

Title: Causal aspects of quantum information in quantum gravity

Speakers: Alex May

Series: Perimeter Institute Quantum Discussions

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Abstract: Quantum information science was initially motivated by questions about information processing. For example, what are the consequences of quantum mechanics for computation? Or for cryptography? More recently, quantum information has also become a perspective through which we can study questions in theoretical physics more broadly, including in condensed matter and quantum gravity. While quantum information considers the constraints of quantum mechanics, there are additional constraints on information implied by relativity. In particular, it is impossible to send information faster than the speed of light. In this talk, I consider constraints on information processing imposed by quantum mechanics and relativity together, and the consequences of these constraints for quantum gravity. Doing so reveals novel aspects of how gravitational degrees of freedom can be recorded into a quantum mechanical system, and how an extra dimension can be recorded into ``holographic'' field theories.

Zoom Link: <https://pitp.zoom.us/j/99249375009?pwd=QWZyZzQrNXJwSGYyYTZheGwwaVRndz09>

Causal aspects of quantum information
in quantum gravity

Alex May

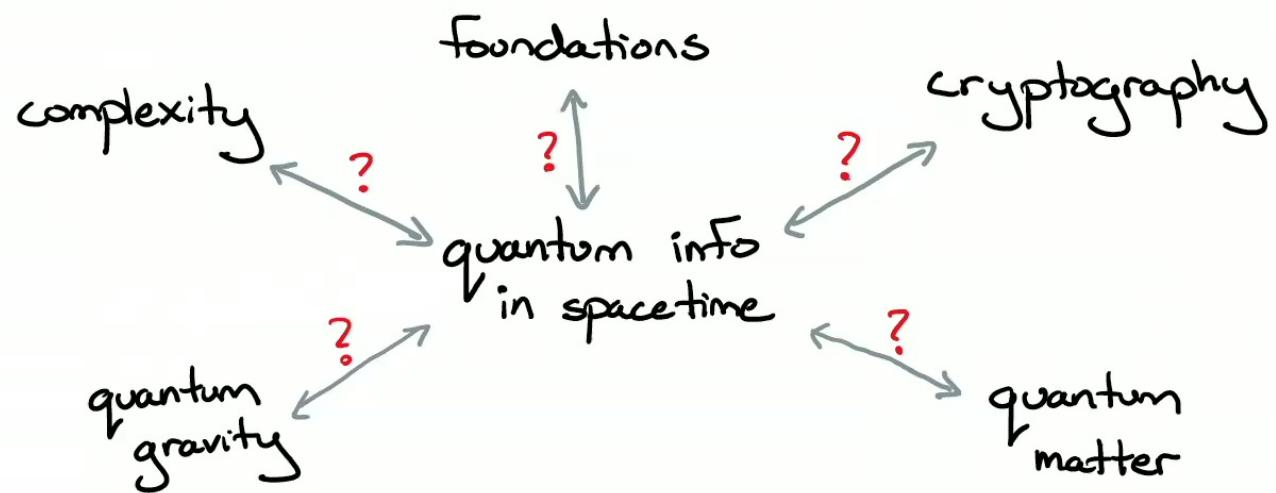
Stanford University

Quantum information processing in spacetime

- Quantum mechanics is not the full structure underlying information processing in our universe

What is the theory of information processing in spacetime?

- Then ask about:



Q. Info. in spacetime and quantum gravity

- We'll focus on:



- This connection will give insight into some basic questions in quantum gravity:
 - (A) How can "holography" be possible?
 - (B) How are gravitational d.o.f. (metric) recorded into quantum d.o.f.?
- Along the way, we'll use insights from cryptography, foundations, and connect to complexity.

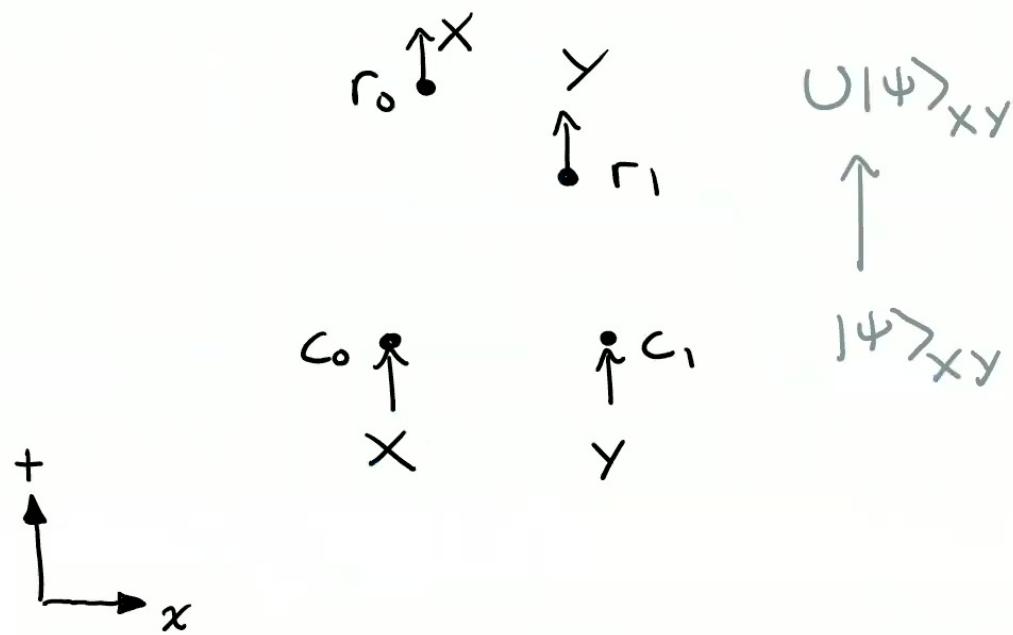
① Quantum information in spacetime

Quantum info.

Inputs: $| \psi \rangle_{xy}$

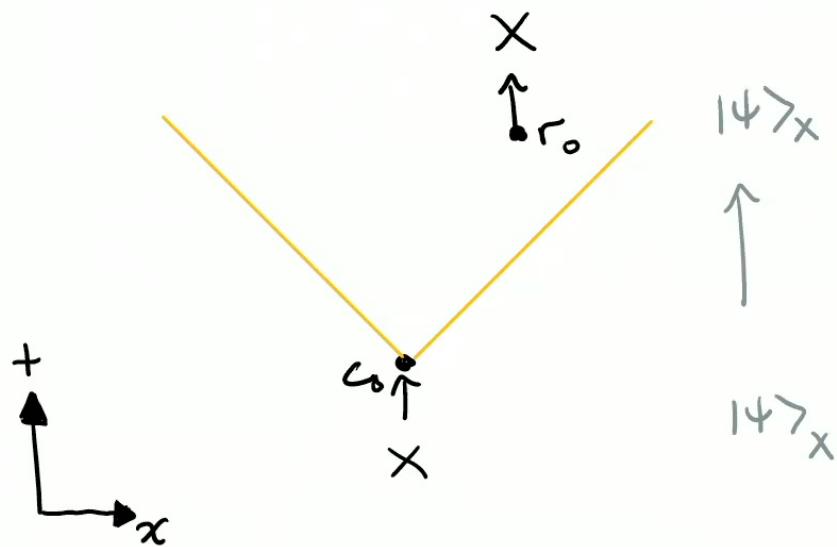
Outputs: $U|\psi\rangle_{xy}$

Quantum tasks (Kent 2012)

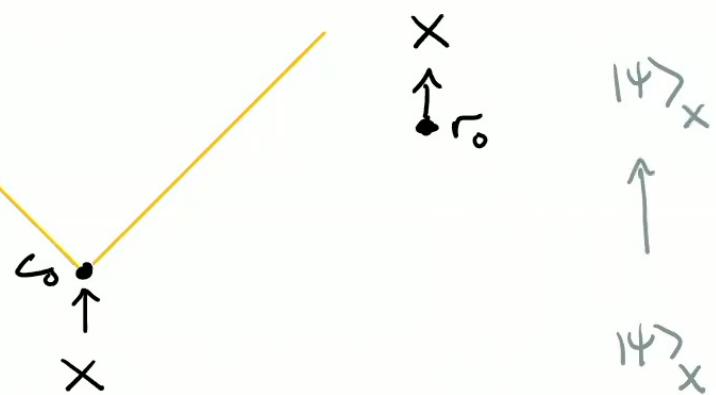


Note: Considering a fixed spacetime background (for now)

Causal relationships matter



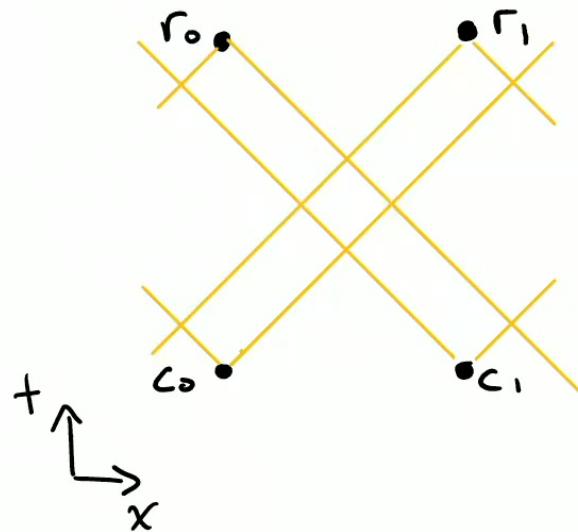
$$P_{\text{success}} = 1$$



$$P_{\text{success}} = 0$$

Causal Features

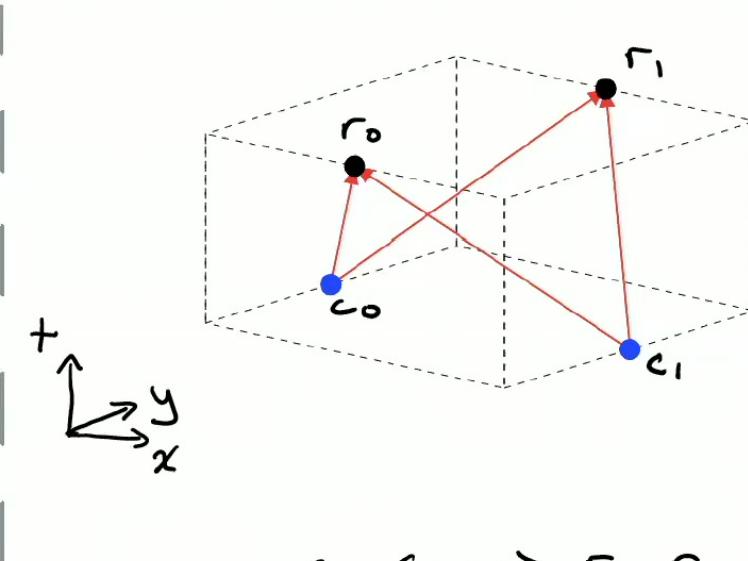
- There are also more subtle aspects of causal structure



$$c_0, c_1 \rightarrow r_0, r_1$$

and

$$\exists J : c_0, c_1 \rightarrow J \rightarrow r_0, r_1$$



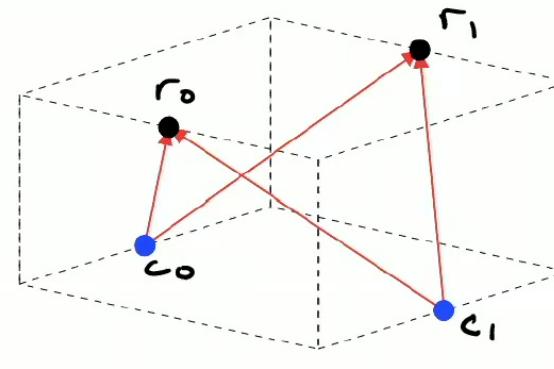
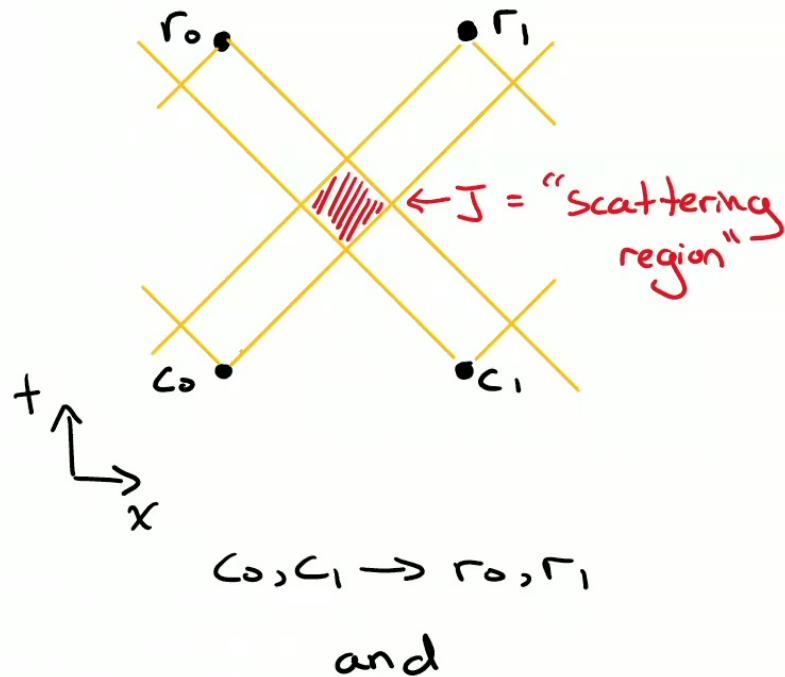
$$c_0, c_1 \rightarrow r_0, r_1$$

but

no such region J

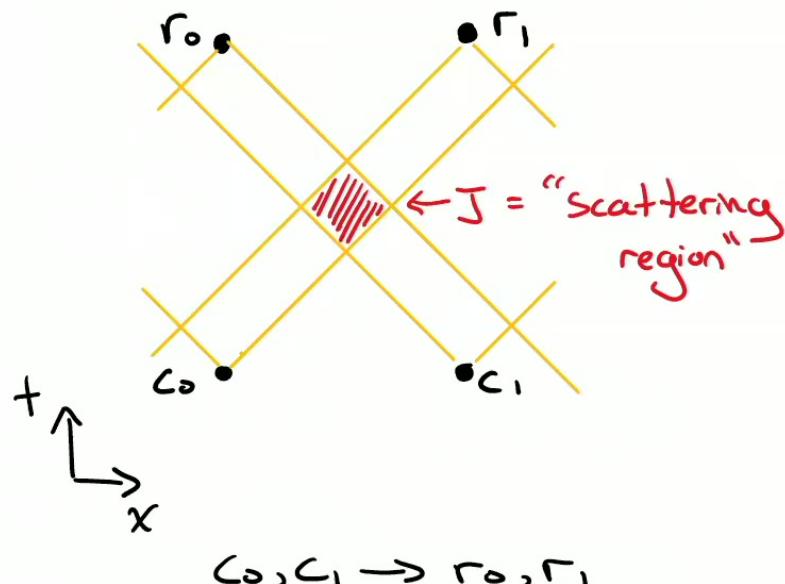
Causal features

- There are also more subtle aspects of causal structure



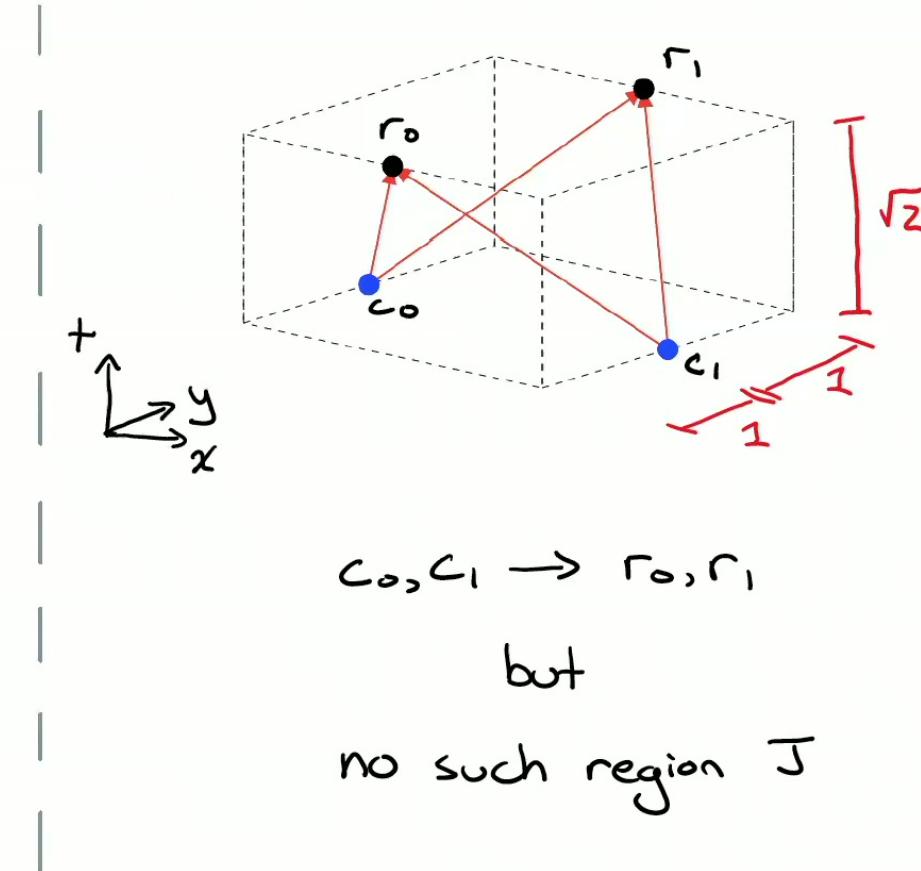
Causal Features

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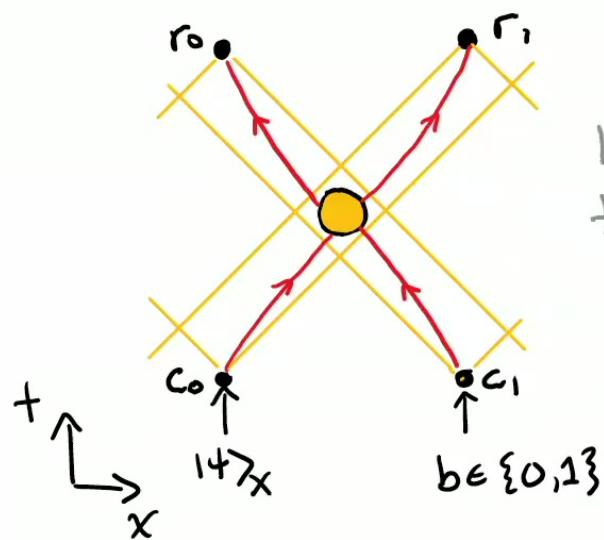
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$$\exists J : c_0, c_1 \rightarrow J \rightarrow r_0, r_1$$

