Title: Strong Gravity

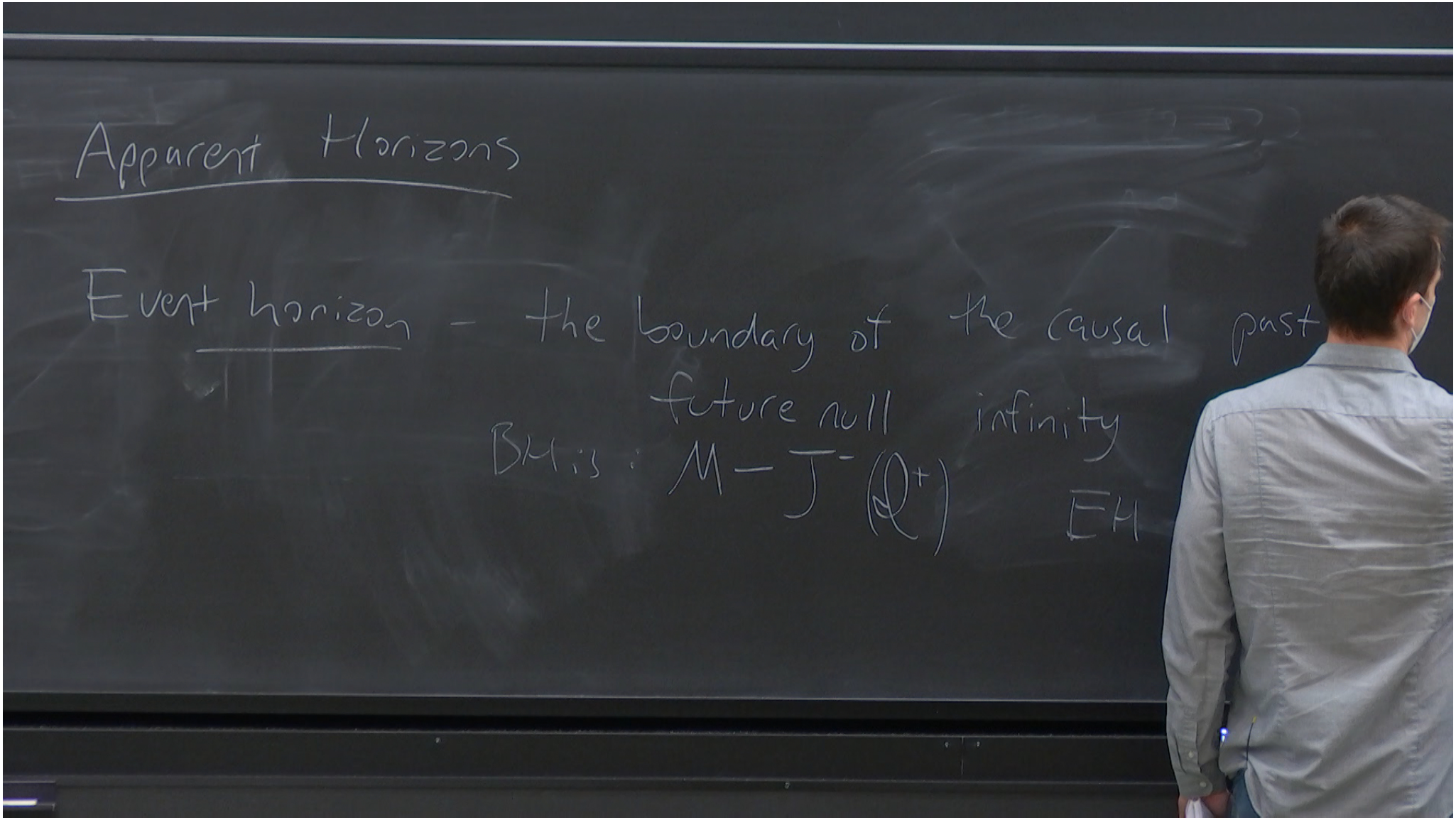
Speakers: William East, Giuseppe Sellaroli

Collection: Strong Gravity (2021/2022)

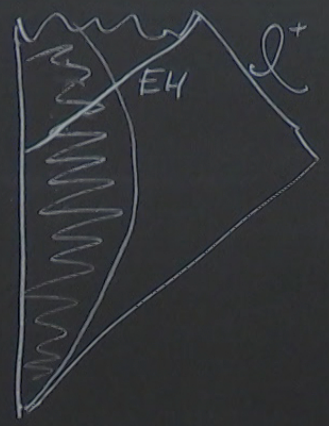
Date: May 06, 2022 - 10:15 AM

URL: https://pirsa.org/22050016

Abstract: This course will introduce some advanced topics in general relativity related to describing gravity in the strong field and dynamical regime. Topics covered include properties of spinning black holes, black hole thermodynamics and energy extraction, how to define horizons in a dynamical setting, formulations of the Einstein equations as constraint and evolution equations, and gravitational waves and how they are sourced.

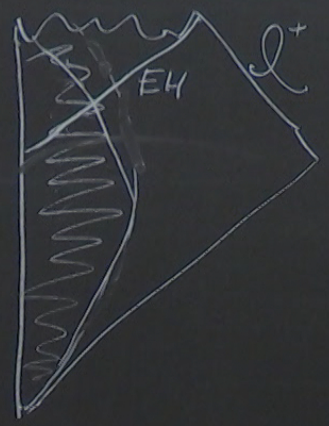

Apparent Horizons
Event horizon - the boundary of the causal past
future null infinity
BH is: $M - J^-(\mathcal{I}^+)$
EH

