Title: Cosmology (2021/2022)

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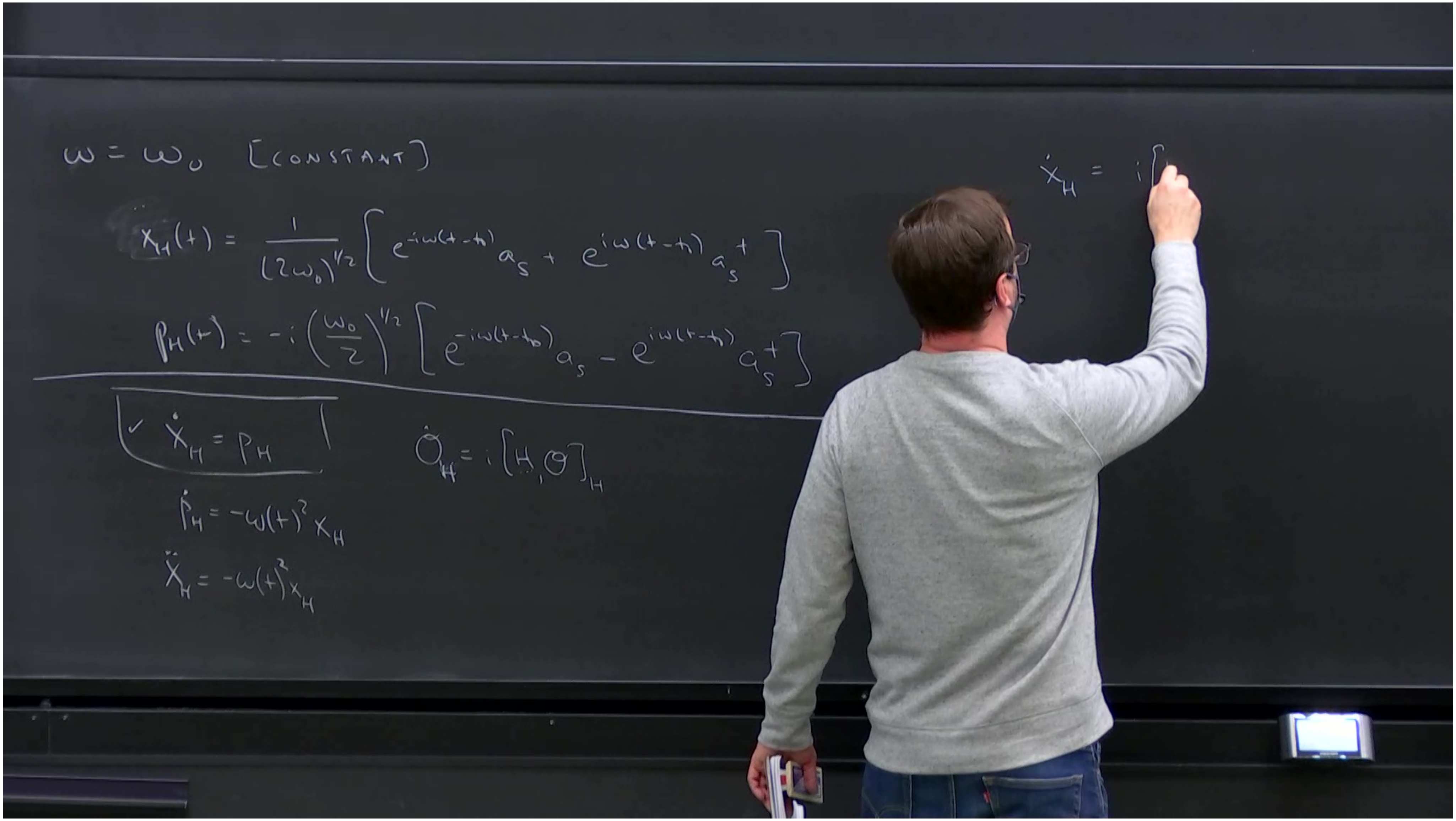
Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.

