Title: Strong Gravity

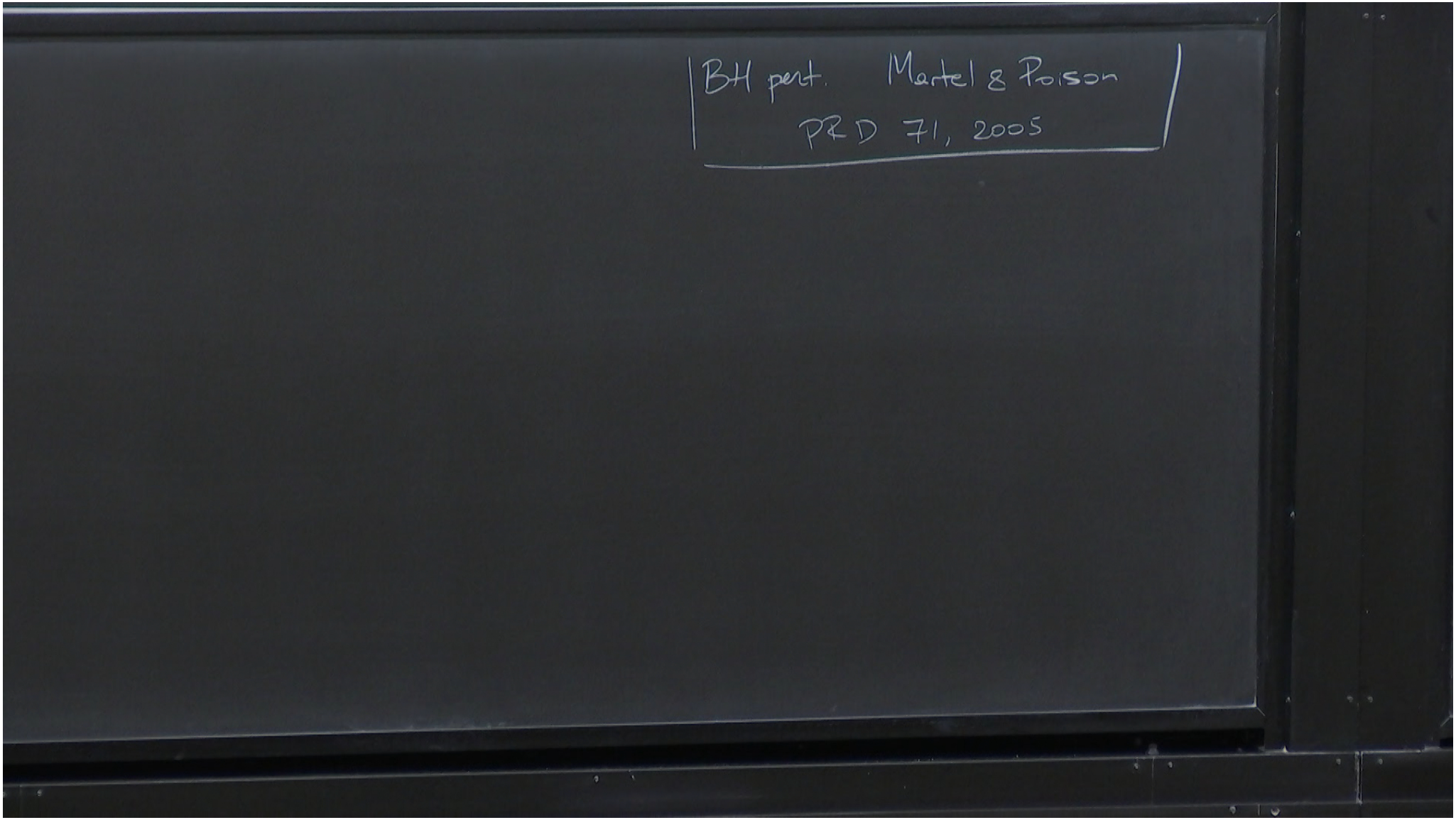
Speakers: William East, Giuseppe Sellaroli

Collection: Strong Gravity (2021/2022)

Date: May 04, 2022 - 10:15 AM

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Abstract: This course will introduce some advanced topics in general relativity related to describing gravity in the strong field and dynamical regime. Topics covered include properties of spinning black holes, black hole thermodynamics and energy extraction, how to define horizons in a dynamical setting, formulations of the Einstein equations as constraint and evolution equations, and gravitational waves and how they are sourced.


BH pert. Martel & Poisson
PRD 71, 2005

Vector S.H : $\begin{cases} Y^{lm}_A = \partial_A Y_{lm} \\ X^{lm}_{B\cdot} = \varepsilon^B_A \partial_B Y_{lm} \end{cases}$

