Title: Cosmological tale: String Cosmology backgrounds from Classical String Geometry

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Series: Quantum Gravity

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Abstract: In this talk, I intend to give a pedagogical, nonetheless biased, introduction to String Cosmology. After briefly reviewing the Lambda-CDM model and motivating inflation, we will remind ourselves that early universe cosmology remains singular and waiting for alternatives. That will be our cue to consider string theory to define our gravity sector and string thermodynamics to define our matter sector. We will learn how that doesn't work unless we consider more string corrections (in fact, an infinity of them!). Once we are stringy enough, we will be able to build a full cosmological model that is non-singular and already poses itself as an alternative to inflation at the background level.

Zoom Link: https://pitp.zoom.us/j/98491655811?pwd=ZzArNjFVMmZIdE1STjd2TVNWSlZtZz09

α'-Cosmological Tale: String Cosmology Backgrounds from Classical String Geometry

Guilherme Franzmann

In collaboration with **Robert Brandenberger** and **Heliudson Bernardo**
(hep-th/1911.00088, hep-th/2002.09856, hep-th/2005.08324, and hep-th/2007.14096)

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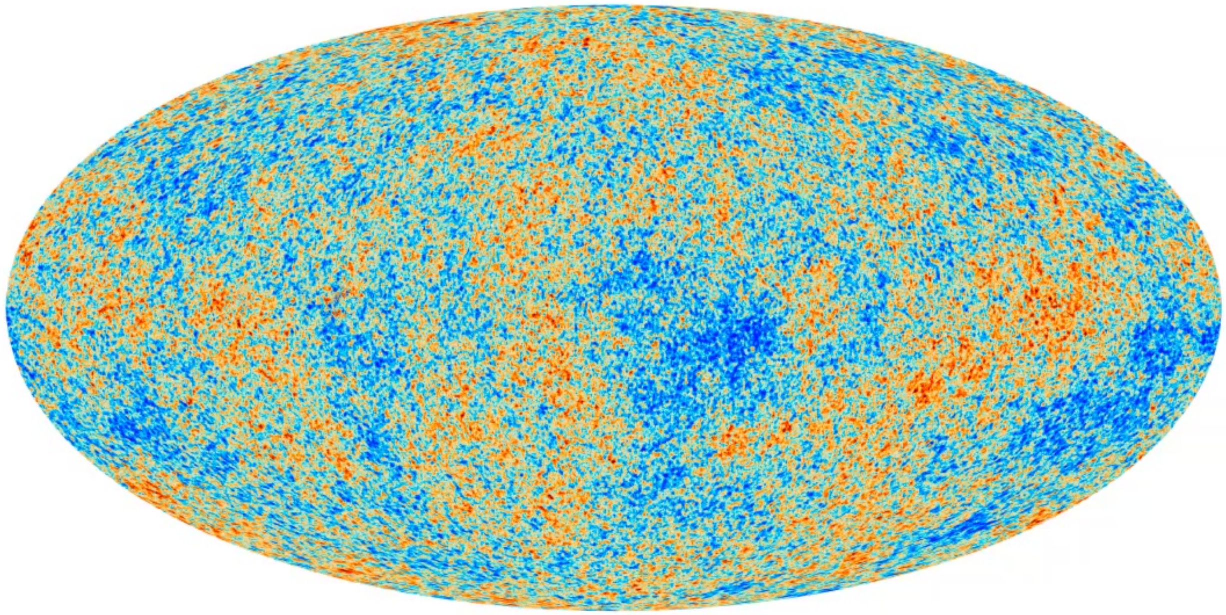

Black Body Radiation with average $T \sim 2.7$ K and fluctuations of order $\Delta T/T \sim 10^{-5}$.

(Planck Collaboration, '13)

ΛCDM Model

- With only **6 parameters** it explains all the current cosmological data
- Among these parameters, two are related to the initial fluctuations that gave rise to the CMB

$$P(k) = \Delta_R^2 k^{n_s - 1},$$

where $\Delta_R^2 \sim 2.5\times10^{-9}$ and $n_s \sim 0.9667$[1]. The small value of $n_s - 1$ encodes the almost scale-invariance of the power spectrum

[1] (Planck Collaboration, '13)

1970: Peebles and Yu showed that in order to recover an almost scale-invariant power spectrum there should be a way to produce standing waves (constant amplitude) fluctuations on super-Hubble scales (H^{-1}) before the CMB was emitted.

How does one obtain such a spectrum? The fact it's almost scale-invariant/thermalized is puzzling and would seem to imply acausal physics.

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Inflation

- First causal scenario that yields such physics by invoking **a phase of quasi-de Sitter expansion**
- It predicts an almost scale-invariant power spectrum for the fluctuations with a red tilt: $n_s - 1 < 0$ (Mukhanov and Chibisov, '81)

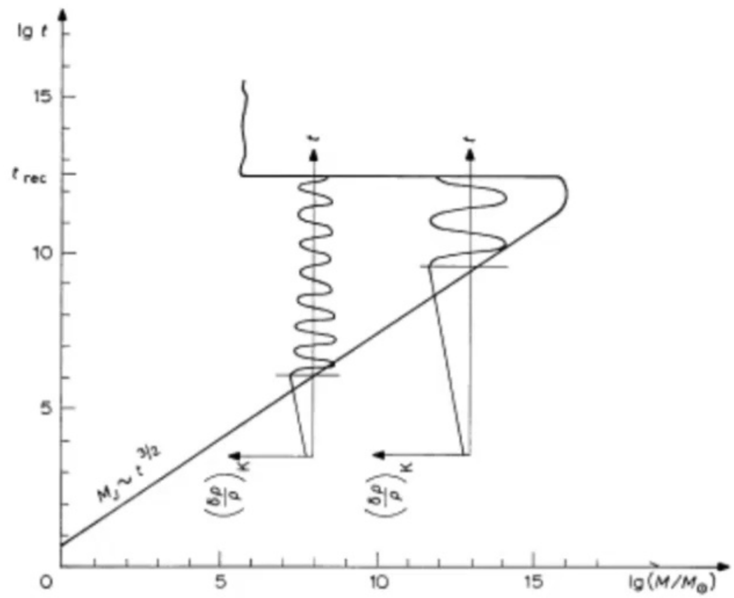

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

(Sunyaev and Zeldovich, '70)