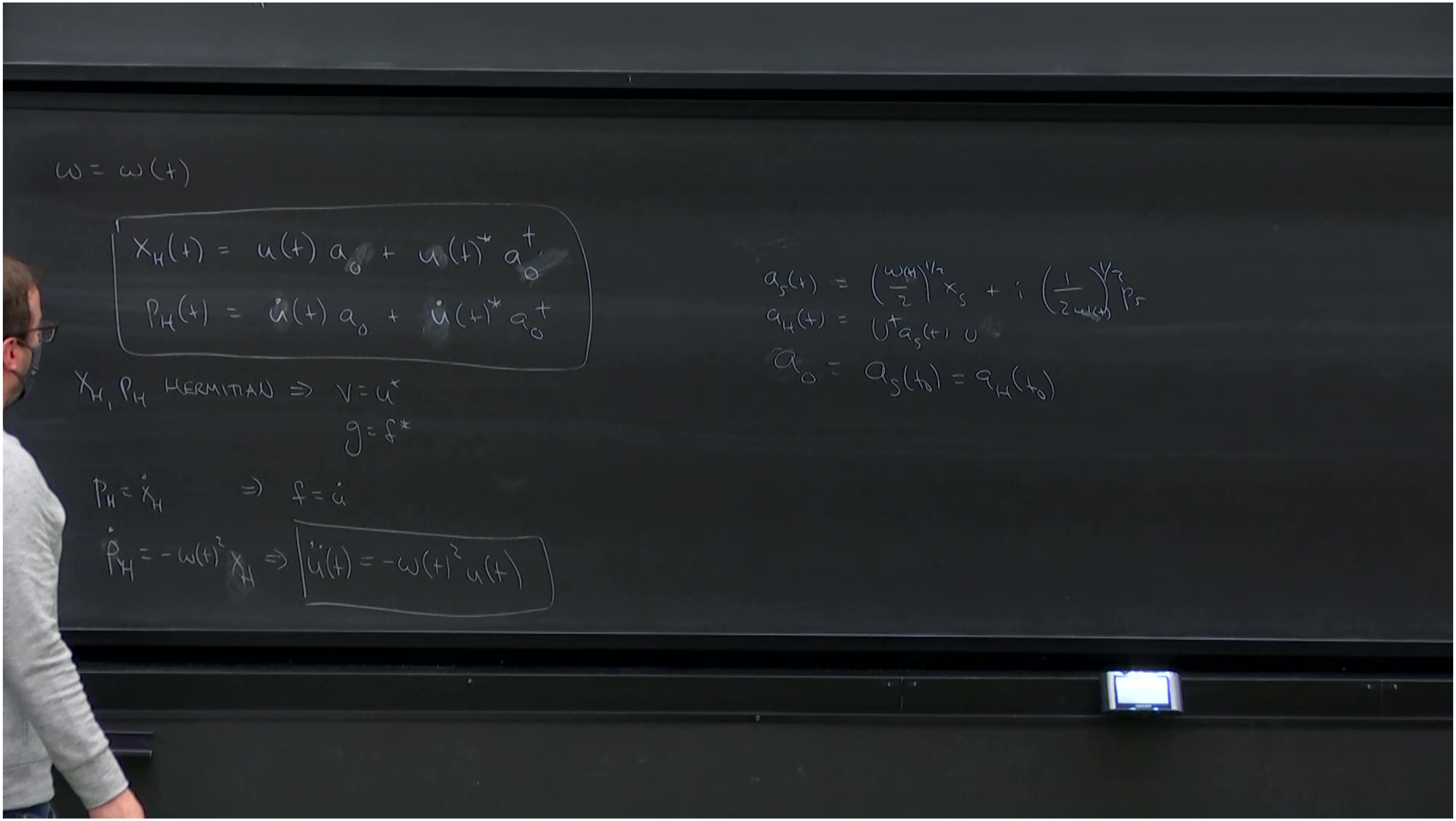

$\omega = \omega_0$ [CONSTANT]
$x_H(t) = \frac{1}{(2\omega_0)^{1/2}} \left[e^{-i\omega(t-t_0)} a_s + e^{i\omega(t-t_1)} a_s^+ \right]$
$p_H(t) = -i \left(\frac{\omega_0}{2}\right)^{1/2} \left[e^{-i\omega(t-t_0)} a_s - e^{i\omega(t-t_0)} a_s^+ \right]$
✓ $\dot{x}_H = p_H$
$\dot{O}_H = i[H, O]_H$
$\dot{p}_H = -\omega(t)^2 x_H$
$\ddot{x}_H = -\omega(t)^2 x_H$
$\dot{x}_H = i[$

$$\dot{x}_H = i\left[H_s, x_s\right]_H$$

$$= i\left[\frac{1}{2}p_s^2 + \cancel{\frac{1}{2}\omega(t)^2 x_s^2}, x_s\right]_H$$

$$= i\left(\frac{1}{2}\right)\left(-2ip_H\right) \quad \text{since } [x,p] = i$$

$$= p_H$$



$\omega = \omega(t)$
$x_H(t) = u(t)\, a_0 + u(t)^* a_0^\dagger$
$p_H(t) = \dot{u}(t)\, a_0 + \dot{u}(t)^* a_0^\dagger$
x_H, p_H HERMITIAN $\Rightarrow$ $v = u^*$
$g = f^*$
$p_H = \dot{x}_H \Rightarrow f = \dot{u}$
$\dot{p}_H = -\omega(t)^2 x_H \Rightarrow \ddot{u}(t) = -\omega(t)^2 u(t)$
$a_S(t) = \left(\frac{\omega(t)}{2}\right)^{1/2} x_S + i \left(\frac{1}{2\omega(t)}\right)^{1/2} p_S$
$a_H(t) = U^\dagger a_S(t)\, U$
$a_0 = a_S(t_0) = a_H(t_0)$

$\omega(t) \to \omega_0$ AT EARLY TIMES

$$u(t) \to \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}$$

$$\langle \psi(t) | \hat{x}_S^2 | \psi(t) \rangle = \langle 0 | \hat{x}_H(t)^2 | 0 \rangle$$

$$= \langle 0 | \left(u(t) a_0 + u(t)^* \cancel{a_0^\dagger} \right) \left(\cancel{u(t) a_0} + u(t)^* a_0^\dagger \right) | 0 \rangle$$

$$= u(t) u(t)^* \underbrace{\langle 0 | a_0 a_0^\dagger | 0 \rangle}_{=1} = |u(t)|^2$$

$$\langle \psi | x^4 | \psi \rangle = 3|u|^4$$

$$[x_H(t), p_H(t)]$$

IN FACT

- SCHRODINGER WAVEFUNCTION IS GAUSSIAN

$$\psi(x) \propto e^{\alpha x^2} \qquad \alpha \in \mathbb{C}$$

$$x_H(t) = u(t)\, a_0 + u(t)^* a_0^\dagger$$

$$- u(t) u(t) \underbrace{\langle 0 | a_0 a_0^\dagger | 0 \rangle}_{=1} = |u(t)|$$

$$[x_H(t), p_H(t)] \overset{?}{=} i$$

$$= \underbrace{u(t)\,\dot{u}(t)^* - u(t)^*\,\dot{u}(t)}_{W(t)}$$

"WRONSKIAN" $\mathrm{Det}\begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix}$

EOM $\ddot{u}(t) = -\omega(t)^2 u(t) \Rightarrow \dot{W}(t) = 0$

IC's $u(t) = \dfrac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}} \Rightarrow W(t) = i$ AT EARLY TIMES