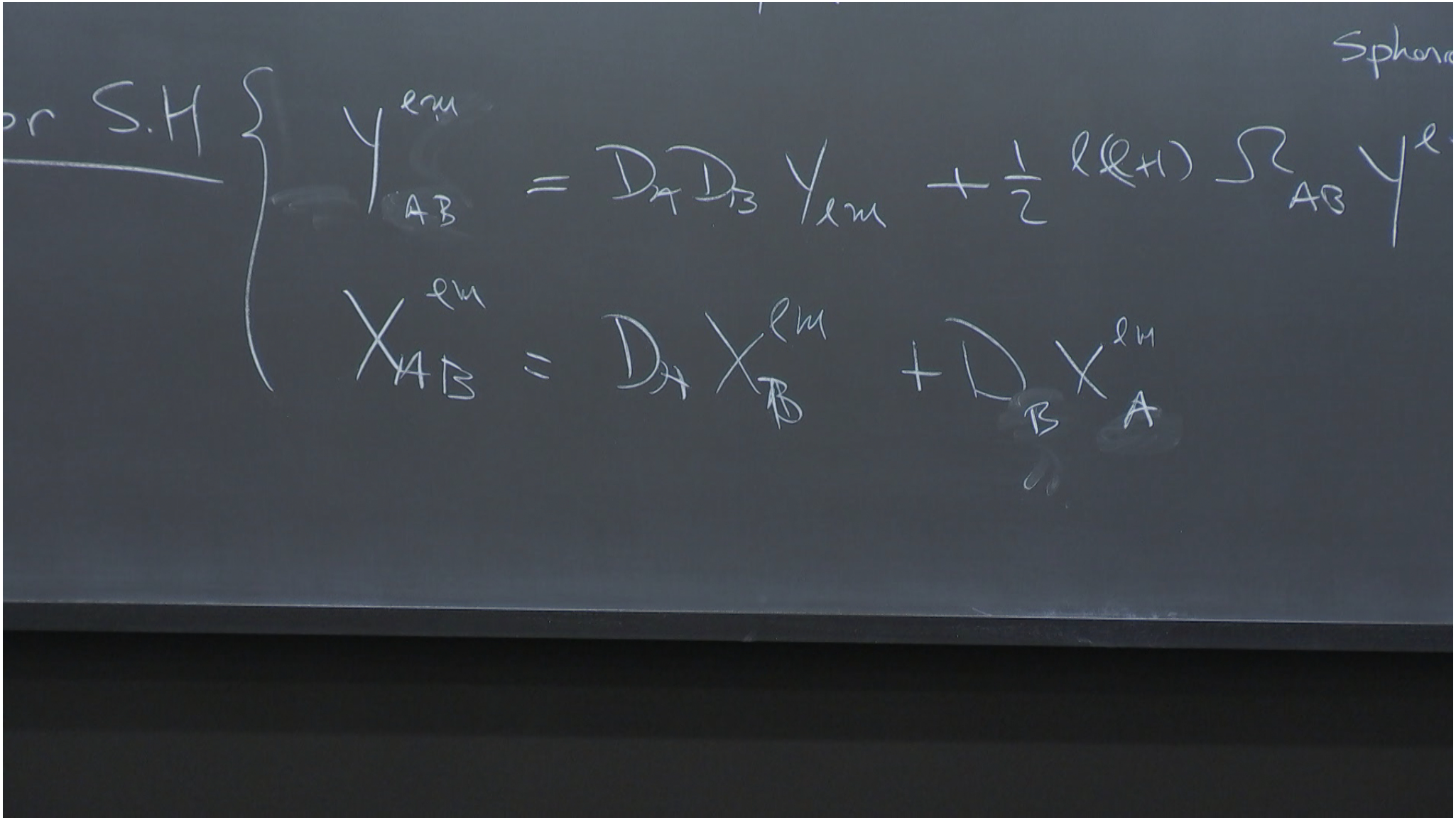
ε_{AB} = L.C assoc

Sphere metric

Tensor S.H $\begin{cases} Y^{lm}_{AB} = D_A D_B Y_{lm} + \frac{1}{2} l(l+1) \Omega_{AB} Y^{lm} \\ X^{lm}_{AB} = D_A X^{lm}_B + D_A X^{lm}_B \end{cases}$

BH pert

F


Spher
or S.H
$Y^{\ell m}_{AB} = D_A D_B Y_{\ell m} + \frac{1}{2} \ell(\ell+1) \Omega_{AB} Y^{\ell}$
$X^{\ell m}_{AB} = D_A X^{\ell m}_{B} + D_B X^{\ell m}_{A}$

$$P_{ab} = \sum h_{ab}^{\ell m} Y_{\ell m}$$

$$P_{aA} = \sum J_a^{\ell m} Y_A^{\ell m} + \sum h_a^{\ell m} X_A^{\ell m}$$

$$P_{AB} = r^2\left(\sum K^{\ell m} \Omega_{AB} Y^{\ell m} + G^{\ell m} Y_{AB}^{\ell m} + h_2^{\ell m} X_{AB}^{\ell m}\right)$$

Regge Wheeler gauge $\quad J_a^{\ell m} = 0 = G^{\ell m} \qquad h_2^{\ell m} = 0$

Pert Eqns $\rightarrow$ 2 eqns. for 2 "Master Variables" $\{\Psi_{even}, \Psi_{odd}\}$

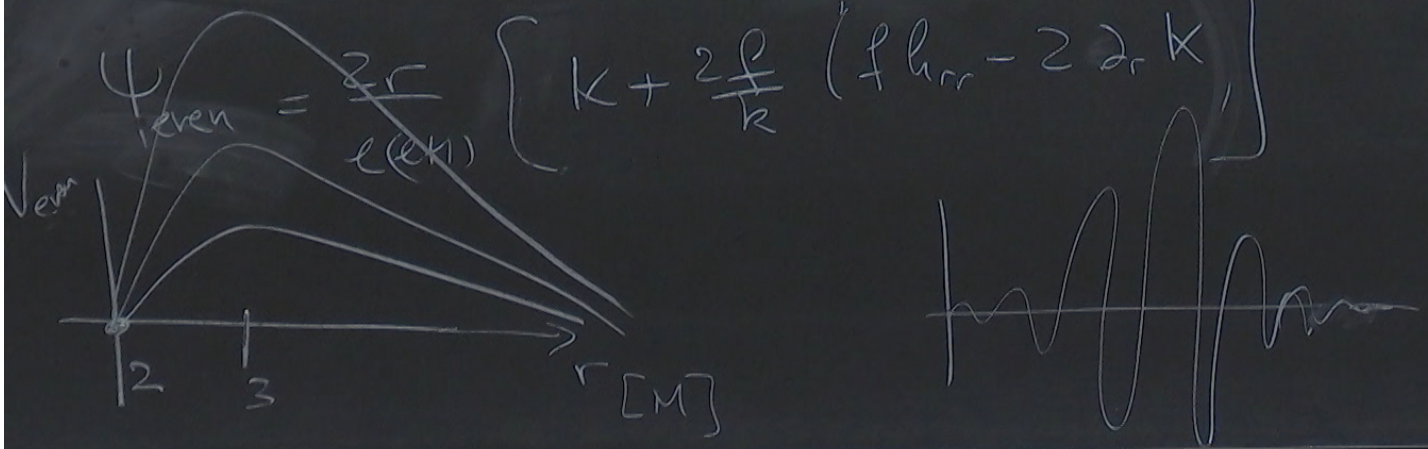
$$\Rightarrow \left[-\frac{\partial^2}{\partial t^2} + f\frac{\partial}{\partial r}\left(f\frac{\partial}{\partial r}\right) - V_{even}\right]\Psi_{even} = S_{even}(T_{ab})$$

