Title: Subsystem symmetry fractionalization in two dimensions

Speakers: Michael Hermele

Series: Quantum Matter

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Abstract:

Unlike conventional global symmetries that act on all of space, subsystem symmetries act only on rigid subspaces such as lines, planes or fractal regions. Such symmetries are of interest for their close connection to fracton physics and in their own right. In this talk, I discuss phenomena that can occur in two-dimensional systems with subsystem symmetry and anyon excitations, where the symmetry fractionalizes on anyon excitations. This turns out to be manifest in the mobility of the fractionalized excitations under dynamics that respect the symmetry. Such symmetry fractionalization was previously thought to be impossible, and indeed there are a number of differences from conventional symmetry fractionalization. I will present a general framework to describe subsystem symmetry fractionalization in two dimensions, and give a number of simple examples of this phenomenon in commuting Pauli Hamiltonians.

This talk is based on arXiv:2203.13244, work in collaboration with David Stephen, Arpit Dua, José Garre-Rubio and Dominic Williamson.

Zoom Link: https://pitp.zoom.us/j/93741883840?pwd=RlMrUGwwVEt5WFdoNGpkMmdSVXp5UT09

Subsystem symmetry fractionalization in two dimensions

Michael Hermele

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CTQM

Quantum Matter Frontier Seminar

With: David Stephen, Arpit Dua, José Garre-Rubio, Dominic Williamson. arXiv:2203.13244

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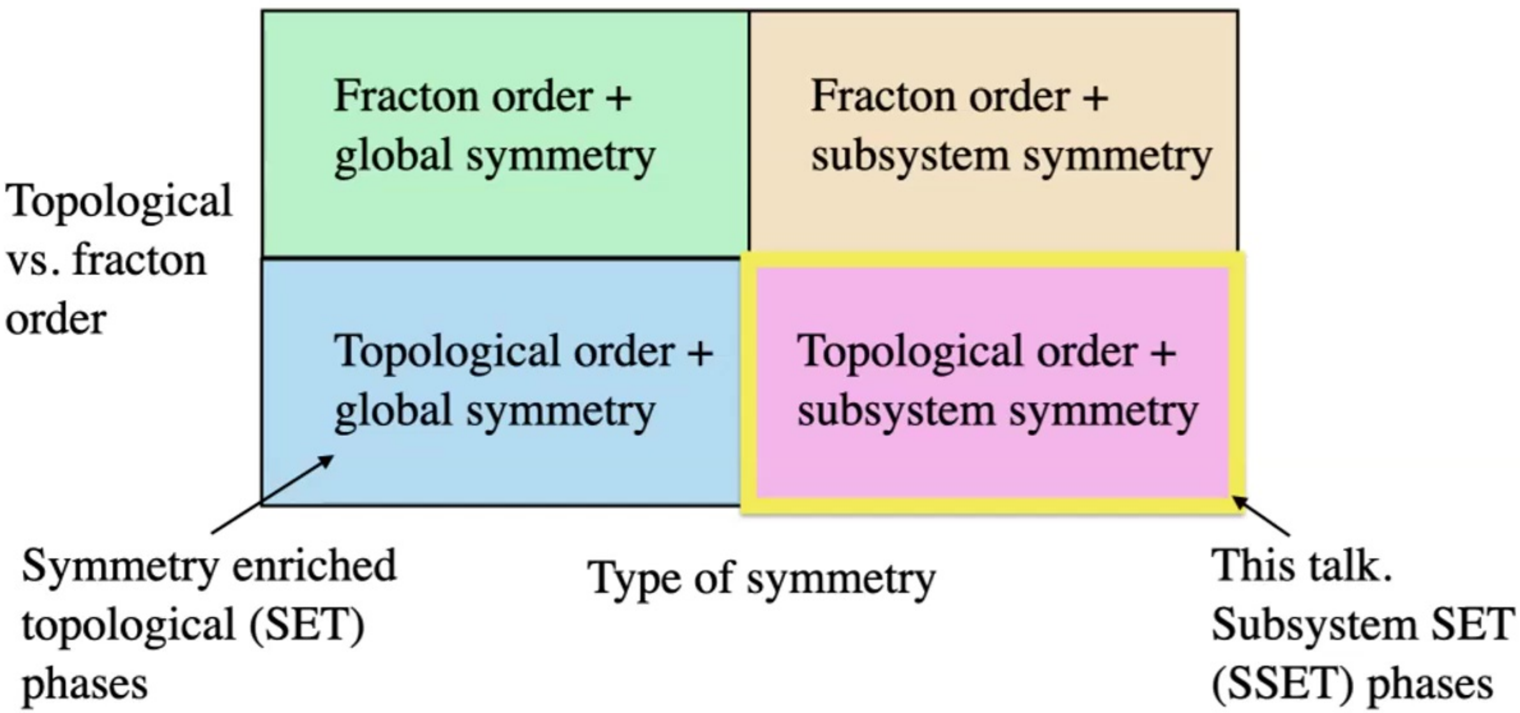
Department of Energy Basic Energy Sciences, Grant # DE-SC0014415

Motivation

General question: how are topologically ordered phases (or fracton phases) enriched in the presence of symmetry?