$\underbrace{\langle 0 | q_0 q | 0 \rangle}_{=1} = |u(t)|$

$$[x_H(t), p_H(t)] \overset{?}{=} i$$

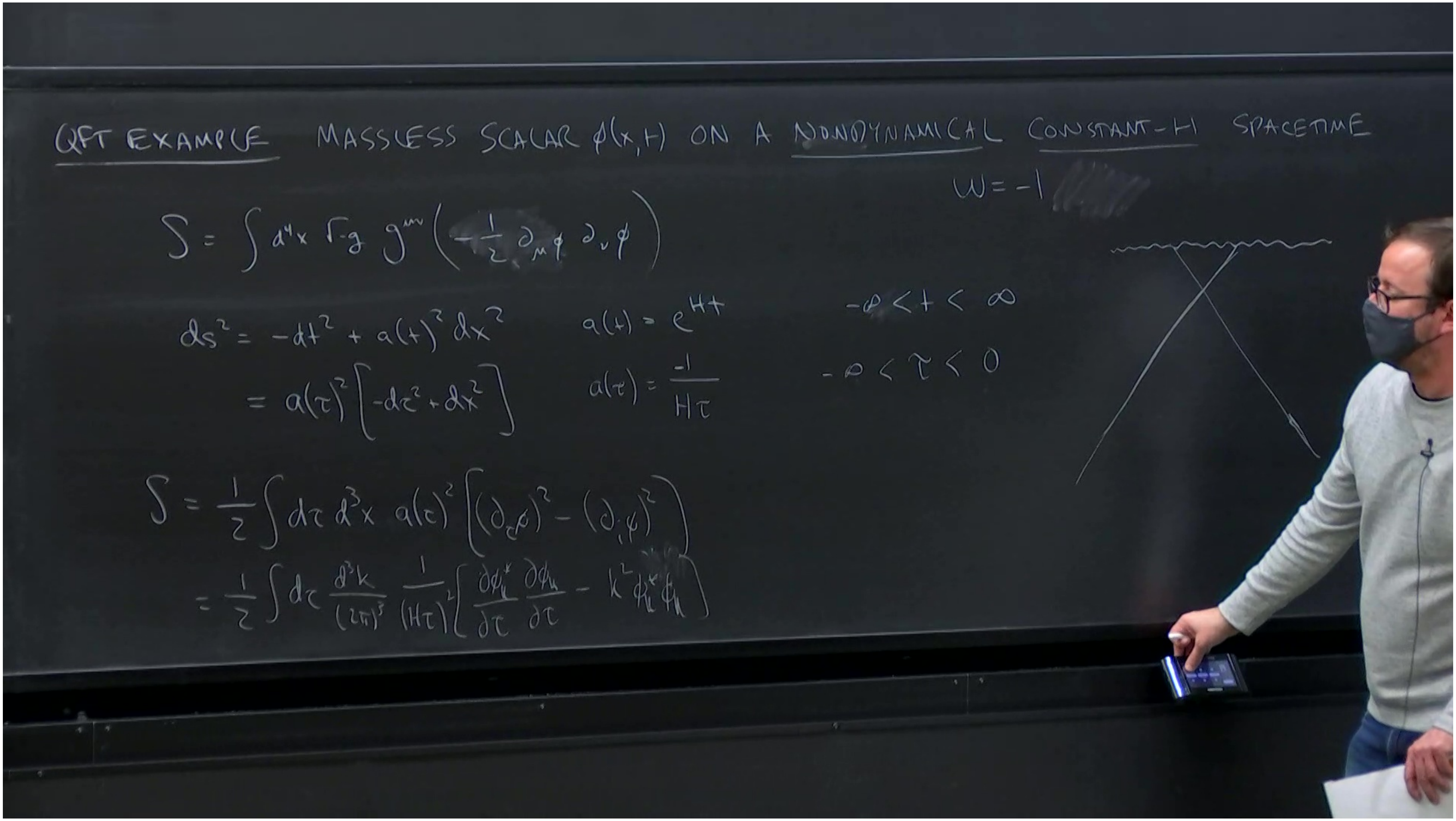
$$= \underbrace{u(t)\,\dot{u}(t)^* - u(t)^*\,\dot{u}(t)}_{W(t)}$$