past of



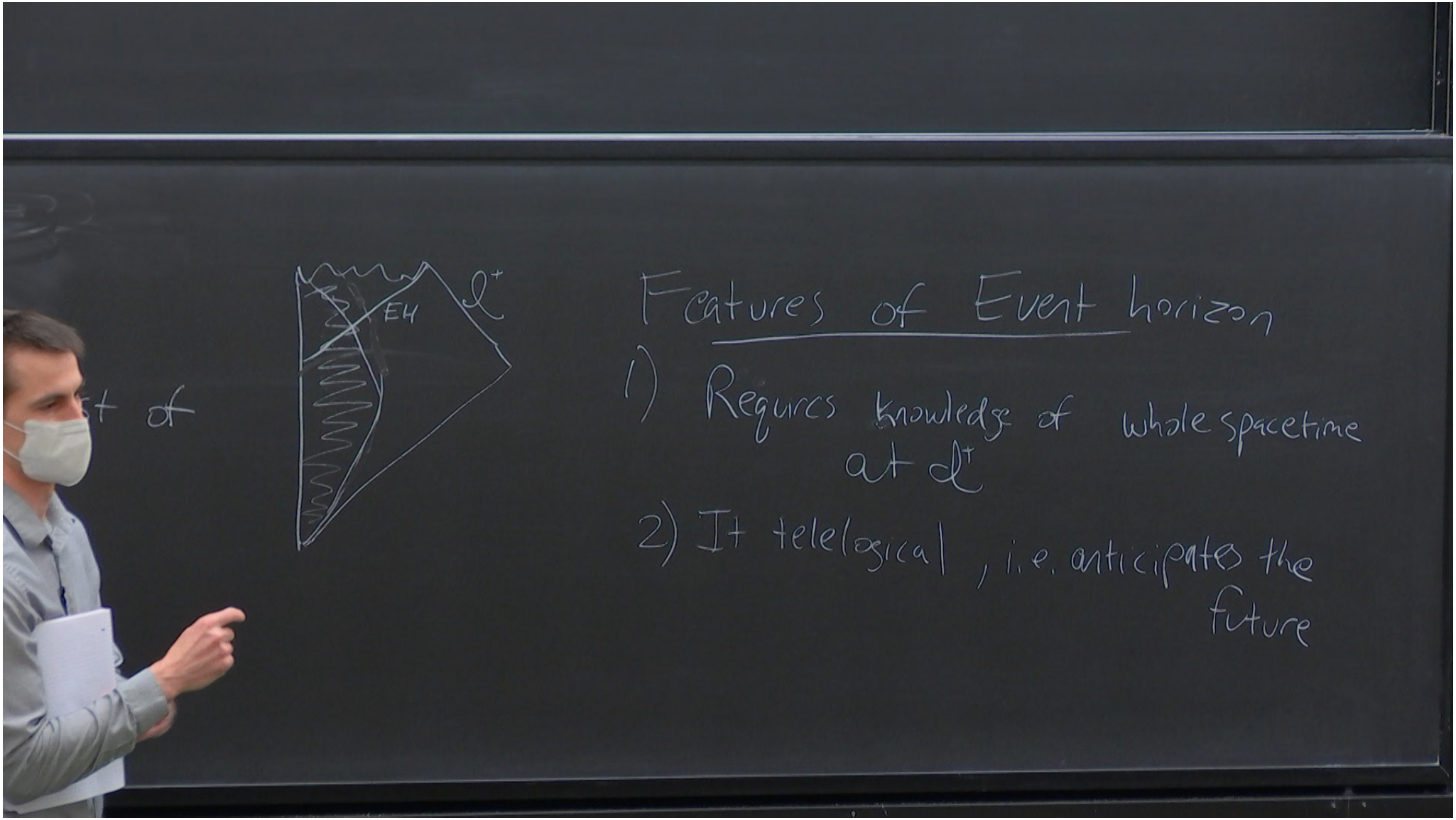

B

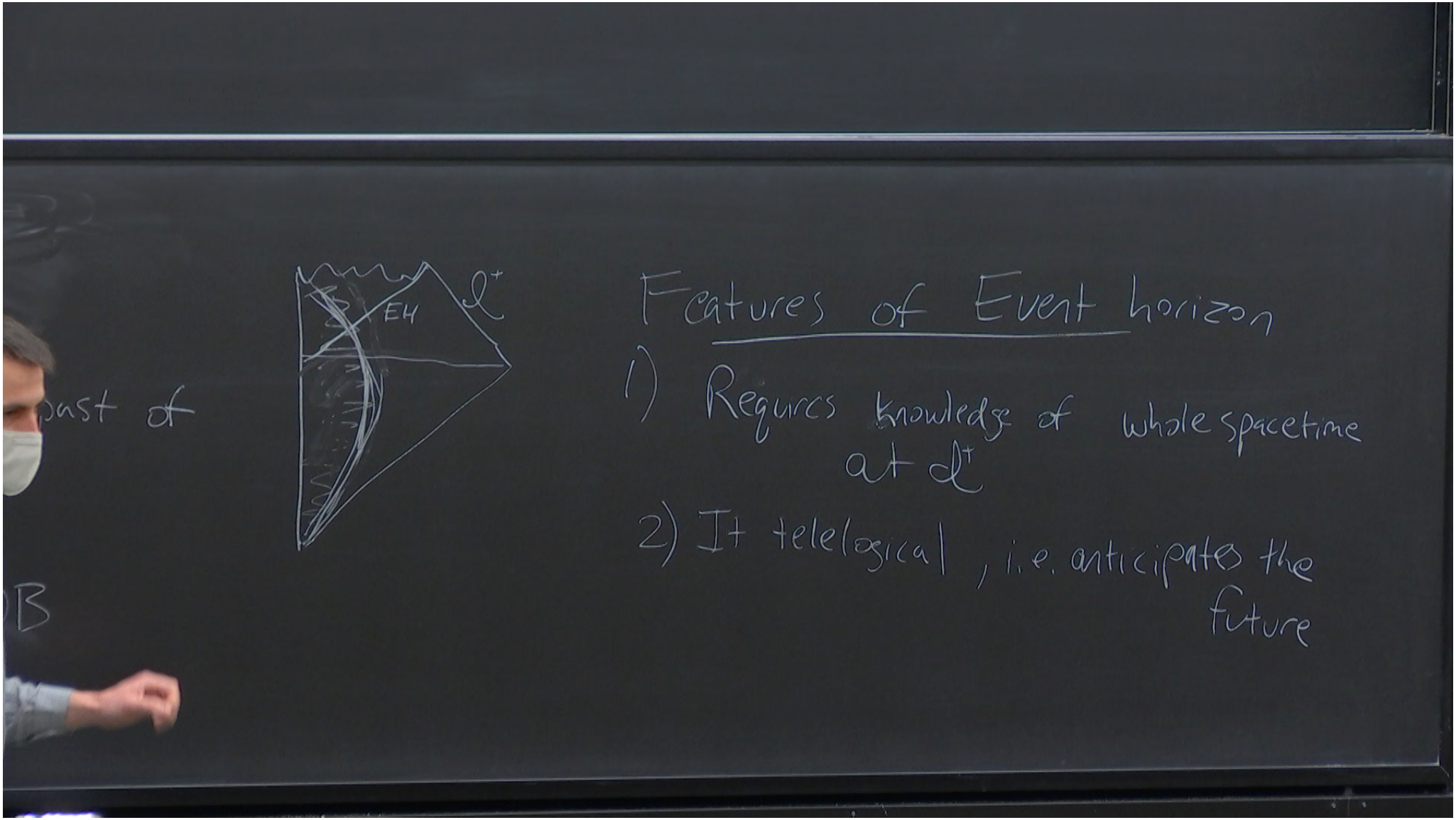
past of

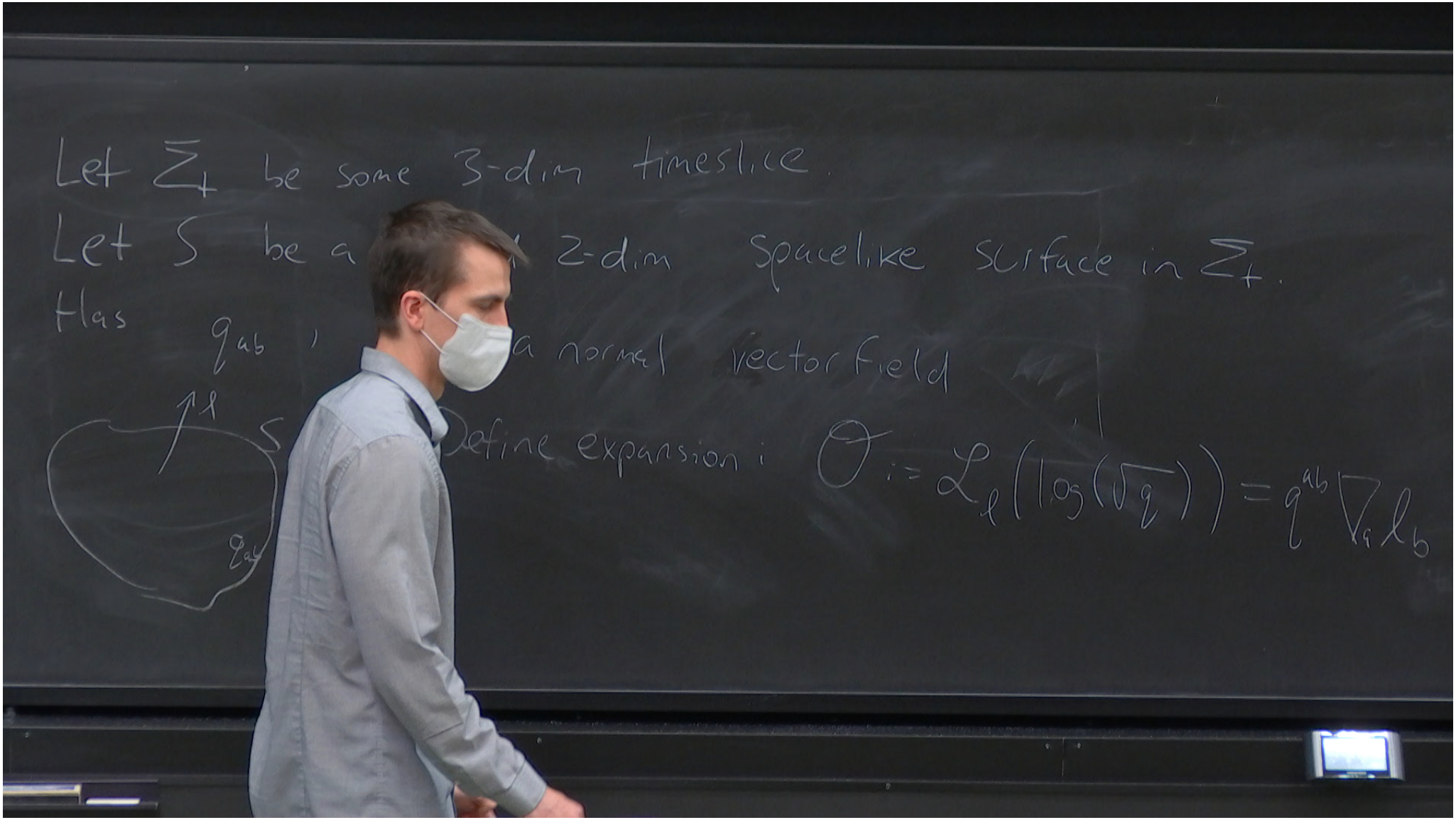


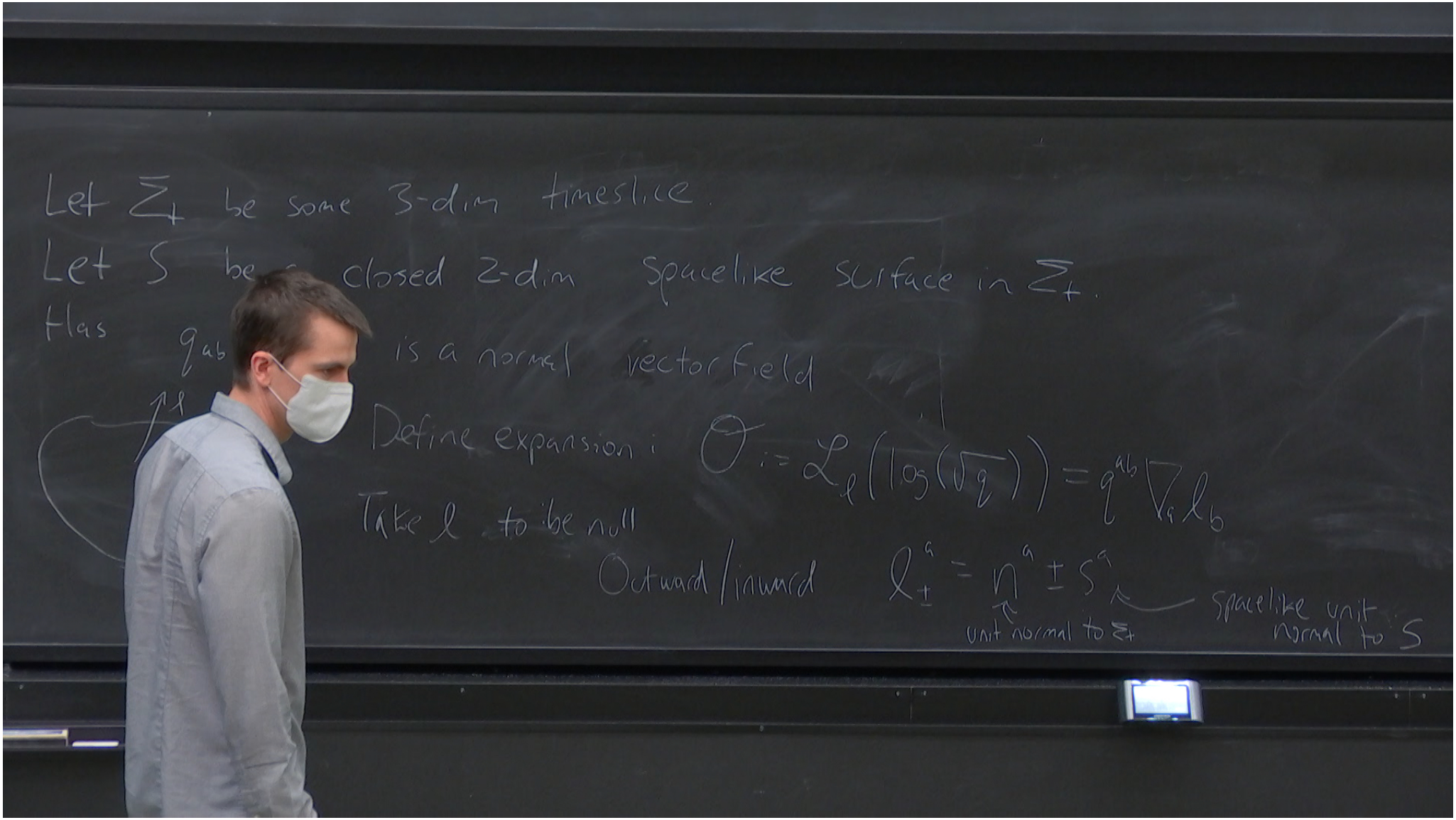

## Features of Event horizon

1) Requires knowledge of whole spacetime at $\mathcal{I}^+$


EH
$\mathcal{I}^+$
Features of Event horizon
1) Requires knowledge of whole spacetime at $\mathcal{I}^+$
2) It telelogical, i.e. anticipates the future


EH
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past of
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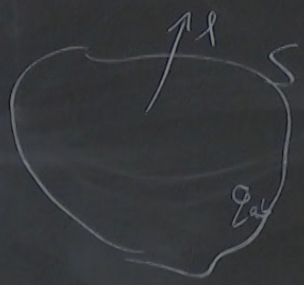

Let $\Sigma_t$ be some 3-dim timeslice.
Let $S$ be a 2-dim spacelike surface in $\Sigma_t$.
Has $q_{ab}$, a normal vector field
Define expansion: $\theta := \mathcal{L}_\ell(\log(\sqrt{q})) = q^{ab}\nabla_a \ell_b$


Let $\Sigma_t$ be some 3-dim timeslice.
Let $S$ be a closed 2-dim spacelike surface in $\Sigma_t$.
Has $q_{ab}$
is a normal vector field
Define expansion: $\theta := \mathcal{L}_\ell(\log(\sqrt{q})) = q^{ab}\nabla_a \ell_b$
Take $\ell$ to be null
Outward/inward $\ell^a_\pm = n^a \pm s^a$
unit normal to $\Sigma_t$
spacelike unit normal to $S$

Let $\bar{\Sigma}_t$ be some 3-dim timeslice.

Let $S$ be a closed 2-dim spacelike surface in $\bar{\Sigma}_t$.