SET phases

Topological order of the 2d toric code

1 (trivial)	e (boson)	m (boson)	$\epsilon = e \times m$ (fermion)

Mutual statistics between e and m: $\Theta_{e,m} = \pi$

Add a Z_2 global symmetry (Ising symmetry), $Z_2 = \{1, a\}$

- Trivial excitations have $U(a) = \pm 1$.
- Non-trivial anyons may have $U(a) = \pm i$, "carry half charge." A sign of this is $U(a)^2 = -1$ on a single anyon, while $U(a)^2 = 1$ on trivial excitations.
- Also possible for symmetry to permute anyons: $e \leftrightarrow m$

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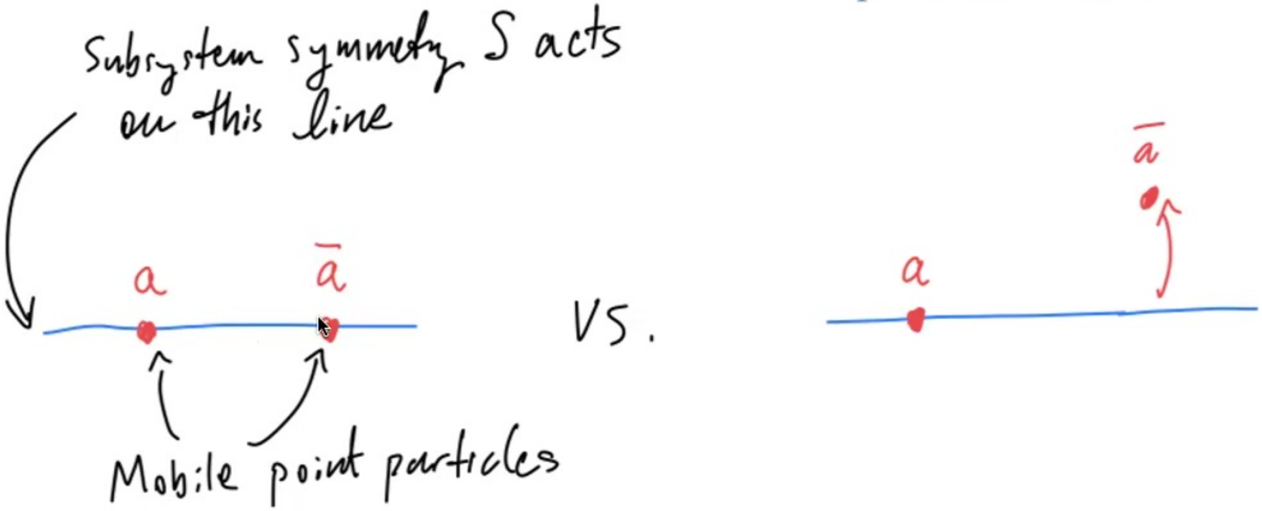
- Symmetries that act on (rigid) subspaces of space (*e.g.* lines, planes, fractals)
- In 3d (and higher), gauging subsystem symmetry can lead to fracton order (Williamson; Vijay, Haah & Fu)
- Can protect subsystem SPT (SSPT) phases with interesting boundary phenomena (Raussendorf, Okay, Wang, Stephen, Nautrup; You, Devakul, Burnell & Sondhi; …)
- SSPT phases are a resource for measurement-based quantum computation (Else, Bartlett & Doherty; Raussendorf, Okay, Wang, Stephen, Nautrup)

Subsystem-symmetry enriched topological (SSET) phases

- Simplest case: 2d topologically ordered phases with subsystem symmetry
- Related to SSPT phases, with *both* subsystem and global (0-form) symmetries, and non-trivial interplay between these
- Also related (in higher dimensions) to systems with both topological and fracton order
- In general, non-trivial SSET phases have interesting symmetry-enforced mobility restrictions

A no-go argument

Stephen, Garré-Rubio, Dua, Williamson



- *a* excitation must transform like a local excitation under *S*
- Conclusion: subsystem symmetry cannot be fractionalized on mobile point-like excitations
- As a consequence, Stephen *et al* studied subsystem symmetry fractionalization on loop excitations of d=3 topological orders
- Refined argument (this talk): not everything is ruled out

Outline

1. Review: fractionalization of ordinary global symmetries in two dimensions (*i.e.* 0-form symmetries)

2. General framework for subsystem symmetry fractionalization

3. Illustration in a solvable model

Setup

- Consider 2d gapped bosonic lattice system with non-trivial topological order (*i.e*. anyon excitations are present), need not be abelian

- System has an unbroken global (0-form) symmetry G, assume unitary and finite, represented by products of on-site unitaries $U(g)$. Assume symmetry does not permute anyon types.

- Fix a region R and focus on "localizable states on R":

Localizable states on R are locally same as ground state for all discs within ∂R
(∂R = thickened boundary of R)

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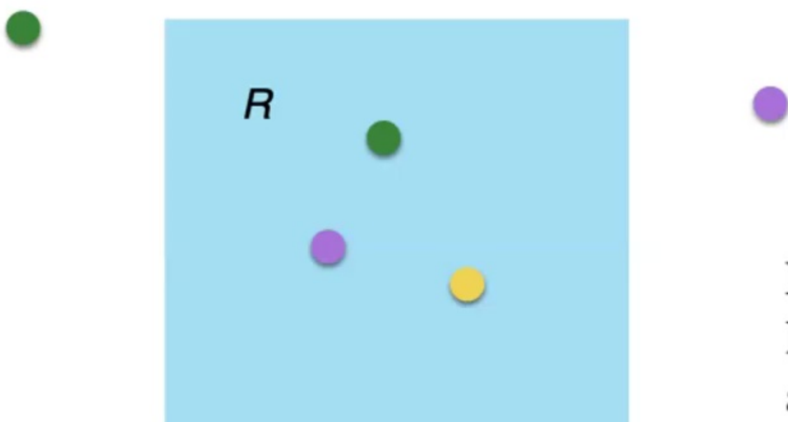

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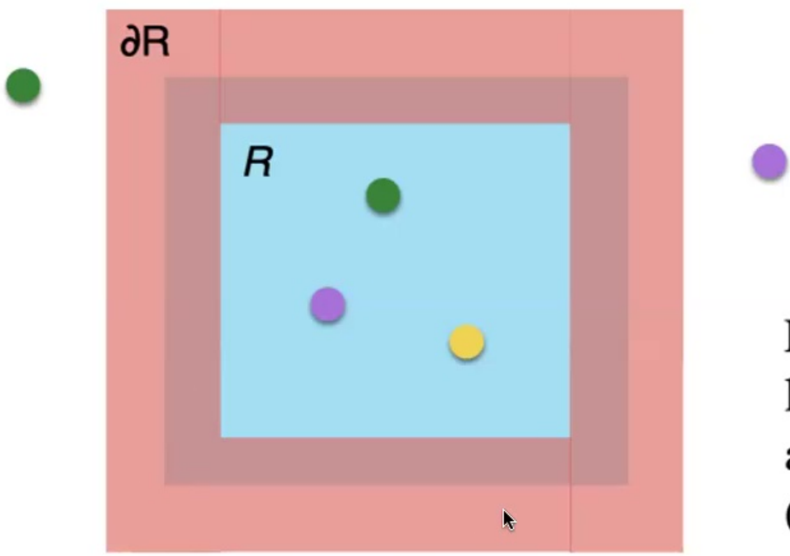