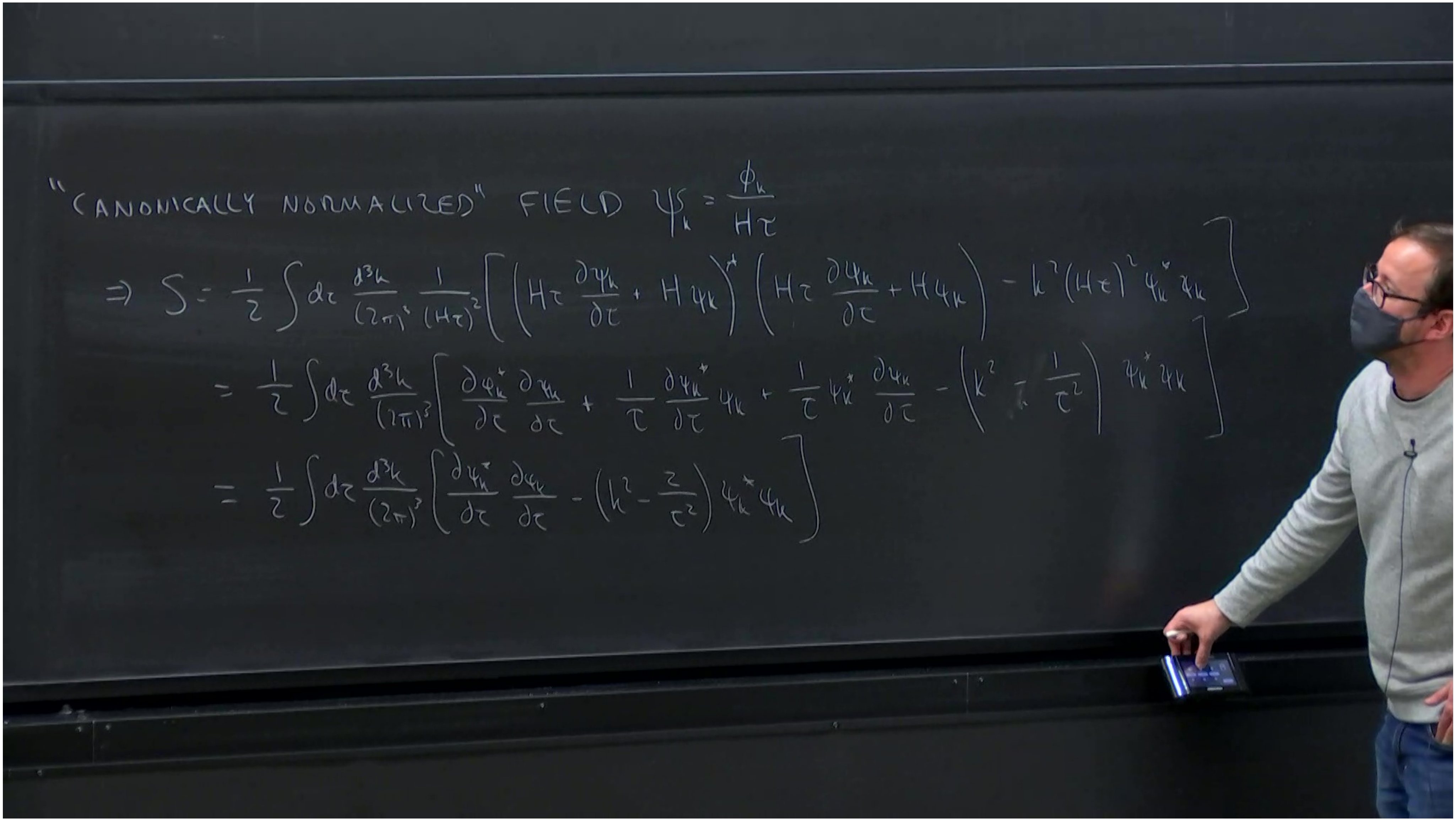
"WRONSKIAN" $\mathrm{Det}\begin{pmatrix} u(t) & u(t)^* \\ \dot{u}(t) & \dot{u}(t)^* \end{pmatrix}$

EOM $\ddot{u}(t) = -\omega(t)^2 u(t) \Rightarrow \dot{W}(t) = 0$

IC's $u(t) = A e^{-i\omega_0 t}$

$$\Rightarrow W(t) = 2i\omega_0 |A|^2 \Rightarrow |A| = \frac{1}{(2\omega_0)^{1/2}}$$


QFT EXAMPLE MASSLESS SCALAR $\phi(x,t)$ ON A NONDYNAMICAL CONSTANT-H SPACETIME
$w=-1$
$S=\int d^4x\sqrt{-g}\,g^{\mu\nu}\left(-\frac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\right)$
$ds^2=-dt^2+a(t)^2dx^2$
$=a(\tau)^2\left[-d\tau^2+dx^2\right]$
$a(t)=e^{Ht}$
$a(\tau)=\frac{-1}{H\tau}$
$-\infty<t<\infty$
$-\infty<\tau<0$
$S=\frac{1}{2}\int d\tau\,d^3x\,a(\tau)^2\left[(\partial_\tau\phi)^2-(\partial_i\phi)^2\right]$
$=\frac{1}{2}\int d\tau\frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left[\frac{\partial\phi_k^*}{\partial\tau}\frac{\partial\phi_k}{\partial\tau}-k^2\phi_k^*\phi_k\right]$



"CANONICALLY NORMALIZED" FIELD $\psi_k = \frac{\phi_k}{H\tau}$
$\Rightarrow S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\frac{1}{(H\tau)^2}\left[\left(H\tau\frac{\partial\psi_k}{\partial\tau} + H\psi_k\right)^*\left(H\tau\frac{\partial\psi_k}{\partial\tau} + H\psi_k\right) - k^2(H\tau)^2\psi_k^*\psi_k\right]$
$= \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} + \frac{1}{\tau}\frac{\partial\psi_k^*}{\partial\tau}\psi_k + \frac{1}{\tau}\psi_k^*\frac{\partial\psi_k}{\partial\tau} - \left(k^2 - \frac{1}{\tau^2}\right)\psi_k^*\psi_k\right]$
$= \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial\psi_k^*}{\partial\tau}\frac{\partial\psi_k}{\partial\tau} - \left(k^2 - \frac{2}{\tau^2}\right)\psi_k^*\psi_k\right]$

$= x_H \quad \Rightarrow \quad f = u$

$\ddot{x}_H = -\omega(t)^2 x_H \Rightarrow \boxed{\ddot{u}(t) = -\omega(t)^2 u(t)}$

SHO

$$S = \frac{1}{2}\int dt\left(\dot{x}^2 - \frac{1}{2}\omega(t)^2 x^2\right)$$

$$p = \frac{\partial L}{\partial \dot{x}} = \dot{x}$$

$$\dot{p} = -\omega(t)^2 x$$

$$x(t) = u(t)\, a_0 + u(t)^* a_0^\dagger \qquad x^* = x$$

$$p(t) = \dot{u}(t)\, a_0 + \dot{u}(t)^* a_0^\dagger$$

QFT

$$S = \frac{1}{2}\int d\tau \frac{d^3k}{(2\pi)^3}\left[\frac{\partial \psi_k^*}{\partial \tau}\frac{\partial \psi_k}{\partial \tau} - \left(k^2 - \frac{2}{\tau^2}\right)\psi_k^* \psi_k\right]$$

$$\Pi = \frac{\delta \mathcal{L}}{\delta \partial_\tau \psi} = \partial_\tau \psi$$