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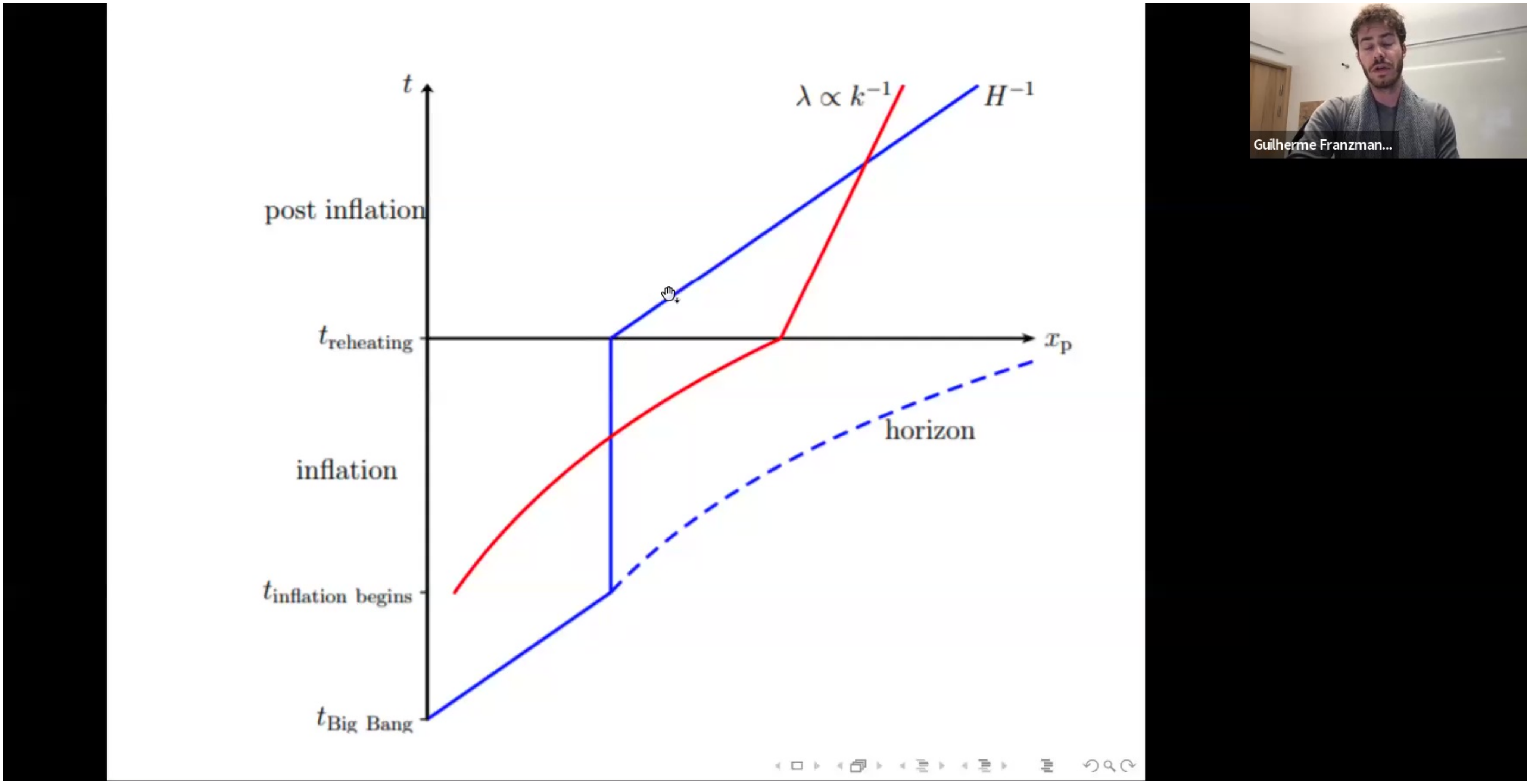

t
$\lambda \propto k^{-1}$
H^{-1}
post inflation
$t_{\text{reheating}}$
x_{p}
horizon
inflation
$t_{\text{inflation begins}}$
$t_{\text{Big Bang}}$
Guilherme Franzman...

Problems with Inflation

- Non-fundamental scalar field
- Eternal inflation and multiverse (Brandenberger, Costa, GF, '15)
- Trans-planckian problem (Martin and Brandenberger, '01)
- **Singularity** (Borde and Vilenkin '94)


INFLATION


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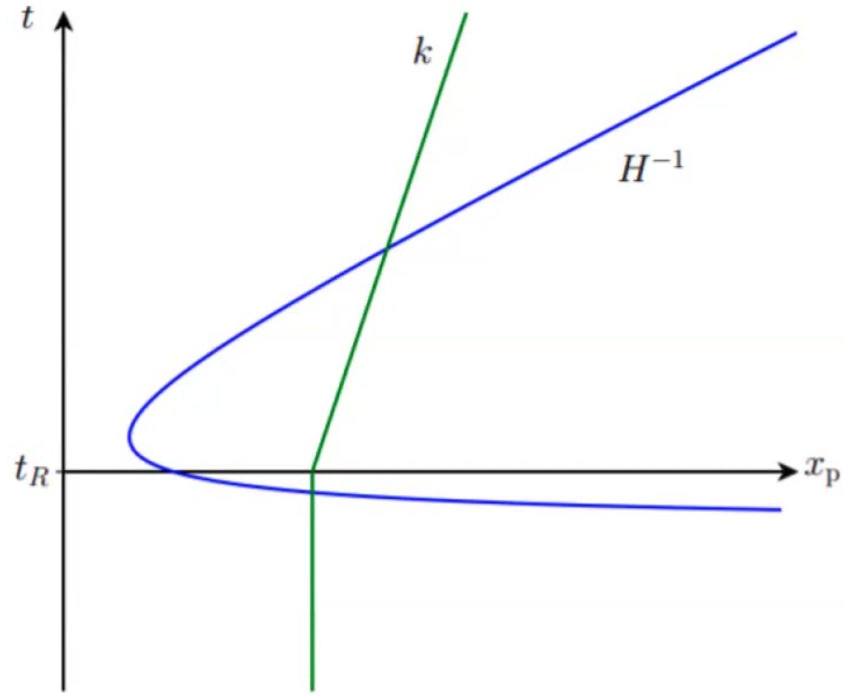
Alternatives? Yes!



Emergent Scenario together with holographic scaling.

Summary so far

- Standard Cosmological Model remains incomplete
- Singularities are classical, thus finding a good QG theory should do it: **string theory**[2]
- Strings allow for new degrees of freedom and introduce new symmetries/dualities

[2] No convincing embedding of inflation in string theory so far, e.g. swampland conjectures (Vafa et al., '18) and TCC (Bedroya et al., '19)

End of the first act.

Questions?

Summary so far

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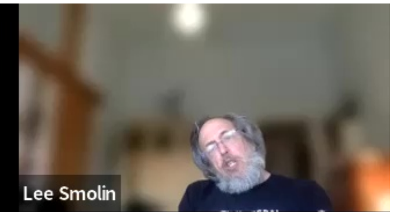


Second act!

- String Theory remains (*not biased at all...*) the main candidate for a theory of Quantum Gravity
- Perturbative structure of the theory is well known
  - Expansion in g_s: dimensionless string coupling (*QFTish*)
  - Expansion in $\sqrt{\alpha'}$: string length (*stringish*)
- Low energy EFT (supergravities, GR limit) valid for small curvature: $\sqrt{\alpha'}/R_c \ll 1$
- Singularities happen for $\mathcal{R} \to \infty$, so not only α'-corrections but non-perturbative regime is fundamental (Wang, Wu, Yang and Ying, '19)

Hero's Journey

- B-supergravity action + cosmological ansatz: global $O(d,d)$ (Meissner and Veneziano, '91) generalizing scale-factor duality (Veneziano, '91)
- $O(d,d)$ Covariant String Cosmology developed but lowest order in α' (Gasperini and Veneziano, '92)
- Impressively, $O(d,d)$ symmetry present to all orders in α' as long as fields are spatially independent (Sen, '92)
- $O(d,d)$ transf. receive α' corrections, **but**:
  - one can redefine the fields to keep their form at least in 1st order in α' (Meissner, '97)
  - assuming this is the case to ALL orders (which certainly happens in conventional string field theory variables (Kugo and Zwiebach, '92)), one can classify all $O(d,d)$ invariant α'-corrections **(Hohm and Zwiebach, '19)**
- No matter sources considered though!