Has $q_{ab}$, $\ell$ is a normal vector field



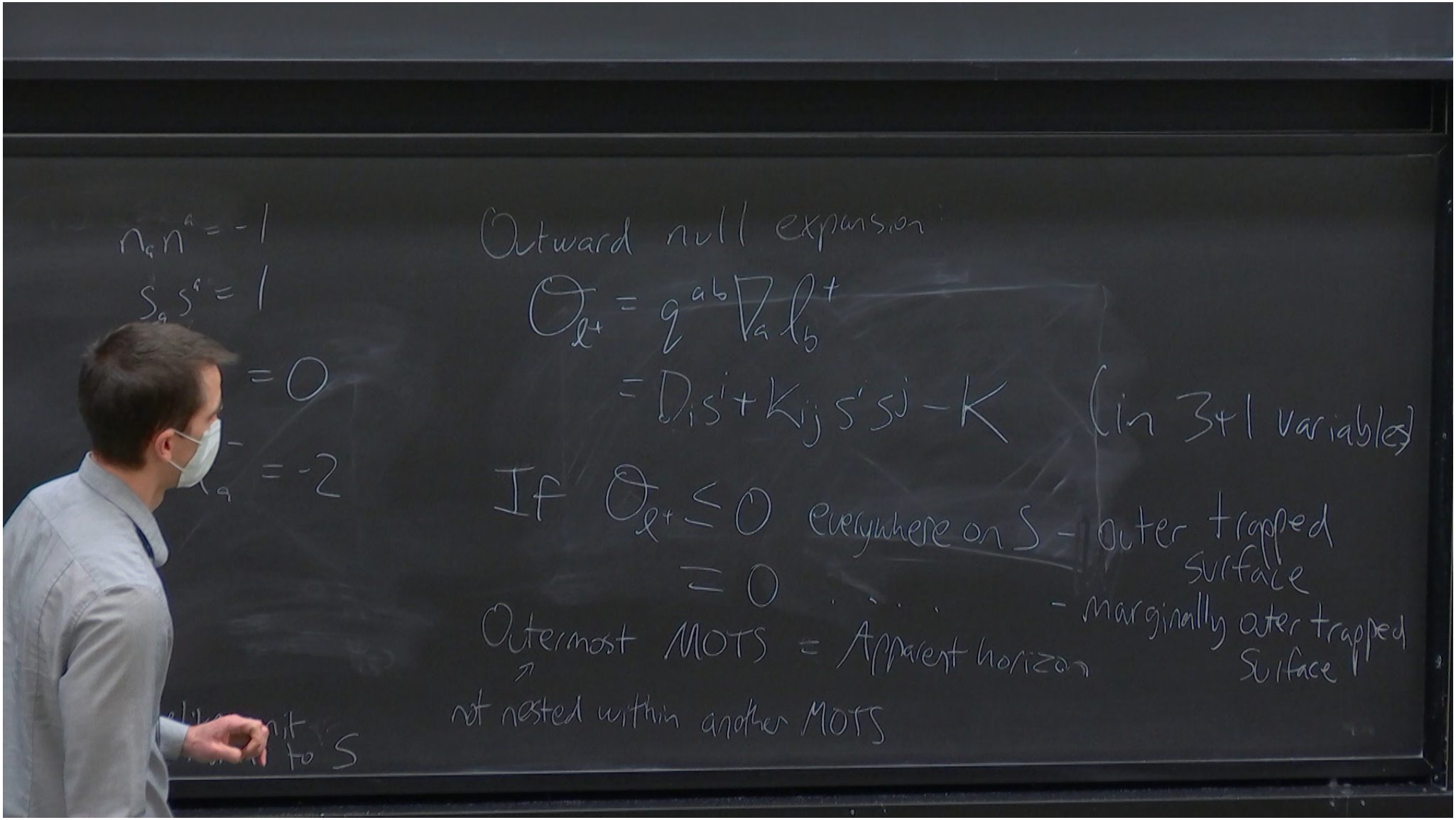

Define expansion: $\Theta := \mathcal{L}_\ell(\log(\sqrt{q}))$

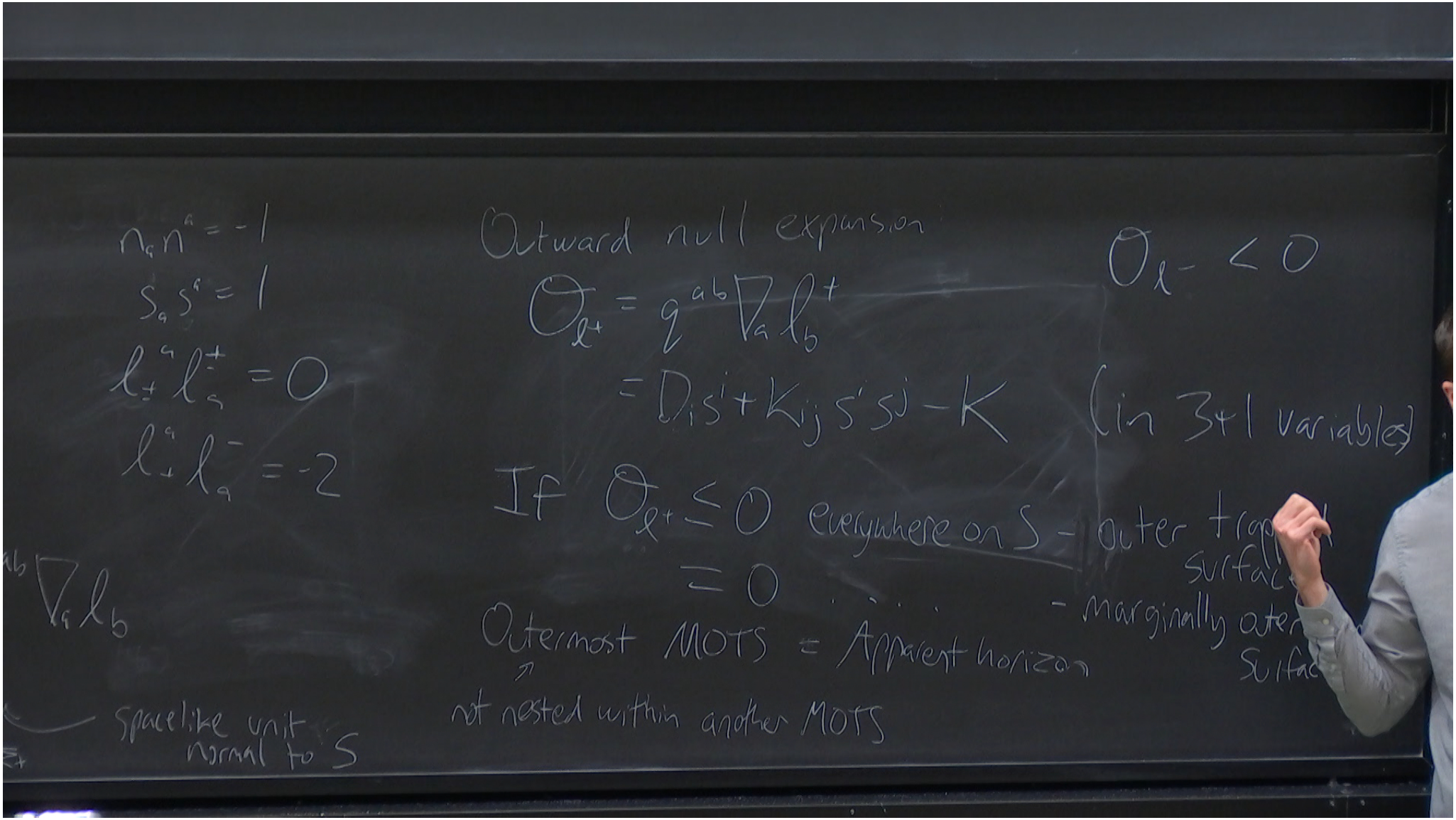
Take $\ell$ to be null

Outward/inward $\ell^a_\pm = n^a$

unit normal

unit ... al to $S$


$n_a n^a = -1$
$s_a s^a = 1$
$= 0$
$= -2$
Outward null expansion
$\Theta_{\ell^+} = q^{ab} \nabla_a \ell_b^+$
$= D_i s^i + K_{ij} s^i s^j - K$ (in 3+1 variables)
If $\Theta_{\ell^+} \leq 0$ everywhere on $S$ - outer trapped surface
$= 0$ ... - marginally outer trapped surface
Outermost MOTS = Apparent horizon
not nested within another MOTS
to $S$


$n_a n^a = -1$
$s_a s^a = 1$
$l_+^a l_{-a} = 0$
$l_+^a l_{-a} = -2$
Outward null expansion
$\theta_{l^+} = q^{ab} \nabla_a l_b^+$
$= D_i s^i + K_{ij} s^i s^j - K$
$\theta_{l^-} < 0$
(in 3+1 variables)
If $\theta_{l^+} \leq 0$ everywhere on $S$ - outer trapped surface
$= 0$ - marginally outer
Outermost MOTS = Apparent horizon
not nested within another MOTS
$\nabla_a l_b$
spacelike unit normal to $S$

$n_a n^a = -1$

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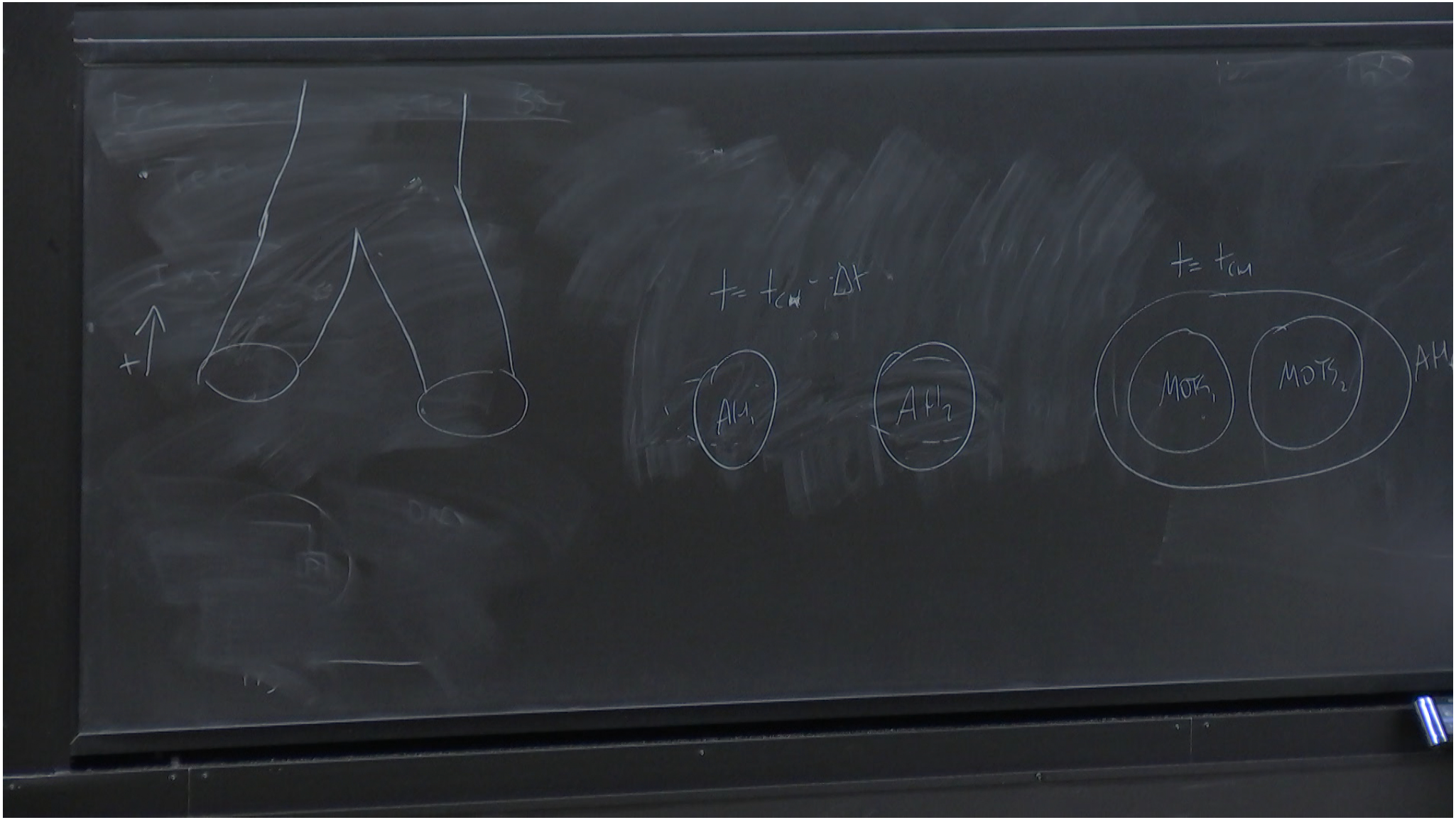
$= 0$ . . . - marginally outer trapped surface