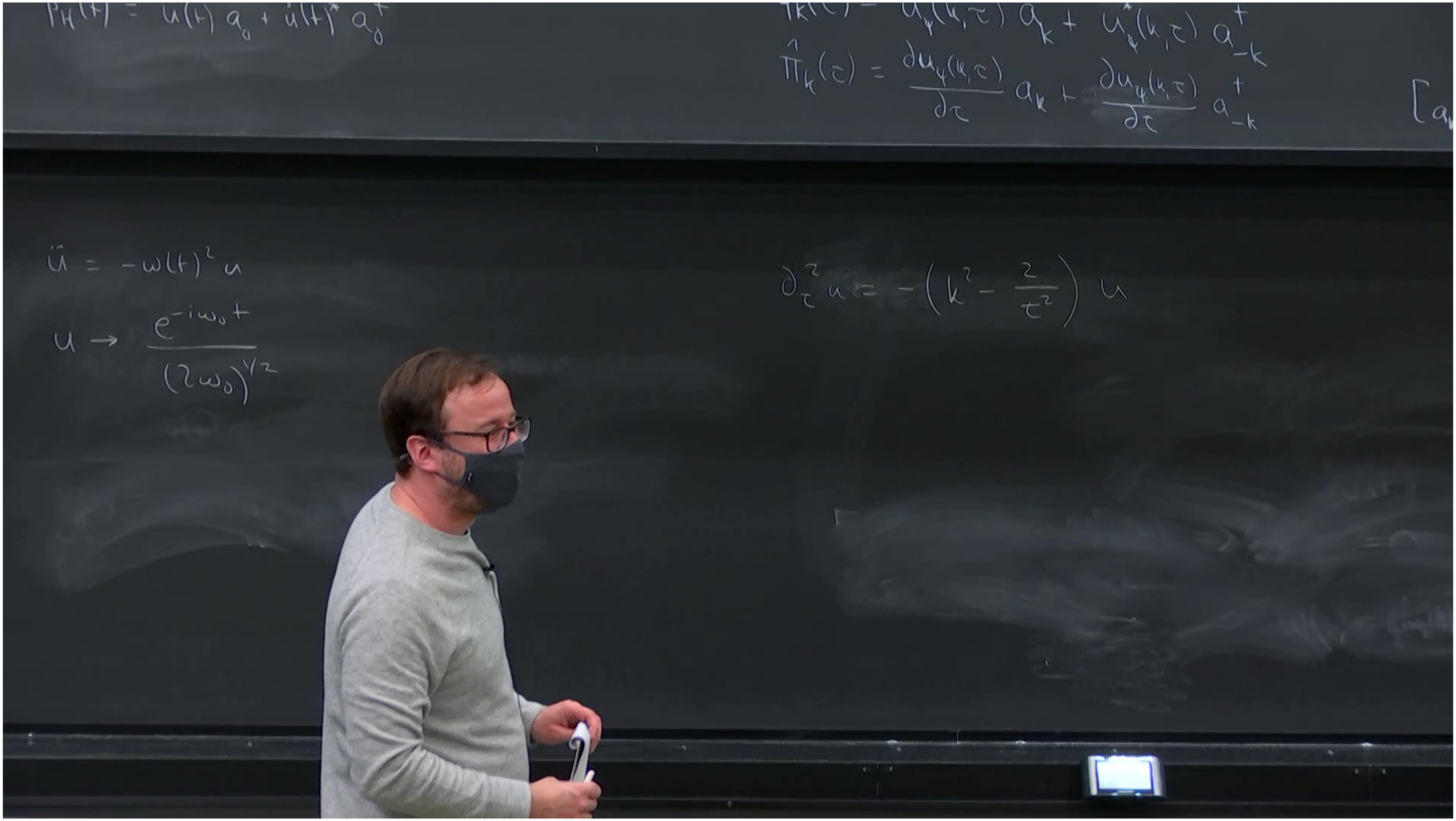
$$\psi(x)^* = \psi(x)$$

$$\Rightarrow \psi(k)^* = \psi(-k)$$

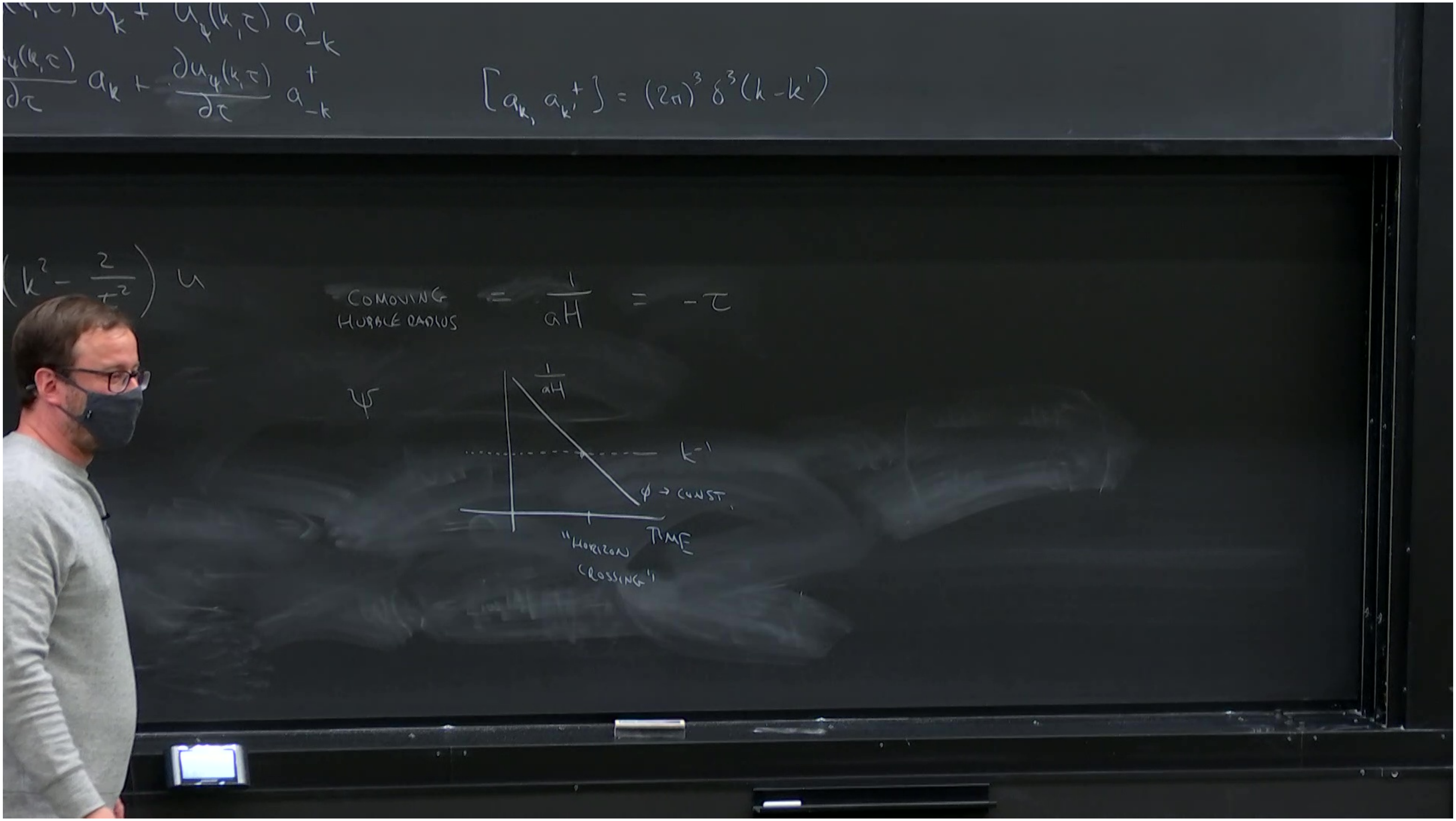
$$\partial_\tau \Pi = -\left(k^2 - \frac{2}{\tau^2}\right)\psi$$

$$\hat{\psi}_k(\tau) = u_\psi(k,\tau)\, a_k + u_\psi^*(k,\tau)\, a_{-k}^\dagger$$

$$\hat{\Pi}_k(\tau) = \frac{\partial u_\psi(k,\tau)}{\partial \tau} a_k + \frac{\partial u_\psi^*(k,\tau)}{\partial \tau} a_{-k}^\dagger$$



$\ddot{u} = -\omega(t)^2 u$
$u \to \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}$
$\partial_\tau^2 u = -\left(k^2 - \frac{2}{\tau^2}\right) u$


$[a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k-k')$
COMOVING HUBBLE RADIUS $= \frac{1}{aH} = -\tau$
ψ
$\frac{1}{aH}$
k^{-1}
$\phi \to$ CONST.
TIME
"HORIZON CROSSING"

$$\hat{P}_H(t) = \dot{u}(t)\, a_0 + \dot{u}(t)^* a_0^\dagger$$

$$\hat{\psi}_k(\tau) = u_\psi(k,\tau)\, a_k + u_\psi^*(k,\tau)\, a_{-k}^\dagger$$

