Title: Quantum Gravity

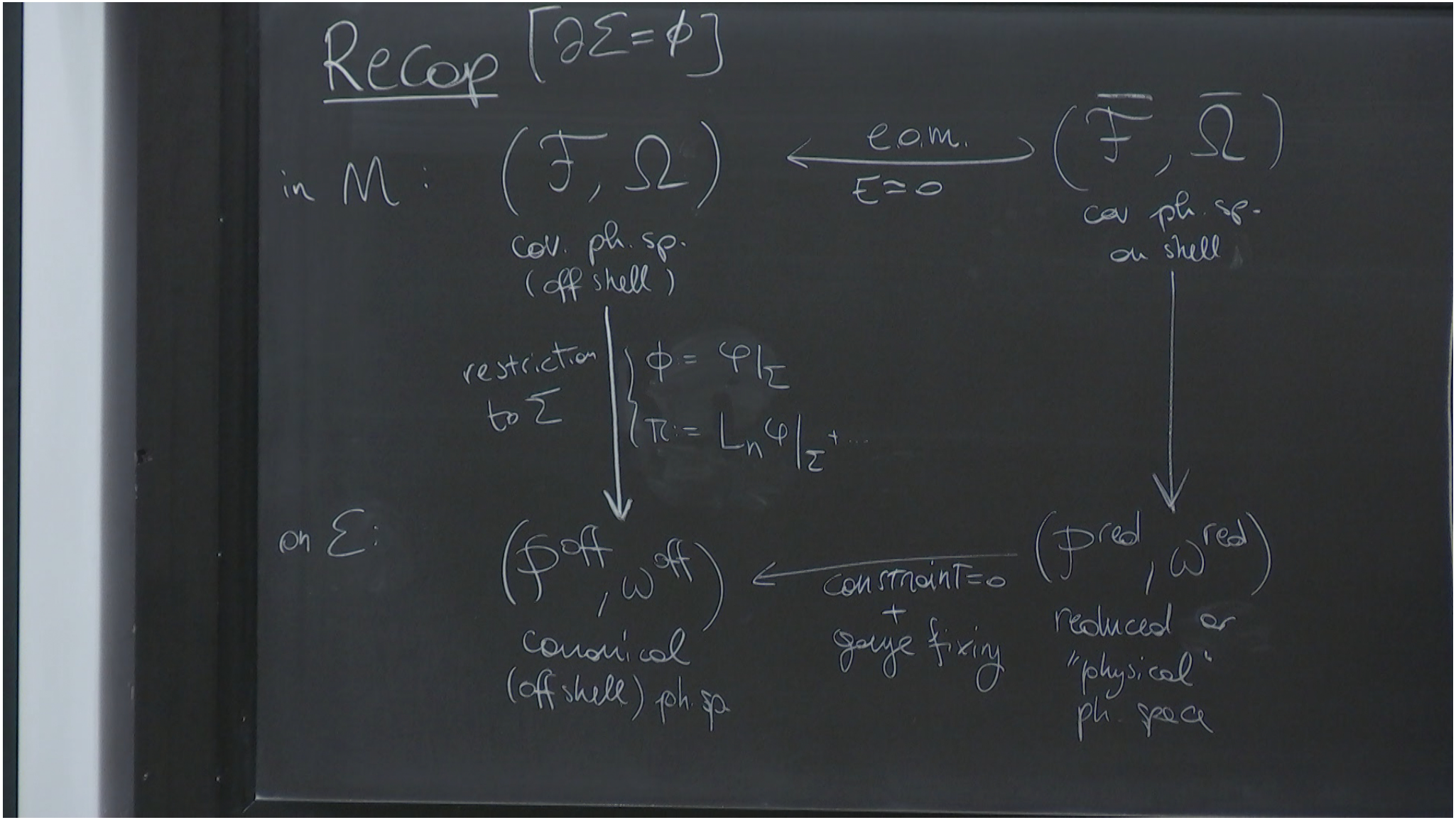
Speakers: Aldo Riello

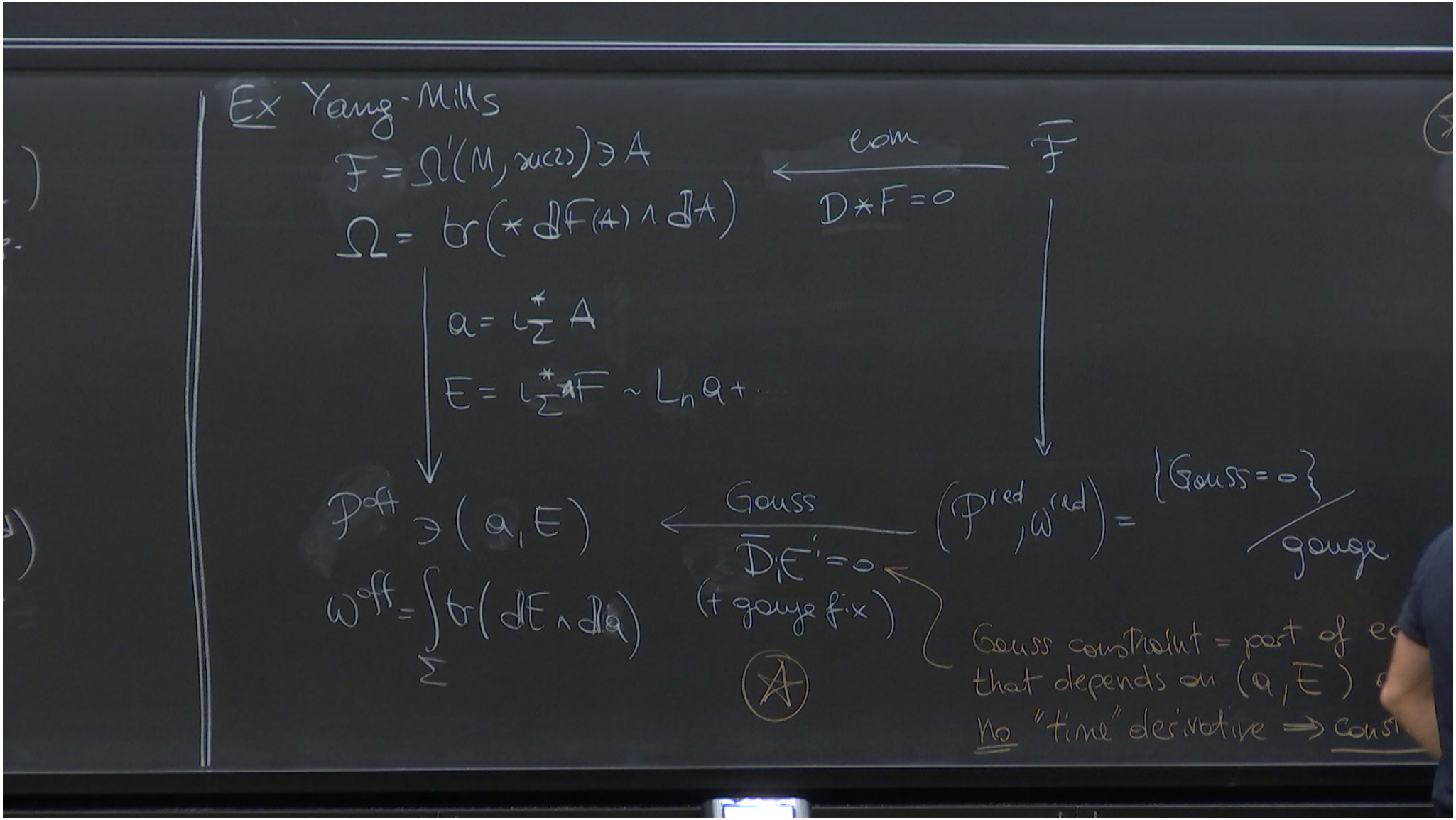
Collection: Quantum Gravity (2021-2022)

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Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.


Recap [∂Σ=∅]
in M:
(F, Ω)
e.o.m.
E=0
(F̄, Ω̄)
cov. ph. sp. (off shell)
cov ph. sp. on shell
restriction to Σ
ϕ := φ|Σ
π := Ln φ|Σ + ...
on Σ:
(P^off, ω^off)
constraint=0 + gauge fixing
(P^red, ω^red)
canonical (off shell) ph sp
reduced or "physical" ph space


Ex Yang-Mills
$\mathcal{F} = \Omega^1(M, su(2)) \ni A$
$\Omega = tr(* d_A F(A) \wedge d_A A)$
eom
$D * F = 0$
$\bar{\mathcal{F}}$
$a = \iota_\Sigma^* A$
$E = \iota_\Sigma^* {}^*F \sim L_n a + \dots$
$\mathcal{P}^{off} \ni (a, E)$
$\omega^{off} = \int_\Sigma tr(dE \wedge d_A a)$
Gauss
$\bar{D}_i E^i = 0$
(+ gauge fix)
