\mathcal{A}(\phi) = \sqrt{\mathrm{N}(\phi_0, \sigma_i)} = \frac{\exp\left[-\frac{(\phi-\phi_0)^2}{4\sigma^2}\right]}{(2\pi\sigma^2)^{1/4}},$

$$\psi(b,T) = \mathcal{N}'\psi(b, T_\phi; \phi_0)\exp\left[-\frac{\sigma^2(X_\phi - T_\phi)^2}{\mathfrak{h}^2}\right].$$

(obviously there are other, more quantum states).

- You saturate the Heisenberg relations: $\sigma_T \sigma_\phi \geq \frac{l_P^2}{6V_c}$

- In the appropriate limit, the peak of the probability follows the classical trajectory:

$$\dot{X}_\alpha = \dot{T}_\alpha$$

Less trivially, it also extends to multifluid situations:

- Base action: $$S = \frac{3V_c}{8\pi G_0}\int dt\left(a^2\dot{b} - Na\left[-(b^2+kc^2)+\sum_i \frac{m_i}{a^{1+3w_i}}\right]\right),$$
- Multi-time setting:
$$H\left[b, a^2; \boldsymbol{\alpha} \to i\frac{l_P^2}{3V_c}\frac{\partial}{\partial \boldsymbol{T}}\right]\psi = 0.$$
$$\psi(b, \boldsymbol{T}) = \mathcal{N}\int d\boldsymbol{\alpha}\mathcal{A}(\boldsymbol{\alpha})\exp\left[-i\frac{3V_c}{l_P^2}\boldsymbol{\alpha}\boldsymbol{T}\right]\psi_s(b;\boldsymbol{\alpha}),$$
- Away from single fluid it becomes obvious that MSS is a dispersive medium:
$$\psi_s(b,\boldsymbol{\alpha}) = \mathcal{N}_D\exp\left[i\frac{3V_c}{l_P^2}P(b,\boldsymbol{\alpha})\right]$$
$$\boldsymbol{\alpha}\cdot\boldsymbol{T} - P(b,\boldsymbol{\alpha}) = 0$$
- The wave packets have a peak that follows the classical trajectory still
$$P(b,\boldsymbol{\alpha}) = P(b;\boldsymbol{\alpha}_0) + \sum_i \left.\frac{\partial P}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}_0}(\alpha_i - \alpha_{i0}) + \ldots$$
$$\psi \approx \mathcal{N}_D e^{i\frac{3V_c}{l_P^2}(P(b;\boldsymbol{\alpha}_0)-\boldsymbol{\alpha}_0\cdot\boldsymbol{T})}\prod_i \psi_i(b,T_i)$$
$$\psi_i(b,T_i) = \int d\alpha_i \mathcal{A}(\alpha_i)e^{-i\frac{3V_c}{l_P^2}(\alpha_i-\alpha_{i0})(T_i - \frac{\partial P}{\partial \alpha_i})}$$

- An effective X can be defined. In the Gaussian approximation the classical trajectories would still be followed:

$$T_i = \left.\frac{\partial P(b)}{\partial \alpha_i}\right|_{\boldsymbol{\alpha}_0}.$$

- Indeed the CS functional and its generalizations are the linearizing variables (cf. DSR-like theories, with MDRs) of the dispersive medium in mono-fluid situations: the packets do not feel dispersion in terms of X, not b.

