Title: Engineering theories of AdS_3 via symmetric product orbifolds

Speakers: Alejandra Castro

Series: Quantum Fields and Strings

Date: May 03, 2022 - 2:00 PM

URL: https://pirsa.org/22050021

Abstract: I will discuss the large-N limit of two-dimensional symmetric product orbifolds. The goal is to single out which symmetric product orbifold theory could lead to a strongly coupled theory, whose dual could be a semi-classical theory of AdS_3 gravity, by quantifying the large-N limit of the BPS spectrum and the behaviour under exactly marginal deformations. From this analysis, I will propose an infinite family of new holographic CFTs.

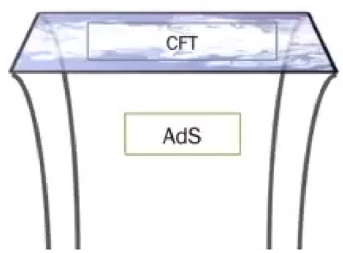
Zoom Link: https://pitp.zoom.us/j/99948208493?pwd=RDRYclc2a05kMmNZL3ZhMWxMdWdNUT09

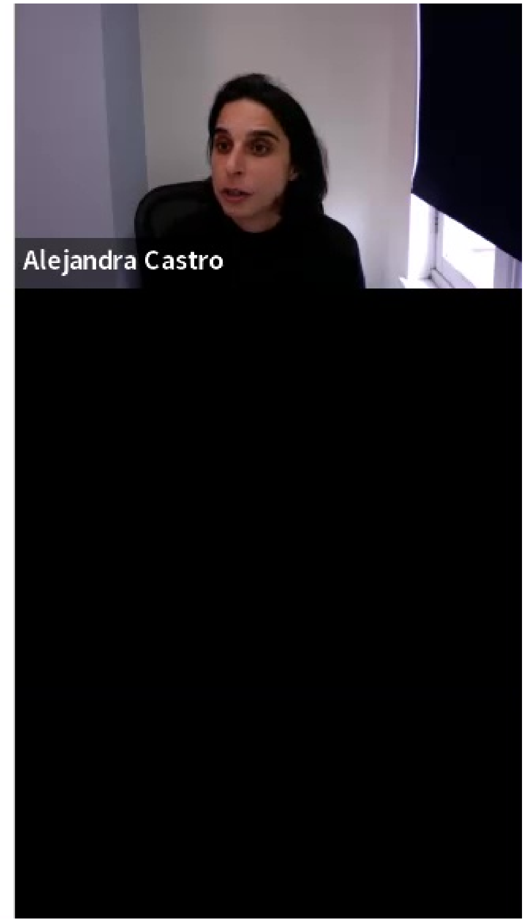
ENGINEERING ADS$_3$ GRAVITY

Alejandra Castro, University of Amsterdam

Perimeter Institute, May 2022

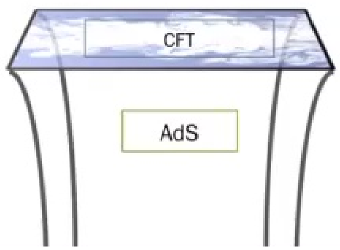
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

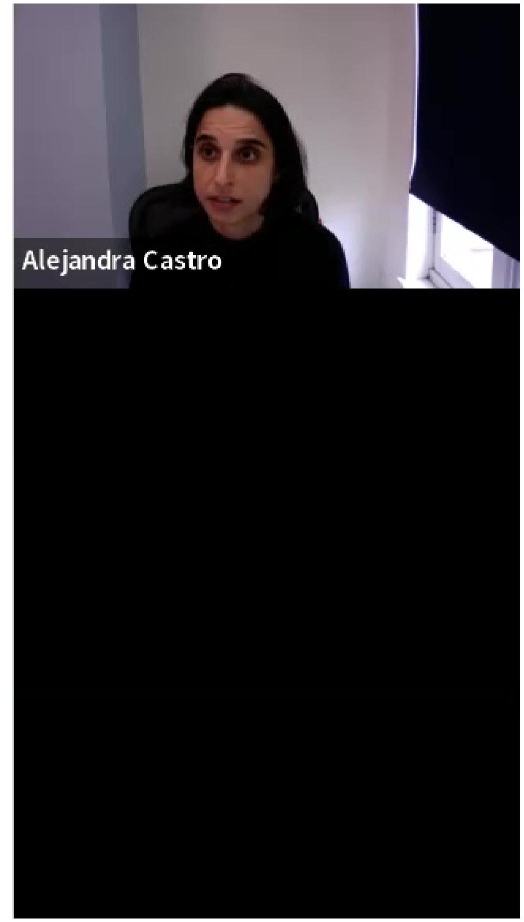




AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

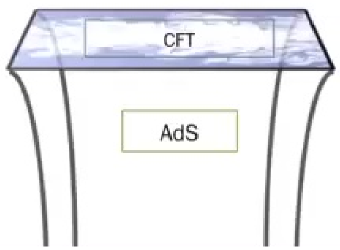
- Which CFTs capture classical (geometric) properties of gravity?

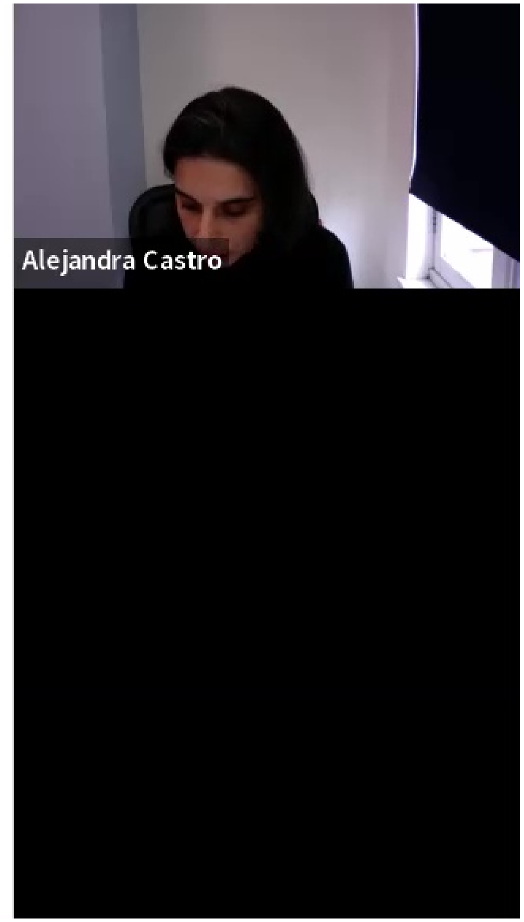




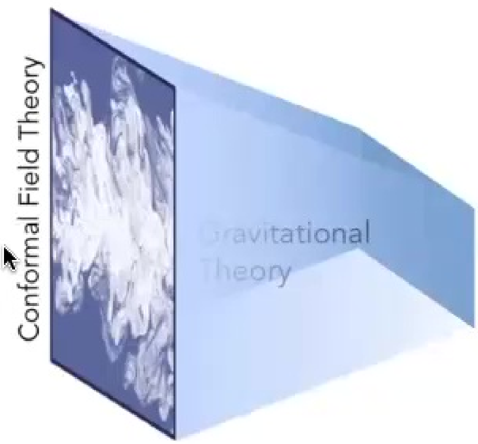
AdS/CFT provides a non-perturbative, UV-complete definition of quantum gravity in Anti-de Sitter space.

- Which CFTs capture classical (geometric) properties of gravity?
- What are possible theories of quantum gravity can be concretely designed?
- What are the materials needed to assemble them?

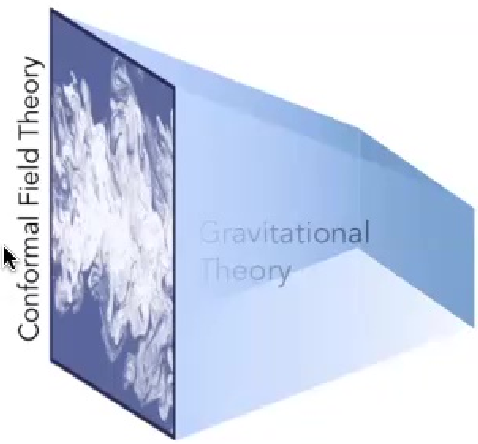




- Define gravity via the dual CFT.
- Quantify the space of consistent, UV complete, QG theories.
- Decode the fundamental mechanism of holography.

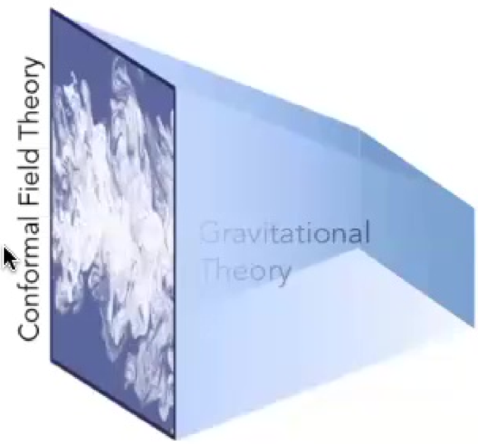


- Define gravity via the dual CFT: necessary conditions.
- Quantify the space of consistent, UV complete, QG theories.
- Decode the fundamental mechanism of holography: sufficient conditions.



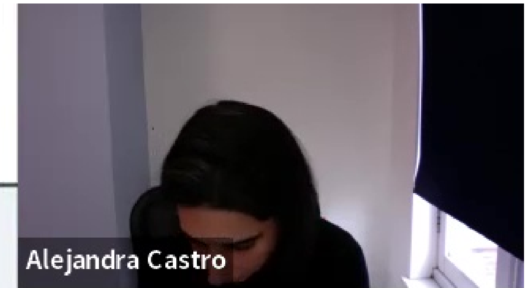
- Define gravity via the dual CFT: necessary conditions.
- Quantify the space of consistent, UV complete, QG theories.
- Decode the fundamental mechanism of holography: sufficient conditions.

We will focus on the difficulties you encounter in AdS_3/CFT_2.
Not universal, but it illustrates the challenges.

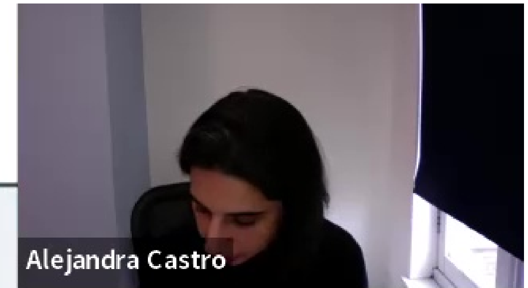


Landscape of Holographic CFTs

Deformations of Symmetric Product Orbifolds



Landscape of Holographic CFTs



- Define gravity via the dual CFT: necessary conditions.
- Build precise and new examples of potential holographic CFTs.

Based on:
arXiv: 1910.05353 [hep-th]
arXiv: 1910.05342 [hep-th]
with A. Belin, C. Keller and B. Mühlmann

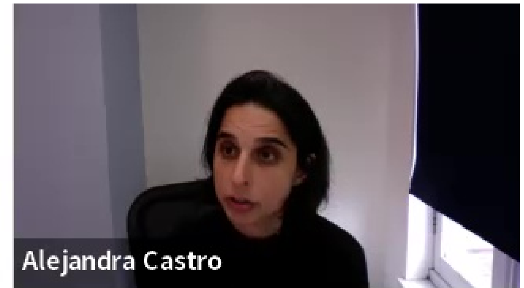
arXiv: 2002.07819 [hep-th]
with A. Belin, N. Benjamin, C. Keller and S. Harrison

- Signals of strong coupling starting from Symmetric Product Orbifolds.
- Do we have sufficient conditions?