$(\mathcal{P}^{red}, \omega^{red}) = \frac{\{\text{Gauss} = 0\}}{\text{gauge}}$
Gauss constraint = part of eq
that depends on (a, E)
no "time" derivative ⇒ consi

★ "Constraints generate gauge transformations"

- in $\mathcal{F}$: $i_{\rho(\xi)}\Omega = -\mathrm{d}J(\xi)$
- in $\mathcal{F}^{\mathrm{off}}$: $i_{\rho(\xi)}\omega^{\mathrm{off}} = -\mathrm{d}\,\mathrm{Gauss}(\xi) \approx 0$

↳ if ω^{red} has to be non-degenerate, setting $\mathrm{Gauss} = 0$, requires quotienting/fixing gauge

$\mathrm{d}) = \dfrac{\{\mathrm{Gauss} = 0\}}{\mathrm{gauge}}$

constraint = part of eom
depends on (q, E) and
ne" derivative ⇒ constraint on initial data

GR

in $\mathcal{F}$: $\Theta_\Sigma = \int_\Sigma \sqrt{h}\,(K^{ij} - K h^{ij})\,\delta h_{ij} - 2\delta \int_\Sigma \sqrt{h}\,K - \int_{\partial\Sigma} \sqrt{\sigma}\,\beta$ $\quad \beta = dn^a - g^{ab} d(n_b)$