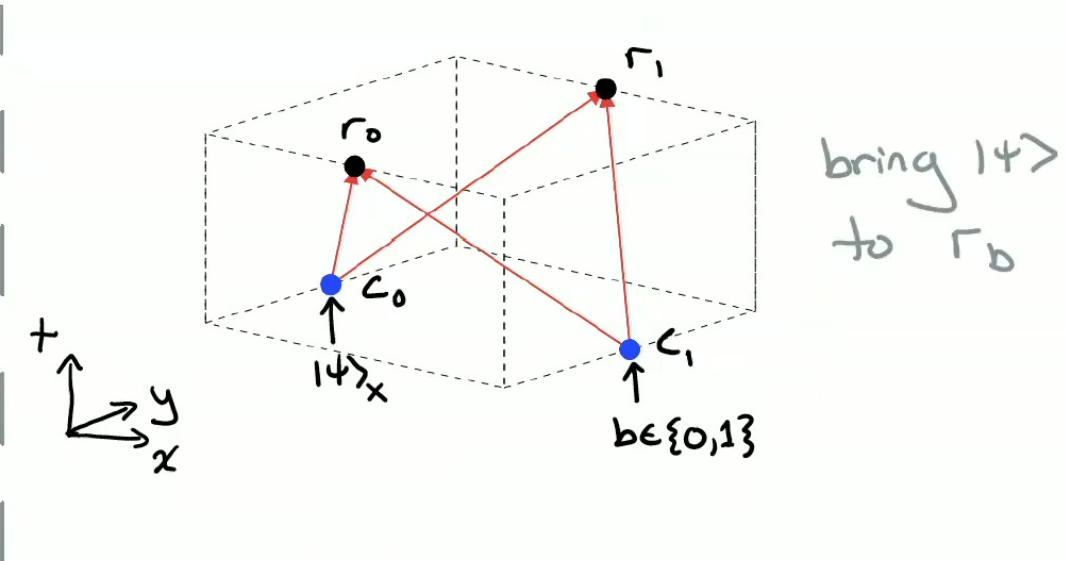


Causal features and quantum tasks

- Existence of this causal feature is relevant for performing "tasks"



bring $|4\rangle$
to r_b

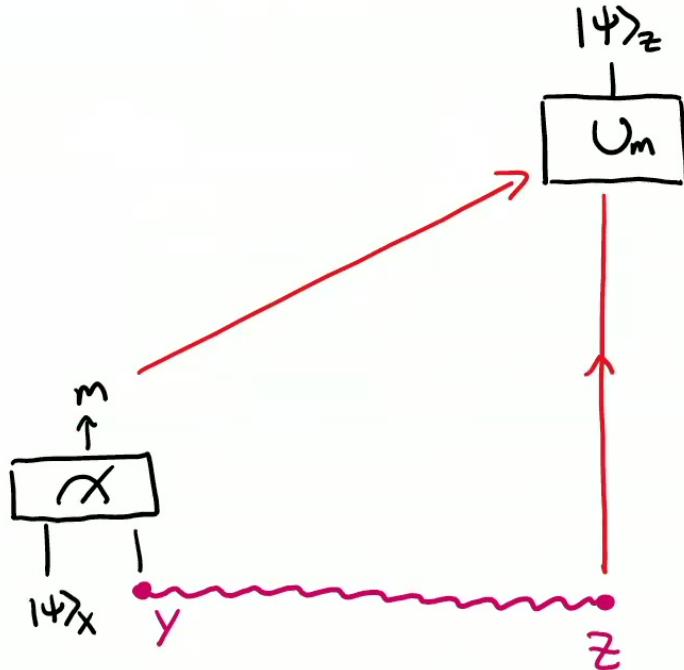


bring $|4\rangle$
to r_b

Use scattering region
to complete task

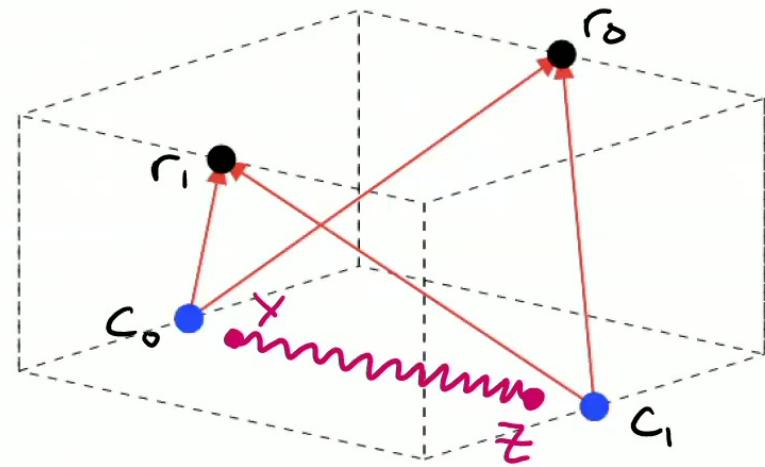
? Can we complete this task ?

Teleportation strategy



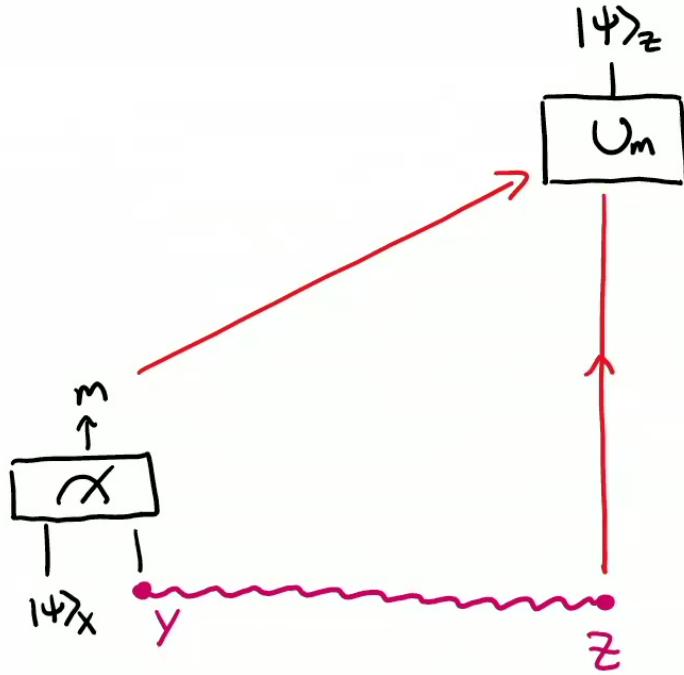
See Bennett et al.
Kent (2012)

Low Battery
Your Mac will sleep soon unless plugged into a power outlet.



$$|\Psi\rangle_{yz} = \underbrace{\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)}_{\text{entangled state}} \neq |\Psi\rangle_y |\phi\rangle_z$$

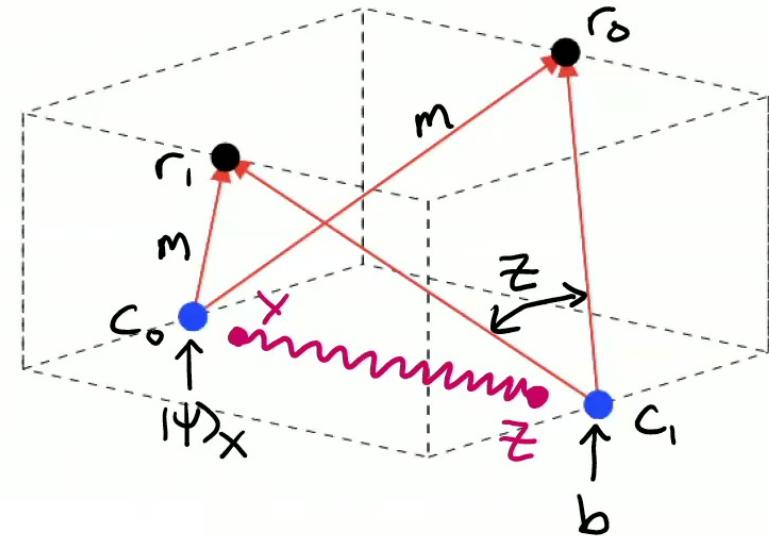
Teleportation strategy



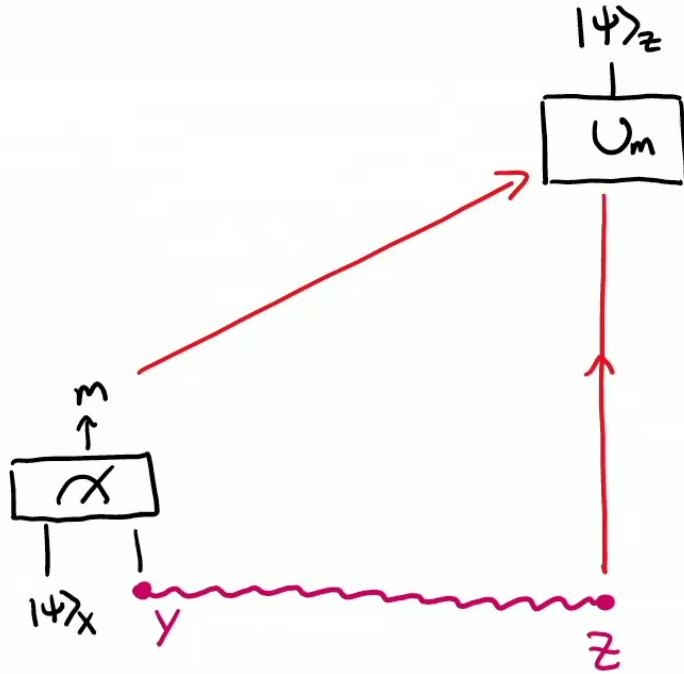
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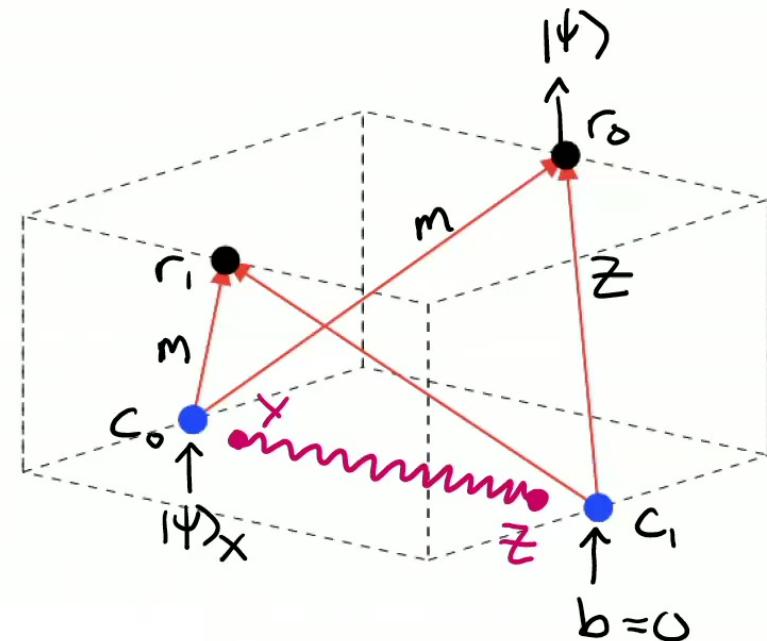
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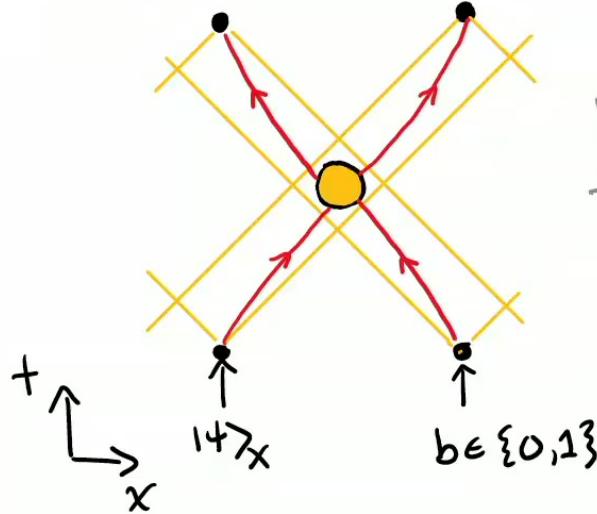
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Causal features and quantum tasks

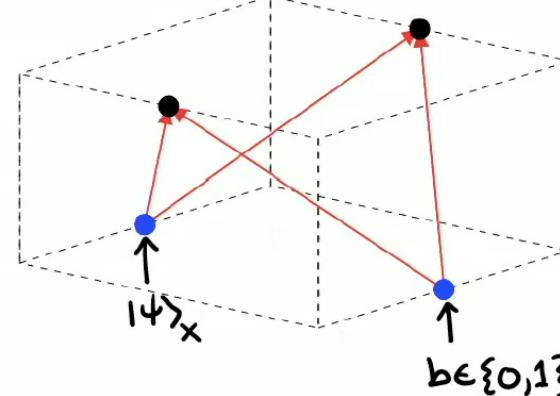
Low Battery
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- Causal structures and entanglement can be interchangeable resources



bring $|+\rangle$
to Γ_b

Use scattering region
to complete task



bring $|+\rangle$
to Γ_b

Use entanglement to complete task



Low Battery

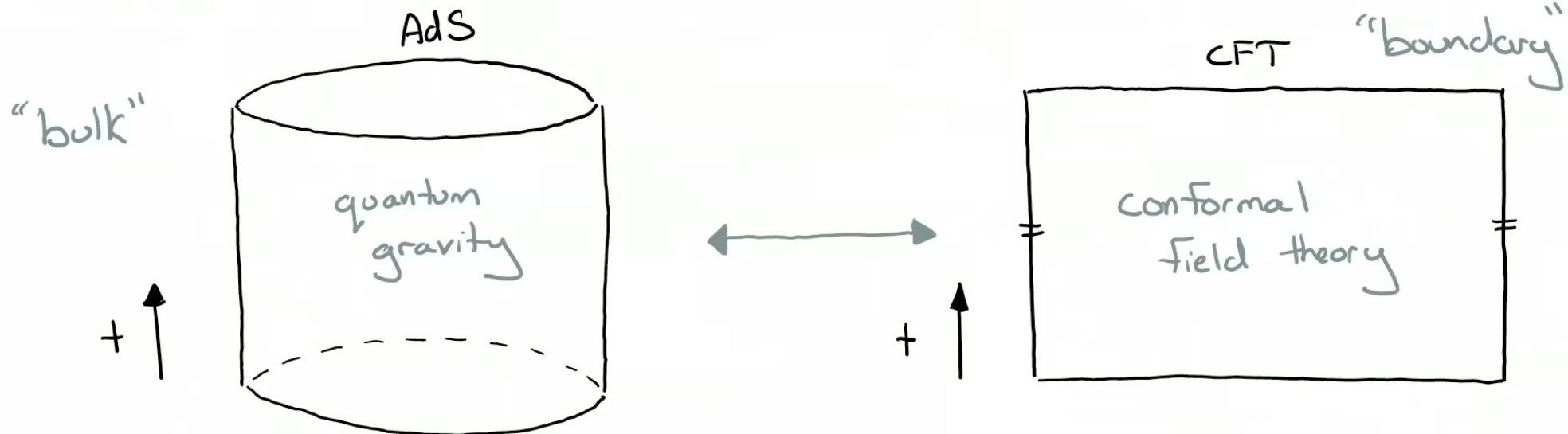
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② Quantum gravity + AdS/CFT

AdS/CFT Maldacena (1997)