Based on:
arXiv: 2204.07590 [hep-th]
L. Apolo, A. Belin, S. Bintanja, C.Keller

Work in progress
with N. Benjamin, S. Bintanja and J. Hollander

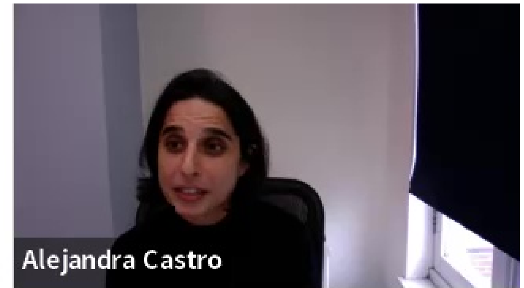
Deformations of Symmetric Product Orbifolds



LANDSCAPE OF HOLOGRAPHIC CFTS

Meeting neccesary conditions

AdS_3 Gravity



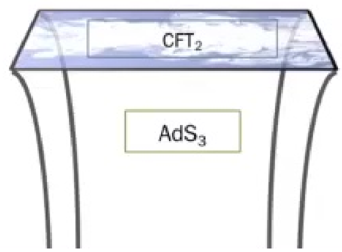
Basic Requirements:

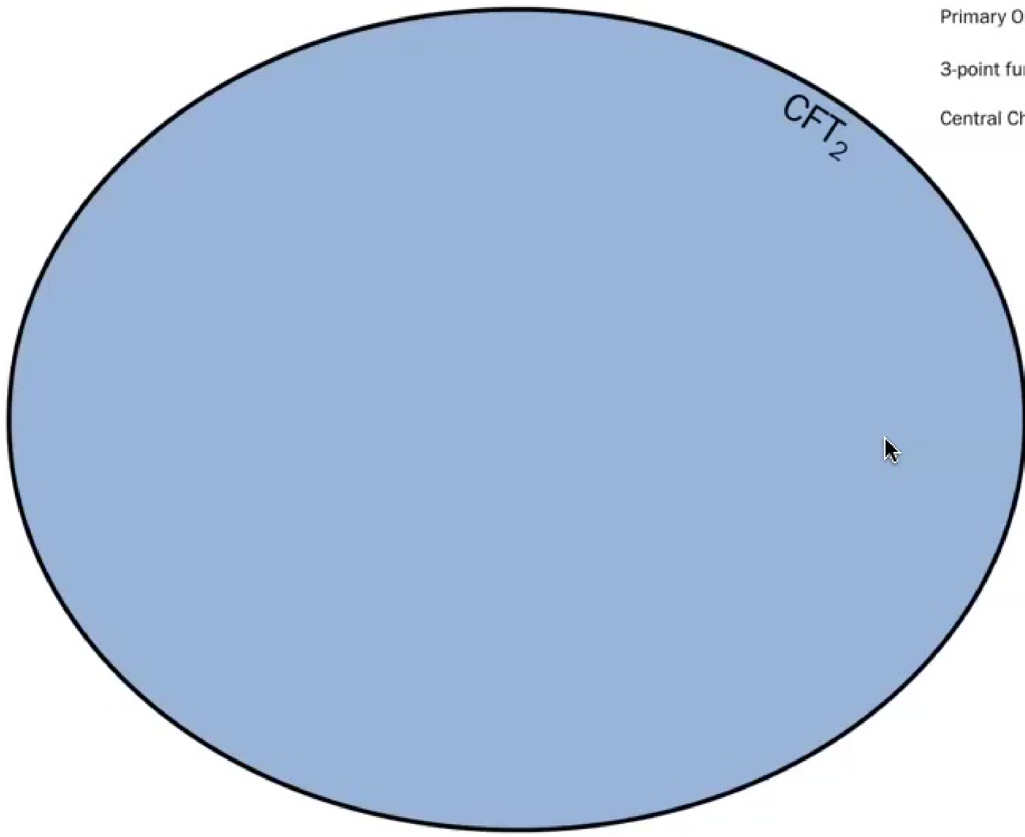
Gravitational theories that have the property that their low-energy description is given by a local EFT.

The theory:

$$I_{3\mathrm{D}} = \frac{1}{16\pi G_N}\int d^3x\sqrt{-g}\left(R+\frac{2}{\ell^2}\right) + \mathrm{matter}$$

Universal entry in AdS_3/CFT_2: $\quad c = \dfrac{3\ell}{2G_N} \gg 1$

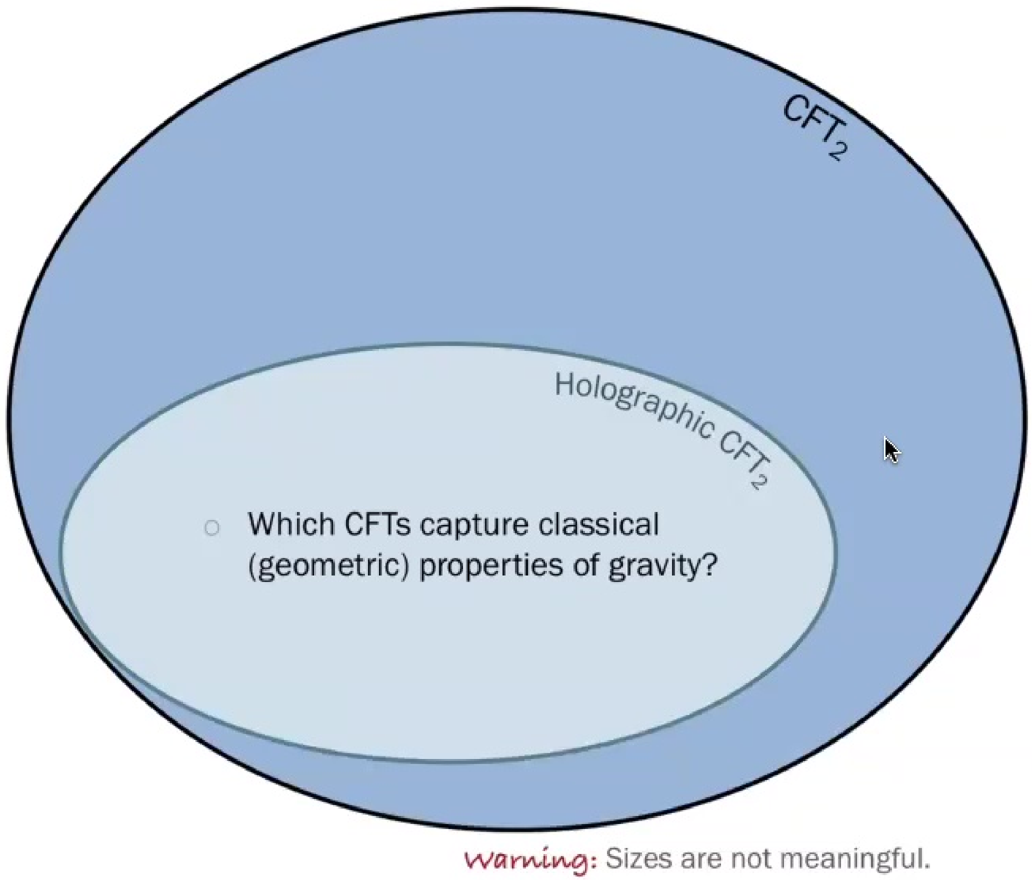




Primary Operators: Δ_J (Conformal dimensions)

3-point functions: C_{IJK} (OPE coefficients)

Central Charge: c


CFT$_2$
Holographic CFT$_2$
Which CFTs capture classical (geometric) properties of gravity?
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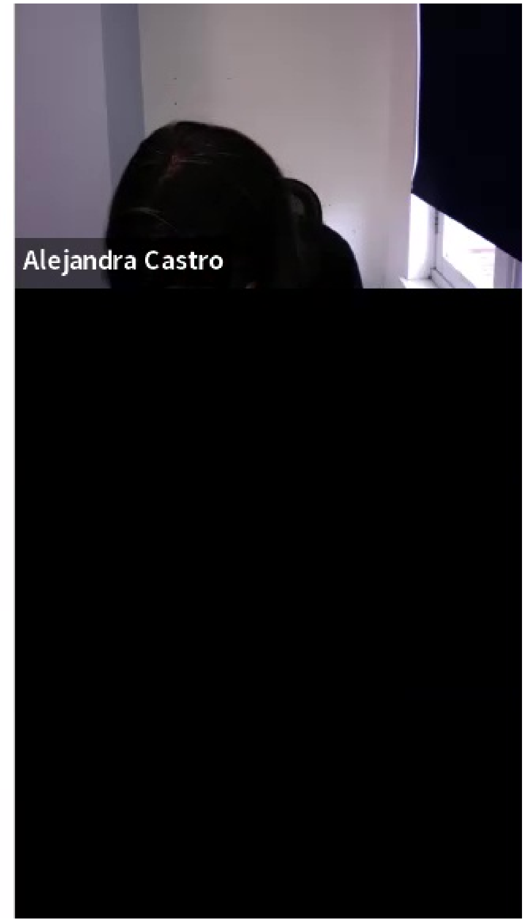
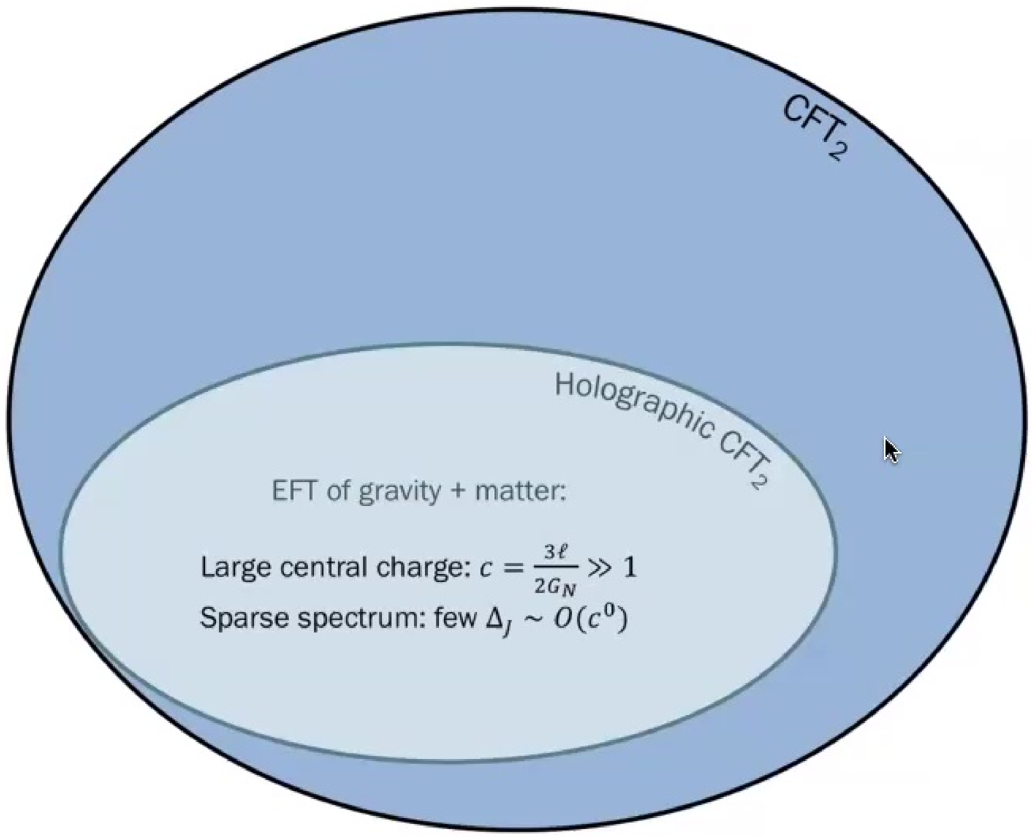