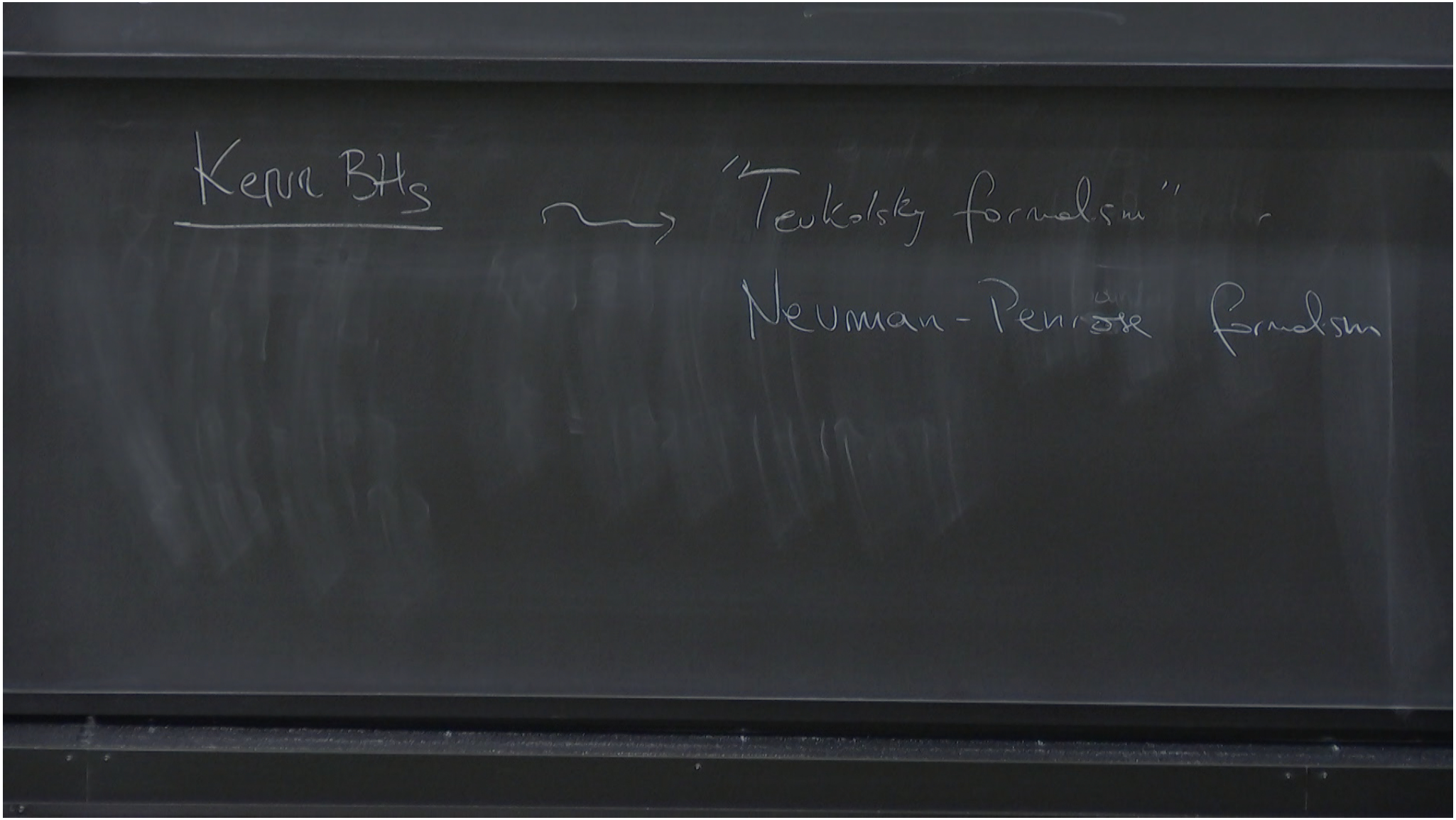
$$\Psi_{even} = \frac{2r}{\ell(\ell+1)}\left[K + 2\frac{f}{k}\left(f h_{rr} - 2 r_* K'\right)\right]$$

Pert Eqns → 2 eqns. for 2 "Master Variables" $\{\Psi_{even}, \Psi_{odd}\}$

$$\left[-\frac{\partial^2}{\partial t^2} + f\frac{\partial}{\partial r}\left(f\frac{\partial}{\partial r}\right) - V_{even}\right]\Psi_{even} = S_{even}(T_{ab})$$

$$\Psi_{even} = \frac{2r}{\ell(\ell+1)}\left[K + \frac{2f}{k}\left(f h_{rr} - 2 r_* K'\right)\right]$$




Kerr BHs
"Teukolsky formalism"
Neuman-Penrose formalism

[M]

ades of BHs"

$\omega = \omega_R + i\omega_I$ | r_* tortoise coord.



Bdry Condition

i) $r \to r_+ = r_{BH}$
$r_* \to -\infty$

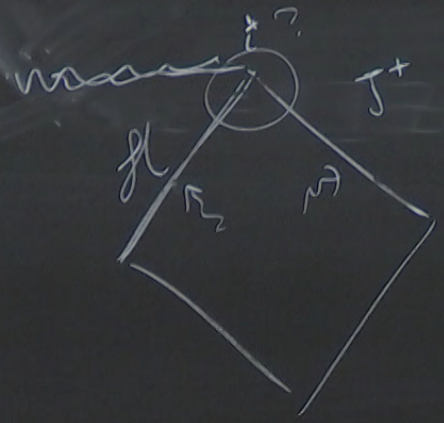
ii) $r \to \infty$
$r_* \to \infty$

$h \sim e^{-i\omega(t+r_*)}$ $\quad t \in [2\pi, \infty)$
$r_* (-\infty, \infty)$

$h \sim e^{-i\omega(t-r_*)}$

"QNM modes of BHs"

$\omega = \omega_R + i\omega_I$



Bdry Condition

i) $r \to r_+ = r_{BH}$
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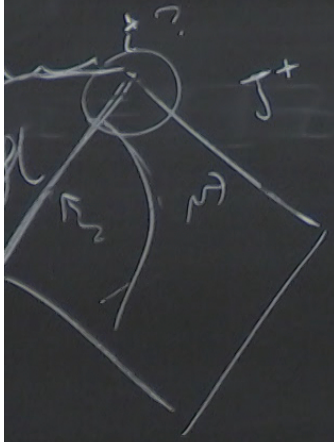
$h \sim e^{-i\omega(t+r_*)}$

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QNModes of BHs"

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r_* tortoise co



Bdry Condition

i) $r \to r_+ = r_{BH}$
$r_* \to -\infty$

ii) $r \to \infty$
$r_* \to \infty$

$h \sim e^{-i\omega(t+r_*)}$ $r_* \in [2M, \infty)$
$r_* (-\infty, \infty)$

$h \sim e^{-i\omega(t-r_*)}$

$\{l, m\}$

QNMs – Can't provide a full description.