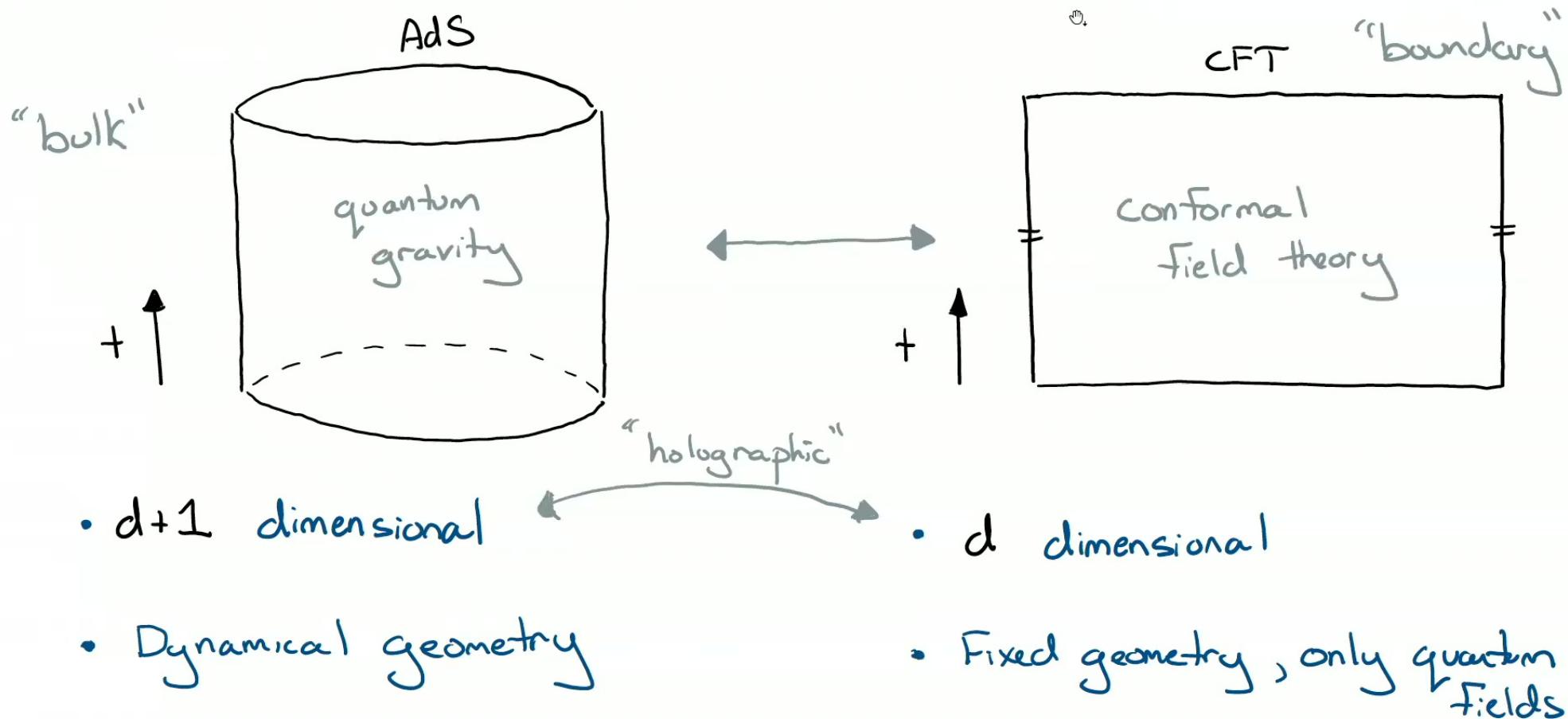
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- We will study our quantum gravity questions in context of AdS/CFT



AdS/CFT Maldacena (1997)

- We will study our quantum gravity questions in context of AdS/CFT

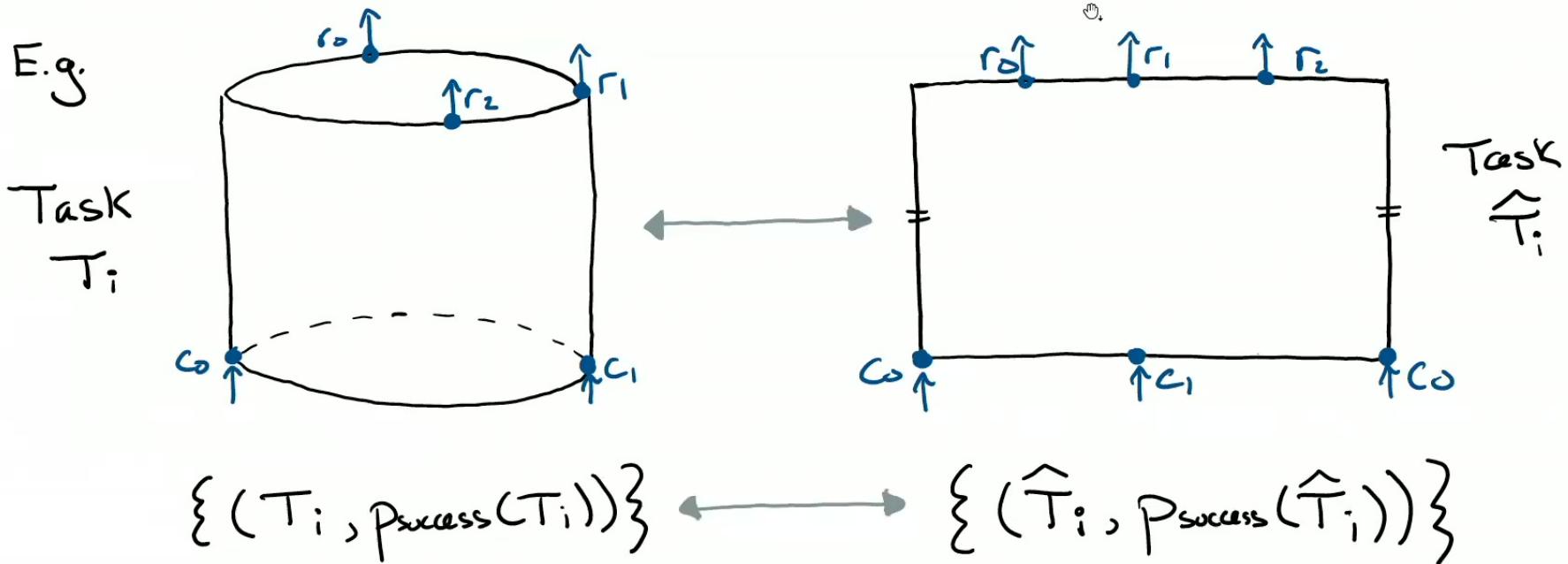


AdS/CFT correspondence

- AdS/CFT is a consistent theory of quantum gravity, valid at all energy scales.
- There are a few limitations to keep in mind, most importantly:
we don't live in an asymptotically AdS space!
- Still, there's a lot to learn from AdS/CFT:
 - How can QM encode a higher dimensional theory?
 - How can gravitational physics be recorded into QM?

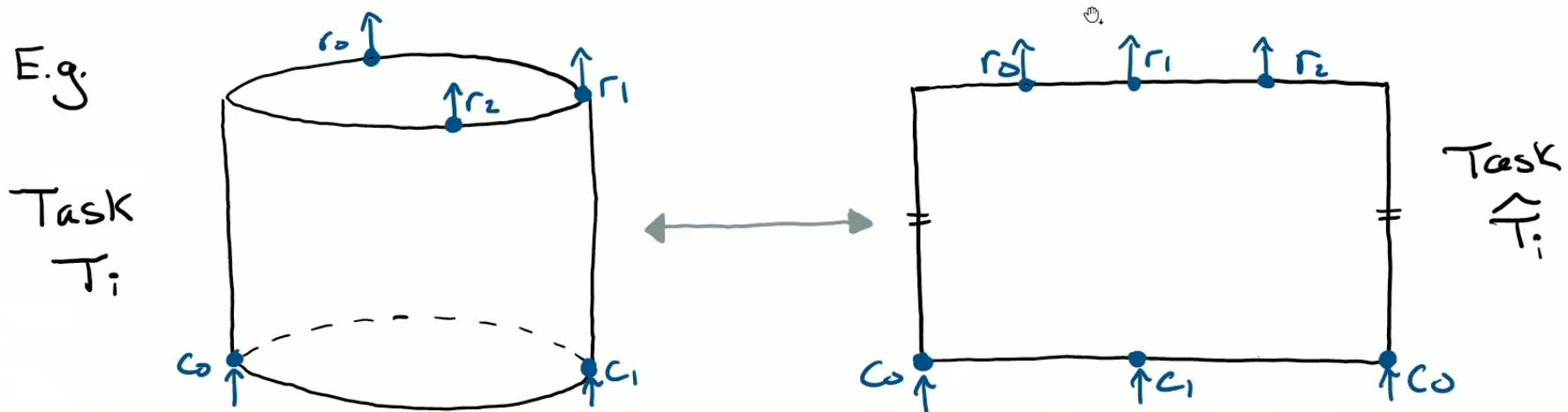
Quantum tasks in holography AM (2019)

- To address \textcircled{A} and \textcircled{B} , study quantum tasks in AdS/CFT



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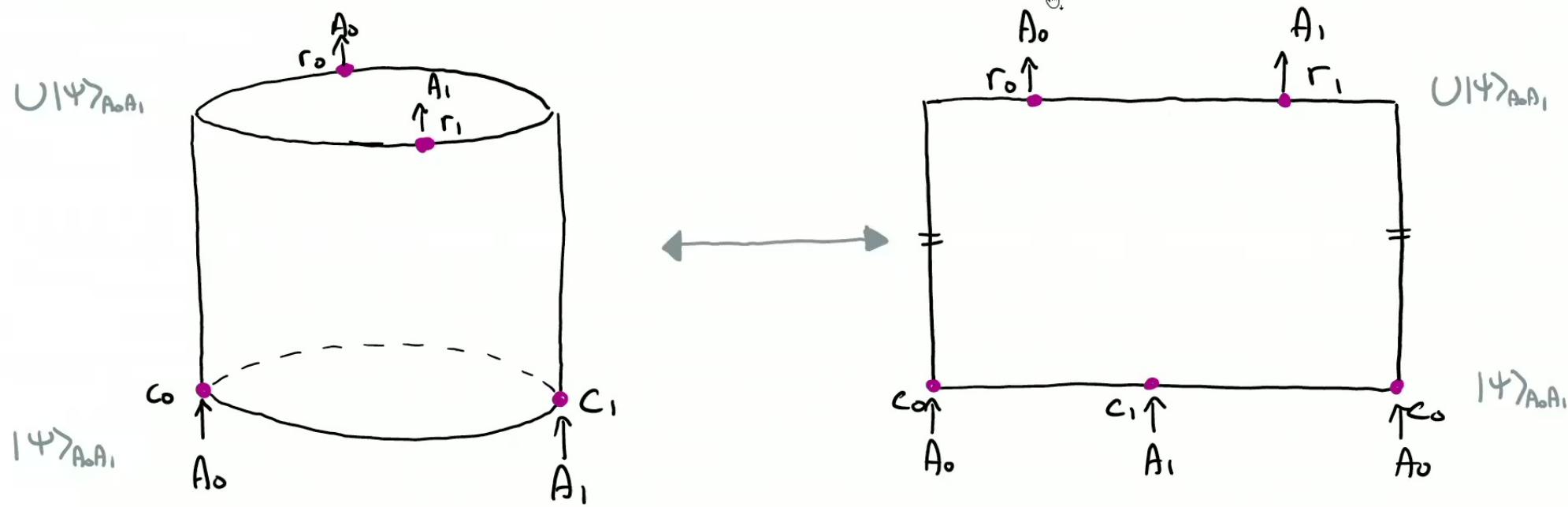
$$\{(T_i, p_{\text{success}}(T_i))\} \longleftrightarrow \{(\hat{T}_i, p_{\text{success}}(\hat{T}_i))\}$$

$$p_{\text{succ}}(T) = p_{\text{succ}}(\hat{T})$$

Operational perspective
on AdS/CFT, inspired
by Kent (2012)

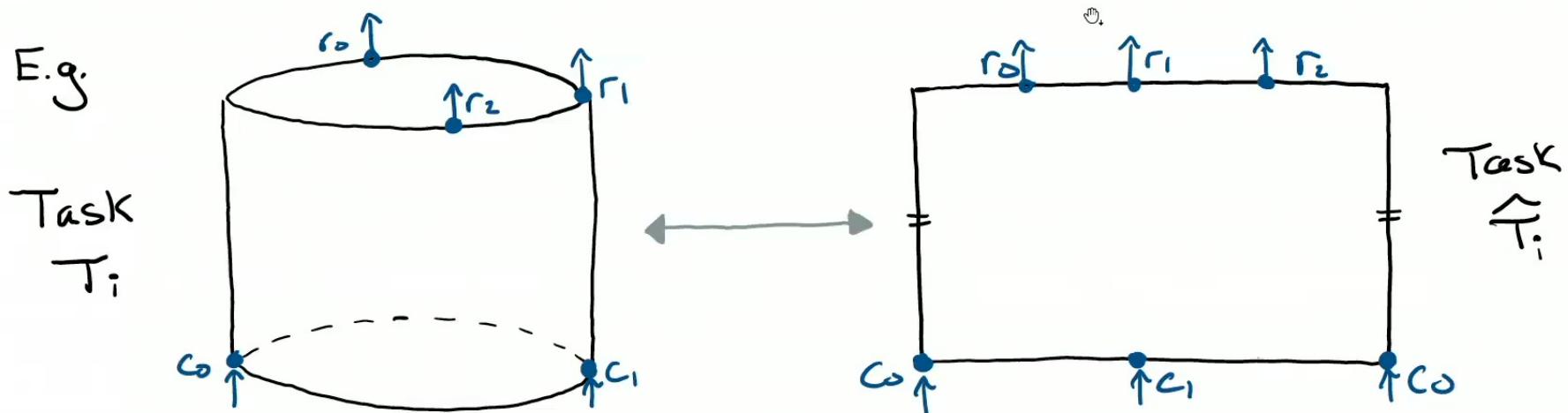
A puzzle (sharpened ④)

- To better understand how a theory can be "holographic", consider this task:



Quantum tasks in holography AM (2019)