Alejandra Castro



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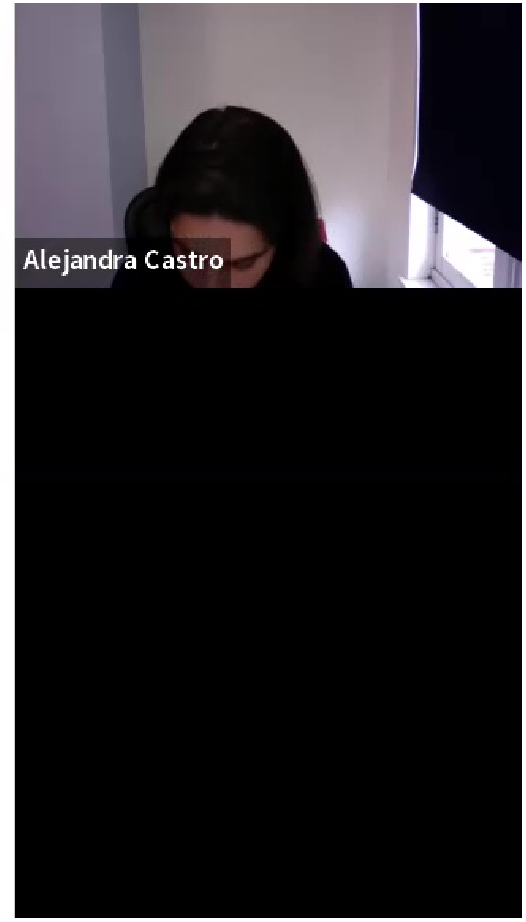

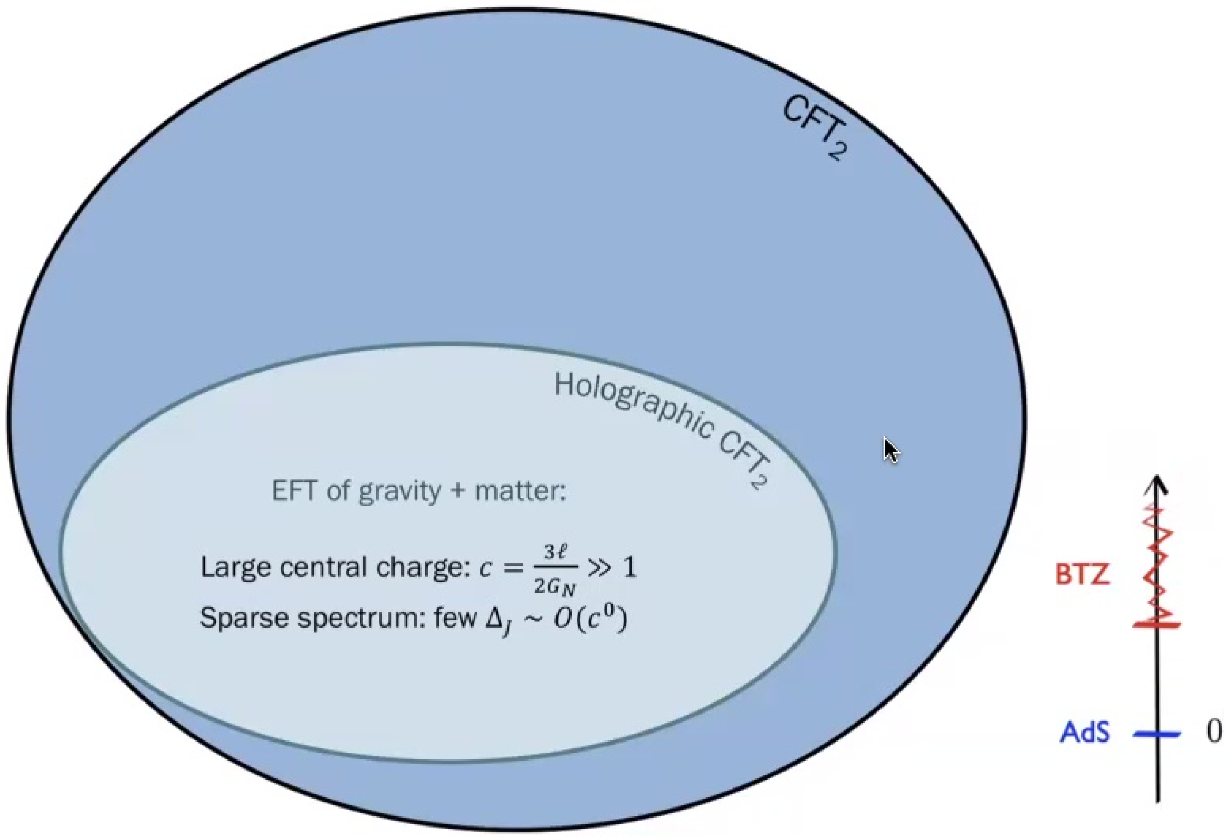

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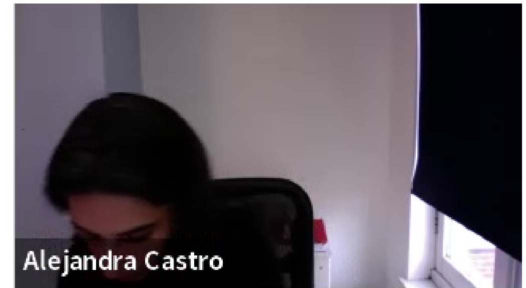
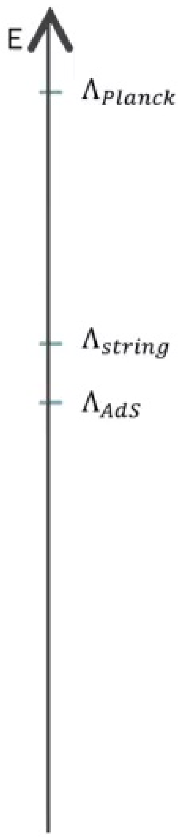


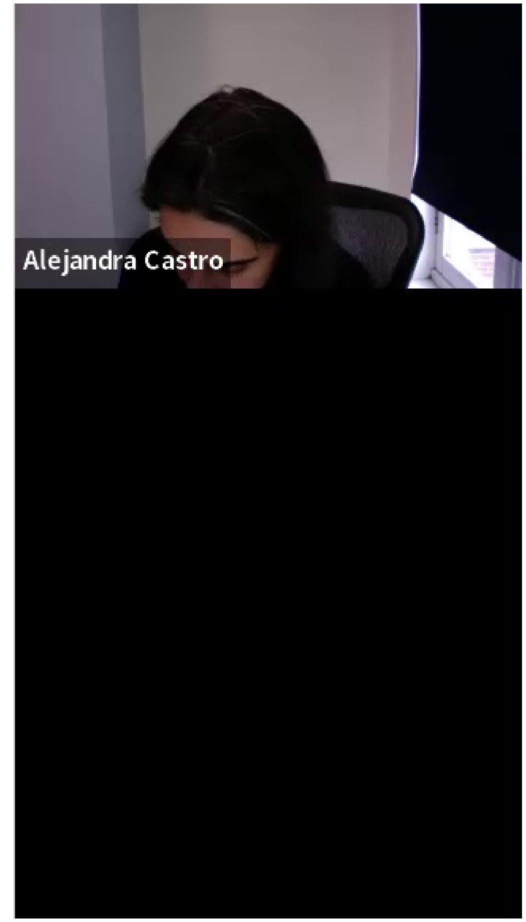


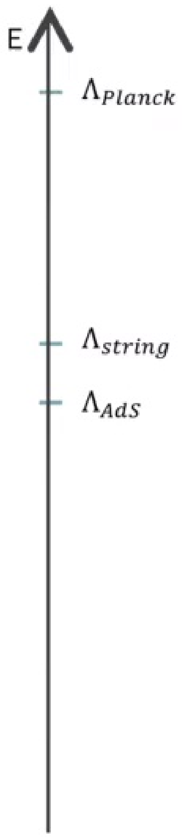
Expectations of the string spectrum

$$\Lambda_{string} \sim \Lambda_{AdS}$$

$$d(\Delta) \sim e^{c_H \Delta} \quad \text{where} \quad \Delta \sim \frac{E}{\Lambda_{AdS}} \gg 1$$

Hagedorn (fast) growth





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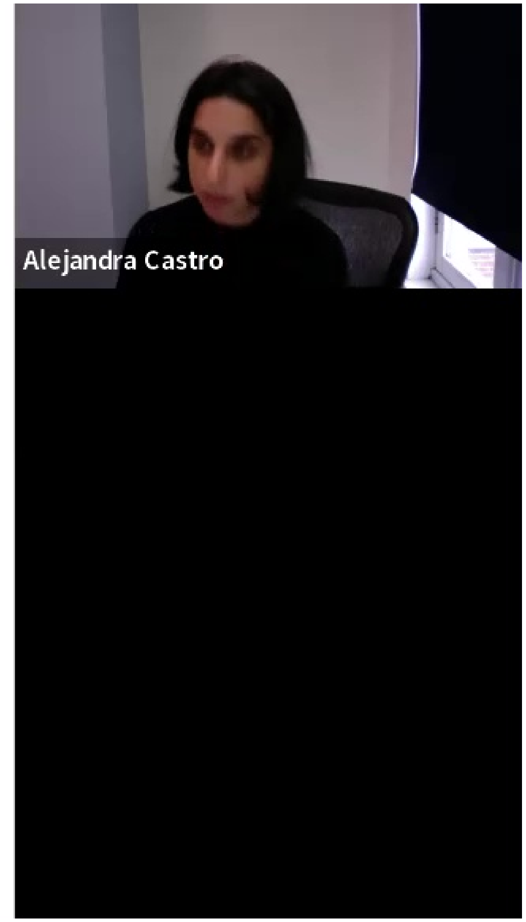
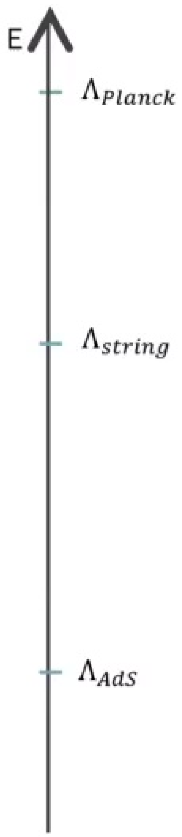




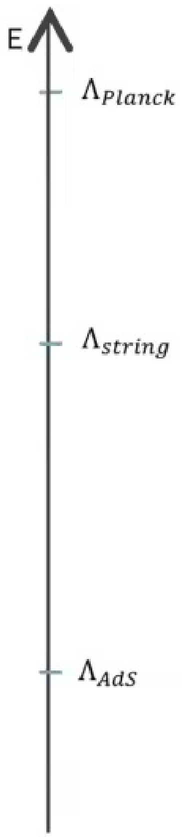
Expectations of the EFT spectrum

$$\Lambda_{string} \gg \Lambda_{AdS}$$

$d(\Delta) \sim e^{c_s \Delta^{\gamma}}$ where $\Delta \sim \frac{E}{\Lambda_{AdS}}$, and $\gamma < 1$.