Objectives

1. To construct a full α'-corrected manifestly $O(d,d)$ covariant formulation of String Cosmology in the presence of matter
2. To build a non-singular very early universe model including all stringy corrections

Lowest Order Vacuum Action

- Gravitational sector of all string theories include: $G_{\mu\nu}$, $B_{\mu\nu}$ and ϕ (massless level of bosonic theory) [Weyl Anomaly]

$$S_0 = \frac{1}{2\kappa^2}\int d^D x \sqrt{-G}\, e^{-2\phi}\left(R + 4G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

- For a **cosmological background:** $G_{00} = -n^2(t)$, $G_{0i} = 0$, $G_{ij} = g_{ij}(t)$, $B_{00} = 0$, $B_{0i} = 0$, $B_{ij} = b_{ij}(t)$ and $\phi = \phi(t)$

$$S_0 = -\frac{1}{2\kappa^2}\int d^d x dt\; n e^{-\Phi}\left[(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2\right]$$

where $\mathcal{D} = 1/n\partial_t$, $\Phi = 2\phi - \ln\sqrt{\det g}$ and $D = d + 1$ (Meissner and Veneziano, '91)

- The action is manifestly invariant under the global $O(d,d)$ and time reparametrization

$$\mathcal{S} = \eta\mathcal{H} = \begin{pmatrix} bg^{-1} & g - bg^{-1}b \\ g^{-1} & -g^{-1}b \end{pmatrix}$$

where

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}$$

η being an $O(d,d)$ metric and $\mathcal{H} \in O(d,d)$ while Φ is a scalar under $O(d,d)$. The **EOM** are (Gasperini and Veneziano, '91)

$$2\mathcal{D}^2\Phi - (\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = 0 \quad (\Phi)$$

$$\mathcal{D}^2\mathcal{S} + \mathcal{S}(\mathcal{D}\mathcal{S})^2 - (\mathcal{D}\Phi)(\mathcal{D}\mathcal{S}) = 0 \quad (\mathcal{S})$$

$$(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = 0 \quad (n)$$

Vanilla Matter Coupling

- Consider matter action (Gasperini and Veneziano, '91) to be $O(d,d)$ invariant (true for a gas of ○-strings (Gasperini, '07) and (Tseytlin and Vafa, '92))

$$S_T = -\frac{1}{2\kappa^2}\int d^d x dt\, n e^{-\Phi}\left[(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2\right] + \boxed{S_m[\Phi, \mathcal{S}, n, \chi]}$$

- We also include a dependence on the shifted dilaton field $\Phi(t)$ in the matter action (Gasperini, '07)
- The **EOM** are

$$(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = \boxed{-2\kappa^2 e^{\Phi}\frac{\delta S_m}{\delta n} = 2\kappa^2 e^{\Phi}\bar{\rho}}$$

$$\mathcal{D}^2\mathcal{S} + \mathcal{S}(\mathcal{D}\mathcal{S})^2 - \mathcal{D}\Phi\mathcal{D}\mathcal{S} = \boxed{2\kappa^2 e^{\Phi}\mathcal{S}\eta\bar{\mathcal{T}}}$$

$$2\mathcal{D}^2\Phi - (\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = \boxed{-2\kappa^2\frac{e^{\Phi}}{n}\frac{\delta S_m}{\delta\Phi} \equiv \kappa^2 e^{\Phi}\bar{\sigma}}$$

- The **EOM** are

$$2\mathcal{D}^2\Phi - (\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = \kappa^2 e^{\Phi}\bar{\sigma}$$
$$\mathcal{D}^2\mathcal{S} + \mathcal{S}(\mathcal{D}\mathcal{S})^2 - \mathcal{D}\Phi\mathcal{D}\mathcal{S} = 2\kappa^2 e^{\Phi}\mathcal{S}\eta\bar{\mathcal{T}}$$
$$(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2 = 2\kappa^2 e^{\Phi}\bar{\rho}$$

- The **continuity equation** is

$$\mathcal{D}\bar{\rho} + \frac{1}{4}\mathrm{tr}\left[\mathcal{S}(\mathcal{D}\mathcal{S})\eta\bar{\mathcal{T}}\right] = \boxed{\frac{1}{2}\bar{\sigma}\mathcal{D}\Phi}$$

- These equations define $O(d,d)$ covariant String Cosmology to lowest order in α'. They reduce to the Pre-Big Bang equations once evaluated in components of the $O(d,d)$ tensors.

An Intermission: String Cosmology

- **FLRW + perfect fluid:** $g_{ij} = a^2(t)\delta_{ij}$ and $b_{ij} = 0$ $(\bar{\sigma} = 0)$

  (Tseytlin, Vafa, '91)

$$\dot{\Phi}^2 - (D-1)\,H^2 = e^{\Phi}\bar{\rho}$$

$$\dot{H} - H\dot{\Phi} = \frac{1}{2}e^{\Phi}\bar{p}$$

$$\ddot{\Phi} - \dot{\Phi}^2 - (D-1)\,H^2 = 0,$$

where $H = \dot{a}/a$

- These equations are invariant under the scale-factor duality

  (Veneziano and Meissner, '91)

$$a \to 1/a \qquad \bar{\rho} \to \bar{\rho}$$

$$\Phi \to \Phi \qquad \bar{p} \to -\bar{p}$$

String Gas Cosmology

(Brandenberger, Vafa, '89)

- We consider a **thermodynamical gas of closed strings**. Since we know the string's spectrum, we can write

$$\rho = \frac{1}{a^{D-1}} \sum_s N_s E_s$$

$$l_s^2 E_s^2 = \underbrace{2\left(N + \tilde{N} - 2\right)}_{\text{oscillatory}} + \underbrace{\frac{n^2}{a^2}}_{\text{momentum}} + \underbrace{m^2 a^2}_{\text{winding}}$$

- The mass spectrum is invariant under:

$$\begin{cases} a & \leftrightarrow \frac{1}{a} \\ n & \leftrightarrow m \end{cases}$$

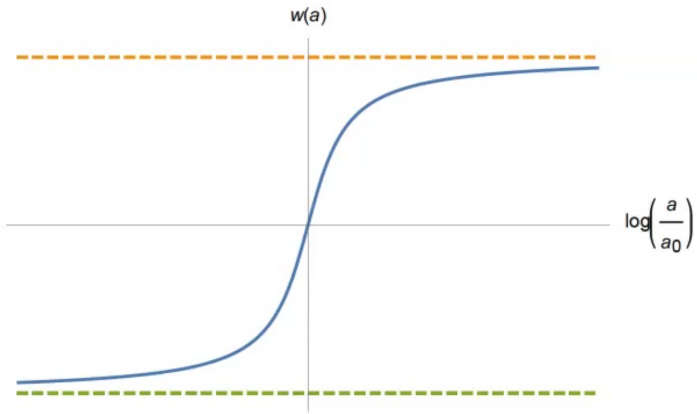
- The pressure is given by,

$$p = -\frac{\partial(\rho V)}{\partial V} = -\frac{1}{D-1} a^{1-D} \sum_s \frac{N_s}{l_s^2} \left(-\frac{n^2}{a^2} + m^2 a^2 \right)$$

too complicated! (Nayeri, '06)