- solns are labeled by $(\ell, m$ ... $\overline{F}^{(r)}_{\ell m}$, $W_{\ell m}$

- ... are not a basis.

n't provide a full description.

ns. are labeled by (ℓ, m, n) , $\overline{F^{(r)}_{\ell m n}}$, $W_{\ell m n}$

. are not a basis.

- solns. are labeled by (ℓ, m, n) $\ell m n$)

- " are not a basis.

$\ell = m = 2 \quad n = 0$

$MW = 0.3785 - 0.0905\,i$

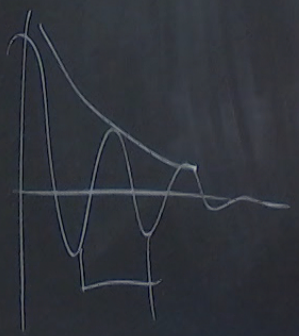
Ms - Can't provide a full description.

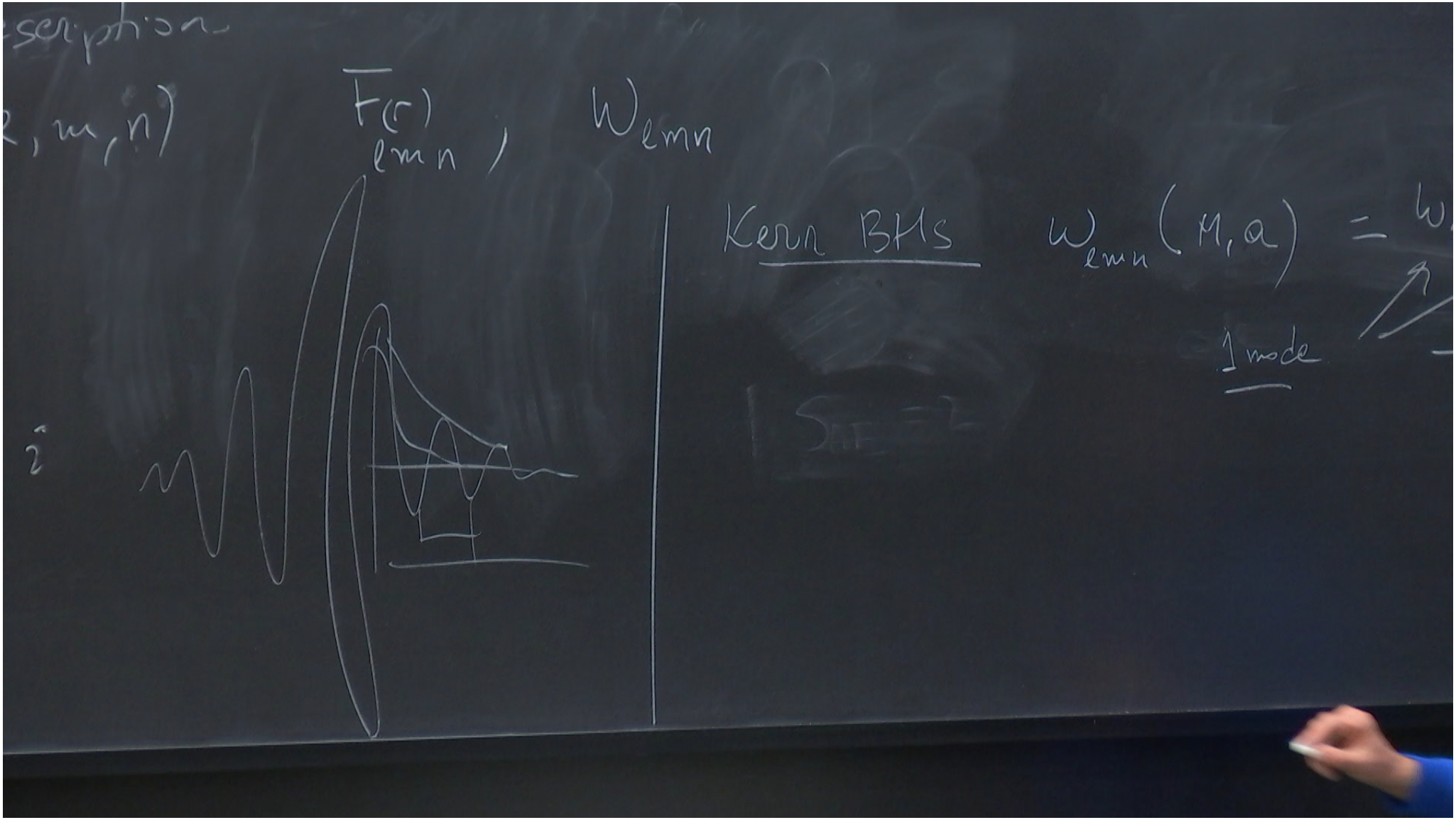
- solns are labeled by (ℓ, m, n) $\quad \overline{F}(r)_{\ell m n}$, $W_{\ell m n}$

