Title: Holomorphic Floer Theory and the Fueter Equation

Speakers: Semon Rezchikov

Series: Mathematical Physics

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Abstract: The Lagrangian Floer homology of a pair of holomorphic Lagrangian submanifolds of a hyperkahler manifold is expected to simplify, by work of Solomon-Verbitsky and others. This occurs in part because, in this setting, the symplectic action functional, the gradient flow of which computes Lagrangian Floer homology, is the real part of a holomorphic function. As noted by Haydys, thinking of this holomorphic function as a superpotential on an infinite-dimensional symplectic manifold gives rise to a quaternionic analog of Floer's equation for holomorphic strips: the Fueter equation. I will explain how this line of thought gives rise to a `complexification' of Floer's theorem identifying Fueter maps in cotangent bundles to Kahler manifolds with holomorphic planes in the base. This complexification has a conjectural categorical interpretation, giving a model for Fukaya-Seidel categories of Lefshetz fibrations, which should have algebraic implications for the study of Fukaya categories. This is a report on upcoming joint work with Aleksander Doan.

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Monday, March 14, 2022 3:04 PM

Holomorphic Floer Theory and the Fueter Equation.

(joint w/ Aleksander Doan — Columbia, Trinity College)

April 22, 2022

Semon Rezchikov
Harvard University

Symplectic Geometry:

M^{2n} $\omega \in \Omega^2(M, \mathbb{R})$ $d\omega$, $\omega^n \neq 0$

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Symplectic Geometry:

M^{2n}, $\omega \in \Omega^2(M, \mathbb{R})$, $d\omega$, $\omega^n \neq 0$

$I : TM \to TM$ $\quad I^2 = -1$. $\omega(-, I-) = g$ (nearly Kähler)

$\nabla I = 0$. (Kähler)

$L^n \subset M^{2n}$ $\quad \omega|_L = 0$ Lagrangian Submanifold.

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Symplectic Geometry:

$$M^{2n}, \quad \omega \in \Omega^2(M, \mathbb{R}), \quad d\omega, \quad \omega^n \neq 0$$

$$I : TM \to TM \quad I^2 = -1. \quad \omega(-, I-) = g \quad \text{(nearly Kähler)}$$

$$\nabla I = 0. \quad \text{(Kähler)}$$

$$L^n < M^{2n} \quad \omega|_L = 0 \quad \text{Lagrangian Submanifold.}$$

It is interesting to search for an "appropriate" complexification of symplectic geometry.

Some options:

$$① \; M^{2n} \quad \Omega \in \Omega^{2,0}(M, \mathbb{C}) \quad \Omega^n \neq 0 \qquad d\Omega = 0$$

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complexification of symplectic geometry.

Some options;

① $M^{2n}_{\mathbb{C}}$, $\Omega \in \Omega^{2,0}_{hol}(M,\mathbb{C})$, $\Omega^n \neq 0$. $d\Omega = 0$

"holomorphic symplectic."

L^n, $\Omega|_L = 0$ complex lagrangian.

no metric

② Holonomy $Sp(n)$ — hyperkähler. $IJ = -JI = K$.

$I, J, K, \omega_I, \omega_J, \omega_K, g.$

$\omega_J + i\omega_K = \Omega \implies (I, \Omega)$ hol'c symplectic.

"holomorphic symplectic."

$L^{\wedge}$, $\Omega|_L = 0$ complex lagrangian.

no metric

(2) Holonomy $Sp(n)$ — hyperkähler. $IJ = -JI = K$.

$I, J, K, \omega_I, \omega_J, \omega_K, g.$

$\omega_J + i\omega_K = \Omega \Rightarrow (I, \Omega)$ hol'c symplectic.

Examples are rich but rare due to integrability requirements. any

Relaxing these is nontrivial.

$\Omega^{2,0}(M, \mathbb{C})$ $d\Omega = 0$?

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(2) Holonomy $Sp(n)$ — hyperkähler.

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Examples are rich but rare due to integrability requirements.

any

Relaxing these is nontrivial.

$\left\{ \text{e.g.: } I, \underset{\Omega}{\Omega^{2,0}}(M, \mathbb{C}), d\Omega = 0 \right\}$

$\implies I$ integrable!

Symplectic Floer Homology —

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Symplectic Floer Homology

$L_0, L_1 \subset M \qquad P = \{ \gamma : [0,1] \to M : \gamma(0) \in L_0, \gamma(1) \in L_1 \}$

$$\text{"}dA\text{"} = \int_0^1 \omega(-, \dot\gamma)\, dt$$

is a closed 1-form on P with natural perturbations

$$\pm \int_\gamma H\, dt$$

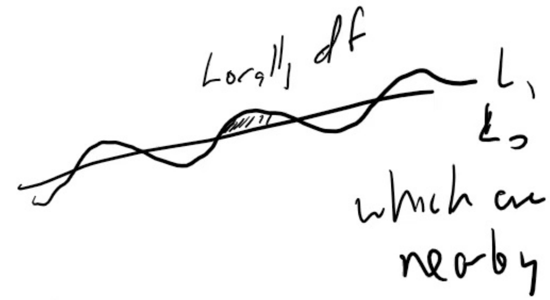
If $\omega = d\lambda \qquad A = \int_\gamma \lambda$.

