Title: Quantum Gravity

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Collection: Quantum Gravity (2021-2022)

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Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

ADM ph. space

(off shell, canonical)

(π^{ij}, h_{ij})

h_{ij}: metric on Σ

the (a posteriori) geometric interpretation of π^{ij} is given by the "Legendre transf":

$$\pi^{ij} = \sqrt{h}\,(K^{ij} - K h^{ij})$$

$$K_{ij} = \frac{1}{2} L_n h_{ij}$$

"a posteriori" b.c. on $\mathcal{P}_{ADM}$ there is no spacetime, just space, until we solve the dyn.

Dynamics & Syms are encoded in Dirac's Hypersurface Deformation "Algebra" (HDA)

$$\{V(X), V(Y)\} = V([X,Y])$$

$$\{V(X), H(N)\} = H(L_X N)$$

$$\{H(N), H(M)\} = V(Z)$$

$$\hookrightarrow Z^i = h^{ij}(N\partial_j M - M\partial_j N)$$

ation

$X \in \mathfrak{X}(\Sigma)$ v.f.
(infinitesimal diffeo of Σ)

$N \in C^\infty(\Sigma)$ "lapse"

The a posteriori geom interpret.
is $\xi^a = X^a + N n^a$ 4d diffeo

tangent to Σ normal to Σ

$(\star)$

$$V(X) = \int_\Sigma X^i v_i \quad , \quad v_i = \bar{\nabla}_j \pi^j{}_i$$

$$H(N) = \int_\Sigma N \mathcal{H} \quad , \quad \mathcal{H} = \frac{1}{2\sqrt{h}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) - \frac{\sqrt{h}}{2}\left(\bar{R}(h) - 2\Lambda\right)$$