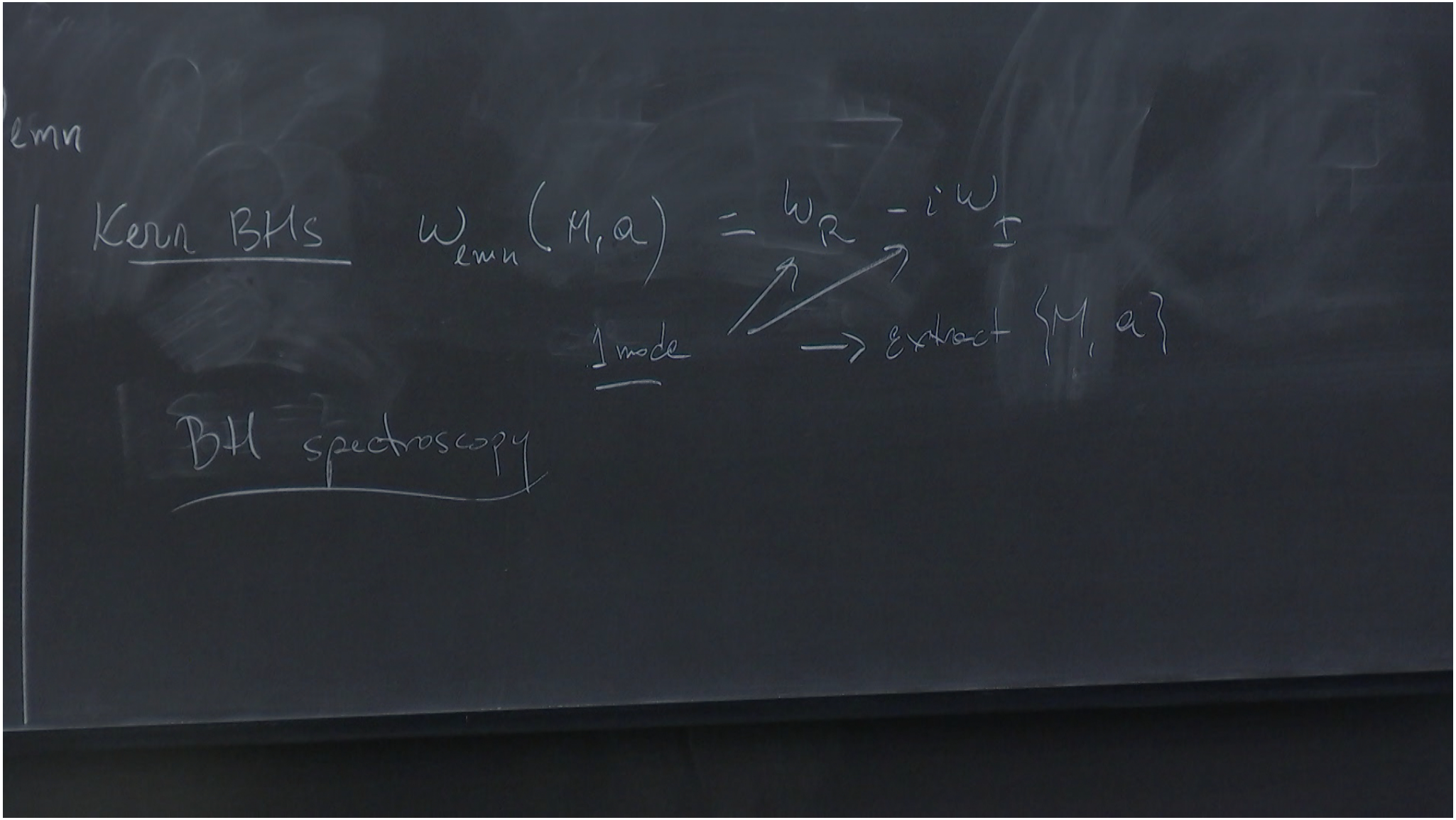
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$m = 2 \quad n = 0$

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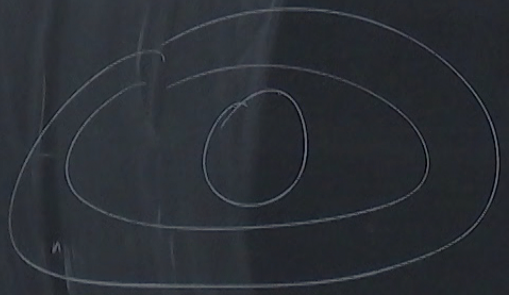



