Title: Comments on the volume of the black hole interior in 2d gravity

Speakers: Gabor Sarosi

Series: Quantum Fields and Strings

Date: April 19, 2022 - 2:00 PM

URL: https://pirsa.org/22040122

Abstract: The volume of the interior of a two-sided eternal black hole classically grows forever. I will derive a microscopic formula for this volume in JT gravity by summing the non-perturbative contribution of higher topologies. The non-perturbative corrections lead to the saturation of the volume of the interior at times exponential in the entropy of the black hole. I will connect the microscopic formula for the volume with properties of a "thermo-averaged" density matrix, in particular with its second Renyi entropy, which I argue to measure the number of nearly orthogonal states visited by time evolution. I will discuss various problems with this interpretation.

Zoom Link: https://pitp.zoom.us/j/99880539557?pwd=VWZiMFI5Rk5pdnNZRE5kalRvUk1Zdz09

Comments on the volume of the black hole interior in 2d gravity

Gabor Sarosi (CERN)

Based on:
2107.06286
with Luca Iliesiu & Mark Mezei

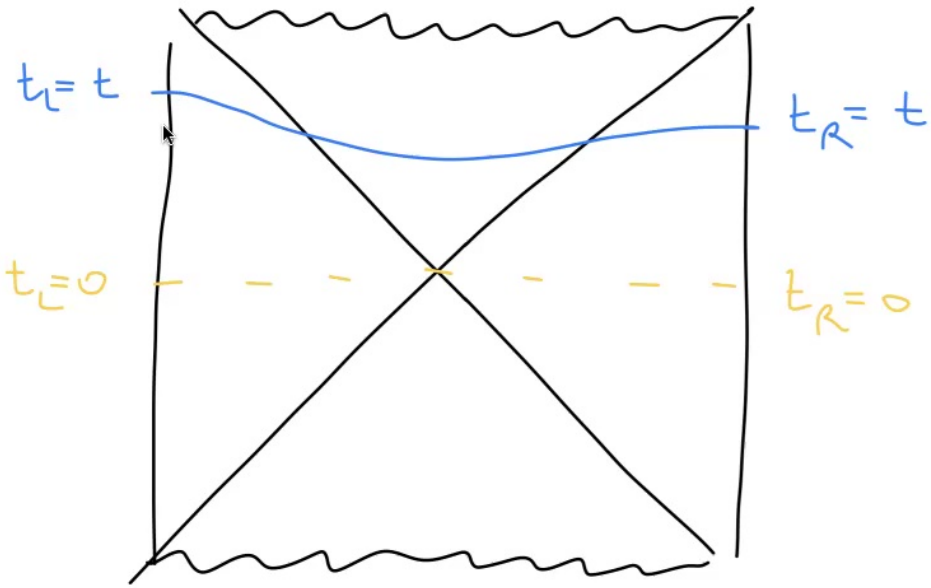
and w.i.p. with
with Luca Iliesiu & Mark Mezei & Alexey Milekhin

Plan

- Motivation
- The volume of the interior in JT gravity
- Microscopic interpretation

Growth of the interior

Black hole interior: an expanding cosmology



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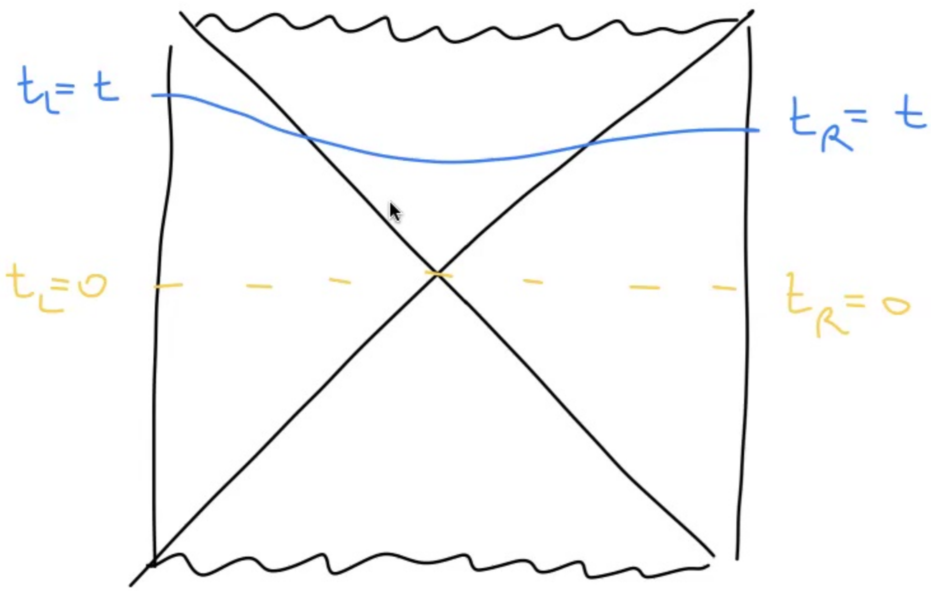
Growth of the interior

Black hole interior: an expanding cosmology



volume of maximal Cauchy slice $\propto Mt$

Question: what is the microscopic origin of this “creation of space”?