Lagrangian Floer Homology.

Floer: "finite dimensional model"

$f: Q \longrightarrow \mathbb{R}$ C^2-small Morse

$\rightsquigarrow$ $L_0 = Q,\ L_1 \subset \Gamma(df) \subset T^*Q.$



$HM(f) = HF(L_0, L_1)$

via bijection between Morse trajectories and Floer trajectories.

Complexify Ω:

$(M, I, \Omega \in \Omega^{2,0}_{I\text{-hol}}(\mathbb{C}))$

L_0, L_1 — complex lagn. under I

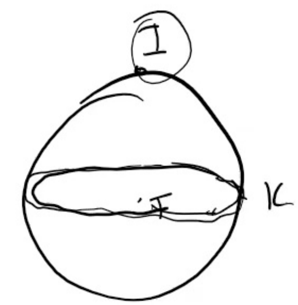
$\mathcal{P}$ is ∞-dim' complex manifold.

$\omega = \mathrm{Re}\,\Omega \Rightarrow \mathcal{A} = \mathrm{Re}\,\mathcal{A}_{\mathbb{C}} \leftarrow I$-holomorphic.

$\omega_\theta = \mathrm{Re}(e^{i\theta}\Omega)$

hol'c function!

$\int_\gamma \Omega$



S^2_{dim}

In this setting $HF_{\omega_\theta}(L_0, L_1)$ simplifies.

$\omega_\theta = \cos$

$$\Omega = \omega_J + i\,\omega_K .$$

critical values of $\boxed{A_{\mathbb{C}}}$.

$\mathrm{Re}(e^{i\theta}A_{\mathbb{C}})$

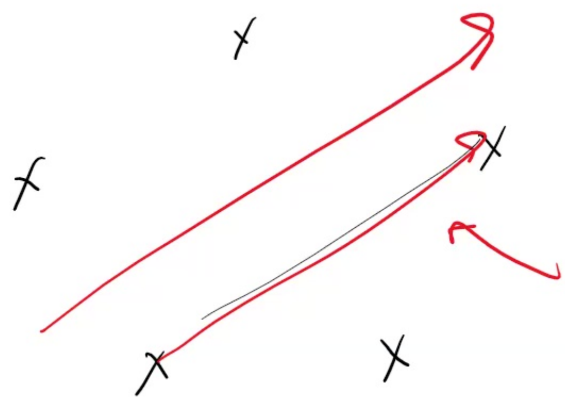


image of gradient flow.

Two points rarely lie on a line!

If L_0, L_1 intersect transversely we should have

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What is complex Morse theory?

Idea: $M \xrightarrow{W} \mathbb{C} \rightsquigarrow$ category.

Obj: critical points

Morphisms: Gradient flows of $\mathrm{Re}(e^{i\theta}W)$

Composition.

$\nabla \mathrm{Re}(W) = I \nabla \mathrm{Im}(W)$

countg.

$\partial_{\bar{z}} u$

$u: \mathbb{C} \to M$. $\quad \partial_s u + I \partial_\tau u = \nabla \mathrm{Re}(e^{i\theta}W)$.

somehow define com

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Obj: critical points

Morphisms: Gradient flows of $Re(e^{i\theta}W)$

Composition

$$\nabla Re(W) = I \nabla Im(W)$$

conditions.

$$u : \mathbb{C} \to M.$$

$\partial_{\bar{z}} u$

$$\partial_s u + I \partial_\tau u = \nabla Re(e^{i\theta} W).$$

somehow define com

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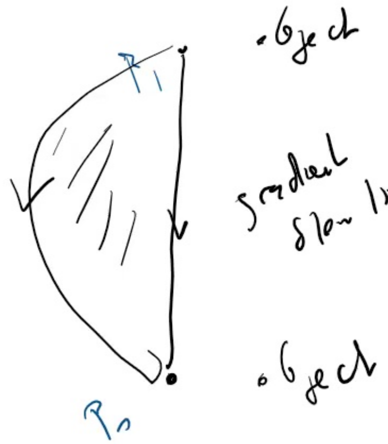

Donaldson-Thomas/Segal, Gaiotto-Moore-Witten; Kontsevich

Haydys

"Algebra of the Infrared

cond'n.