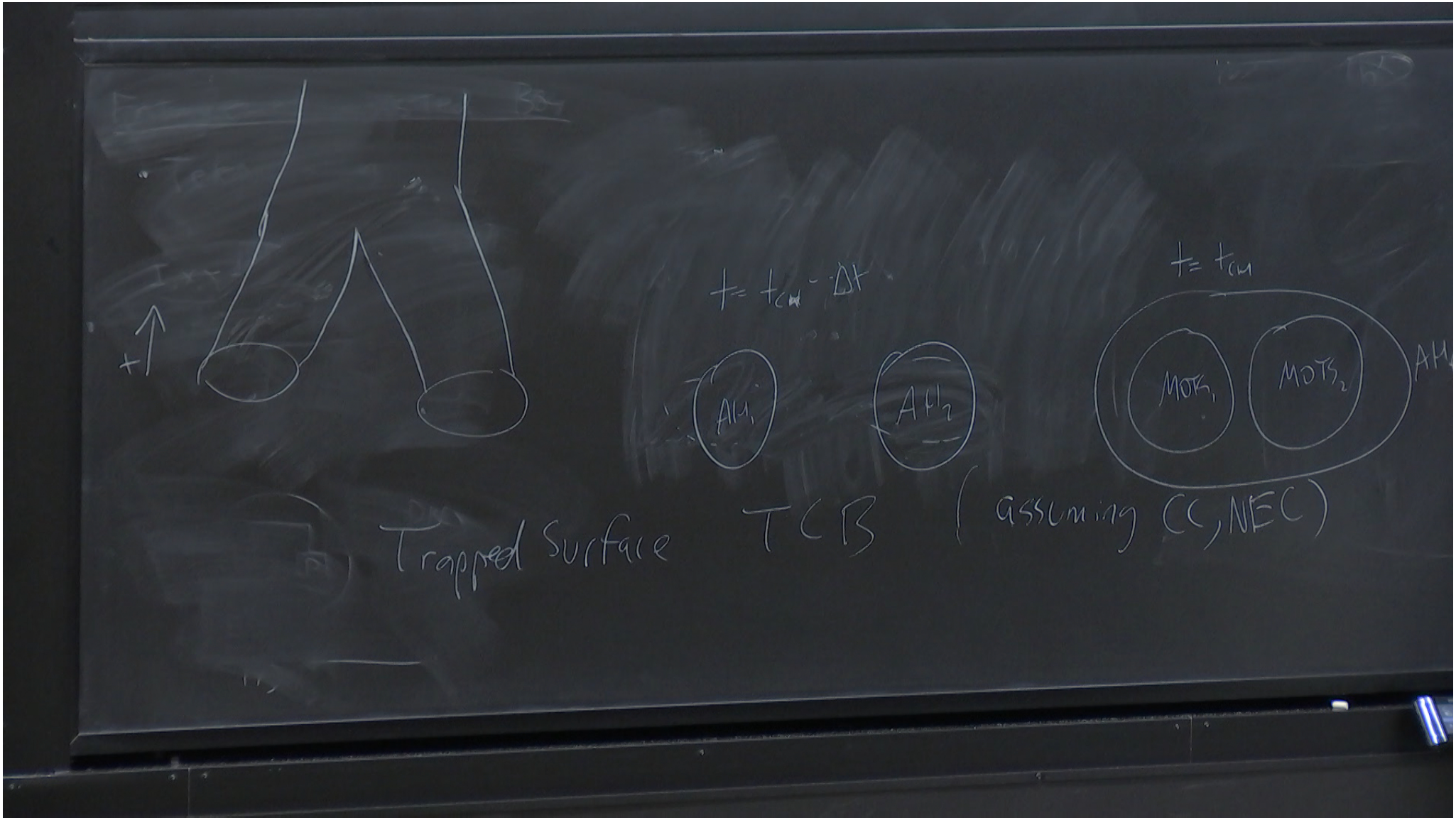
Outermost MOTS = Apparent horizon

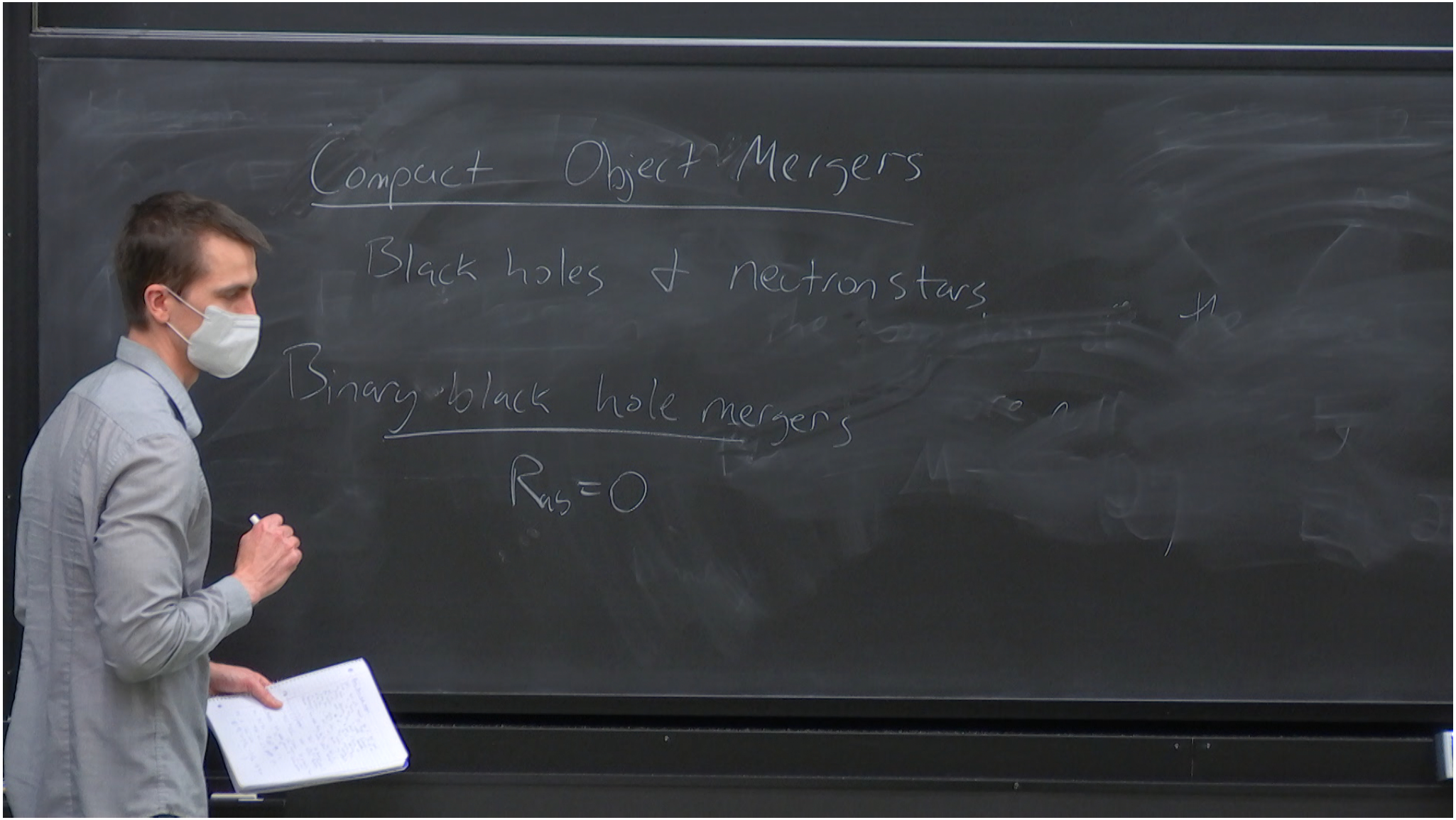
↗ not nested within another MOTS

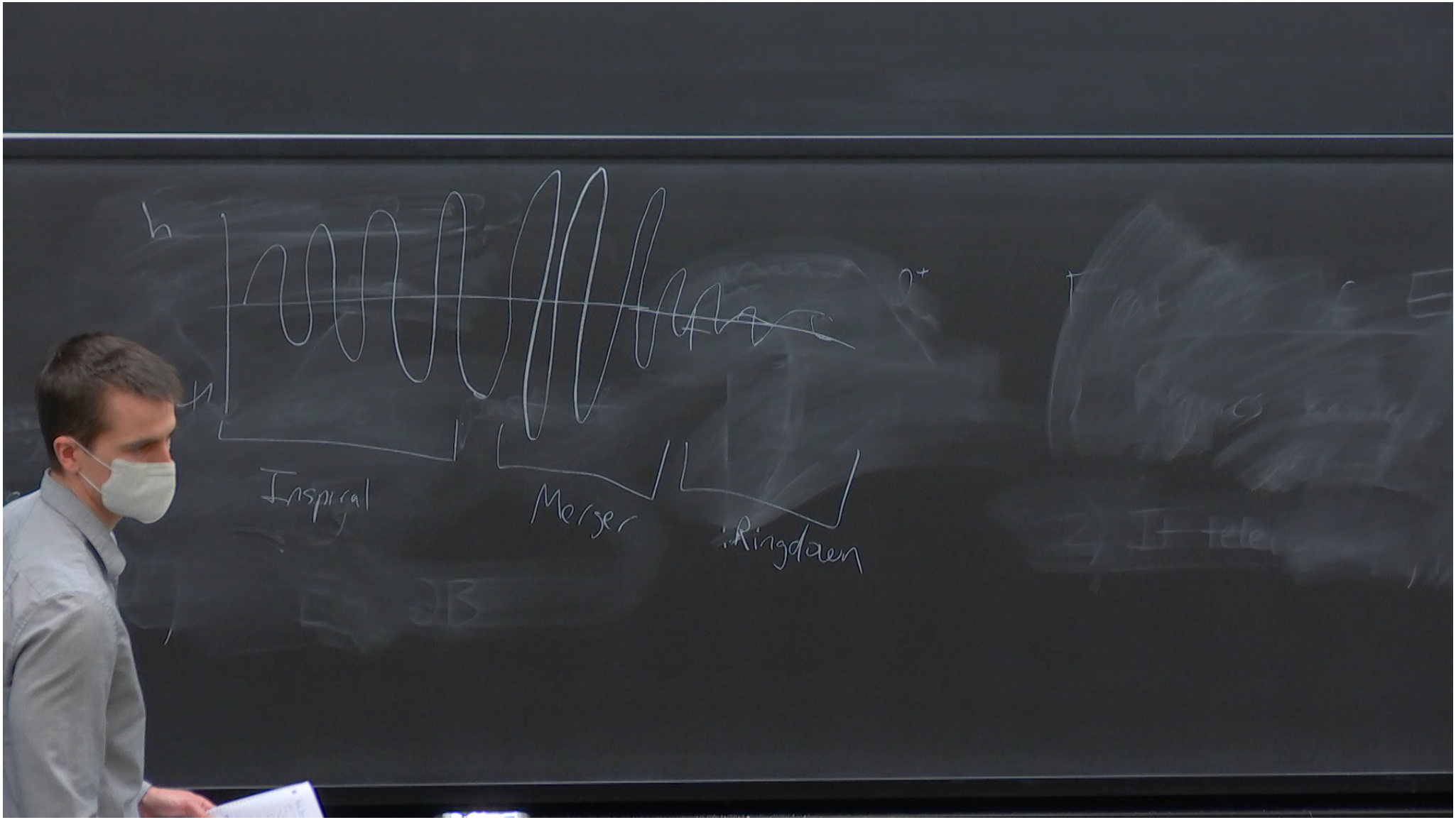
$q^{ab} \nabla_a l_b$

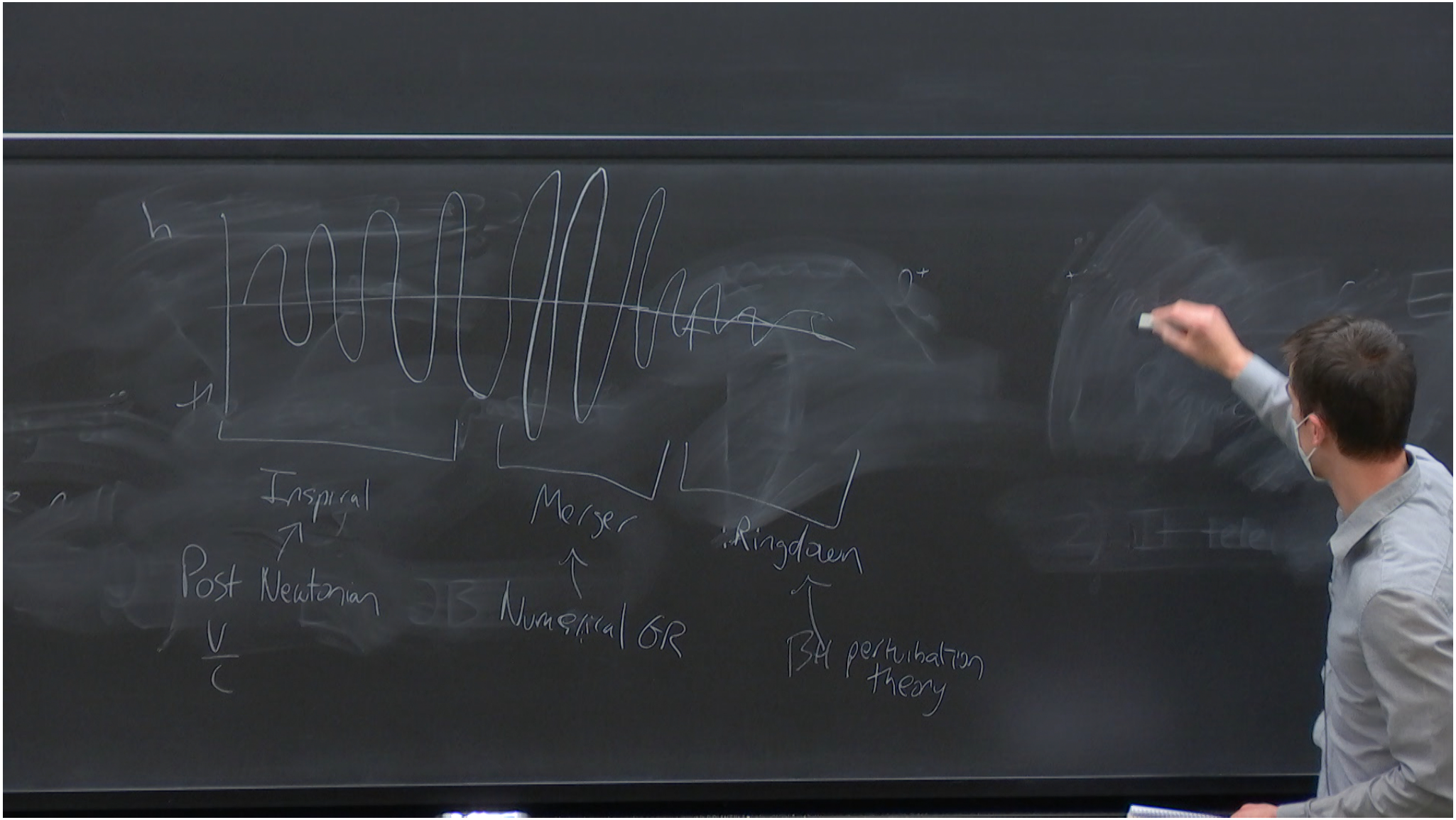
$s_a$ spacelike unit normal to $S$