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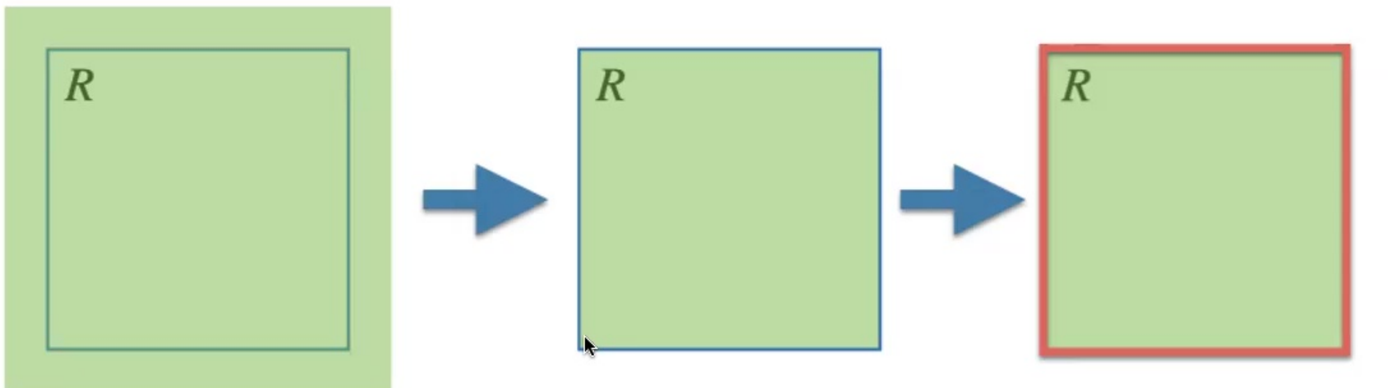
Symmetry localization

Essin & Hermele; Mesaros & Ran; Barkeshli, Bonderson, Cheng, Wang



Symmetry operator $U(g)$

Truncated symmetry $U_R(g)$

Symmetry localization $V_R(g)$

- Symmetry localization $V_R(g)$ is constructed so as not to create excitations in ∂R (*i.e.* preserves localizable states)
- Only exists when symmetry does not permute anyon types and is not spontaneously broken

Symmetry fractionalization from localized operators

- Multiply two localized operators: $V_R(g_1)V_R(g_2) = \omega(g_1, g_2)V_R(g_1g_2)$

  ↑ Supported on ∂R

- $\omega(g_1,g_2)$ is an (abelian) anyon string operator on ∂R

- Can show: $\omega(g_1, g_2)\omega(g_1g_2, g_3) = \omega(g_1, g_2g_3)\omega(g_2, g_3)$ (2-cocycle condition)

- Ambiguity in $\omega(g_1,g_2)$ from $V_R(g) \rightarrow \lambda(g)V_R(g)$

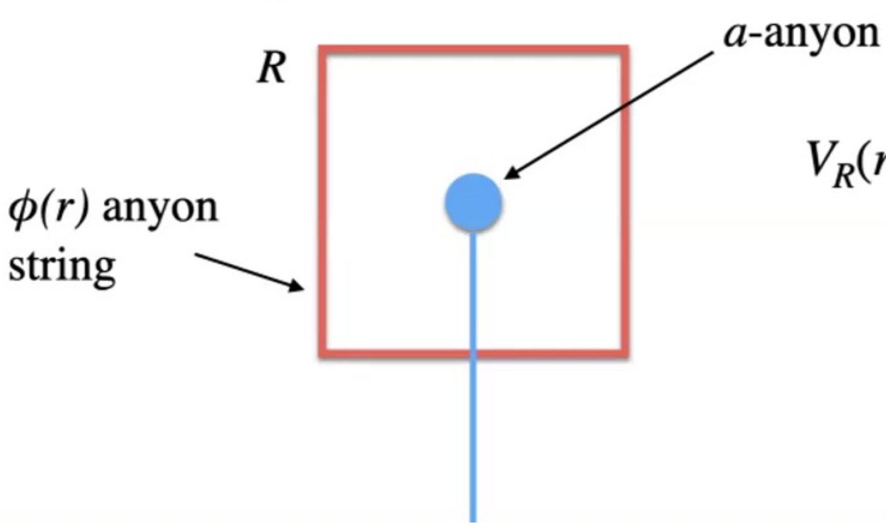
  ↖ abelian anyon string

$$\Longrightarrow \omega(g_1, g_2) \rightarrow \frac{\lambda(g_1)\lambda(g_2)}{\lambda(g_1g_2)}\omega(g_1, g_2)$$ (Transformation by 2-coboundary)

Therefore extract cohomology class $[\omega] \in H^2(G, \mathscr{A})$,
where $\mathscr{A}$ is group of abelian anyons

Generators and relations

- Describe symmetry group in terms of generators s_i, relations $r = s_1 s_2 \cdots s_n$
- For each relation $V_R(r) = V_R(s_1) \cdots V_R(s_n)$ is an anyon string on ∂R, with anyon type $\phi(r)$. These anyon types determine a fractionalization class $[\omega] \in H^2(G, \mathscr{A})$
- Physically, $\phi(r)$ tells us how the relation r acts projectively on anyons within R:

