Title: Quantum Gravity

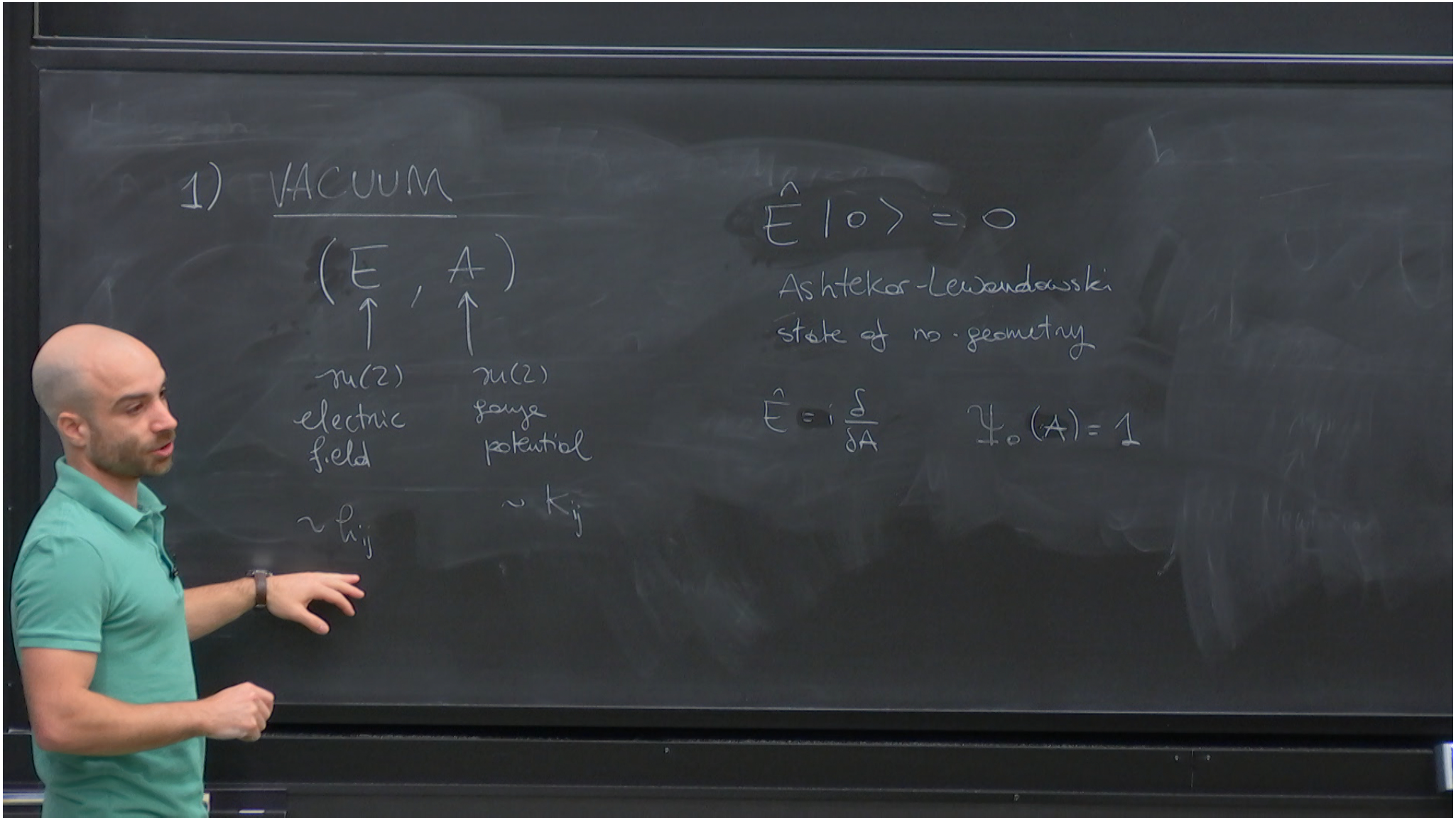
Speakers: Aldo Riello

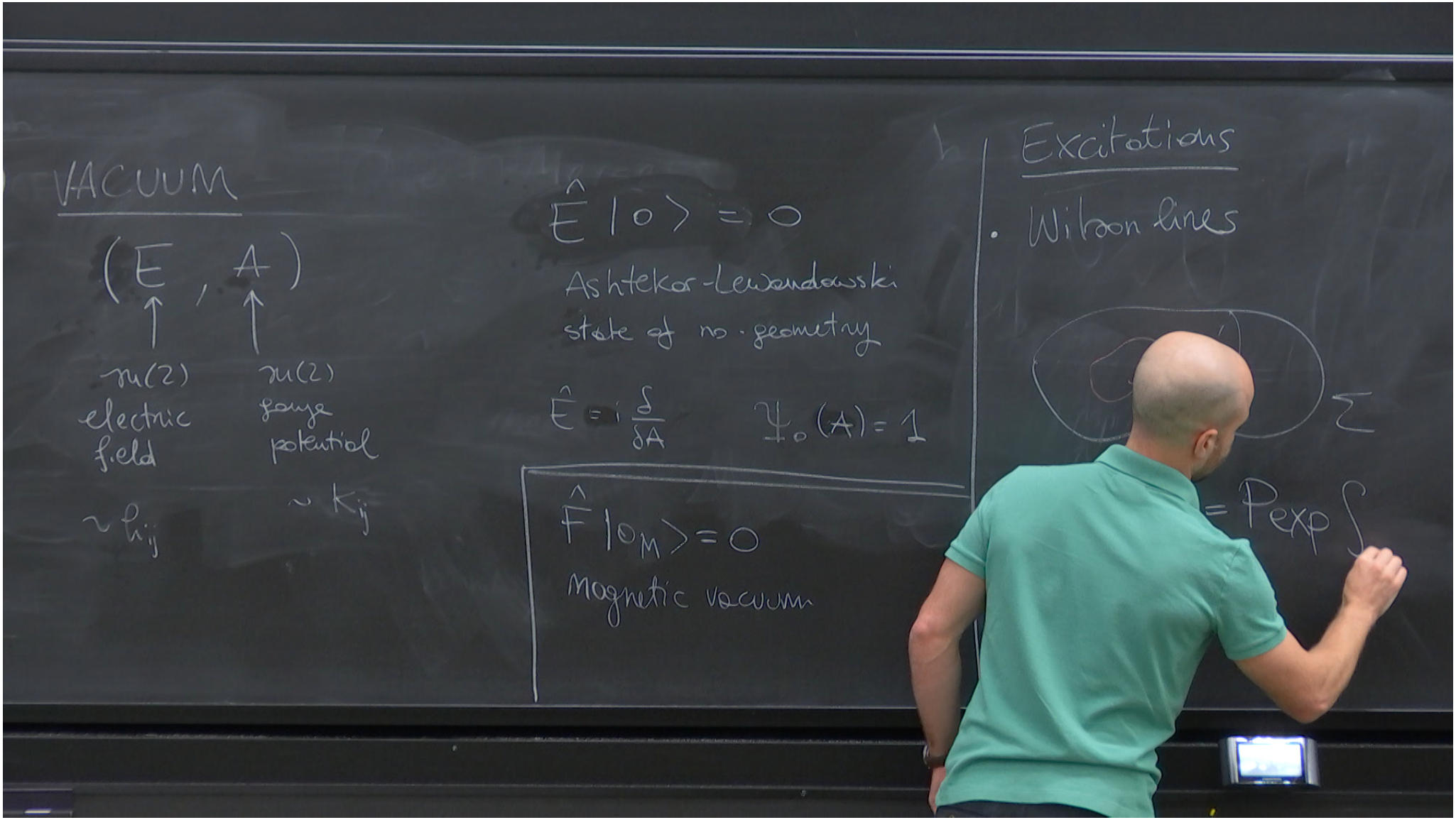
Collection: Quantum Gravity (2021-2022)