AFTER DECONSTANTIZATION:

- Your WdW equation becomes a Schrodinger equation:

$$[\alpha, T_\alpha] = i\mathfrak{h}$$

$$\left[-i\mathfrak{h} h_\alpha(b)\frac{\partial}{\partial b} - i\mathfrak{h}\frac{\partial}{\partial T_\alpha}\right]\psi(b, T_\alpha) = 0$$

- You gain a time factor in your monochromatic solutions (with the spatial factor satisfying the original WdW eqn)

$$\psi = \psi_s(b;\phi)\exp\left[-i\frac{3V_c}{l_P^2}\phi T_\phi\right]$$

$$\psi(b, T_\phi) = \mathcal{N}\exp\left[i\frac{3V_c}{l_P^2}\phi(T_\phi - X_\phi(b))\right]$$

- You get to superpose plane waves, obtaining normalizable physical states:

$$\psi(b) = \int \frac{d\phi}{\sqrt{2\pi\mathfrak{h}}}\mathcal{A}(\phi)\exp\left[\frac{i}{\mathfrak{h}}\phi(X_\phi(b) - T_\phi)\right]$$

But what about departures from the classical limit?

- They certainly happen near the Big Bang: I will show how this can solve minimally the singularity problem. (S.Gielen + JM, 2204.01771)

- They can happen "here and now": (S.Gielen + JM. 2201.03596)

A generalization for radiation of the Chern-Simons wave function and its unimodular extension.

- The Hamiltonian is particularly simple:

$$-(b^2+k)a^2+m=0.$$

- So $[b,a^2]=[m,T]=i\mathfrak{h}:=\frac{8\pi i G\hbar}{3V_c}$ leads to: $i\mathfrak{h}\frac{\partial}{\partial T}\psi(b,T)=-i\mathfrak{h}(b^2+k)\frac{\partial}{\partial b}\psi(b,T)$

- And so solutions of the form:

$$\psi(q,T)=\int\frac{d\alpha}{\sqrt{2\pi\mathfrak{h}}}\mathcal{A}(\alpha)\exp\left[\frac{i}{\mathfrak{h}}\alpha(X-T)\right]$$

$$\alpha_b=m\,,\quad X_b=\int^b\frac{d\tilde{b}}{(\tilde{b}^2+k)}$$

(i.e. a new CS functional, adapted to radiation)

The probability distribution at “late” times:

- T is minus conformal time while $|T| \gg \sigma_T$

$$\dot{T} = -\frac{N}{a}$$

- The peak gets sharper as the Universe expands:

$$\sigma_b \approx \sigma(X_b)/|X_b'| = b^2\sigma_T$$

- The dispersive nature of the medium actually sharpens the wave function so that they become quasi-coherent in b

$$\mathcal{P}(b,\eta) \approx \frac{\exp\left[-\frac{\left(b-\frac{1}{\eta}\right)^2}{2\sigma_b^2}\right]}{\sqrt{2\pi\sigma_b^2}}, \quad \sigma_b \approx \sigma_T/\eta^2.$$

Less trivially, it also extends to multifluid situations:

- Base action: $S = \frac{3V_c}{8\pi G_0}\int dt\left(a^2\dot{b} - Na\left[-(b^2+kc^2)+\sum_i \frac{m_i}{a^{1+3w_i}}\right]\right),$
- Multi-time setting:

$$H\left[b, a^2; \boldsymbol{\alpha} \to i\frac{l_P^2}{3V_c}\frac{\partial}{\partial \boldsymbol{T}}\right]\psi = 0.$$

$$\psi(b, \boldsymbol{T}) = \mathcal{N}\int d\boldsymbol{\alpha}\,\mathcal{A}(\boldsymbol{\alpha})\exp\left[-i\frac{3V_c}{l_P^2}\boldsymbol{\alpha T}\right]\psi_s(b;\boldsymbol{\alpha}),$$

- Away from single fluid it becomes obvious that MSS is a dispersive medium:

$$\psi_s(b,\boldsymbol{\alpha}) = \mathcal{N}_D \exp\left[i\frac{3V_c}{l_P^2}P(b,\boldsymbol{\alpha})\right]$$

$$\boldsymbol{\alpha}\cdot\boldsymbol{T} - P(b,\boldsymbol{\alpha}) = 0.$$

- The wave packets have a peak that follows the classical trajectory still

$$P(b,\boldsymbol{\alpha}) = P(b;\boldsymbol{\alpha}_0) + \sum_i \frac{\partial P}{\partial \alpha_i}\bigg|_{\boldsymbol{\alpha}_0}(\alpha_i - \alpha_{i0}) + \ldots$$