$w = -1/d$	$w = 0$	$w = 1/d$
$a(t) \sim t^{-2/D}$	$a(t) \sim \text{const.}$	$a(t) \sim t^{2/D}$
$\rho(a) \sim a^{-(D-2)}(t)$	$\rho(t) \sim \text{const.}$	$\rho(a) \sim a^{-D}(t)$
$\dot{\phi}(t) \sim -4d/Dt$	$\dot{\phi}(t) \sim -2/t$	$\dot{\phi}(t) = 0$

$$w(a) = \frac{2}{\pi d} \arctan\left[\beta \ln\left(\frac{a}{a_0}\right)\right]$$



$$w\left(a^{-1}\right) = -w(a)$$

(Brandenberger, Costa, GF, Weltman, '18)

Including α'-corrections

- The vacuum lowest order action was

$$S_0 = -\frac{1}{2\kappa^2}\int d^dx dt\, n e^{-\Phi}\left[(\mathcal{D}\Phi)^2 + \frac{1}{8}\mathrm{tr}(\mathcal{D}\mathcal{S})^2\right]$$

- It's been shown that using field redefinitions the corrections α'^k take the form (Hohm and Zwiebach, '19)

$$S_k = \frac{1}{2\kappa^2}\int d^dx I_k, \qquad I_k = \alpha'^k \int dt n e^{-\Phi} X\{\mathcal{D}\mathcal{S}\}$$

first order and linear dependence on $\mathcal{S}$

- Induction: assume it for $(k-1)$th order and use field redefinitions at kth order

$$\Phi \to \Phi + \alpha'^k \delta\Phi, \qquad \mathcal{S} \to \mathcal{S} + \alpha'^k \delta\mathcal{S}$$

to bring the kth-correction into that form

$w = -1/d$	$w = 0$	$w = 1/d$
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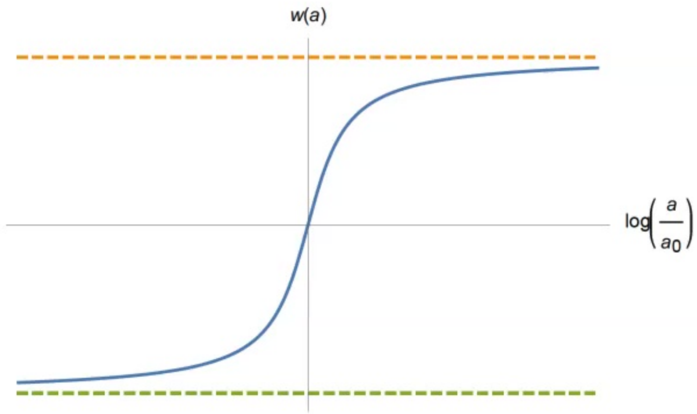

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to bring the kth-correction into that form

- **Considering then lapse redefinitions,** only the single trace higher order terms contribute and the corrected action to all orders in α' is

$$S_0^{(\alpha')} = \frac{1}{2\kappa^2} \int dt \, n e^{\Phi} \left[-(\mathcal{D}\Phi)^2 + \sum_{k=1}^{\infty} \alpha'^{k-1} c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} \right]$$

- Large simplification over what could have been expected from higher-derivative corrections: not only does the dilaton contribution have the same structure, but there are no extra time derivatives of $\mathcal{D}\mathcal{S}$

- **Drawback:** $\{c_k\}$ for $k \neq 1$ ($c_1 = -1/8$) not known and take different values for different string theories ($c_2 = 1/64$ for heterotic strings and $c_2 = 0$ for type II strings, for instance)

Vacuum α'-corrected String Cosmology

FRWL ansatz:

$$\dot{\Phi}^2 + HF'(H) - F(H) = 0$$

$$\frac{d}{dt}F'(H) - \dot{\Phi}F'(H) = 0$$

$$2\ddot{\Phi} - \dot{\Phi}^2 + F(H) = 0,$$

where

$$F(H) \equiv 2d\sum_{k=1}^{\infty}(-\alpha')^{k-1}c_k 2^{2k}H^{2k}$$

and $'$ denotes a derivative with respect to the Hubble parameter, $H \equiv \dot{a}/a$.

How does one include **matter** in such a framework? Should we start with matter in the lowest order action and correct it as well or add it directly to the corrected action? It turns out both procedures are equivalent (Bernardo, Brandenbeger, GF, '19).

α'-Cosmology

(Bernardo, Brandenbeger, GF, '19)

The **action** is

$$S_T^{(\alpha')} = \frac{1}{2\kappa^2}\int d^d x dt n e^{-\Phi}\left[-(\mathcal{D}\Phi)^2 + \sum_{k=1}^{\infty} \alpha'^{k-1} c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k}\right] + S_m[\Phi, n, \mathcal{S}, \chi]$$

The **EOM** are

$$2\mathcal{D}^2\Phi - (\mathcal{D}\Phi)^2 - \sum_{k=1}^{\infty} \alpha'^{k-1} c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} = \kappa^2 e^{\Phi}\bar{\sigma}$$

$$(\mathcal{D}\Phi)^2 - \sum_{k=1}^{\infty} \alpha'^{k-1}(2k-1) c_k \mathrm{tr}(\mathcal{D}\mathcal{S})^{2k} = 2\kappa^2 \bar{\rho} e^{\Phi}$$

$$\mathcal{D}\left(e^{-\Phi}\sum_{k=1}^{\infty} \alpha'^{k-1} 4k c_k \mathcal{S}(\mathcal{D}\mathcal{S})^{2k-1}\right) = -\kappa^2 \eta \bar{\mathcal{T}}$$

implying the same continuity equation as expected.

End of the second act.

Questions?