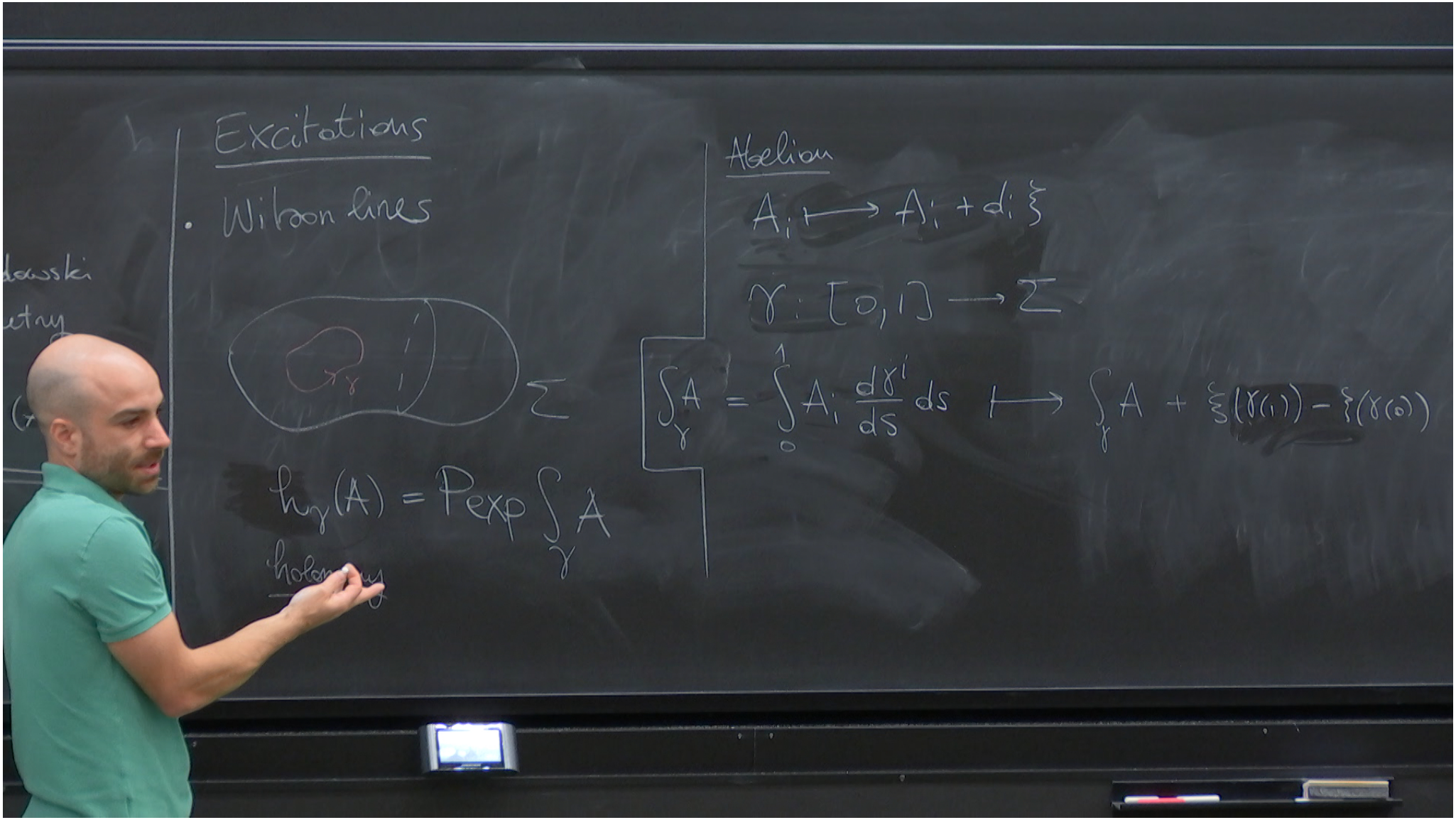
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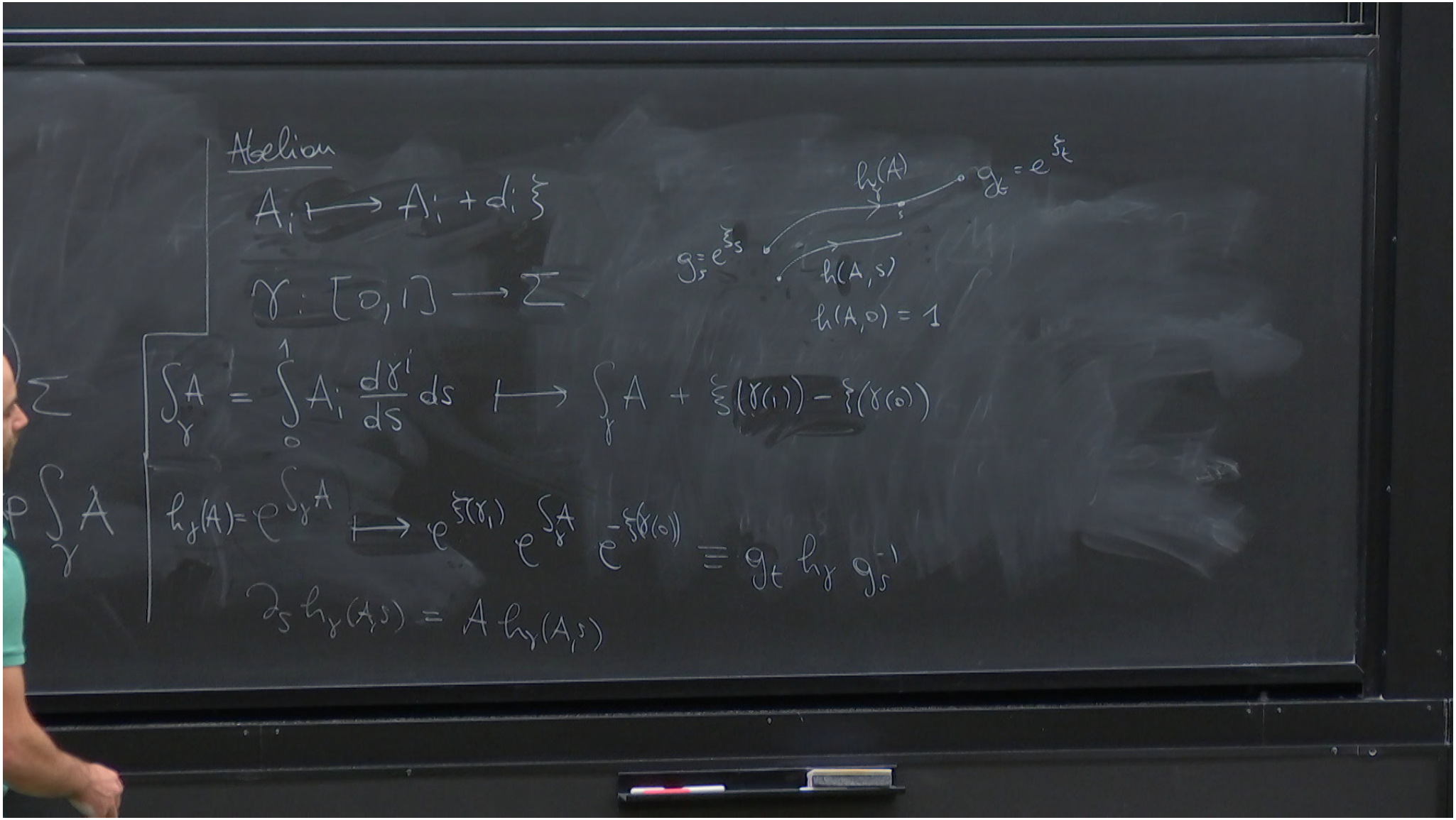
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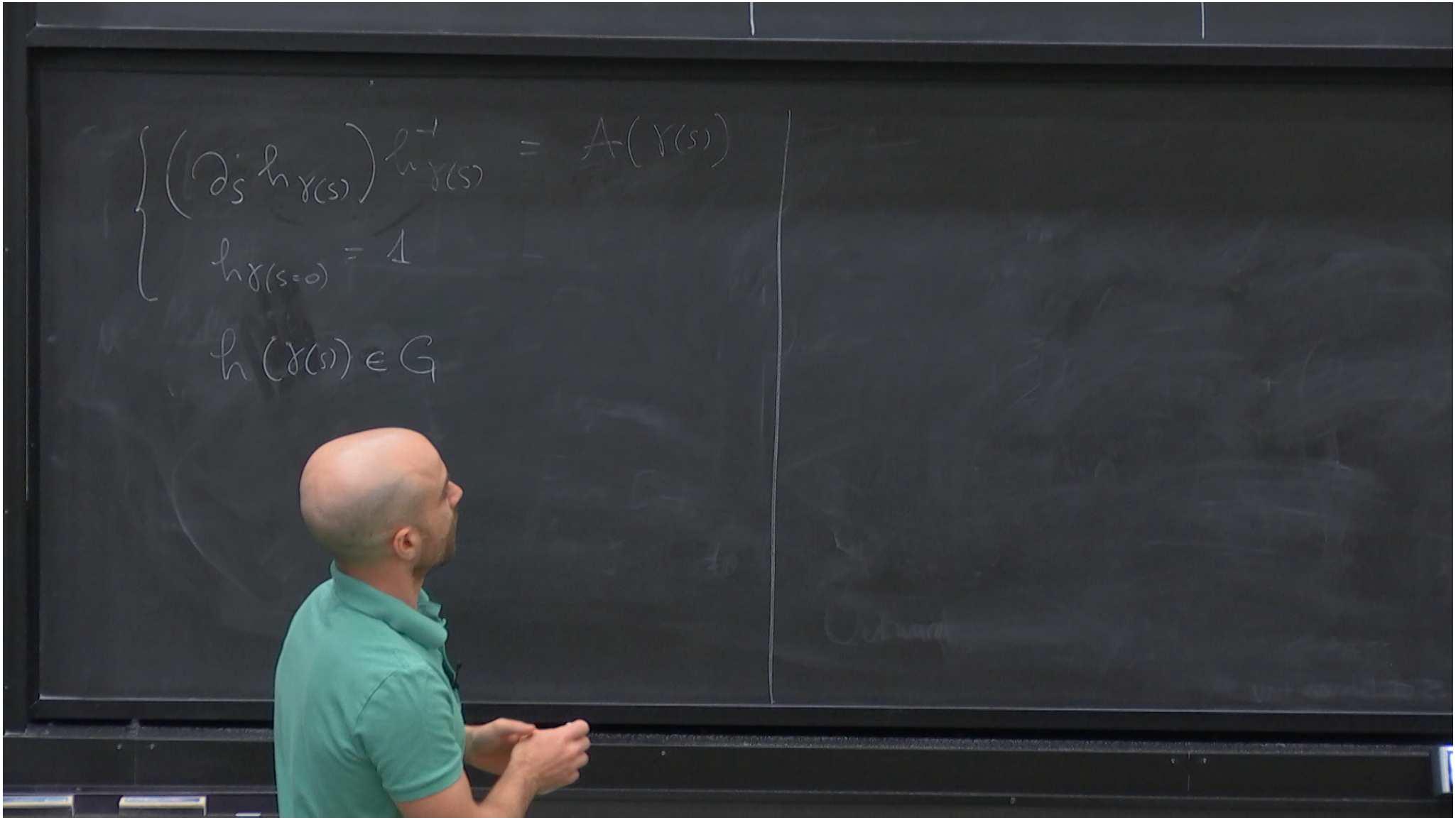
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