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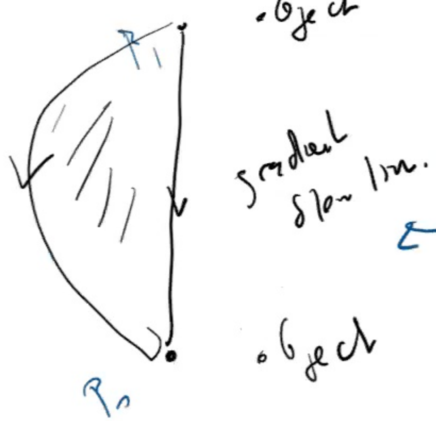

Donaldson-Thomas/Segal, Gaiotto-Moore-Witten; Kontsevich

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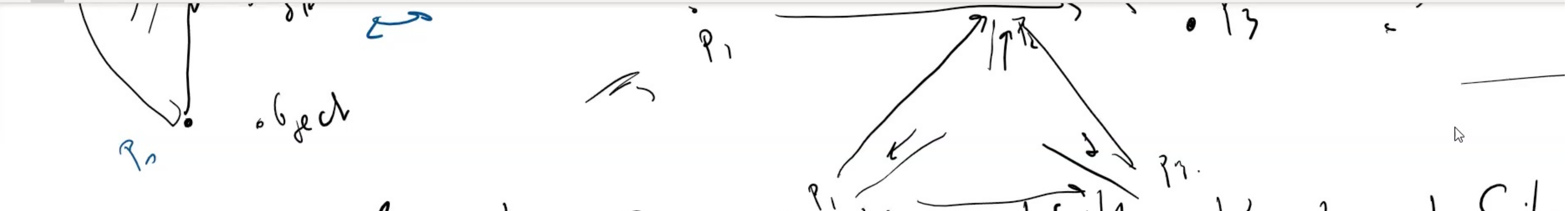

naldson-Thomas/Segal, Gaiotto-Moore-Witten, Kontsevich-Soibe

Haydys

"Algebra of the Infrared."

" ζ-instanton equation "

Lefshetz Fibration

Rigorous definition of Category: FS(W).

Lagrangian submanifold

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↑ crit pt Donghao Wang ('21)

Do this with $\mathcal{A}_{\mathbb{C}}$.

"$\partial_s u + I \partial_t u = \nabla \mathrm{Re}\, \mathcal{A}_{\mathbb{C}}$" $\longrightarrow$ Fueter equation.

$u : \mathbb{R}_s \times \mathbb{R}_t \times [0,1]_\tau \longrightarrow M.$ $K = IJ.$

$\partial_s u + I \partial_t u + J \partial_\tau u$.

$K \underline{\partial_\tau u + J \partial_s u} - I \partial_t u = 0.$

PDE underlying A-twist of 3d $\mathcal{N}=4$

Ahn Ki

$$K\partial_\tau u + J\partial_s u - I\partial_t u = 0.$$

PDE underlying A-twist of 3d $\mathcal{N}=4$
(Anselmi-Fre + dimensional reduction)

Arise as limits of G_2-instantons. (Worpulski Haydys, Doan).

Investigated by Hohloch-Noetzel-Salamon, Ginzburg-Hein.

↳ 3d vs 4d.

Should get

$\mathrm{Fuet}(L_0, L_1)$ a category for L_0, L_1 holo Lags in Hyperkahler.

$\mathrm{Hom}(L_0, L_1)$ in a 2-category $\mathrm{Fuet}(M)$.

Ahu Ki

Gammage

Bousse

Formal

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(Anselmi-Fre + dimensional reduction)

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Hom(L_0, L_1) in a 2-category Fuet(M).

$HH_*(\mathrm{Hom}(L_0, L_1)) = HF(L_0, L_1)$ ↑ simple.

Bousse

formal

DT

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$HH_x(Hom(L_0, L_1)) = HF(L_0, L_1)$ ↑ simple.

Local model ("Floer's theorem")

$W : M \longrightarrow \mathbb{C}$

$FS(W) = \mathrm{Fuct}(\partial, \Gamma(dW))$

T^*M

$$\boxed{FS(W)} = \mathrm{Funct}\big(\mathcal{O}, \Gamma(dW)\big)$$

$$T^*M$$

Cons

Problem 1: T^*M is not Hyper-Kahler.

Solution: "Taming triples".

$I, J, K, \omega_I, \omega_J, \omega_K,$ + inequali

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Problem 1: T^*M is not Hyper-Kähler.

Solution: "Taming triples".

$$\omega(-, I-) = g + \omega(I-, \cdot)$$

$$I^*\omega = \omega.$$

$I, J, K, \omega_I, \omega_J, \omega_K$, + inequality

$$E(u) \leq \int \omega_I \, d\tau + \omega_K \, dt.$$

Fueter map

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$I, J, K, \omega_I, \omega_J, \omega_K,$ + inequal

$$E(u) \leq \int \omega_I \, d\tau + \omega_K \, dt.$$

↑ Fueter map

∞-dim space of taming triples

$\Rightarrow$ makes transversality possi family triples

makes transversality possible.

must be noncompact; so is $\mathcal{O}, \Gamma(dW)$.

