

Title: Quantum Gravity

Speakers: Aldo Riello

Collection: Quantum Gravity (2021-2022)

Date: May 05, 2022 - 10:15 AM

URL: <https://pirsa.org/22050008>

Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

ADM ph space
(off shell, canonical)

(π^i, h_{ij})
 ↑
 metric on Σ

the (a posteriori) geometric interpretation of π^i is given by the "Legendre transf"

$$\pi^i = \sqrt{h} (K^i - K h^i)$$

$K_{ij} = \frac{1}{2} L_n h_{ij}$
"a posteriori" bc.
on \mathcal{P}_{ADM} there is no spacetime, just space, until we solve the dyn.

Dynamics & Syms are encoded
in Dirac's Hypersurface Deformation

"Algebra" (HDA)

$$\{V(X), V(Y)\} = V([X, Y])$$

$$\{V(X), H(N)\} = H(L_X N)$$

$$\{H(N), H(M)\} = V(Z)$$

$$\hookrightarrow Z^i = \underbrace{h^{ij}}_{\text{circled}} (N \partial_j M - M \partial_j N)$$

ation

$X \in \mathcal{X}(\Sigma)$ v.f.
(infinitesimal diffeo of Σ)

$N \in C^\infty(\Sigma)$ "lapse"

The a posteriori geom interpret.

is $\xi^e = X^e + N n^e$ 4d diffeo
↑ tangent to Σ ↑ normal to Σ

$$V(X) = \int_{\Sigma} X^i \mathcal{V}_i, \quad \mathcal{V}_i = \bar{\nabla}_j \pi^j_i$$

$$H(N) = \int_{\Sigma} N \mathcal{H}, \quad \mathcal{H} = \frac{1}{2\sqrt{h}} (\pi^{ij} \pi_{ij} - \frac{1}{2} \pi^2) - \frac{\sqrt{h}}{2} (\bar{R}(h) - 2\Lambda)$$

(★)



Rmk 1

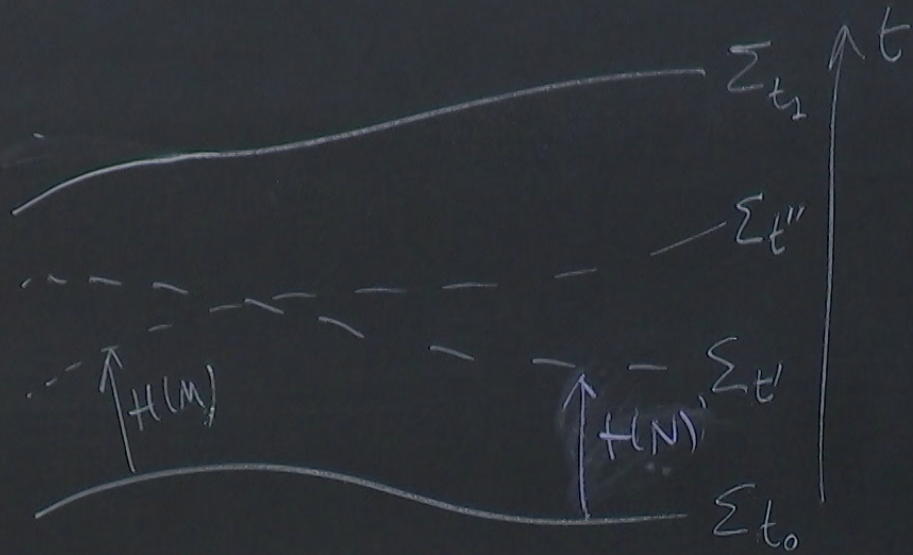
geometrical meaning of the HDA is

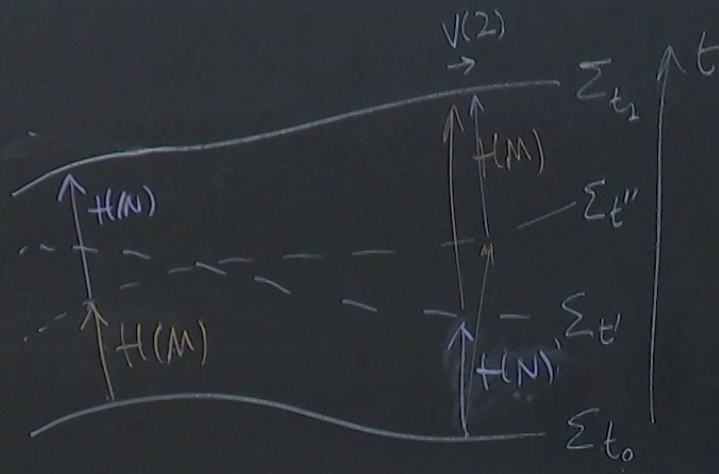
- (i) 3d diffeos one symm. (V)
- (ii) refoliation invariance of spacetime (H)