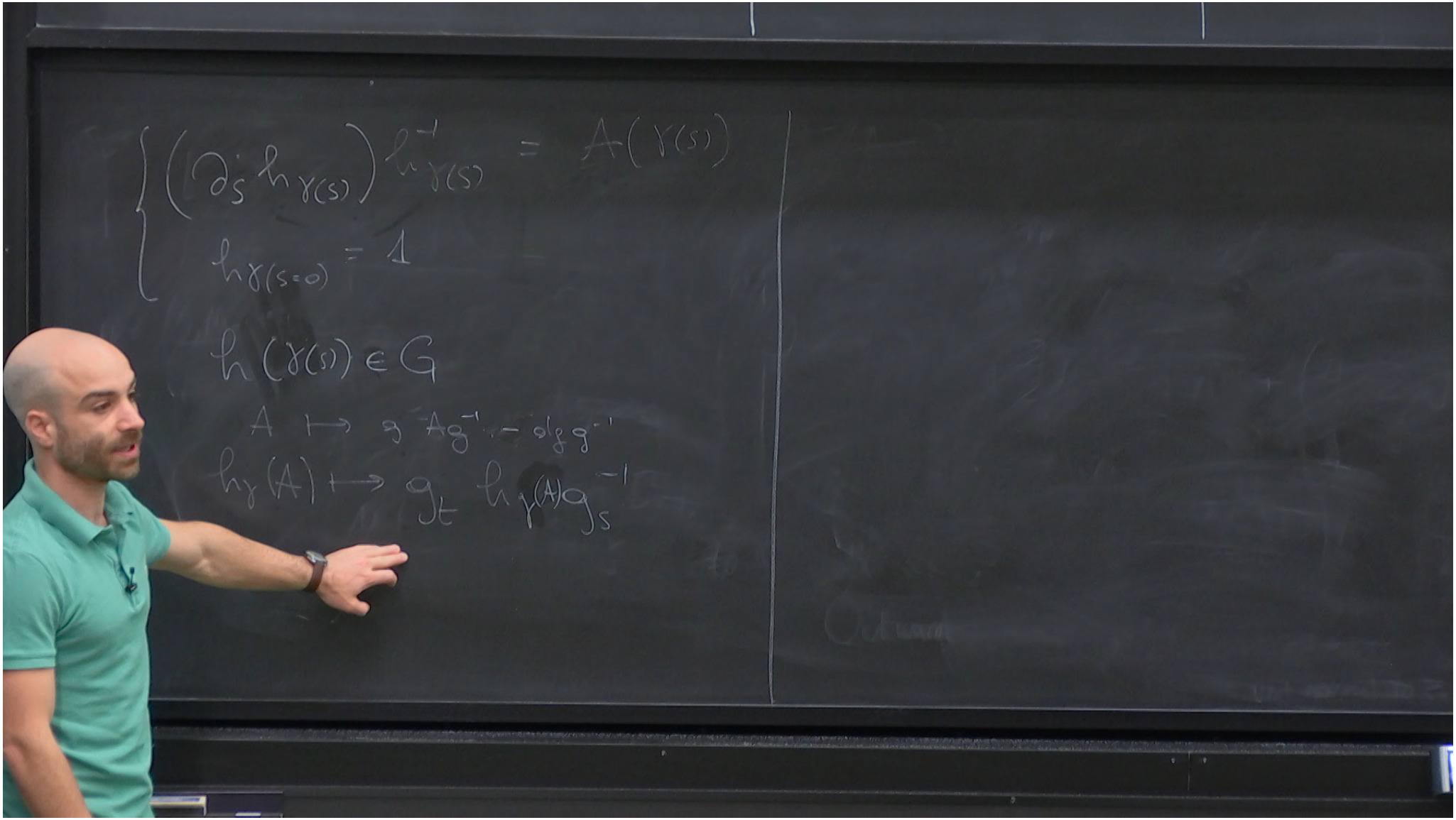

1) VACUUM
(E , A)
su(2) electric field
su(2) gauge potential
~ E_{ij}
~ K_{ij}
$\hat{E}|0\rangle = 0$
Ashtekar-Lewandowski state of no-geometry
$\hat{E} = i\frac{\delta}{\delta A}$
$\Psi_0(A) = 1$