- To address ① and ②, study quantum tasks in AdS/CFT



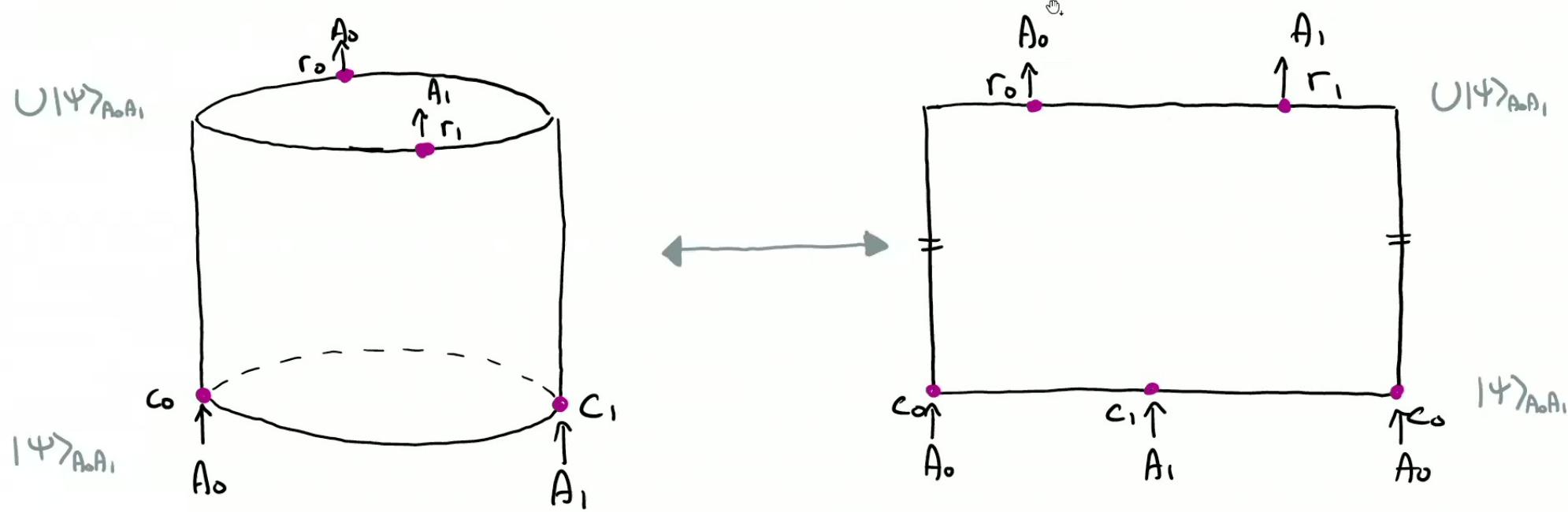
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Operational perspective
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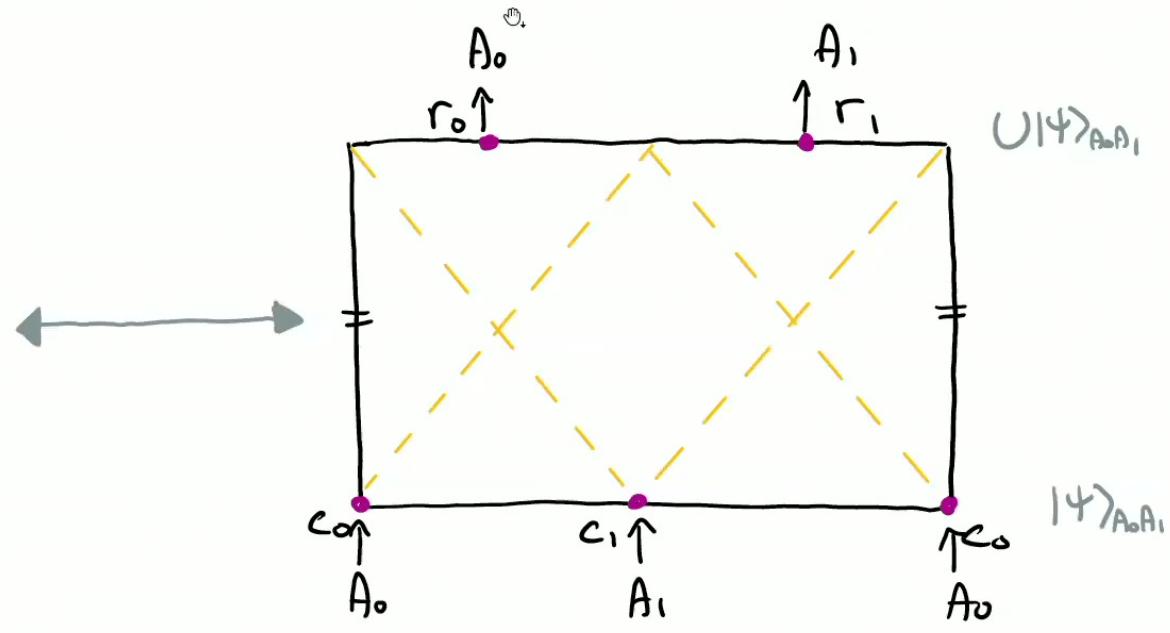
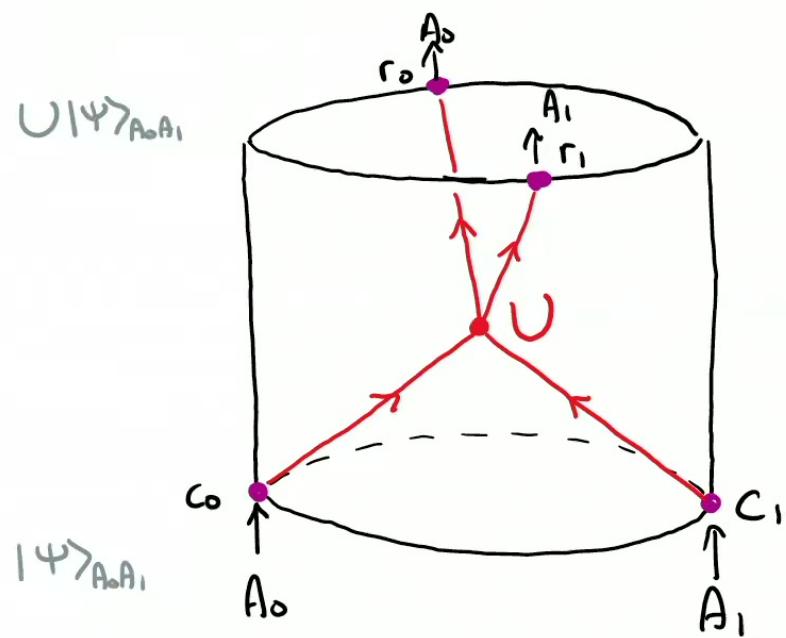
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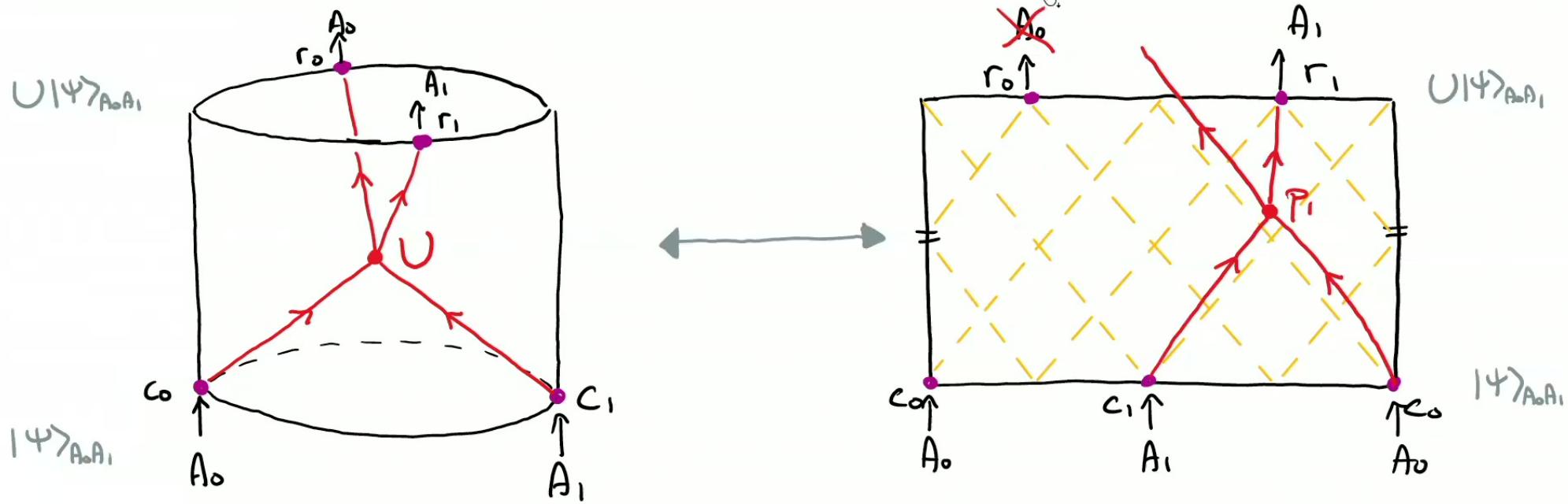
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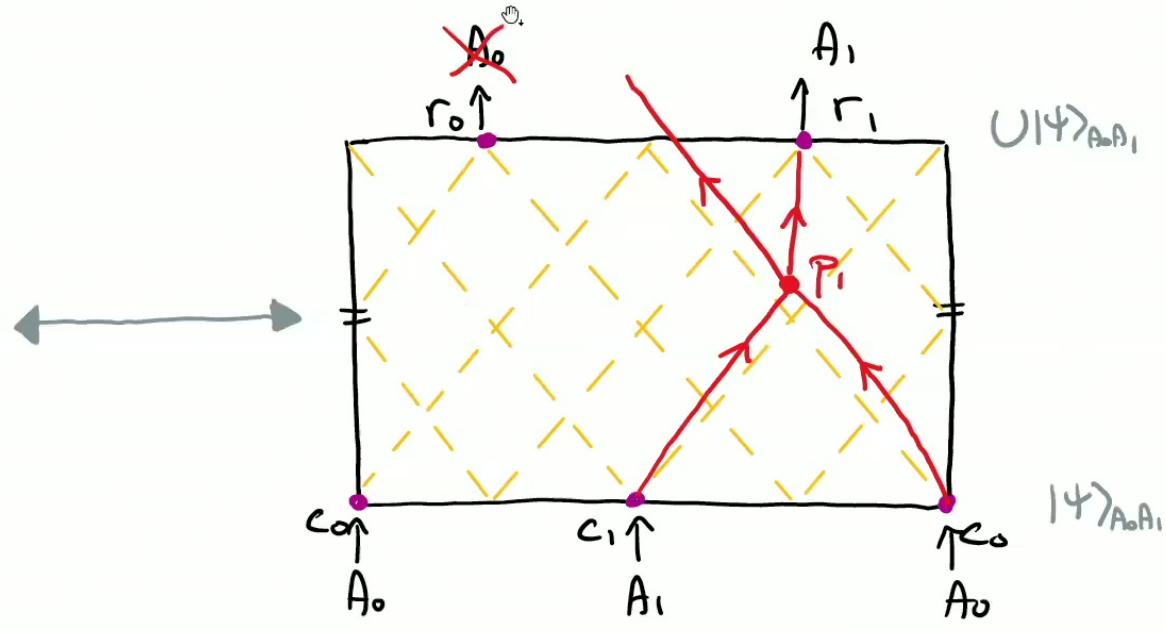
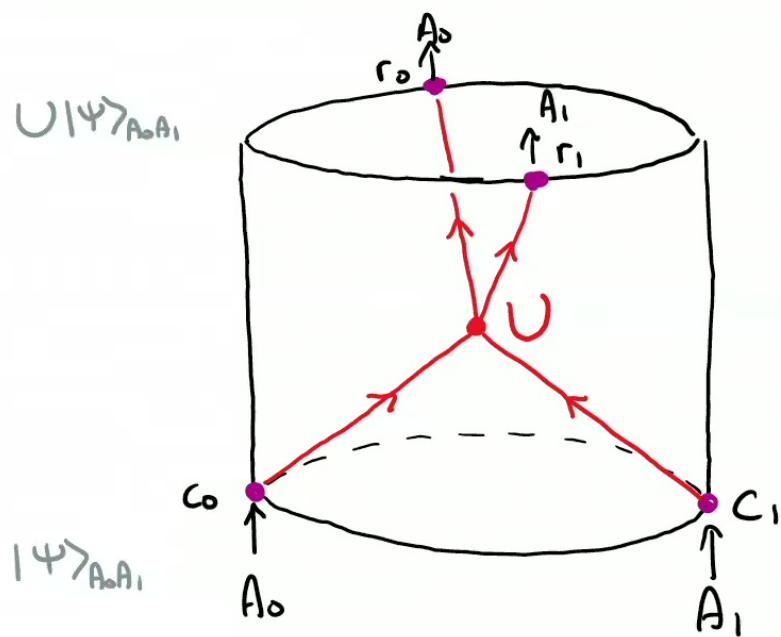
A puzzle (sharpened ④)

- To better understand how a theory can be "holographic", consider this task:



A puzzle (sharpened ④)

- To better understand how a theory can be "holographic", consider this task:



- What gives? Did we misunderstand AdS/CFT?

↳ Q.I. in spacetime will be key to understanding this!

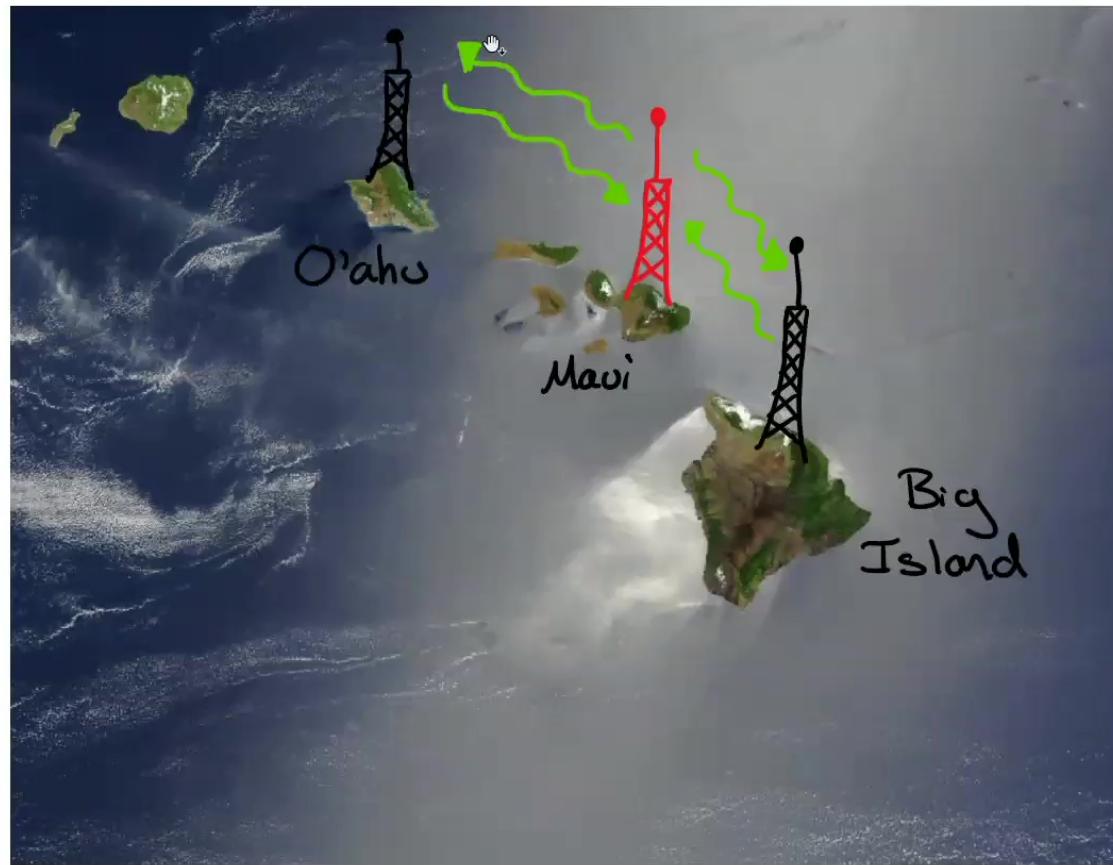


- ③ Position-verification and holography
- (Ⓐ How can "holography" be possible?)

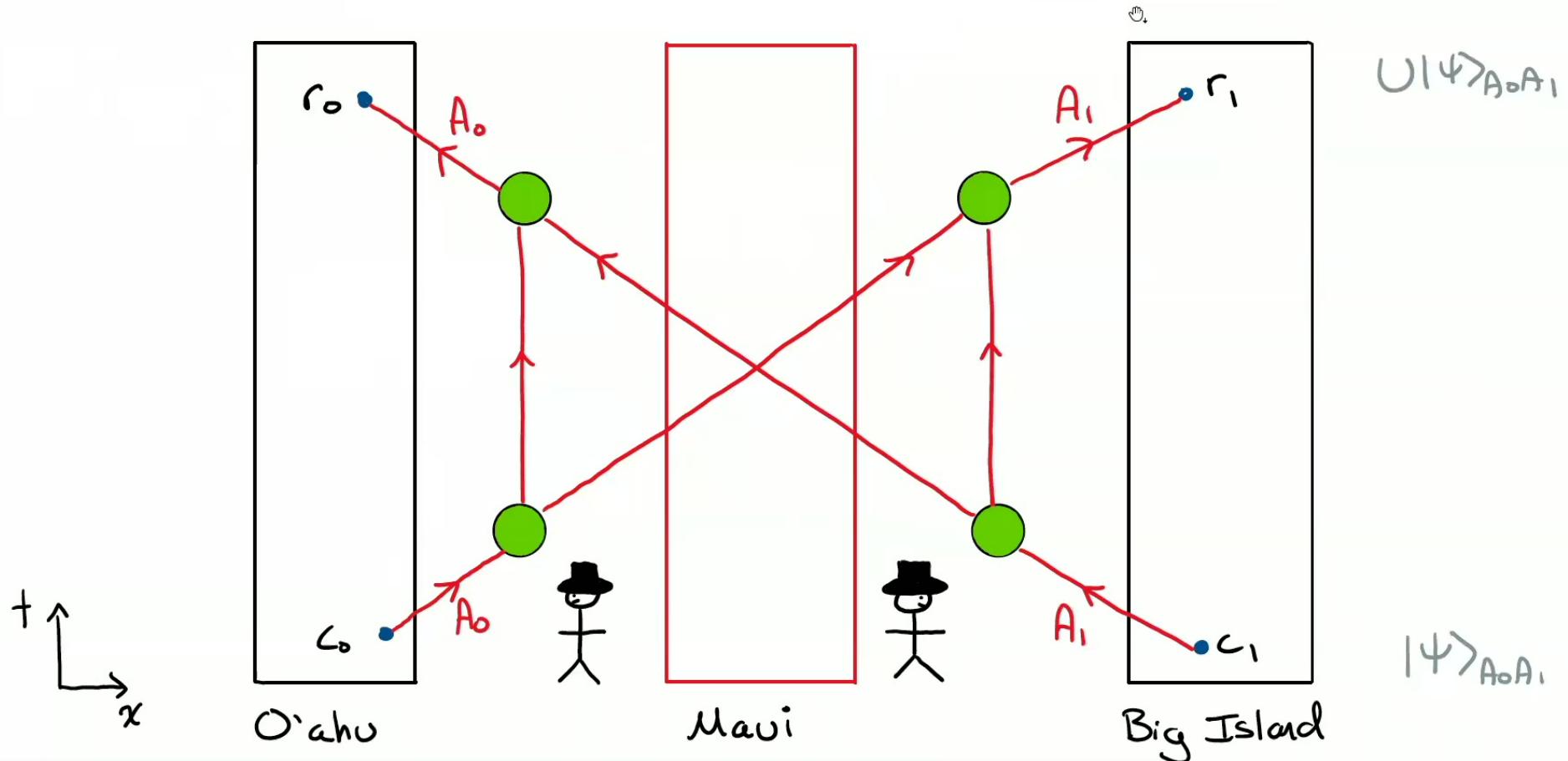
The same puzzle, elsewhere

(Kent , Monroe , Spiller 2011)

- The key to resolving this lies in an unexpected place, quantum cryptography.
- In position-verification, Bob tries to verify Alice's location in space by interacting with her remotely.