$$\Lambda_{string} > E \gg \Lambda_{AdS}$$

Supergravity (slow) growth



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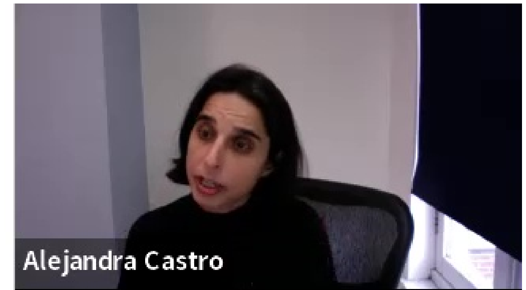
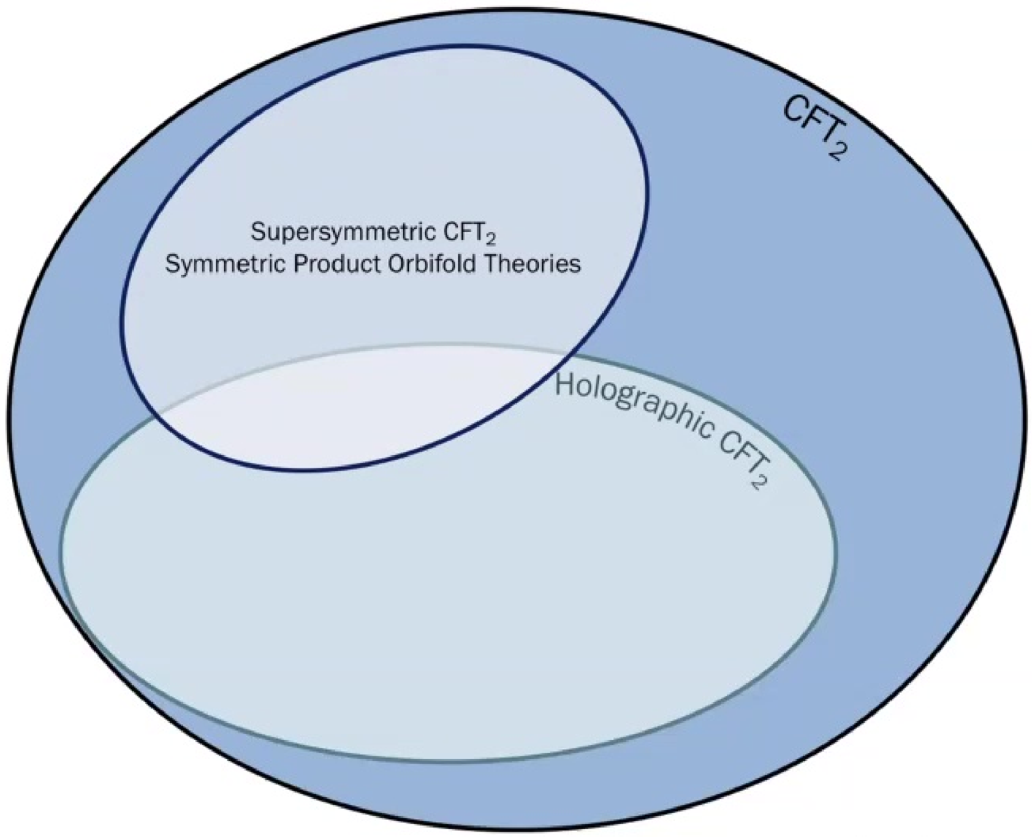
$$\Lambda_{string} > E \gg \Lambda_{AdS}$$

Supergravity (slow) growth

$$AdS_3 \times M_{D-3} \Rightarrow \quad \gamma = \frac{D-1}{D} < 1$$





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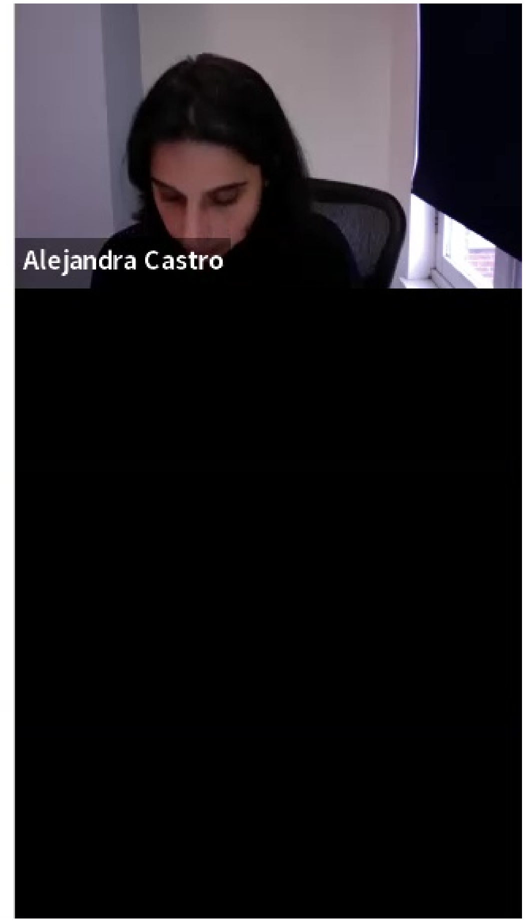
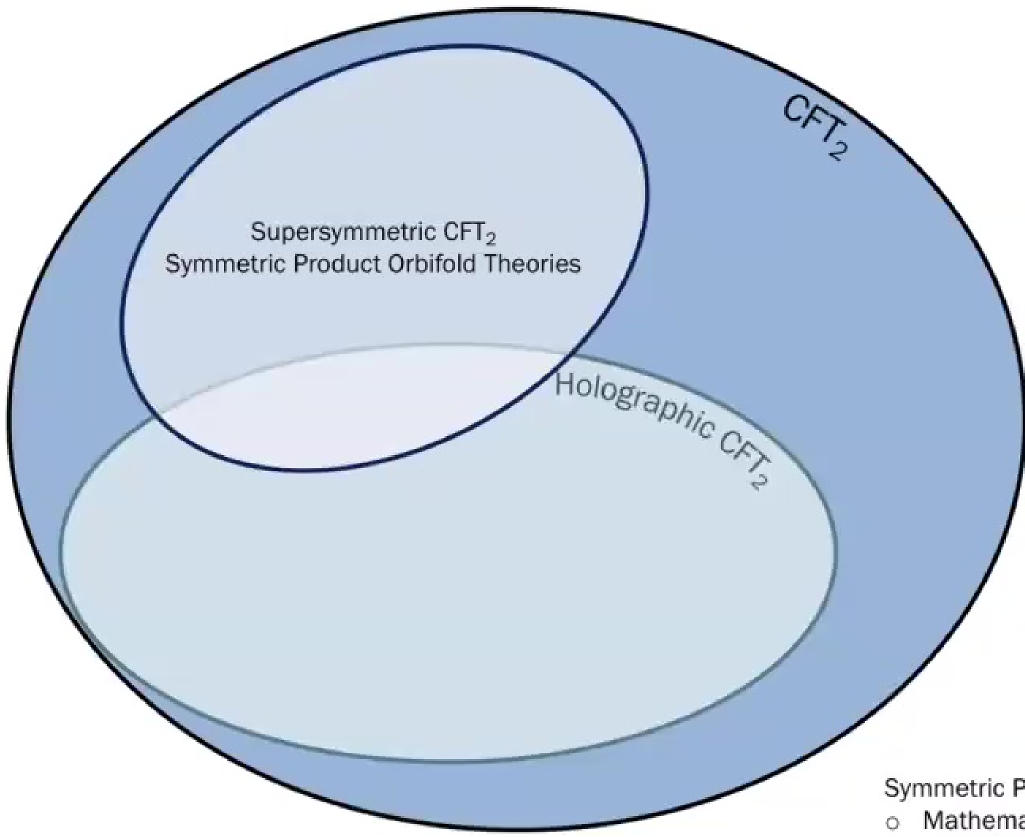
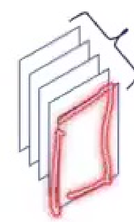
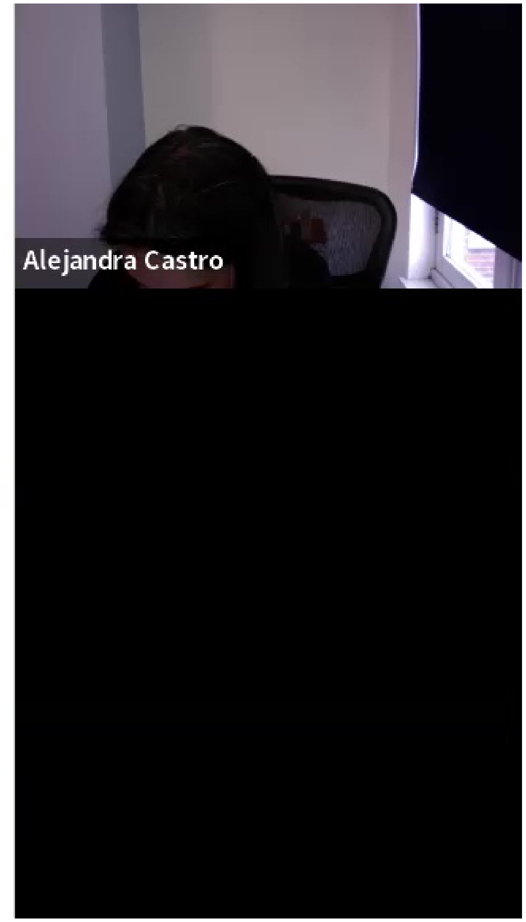






Symmetric Product Orbifolds:
- Mathematical control
- Demystify known string theory setups

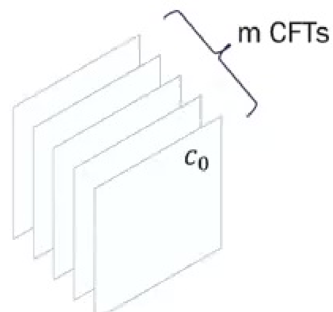


We want CFTs with large central charge
We want control on the spectrum

Search within symmetric product theories.

$$\mathcal{C}_m = \mathcal{C}^{\otimes m}/S_m$$

Total central charge: $c = c_0 m \sim \frac{3\ell}{2G_N} \gg 1$ as $m \to \infty$



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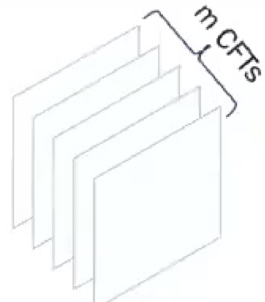


$$\mathcal{Z}(\tau,\rho) = \sum_m Z(\tau;\mathcal{C}_m)\,p^m = \prod_{m>0,n\in\mathbb{Z}} \frac{1}{(1-p^m q^n)^{d(mn)}} = \sum_{m,n} d_m(n) p^m q^n$$

[DMVV]

We want CFTs with large central charge
We want control on the spectrum

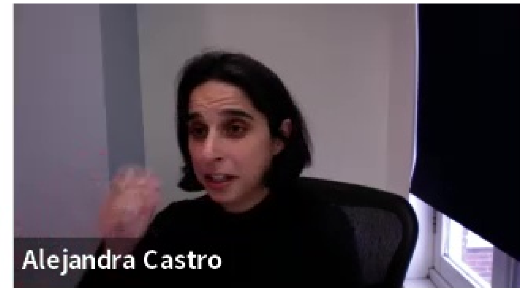
Search within symmetric product theories.