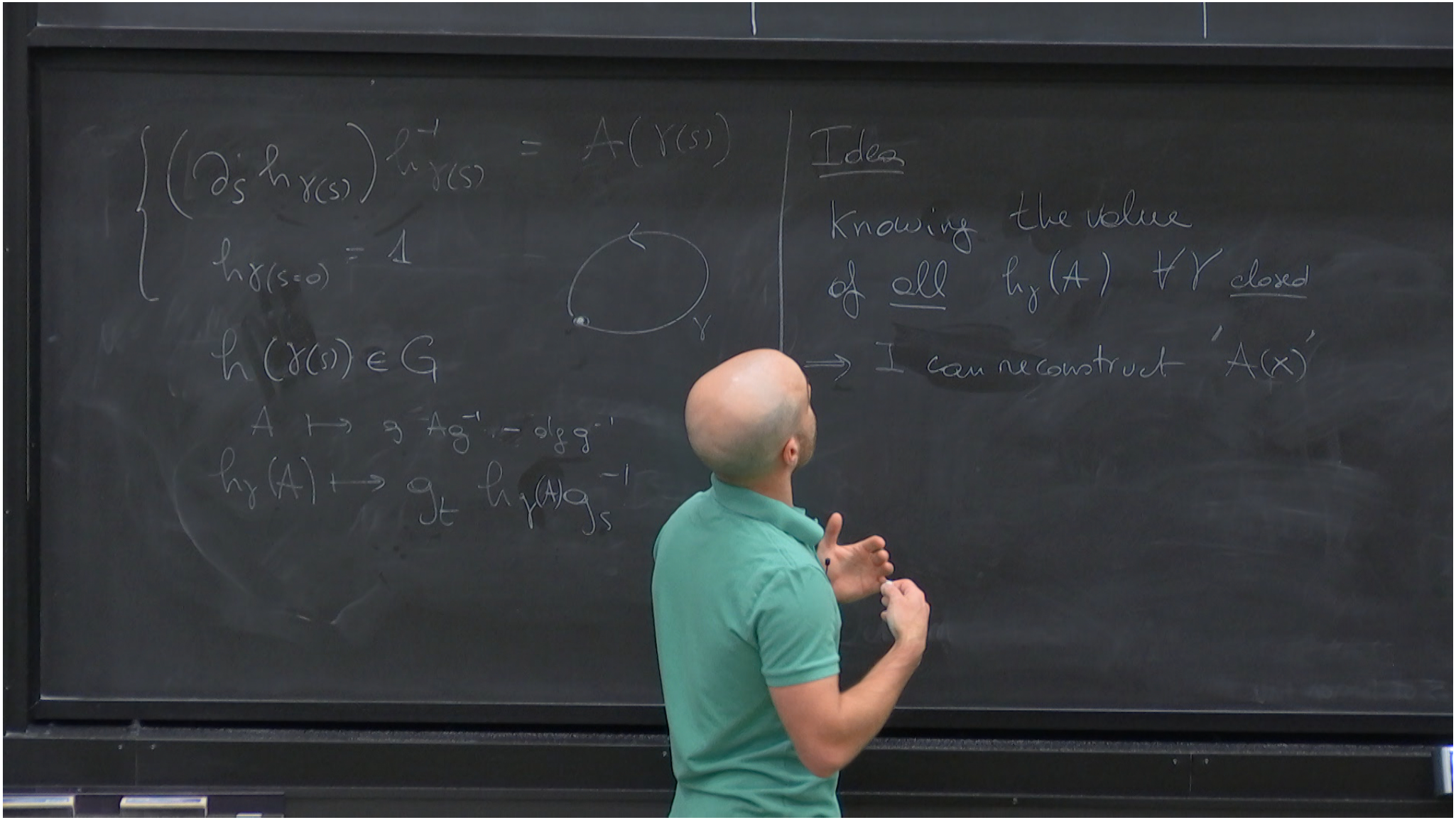

VACUUM
(E, A)
su(2) electric field
su(2) gauge potential
$\sim F_{ij}$
$\sim K_j$
$\hat{E}|0\rangle = 0$
Ashtekar-Lewandowski state of no-geometry
$\hat{E} = -i\frac{\delta}{\delta A}$
$\Psi_0(A) = 1$
$\hat{F}|0_M\rangle = 0$
Magnetic vacuum
Excitations
Wilson lines
$= P\exp\int$


Excitations
• Wilson lines
Abelian
$A_i \longmapsto A_i + d_i \xi$
$\gamma : [0,1] \longrightarrow \Sigma$
$\int_\gamma A = \int_0^1 A_i \frac{d\gamma^i}{ds} ds \longmapsto \int_\gamma A + \xi(\gamma(1)) - \xi(\gamma(0))$
Σ
$h_\gamma(A) = \mathrm{Pexp} \int_\gamma A$
holonomy

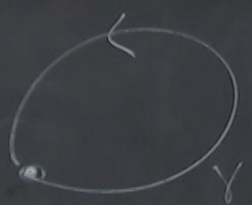

Abelian
$A_i \longmapsto A_i + \partial_i \xi$
$\gamma : [0,1] \longrightarrow \Sigma$
$\int_\gamma A = \int_0^1 A_i \frac{dx^i}{ds} ds \longmapsto \int_\gamma A + \xi(\gamma(1)) - \xi(\gamma(0))$
$h_\gamma(A) = e^{\int_\gamma A} \longmapsto e^{\xi(\gamma_1)} e^{\int_\gamma A} e^{-\xi(\gamma(0))} \equiv g_t h_\gamma g_s^{-1}$
$\partial_s h_\gamma(A,s) = A\, h_\gamma(A,s)$
$h_\gamma(A)$
$g_t = e^{\xi_t}$
$g_s = e^{\xi_s}$
$h(A,s)$
$h(A,0) = 1$


$\begin{cases} (\partial_s h_{\gamma(s)}) h^{-1}_{\gamma(s)} = A(\gamma(s)) \\ h_{\gamma(s=0)} = 1 \end{cases}$
$h(\gamma(s)) \in G$


$\begin{cases} (\partial_s h_{\gamma(s)}) h^{-1}_{\gamma(s)} = A(\gamma(s)) \\ h_{\gamma(s=0)} = 1 \end{cases}$
$h(\gamma(s)) \in G$
$A \mapsto g A g^{-1} - \partial_s g \, g^{-1}$
$h_\gamma(A) \mapsto g_t \, h_{\gamma(A)} g_s^{-1}$


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γ
Idea
Knowing the value
of all $h_\gamma(A)$ $\forall \gamma$ closed
$\Rightarrow$ I can reconstruct '$A(x)$'

$= A(\gamma(s))$



$\dots - \partial_s g\, g^{-1}$

$h_{\gamma(A)} g_s^{-1}$

Idea

knowing the value of <u>all</u> $h_\gamma(A)$ $\forall \gamma$ <u>closed</u>

$\Rightarrow$ I can reconstruct '$A(x)$'

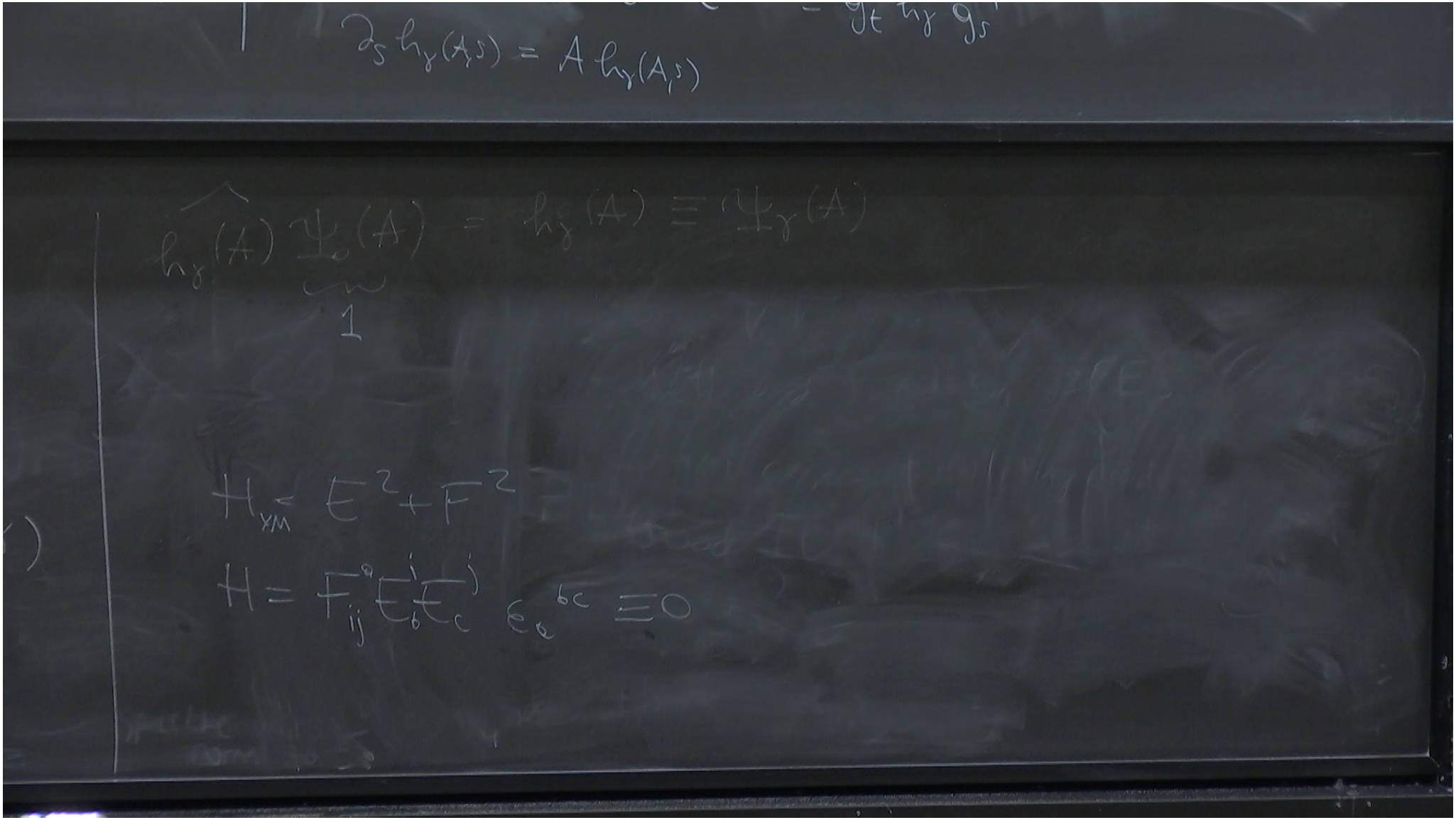
$\Rightarrow \Psi(A)$ can be written as $\Psi(\gamma)$