Rmk 1

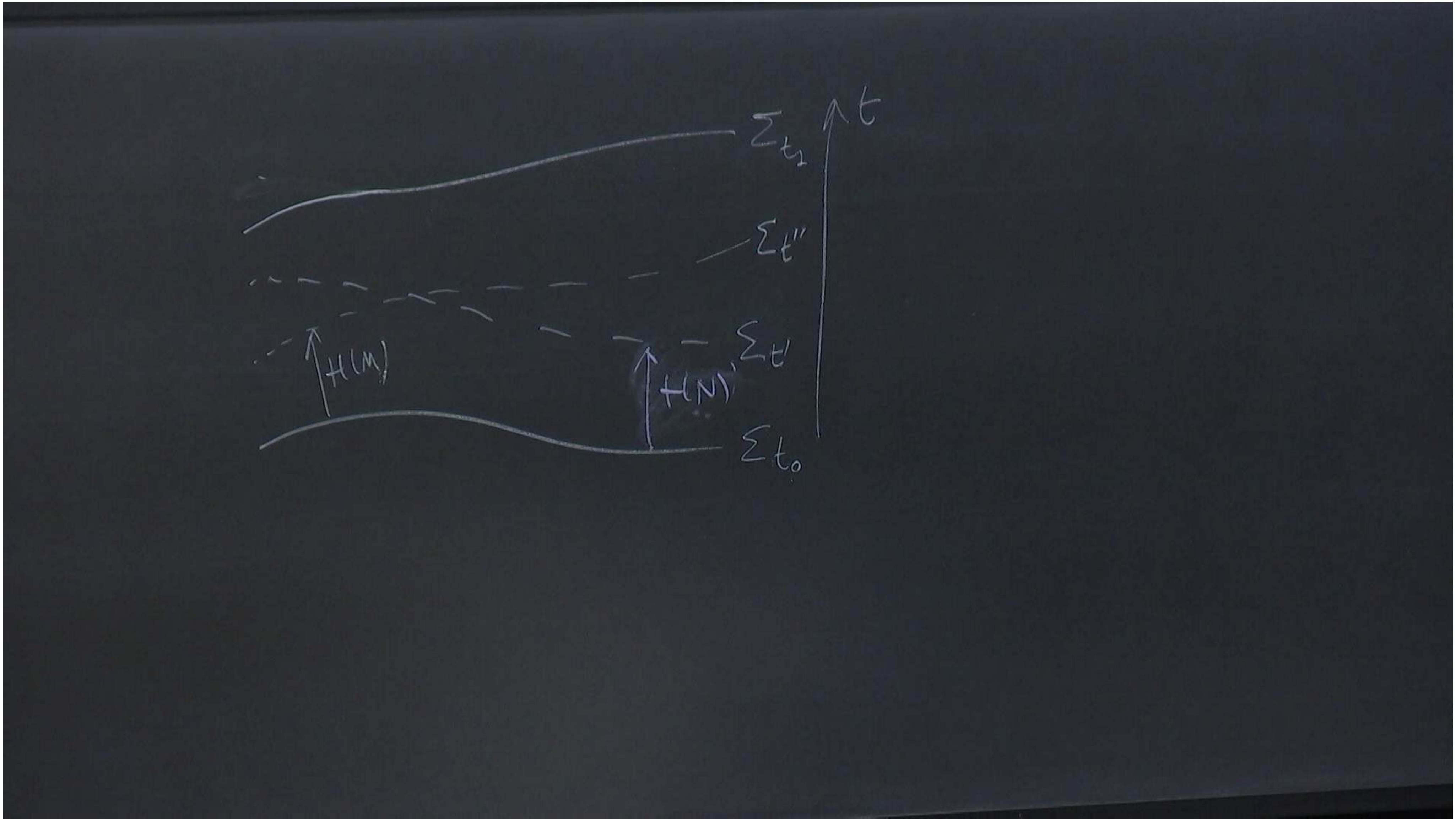
geometrical meaning of the HDA is

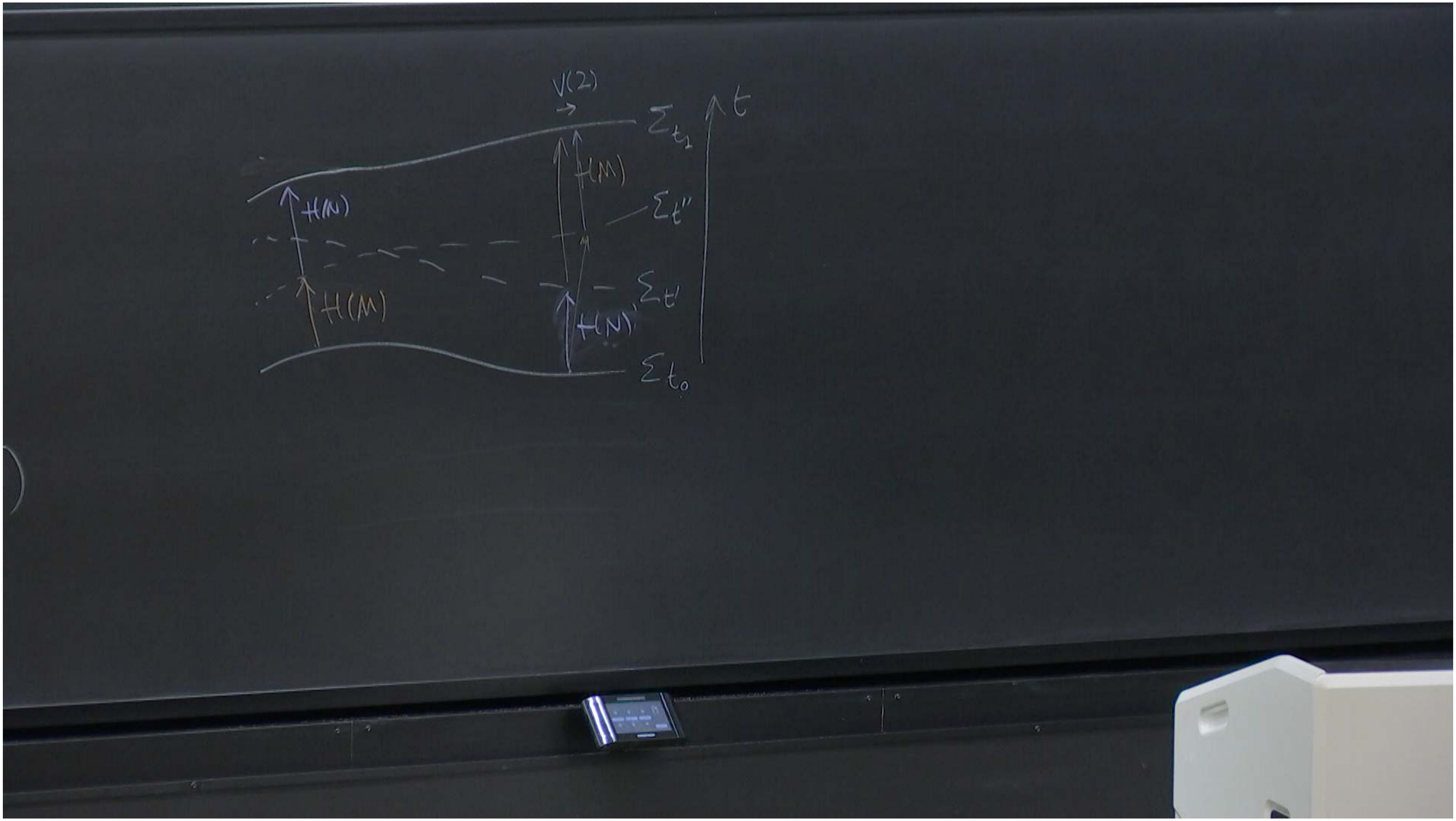
(i) 3d diffeos are symm. (V)