Kerr BHs
1 mode

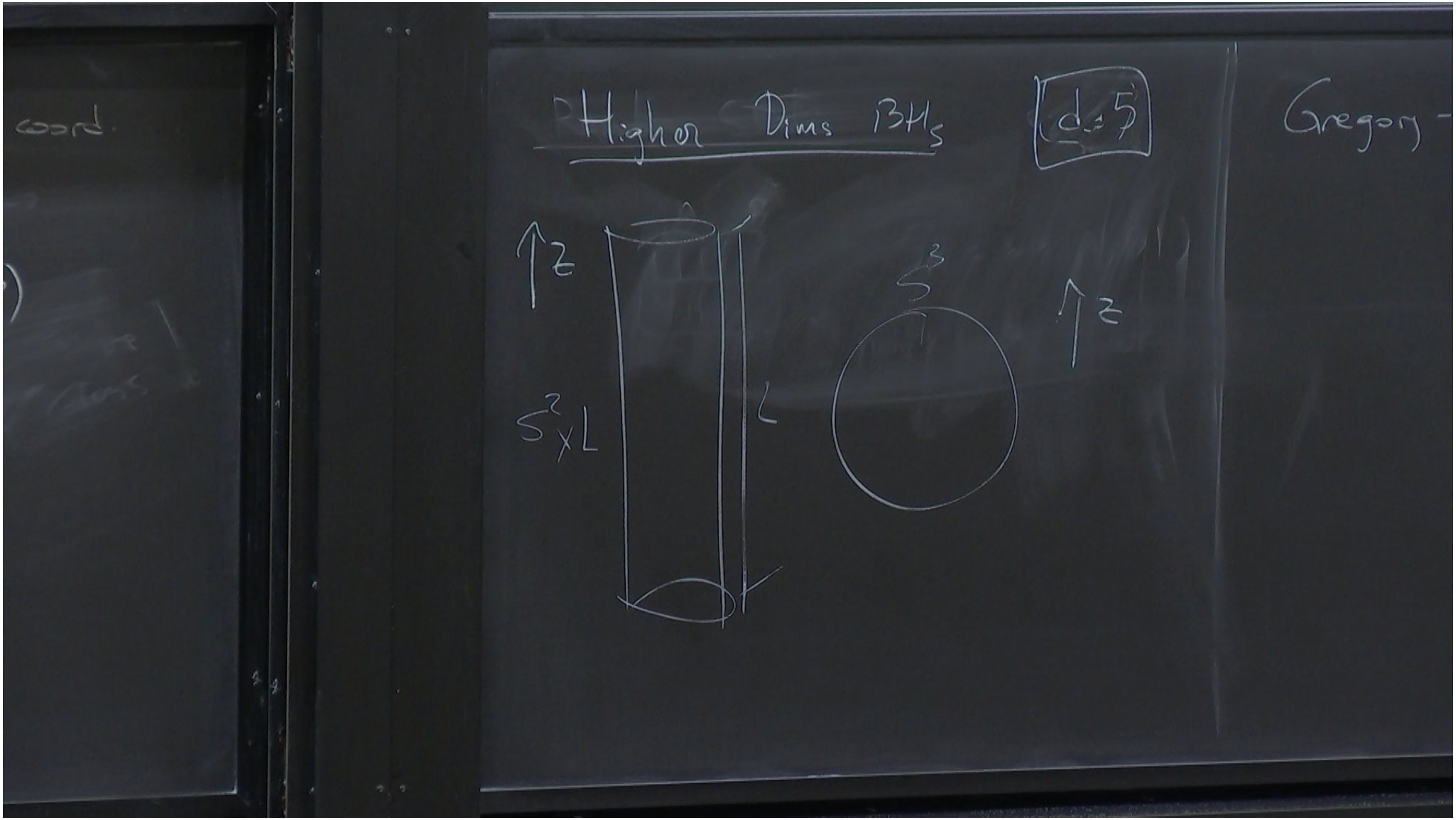

lmn
Kerr BHs $\omega_{lmn}(M, a) = \omega_R - i\omega_I$
1 mode
→ extract $\{M, a\}$
BH spectroscopy

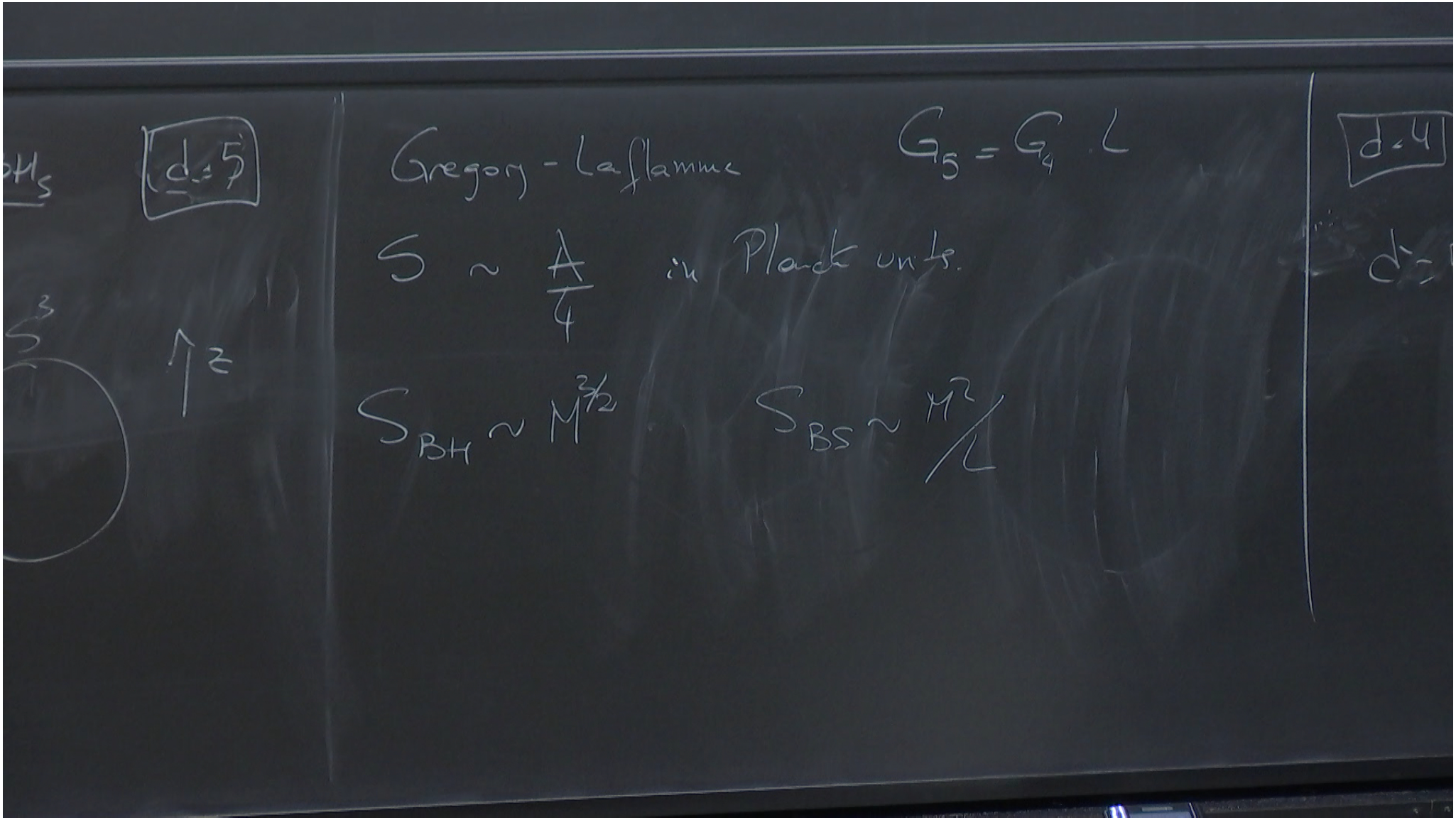
d=4 BH topology is S^2

d=5 $S^2 \times S^1$ S^3



No Kerr bound ($d \geq 6$)


Higher Dims BHs
d=5
Gregory
z
$S^3 \times L$
L
S^3
z


d=5
Gregory - Laflamme
$G_5 = G_4 \cdot L$
$S \sim \frac{A}{4}$ in Planck units.
$S_{BH} \sim M^{3/2}$
$S_{BS} \sim M^2/L$
d=4

$d=5$ Gregory - Laflamme $G_5 = G_4 L$

$S \sim \frac{A}{4}$ in Planck units.

z

$S_{BH} \sim M^{3/2}$ $S_{BS} \sim M^2 L$



20's → Wiseman

$$ds^2_{BS} = \text{Sch} + dz^2$$

$$ds^2_{(S^3)\ BH} = -\left(1 - \left(\frac{r_+}{r}\right)^2\right) dt^2 + \left(\quad\right)^{-1} dr^2 + r^2 d\Omega^2_3$$

$g = \overset{BS}{g} + h \qquad \left(\delta_a^c \delta_b^d \Box + 2 R_a{}^c{}_b{}^d\right) h_{cd} = 0$