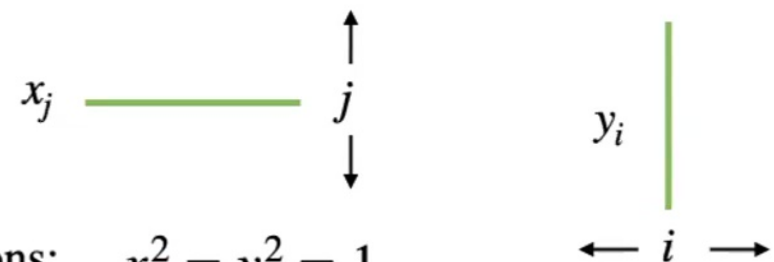


$$V_R(r)\,|\psi\rangle = \exp\left(i\Theta_{a,\phi(r)}\right)\,|\psi\rangle$$

Mutual statistics between a and $\phi(r)$

Subsystem symmetry

- Introduce general framework in the context of a specific subsystem symmetry
- This is a linear Z_2 subsystem symmetry on perpendicular rows/columns
- Row generators x_j and column generators y_i, with $i, j \in \mathbb{Z}$



- Relations:

$$x_i^2 = y_j^2 = 1$$

$$x_j x_{j'} x_j^{-1} x_{j'}^{-1} = 1$$

$$y_i y_{i'} y_i^{-1} y_{i'}^{-1} = 1$$

$$x_j y_i x_j^{-1} y_i^{-1} = 1$$

$$\prod_j x_j \prod_i y_i^{-1} = 1$$

"Global relation"

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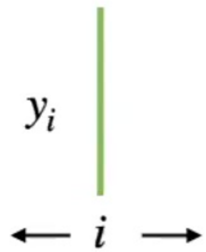

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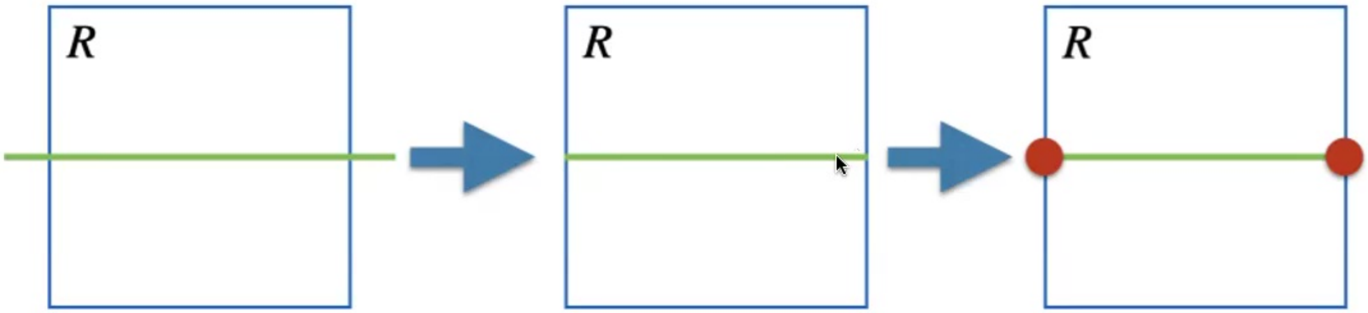
"Global relation"

Symmetry localization

Localization of row symmetries x_j:



Symmetry $U(x_j)$ Truncated $U_R(x_j)$ Localization $V_R(x_j)$

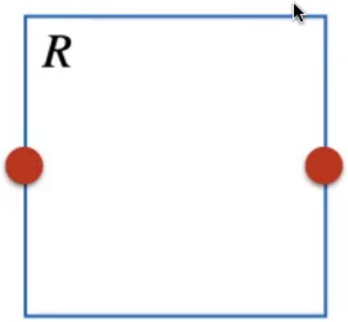
Localization of column symmetries is analogous

We assume these localizations exist.

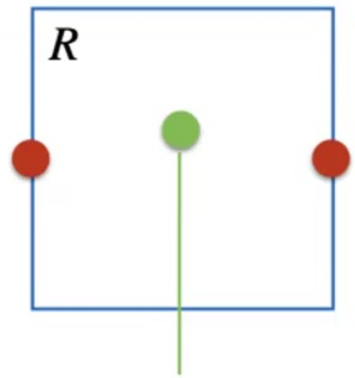
Note: can argue subsystem symmetry cannot permute anyon types.
This is necessary but not sufficient for localizations to exist.

No-go argument revisited

Consider the relation $r = x_j^2$ and its localization $V_R(r) = V_R(x_j)^2$



$V_R(r)$ is only supported on two small disc-shaped regions within ∂R



Anyon string operator can always avoid support of $V_R(r)$

Fractionalization of this relation is trivial

Similar arguments go through for all of:

$$x_i^2 = y_j^2 = 1$$

$$x_j x_{j'} x_j^{-1} x_{j'}^{-1} = 1$$

$$y_i y_{i'} y_i^{-1} y_{i'}^{-1} = 1$$

$$x_j y_i x_j^{-1} y_i^{-1} = 1$$

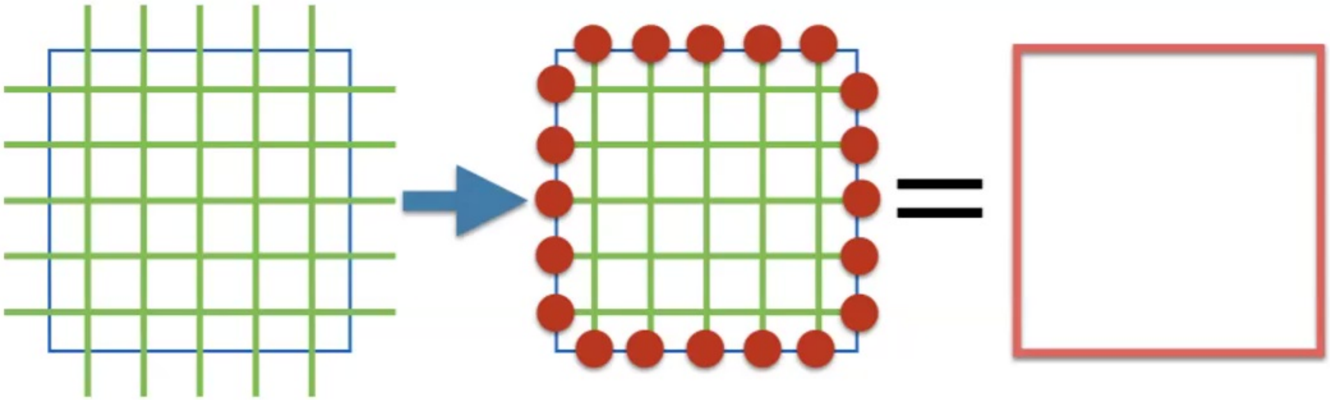
"non-fractionalizable relations"