C^0 estimates.

technical reasons when $M \}$

Kähler.

family triples

makes transversality possible.

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C^0 estimates.

technical reasons when M }.
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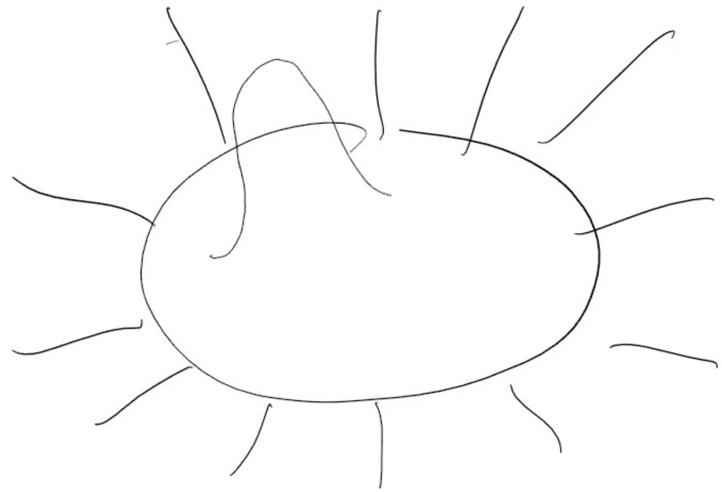

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$\Rightarrow$ makes transversality possible.

Problem 2 M must be noncompact; so

Need a-priori C^0 estimates.

{also needed for technical reasons when M not Kähler.}

Solution: Theory of IJK-convex functions

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Solution: Theory of IJK-convex functions

Def: $\rho : X \longrightarrow \mathbb{R}$ is IJK convex

$X \times \mathbb{R}^3$.

$$u^x \omega^\rho_I \wedge dt + u^x \omega^\rho_J \wedge ds + u^x \omega^\rho_K \wedge dt < 0.$$

For all fueter maps.

$\{ \omega_I = -d I d\rho$.

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Def: $\rho : X \longrightarrow \mathbb{R}$ is IJK convex

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For all fueter maps. $\left\{ \omega_I = -d I d\rho \right.$

Def X has a conical end if outsi

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$X \times \mathbb{R}$ $\mathbb{R} \times M$

$$u^X \omega^\rho_I \wedge dt + u^X \omega^\rho_J \wedge ds + u^X \omega^\rho_K \wedge dt < 0.$$

For all fueter maps. $\{ \omega_I = -d(d\rho \cdot I) \}$

Def X has a conical end if outside K compact, ρ has no critical points and is convex

L is conical if outside K compact,

$$d\rho|_{JTL} = d\rho|_{KTL} = 0.$$

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then Fueter maps ... are constrained to a compact set.

Thm Let $W : M \to \mathbb{C}$ be a Lefschetz fibration (M compact, or $|\nabla W| = 1$ outside compact set)

$T^*M = X$

For each small $\varepsilon > 0$ there is a triple I_τ, J_τ, K_τ on $T^*M = X$

Floer planes in M w ∂ ... I on M $\longleftrightarrow$ Fueter maps to T^*M w ∂ on L_0, L_1 (0, $\Gamma(dW)$).

$\partial_s u - J\partial_t u - \varepsilon \nabla \mathrm{Im} W) = 0$

$u : \mathbb{R}^2_{s,t} \to M$

$\lim_{s \to \pm\infty} u = p \in \mathrm{Crit}(W)$

$\lim_{t \to \pm\infty} u = \gamma, \quad \dot\gamma = \nabla \mathrm{Re}\, W$

$U : \mathbb{R}^2_{s,t} \times [0,1]_\tau \to T^*M.$

$\boxed{\partial_s u + I\partial_t u + J\partial_s u = 0}$

$U|_{\mathbb{R}^2 \times \{0\}} \subset L_0 \quad U|_{\mathbb{R}^2 \times 1} \subset L_1$

$\lim_{s \to \pm\infty} \in L_0 \cap L_1. \quad \lim_{t \to \pm\infty} = \{u : \mathbb{R}_s \times [0,1]_\tau \to M ; \partial_s u + J\partial_\tau$

$TX = TM \oplus T^*M$

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$T^*M = X$

(M compact, or $|\nabla W| \geq 1$ outside compact set)

$w\ \partial$ $\qquad I$ on M

For each small $\varepsilon > 0$ there is a triple I_τ, J_τ, K_τ on $T^*M = X$

Floer planes in M $\hookrightarrow$ Fueter maps to T^*M $w\ \partial$ on L_0, L_1

$$\partial_s u - J\partial_t u - \varepsilon \nabla(\operatorname{Im} W) = 0 \qquad L_0 = 0,\quad L_1 = \Gamma(dW).$$