Final act! - FLRW α'-Cosmology

- **Friedmann-like equations** are

$$\dot{\Phi}^2 + HF'(H) - F(H) = 2\kappa^2 e^{\Phi}\bar{\rho}$$
$$\dot{H}F''(H) - \dot{\Phi}F'(H) = -2\kappa^2 e^{\Phi} d\bar{p}$$
$$2\ddot{\Phi} - \dot{\Phi}^2 + F(H) = \kappa^2 e^{\Phi}\bar{\sigma}$$

$$\dot{\bar{\rho}} + dH\bar{p} = \frac{1}{2}\dot{\Phi}\bar{\sigma}$$

- **Invariant under the scale factor duality** $a \to 1/a$, which implies

$$H \to -H, \quad \Phi \to \Phi, \quad \bar{\rho} \to \bar{\rho}, \quad \bar{p} \to -\bar{p}, \quad \bar{\sigma} \to \bar{\sigma}$$

assuming matter action is duality invariant. This is a remnant of the $O(d,d)$ transformation for the FLRW background

Cosmological Solutions - constant dilaton

- Why? Rolling dilaton leads to violations of the weak equivalence principle (late-time cosmology) and modulates the string coupling, so better to have it under control
- For **constant dilaton**

$$\dot{H} = \frac{1}{2d}[F(H) - d^2H^2]$$

$$(F'' - 2d^2w)(F - d^2H^2) + 2d^2HF'(1+w) = 0$$

given $w(t)$, find $F(H)$ and $H(t)$ and using the continuity equation, $\rho(t)$

- Thus, we have a systematic way to generate solutions given an equation of state

- **Perturbatively...**

$$F(H) = -dH^2 + \beta_1 \alpha' H^4 + \dots$$

$$2d^2 H^2 (1-d)(1-dw) \\ -2d\beta_1 \alpha' H^4 (7 - 2d - 3dw) + \dots = 0$$

- No constant EOS solution perturbatively anymore!
- One can perturb the EOS

$$w(t) = w_0 - 32 d c_2 w_2 \alpha' H^2 + 128 d c_3 w_3 \alpha'^2 H^4 - 512 d c_4 w_4 \alpha'^3 H^6 + \dots$$

and find **a <u>unique</u> time-dependent solution**, as expected

De Sitter!

- **Asymptotically** appealing to early- and late-time cosmology
- Imposing $H = H_0$ into the equations leads to two conditions:

$$\frac{\dot{w}}{w} - dH_0 w = -\frac{F'(H_0)}{2dw}$$

$$F(H_0) = \frac{H_0}{2} F'(H_0), \quad w \neq 0.$$

while the shifted dilaton evolves as

$$\dot{\Phi} = dH_0 w - \frac{\dot{w}}{w} \pm \sqrt{\left(\frac{\dot{w}}{w} - dH_0 w\right)^2 - F(H_0)}$$

- Constant equation of state: $\dot{\Phi} = 0$ or $\dot{\Phi} = dH_0 w_0$

- $\boxed{\dot{\Phi} = 0}$
  - either results in a vacuum solution or in pressureless matter (Hagedorn phase?)
  - running dilaton: $\dot{\phi} = dH_0/2$
  - H_0 expected to be defined by string scale
- $\boxed{\dot{\Phi} = dH_0 w_0}$
  - possible to implement through any w
  - running dilaton: $\dot{\phi} = dH_0(1+w)/2$
  - $H_0^2 = k^2 e^{\Phi_0} \bar{\rho}_0 / d^2 w_0^2$, so Hubble constant not set by the string scale
- From the last case, **we have a constant dilaton solution if $w_0 = -1$**

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Einstein Frame

Let's write the Hubble parameter in the Einstein frame, where we typically do cosmology:

$$H_E(t_E) \sim -a(t)^{\frac{d}{d-1}} e^{\frac{\Phi}{d-1}} (\dot{\Phi} + H)$$

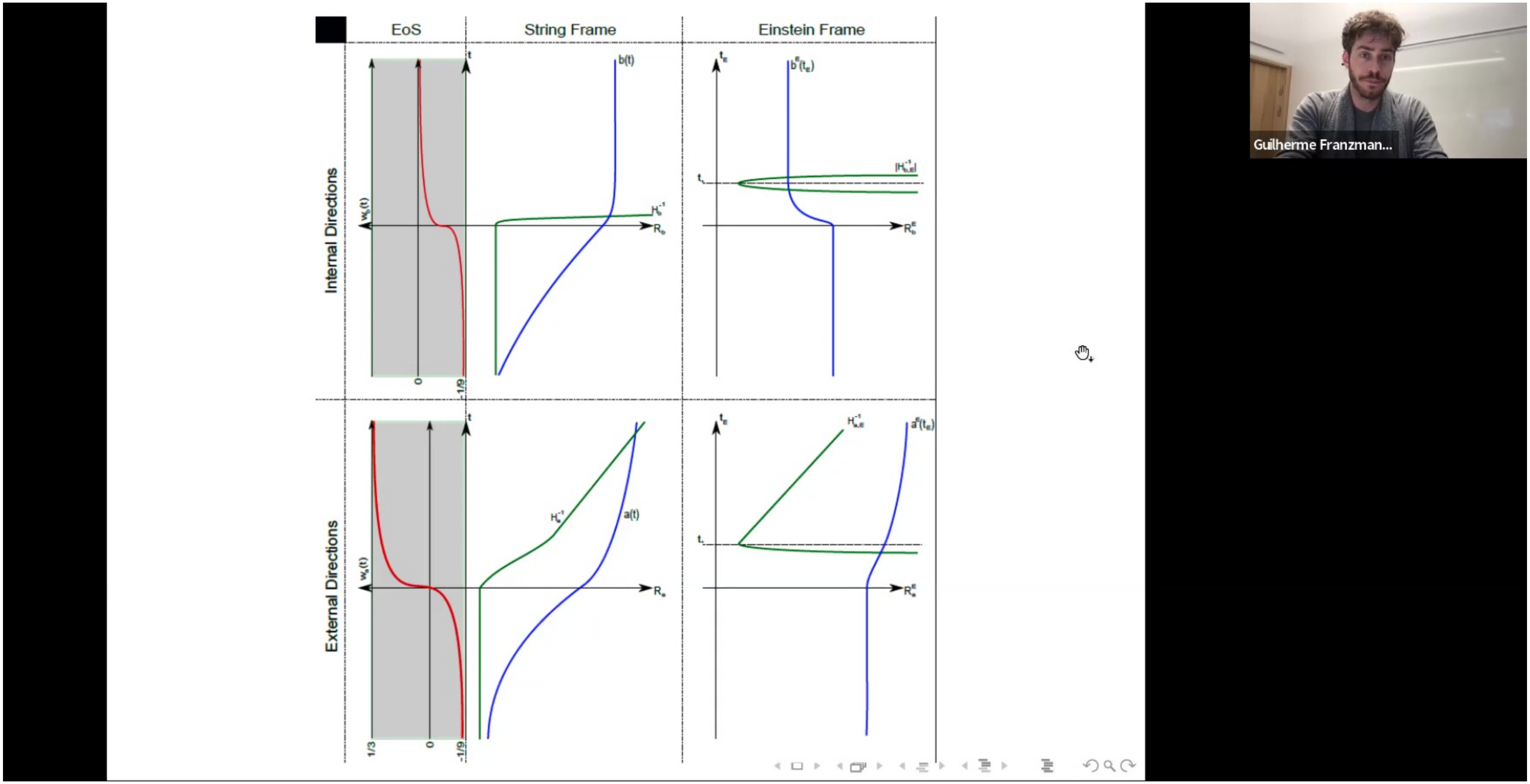
$$\frac{dH_E(t_E)}{dt_E} \sim \ddot{\Phi} + \dot{H} + \frac{1}{d-1}(dH + \dot{\Phi})(H + \dot{\Phi})$$