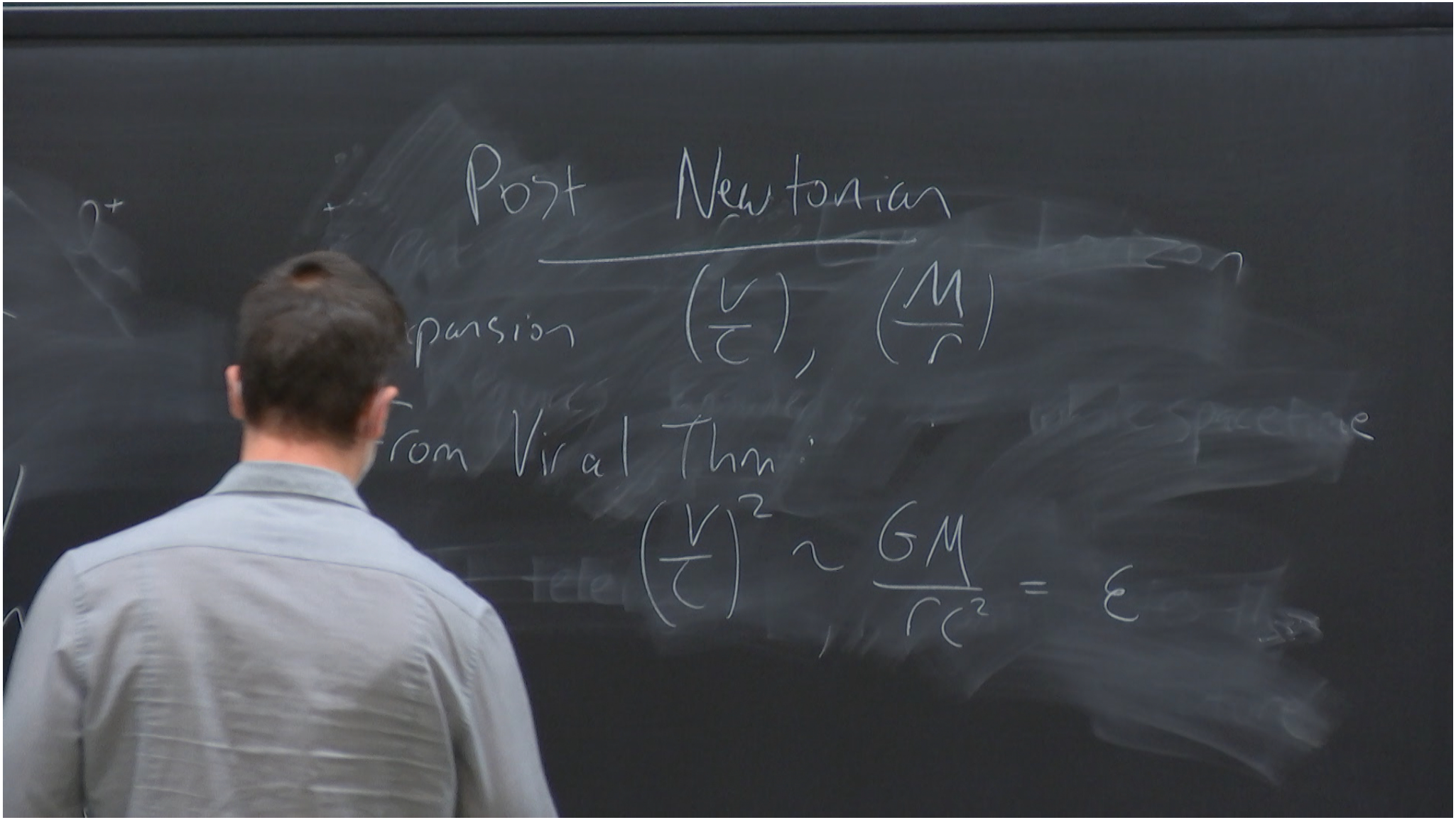

+
$t = t_{CM} - \Delta t$
$AH_1$
$AH_2$
$t = t_{CM}$
$MOTs_1$
$MOTs_2$
$AH_1$

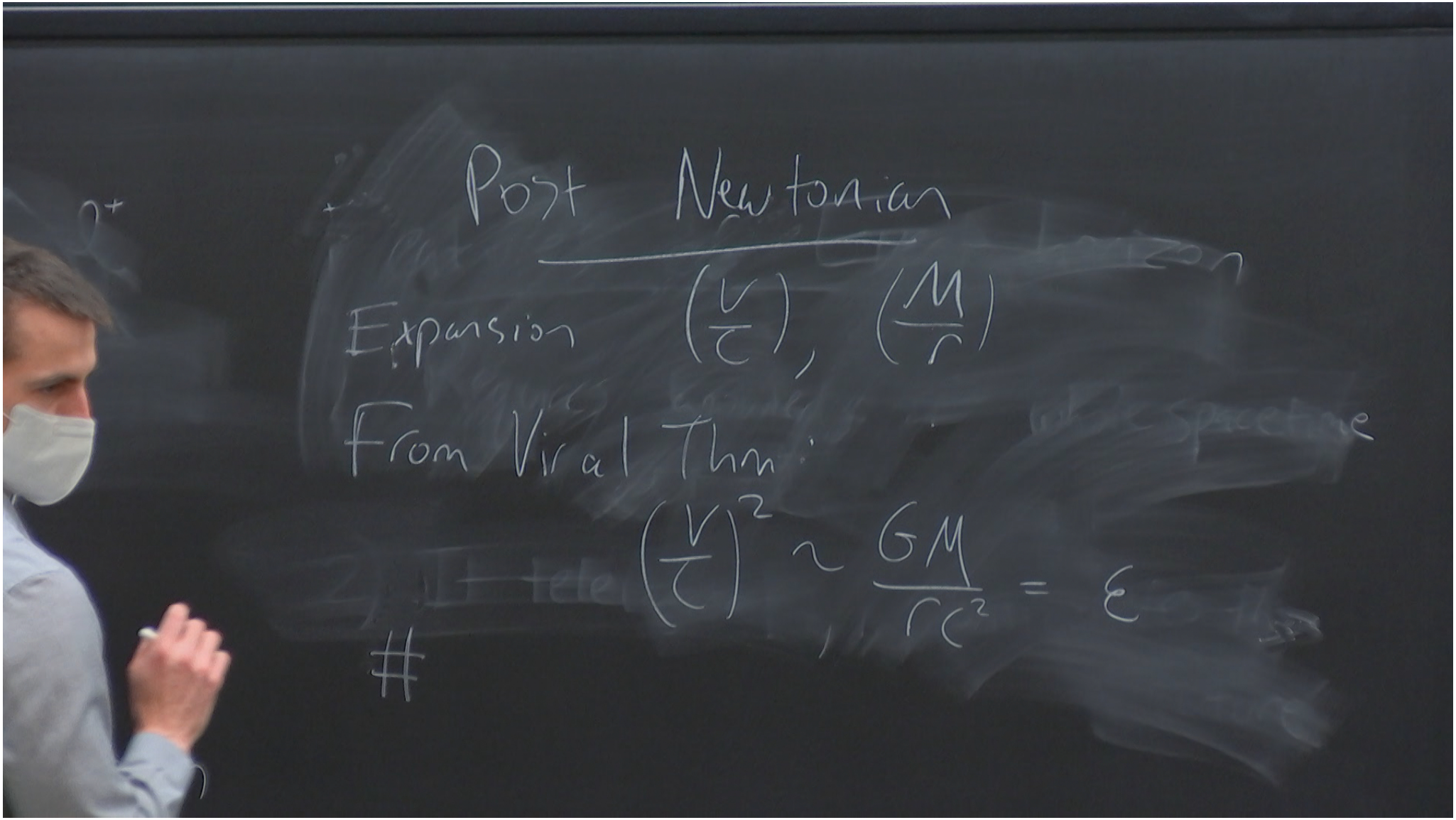

$t = t_{cm} - \Delta t$
$t = t_{cm}$
$AH_1$
$AH_2$
MOTS$_1$
MOTS$_2$
Trapped Surface TCB (assuming CC, NEC)