$u: \mathbb{R}^2_{s,t} \to M$

$$U: \mathbb{R}^2_{s,t} \times [0,1]_\tau \to T^*M. \qquad \boxed{\partial_s u + I\partial_t u + J\partial_s u = 0}$$

$\lim_{s\to\pm\infty} u = p \in \operatorname{Crit}(W)$ $\qquad U|_{\mathbb{R}^2\times\{0\}} \subset L_0 \quad U|_{\mathbb{R}^2\times 1} \subset L_1.$

$\lim_{t\to\pm\infty} u = \gamma, \quad \dot\gamma = \nabla \operatorname{Re} W$ $\qquad \lim_{s\to\pm\infty} \in L_0 \cap L_1. \quad \lim_{t\to\pm\infty} = \{u: \mathbb{R}_s \times [0,1]_\tau \to M;\ \partial_s u + J\partial_\tau$

Proof sketch: metric on M $\rightsquigarrow$ $TX = TM \oplus T^*M$

$$\rightsquigarrow \underbrace{\begin{pmatrix} I & \\ & -I \end{pmatrix}}_{I}, \quad \underbrace{\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}}_{J}, \quad \underbrace{\begin{pmatrix} & I \\ I & \end{pmatrix}}_{K}.$$

$\pi: X \to M.$

$u: \mathbb{R}^2 \to M$ defn $\qquad U: [0,1]_\tau \times \mathbb{R}^2 \to X$

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let $W: M \to \mathbb{C}$ be a Lefschetz fibration

compact, or $|\nabla W| \geq 1$ outside compact set)

$T^* M = X$

I on M

small $\varepsilon > 0$ there is a triple I_τ, J_τ, K_τ on $T^* M = X$

planes in M $\longleftrightarrow$ Fueter maps to $T^* M$

$\partial_t u - \varepsilon \nabla(\mathrm{Im} W) = 0$ $\longleftrightarrow$ w ∂ on L_0, L_1

0, $\Gamma(dW)$.

$u: \mathbb{R}^2_{s,t} \to M$

$U: \mathbb{R}^2_{s,t} \times [0,1]_\tau \to T^* M$. $\boxed{\partial_s u + I \partial_t u + J \partial_s u = 0}$

$= p \in \mathrm{Crit}(W)$

$U|_{\mathbb{R}^2 \times \{0\}} \subset L_0$, $U|_{\mathbb{R}^2 \times 1} \subset L_1$.

$= \gamma$, $\dot\gamma = \nabla \mathrm{Re}\, W$

$\lim_{s \to \pm\infty} \in L_0 \cap L_1$, $\lim_{t \to \pm\infty} = \{u: \mathbb{R}^2_s \times [0,1]_\tau \to M;\ \partial_s u + J \partial_\tau u = 0\}$,

sketch: metric on M $\rightsquigarrow$ $TX = TM \oplus T^* M$

$\rightsquigarrow \begin{pmatrix} I & \\ & -I \end{pmatrix}, \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}, \begin{pmatrix} I & I \\ & \end{pmatrix}$

I, J, K.

$\pi: X \to M$

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outside compact set)

is a triple I_τ, J_τ, K_τ on $T^*M = X$

Fueter maps to T^*M

$\hookrightarrow$ w ∂ on L_0, L_1

0 $\Gamma(dW)$.

$u: \mathbb{R}^2_{s,t} \times [0,1]_\tau \to T^*M.$ $\boxed{\partial_s u + I\partial_t u + J\partial_s u = 0}$

$u|_{\mathbb{R}^2 \times \{0\}} \subset L_0$ $u|_{\mathbb{R}^2 \times 1} \subset L_1$

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$TX = TM \oplus T^*M$

$\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$, $\begin{pmatrix} \pm & I \\ & \end{pmatrix}$

J K.

$\pi : X \to M.$

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...iangle I_τ, J_τ, K_τ on $T^*M = X$

Fueter maps to T^*M

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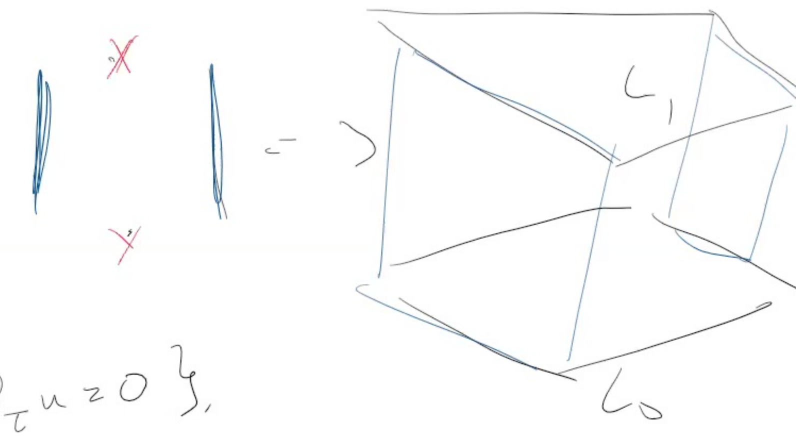

$u|_{\mathbb{R}^2 \times \{0\}} \subset L_0$ $u|_{\mathbb{R}^2 \times 1} \subset L_1$.