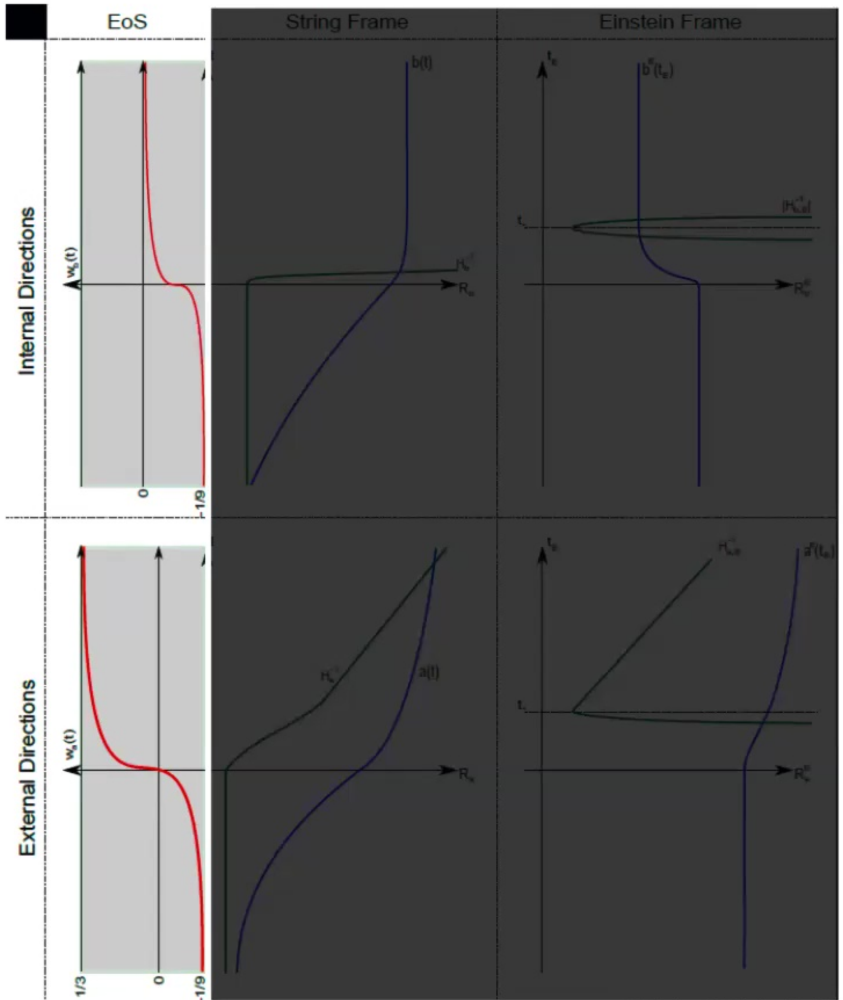
We immeditally see that if $\dot{\Phi} = -H_0$ we have dS in the String frame and a static universe in Einstein frame. But we have just seen that $\dot{\Phi} = dH_0 w_0$, so that would imply precisely a **winding equation of state**: $w = -1/d$.

Rose-Tinted Model of the Early Universe

(Bernardo, Brandenbeger, GF - May '20)

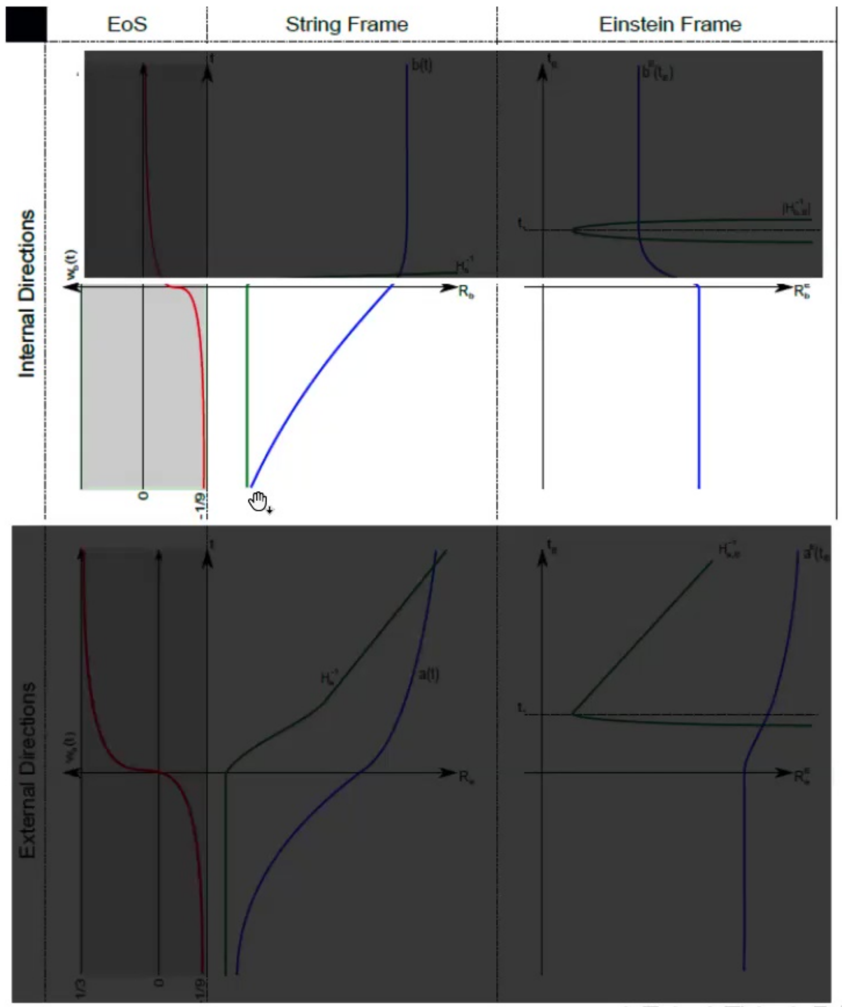
- First we need to improve the formalism to tackle a class of **anisotropic models**, since critical strings demand higher dimensional spaces with internal directions that should be small (and static) in order to make contact with our $4D$ universe
- Given we **cannot treat a continuous evolution of the EoS**, we model the dynamics by different stages with a constant EoS: winding, pressureless, radiation
- We start off with a compactified dS_{10} solution in the String frame with a winding EoS (static in Einstein frame) with all directions expanding until their physical radius become of the string size


EoS
String Frame
Einstein Frame
Internal Directions
External Directions
b(t)
a(t)
t
0
1/3
-1/9
Guilherme Franzman...



EoS
String Frame
Einstein Frame
Internal Directions
External Directions
b(t)
a(t)
1/3
0
-1/9

Guilherme Franzman...