Comments on mathematical structure

- Above arguments determine a *subgroup* of $(G, \mathscr{A})$ 2-cocycles, where G is the (abstract) group of subsystem symmetries
- No coboundary transformations allowed!
- Symmetry fractionalization is *not* characterized by a cohomology class in $H^2(G, \mathscr{A})$

Non-trivial fractionalization

Consider the remaining (global) relation $r = \prod_j x_j \prod_i y_i^{-1}$

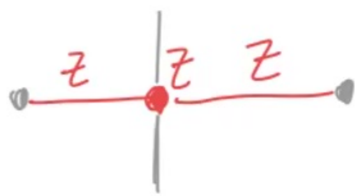


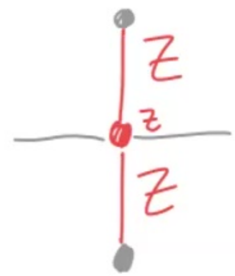
$$V_R(r) = \prod_j V_R(x_j) \prod_i V_R(y_i)^{-1}$$

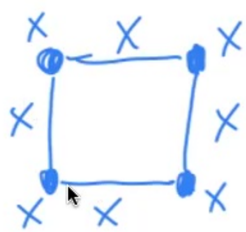
To show fractionalization of this relation is actually possible, need to exhibit a model

Model

$$H = -\sum_v A_v^x - \sum_v A_v^y - \sum_p B_p$$

$A_v^x =$


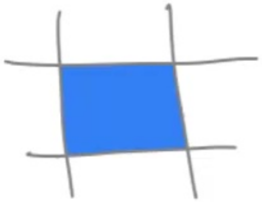
$A_v^y =$


$B_p =$


$$[A_v^x, B_p] = [A_v^y, B_p] = 0 .$$

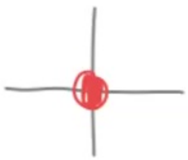
- Can be disentangled to standard Z_2 toric code + trivial system

Excitations



m-particles

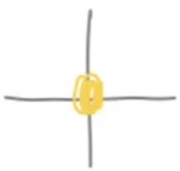
$B_p = -1$



$A_v^x = -1$ $\quad$ $A_v^y = +1$ $\quad$ e_x

$A_v^x = +1$ $\quad$ $A_v^y = -1$ $\quad$ e_y

Two flavors of e-particles

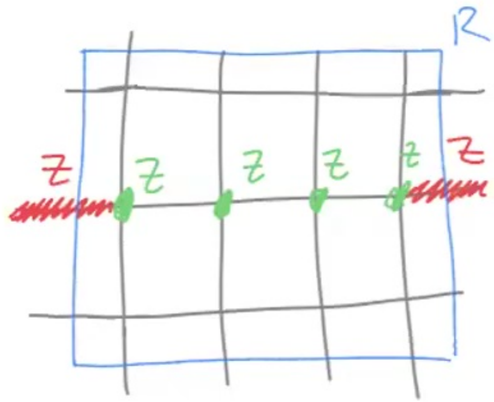


$A_v^x = -1$

$A_v^y = -1$

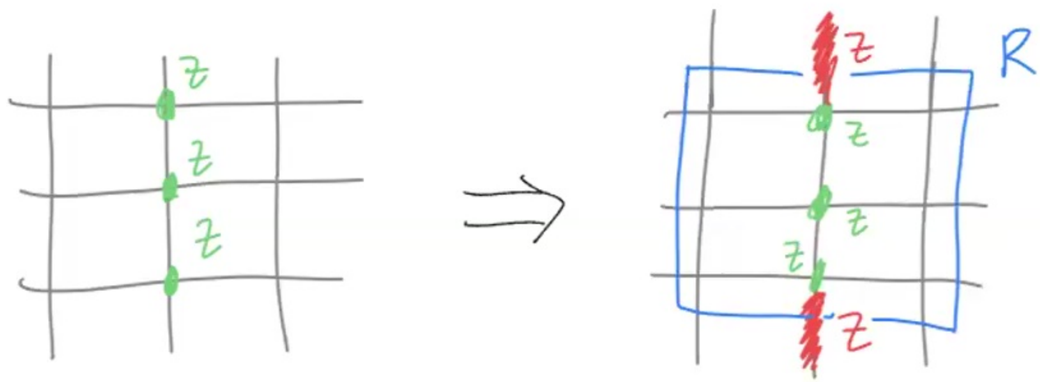
- Local excitation created by X_v.
- Carries a single row and column charge.