$\lim_{s \to \pm\infty} \in L_0 \cap L_1$. $\lim_{t \to \pm\infty} = \{u : \mathbb{R}_s \times [0,1]_\tau \longrightarrow M;\ \partial_s u + J_\tau \partial_\tau u = 0\}$.

$X = TM \oplus T^*M$

$-1)$, $(\pm \quad I)$

J, K.

$\pi: X \to M$.

$u: [0,1]_\tau \times \mathbb{R}^2 \longrightarrow X$

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Proof sketch: metric on M $\rightsquigarrow$ $TX = TM \oplus T^*M$

$\rightsquigarrow \underbrace{\begin{pmatrix} I & \\ & -I \end{pmatrix}}_{I}, \underbrace{\begin{pmatrix} & -1 \\ 1 & \end{pmatrix}}_{J}, \underbrace{\begin{pmatrix} & I \\ I & \end{pmatrix}}_{K}.$

$\pi: X \to M.$

given $u: \mathbb{R}^2 \to M$, define $U: [0,1]_\tau \times \mathbb{R}^2 \to X$

$\pi^* \operatorname{Re} W.$

$U(\tau, s, t) = \phi_\tau(u(s,t))$

$\omega_{can} \rightsquigarrow \phi_{\pi^* \operatorname{Re} W}$

$U = (u, \xi) \quad (M, u^* T^* M)$

$\phi_{\tau *} I, \ \phi_{\tau *} J, \ \phi_* K.$

Hamiltonian flow

Fueter eqn & integration by parts. $\Rightarrow$ $I_\tau, J_\tau, K_\tau.$

$0 \geq \|\nabla_\tau \xi\|^2_{L^2} + \|\nabla^{0,1}_I \xi\|^2_{L^2} - C(\varepsilon + \|\xi\|_{C^0}) (\|\nabla_\tau \xi\|_{L^2} \|\nabla^{0,1} \xi\|_{L^2}).$

convexity controls this.

$\Rightarrow \xi = \nabla^{0,1} = \partial_\tau u = 0$

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$\rightsquigarrow \quad TX = TM \oplus T^*M$

on M

$-I)$, $(\;, \; ^{-1})$, $(I \quad I)$

$J \qquad K$

I

define $\quad U : [0,1]_\tau \times \mathbb{R}^2 \to X$

$U(\tau, s, t) = \phi_\tau(u(s,t))$

$\pi : X \to M.$

$\pi^* \operatorname{Re} W.$

$\omega_{can} \rightsquigarrow \phi_{\pi^* \operatorname{Re} W}.$

Hamiltonian flow

$M, \omega, T^* M)$

$\phi_{\tau *} I, \; \phi_{\tau *} J, \; \phi_* K.$

ion by parts. $\Rightarrow$ $\quad I_\tau, J_\tau, K_\tau.$

$\|\nabla^{0,1}_I \xi\|^2_{L^2} - C(\varepsilon + \|\xi\|_{C^0})(\|\nabla_\tau \xi\|_{L^2} \|\nabla^{0,1} \xi\|_{L^2}).$

convexity controls this.

$\rho : M \to$

$\Rightarrow \xi = \nabla^{0,1} = \partial_\tau u = 0.$

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$\rightsquigarrow \begin{pmatrix} I & \\ & -I \end{pmatrix}, \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}, \begin{pmatrix} I & \\ & \end{pmatrix}$

$I \qquad J \qquad K$

$\pi : X \to M$

given $u : \mathbb{R}^2 \to M$, define $U : [0,1]_\tau \times \mathbb{R}^2 \to X$

$\pi^* \mathrm{Re}\, W$

$U(\tau, s, t) = \phi_\tau(u(s,t))$

$\omega_{can} \to \phi_{\pi^* \mathrm{Re} W}$

Hamiltonian flow

$U = (u, \xi) \quad (M, u^* T^* M)$

$\phi_{\tau *} I, \ \phi_{\tau *} J, \ \phi_* K.$

Fueter eqn & integration by parts. $\Rightarrow$ I_τ, J_τ, K_τ.

$$0 \geq \|\nabla_\tau \xi\|^2_{L^2} + \|\nabla^{0,1}_I \xi\|^2_{L^2} - C(\varepsilon + \|\xi\|_{C^0})(\|\nabla_\tau \xi\|_{L^2} \|\nabla^{0,1} \xi\|_{L^2}).$$

Convexity controls this.