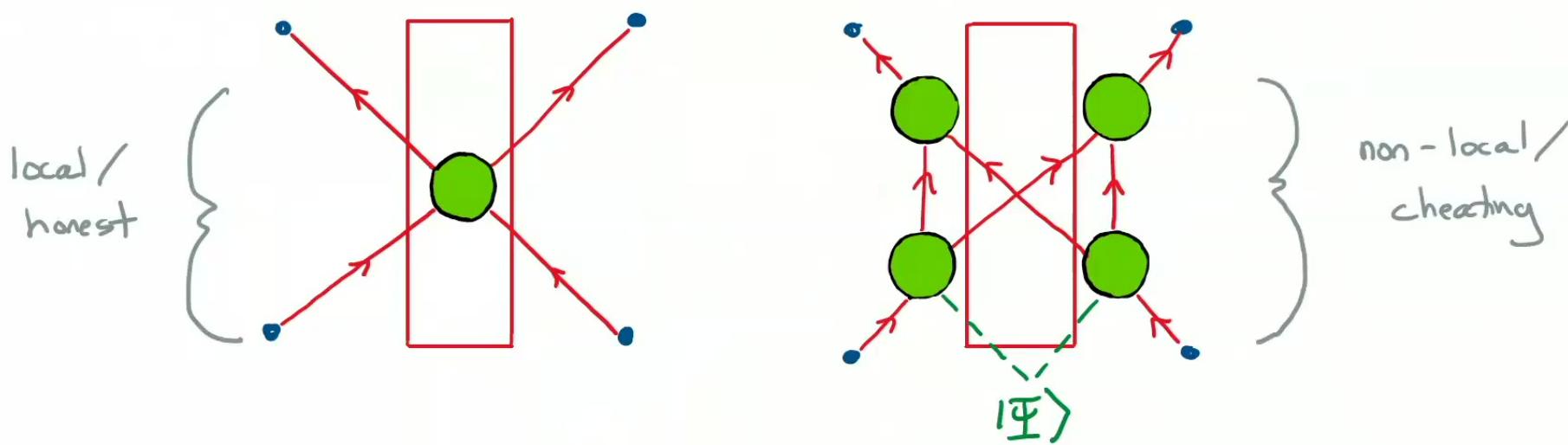


Cheating (non-local) strategy



Position-Verification

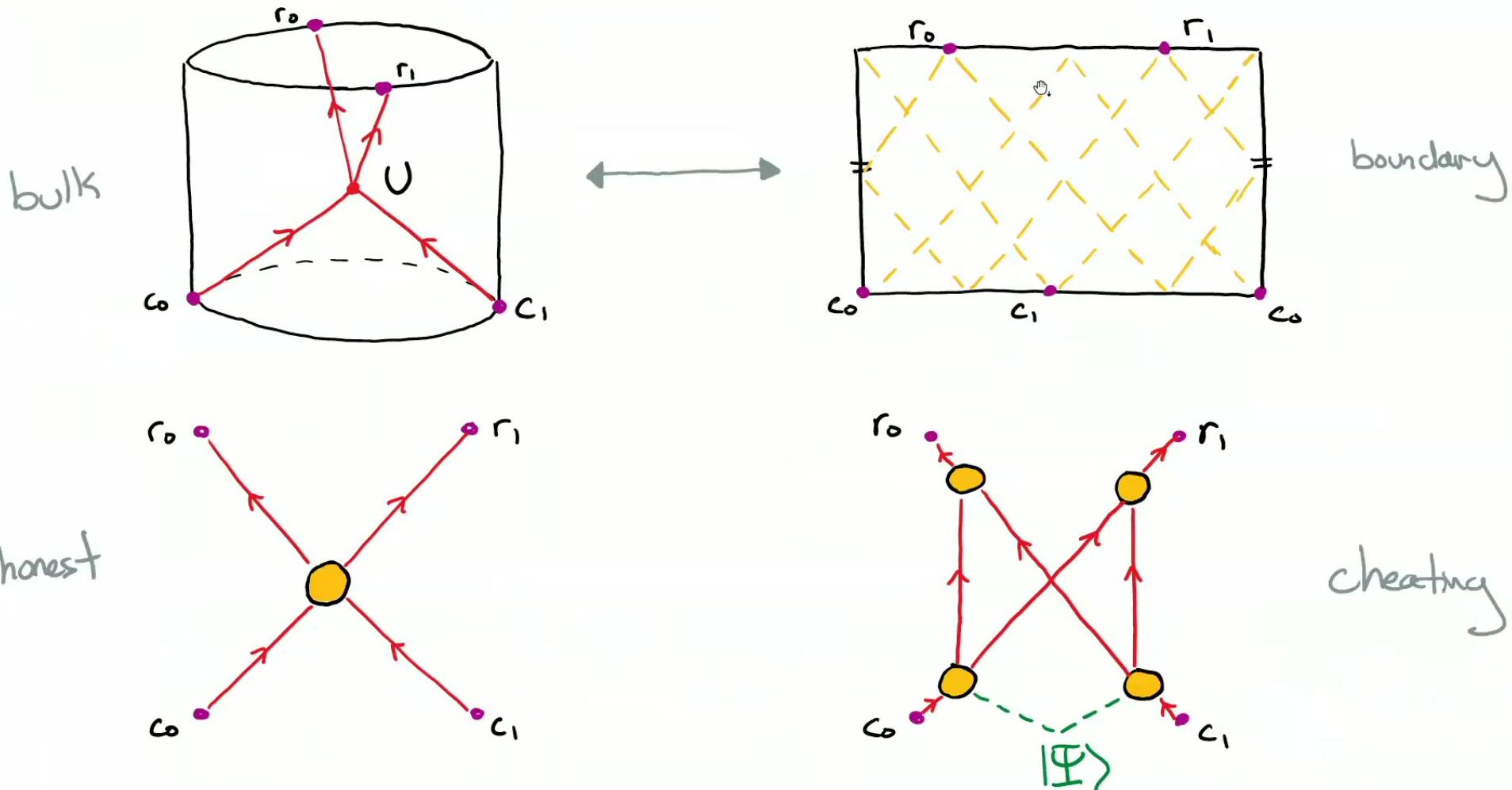
- Is there a choice of unitary $U_{A|A}$, which forces Alice to act honestly?



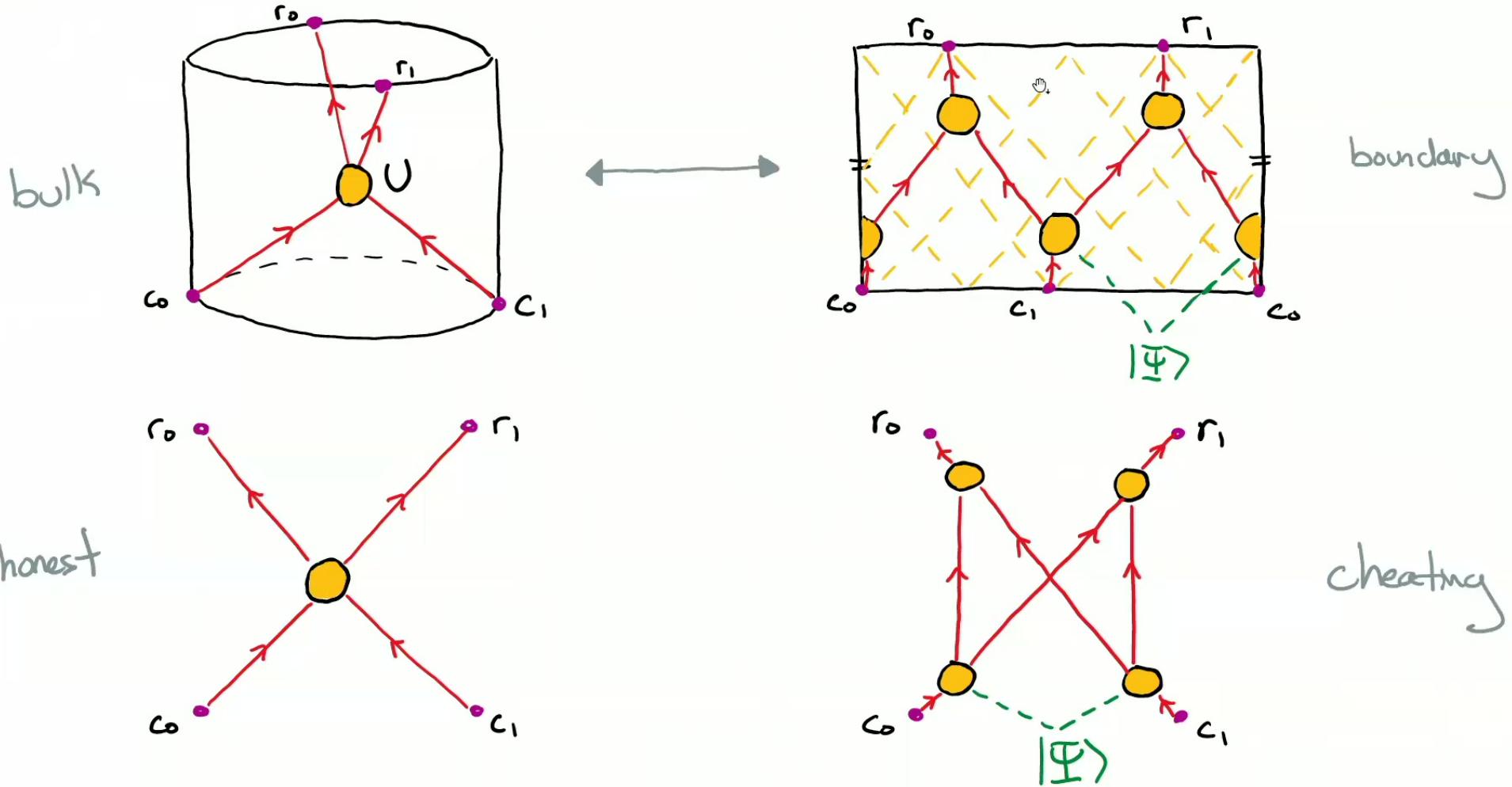
Always possible to do a unitary $U_{A|A}$, in the non-local form, using shared entanglement

Buhrman et al.
(2014)

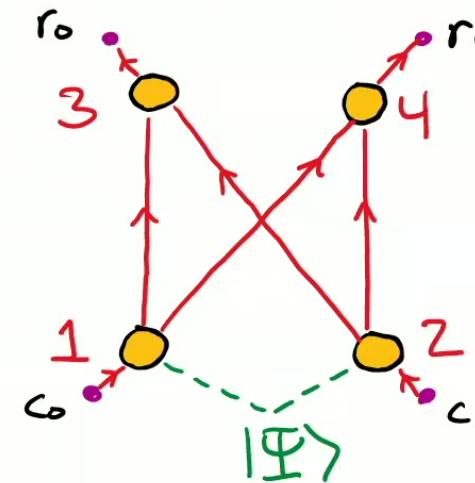
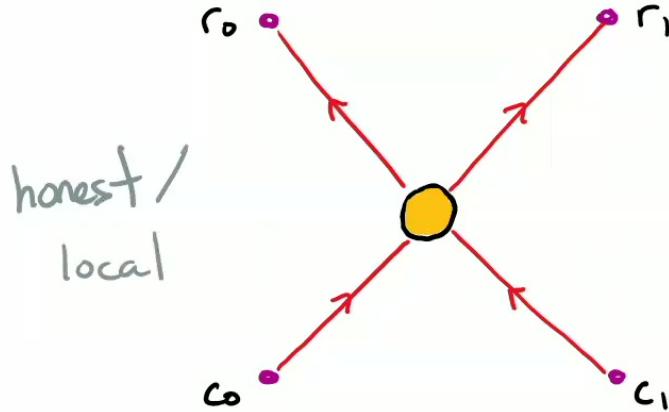
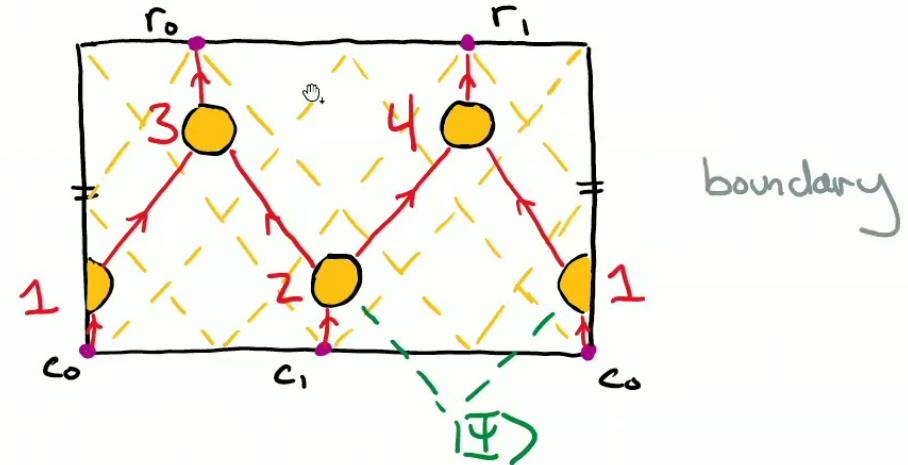
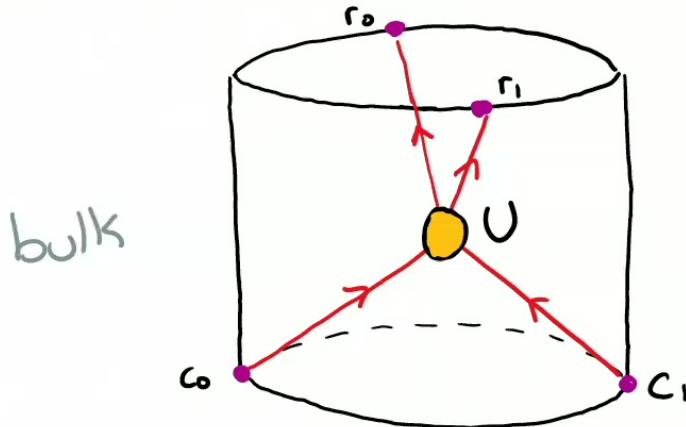
Position verification in AdS/CFT (AM 2019)



Position verification in AdS/CFT (AM 2019)

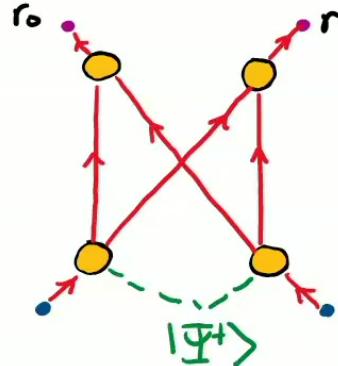
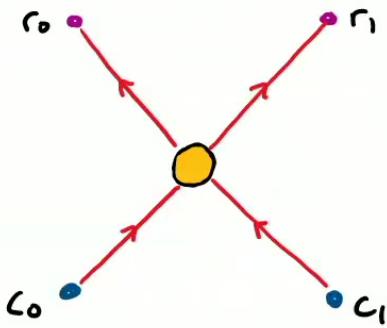
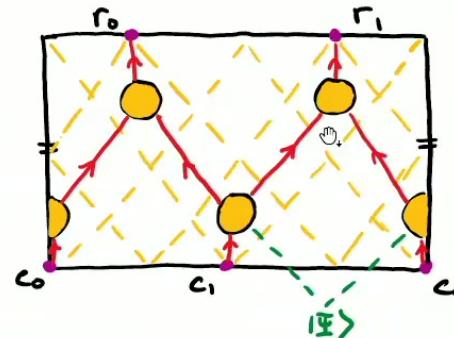
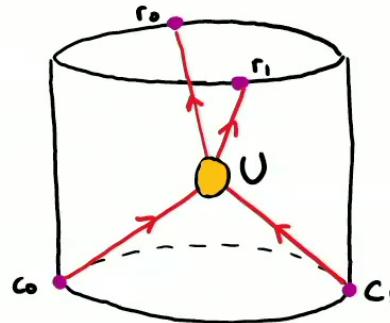


Position verification in AdS/CFT (AM 2019)



cheating/
non-local

Position verification in AdS/CFT (AM 2019)



Resolves earlier puzzle:

Entanglement based, non-local procedure allows local bulk interactions to be reproduced in boundary

Geometry and correlation in AdS/CFT

(B) How can gravitational d.o.f. be recorded
into quantum mechanics?