Subsystem symmetry and localizations



$U(x_j) = \prod Z$

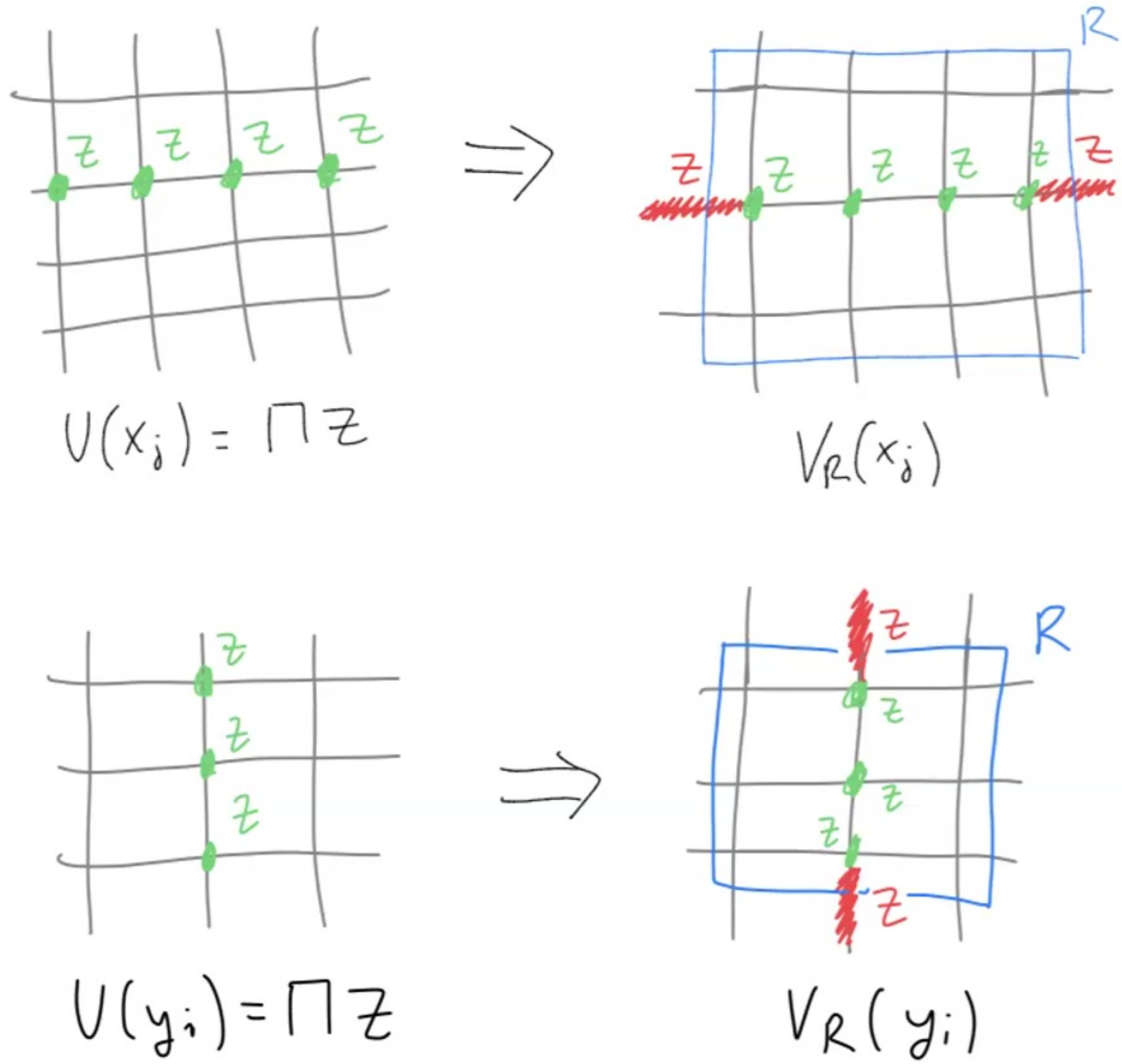
$V_R(x_j)$



$U(y_i) = \prod Z$

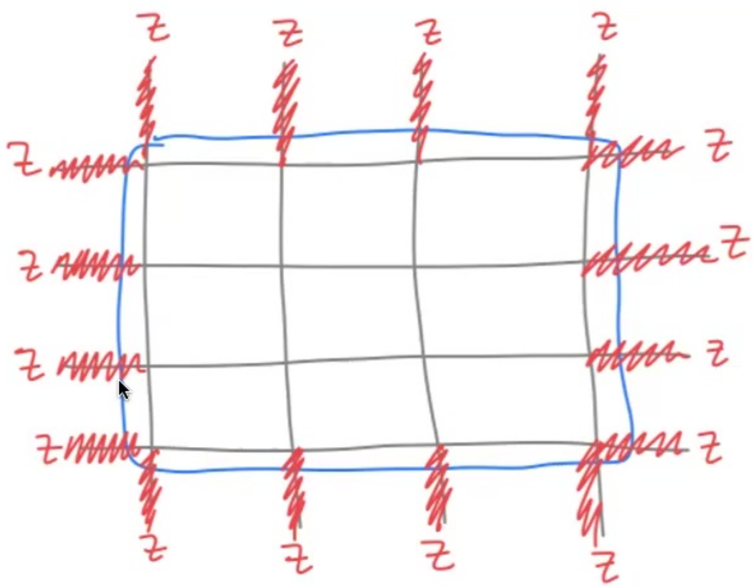
$V_R(y_i)$

Subsystem symmetry and localizations



Symmetry fractionalization

The localized global relation gives a *m*-string



⇒ *e*-particles are charged under an *odd* number of row+column generators, impossible for local excitations