This is not special to GR, but to any theory formulated in a refoliation inv.

way. (E.g. Dirac's parametrized scalar field)

↳ in this case V, H would be functions of (τ, ϕ) with h_{ij} as a background structure





Rmk 2

The HDA is closely related to 4d diffeo invariance, but it is not the algebra of 4d diffeos. : the 3+1 split has important consequences: Normal diffeos are not Hamiltonian sym & their action is "replaced" by $\{H(N), \cdot\}$

(Cf. with parametrized when reparam. invariance (=1d diffeo!) is traded for the flow of $\{C, \cdot\}$ = dynamics)

ted to 4d
(not) the
re 3+1 split
normal diffeos
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, · }
where reparam.
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dynamics

Rmk 3

(Teitelboim 73)
the only consistent way to implement
the HDA or PADM requires

$$V_i = 0 = \mathcal{H}$$

This is ultimately due to the
dependence of $\mathcal{Z} = \mathcal{Z}(h)$

When $V_i = 0 = \mathcal{H}$, the flow of

$H(N)$ can be interpreted as
"reconstructing" a spacetime

Rmk 3

(Teitelboim 73)
the only consistent way to implement
the HDA on PADM requires

$$V_i = 0 = \mathcal{H}$$

This is ultimately due to the
dependence of $\mathcal{Z} = \mathcal{Z}(h)$, $\{h_{ij}\} \neq 0$

When $V_i = 0 = \mathcal{H}$, the flow of

$\mathcal{H}(N)$ can be interpreted as
"reconstructing" a spacetime

Rmk 4

Key

(Hojman, Kuchař, Teitelboim 76)

modulo minor technical assumptions,
the functional form of (\star) on \mathcal{P}^{ADM}

is unique

\Rightarrow HDA fixes GR completely once
we declare h_{ij} is one of our (2)
canonical variables

(Hamiltonian version of Lovelock thm)

Physical Interpretation

① Since $V(X)$ generates 3d diff.
implementation of $\mathcal{H}_i = 0$ means
that 3d diffs are "pure gauge"

⇒ the objects of GR's 3+1 dynamics
are "abstract" 3-geometries

→ GEOMETRODYNAMICS
unfolds over Superspace = $\frac{\text{Riem}(\Sigma)}{\text{Diff}(\Sigma)}$
(Wheeler) $\text{Riem}(\Sigma) \ni h_{ij}$

(h) - 2A)

and is generated by $\{H(N), \dots\}$

② Although much more challenging to deal with, abstractly $H(N)$ is analogous to $C = P_t + h(p, q)$

Main differences

(i) $N(x)$ has no analogue in para. part.
b.c. that is a zero + 1 field theory

Interpr = rate of clicking of a "lousy clock" at position $x \in \Sigma$

diff.
means
"gauge"

dynamics

tries

$$S = \frac{\text{Riem}(\Sigma)}{\text{Aff}(\Sigma)}$$

$\Rightarrow h_{ij}$

(ii) No privileged time in GR

→ technically $H(N)$ cannot
be written as $\sim P(\tau) + h(\dots)$

↑ linear in some
privileged time
variable τ

(iii) Major consequences for
quantization even at
formal level

$$\hat{H}(N)\Psi = 0$$

→ (canonically) ...
be written as $\sim P(\tau) + h(\dots)$
↑ linear in some
privileged time
variable τ

(iii) Major consequences for
quantization even at
formal level

$$\hat{H}(N)\Psi = 0$$

"clock"

\hat{h}_i acts by mult.
 $\hat{\pi}^i$ acts by funct. der
 $= i \frac{\delta}{\delta h_i}$
operator ordering
& anomalies!

(off shell, canonical)

$$(\pi^i, h_{ij})$$

↑ metric on Σ

the (a posteriori) geometric interpretation of π^i is given by the "Legendre transf"

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"a posteriori" bc.
on PADM there is no spacetime, just space, until we solve the dyn.

$$P_{ab} = \begin{pmatrix} N^2 & X_i \\ X_i & h_{ij} \end{pmatrix}$$

in Dirac's Hypersurface De
"Algebra" (HDA)

$$\{V(X), V(Y)\} = V([X, Y])$$

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