$$\mathcal{C}_m = \mathcal{C}^{\otimes m}/S_m$$

$$\Lambda_{string} \sim \Lambda_{AdS}$$

Hagedorn (fast) growth

[Keller, 2011]



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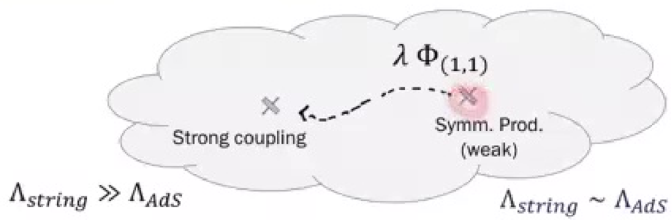
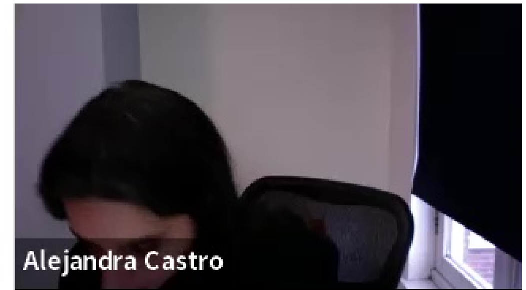


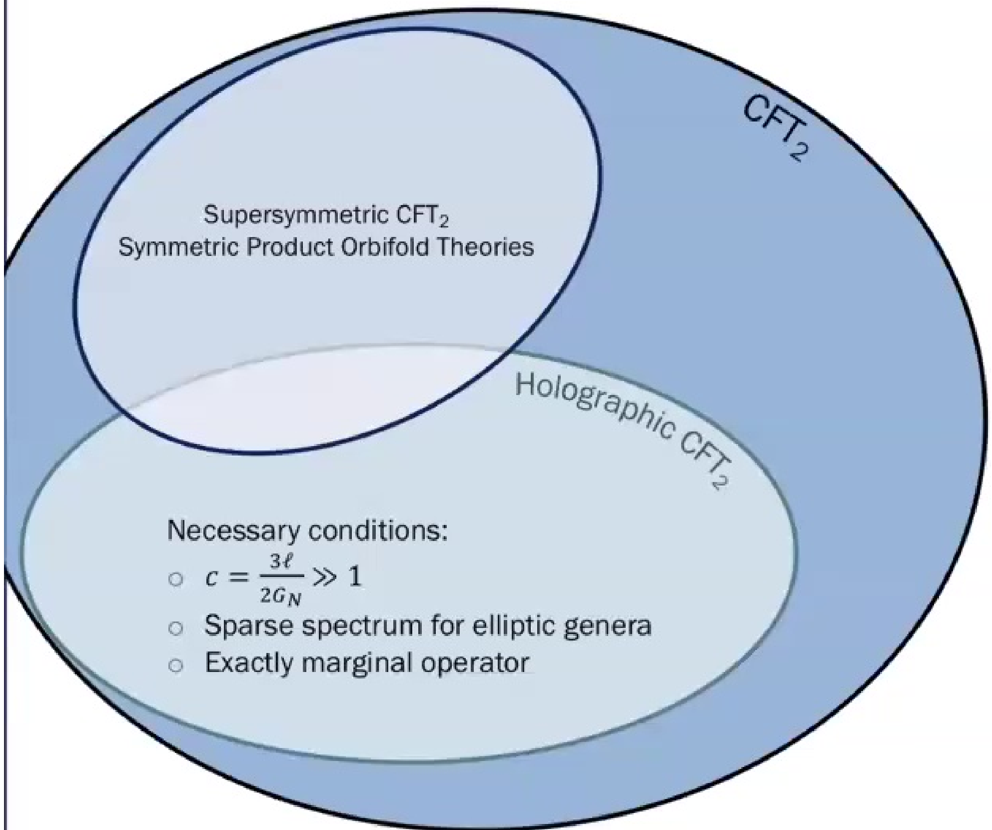


We want CFTs with large central charge
We want control on the spectrum

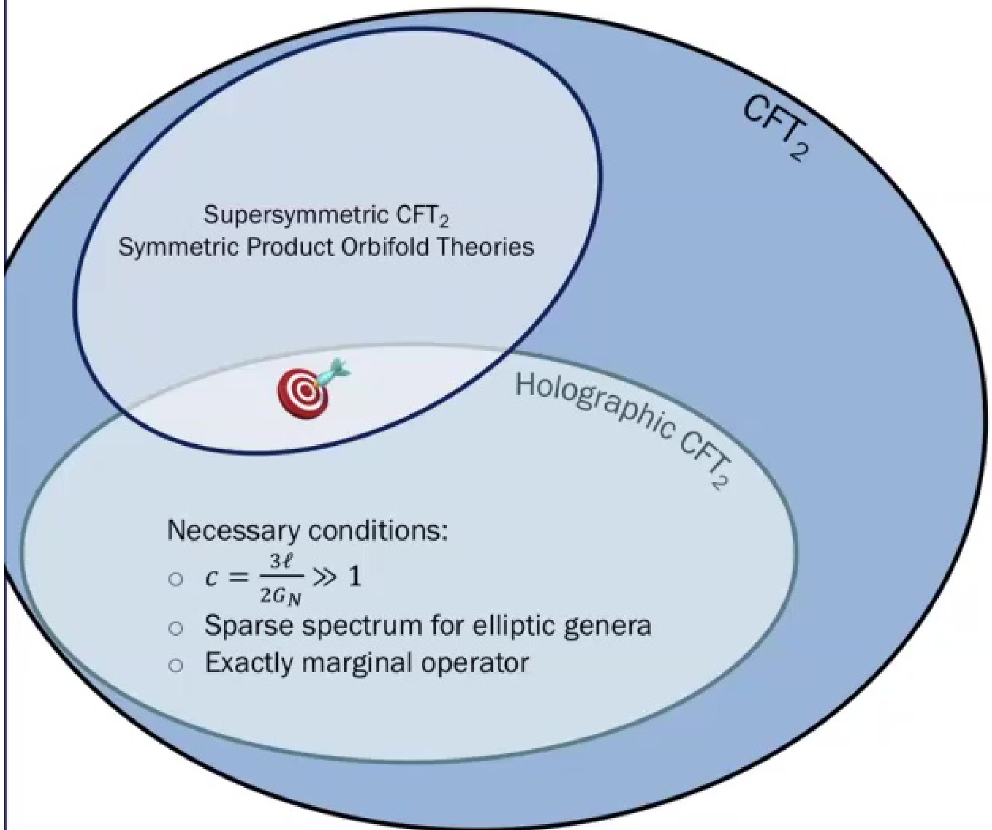
Search within symmetric product theories.

Only hope: Add marginal deformation to make the spectrum sparse

Focus on quantities that are protected.
Those should have gravitational features.



Which CFTs capture classical (geometric) properties of gravity?



¼-BPS spectrum: Make use of modular forms and their complete classification. We developed methods to extract the key portions to test conditions.

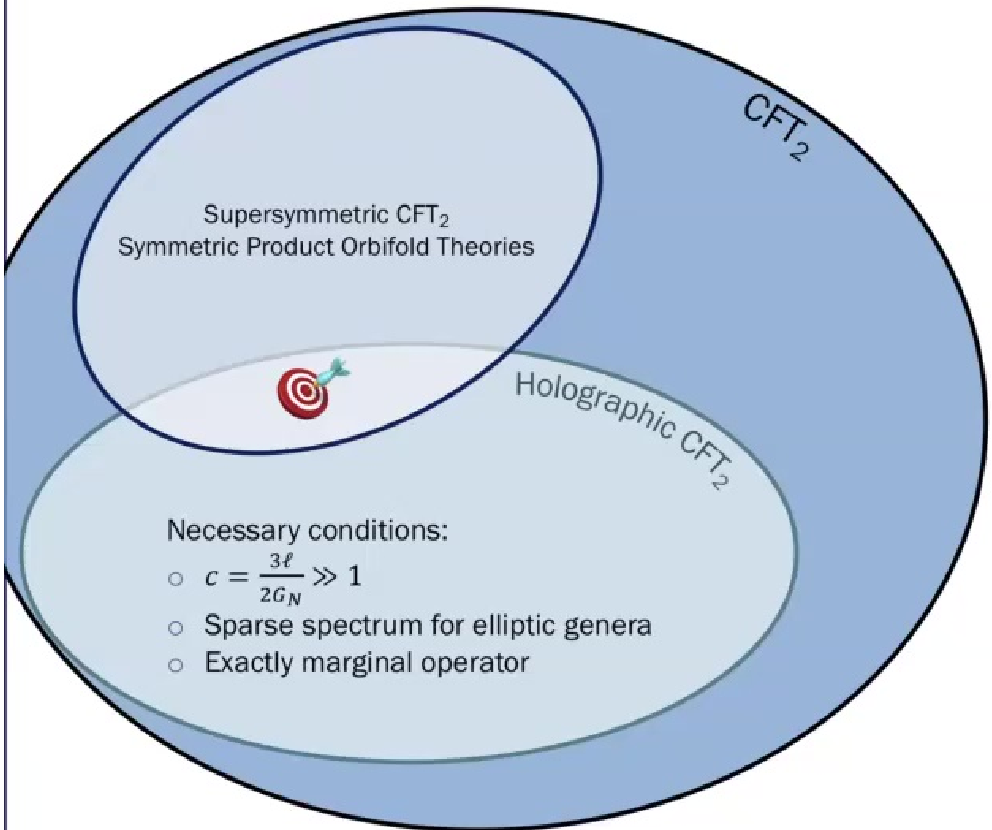
We can fully diagnose if a Symmetric Product theory meets necessary conditions for the elliptic genera.

Slow growth: always of the form

$d(\Delta) \sim e^{\sqrt{\Delta}}$ where $\Delta \sim \frac{E}{\Lambda_{AdS}}$

$\Lambda_{string} > E \gg \Lambda_{AdS}$

Seed theory: $1 \leq c_0 \leq 6$



¼-BPS spectrum: Make use of modular forms and their complete classification. We developed methods to extract the key portions to test conditions.

We can fully diagnose if a Symmetric Product theory meets necessary conditions for the elliptic genera.