Entropy and correlation

- Given a quantum state, want to quantify correlations among subsystems:

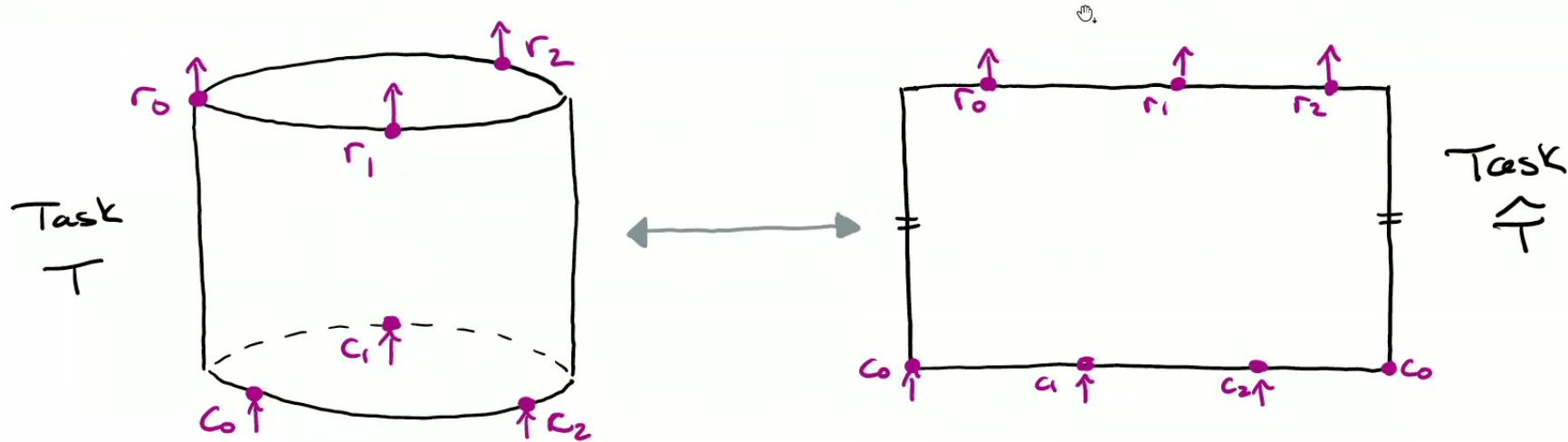
Given: $|\Psi\rangle_{ABC}$

Define: $S(A) \equiv -\text{tr}(p_A \log p_A)$ = "how correlated A is with everything else"
von Neumann entropy

→ $I(A:B) \equiv S(A) + S(B) - S(AB)$ = "how correlated A is with B"
mutual information

Boundary constraints from bulk causal structure

- To address ③, return to our general perspective:



Use:

Bulk
"causal feature" $\Rightarrow P_{\text{exc}}(T) \geq \alpha \Rightarrow P_{\text{exc}}(\hat{T}) \geq \alpha \Rightarrow$ something about CFT?

exists

2→2 Connected wedge theorem (AM 2019)

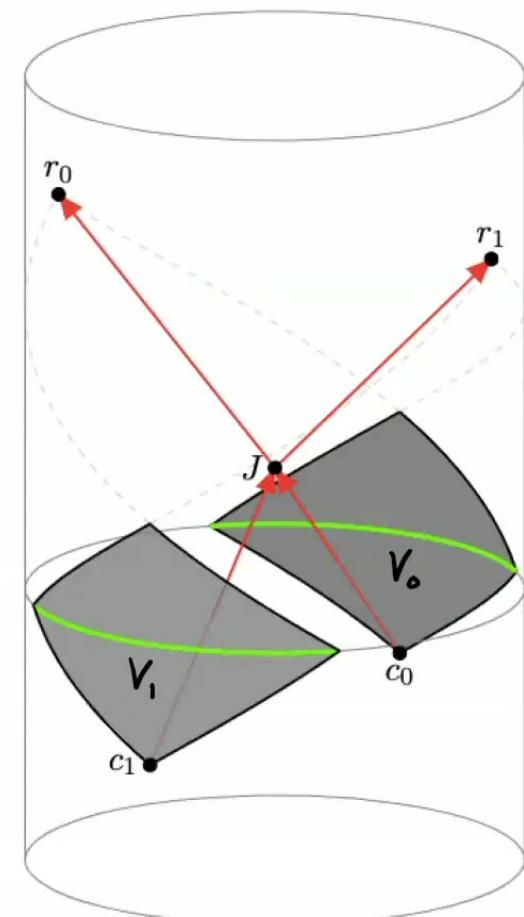
If c_0, c_1, r_0, r_1
have a scattering
region



V_0 and V_1 are
strongly correlated:
 $I(V_0:V_1) = O(\sqrt{G_N})$

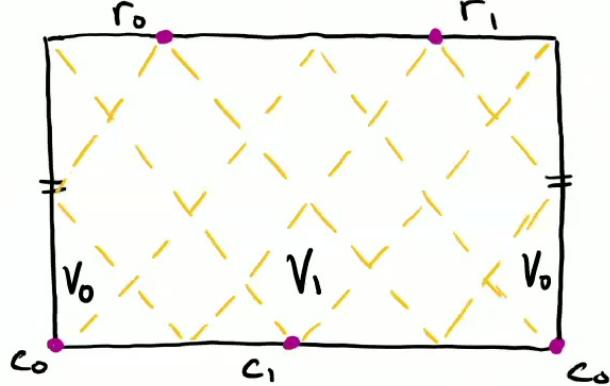
necessity of entanglement

scattering
region
exists $\Rightarrow P_{\text{succ}}(T)=1 \Rightarrow P_{\text{succ}}(\tilde{T})=1 \Rightarrow I(V_0:V_1) = O(\sqrt{G_N})$



2→2 Connected wedge theorem (AM 2019)

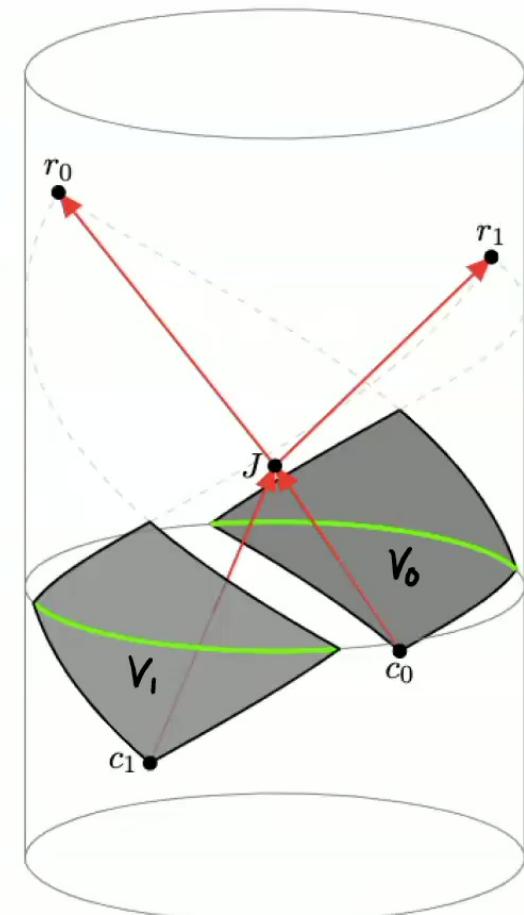
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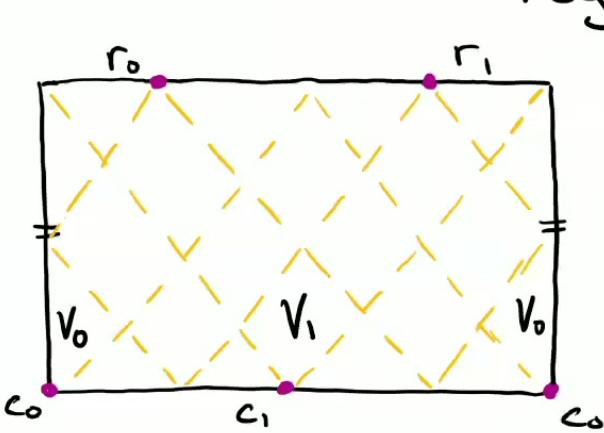
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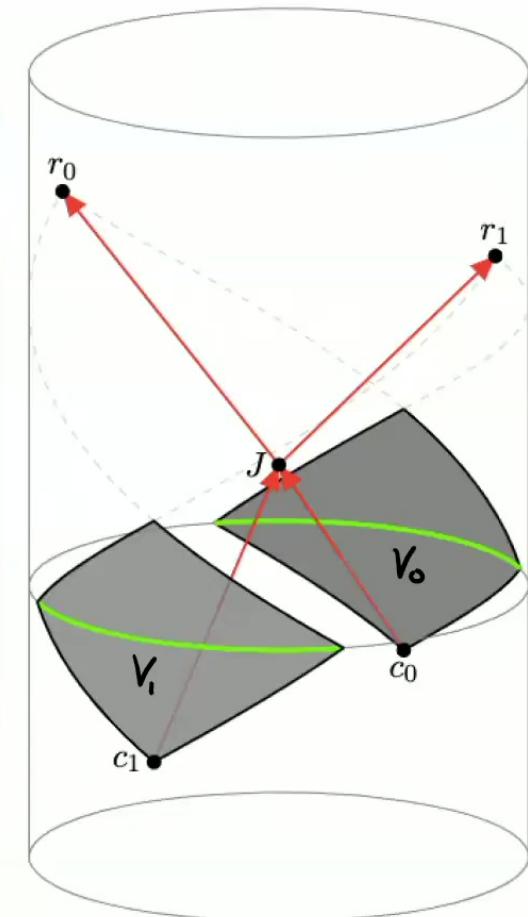
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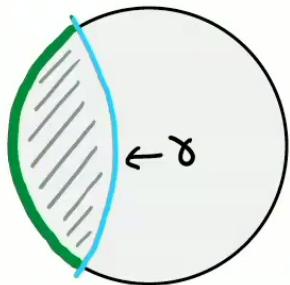
V_0 and V_1 are
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Gravitational data recorded
into boundary correlation

Entanglement in AdS/CFT

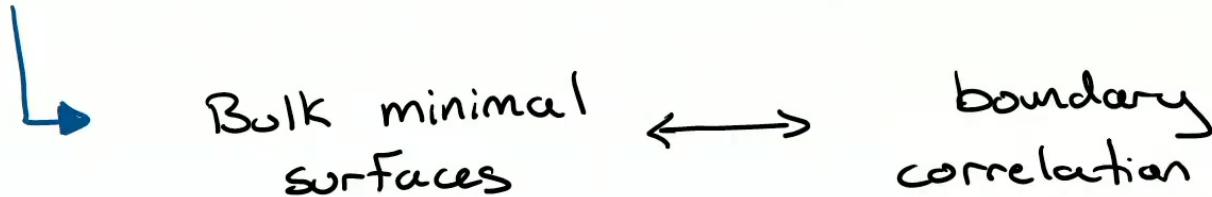
- Mutual information : $I(V_0; V_1) = S(V_0) + S(V_1) - S(V_0 \cup V_1)$
- In AdS/CFT, the entropy plays another role:



$$S(R) = \min_{\gamma} \frac{\text{area}(\gamma)}{4G_N}$$

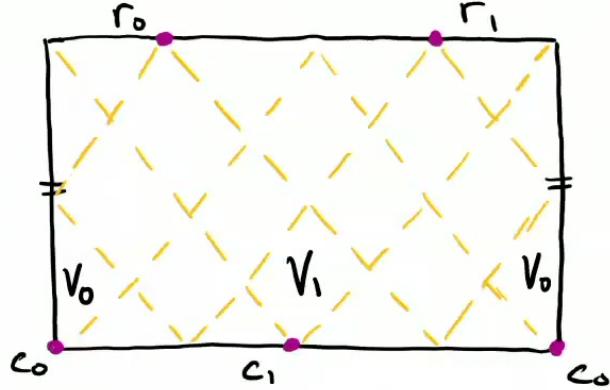
Ryu-Takayanagi
formula
(2006)

- Bulk gravitational data recorded into boundary correlation

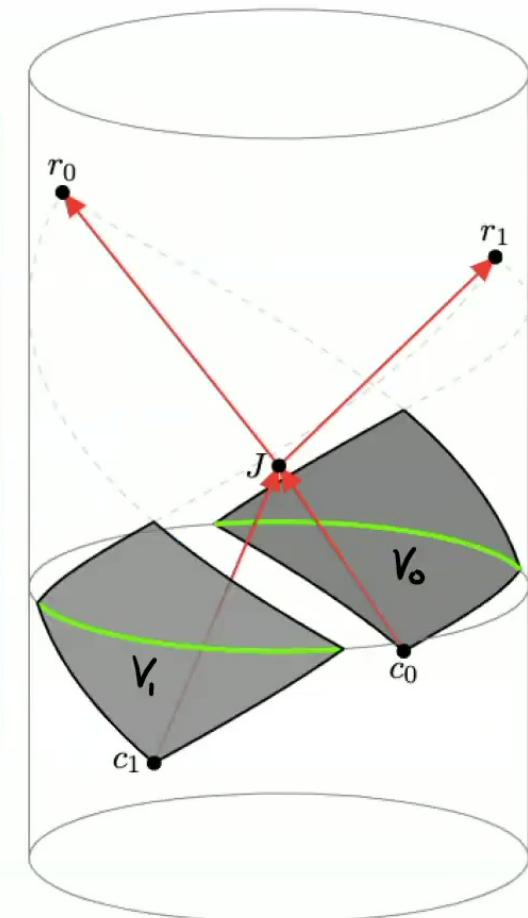


2→2 Connected wedge theorem (AM 2019)

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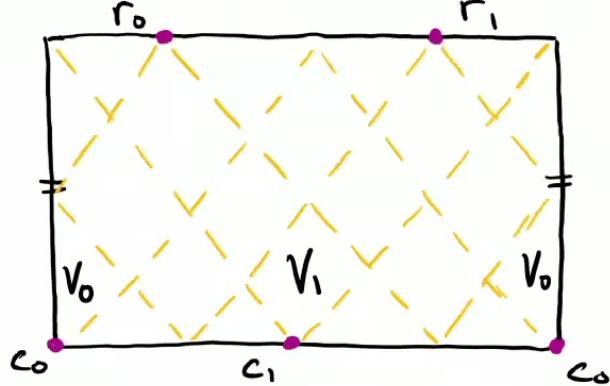
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Gravitational data recorded
into boundary correlation

2→2 Connected wedge theorem (AM 2019)

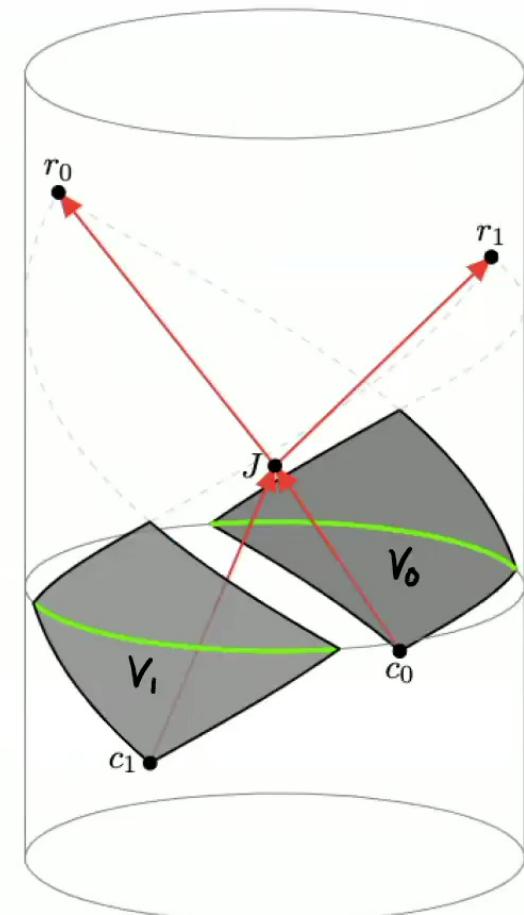
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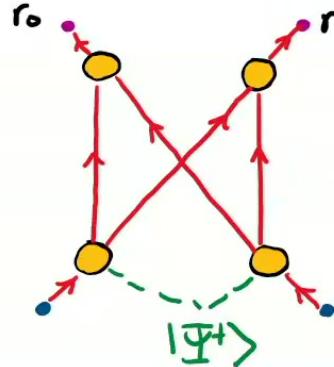
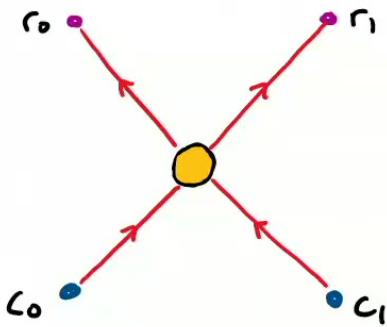
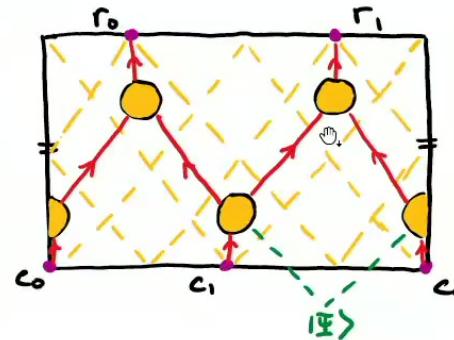
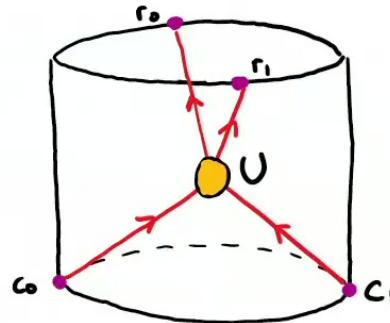
V_0 and V_1 are
strongly correlated:
 $I(V_0:V_1) = O(\sqrt{G_N})$

necessity of entanglement

scattering
region
exists $\Rightarrow P_{\text{succ}}(T) = 1 \Rightarrow P_{\text{succ}}(\tilde{T}) = 1 \Rightarrow I(V_0:V_1) = O(\sqrt{G_N})$



Position verification in AdS/CFT (AM 2019)



Resolves earlier puzzle:

Entanglement based, non-local procedure allows local bulk interactions to be reproduced in boundary

Entropy and correlation

- Given a quantum state, want to quantify correlations among subsystems:

Given: $|\Psi\rangle_{ABC}$

Define: $S(A) \equiv -\text{tr}(p_A \log p_A)$ = "how correlated A is with everything else"
von Neumann entropy

$\rightarrow I(A:B) \equiv S(A) + S(B) - S(AB)$ = "how correlated A is with B"
mutual information

2→2 Connected wedge theorem (AM 2019)

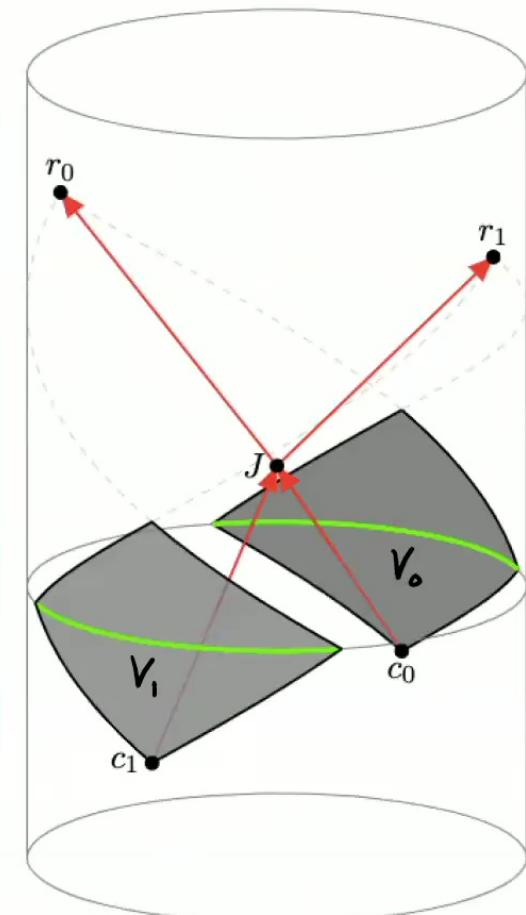
If c_0, c_1, r_0, r_1
have a scattering
region