$\{t, r, z, \theta, \phi\}$

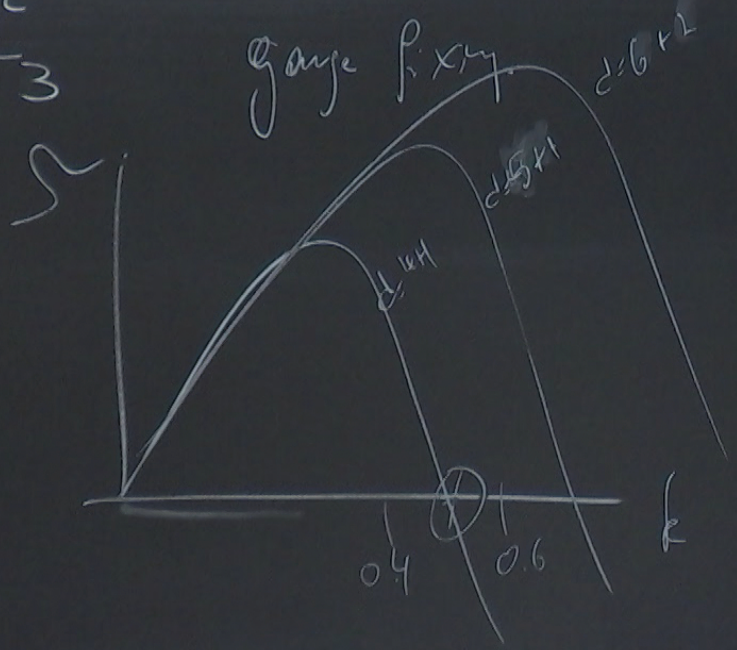
S^2_3 gauge fixing.

$g = \overset{BS}{\bar{g}} + h$ $\quad \left(\delta_a^c \delta_b^d \Box + 2 R_a{}^c{}_b{}^d\right) h_{cd} = 0$

$\{t, r, z, \theta, \phi\}$

$h \sim e^{\Omega t} \; e^{i k z}$

$= d\Omega_3^2$

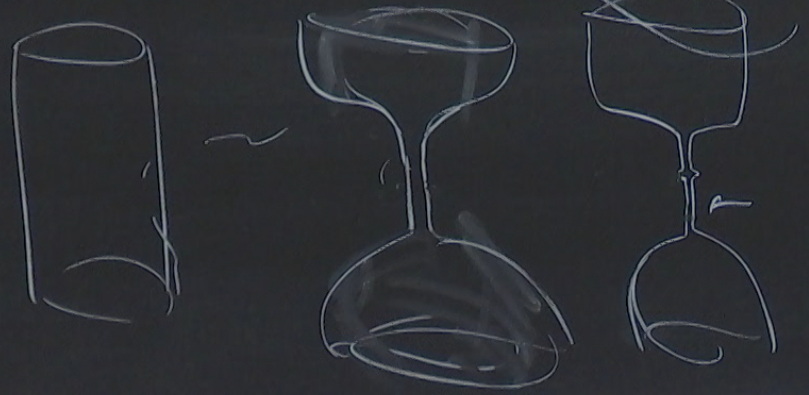


$$ds^2_{BS} = \text{Sch} + dz^2$$

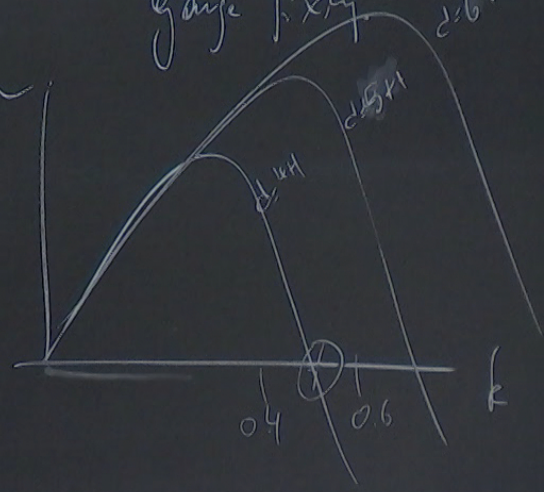
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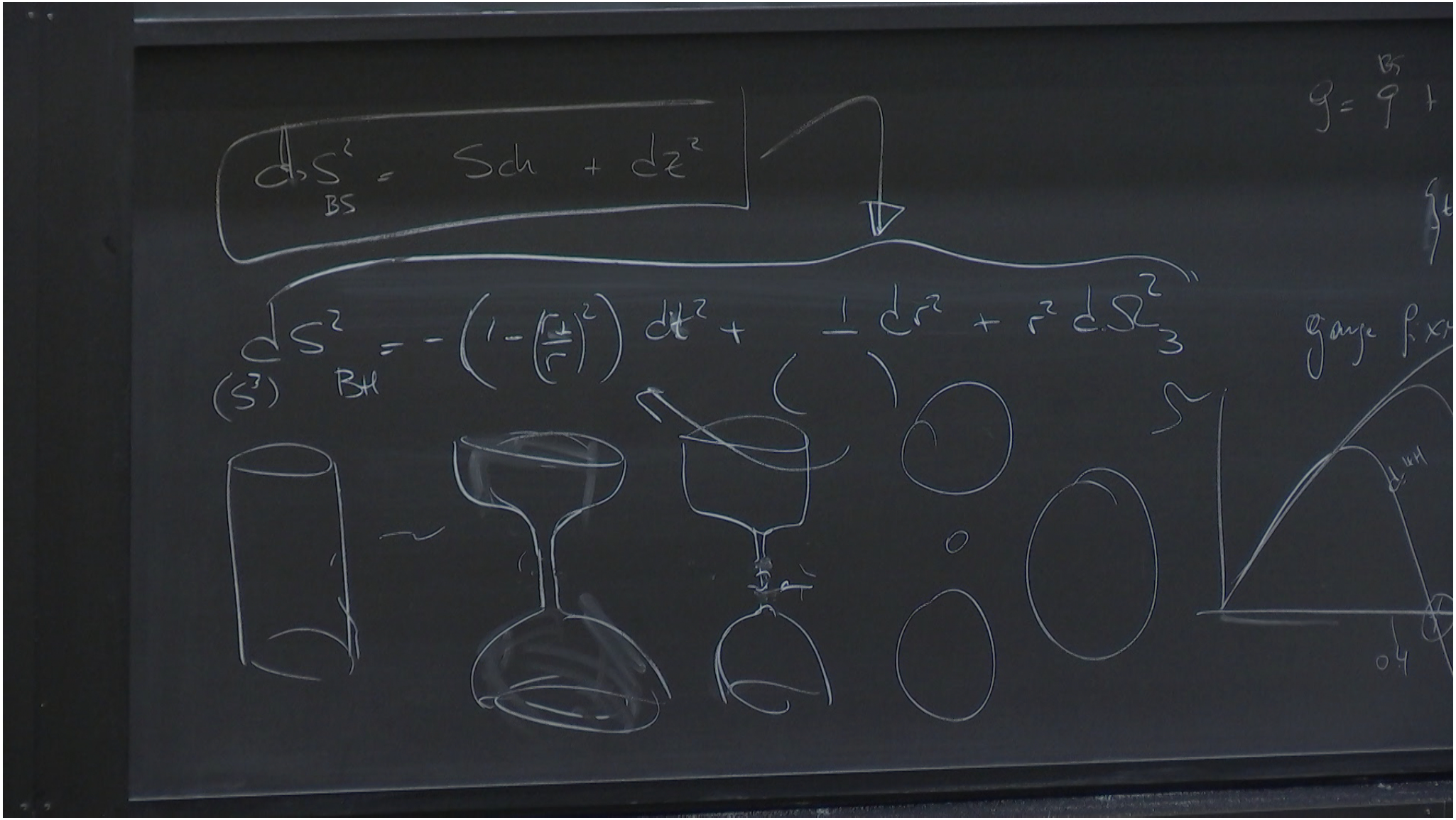
$\{t, r, z, \Omega, \phi\}$