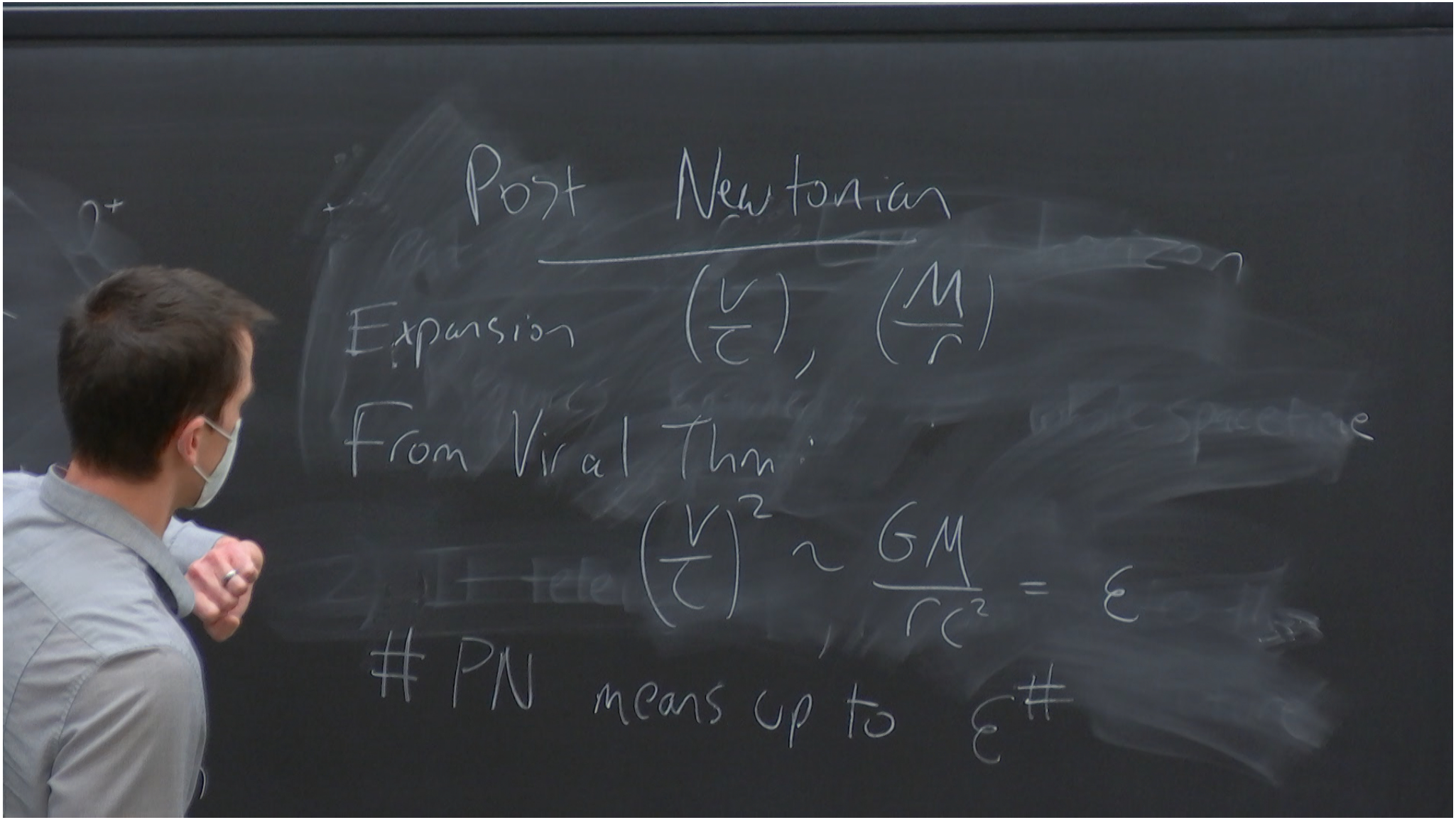

Compact Object Mergers
Black holes & neutron stars
Binary black hole mergers
$R_{Sch} = 0$