$$\psi \approx \mathcal{N}_D e^{i\frac{3V_c}{l_P^2}(P(b;\boldsymbol{\alpha}_0)-\boldsymbol{\alpha}_0\cdot\boldsymbol{T})}\prod_i \psi_i(b,T_i)$$

$$\psi_i(b,T_i) = \int d\alpha_i\,\mathcal{A}(\alpha_i)e^{-i\frac{3V_c}{l_P^2}(\alpha_i-\alpha_{i0})(T_i-\frac{\partial P}{\partial \alpha_i})}$$

The probability distribution at "late" times:

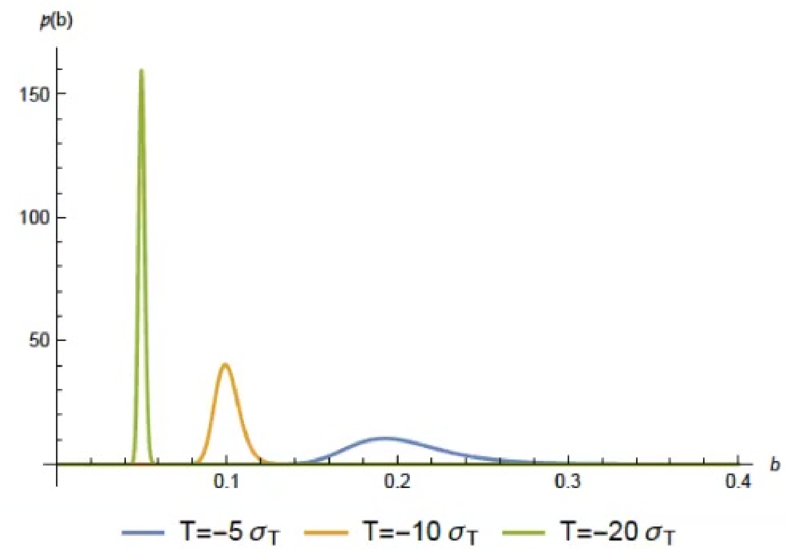

FIG. 1. As $|T| \gg \sigma_T$ the distribution $\mathcal{P}(b)$ quickly becomes near-Gaussian in b, with $\sigma(b)/b \ll 1$. We can identify $T = -\eta$ since $\sigma(T)/|T| \ll 1$, so that in the expanding branch ($T < 0, \eta > 0$) the ever-sharper peak follows the classical trajectory $b = 1/\eta$.

The probability distribution at the "end of the world"…

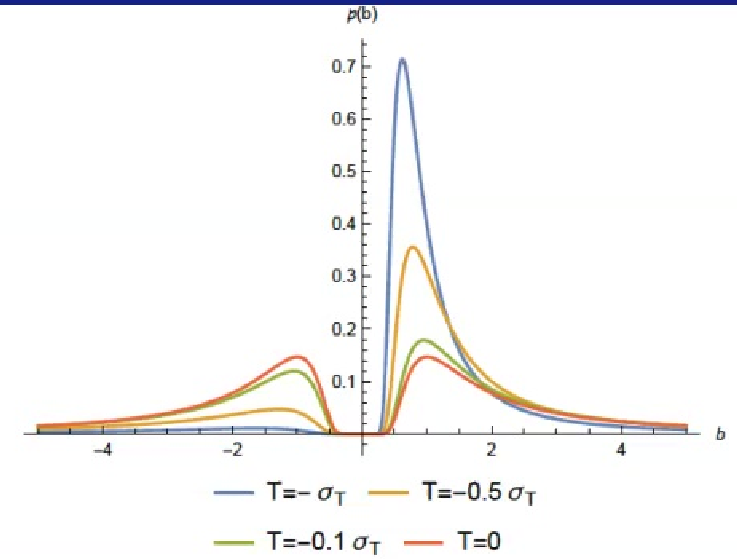

FIG. 2. For $|T| \lesssim \sigma_T$ the distribution $\mathcal{P}(b)$ is very distorted, and its peak does not go to infinity but saturates at $b = b_P$. As $T \to 0^-$ this peak lowers, and a secondary peak in the contracting zone becomes more prominent. At $T = 0$ the two peaks have the same height, but nothing is singular. For $T > 0$ a symmetric film is played, eventually linking up to a semi-classical contracting phase (not displayed). We thus have a quantum bounce.