$$\hat{\Pi}_k(\tau) = \frac{\partial u_\psi(k,\tau)}{\partial \tau} a_k + \frac{\partial u_\psi^*(k,\tau)}{\partial \tau} a_{-k}^\dagger$$

$$\ddot{u} = -\omega(t)^2 u$$

$$u \to \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}$$

$$\partial_\tau^2 u = -\left(k^2 - \frac{2}{\tau^2}\right) u$$

$$u(k,\tau) \to \frac{e^{-ik_0\tau}}{(2k_0)^{1/2}}$$

COMOVIN

HUBBLE RA

ψ

$$\partial_\tau^2 u = -\left(k^2 - \frac{2}{\tau^2}\right) u$$

$$u(k,\tau) \to \frac{e^{-ik\tau}}{(2k)^{1/2}}$$

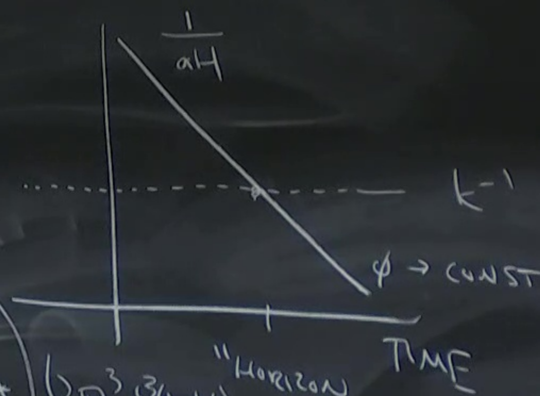
$$\boxed{u(k,\tau) = \frac{1}{(2k)^{1/2}}\left(1 - \frac{i}{k\tau}\right)e^{-ik\tau}}$$

HARD TO FIND

COMOVING HUBBLE RADIUS $= \dfrac{1}{aH} = -\tau$

ψ

$$[\psi_k, \pi_{k'}] = \underbrace{\mathrm{Det}\begin{pmatrix} u & u^* \\ \partial_\tau u & \partial_\tau u^* \end{pmatrix}}\,(2\pi)^3\,\delta^3(k+k')$$