(York; $\partial\Sigma = \emptyset$)

$2K_{ij} = \mathcal{L}_n h_{ij}$

"Legendre transform" $\pi^{ij} = \sqrt{h}\,(K^{ij} - K h^{ij})$

on Σ: ω_{ADM}

(off shell)

GR

in $\mathcal{F}$: $\Theta_\Sigma = \int_\Sigma \sqrt{h}\,(K^{ij} - K h^{ij})\,\delta h_{ij} - 2\delta \int_\Sigma \sqrt{h}\,K - \int_{\partial\Sigma} \sqrt{\sigma}\,\beta$ $\quad \beta = dn^a - g^{ab} d(n_b)$

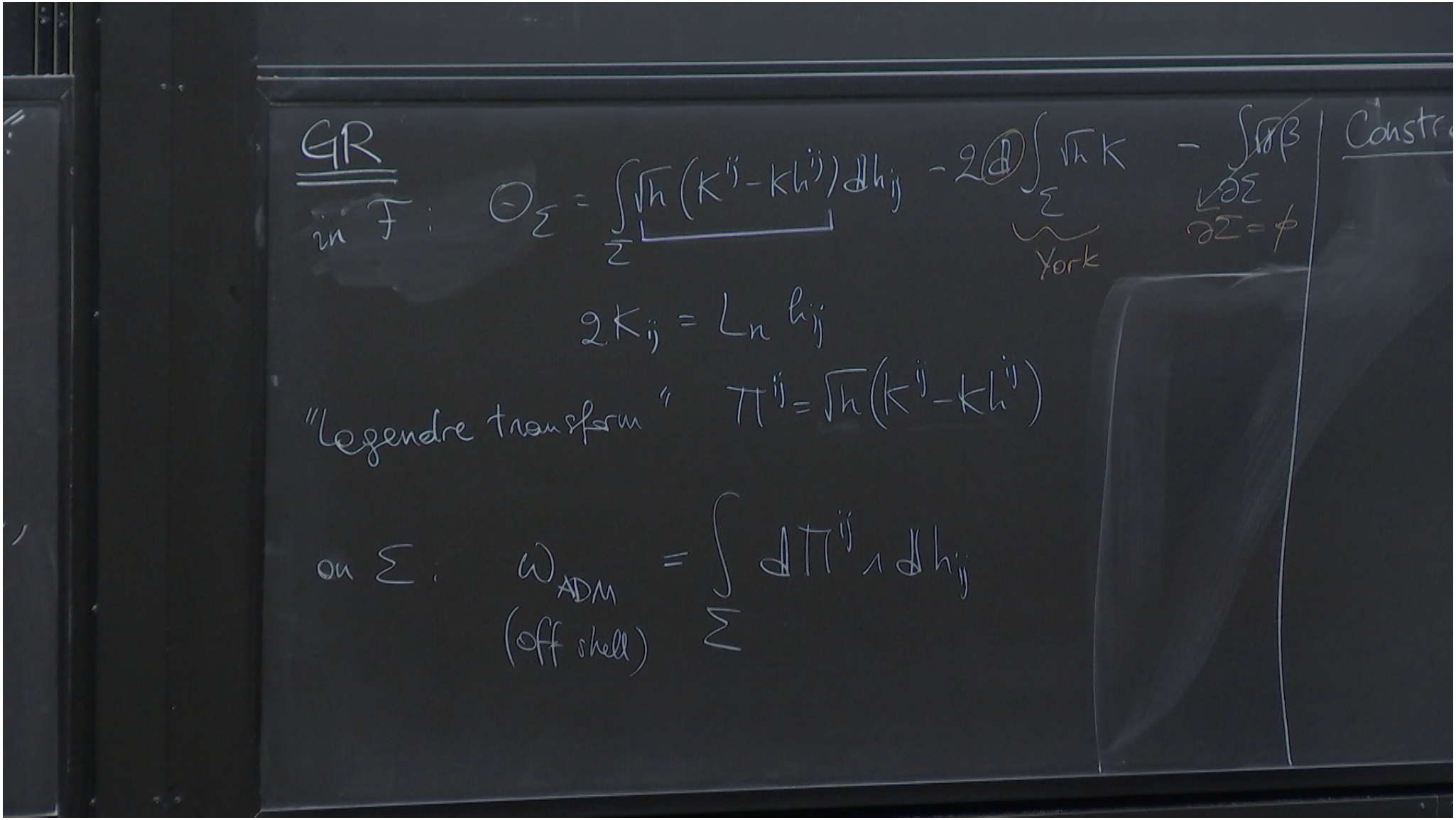
(the middle integral: York; $\partial\Sigma = \phi$)