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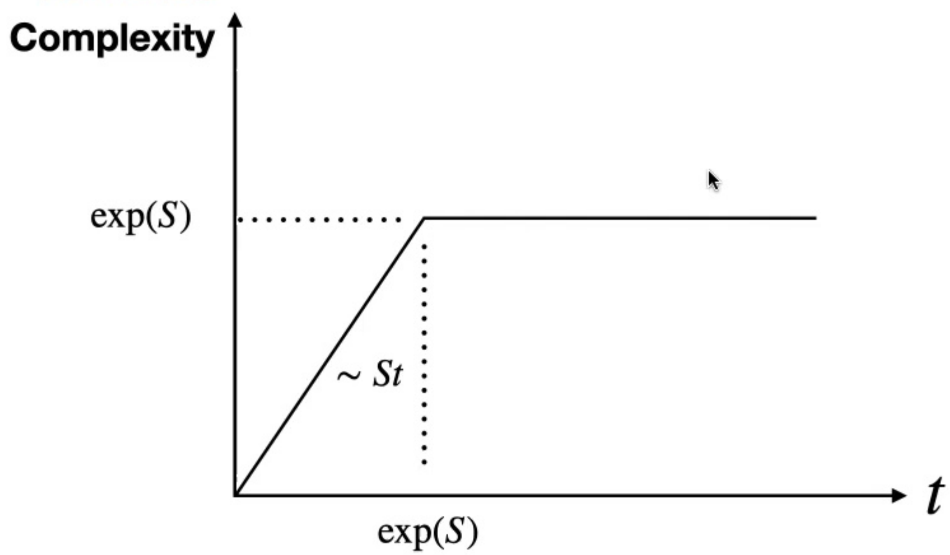
Growth of the interior

Time dependence

$$|\psi\rangle_{\text{target}} = e^{-iHt}\,|TFD\rangle$$

[Susskind,...]



semi-classical gravity

Today, we try to see this in gravity

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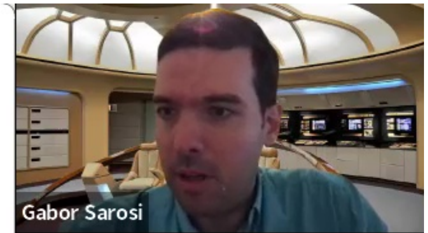
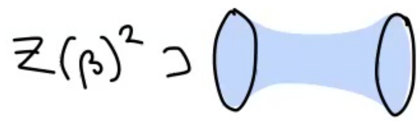


Growth of the interior

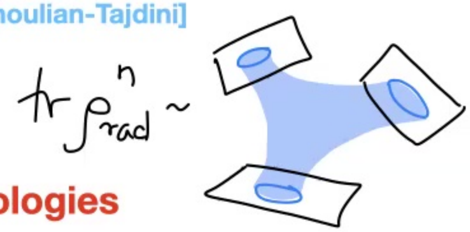
Why do we have a chance of seeing something like this in gravity?

Because Euclidean gravity seems to have an unreasonably large regime of validity, e.g.

Universal energy level repulsion of chaotic systems
[Saad-Shenker-Stanford,Cotler-Jensen,...]



Entropy Page curve of an evaporating black hole
[Penington-Shenker-Stanford-Yang,Amheiri-Hartman-Maldacena-Shaghoulian-Tajdini]



Both are about corrections coming from summing topologies

Today: how do these effects affect the volume of the interior?

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Plan

- Motivation
- The volume of the interior in JT gravity
- Microscopic interpretation

Jackiw-Teitelboim (JT) gravity

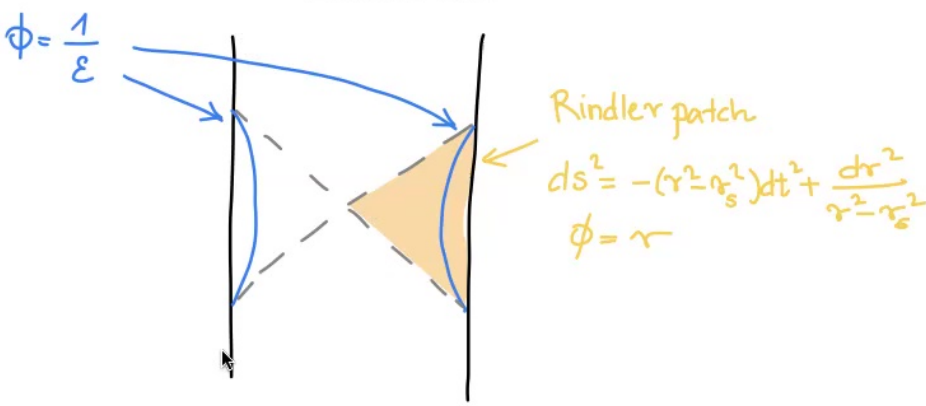
Dilaton-gravity in two dimensions:

$$I_{JT} = -S_0\chi(\mathscr{M}) - \frac{1}{2}\int\sqrt{g}\phi(R+2) - \int_{\partial\mathscr{M}}\sqrt{h}\phi(K-1)$$