Slow growth: always of the form

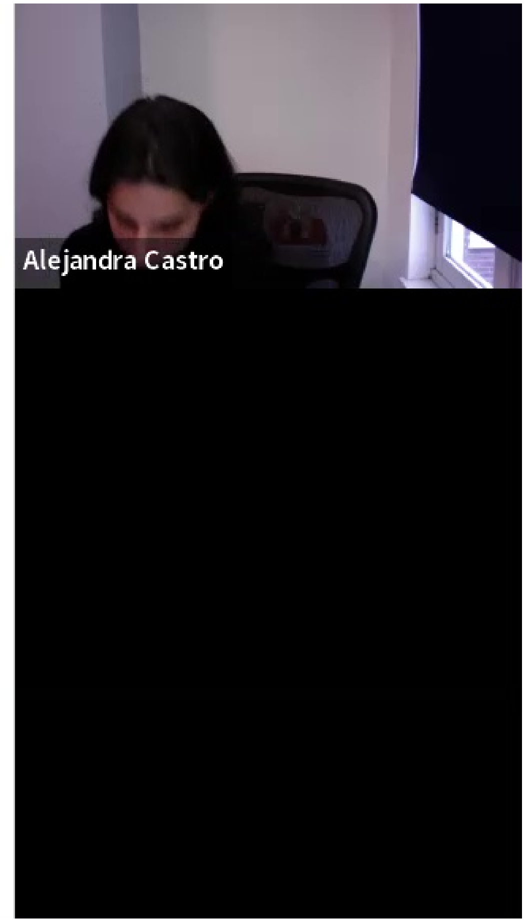
$d_{HF}(\Delta) \sim e^{\sqrt{\Delta}}$ where $\Delta \sim \frac{E}{\Lambda_{AdS}}$

$d_{el} \sim e^{\Delta^{3/4}}$

$\Lambda_{string} > E \gg \Lambda_{AdS}$

Seed theory: $1 \leq c_0 \leq 6$

$\gamma = \frac{D-1}{D}$

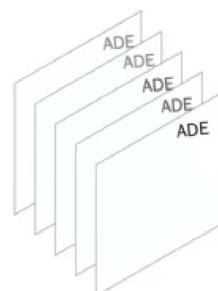


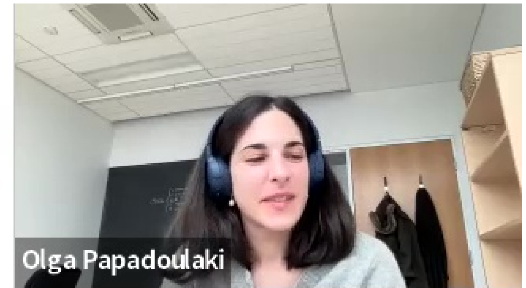
Materials: promising CFTs

Use as a seed N=2 Virasoro Minimal Models:

- CFT_2 whose spectrum is built from a finite number of irreducible representations of the N=2 super-Virasoro algebra.
- Obey ADE classification.
- Central charge

$$c_0 = \frac{3k}{k+2} < 3 \quad \text{where} \quad k = 1, 2, \ldots$$



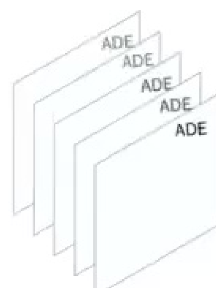


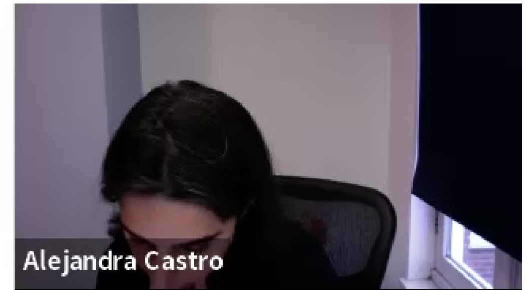
Materials: promising CFTs

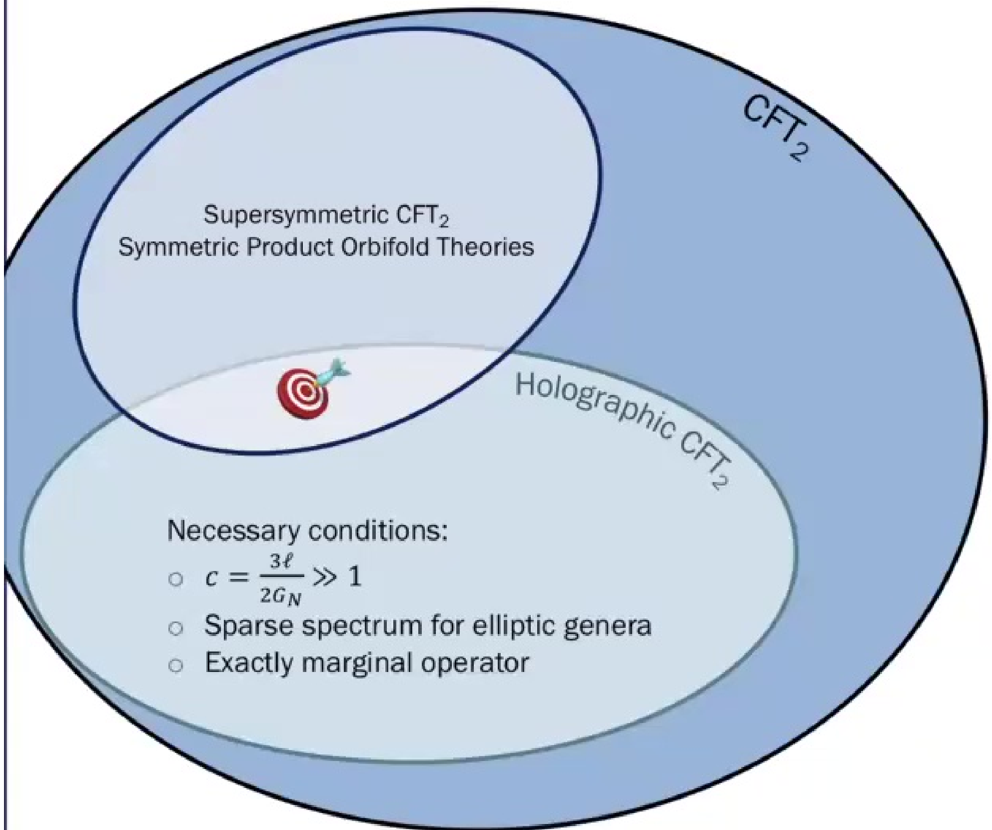
Series	k	untwisted moduli	twisted moduli	single trace twisted
A_2	1	1	28	1 twist 5, 1 twist 7
A_3	2	3	26	1 twist 3, 1 twist 4, 1 twist 5
A_5	4	9	24	1 twist 2, 1 twist 3, 1 twist 4
A_{k+1}	odd, ≥ 3	$P(k+2)-2$	9	1 twist 3
A_{k+1}	even, ≥ 6	$P(k+2)-2$	$10+\sum_{r=1}^{\frac{k}{2}+2} P(r)$	1 twist 2, 1 twist 3
D_4	4	6	20	1 twist 2, 2 twist 3, 1 twist 4
$D_{\frac{k}{2}+2}$	0 mod 4, ≥ 8	$P(\frac{k}{2}+1)+P(\frac{k}{4}+1)$	$8+\sum_{r=1}^{\frac{k}{4}+1} P(r)$	1 twist 2, 1 twist 3
$D_{\frac{k}{2}+2}$	2 mod 4, ≥ 6	$P(\frac{k}{2}+1)$	7	1 twist 3
E_6	10	4	5	1 twist 2
E_7	16	6	5	1 twist 2
E_8	28	6	5	1 twist 2

Necessary conditions:

- $c = \frac{3\ell}{2G_N} \gg 1$
- Sparse spectrum for elliptic genera
- Exactly marginal operator





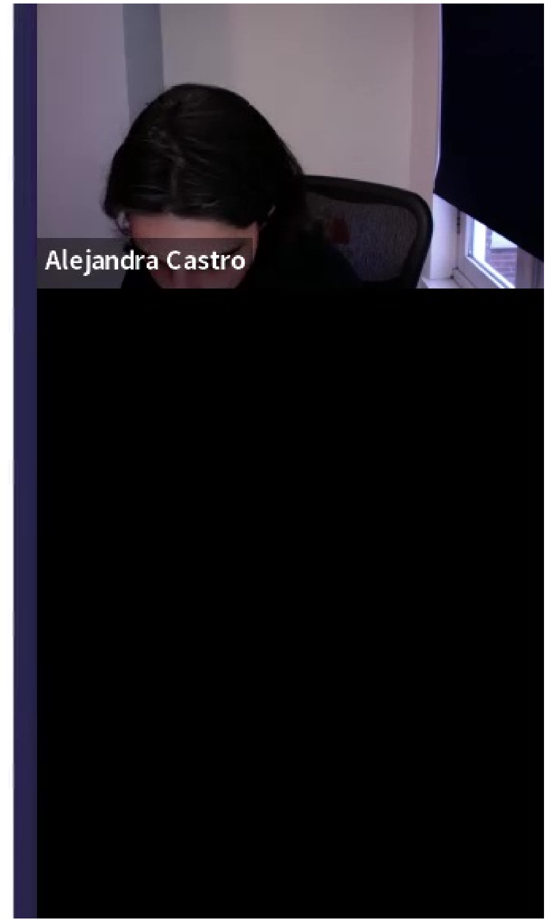


What are necessary vs sufficient conditions to have a classical gravitational dual?

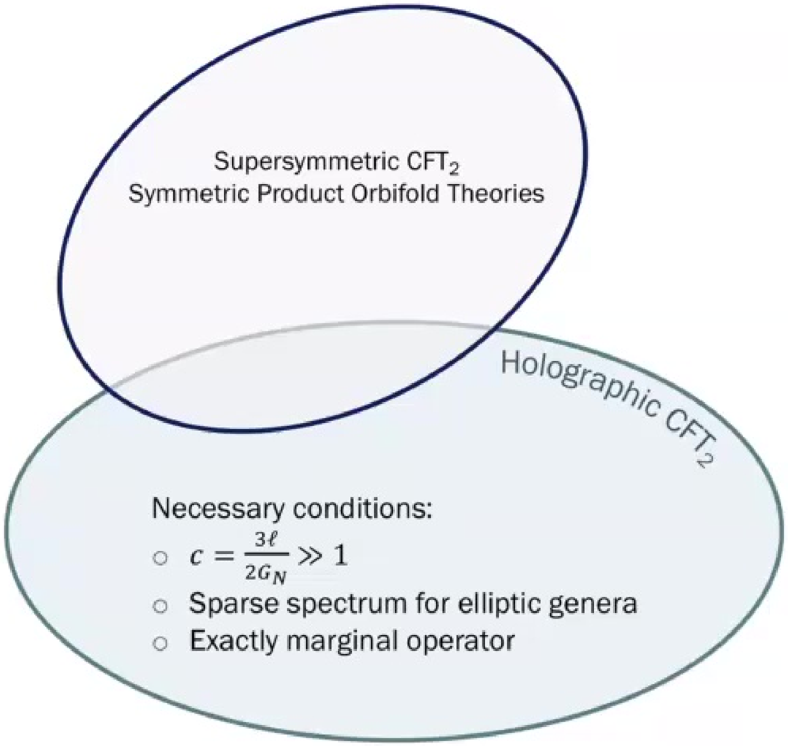
Once we have sufficient conditions, what are the properties of the low energy EFT?

DEFORMATIONS OF SYMMETRIC PRODUCT ORBIFOLDS

Signals of strong coupling starting from Symmetric Product Orbifolds

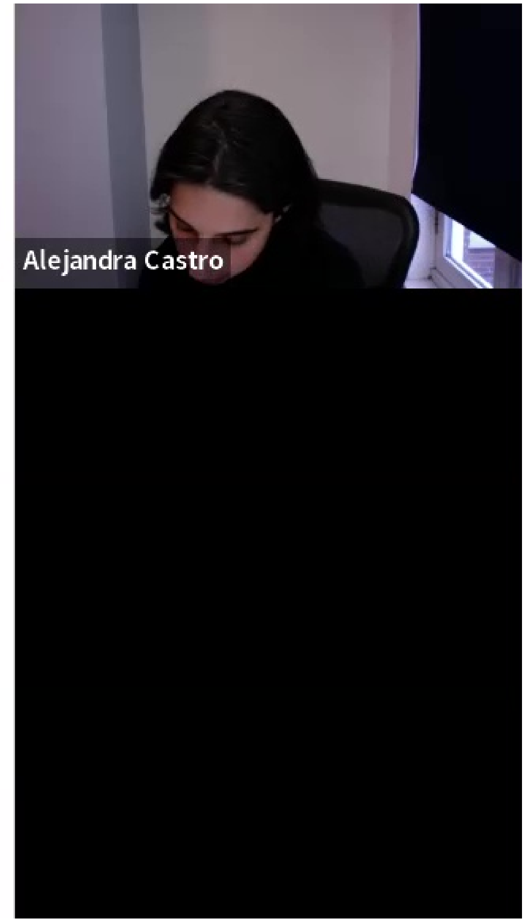


- Have we impossed sufficient conditions?