(ii) refoliation invariance of spacetime (H)

This is not special to GR, but to any theory formulated in a refoliation inv. way. (E.g. Dirac's parametrized scalar field)

↳ in this case V, H would be functions of (π, ϕ) with h_{ij} as a background structure


Σ_{t_1}
t
$\Sigma_{t''}$
$\Sigma_{t'}$
$H(M)$
$H(N)$
Σ_{t_0}



$\vec{V}(2)$
Σ_{t_1}
t
$\mathcal{H}(M)$
$\mathcal{H}(N)$
$\Sigma_{t''}$
$\Sigma_{t'}$
$\mathcal{H}(M)$
$\mathcal{H}(N)$
Σ_{t_0}

Rmk 2

The HDA is closely related to 4d diffeo invariance, but it is not the algebra of 4d diffeos: the 3+1 split has important consequences: Normal diffeos are not Hamiltonian sym & their action is "replaced" by $\{H(N), \cdot\}$

(Cf. with parametrized when reparam. invariance (= 1d diffeo!) is traded for the flow of $\{C, \cdot\}$ = dynamics)

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Rmk 3

(Teitelboim 73)

the only consistent way to implement the HDA on $\mathcal{P}_{ADM}$ requires

$$\mathcal{V}_i = 0 = \mathcal{H}$$

This is ultimately due to the dependence of $Z = Z(h)$

When $\mathcal{V}_i = 0 = \mathcal{H}$, the flow of $\mathcal{H}(N)$ can be interpreted as "reconstructing" a spacetime

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Rmk 3

(Teitelboim 73)

the only consistent way to implement the HDA on P_{ADM} requires

$$V_i = 0 = \mathcal{H}$$

This is ultimately due to the dependence of $Z = Z(h)$, $\{h_{ij}, \cdot\} \neq 0$

When $V_i = 0 = \mathcal{H}$, the flow of $\mathcal{H}(N)$ can be interpreted as "reconstructing" a spacetime

Rmk 4

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$\{h_{ij}, \cdot\} \neq 0$
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Rmk 4
(Hojman, Kuchař, Teitelboim '76)

modulo minor technical assumptions, the functional form of (☆) on $\mathcal{P}^{ADM}$ is unique

$\Longrightarrow$ HDA fixes GR completely once we declare h_{ij} is one of our (2) canonical variables

(Hamiltonian version of Lovelock thm)

$(h) - 2\Lambda)$

Physical Interpretation

(1) Since $V(X)$ generates 3d diff.
implementation of $V_i = 0$ means
that 3d diffeos are "pure gauge"

$\Rightarrow$ the objects of GR's 3+1 dynamics
are "abstract" 3-geometries

$\longrightarrow$ GEOMETRODYNAMICS
unfolds over Superspace $= \mathrm{Riem}(\Sigma) / \mathrm{Diff}(\Sigma)$
(Wheeler)

$\mathrm{Riem}(\Sigma) \ni h_{ij}$

and is generated by $\{H(N), \cdot\}$

diff.

means

"gauge"

dynamics

etries

$= \text{Riem}(\Sigma) / \text{Diff}(\Sigma)$

$) \ni h_{ij}$

(2) Although much more challenging to deal with, abstractly $H(N)$ is analogous to $C = P_t + h(p,q)$

Main differences

(i) $N(x)$ has no analogue in part. part.

b.c. that is a zero + 1 field theory

interpr = rate of clicking of a "lousy clock" at position $x \in \Sigma$

y to

is

q)

in pom. part.

eld theory

f a "lousy clock"