$\partial_s h_\gamma(A,s) = A\, h_\gamma(A,s)$

$\widehat{h_\gamma(A)}\, \Psi_0(A) = h_\gamma(A) \equiv \Psi_\gamma(A)$

Ψ_0

1


$\partial_s h_\gamma(A,s) = A\, h_\gamma(A,s)$
$\widehat{h_\gamma(A)}\, \Psi_0(A) = h_\gamma(A) \equiv \Psi_\gamma(A)$
1
$H_{YM} = E^2 + F^2$
$H = F_{ij}^a E_b^i E_c^j \epsilon_a^{bc} \equiv 0$

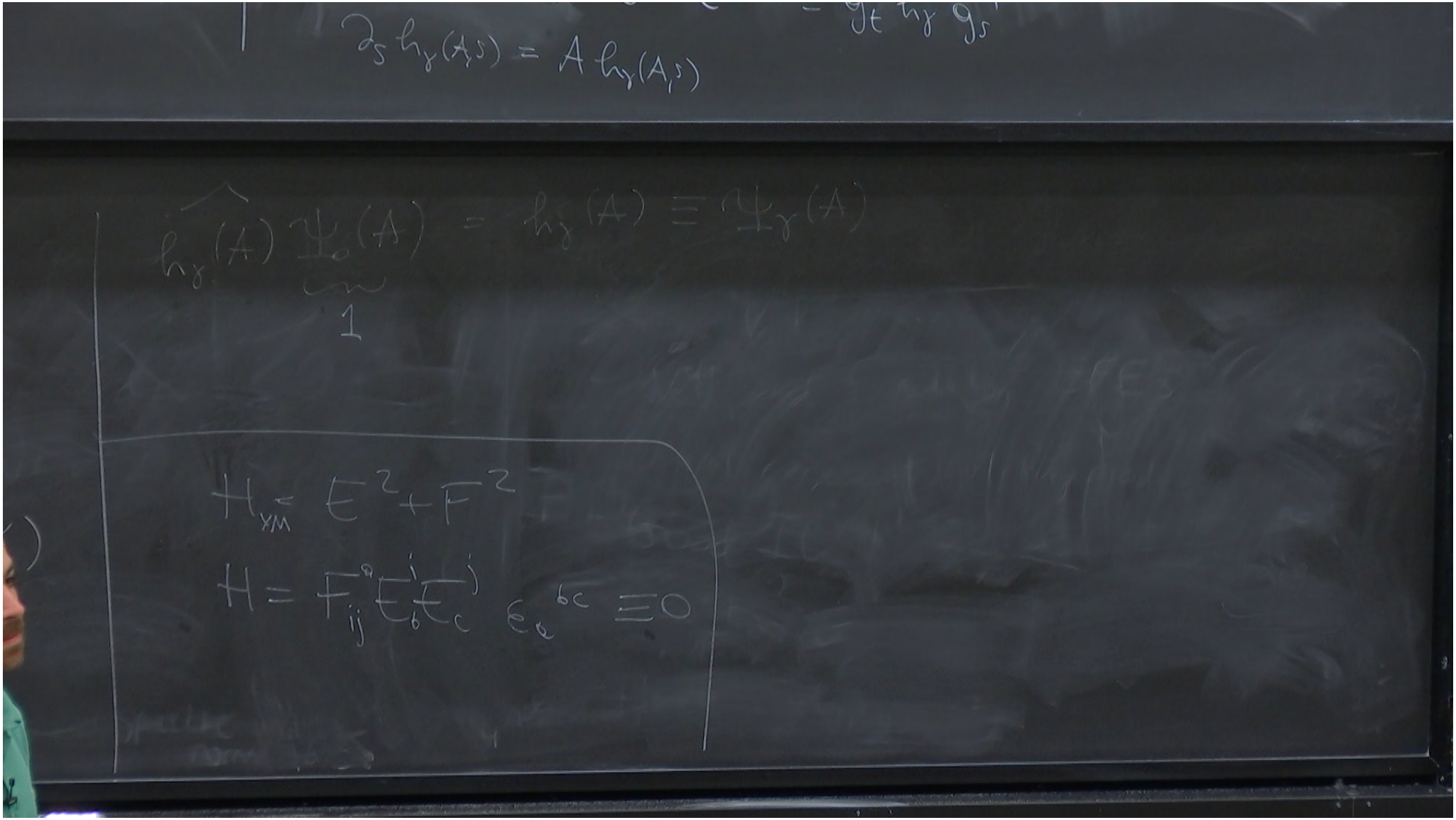
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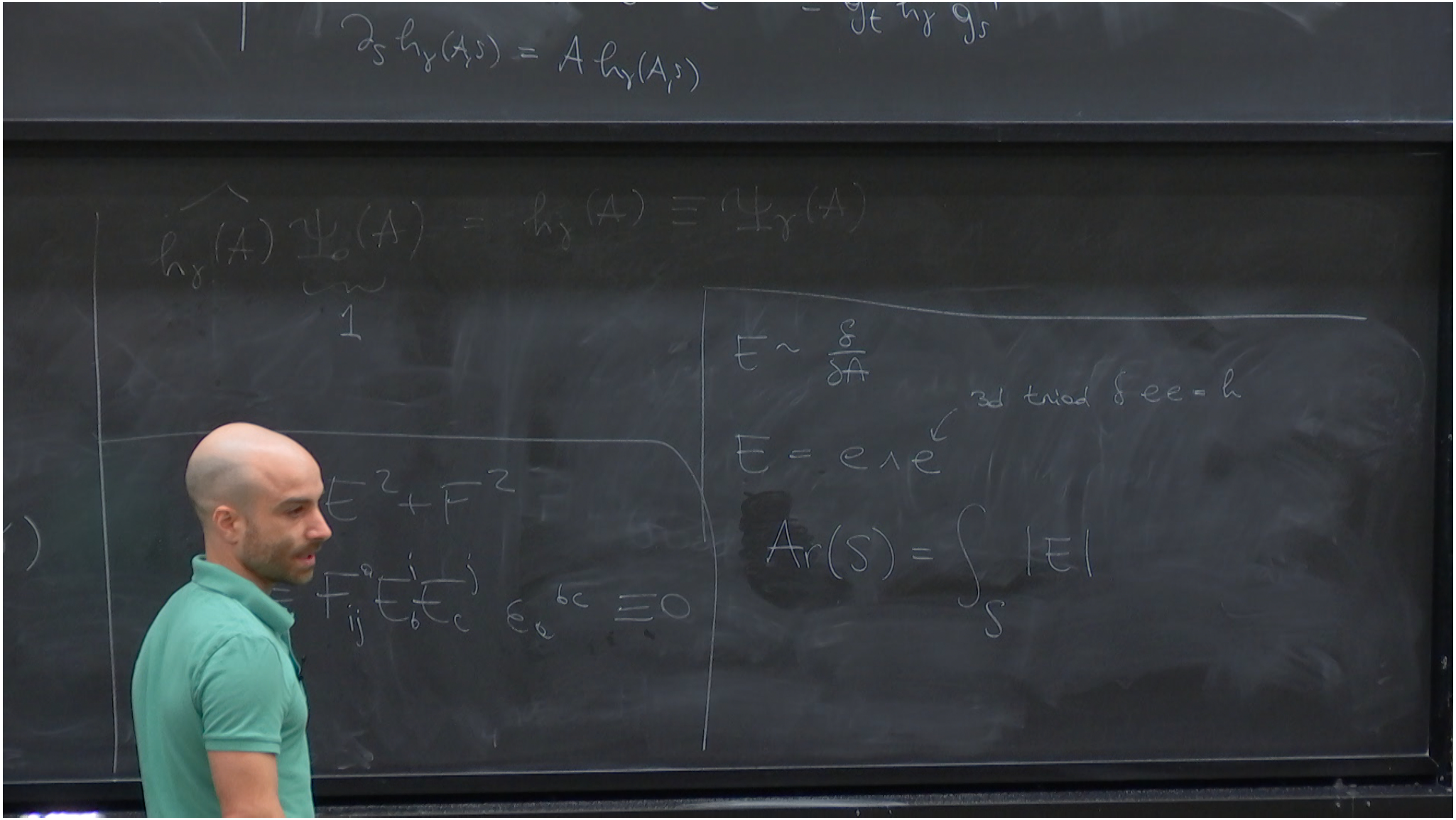
$\widehat{h_\gamma(A)}\, \Psi_0(A) = h_\gamma(A) \equiv \Psi_\gamma(A)$