$\Rightarrow \xi = \nabla^{0,1} = \partial_\tau u = 0$

$\|\xi\|^2 +$

$\hookrightarrow IJ,$

An algebraic aspect

$\mathrm{Fuet}(L_0, L_1)$ is a Hom.-should compose

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An algebraic aspect

Fuet(L_0, L_1) is a Hom. — should compose

$$\mathrm{Fuet}(\Gamma(dW_1), \Gamma(dW_2)) = \mathrm{Fuet}(L_0, \Gamma(dW_1 - dW_2))$$

$$\rightsquigarrow \mathrm{Fuet}(L_0, \Gamma(dW_1)) \otimes \mathrm{Fuet}(L_0, \Gamma(dW_2)) \to \mathrm{Fuet}(L_0, \Gamma(dW_1 + dW_2))$$

$$FS(W_1) \otimes FS(W_2) \to FS(W_1 + W_2)$$

Kapustin - Rozansky.

$$MF(W_1) \otimes M(W_2)$$
$$\downarrow$$
$$MF(W_1 + W_2).$$

$L_0 = (\mathbb{C}^\times)^n$ — mirror symm — Toric vars

$L_0 = \mathbb{C}^n$, $W = ADE$. — MF

Prospects

- Differential Geometry / PDE

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$$FS(W_1) \otimes FS(W_2) \rightarrow FS(W_1 + W_2)$$

Kapustin - Rozansky.

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$L_0 = \mathbb{C}^n, W = ADE.$ — MF

Prospects

- Differential Geometry / PDE

An algebraic aspect

$Fuk(L_0, L_1)$ is a Hom. — should compose

$Fuk(\Gamma(dW_1), \Gamma(dW_2)) = Fuk(L_0, \Gamma(dW_1 - dW_2))$

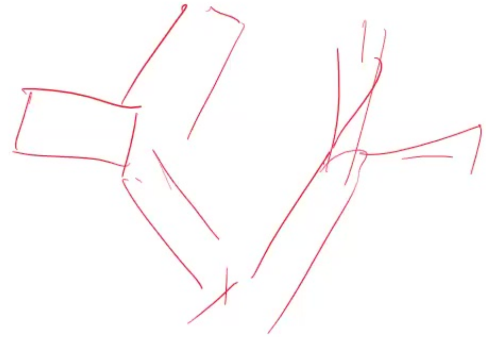
$\rightarrow Fuk(L_0, \Gamma(dW_1)) \otimes Fuk(L_0, \Gamma(dW_2))$
$\rightarrow Fuk(L_0, \Gamma(dW_1 + dW_2))$

$FS(W_1) \otimes FS(W_2) \rightarrow FS(W_1 + W_2)$

$L_0 = (\mathbb{C}^\times)^n$ ~~mirror symm~~ ——— Toric vars

$L_0 = \mathbb{C}^n$, $W = ADE$. ——— MF

DM



Kapustin - Rozansky.

$MF(W_1) \otimes MF(W_2)$
$\downarrow$
$MF(W_1 + W_2)$.

Prospects

- Differential Geometry / PDE

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$L_0 = (\mathbb{C}^\times)^n$ mirror symm. Toric vars

$L_0 = \mathbb{C}^n, W = ADE.$ DM MF

Kapustin - Rozansky.

$MF(W_1) \otimes MF(W_2)$

$MF(W_1 + W_2)$.

Prospects

- Differential Geometry / PDE
- Categorical Aspects
- Quaternionic Weinstein Domains?
- Wrapping?

$$FS(W_1) \otimes FS(W_2) \to FS(W_1 + W_2)$$

Kapustin - Rozansky.

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$L_0 = (\mathbb{C}^\times)^n$ — mirror symm — Toric vars

$L_0 = \mathbb{C}^n,\ W = ADE.$ — ∞ MF

DM

Prospects

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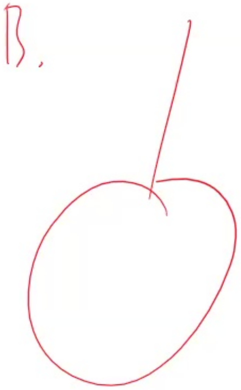
- Wrapping?

$T^*G/B.$

$\mathbb{C}^\times$



$$FS(W_1) \otimes FS(W_2) \to FS(W_1 + W_2)$$

Kapustin - Rozansky.

$$MF(W_1) \otimes MF(W_2) \simeq MF(W_1 + W_2).$$

$L_0 = (\mathbb{C}^\times)^n$ — ~~mirror symm~~ — Toric vars

$L_0 = \mathbb{C}^n$, $W = ADE$. — DM — $\simeq$ MF

Prospects

- Differential Geometry / PDE
- Categorical Aspects
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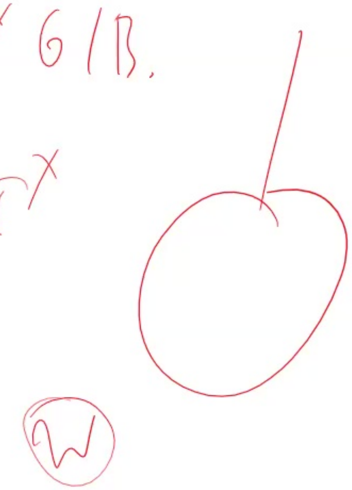
∂_s

$T^\times G/B$.