EoS
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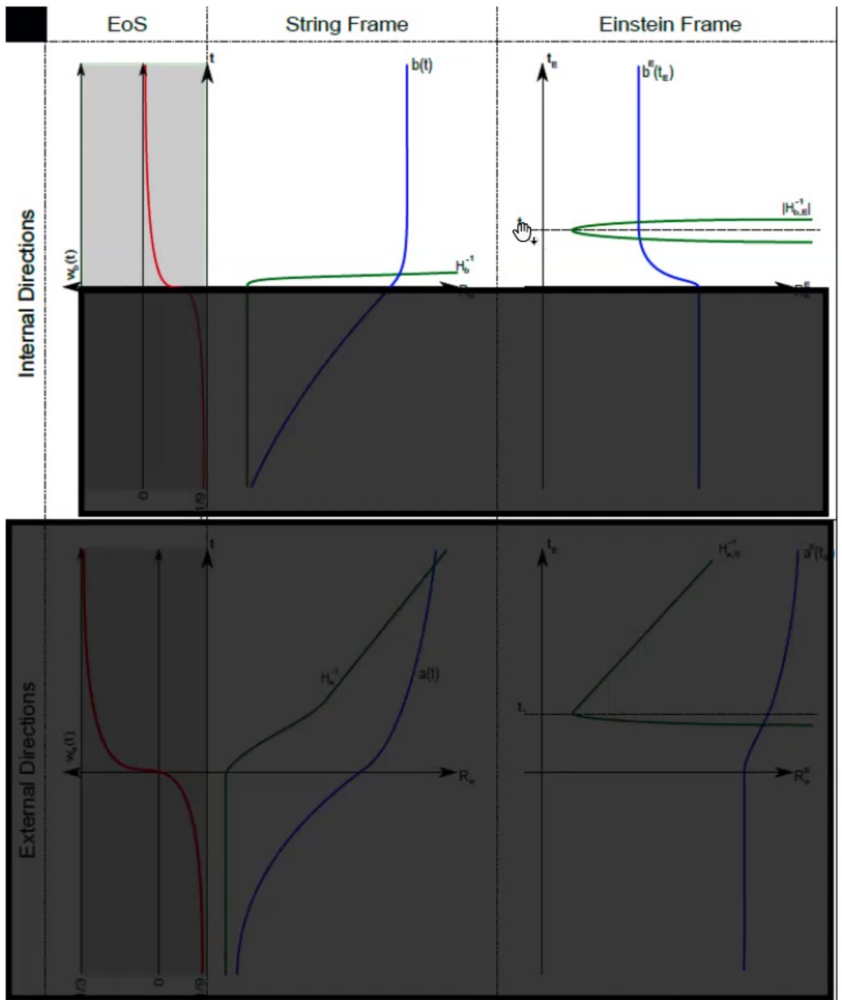
External Directions
0
-1/9
1/3
1/9


Guilherme Franzman...


EoS
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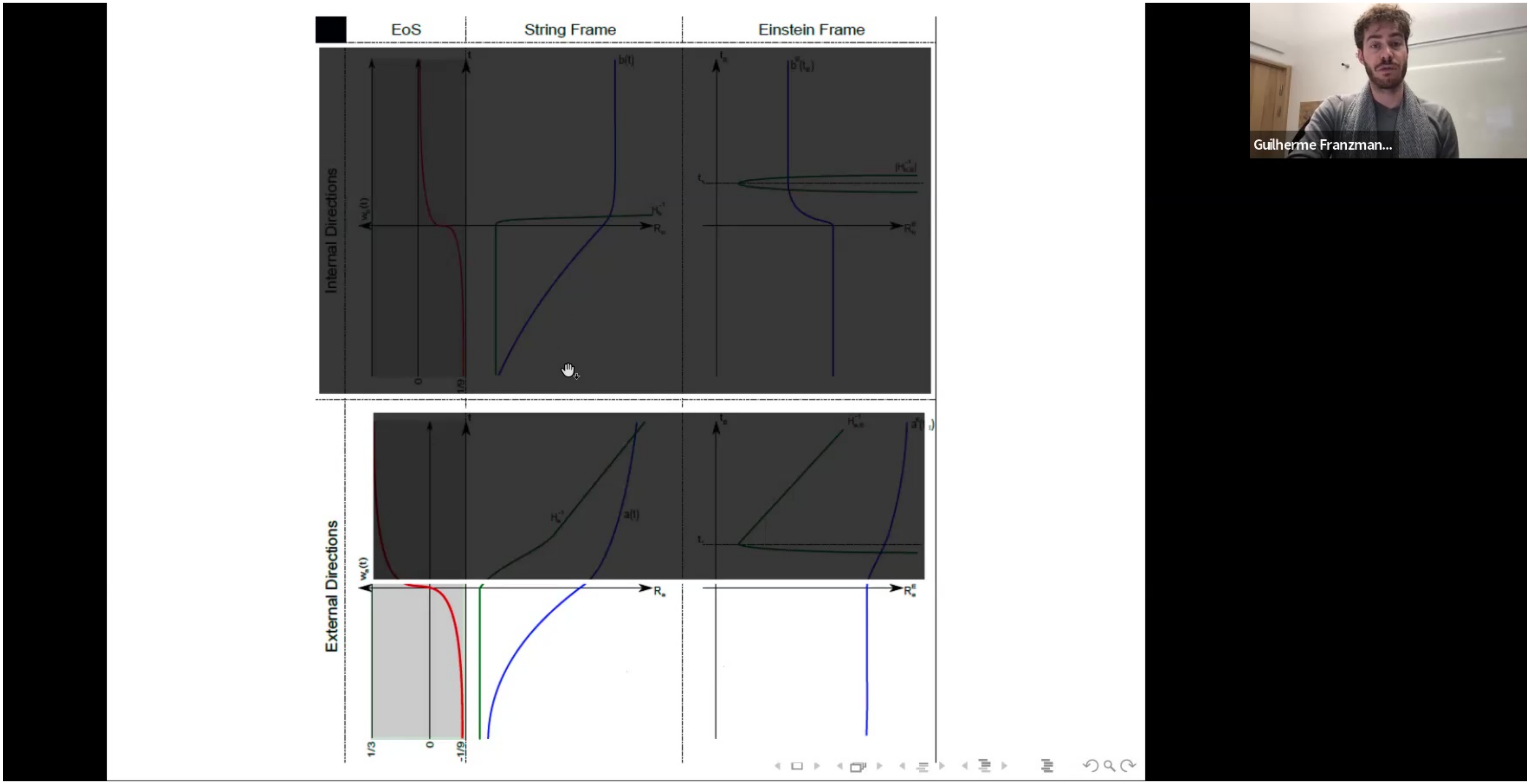
b(t)