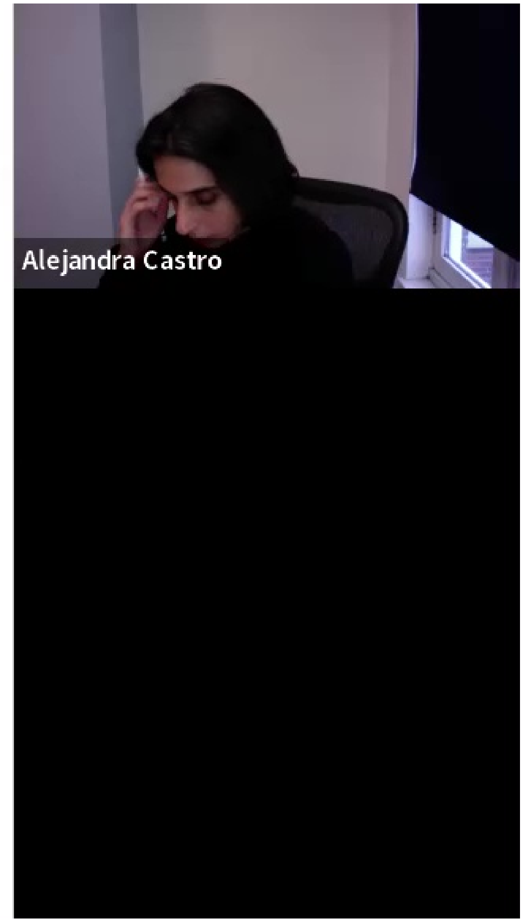
Important features:

- Sparse elliptic genera seems to always guarantee an exactly marginal operator.
- However, we have examples of theories with Hagedorn elliptic genera and also an exactly marginal operator.



Theory	Sparse?	Moduli?	Composition
$A_6 \otimes A_{41}$	✓	✓	(11,88), (22,22)
$A_7 \otimes A_{23}$	✓	✓	(11,55),(22,22)
$A_8 \otimes A_{17}$	✓	✓	(11,44),(22,22)
$A_9 \otimes A_{14}$	✓	✓	(22,22)
$A_{11} \otimes A_{11}$	✓	✓	(11,33),(33,11),(22,22)
$A_6 \otimes D_{22}$	✗	✗	
$A_7 \otimes D_{13}$	✗	✓	(11,55)
$A_{23} \otimes D_5$	✗	✓	(55,11)
$A_8 \otimes D_{10}$	✗	✗	
$A_{14} \otimes D_6$	✗	✗	
$A_{11} \otimes D_7$	✓	✓	(11,33),(33,11)
$A_8 \otimes E_7$	✗	✗	
$A_{11} \otimes E_6$	✗	✓	(33,11)
$D_5 \otimes D_{13}$	✗	✓	(11,55)
$D_7 \otimes D_7$	✓	✓	(11,33),(33,11)
$D_7 \otimes E_6$	✓	✓	(33,11)
$E_6 \otimes E_6$	✗	✗	
$A_2 \otimes A_5 \otimes A_5$	✓	✓	(11,11,22),(11,22,11)
$A_2 \otimes A_5 \otimes D_4$	✓	✓	(11,22,11)
$A_2 \otimes D_4 \otimes D_4$	✗	✗	
$A_3 \otimes A_3 \otimes A_5$	✓	✓	(11,11,22)
$A_3 \otimes A_3 \otimes D_4$	✗	✗	

Examples of theories where the seed has $c_0 = 5$

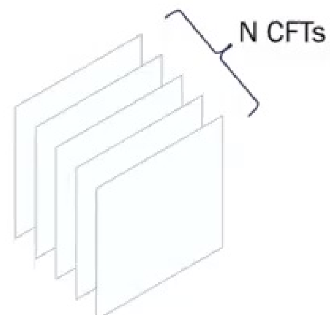


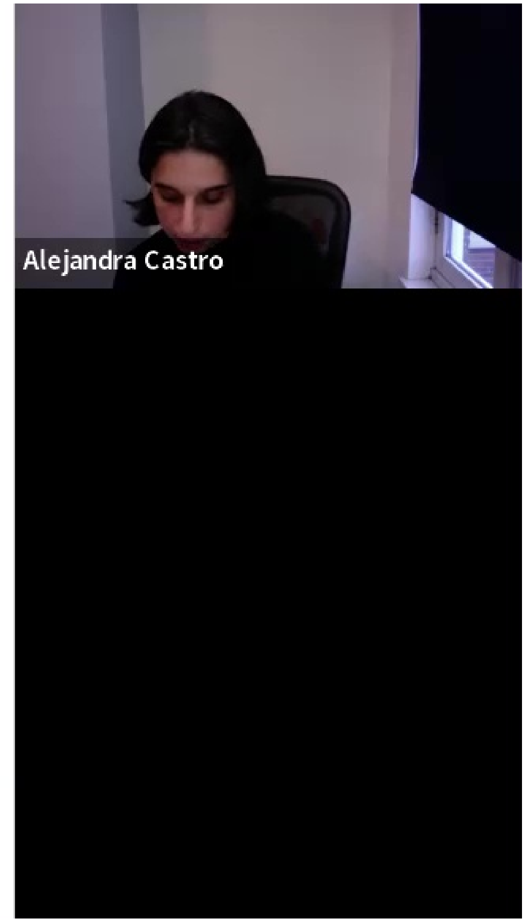
Marginal Operator

Three requirements on this operator:

- ½-BPS: Supersymmetry protects the deformation everywhere in the conformal manifold.
- Twisted: break the orbifold structure of the symmetric product.
- Single-trace: have an effect at leading order at large-N.

$$S \to S + \lambda\sqrt{N} \int d^2z \, \Phi_{1,1}(z,\bar{z})$$



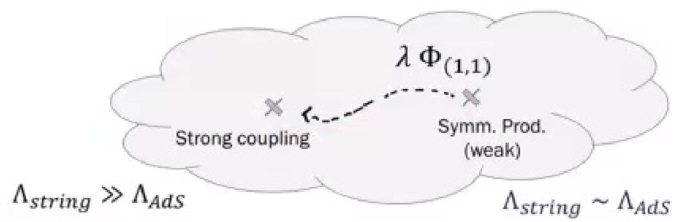


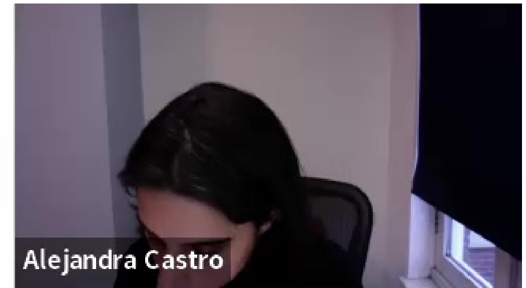
Marginal Operator

$$S \to S + \lambda\sqrt{N}\int d^2z\,\Phi_{1,1}(z,\bar{z})$$

Expectations of this operator:

- to induce anomalous dimensions on most operators, $\langle \mathbb{O}_a(z)\mathbb{O}_a(z')\rangle_\lambda = \dfrac{1}{(z-z')^{2(h+\mu_a(\lambda))}(\bar{z}-\bar{z}')^{2(\bar{h}+\bar{\mu}_a(\lambda))}}$
- reduce the Hagedorn growth.





Lifting currents

How do these currents look like? Examples of universal currents, independent of the seed,

Spin-4: only Virasoro on the seed.

$$
\begin{aligned}
W_4(z) &= (TT)(z) - \frac{3}{10}\partial^2 T(z) - \frac{\frac{22}{5c}+N}{N-1}\sum_{i\neq j}(T^{(i)}T^{(j)})(z), \\
&= \sum_{i=1}^{N}\left[(T^{(i)}T^{(i)})(z) - \frac{3}{10}\partial^2 T^{(i)}(z)\right] - \frac{\frac{22}{5c}+1}{N-1}\sum_{i\neq j}(T^{(i)}T^{(j)})(z)
\end{aligned}
$$

Spin-2: only Super-Virasoro on the seed.

$$
\begin{aligned}
W_2 &= T - \frac{3}{2}(JJ) + \frac{3(cN-1)}{2c(N-1)}\sum_{j\neq l} J^{(j)}J^{(l)}, \\
\mathscr{W}^{\pm}_{5/2} &= \partial G^{\pm} - 3(JG^{\pm}) + \frac{3(cN-1)}{c(N-1)}\sum_{j\neq l} J^{(j)}G^{\pm(l)}, \\
\mathtt{w}_3 &= \frac{3}{2}\left[(G^{+}G^{-}) - 2(TJ) - \partial T + \frac{2}{3}\partial^2 J\right] - \frac{3(cN-1)}{2c(N-1)}\sum_{j\neq l}\left[G^{+(j)}G^{-(l)} - 2T^{(j)}J^{(l)}\right].
\end{aligned}
$$

Conformal perturbation theory

$$\langle \mathbb{O}_a(z)\mathbb{O}_a(z')\rangle_\lambda = \frac{1}{(z-z')^{2(h+\mu_a(\lambda))}(\bar{z}-\bar{z}')^{2(\bar{h}+\bar{\mu}_a(\lambda))}}$$

For currents, obtaining anomalous dimension means:

- Evaluating 4-pt functions: first correction at $O(\lambda^2)$.
- Mapping from base to cover space: non-generic behaviour of $\Phi_{1,1}(z,\bar{z})$ as a function of twist and c_0.
- Null currents at specific values of c_0.