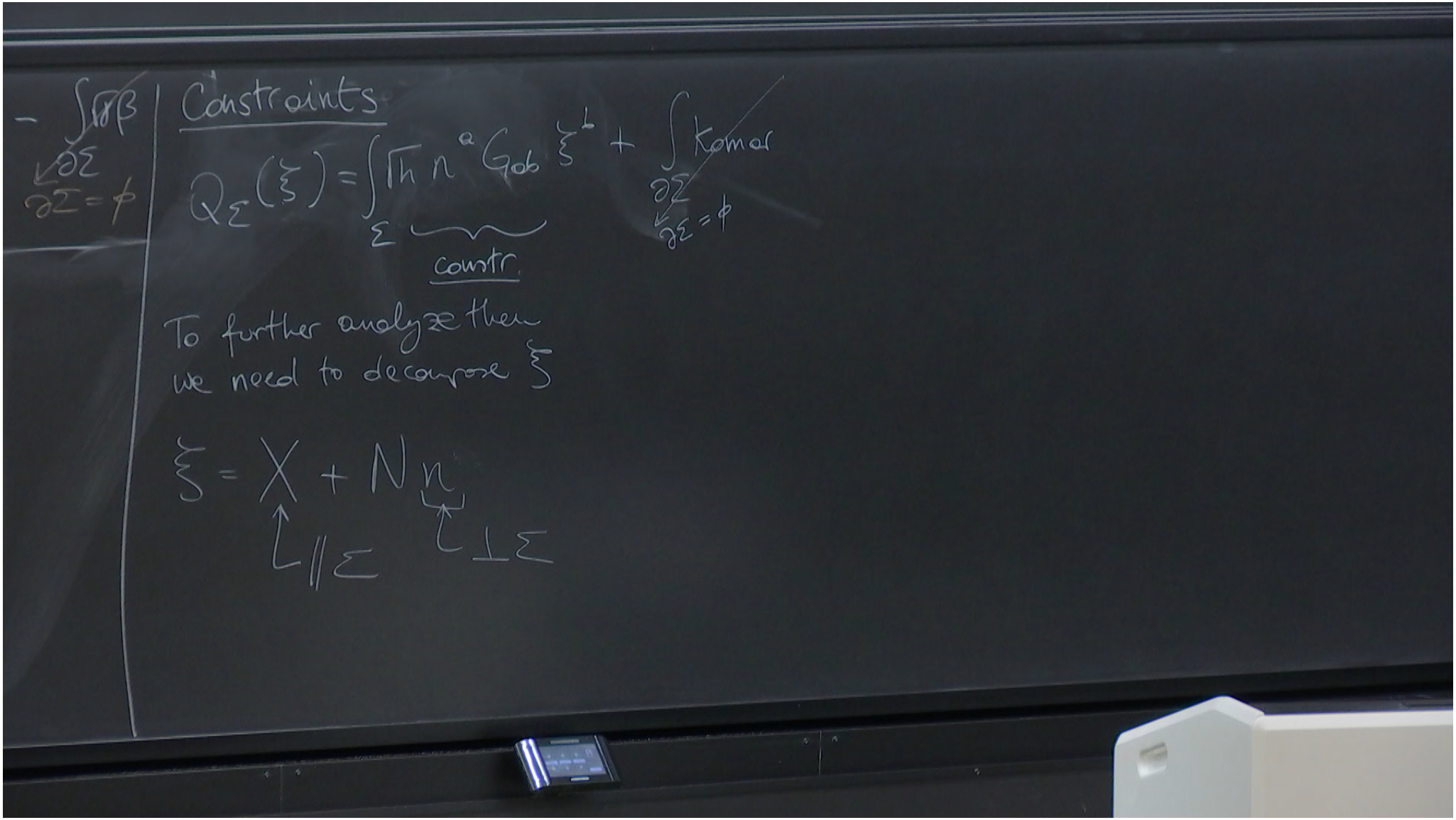
$$2K_{ij} = \mathcal{L}_n h_{ij}$$

"Legendre transform" $\quad \pi^{ij} = \sqrt{h}\,(K^{ij} - K h^{ij})$

on Σ: $\quad \omega_{ADM} = \int_\Sigma \delta\pi^{ij} \wedge \delta h_{ij}$

(off shell)


GR
in $\mathcal{F}$: $\Theta_\Sigma = \int_\Sigma \sqrt{h}\,(K^{ij} - K h^{ij})\,d h_{ij}$ $- 2 \int_\Sigma \sqrt{h}\,K$ $- \int_{\partial\Sigma} \sqrt{\sigma}\,\beta$
York
$\partial\Sigma = \phi$
$2K_{ij} = \mathcal{L}_n h_{ij}$
"Legendre transform" $\pi^{ij} = \sqrt{h}\,(K^{ij} - K h^{ij})$
on Σ: $\omega_{ADM} = \int_\Sigma d\pi^{ij} \wedge d h_{ij}$
(off shell)
Constr


Constraints
$Q_\Sigma(\xi) = \int_\Sigma \sqrt{h}\, n^a G_{ab} \xi^b + \int_{\partial\Sigma} \text{Komar}$
$\partial\Sigma = \emptyset$
constr.
To further analyze them we need to decompose ξ
$\xi = X + N n$
$\parallel \Sigma$
$\perp \Sigma$