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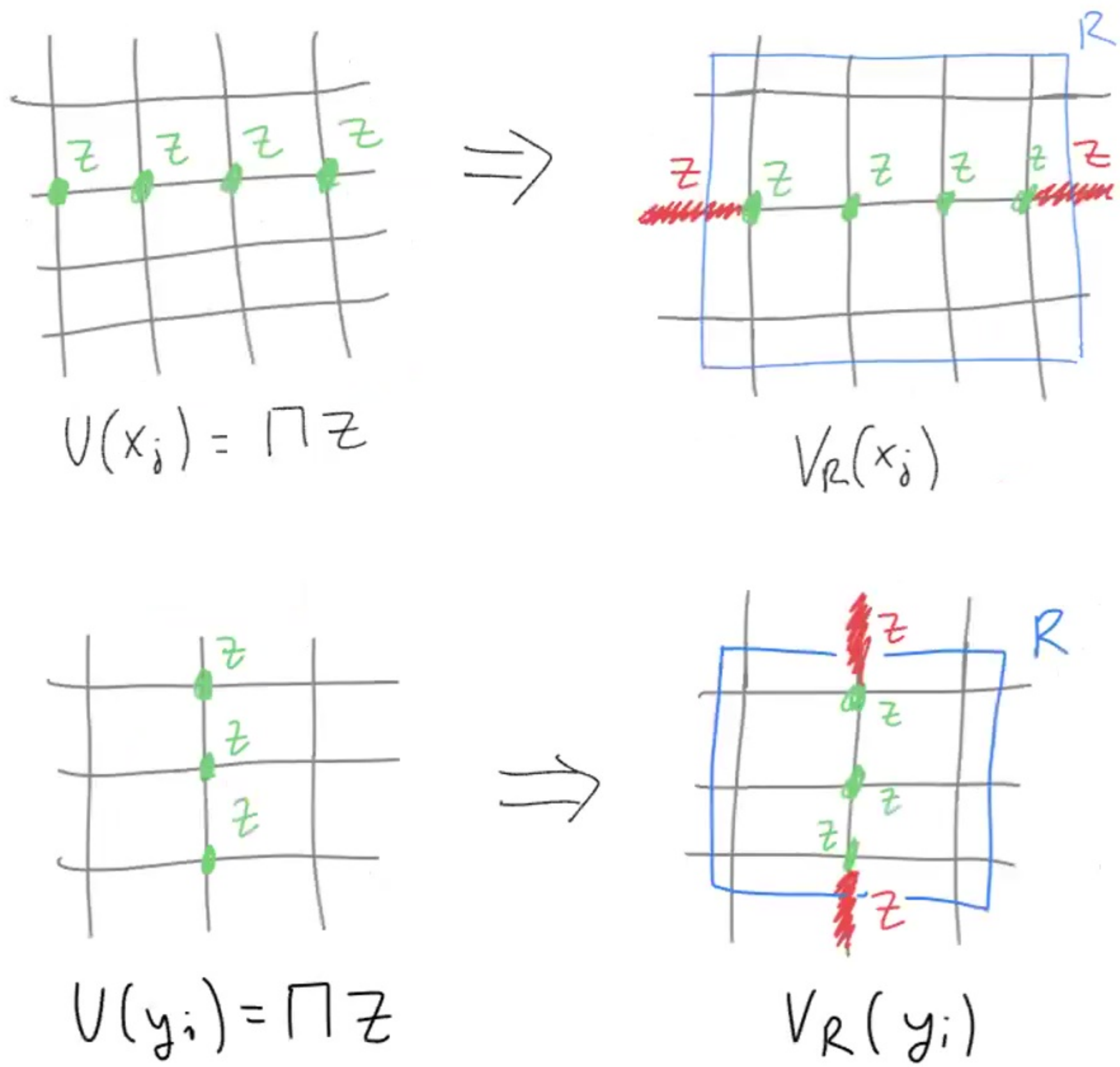

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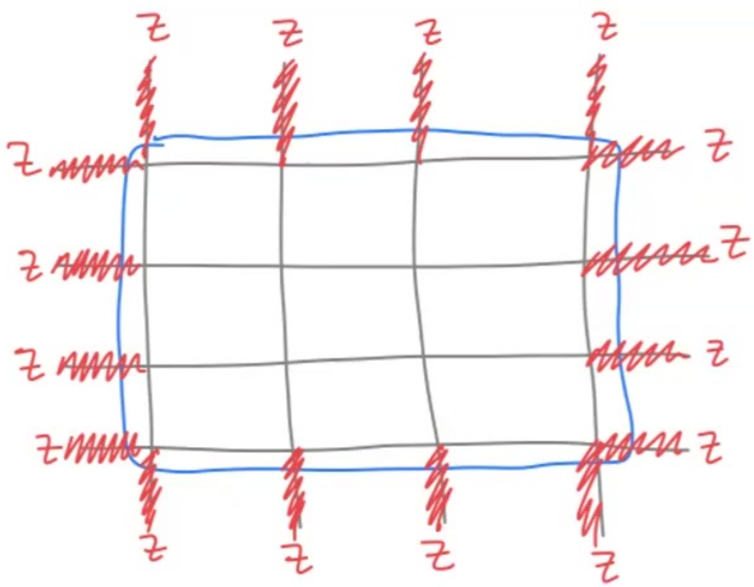
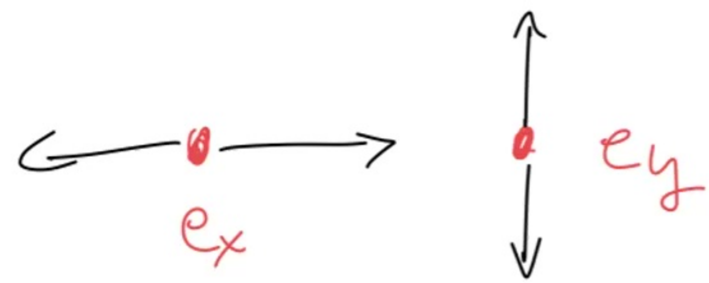


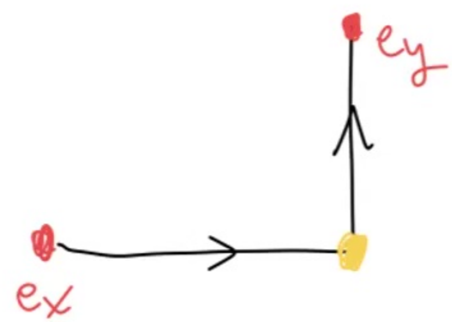
$\Rightarrow$ e-particles are charged under an *odd* number of row+column generators, impossible for local excitations

Consequence: symmetry-enforced lineons

The e-particles are symmetry-enforced lineons



For e_x to turn a corner, convert to e_y, need to leave behind a local excitation charged under the subsystem symmetry