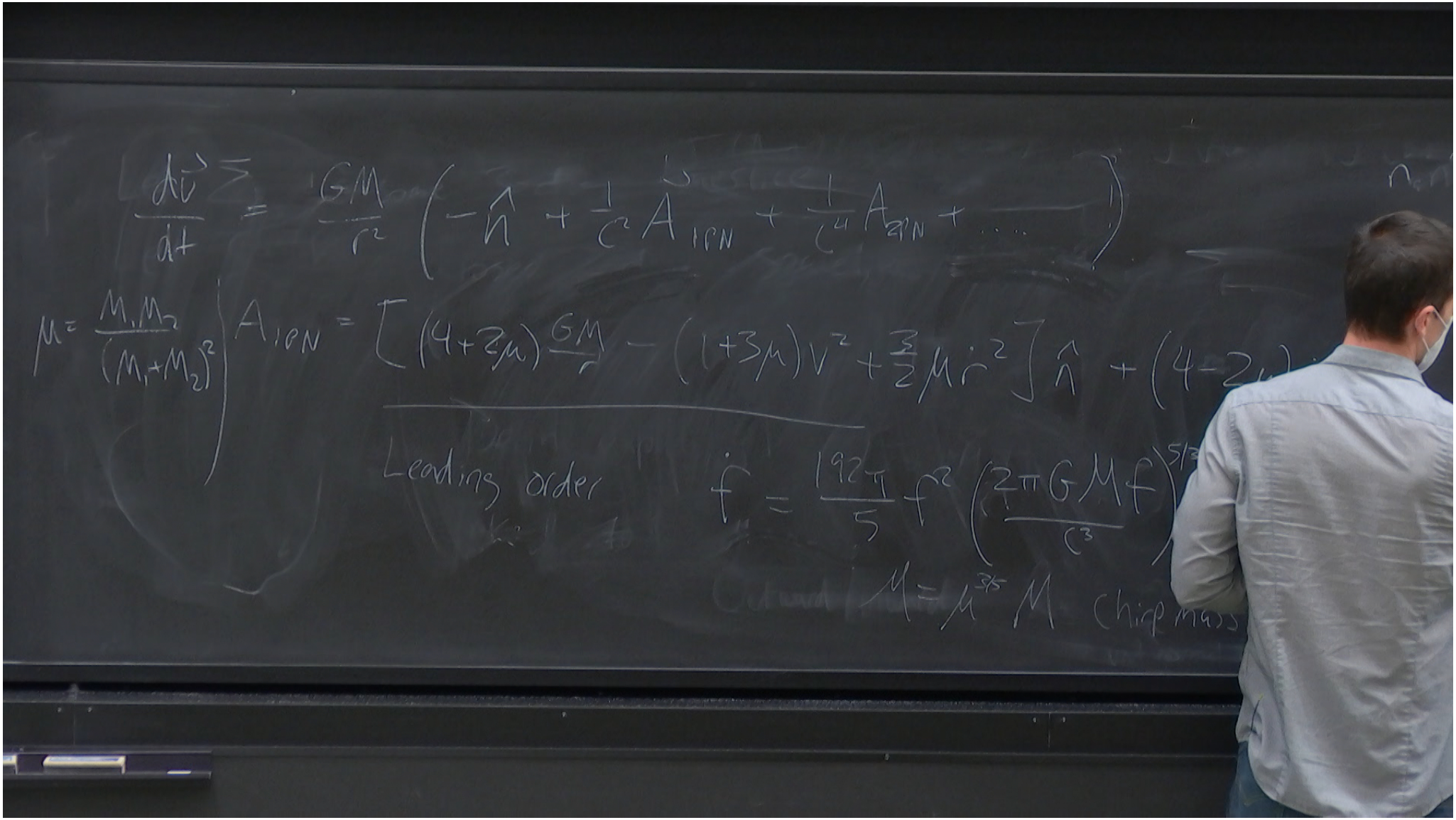

h
Inspiral
Merger
Ringdown

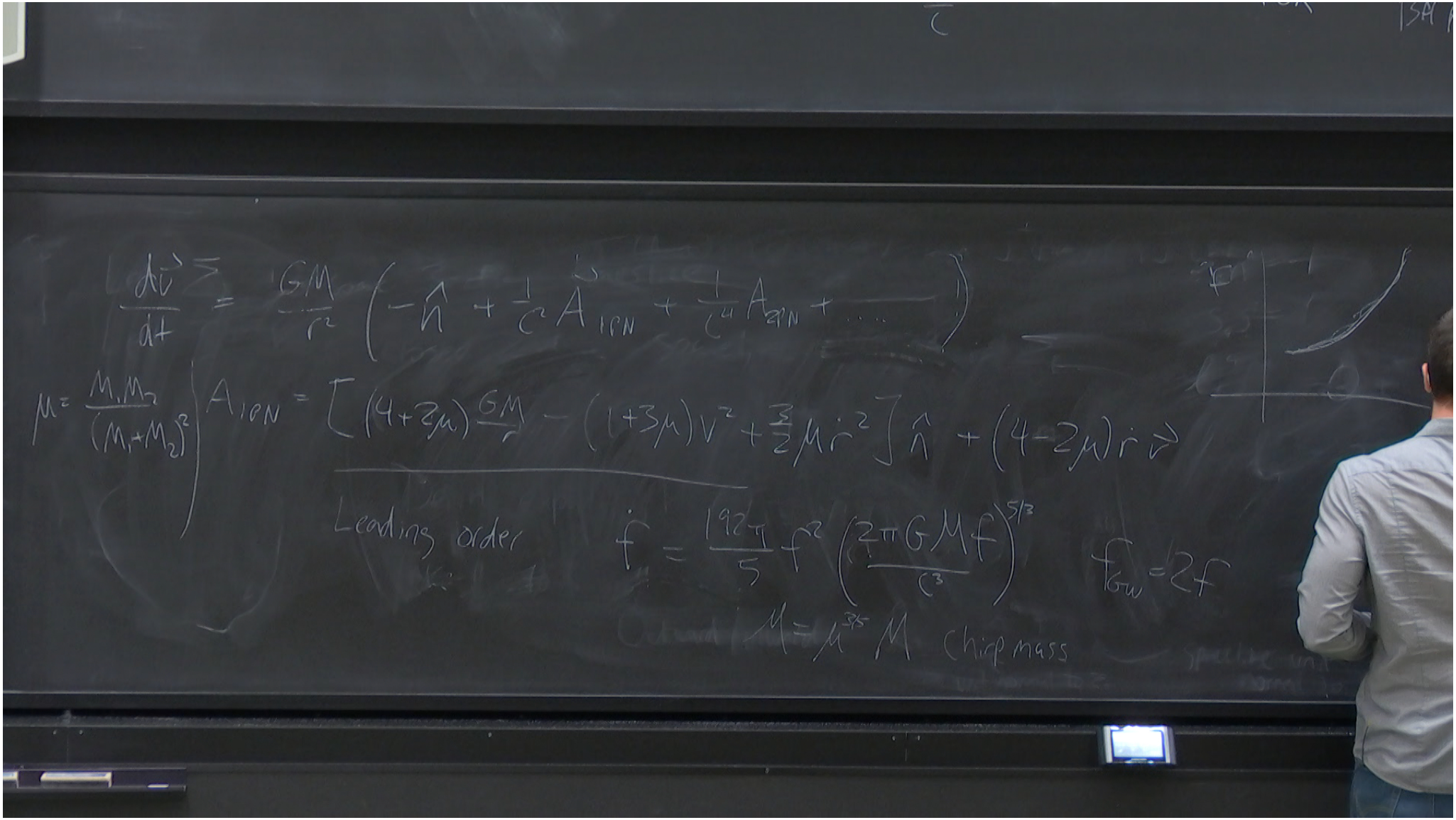

h
Inspiral
Merger
Ringdown
Post Newtonian
Numerical GR
BH perturbation theory

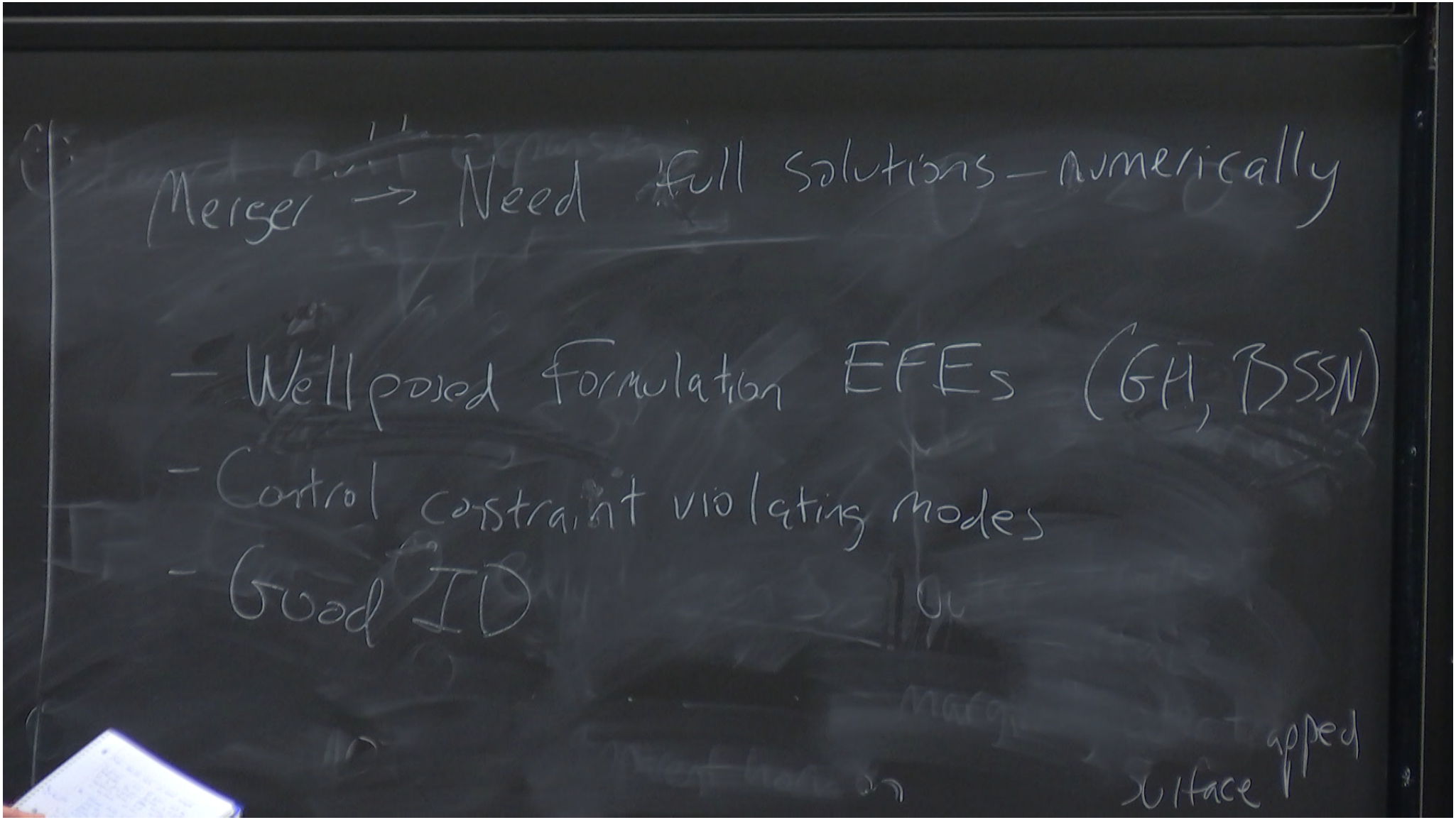

Post Newtonian
pansion $\left(\frac{v}{c}\right), \left(\frac{M}{r}\right)$
rom Viral Thm:
$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{rc^2} = \epsilon$


Post Newtonian
Expansion $\left(\frac{v}{c}\right)$, $\left(\frac{M}{r}\right)$
From Viral Thm:
$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{rc^2} = \varepsilon$
#


Post Newtonian
Expansion $\left(\frac{v}{c}\right)$, $\left(\frac{M}{r}\right)$
From Viral Thm:
$\left(\frac{v}{c}\right)^2 \sim \frac{GM}{rc^2} = \varepsilon$
# PN means up to $\varepsilon^{\#}$

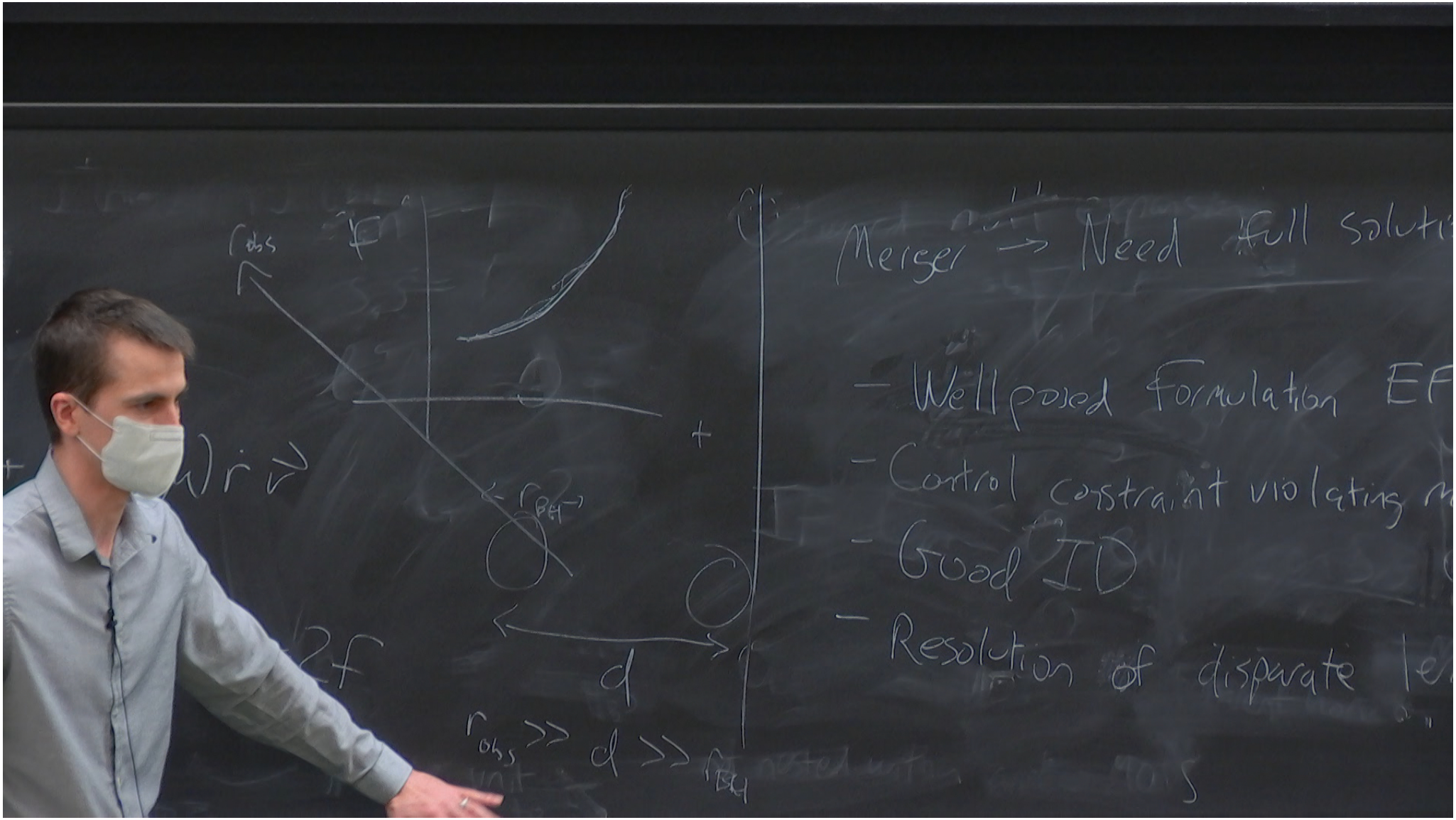

$\frac{d\vec{v}}{dt} = \frac{GM}{r^2}\left(-\hat{n} + \frac{1}{c^2}\vec{A}_{1PN} + \frac{1}{c^4}A_{2PN} + \dots\right)$
$\mu = \frac{M_1 M_2}{(M_1+M_2)^2}$
$A_{1PN} = \left[(4+2\mu)\frac{GM}{r} - (1+3\mu)v^2 + \frac{3}{2}\mu\dot{r}^2\right]\hat{n} + (4-2\mu)$
Leading order
$\dot{f} = \frac{192\pi}{5} f^2 \left(\frac{2\pi G M f}{c^3}\right)^{5/3}$
$\mathcal{M} = \mu^{3/5} M$ chirp mass