LOOKS DIVERGENT AS $\tau \to 0$

$$\psi = \frac{\phi}{H\tau}$$

$$\hat{\phi}_k(\tau) = H\tau\, \hat{\psi}_k(\tau)$$

$$= u_\phi(k,\tau)\, a_k + u_\phi^*(k,\tau)\, a^\dagger_{-k}$$

$$u_\phi(k,\tau) = H\tau\, u_\psi(k,\tau)$$

$$= \frac{H}{(2k^3)^{1/2}} (1 + ik\tau) e^{-ik\tau}$$

$$\mathrm{Det}\begin{pmatrix} u_\phi & u_\phi^* \\ \frac{\partial_\tau u_\phi}{(H\tau)^2} & \frac{\partial_\tau u_\phi^*}{(H\tau)^2} \end{pmatrix} = i$$

$$\frac{\partial}{\partial\tau}\left[\frac{1}{\tau^2}\frac{\partial\phi}{\partial\tau}\right] = -k^2 \frac{\phi}{\tau^2}$$

SIZE OF FLUC

SIZE OF FLUCTUATIONS

$$\langle \psi(\tau) | \hat{\phi}_k \, \hat{\phi}^*_{k'} | \psi(\tau) \rangle = \langle 0 | \hat{\phi}_k(\tau) \, \hat{\phi}_{k'}(\tau)^\dagger | 0 \rangle$$

$$= \langle 0 | \left(u_\phi(k,\tau)\, a_k + \cancel{u^*_\phi(k,\tau)\, a^\dagger_{-k}} \right) \left(u^*_\phi(k',\tau)\, a^\dagger_{k'} + \cancel{u_\phi(k',\tau)\, a_{-k'}} \right) | 0 \rangle$$

$$= u_\phi(k,\tau)\, u^*_\phi(k',\tau) \langle 0 | a_k a^\dagger_{k'} | 0 \rangle \qquad [a_k, a^\dagger_{k'}] = (2\pi)^3 \delta^3(k-k')$$

$$= |u_\phi(k,\tau)|^2 (2\pi)^3 \delta^3(k-k')$$

a_{-k} $\quad [a_k, a_{k'}^\dagger] = (2\pi)^3 \delta^3(k-k')$

AT LATE TIMES $\tau \to 0$

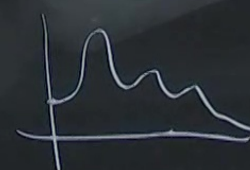
$$\langle \phi_k \phi_{k'}^\dagger \rangle = \left| u_\phi(k, \tau=0) \right|^2 (2\pi)^3 \delta^3(k-k')$$

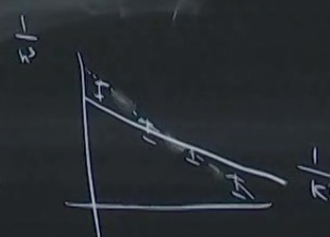
$$= \frac{A}{2k^3} (2\pi)^3 \delta^3(k-k')$$