Vector constr

$$V(X) \equiv Q_\Sigma(X) = \int_\Sigma (\bar{\nabla}_i \pi^i{}_j) X^j$$

↑ tensor density

Scalar or Hamiltonian constr.

$$H(N) = Q_\Sigma(Nn) = \int_\Sigma N \left[\frac{1}{2\sqrt{h}} \underbrace{\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2 \right)}_{\text{"kinetic"}} - \underbrace{\frac{1}{2}\sqrt{h}\left({}^3\bar{R} - 2\Lambda \right)}_{\text{"potential" it's a function of } h \text{ only}} \right]$$

Hamiltonian properties

in $\mathcal{F}$: $i_{\varphi(\xi)}\Omega_\Sigma = -\delta Q_\Sigma(\xi) + \int_\Sigma \xi \cdot E + (\partial\Sigma)$

in $\mathcal{P}^{ADM}$: $i_{\varphi(X)}\omega^{ADM} = -\delta V(X)$ ← "like YM"

" $i_{\varphi(N)}\omega^{ADM} = -\delta H(N) + \int_\Sigma N(E)$ "

cannot be written in terms of (π, h) only!

⇒ Cannot be written on $\mathcal{P}^{ADM}$

(Lee-Wald 1991)

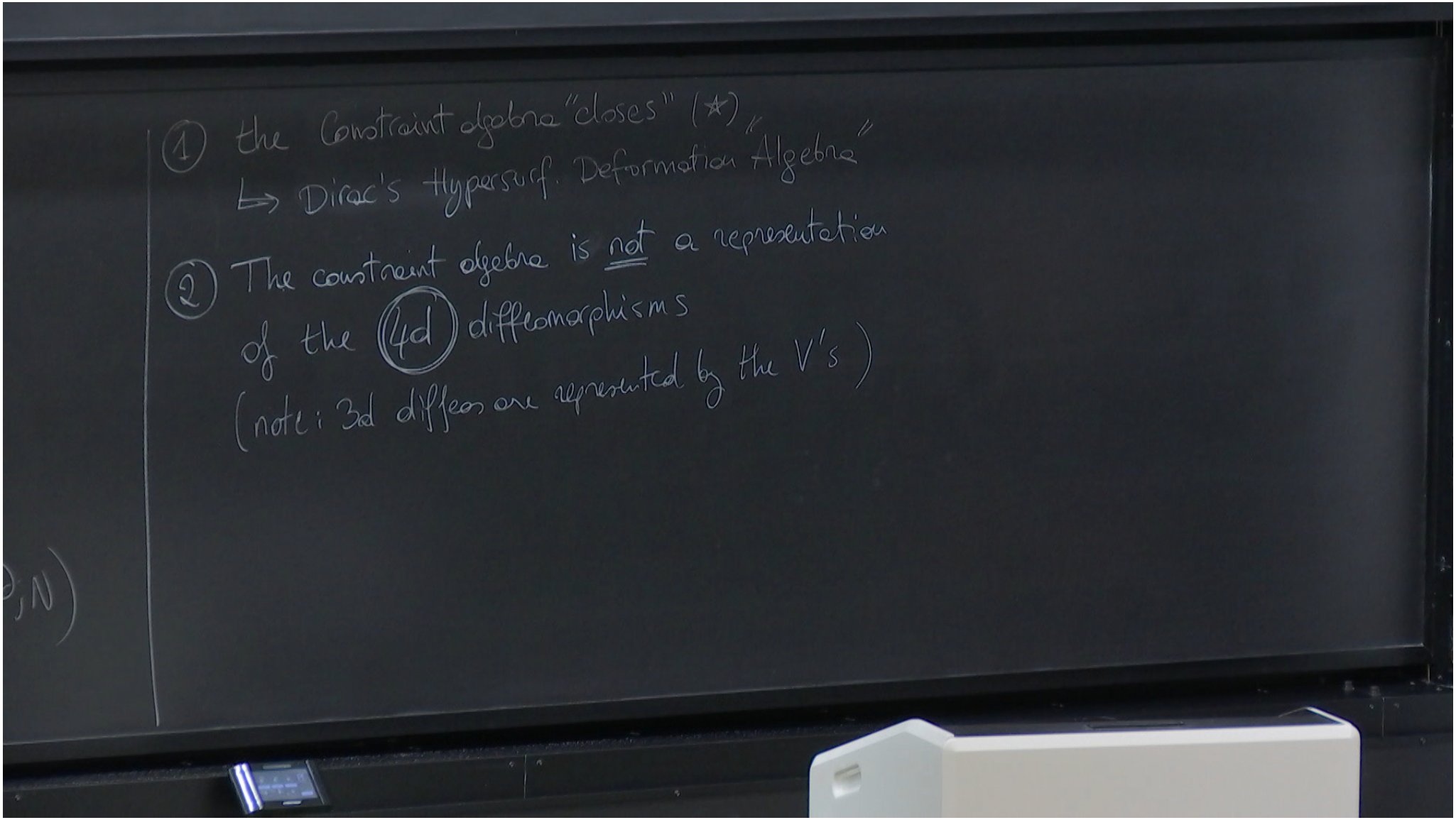
To consistently set Constr = 0 on p°ff

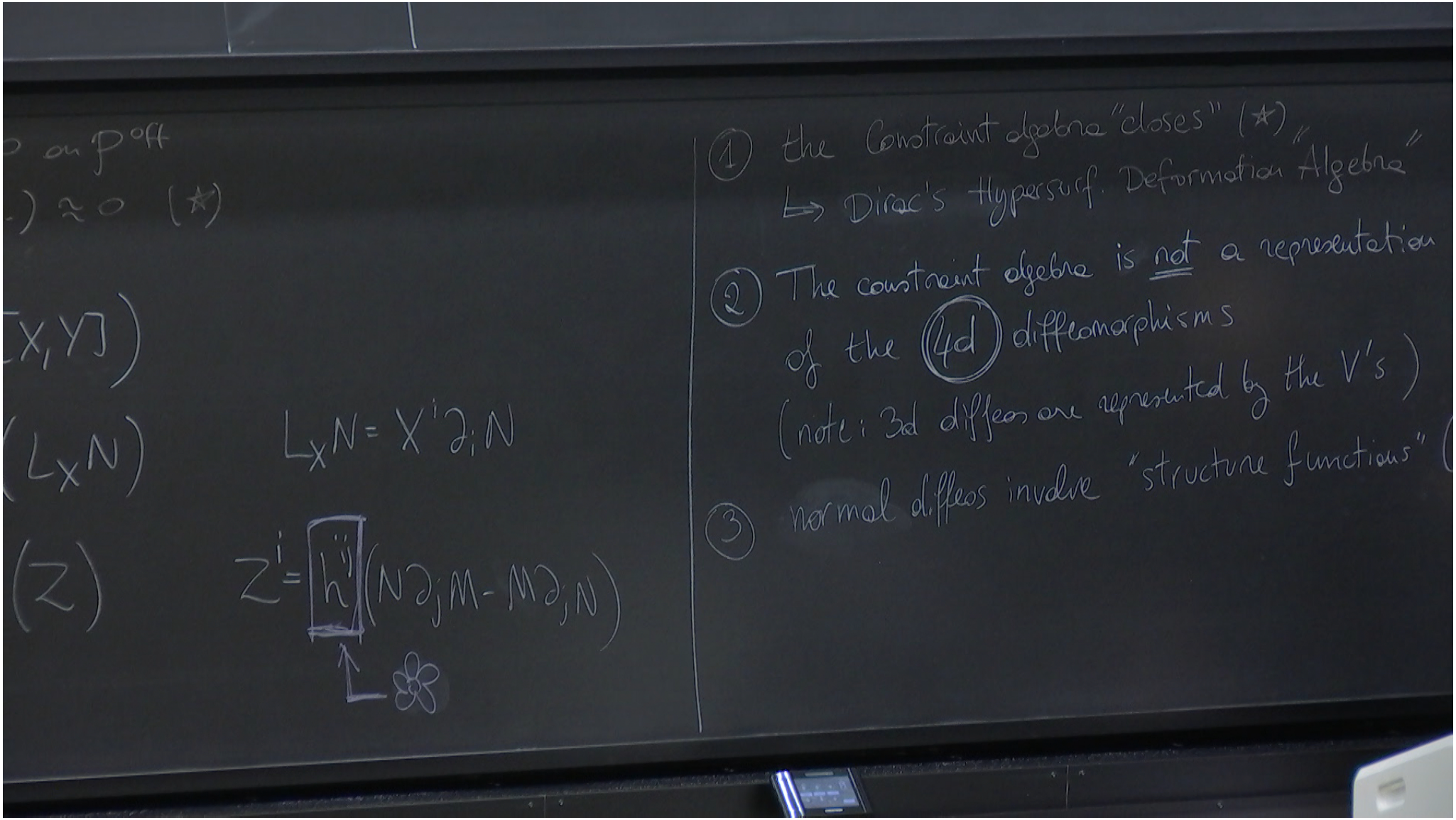
we need $\{C(\xi), C(\eta)\} = C(\dots) \approx 0$