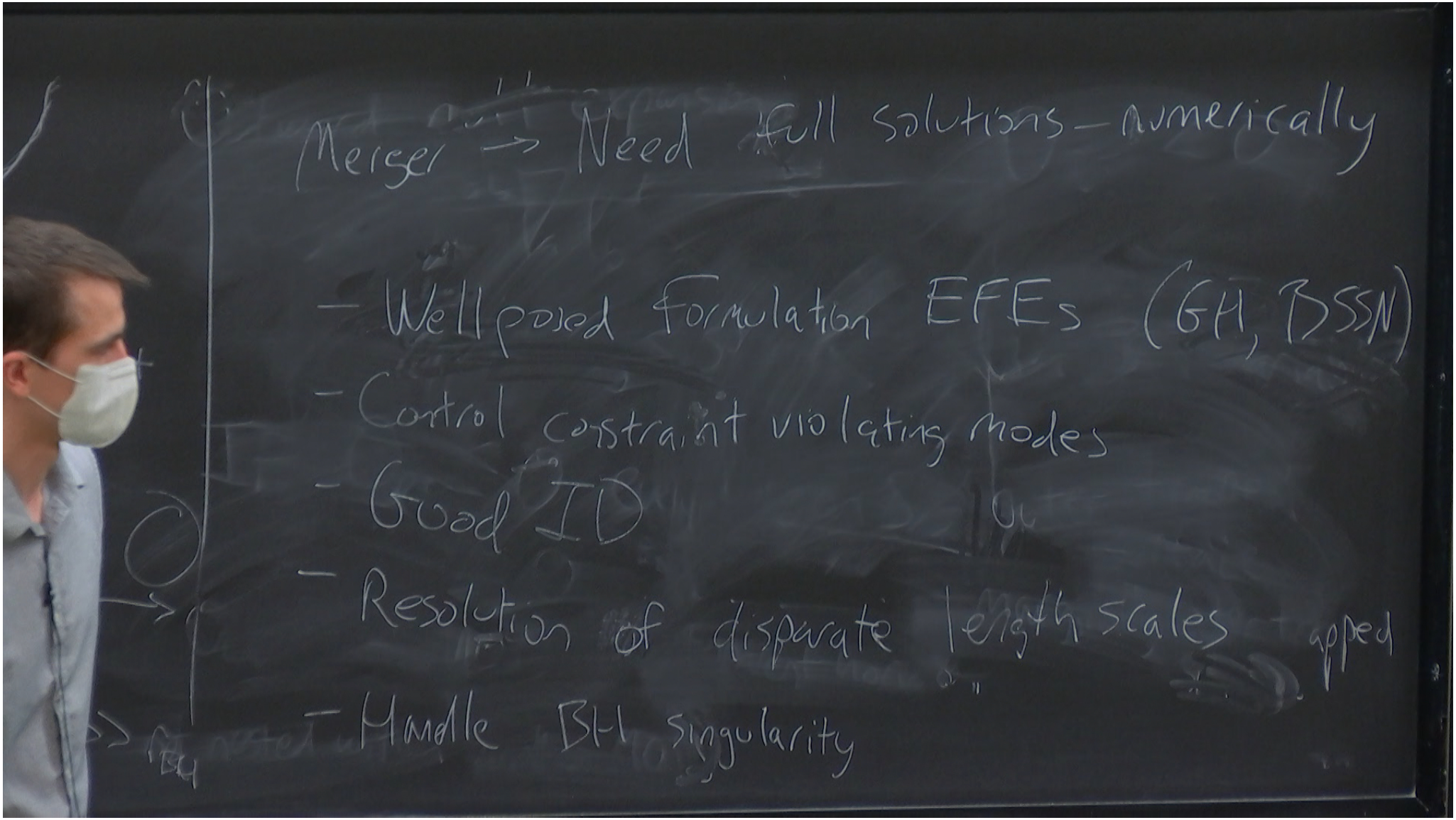

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Leading order
$\dot{f} = \frac{192\pi}{5} f^2 \left(\frac{2\pi G\mathcal{M} f}{c^3}\right)^{5/3}$
$f_{GW} = 2f$
$\mathcal{M} = \mu^{3/5} M$ chirp mass

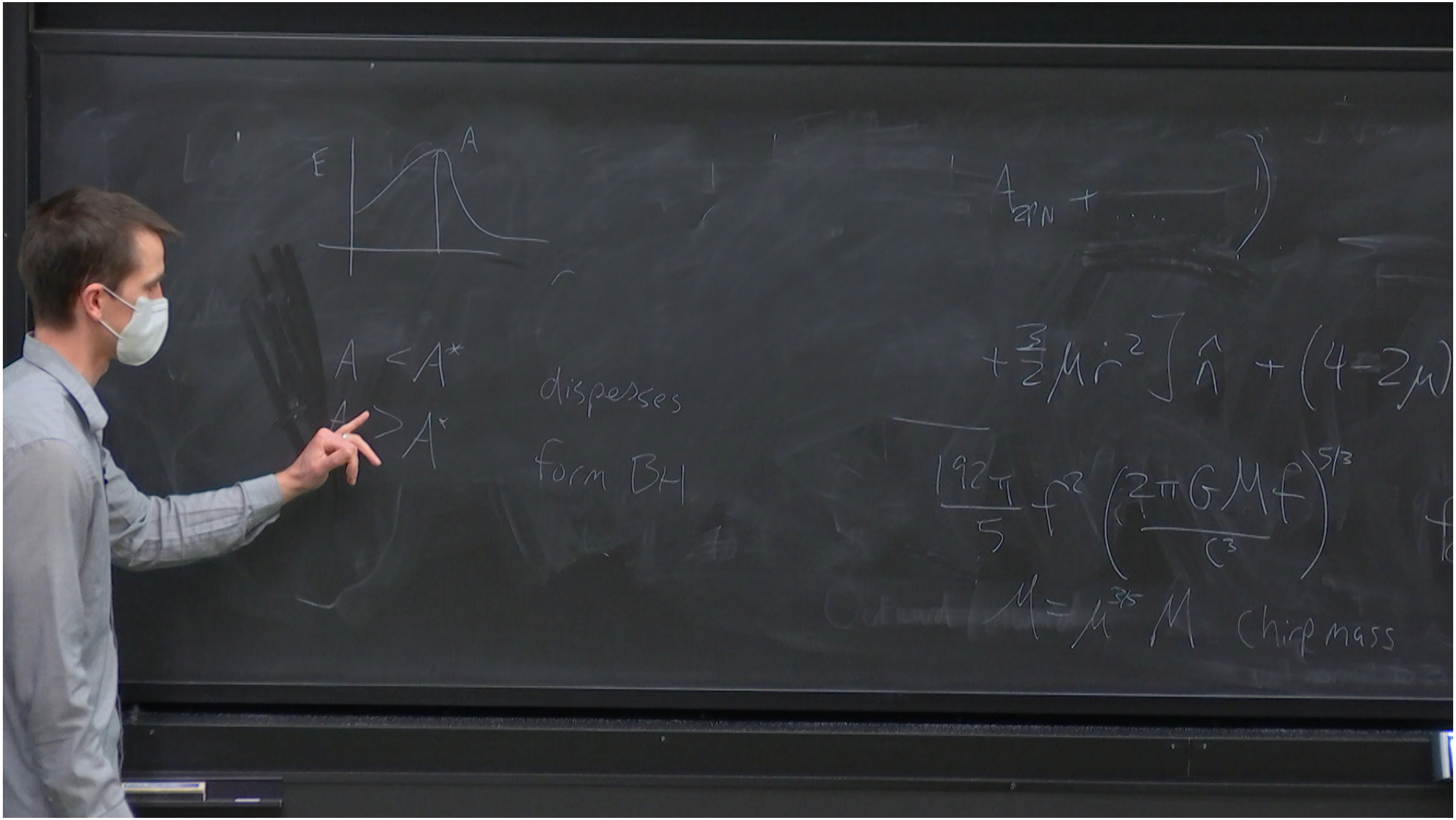

Merger → Need full solutions – numerically
– Well posed formulation EFEs (GH, BSSN)
– Control constraint violating modes
– Good ID

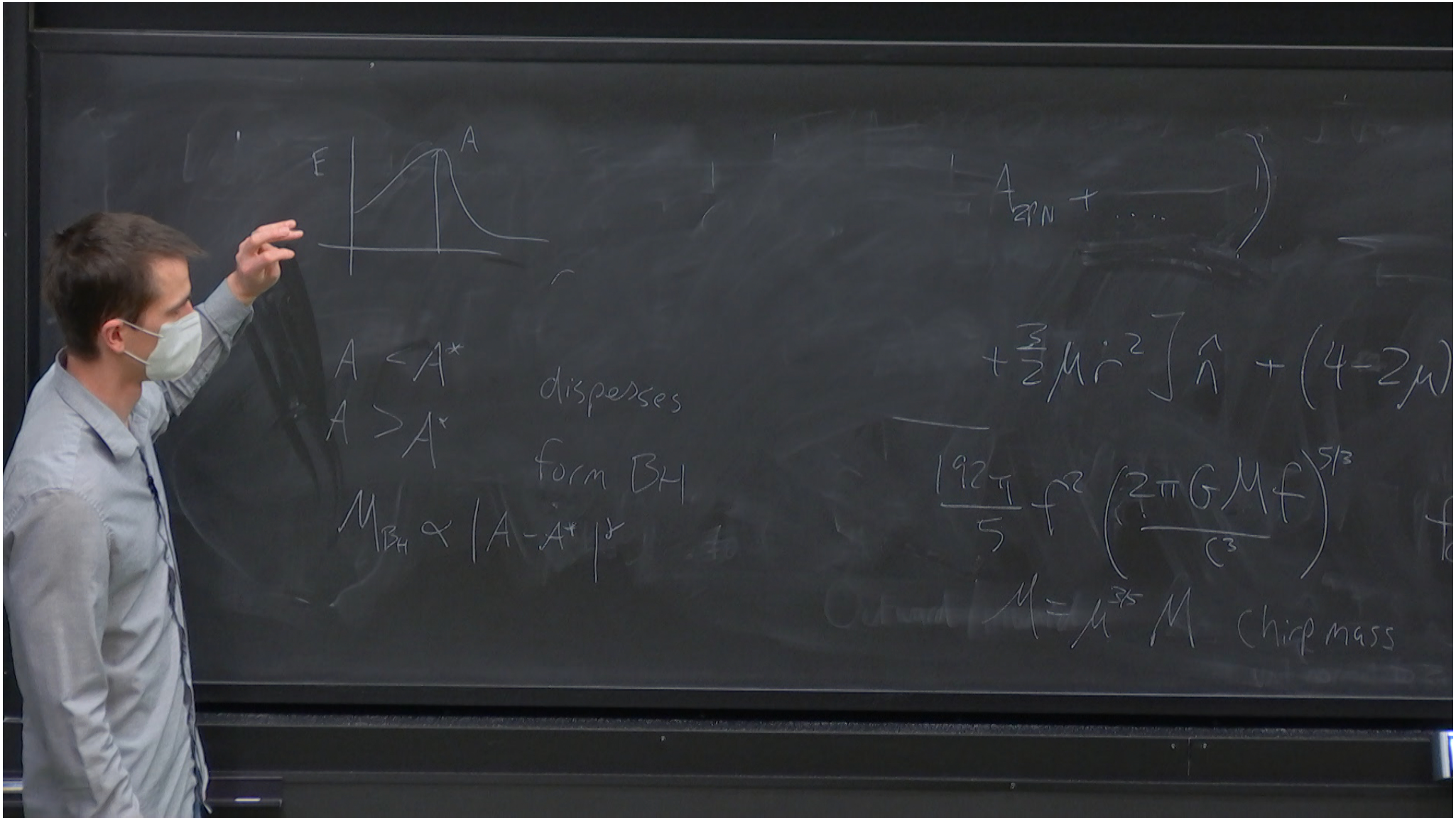
Merger → Need full solutions – numerically

- Wellposed formulation EFEs (GH, BSSN)
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- Good ID
- Resolution of disparate length scales (apped


Merger → Need full solutio
- Well posed formulation EF
- Control constraint violating m
- Good ID
- Resolution of disparate le
$r_{obs}$
$r_{BH}$
d
$r_{obs} >> d >> r_{BH}$


Merger → Need full solutions – numerically
– Well posed formulation EFEs (GH, BSSN)
– Control constraint violating modes
– Good ID
– Resolution of disparate length scales
– Handle BH singularity


E
A
A < A*
A > A*
disperses
form BH
+ ... )
+ 3/2 μ r² ∫ n̂ + (4 − 2μ)
192π/5 f² (2πGMf/c³)^5/3
M = μ^3/5 M
Chirp mass


E
A
$A < A^*$ disperses
$A > A^*$ form BH
$M_{BH} \propto |A - A^*|^\gamma$
$\frac{192}{5} f^2 \left(\frac{2\pi G \mathcal{M} f}{c^3}\right)^{5/3}$
$\mathcal{M} = \mu^{3/5} M$ Chirp mass