So we have a quantum solution to the singularity problem.

- It boils down to time being "quantum" and so its uncertainty non-negligible when For $|T| \lesssim \sigma_T$
- Unitarity in quantum the wave function has to go somewhere. (A quantum transition between a contracting and expanding Universe is inevitable.)
- The fact it never goes to infinite curvatures is due in part to the measure .

Near the would be singularity it all changes:

- Instead of going away to infinity, the peak gets stuck at a maximal curvature:

$$b \approx b_P = 1/(\sqrt{2}\sigma_T) = \sqrt{2}\sigma_m/\hbar$$

- A symmetric contracting peak emerges at $b \approx -b_P < 0$
- At T=0 we can actually compute:

$$\mathcal{P}(b, T=0) = \frac{1}{b^2}\frac{\exp\left[-\frac{\left(\frac{1}{b}\right)^2}{2\sigma_T^2}\right]}{\sqrt{2\pi\sigma_T^2}}$$

- For T>0 the situation reverses linking up to a classical contracting Universe.

The current Universe is special in two ways:

- The Universe is currently filled with ingredients with different equations of state but comparable densities:
  - We may be in the process of handing over from one type of clock (a G or a dust clock) to another (a Lambda clock).
- We moved from the $w>-1/3$ regime to $w<-1/3$.
  - We have just come out of a bounce in connection space! (Not in metric space.)

Illustration with radiation and Lambda

$$H = Na\left(-(b^2+k) + \frac{\Lambda}{3}a^2 + \frac{m}{a^2}\right) = 0.$$

$$a_\pm^2 = \frac{\phi}{2}\left(V(b) \pm \sqrt{V(b)^2 - 4m/\phi}\right)$$

$$b^2 = b_0^2 := 2\sqrt{\frac{m}{\phi}} - k\,.$$

$$V(b) \equiv b^2 + k$$

$$h_\pm(b)a_\pm^2 - \phi := \frac{2a_\pm^2}{V(b) \pm \sqrt{V(b)^2 - 4m/\phi}} - \phi = 0$$

$$\psi_0(b;\Lambda,m) = \mathcal{N}\exp\left[i\frac{3V_c}{l_P^2}\phi X(b)\right]$$

$$X(b;\Lambda m,\pm) = \int db\frac{1}{2}\left(g \pm \sqrt{g^2 - \frac{4}{3}\Lambda m}\right)$$

$$X_+(b) \approx X_\phi = \frac{b^3}{3} + kb$$

$$X_-(b) \approx \frac{\Lambda m}{3}X_r$$

$$b^2 \gg \Lambda m$$

Great: but what happens during the Crossover?

As with any reflection there is an evanescent wave

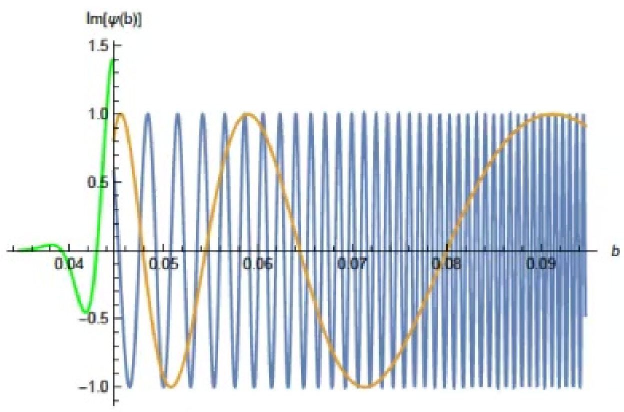

FIG. 6. Imaginary part of the full wave function ψ_s normalized so as to match the asymptotic radiation dominated expression, with parameters $\mathfrak{h} = 1$, $m = 1$, $\phi = 10^6$. The incident (orange) and reflected (blue) waves, when superposed, match the evanescent wave (green) up to second derivatives in this plot.