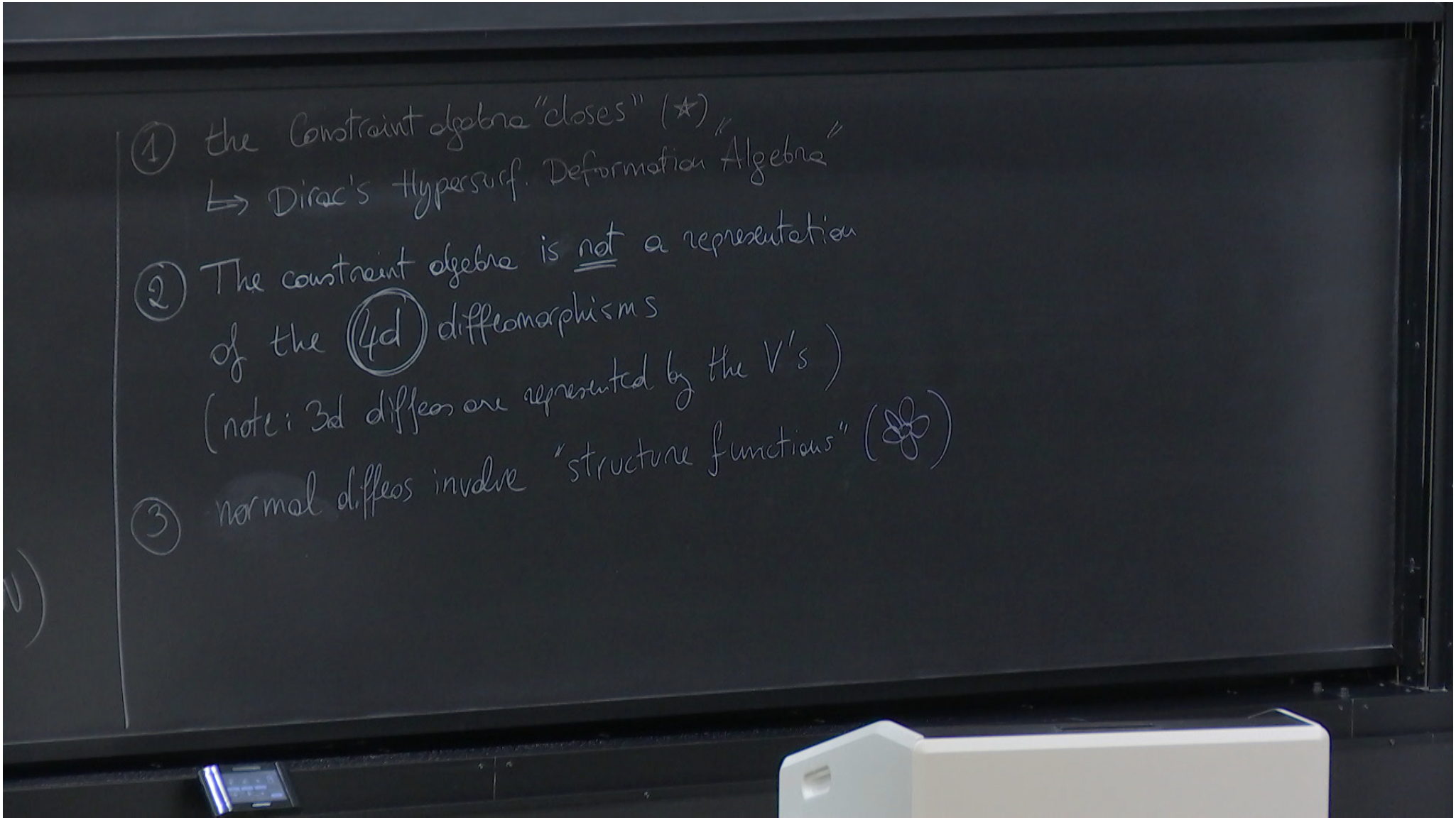
↳ $\{V(X), V(Y)\} = V([X,Y])$

$\{V(X), H(N)\} = H(L_X N)$ $\quad L_X N = X^i \partial_i N$

$\{H(N), H(M)\} = V(Z)$ $\quad Z^i = \boxed{h^{ij}}(N\partial_j M - M\partial_j N)$


① the Constraint algebra "closes" (*)
↳ Dirac's Hypersurf. Deformation Algebra"
② The constraint algebra is not a representation
of the (4d) diffeomorphisms
(note: 3d diffeos are represented by the V's)


on P off
≈ 0 (★)
[X, Y])
(L_X N)
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(Z)
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③ normal diffeos involve "structure functions"