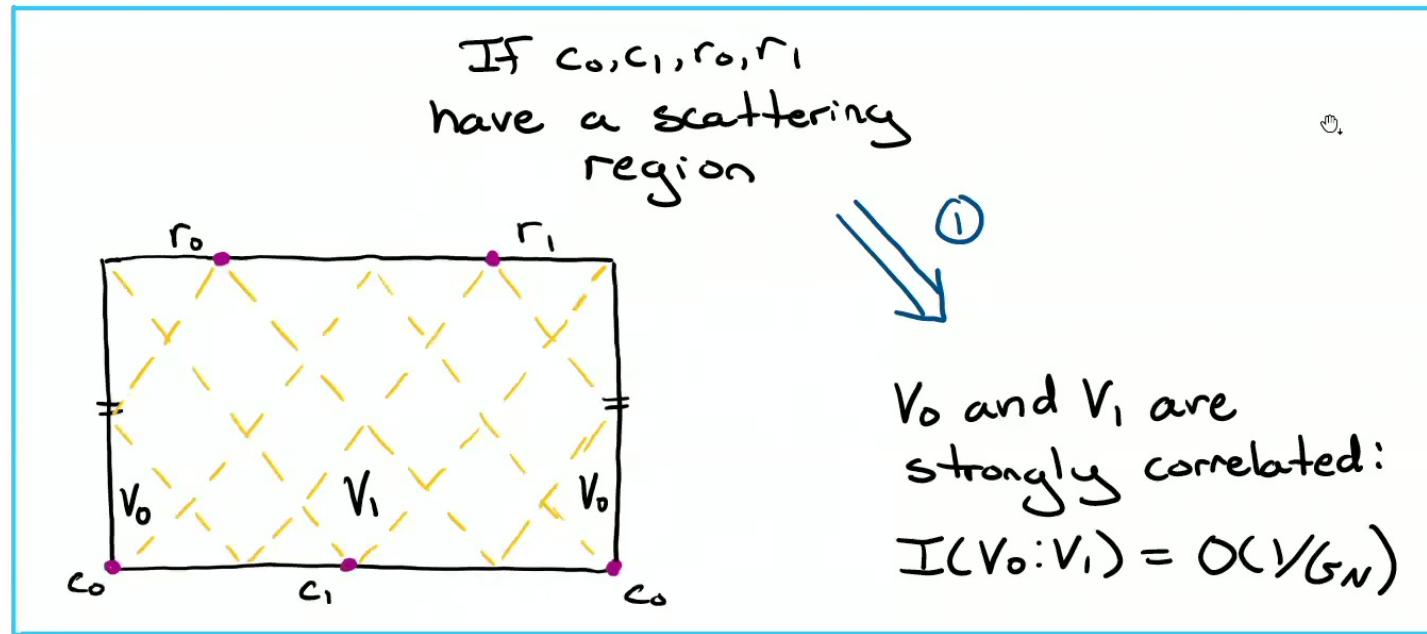
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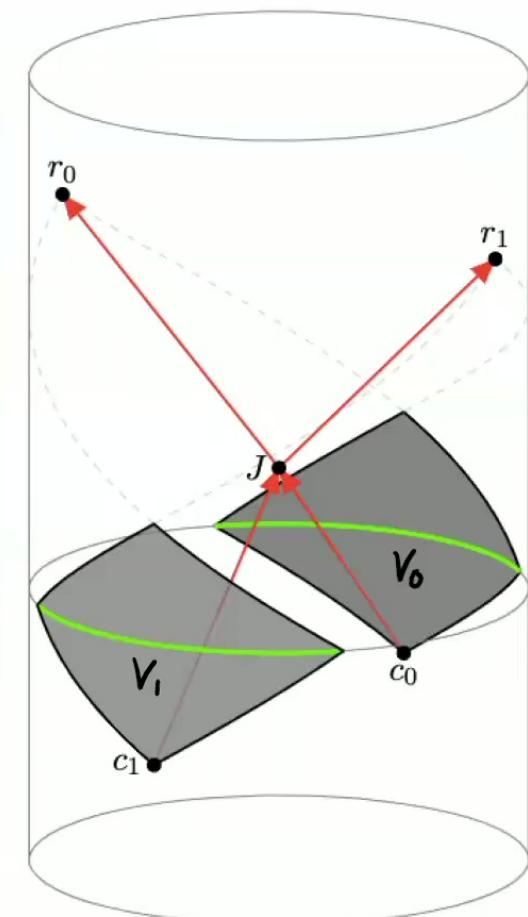
scattering
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exists $\Rightarrow P_{\text{succ}}(T)=1 \Rightarrow P_{\text{succ}}(\tilde{T})=1 \Rightarrow I(V_0:V_1) = O(\sqrt{G_N})$



2→2 Connected wedge theorem (AM 2019)

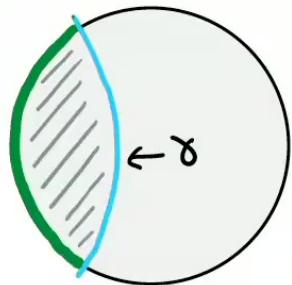


scattering
region
exists $\Rightarrow P_{\text{succ}}(T)=1 \Rightarrow P_{\text{succ}}(\tilde{T})=1 \Rightarrow I(V_0:V_1) = O(\sqrt{G_N})$



Entanglement in AdS/CFT

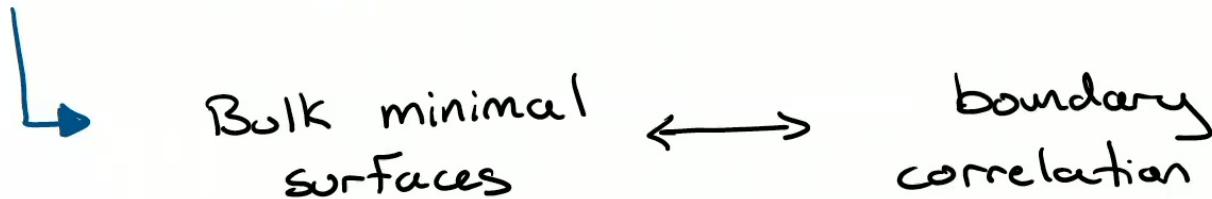
- Mutual information : $I(V_0; V_1) = S(V_0) + S(V_1) - S(V_0 \cup V_1)$
- In AdS/CFT, the entropy plays another role:



$$S(R) = \min_{\gamma} \frac{\text{area}(\gamma)}{4G_N}$$

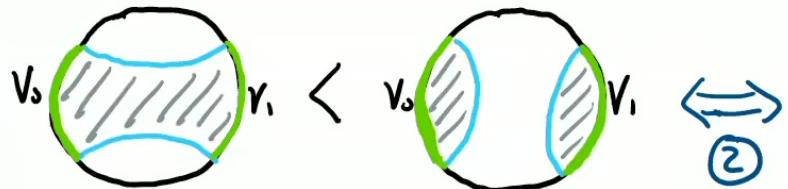
Ryu-Takayanagi
formula
(2006)

- Bulk gravitational data recorded into boundary correlation

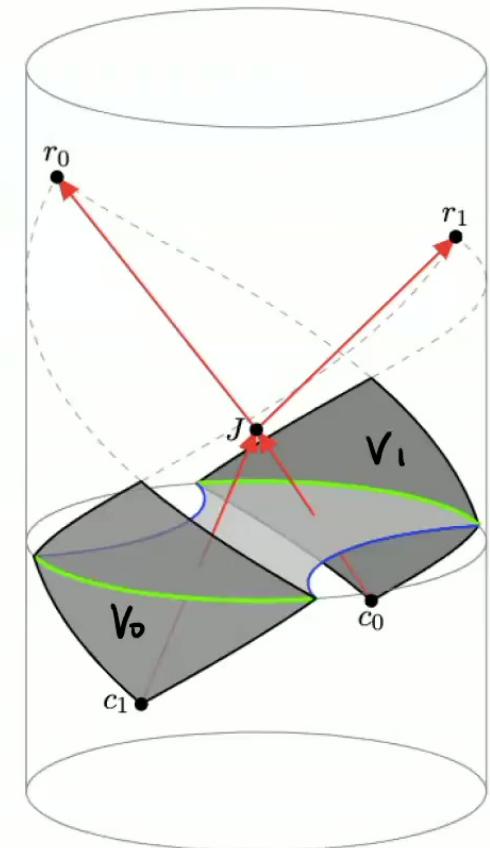


2→2 connected wedge theorem

IF c_0, c_1, r_0, r_1
have a scattering
region



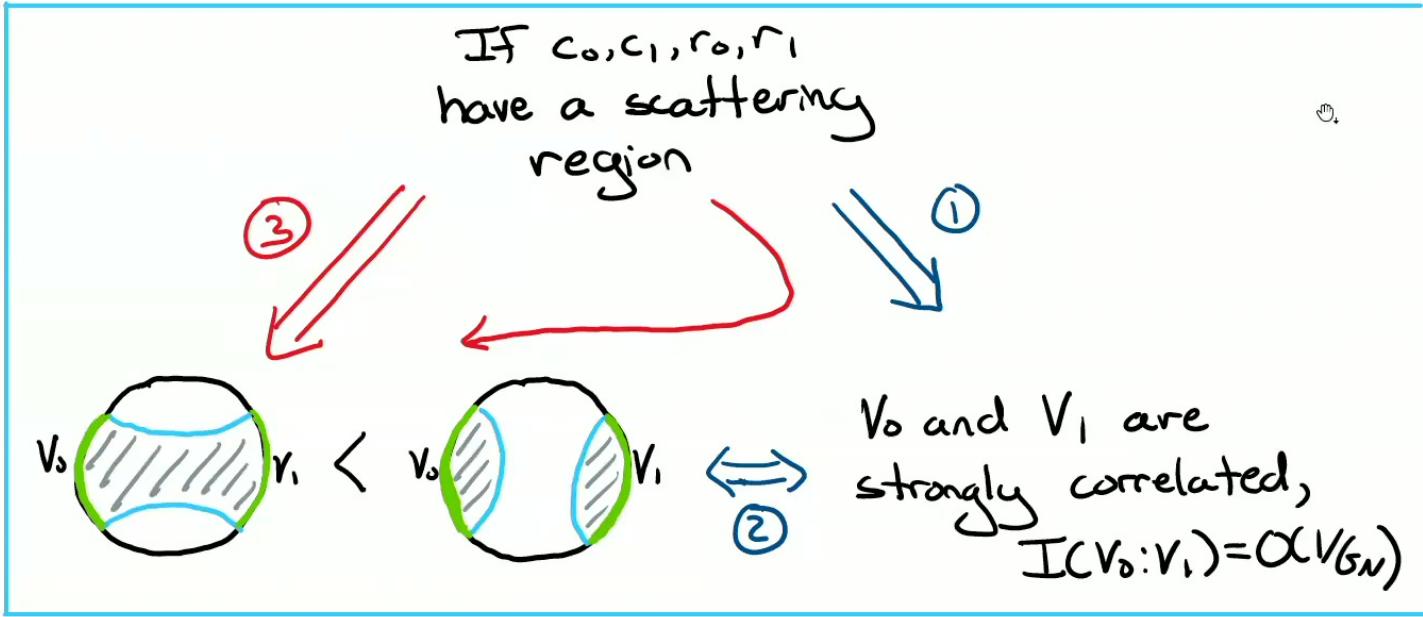
V_0 and V_1 are
strongly correlated,
 $I(V_0:V_1) = O(1/G_N)$



① Quantum tasks (necessity of entanglement)

② Follows from Ryu-Takayanagi formula

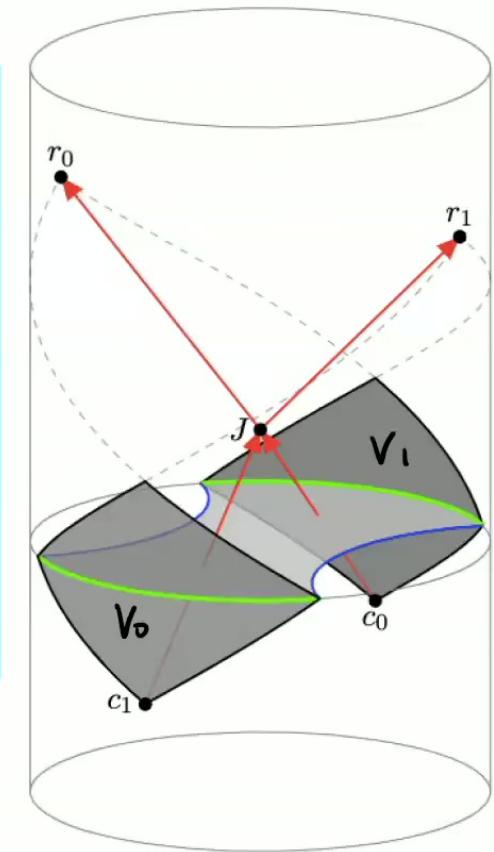
2→2 connected wedge theorem



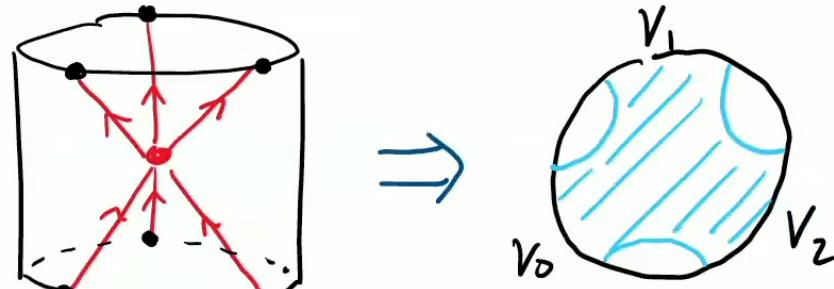
③ Proven directly in G.R.

↳ Assume NEC, use focusing theorem.

A.M., J. Sorce, G. Pennington. (2020)

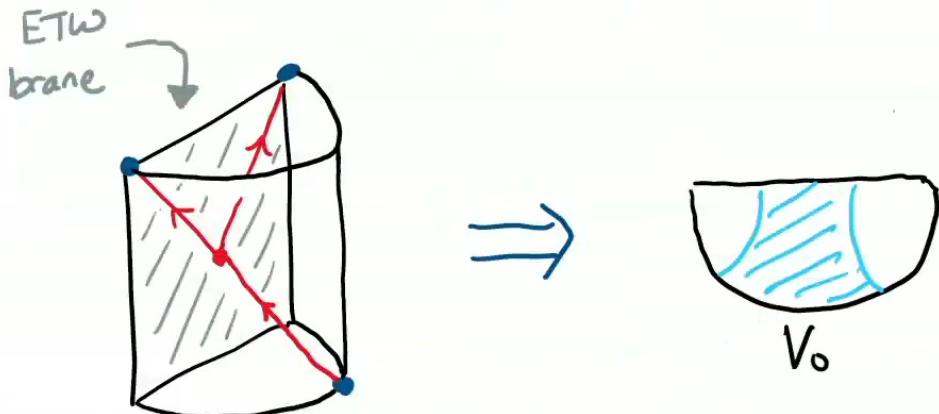


$n \rightarrow n$ connected wedge theorem



AM, J. Sorce, B. Yoshida (upcoming)

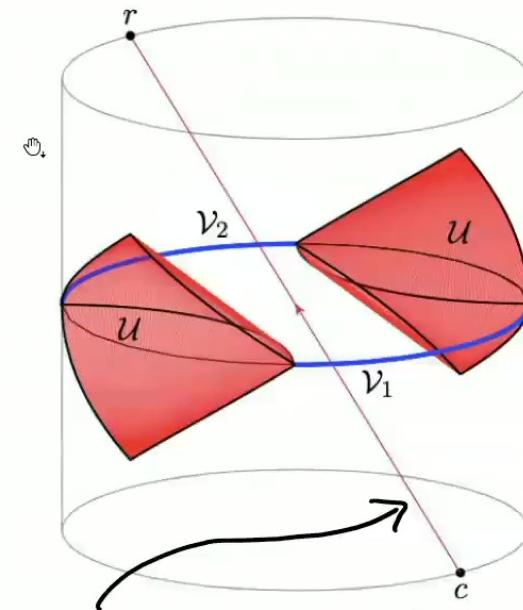
$1 \rightarrow 2$ connected wedge theorem



AM, D. Wakeham (2021)