$\mathbb{C}^\times$



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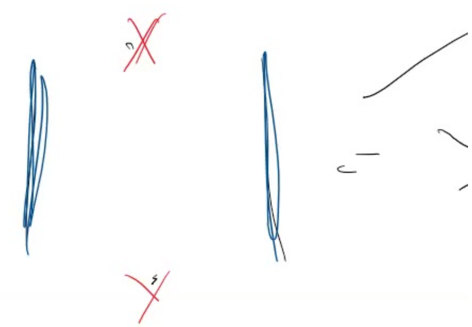
e J_τ, J_τ, K_τ on $T^*M = X$

eter maps to T^*M

∂ on L_0, L_1

$\mathcal{O}$ $\Gamma(dW)$.

$\mathbb{R}^2 \times [0,1]$



$\mathbb{R}^2_{s,t} \times [0,1]_\tau \longrightarrow T^*M$.

$$\boxed{\partial_s u + I\partial_t u + J\partial_s u = 0}$$

$\Sigma \times [0,1]$

$\mathbb{R}^2 \times \{0\} \subset L_0$ $\quad u|_{\mathbb{R}^2\times 1} \subset L_1$,

$\pm\infty \in L_0 \cap L_1$. $\lim_{t \to \pm\infty} = \{u : \mathbb{R}_s \times [0,1]_\tau \longrightarrow M;\ \partial_s u + J_\tau \partial_\tau u = 0\}$,

$TM \oplus T^*M$

$\begin{pmatrix} & I \\ \pm & \end{pmatrix}$

hould compos

$l_0, T(dW_1 - dW_2))$

$dW_2))$

Fuk$(l_0, T(dW_1 + dW_2))$

$S(W_1 + W_2)$

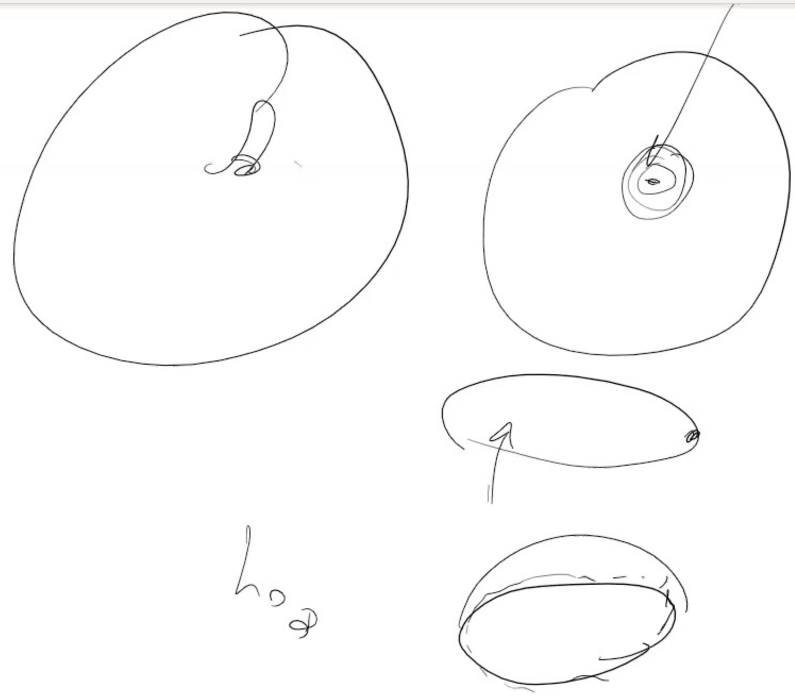
Kapustin - Rozansky.

$MF(W_1) \otimes MF(W_2)$

$MF(W_1 + W_2)$.

vars

·MF

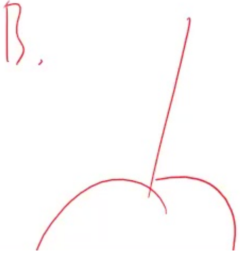


L_0

/PDE

T^*G/B.

$\mathbb{C}^x$



$U = (u, \xi)$ $\quad (M_{\hat{u}}, M)$ $\quad \varphi_\tau \times I,\ \varphi_\tau \times J,\ \varphi_\tau \times K$ $\quad$ Hamiltoni

$\Rightarrow$ I_τ, J_τ, K_τ.

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FS$(W_1) \otimes$ FS$(W_2) \to$ FS$(W_1 + W_2)$

mirror symm.

MF$(W_1) \otimes$ M(W_2)