Drawing lessons and comparissons from computations done for D1D5 CFT:

- $\Phi_{1,1}(z,\bar{z})$ is a twist-2 ground state
- Seed theory $c_0 = 6$

[Lunin,Mathur; Gaberdiel, Peng, Zadeh; Keller, Zadeh; Benjamin, Keller, Zadeh]

Anomalous dimension for spin-2

k	$c=\frac{3k}{k+2}$	n	$\mu_{(2)}$
1	1	5	—
		7	
2	$\frac{3}{2}$	3	$\frac{20\pi^2\lambda^2(3N-2)}{27(N-1)}$
		4	$\frac{187\pi^2\lambda^2(3N-2)}{256(N-1)}$
		5	$\frac{4\pi^2\lambda^2(3N-2)}{5(N-1)}$
3	$\frac{9}{5}$	3	$\frac{44\pi^2\lambda^2(9N-5)}{243(N-1)}$
4	2	2	$\frac{39\pi^2\lambda^2(2N-1)}{64(N-1)}$
		3	$\frac{19\pi^2\lambda^2(2N-1)}{27(N-1)}$
		4	$\frac{207\pi^2\lambda^2(2N-1)}{256(N-1)}$
$5,6,\ldots$	$2<c<3$	3	$\frac{4\pi^2\lambda^2(c^2+12c-9)(cN-1)}{27c^2(c-1)(N-1)}$
$6,8,\ldots$	$2<c<3$	2	$\frac{3\pi^2\lambda^2(24+c)(cN-1)}{64c(c-1)(N-1)}$

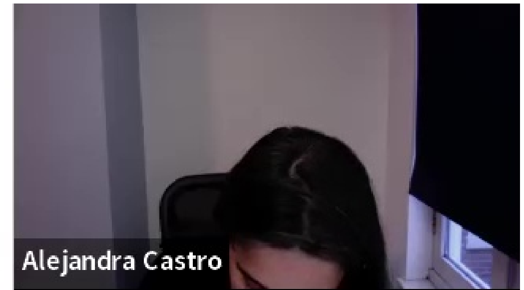
Spin-2 Operator

$$W_2(z) = T(z) - \frac{3}{2}(JJ)(z) + \frac{3(cN-1)}{2c(N-1)}\sum_{i\neq j}^{N} J^{(i)}(z)J^{(j)}(z)$$

$$= T(z) + \frac{3(c-1)}{2c(N-1)}(JJ)(z) - \frac{3(cN-1)}{2c(N-1)}\sum_{i=1}^{N}(J^{(i)}J^{(i)})(z)\,.$$

Anomalous dimension

$$\langle \mathbb{O}_a(z)\mathbb{O}_a(z')\rangle_\lambda = \frac{1}{(z-z')^{2(h+\mu_a(\lambda))}(\bar z-\bar z')^{2(\bar h+\bar\mu_a(\lambda))}}$$



Anomalous dimension for spin-2

k	$c=\frac{3k}{k+2}$	n	$\mu_{(2)}$
1	1	5	—
		7	
2	$\frac{3}{2}$	3	$\frac{20\pi^2\lambda^2(3N-2)}{27(N-1)}$
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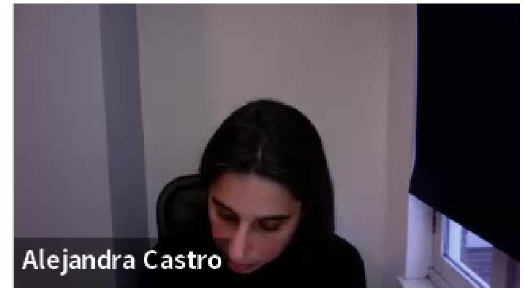
Twist-n deformation

$$S \to S + \lambda\sqrt{N}\int d^2z\, \Phi_{1,1}(z,\bar{z})$$

Deformation on the cover

$$\rightarrow \tilde{\phi}_{(2)} = \varphi^{\mathrm{R}}_{(r,s)}$$

$$\rightarrow \tilde{\phi}_{(3)} = G^{+}_{-1/2}\varphi^{\mathrm{NS}}_{(r,s)}$$



Anomalous dimension for spin-2

k	$c = \frac{3k}{k+2}$	n	$\mu_{(2)}$
1	1	5	—
		7	
2	$\frac{3}{2}$	3	$\frac{20\pi^2\lambda^2(3N-2)}{27(N-1)}$
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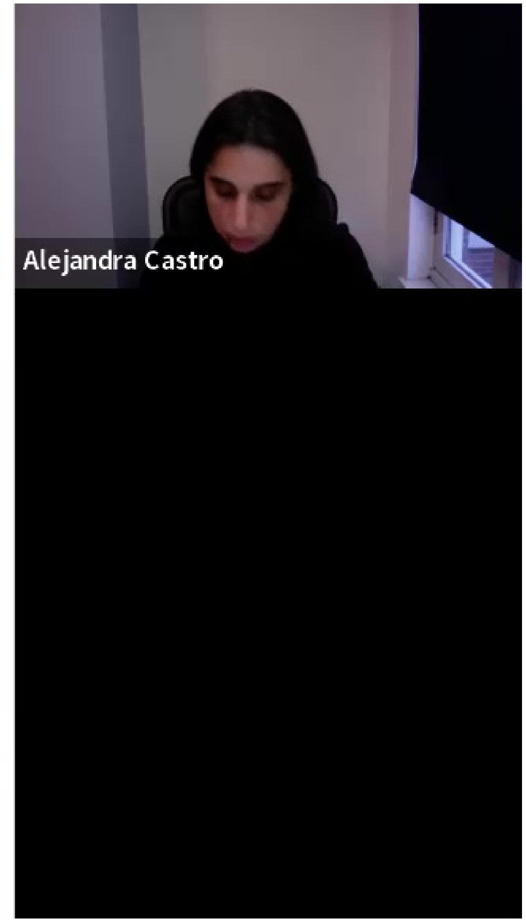
$5, 6, \ldots$	$2 < c < 3$	3	$\frac{4\pi^2\lambda^2(c^2+12c-9)(cN-1)}{27c^2(c-1)(N-1)}$
$6, 8, \ldots$	$2 < c < 3$	2	$\frac{3\pi^2\lambda^2(24+c)(cN-1)}{64c(c-1)(N-1)}$

Remarks

- Sensitivity on the twist and central charge.
- Still, currents are lifting. Good sign!
- Similar results for spin-3: two universal currents.
- For $c_0 = 6$, matches with known results for twist-2 deformations of T^4 sigma model.



Anomalous dimension for spin-2

k	$c=\frac{3k}{k+2}$	n	$\mu_{(2)}$
1	1	5	—
		7	
2	$\frac{3}{2}$	3	$\frac{20\pi^2\lambda^2(3N-2)}{27(N-1)}$
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$6,8,\ldots$	$2<c<3$	2	$\frac{3\pi^2\lambda^2(24+c)(cN-1)}{64c(c-1)(N-1)}$

Next steps

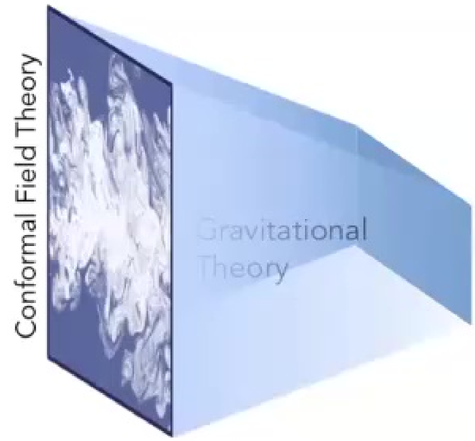
- Include exceptional currents (specific to the seed).
- Regge trajectories.
- Look at other operators, both BPS and non-BPS.
- Contrast theories with theories that are not sparse (for $3 \leq c_0 \leq 6$).

OUTLOOK

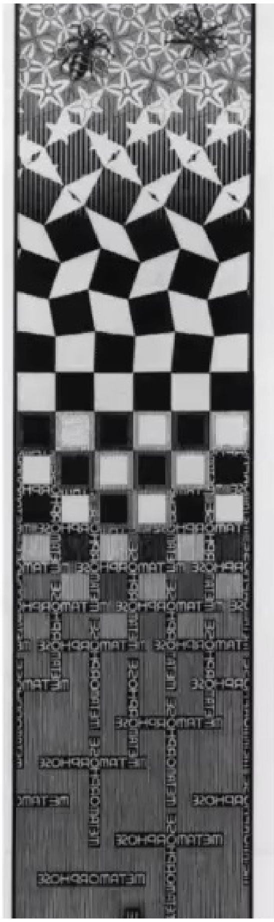
What are the EFT of gravity that admit a UV completion?

Quantify the space of holographic CFT_2:

- Different from known examples
- Systematic and tractable
- Infinite family



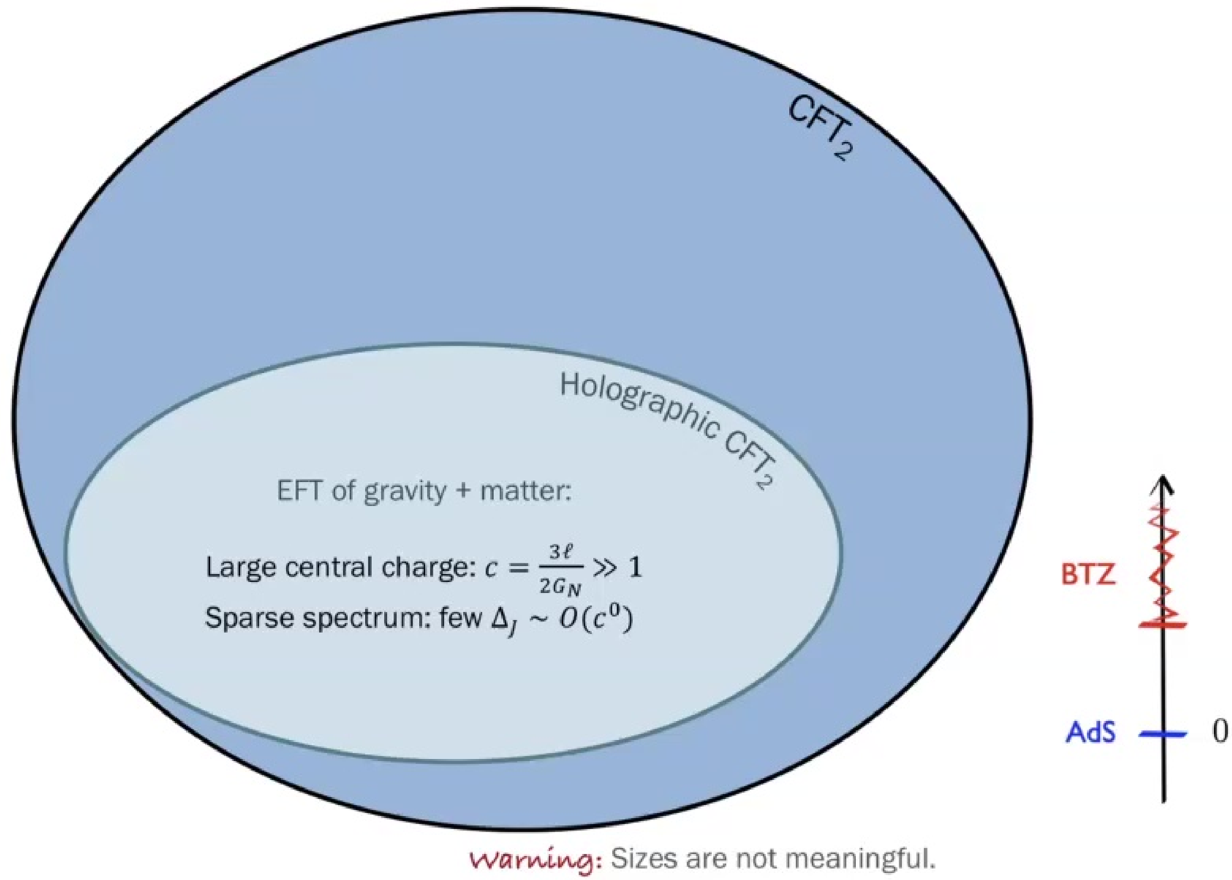
High Energy:
String Theory



Low energy EFT:
Gravity + matter

- Which CFTs capture classical (geometric) properties of gravity?
- What are possible theories of quantum gravity can be designed?
- What are the materials needed to assemble them?

Next steps:
- Seeds with $3 \leq c_0 \leq 6$
- Multi-trace deformations
- String theory description
- N vs k: SUGRA spectrum
- Averages over moduli
- Mock Modularity



CFT$_2$
Holographic CFT$_2$
EFT of gravity + matter:
Large central charge: $c = \frac{3\ell}{2G_N} \gg 1$
Sparse spectrum: few $\Delta_J \sim O(c^0)$
BTZ
AdS
0
Warning: Sizes are not meaningful.

Ji Hoon Lee