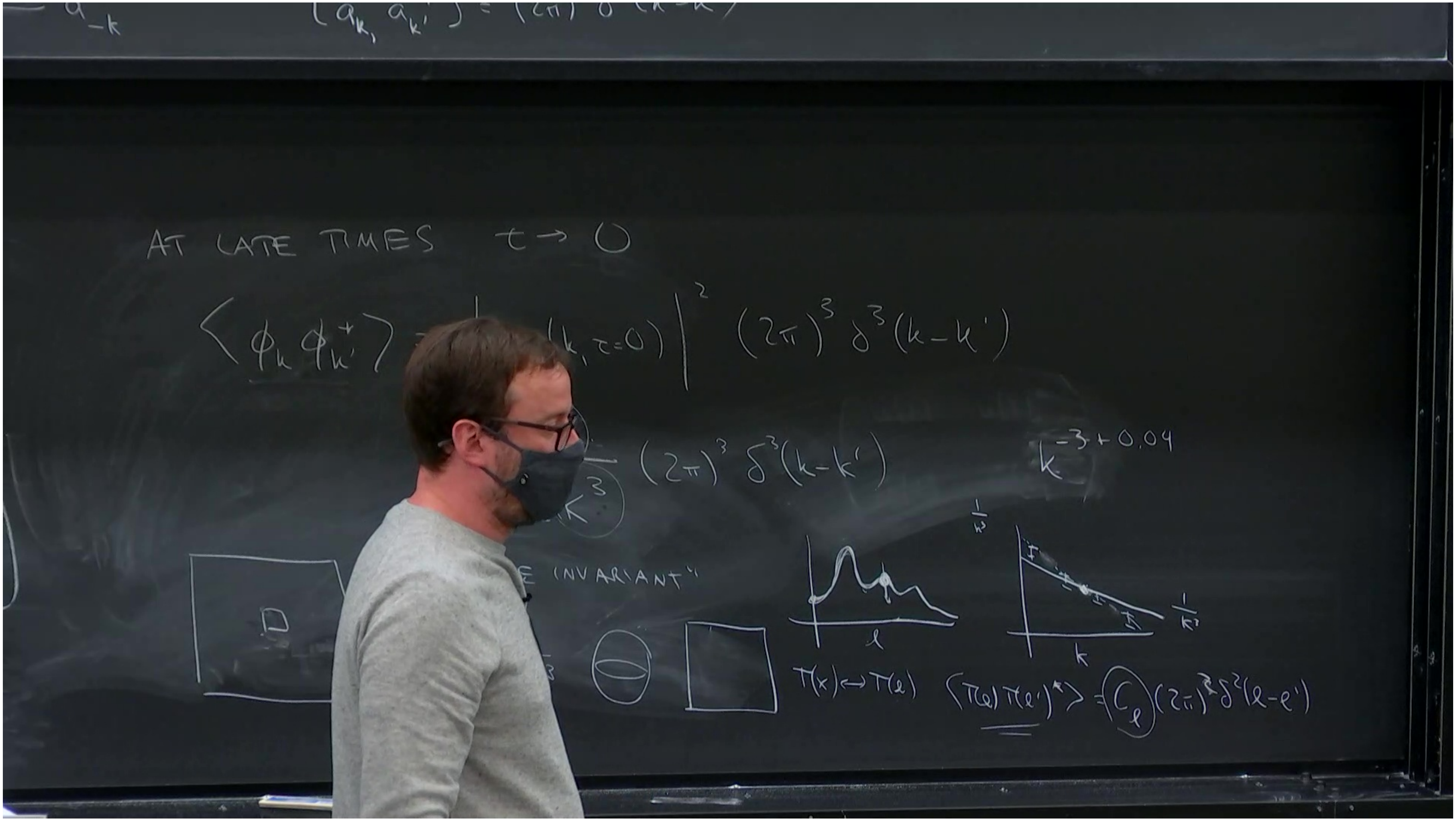
$k^{-3+0.04}$

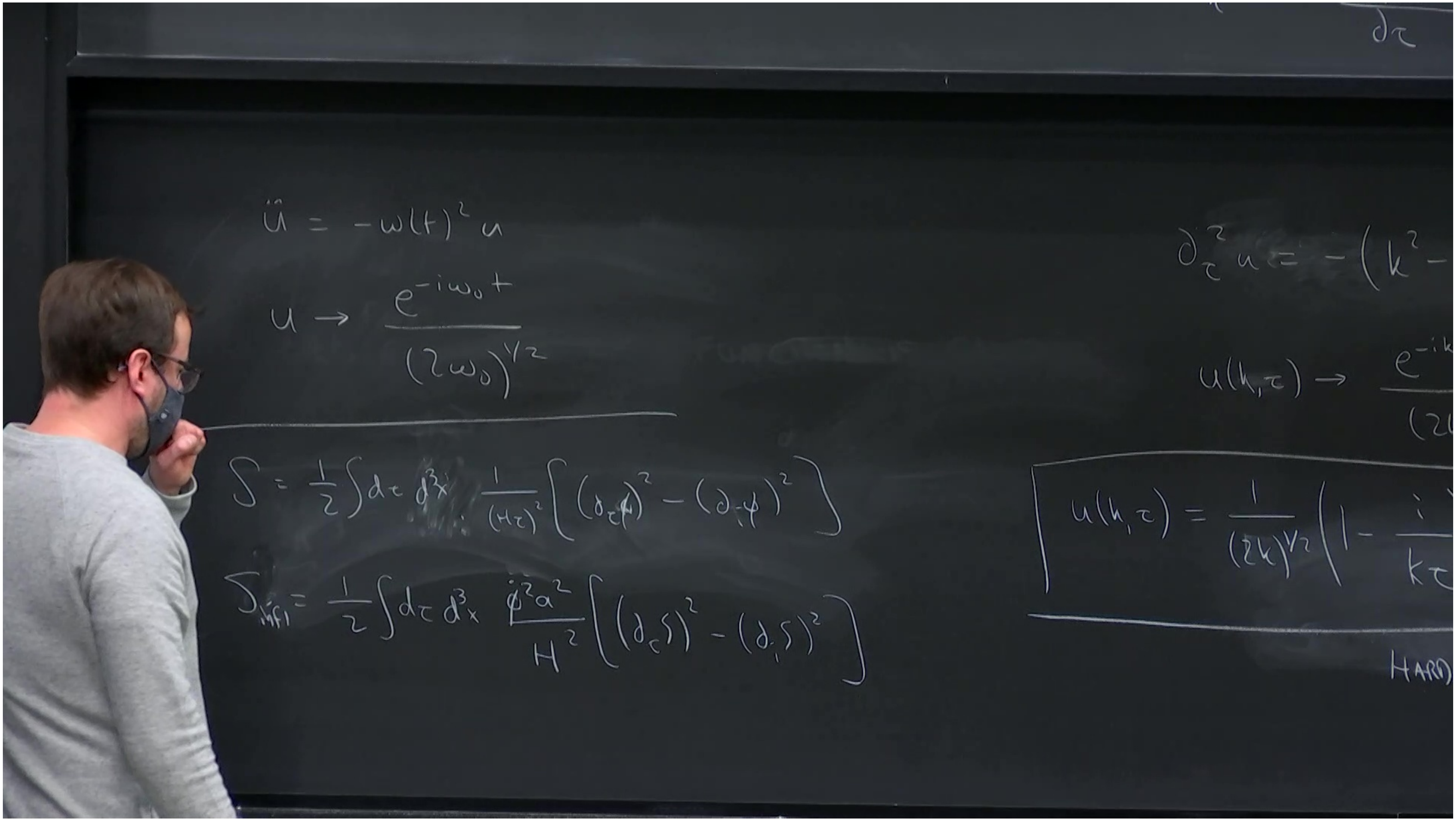
"SCALE INVARIANT"

$\langle \phi_k \phi_{k'}^\dagger \rangle \sim \frac{1}{k^3}$






AT LATE TIMES τ → 0
"INVARIANT"
k^{-3+0.04}
T(x) ⟷ T(ℓ)
⟨T(ℓ) T(ℓ')*⟩ = C_ℓ (2π)² δ²(ℓ-ℓ')



$\ddot{u} = -\omega(t)^2 u$
$u \to \frac{e^{-i\omega_0 t}}{(2\omega_0)^{1/2}}$
$S = \frac{1}{2}\int d\tau\, d^3x\, \frac{1}{(H\tau)^2}\left[(\partial_\tau\phi)^2 - (\partial_i\phi)^2\right]$
$S_{infl} = \frac{1}{2}\int d\tau\, d^3x\, \frac{\dot{\phi}^2 a^2}{H^2}\left[(\partial_\tau\zeta)^2 - (\partial_i\zeta)^2\right]$