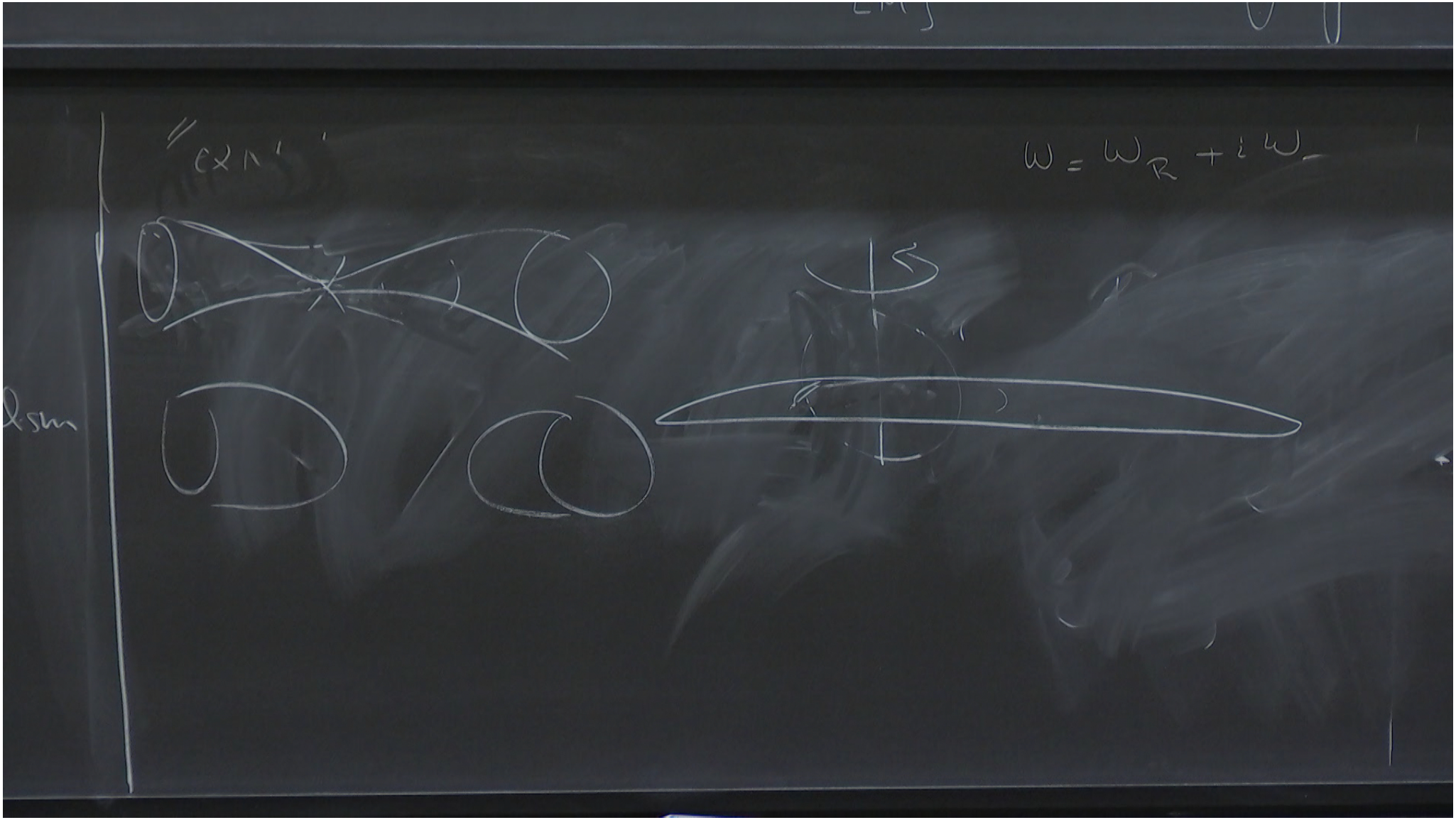
$$ds^2_{(S^3)\ BH} = -\left(1 - \left(\frac{r_+}{r}\right)^2\right) dt^2 + \left(\quad\right) dr^2 + r^2 d\Omega^2_3$$



gauge fixing




$ds^2_{BS} = s\,d\Omega^2 + dz^2$
$ds^2_{(S^3)\,BH} = -\left(1-\left(\frac{r_+}{r}\right)^2\right)dt^2 + \frac{1}{\left(1-\left(\frac{r_+}{r}\right)^2\right)}dr^2 + r^2 d\Omega^2_3$
gauge fix


$W = W_R + i W_-$