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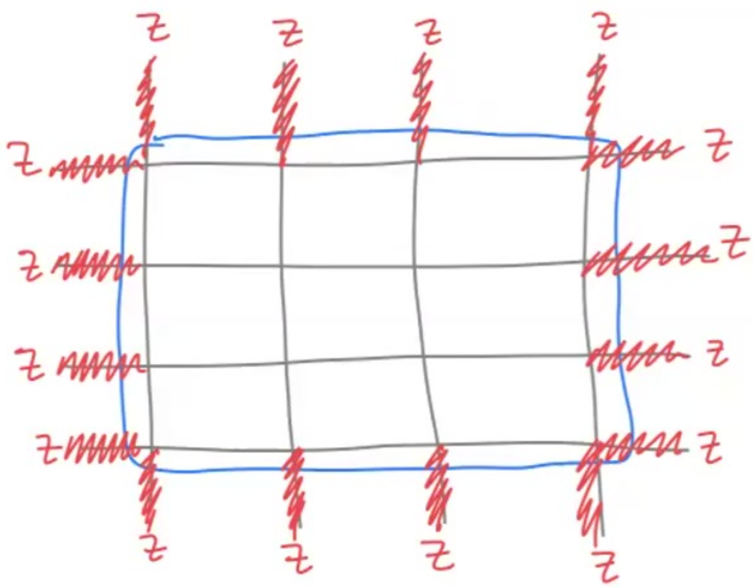


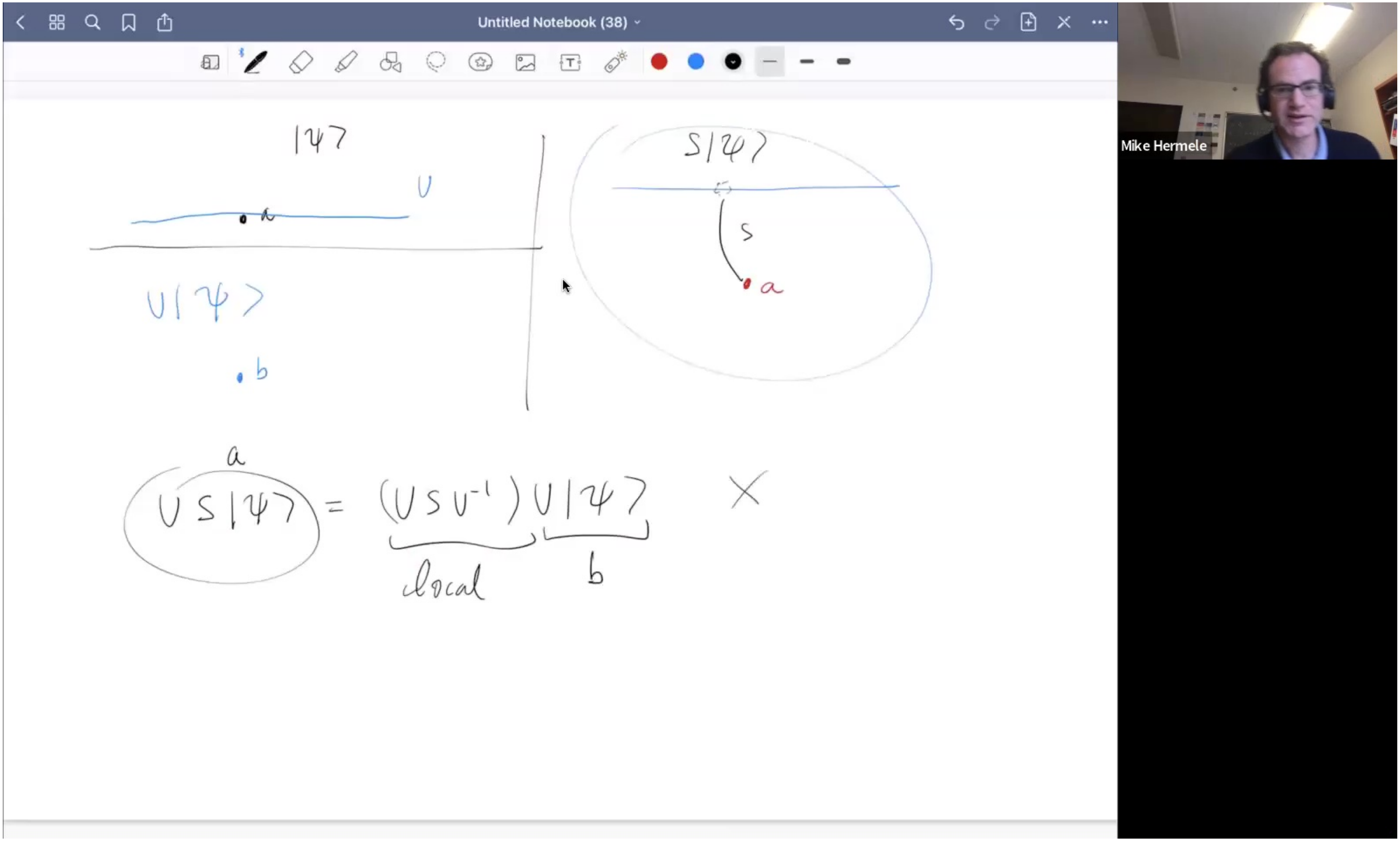
$\Rightarrow$ e-particles are charged under an *odd* number of row+column generators, impossible for local excitations

Other possibilities

- We constructed a different Pauli stabilizer code model where the localized global relation gives an ε-string
- So in this case, an arbitrary assignment of anyon types to the (single) "fractionalizable" relation is possible
- Open question: is anomalous subsystem symmetry fractionalization possible?

Summary / outlook

- Subsystem symmetry can fractionalize on point particles, with consequences for symmetry-enforced restricted mobility
- Exploring/classifying the possibilities still a work in progress: anomalies? three dimensions? interplay between subsystem and global symmetries?
- Nature of the interplay between subsystem symmetry fractionalization and broken higher-form symmetry?


Untitled Notebook (38)
Mike Hermele
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U|ψ⟩
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US|ψ⟩ = (USU⁻¹) U|ψ⟩
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b