As with any reflection, the incident and reflected wave interfere, leading to ringing within the packet

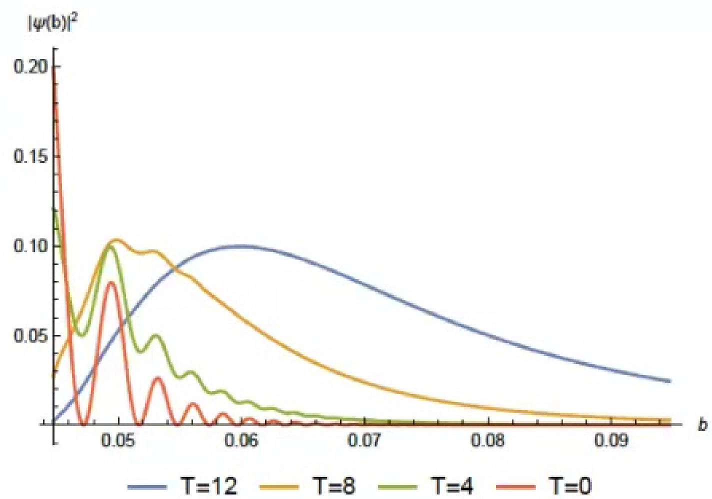

FIG. 8. A plot of $|\psi|^2$ for the same situation as in Fig.7 at $T_m = 12, 8, 4, 0$. (For the particular case of T_m – but not for a generic time – this function is symmetric, so for clarity we have refrained from plotting the equivalent $T_m < 0$.)

The semi-classical probability distribution around the b-bounce

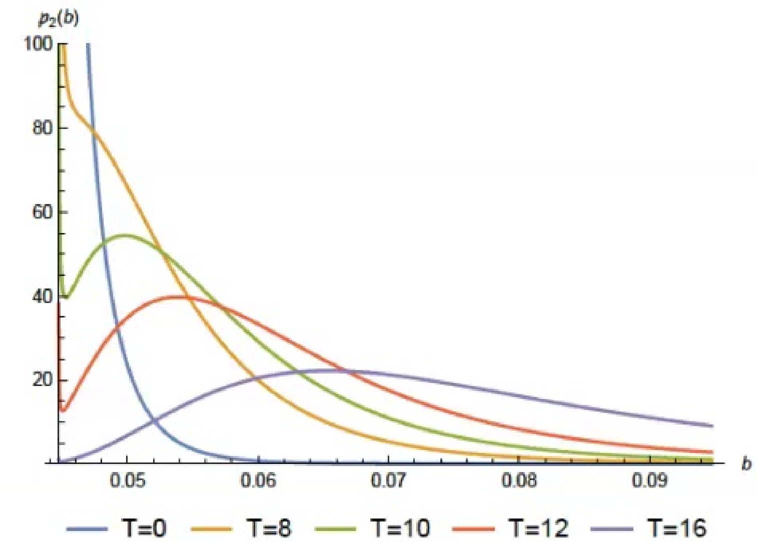

FIG. 9. The probability with the semiclassical measure, for the same situation as in Fig.7, at the various times $T_m = 16, 12, 10, 8, 0$. We have verified explicitly that this probability density, unlike the function plotted in Fig.8, always integrates to unity.

Watch this space… (work in progress)

- The realistic case of a matter to Lambda transition is currently being worked out
- Niayesh and I have solved the cosmological constant problem last week…