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$H_{YM} = E^2 + F^2$
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$\hat{h}_\gamma(A)\,\Psi_0(A) = h_\gamma(A) \equiv \Psi_\gamma(A)$
1
$E \sim \frac{\delta}{\delta A}$
3d triad δ ee = h
$E = e \wedge e$
$Ar(S) = \int_S |E|$
$E^2 + F^2$
$F^a_{ij} E^i_b E^j_c \epsilon^{bc} \equiv 0$

$\partial_s h_\gamma(A,s) = A\, h_\gamma(A,s)$

$\hat{h}_\gamma(A)\, \Psi_0(A) = h_\gamma(A) \equiv \Psi_\gamma(A)$

1

$H_{YM} = E^2 + F^2$

$H = F^a_{ij} E^i_b E^j_c \epsilon^{bc}_a \equiv 0$

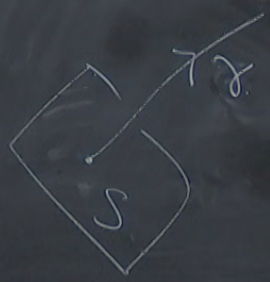
$E \sim \frac{\delta}{\delta A}$

3d triad & $ee = h$

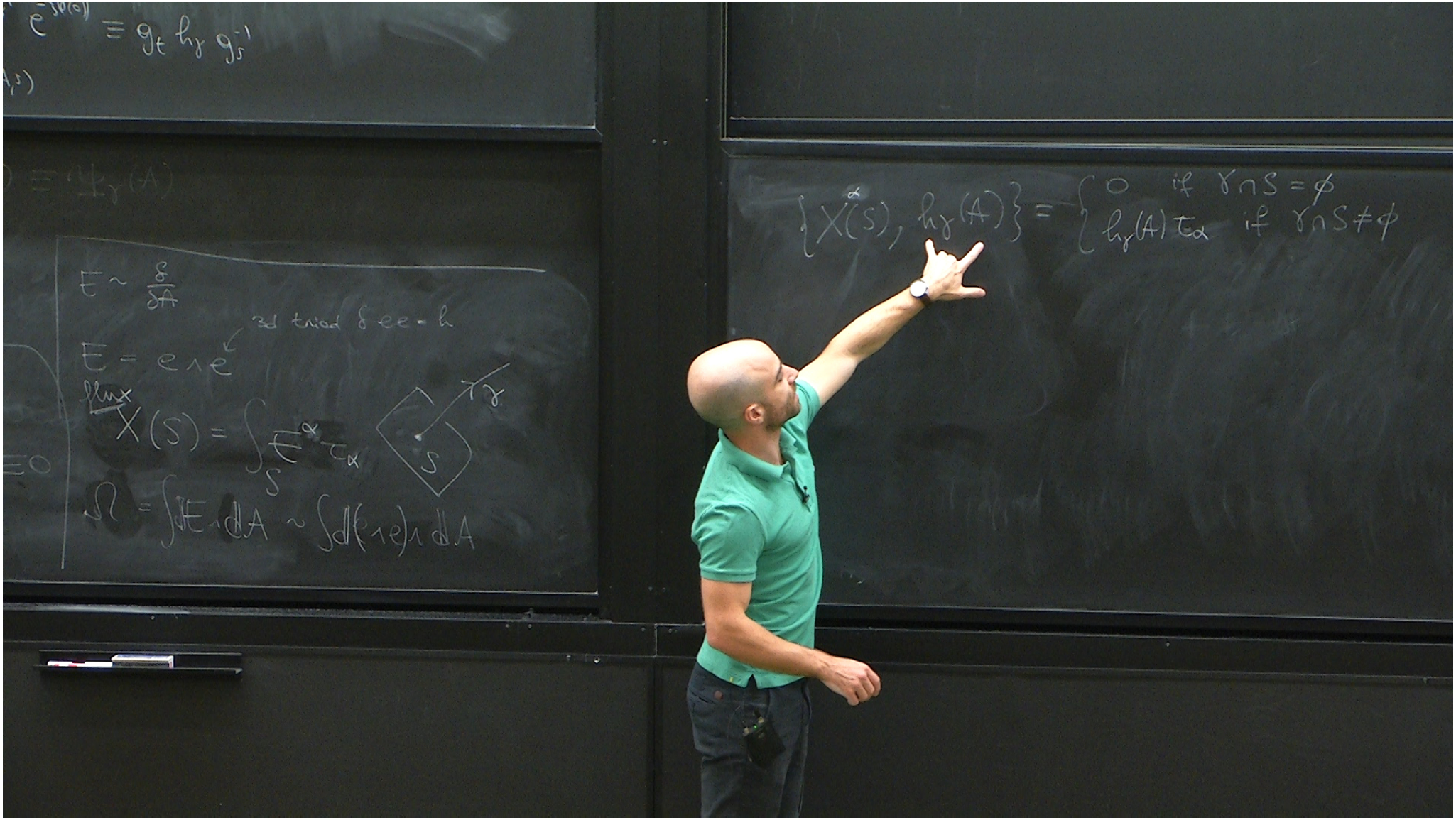
$E = e \wedge e$

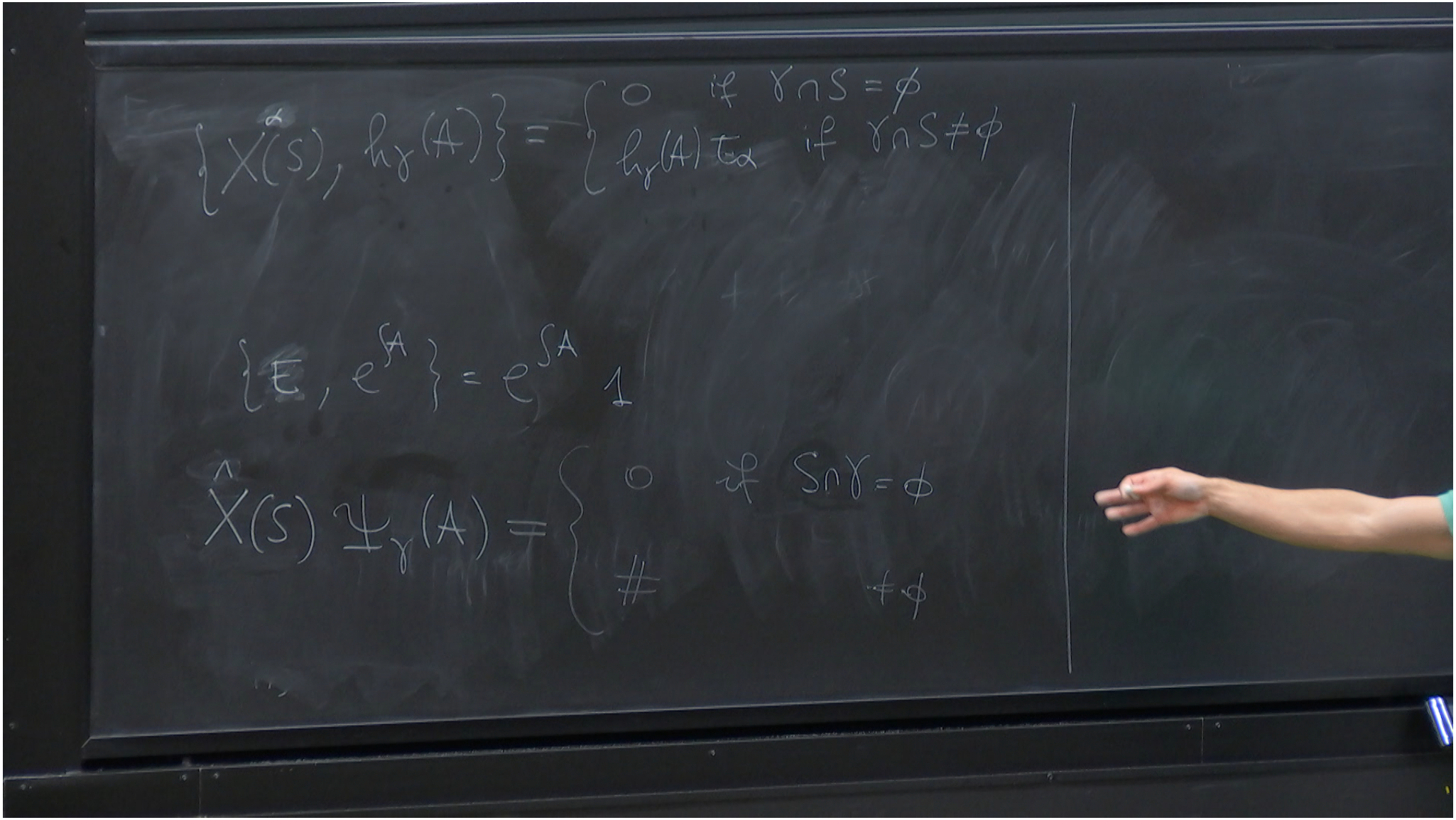
flux $X(S) = \int_S E$

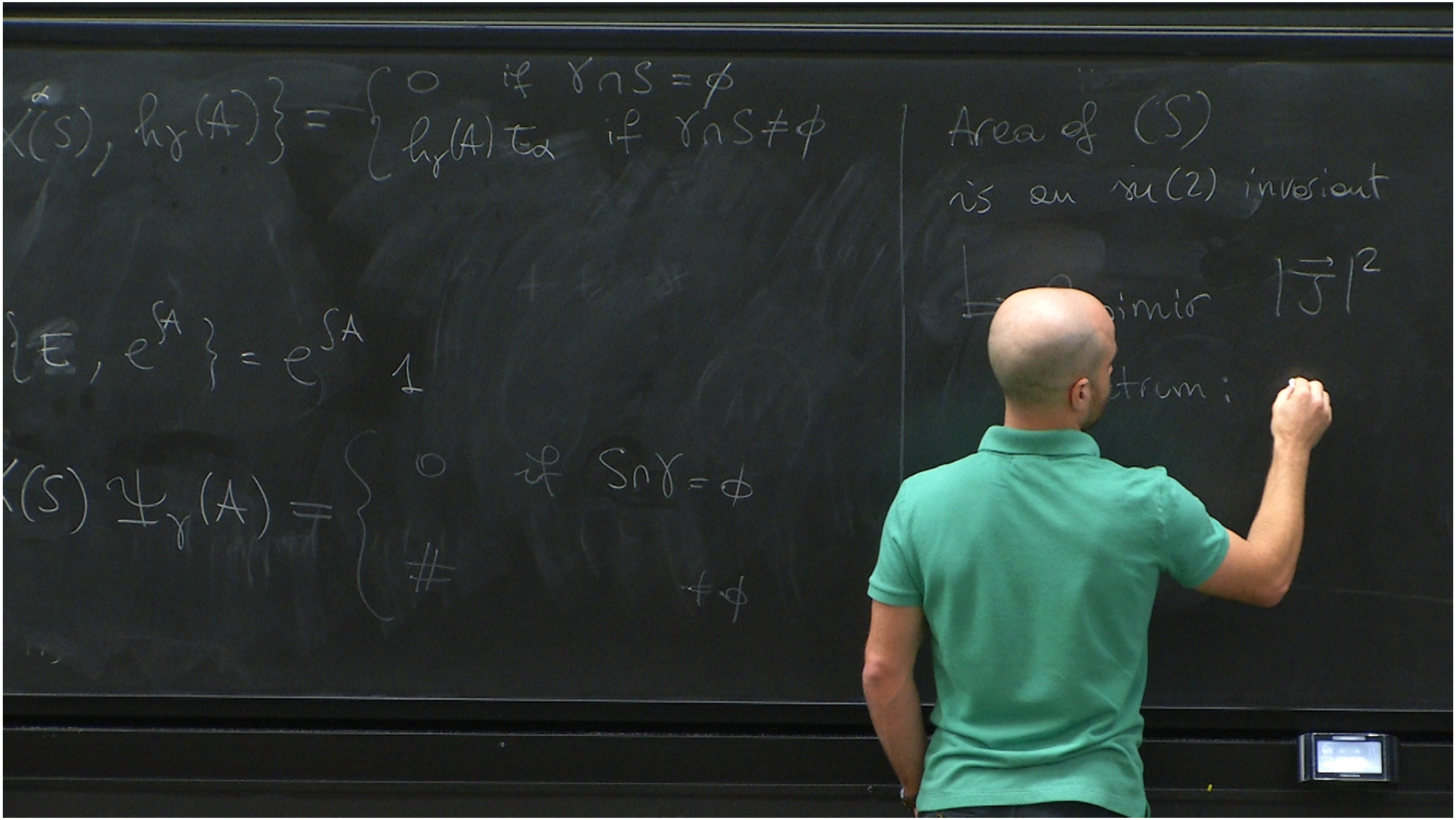
$\Omega = \int E \cdot dA \sim \int d(e \wedge e) \wedge dA$