①

Privacy-duality theorem



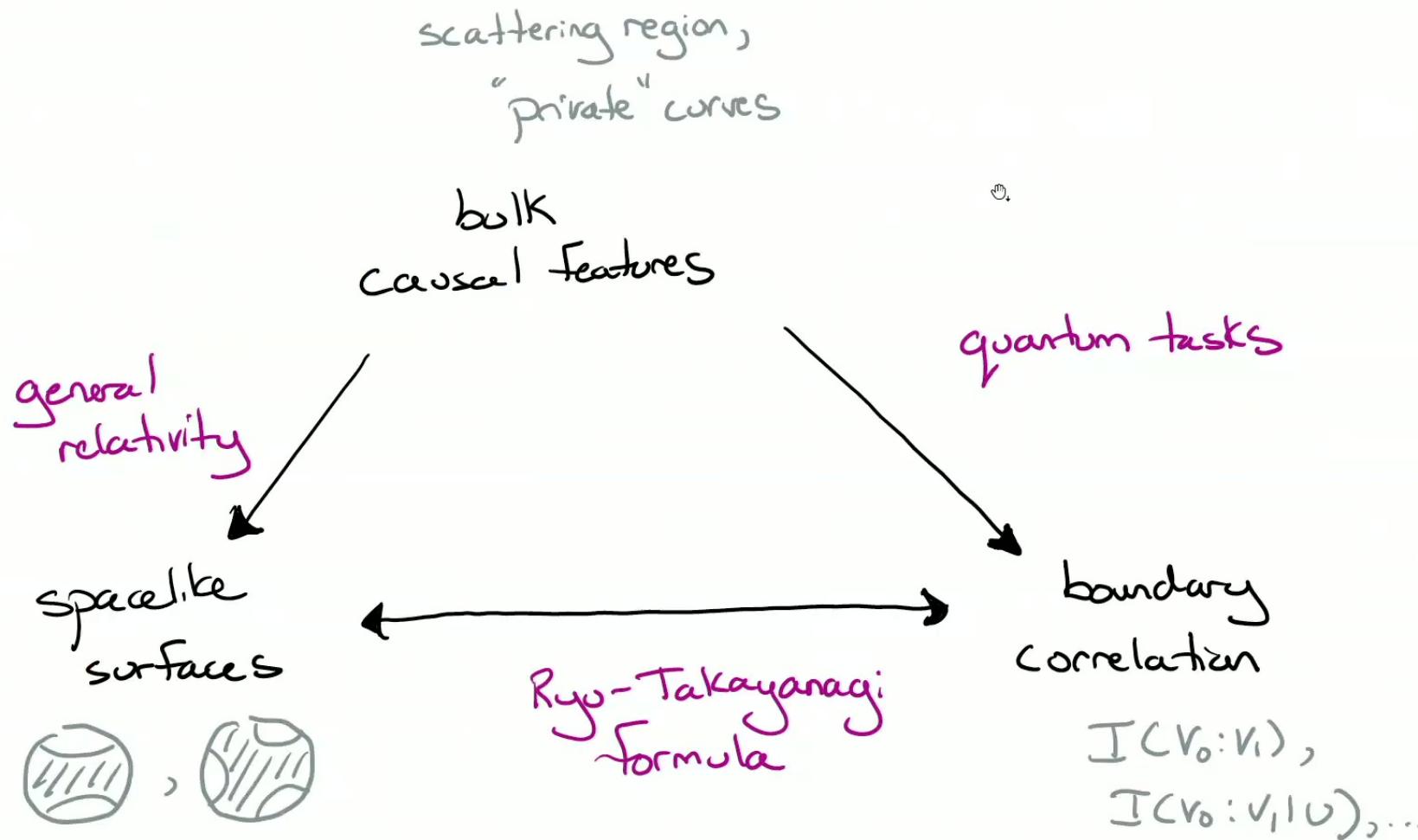
Causal curve from
 $c \rightarrow r$ that avoids U

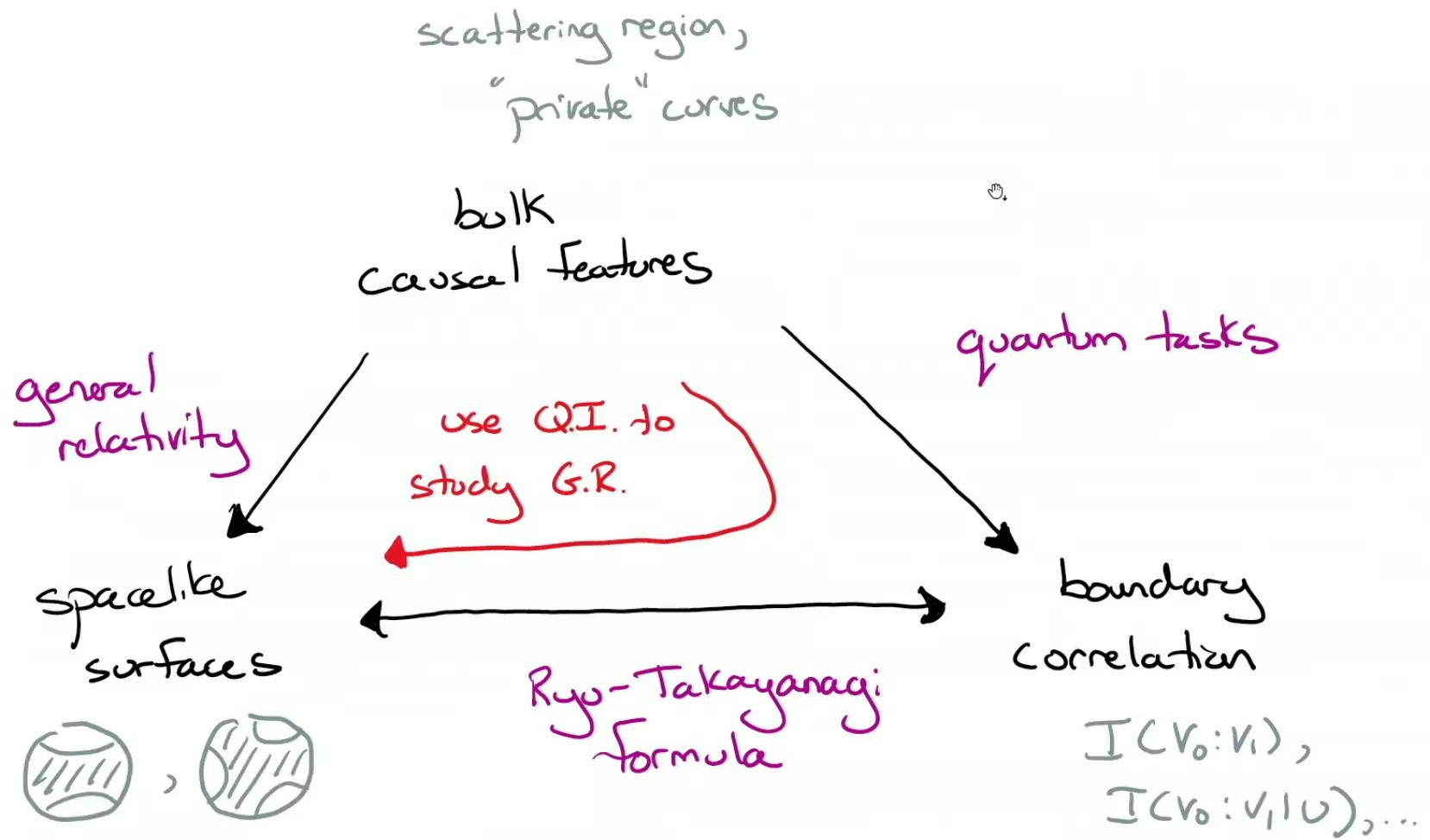


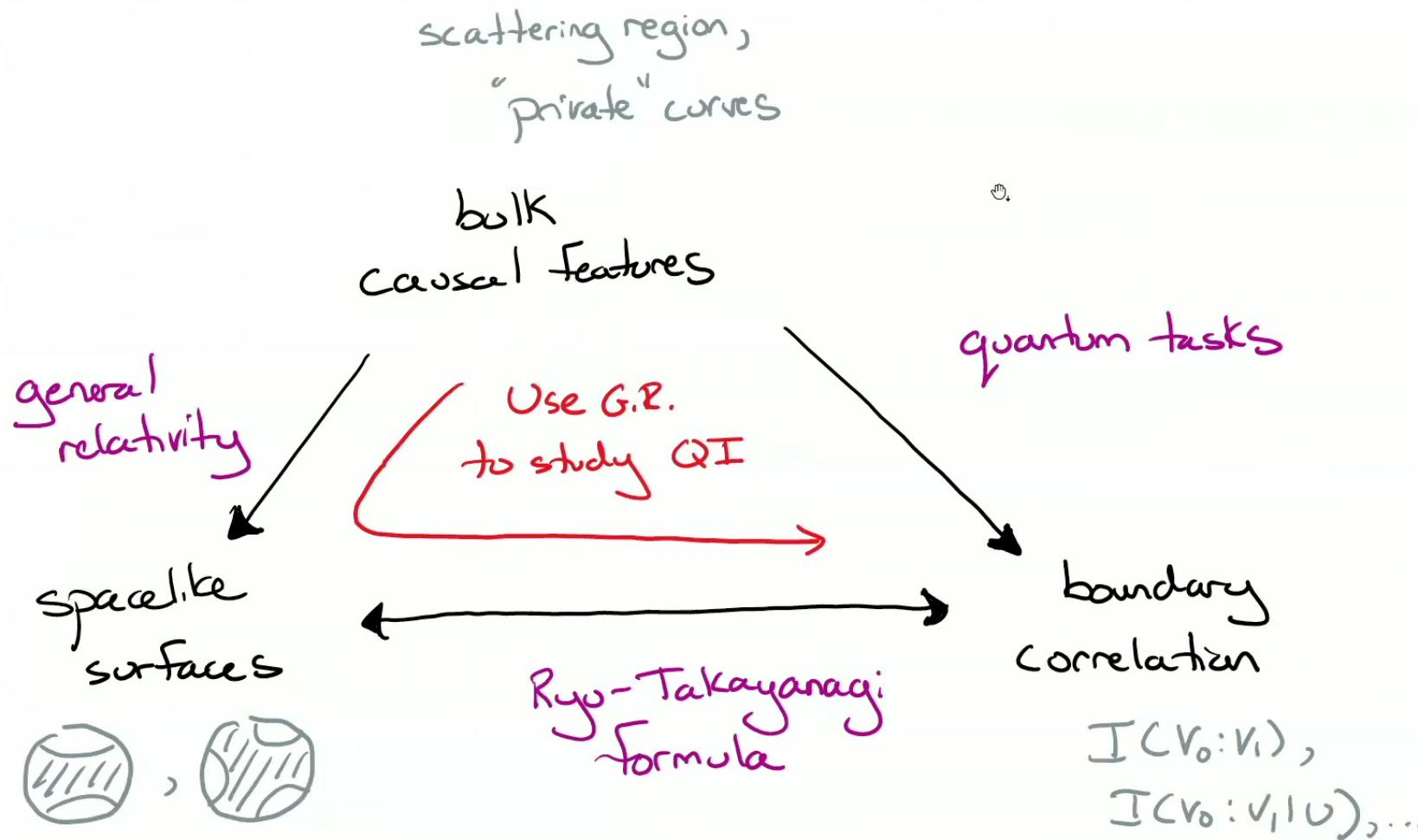
$$I(V_0 : V_1 | U) = O(1/G_n)$$

AM (2021)

③





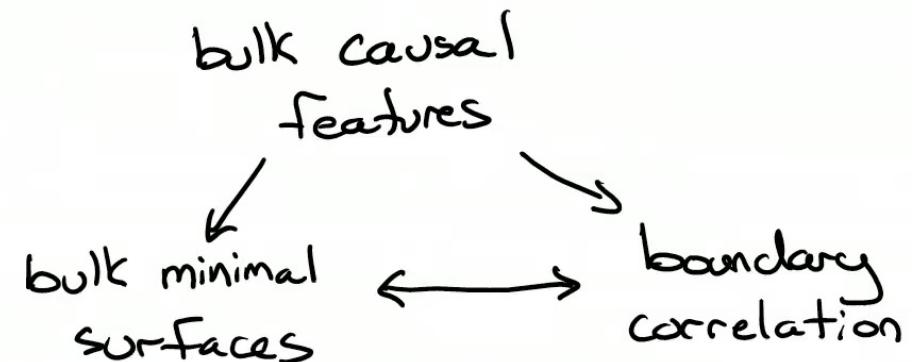




Summary

Summary

- ① Correlation can replace "causal features" when processing information in spacetime.
- ② In holography, higher dimensional theory has causal features lower dimensional theory lacks
↳ ① is needed for holography to be possible!
- ③ "Quantum tasks" approach reveals new aspects of how metric d.o.f. are recorded into QM.





Directions

① Developing QI in spacetime

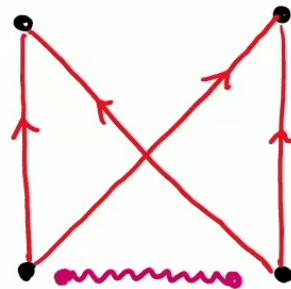
- Recall one of our starting points
- How can we better understand this theory of information?
 - ↳ How are the choice of task, causal structure, and needed correlation related?

What is the theory of information processing in spacetime?

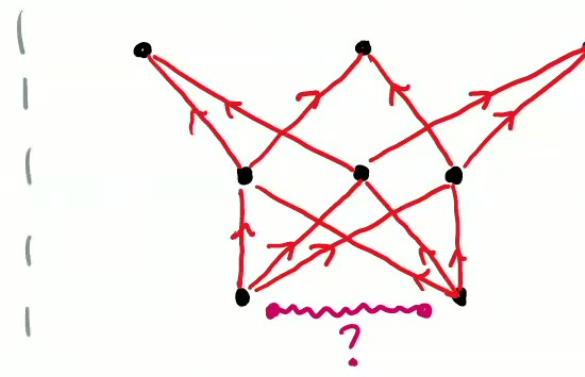
Causal structure and entanglement

Given a causal structure, when is entanglement needed to perform an arbitrary task?

E.g.



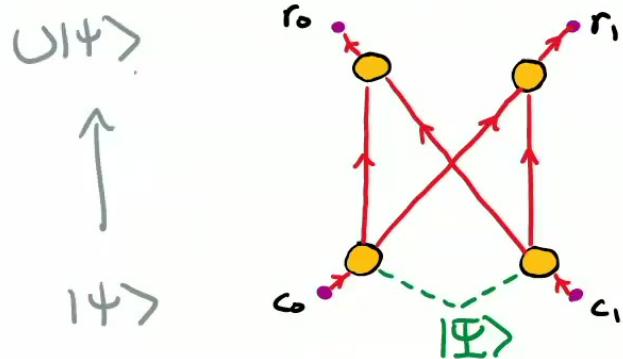
\exists tasks for which entanglement is necessary



Do we ever need entanglement?

Choice of task vs. entanglement cost

- On a fixed causal structure, how is the choice of task related to entanglement needed?



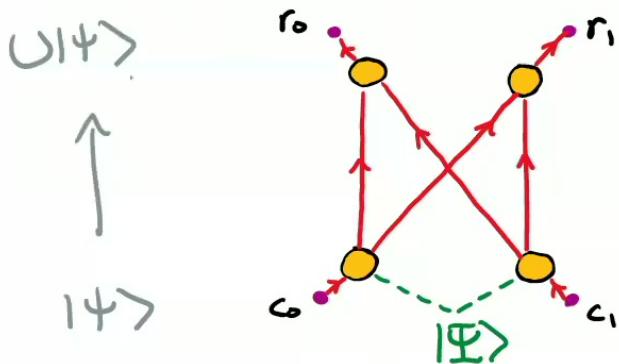
Given \cup determine

$E(\cup) = \#$ EPR pairs needed
to implement \cup "non-locally"

Choice of task vs. entanglement cost

- On a fixed causal structure, how is the choice of task related to entanglement needed?

•



- Some evidence $E(\cup) \sim \text{complexity}$

$$\rightarrow E(f) \leq \begin{matrix} \text{size of a span} \\ \text{program computing } f \end{matrix}$$

special task S. Cree, AM (2022)

Given \cup determine

$E(\cup) = \# \text{ EPR pairs needed}$
to implement \cup "non-locally"

$$\rightarrow E(\cup) = \Theta\left(\begin{matrix} \text{circuit complexity} \\ \text{of } \cup \end{matrix}\right)$$

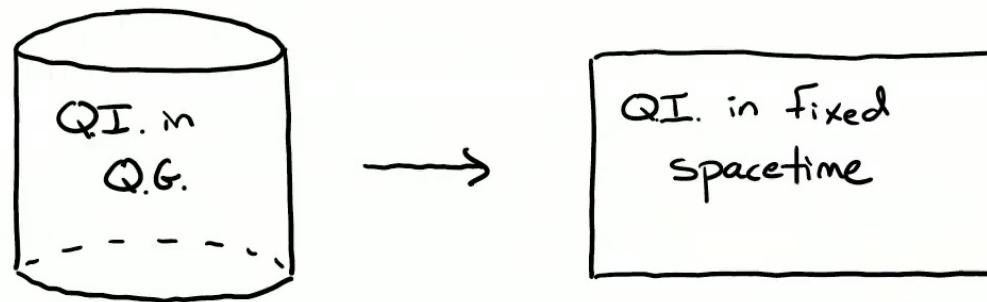
restrict strategy
(Cliffords) AM (2022)

② Information processing in quantum gravity AM (2022)

- We explored applications of quantum information in fixed spacetime.
Can go one step further :

What is the theory of information
processing in quantum gravity?

- We can use AdS/CFT to address this in a precise way!



Summary

Thanks!

- ① Correlation can replace "causal features" when processing information in spacetime.
- ② In holography, higher dimensional theory has causal features lower dimensional theory lacks
 - ↳ ① is needed for holography to be possible
- ③ "Quantum tasks" approach reveals new aspects of how metric d.o.f. are recorded into QM.