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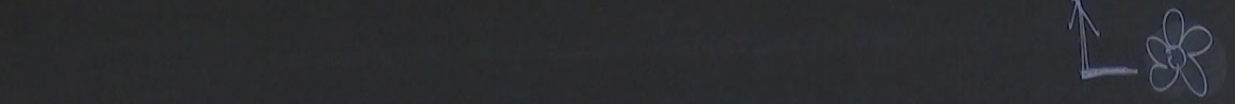
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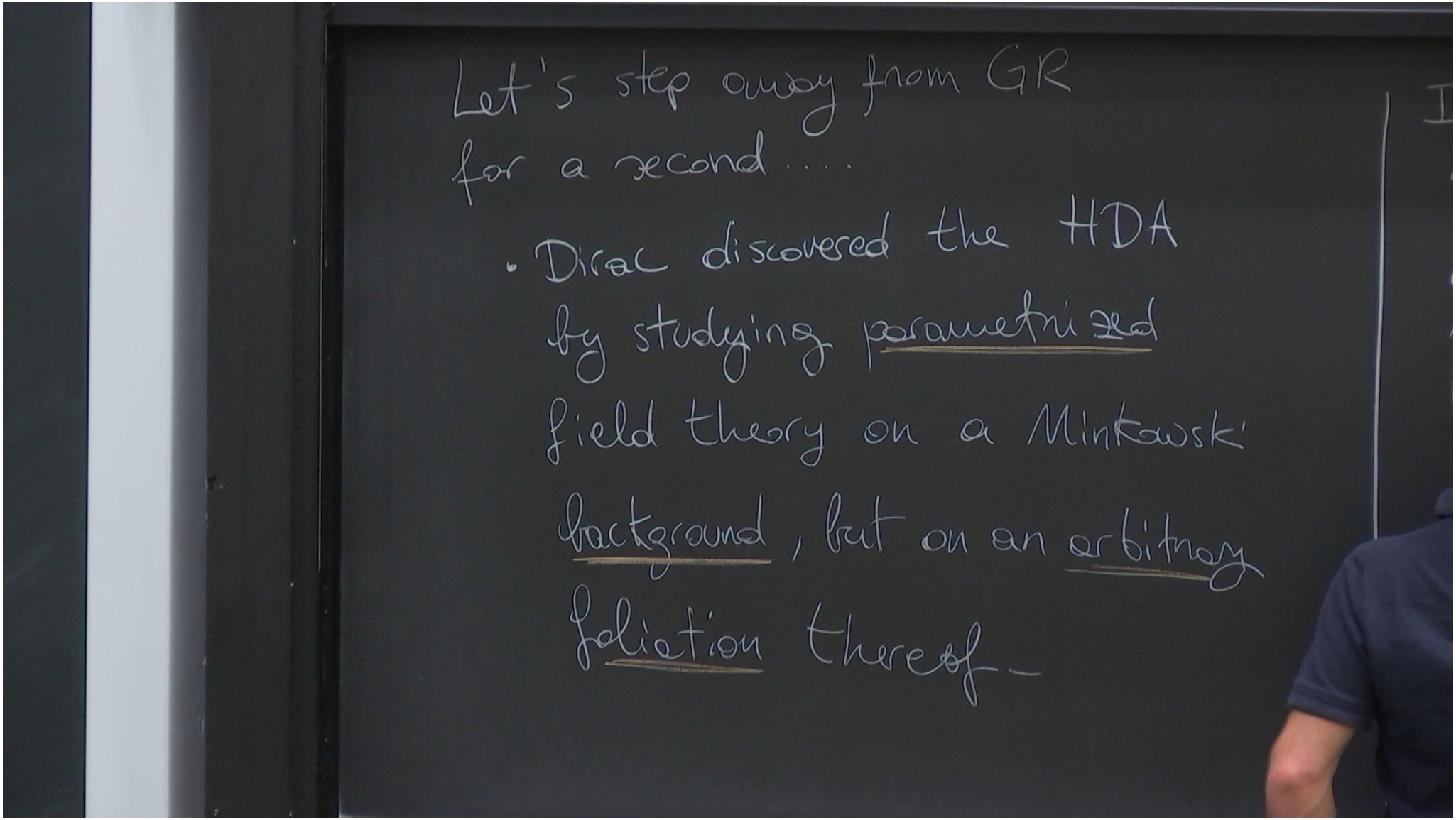
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Let's step away from GR
for a second....
• Dirac discovered the HDA
by studying parametrized
field theory on a Minkowski
background, but on an arbitrary
foliation thereof.

In Dirac's case

$\mathcal{P} = (\phi, \pi_\phi)$

and h_{ij}, K_{ij} were "background" ($\delta h \equiv 0 \equiv \delta k$)

eg $V(X) \approx$ generator of "local" transl.

$H(N) \approx$ — " — normal evolution

→ they satisfy the HDA

- In GR we do (not) have a background SPACETIME

In fact, on β^{ADM} we do not even have an "a priori" geometrical meaning for Π^{ij}

$\Rightarrow$ In <u>Geometrodynamics</u> ($\equiv$ 3+1 GR) the only way to "represent" the HDA

ac's case

ϕ, π)

h_{ij}, K_{ij} were.

ground" ($\not d h = 0 = \not d k$)

X) ≈ generator of "local" transl.

(N) ≈ — n — normal evolution

satisfy the HDA

- In GR we do (not) have a background SPACETIME

In fact, on $\mathcal{F}^{ADM}$ we do not even have an "a priori" geometrical meaning for Π^{ij}

⇒ In Geometrodynamics (≡ 3+1 GR) the only way to "represent" the HDA is to set $V = 0 = H$

...ave a
...SPACETIME
...$\oint_{ADM}$ we
...have an "a priori"
...earning for Π")

...metrodynamics ($\equiv$ 3+1 GR)
...way to "represent" the
...to set $V=0=H$

and, from this, one deduces the existence of a 4d spacetime out of the canonical data

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and, from this, one deduces the existence of a 4d spacetime out of the canonical data

To consistently set Constr = 0 on p$^{\text{off}}$

we need $\{C(\xi), C(\eta)\} = C(\dots) \approx 0$ $(\star)$

↳ $\{V(X), V(Y)\} = V([X,Y])$

$\{V(X), H(N)\} = H(L_X N)$ $\quad L_X N = X^i \partial_i N$

→ $\{H(N), H(M)\} = V(Z)$ $\quad Z^i = h^{ij}(N\partial_j M - M\partial_j N)$