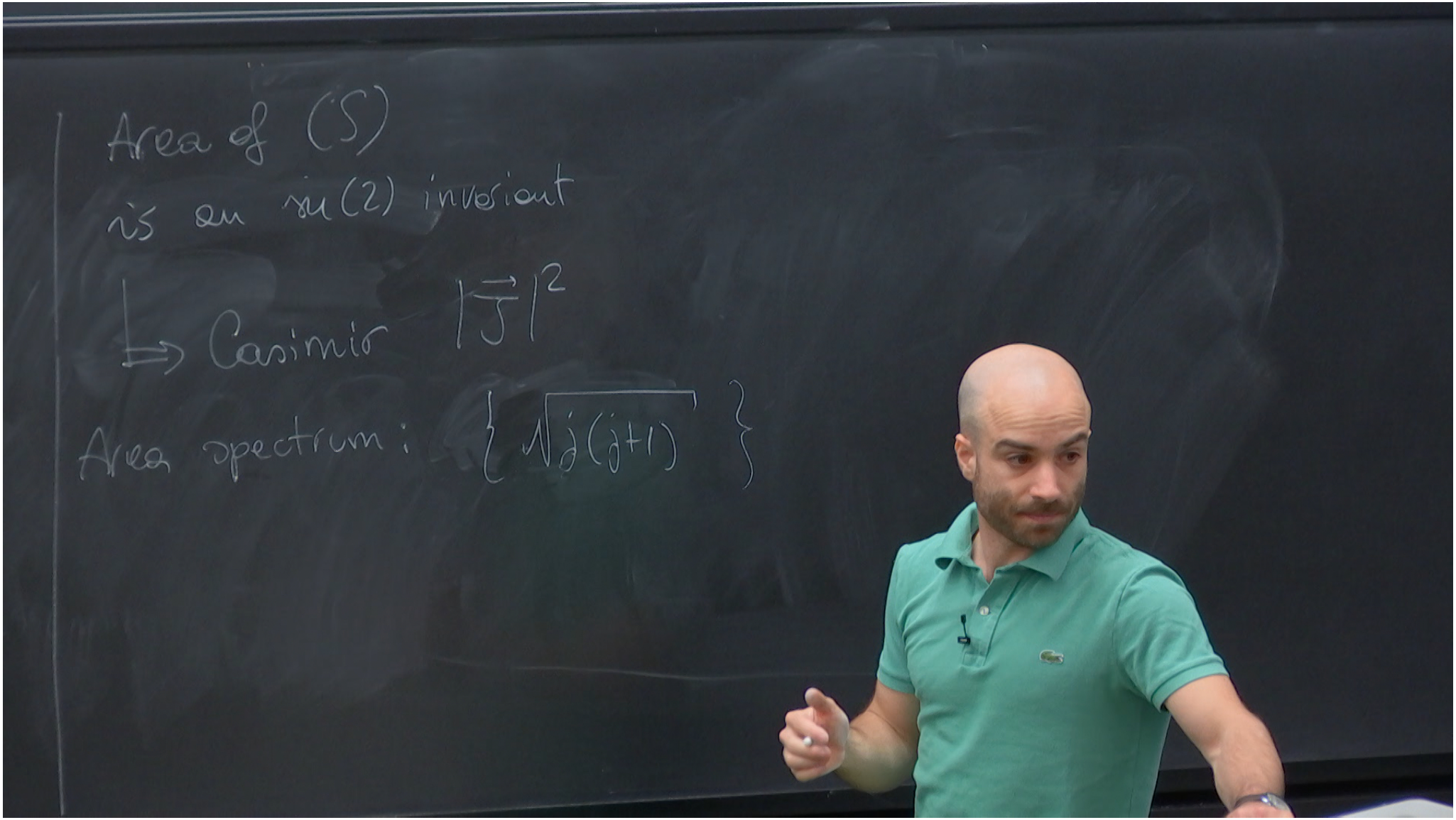


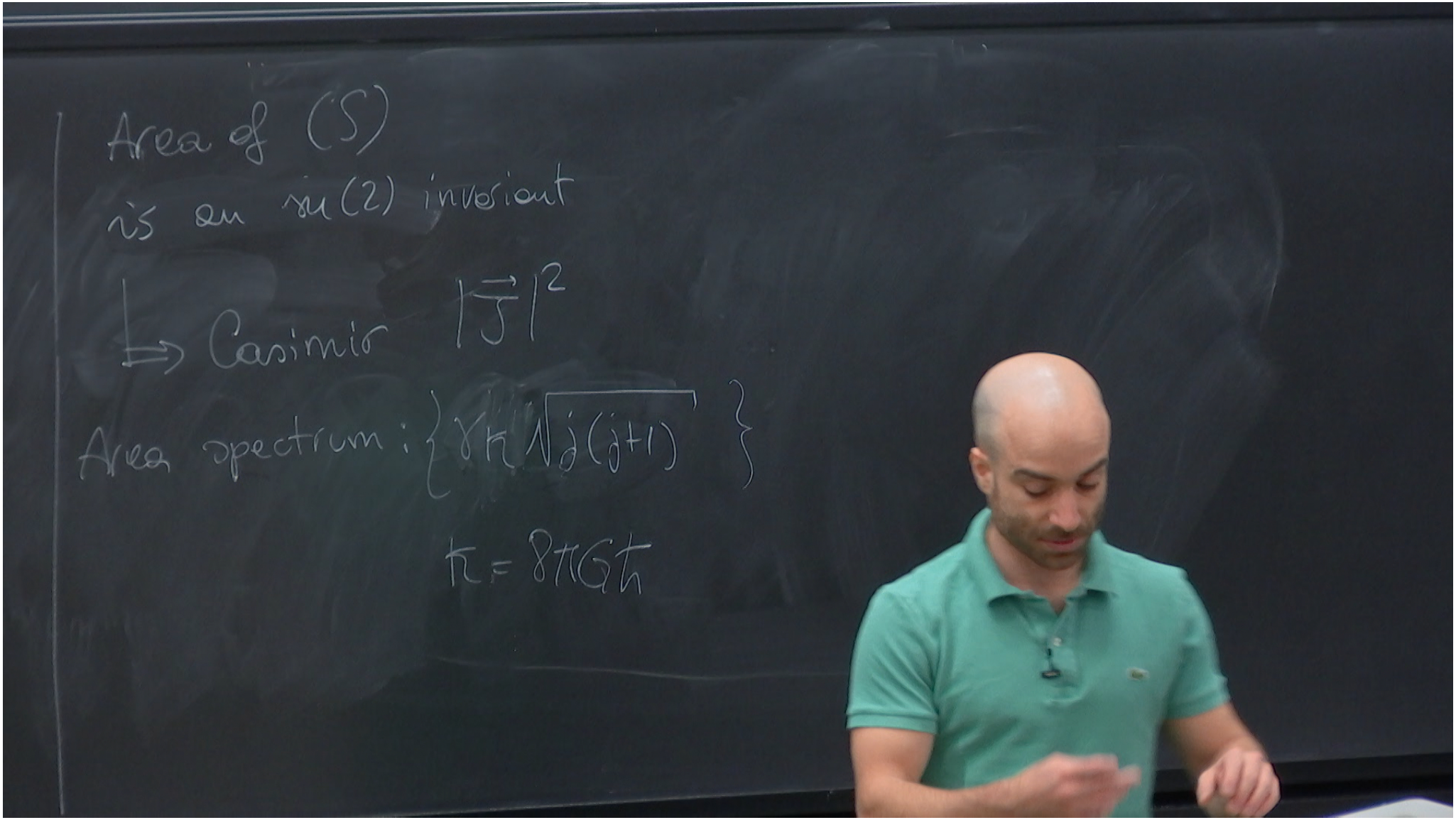
$$\{X(S), h_\gamma(A)\} = \begin{cases} 0 & \text{if } \gamma \cap S = \emptyset \\ \end{cases}$$


$\{X^{\alpha}(s), h_\gamma(A)\} = \begin{cases} 0 & \text{if } \gamma \cap S = \phi \\ h_\gamma(A)\,\tau_\alpha & \text{if } \gamma \cap S \neq \phi \end{cases}$
$E \sim \frac{\delta}{\delta A}$
3d triad & ee = h
$E = e \wedge e$
flux
$X(s) = \int_S E^{\alpha} \tau_\alpha$
$\Omega = \int E \cdot dA \sim \int d(e \wedge e) \wedge dA$
γ
S


$\{\hat{X}^{\alpha}(S), h_\gamma(A)\} = \begin{cases} 0 & \text{if } \gamma \cap S = \phi \\ h_\gamma(A)\, t_\alpha & \text{if } \gamma \cap S \neq \phi \end{cases}$
$\{E, e^{\int A}\} = e^{\int A}\, 1$
$\hat{X}(S)\, \Psi_\gamma(A) = \begin{cases} 0 & \text{if } S \cap \gamma = \phi \\ \# & \neq \phi \end{cases}$


Area of (S)
is an su(2) invariant


Area of (S)
is an su(2) invariant
⟹ Casimir $|\vec{J}|^2$
Area spectrum: $\{\sqrt{j(j+1)}\}$


Area of (S)
is an su(2) invariant
⟹ Casimir $|\vec{J}|^2$
Area spectrum: $\{\gamma \hbar \sqrt{j(j+1)}\}$
$\hbar = 8\pi G\hbar$