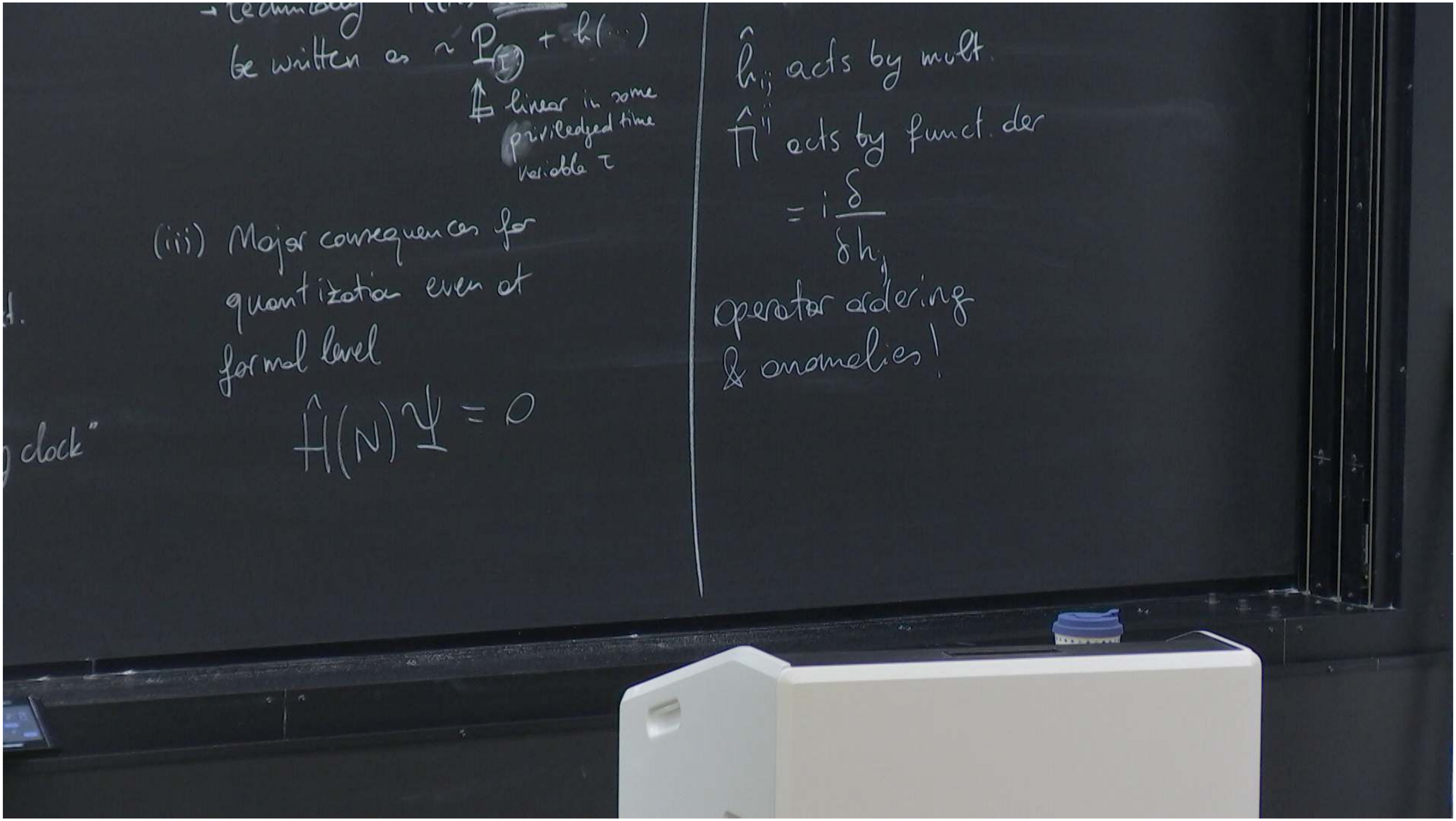
(ii) no priviledged time in GR

→ technically $H(N)$ cannot be written as $\sim P_{(I)} + h(\cdots)$

⇑ linear in some priviledged time variable τ

(iii) Major consequences for quantization even at formal level

$$\hat{H}(N)\Psi = 0$$


be written as ~ $P_{(I)} + h(\cdots)$
linear in some priviledged time variable τ
(iii) Major consequences for quantization even at formal level
$\hat{H}(N)\Psi = 0$
clock"
$\hat{h}_{ij}$ acts by mult.
$\hat{\pi}^{ij}$ acts by funct. der
$= i \frac{\delta}{\delta h_{ij}}$
operator ordering
& anomalies!

(off shell, canonical)

(π^{ij}, h_{ij})

metric on Σ

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$$\pi^{ij} = \sqrt{h}\,(K^{ij} - K h^{ij})$$

$K_{ij} = \frac{1}{2} \mathcal{L}_n h_{ij}$

"a posteriori" b.c. on $\mathcal{P}_{ADM}$ there is no spacetime, just space, until we solve the dyn.

$$g_{ab} = \begin{pmatrix} N^2 & X_i \\ X_i & h_{ij} \end{pmatrix}$$

in Dirac's Hypersurface De[formation] "Algebra" (HDA)

$$\{V(X), V(Y)\} = V([X,Y$$

$$\{V(X), H(N)\} = H(\mathcal{L}_X$$

$$\{H(N), H(M)\} = V(Z)$$

$$\hookrightarrow Z^i = h^{ij}(N\partial_j M - M$$