Guilherme Franzman...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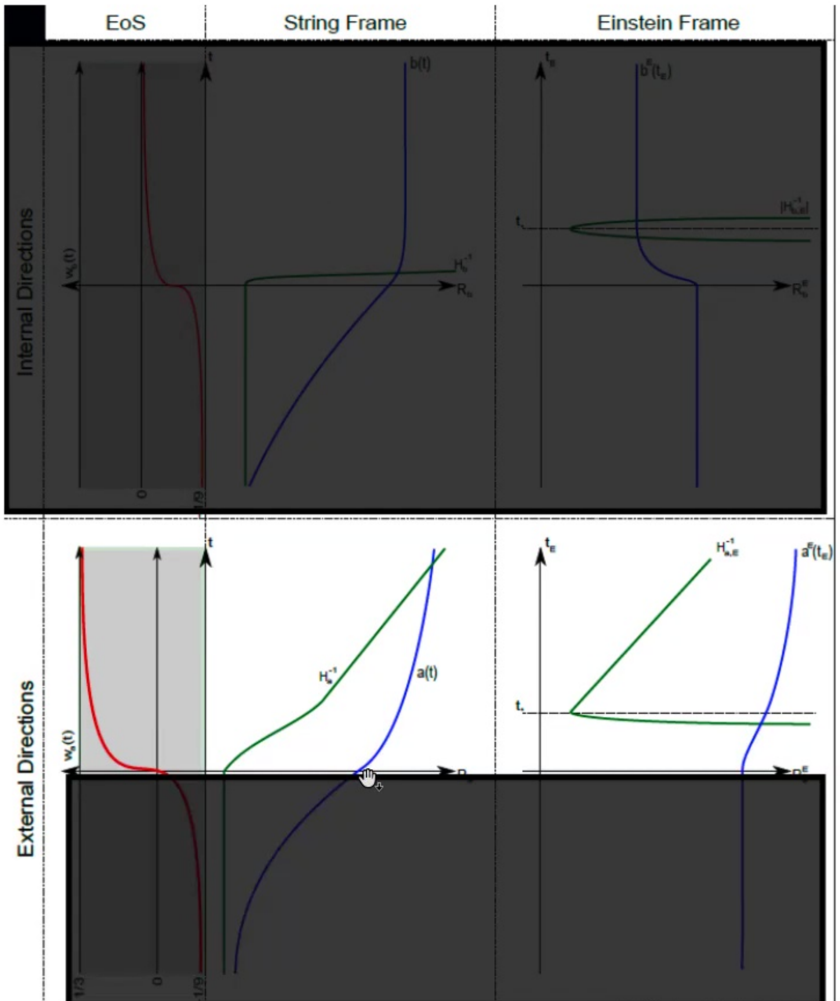
Guilherme Franzman...



EoS
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Einstein Frame
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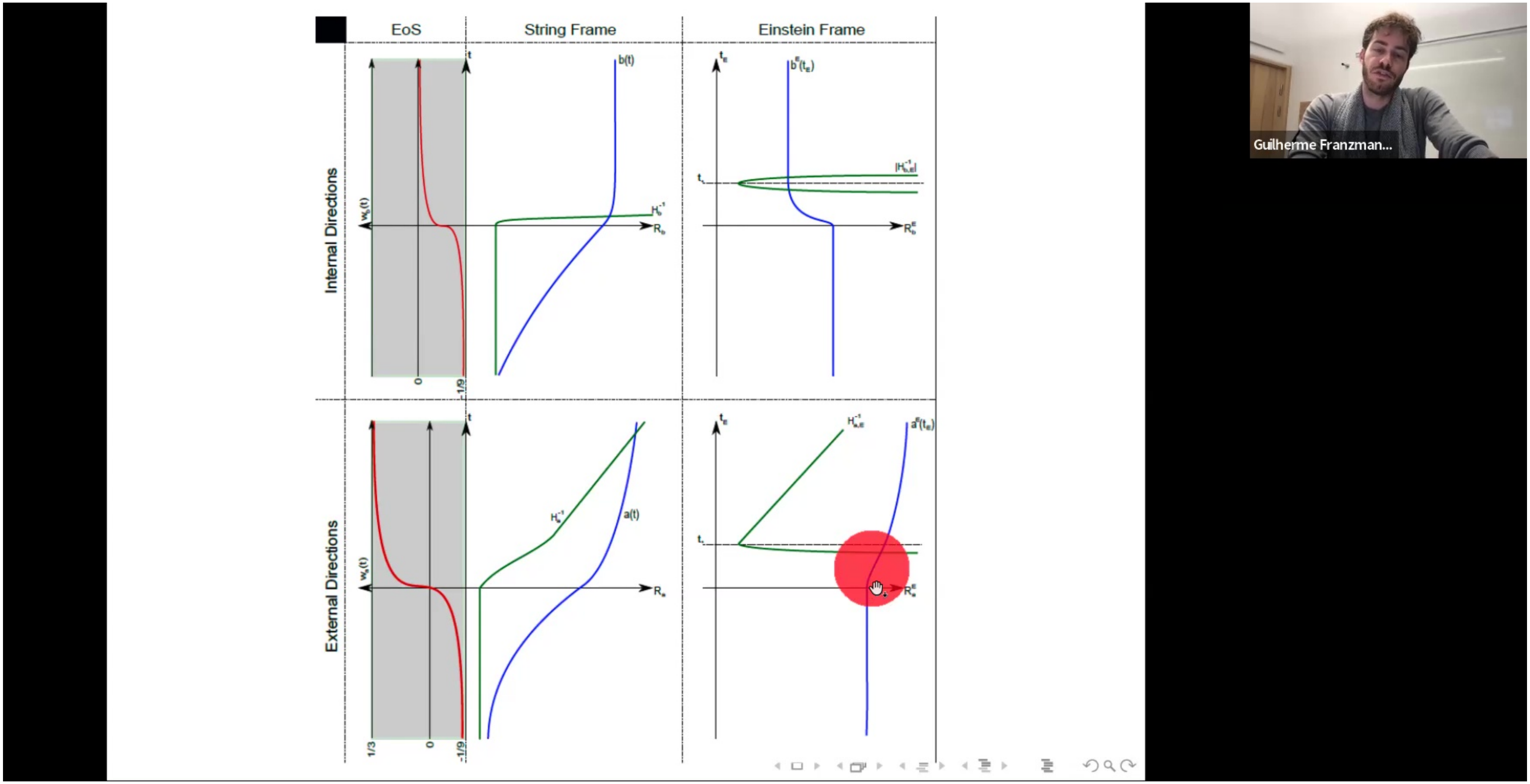
b(t)
a(t)
t

Guilherme Franzman...



EoS
String Frame
Einstein Frame
Internal Directions
External Directions
b(t)
a(t)
1/3
0
-1/9
Guilherme Franzman...

In 5 years at a dinner...

- We have **equations to talk about string cosmology in non-perturbative regimes including the string scale**
- These equations are classical (no loop corrections)
- They depend on an infinite number of unknown coefficients... **BUT**
  - **They are useful to show that likely most perturbative results break down closer to the string scale**
  - We found no obstructions to build controversial solutions like **de Sitter non-perturbatively (both in Einstein and String frames)** and **the emergent scenario may naturally be realized**
- The framework allows many different cosmological solutions, including **the construction of a stringy early universe model which results into 4 large dimensions that may stand as an alternative to inflation**

Beyond the horizon...

- To build other realistic cosmological scenarios (de Sitter with exit mechanism, bouncing, etc) ✓ (Quintin, Bernardo, GF, '21)
- To study energy conditions and non-singular solutions more generally ✓ (Quintin, Bernardo, GF, '21)
- Turning on the 2-form and allowing full $O(d,d)$ symmetry (matter with viscosity) ✓ (Bernardo, Chouha, GF, '21)
- Other compactifications scheme? We made progress only for asymptotically static internal dimensions
- To consider cosmological perturbations in the model (Bernardo, Brandenberger, GF, '20)

The end.

Thank you!