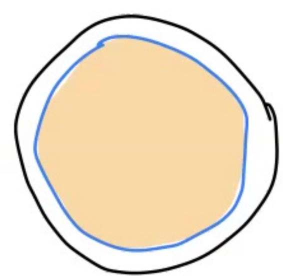
Arises by dimensional reduction
of near-horizon region of near-extremal black holes
[Maldacena-Stanford-Yang,Sarosi,Nayak-Shukla-Soni-Trivedi]

Black hole solution: Global AdS_2 with a cutoff at large constant ϕ

Lorentzian



Euclidean

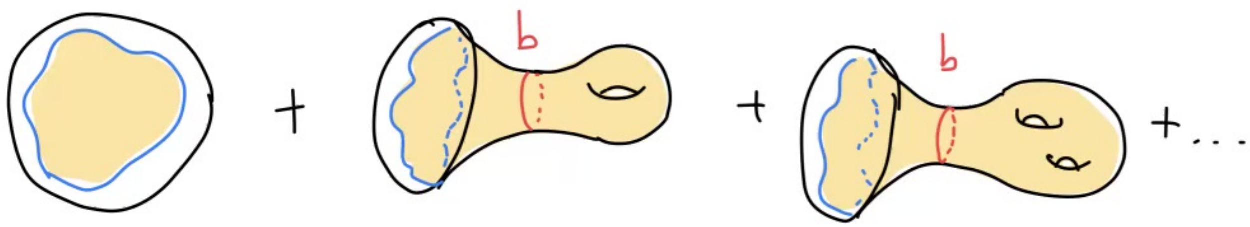


$ds^2 = d\rho^2 + \sinh^2\rho\, d\tau^2$

Jackiw-Teitelboim (JT) gravity

Higher topologies also contribute, weighted by $\chi(\mathscr{M}) = 2g + n - 2$

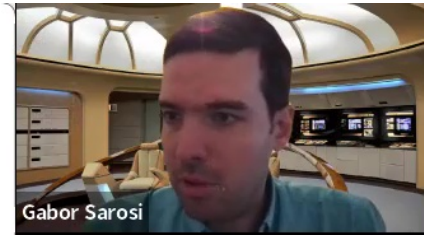
[Saad-Shenker-Stanford]

$Z(\beta) =$ 

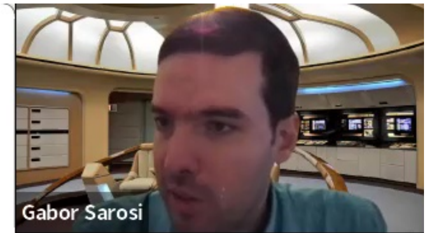
JT gravity is a matrix integral

$$[Z(\beta)]_{\text{JT gravity}} \simeq \int dH e^{-V(H)} \text{Tr}[e^{-\beta H}]$$

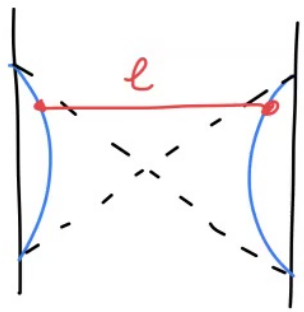
In the genus expansion
and in the double scaling limit
(large matrix, zoomed to
the bottom of the spectrum)



Volume of the interior in JT gravity



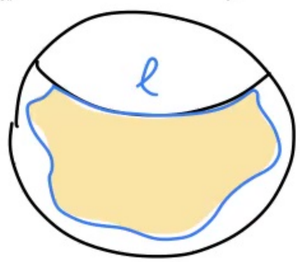
Classical volume



$$\ell_{\text{ren}} = 2\log\left(2\cosh\left[\frac{2\pi}{\beta}t\right]\right) \approx \frac{4\pi}{\beta}t$$

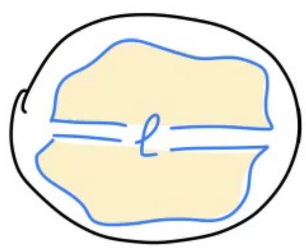
Quantum volume (perturbative): using HH wave function [Yang,Harlow-Jafferis]

$$\psi^{\text{Disk}}_{\beta/2}(\ell) =$$



$$= \int_0^\infty dE\rho_0(E)e^{-\beta E/2}[4e^{-\ell/2}K_{i\sqrt{8E}}(4e^{-\ell/2})]$$

$$\langle \ell \rangle =$$



$$= \frac{e^{-S_0}}{Z_{\text{disk}}}\int e^{\ell} d\ell \, |\psi^{\text{Disk}}_{\frac{\beta}{2}+it}(\ell)|^2 \ell$$

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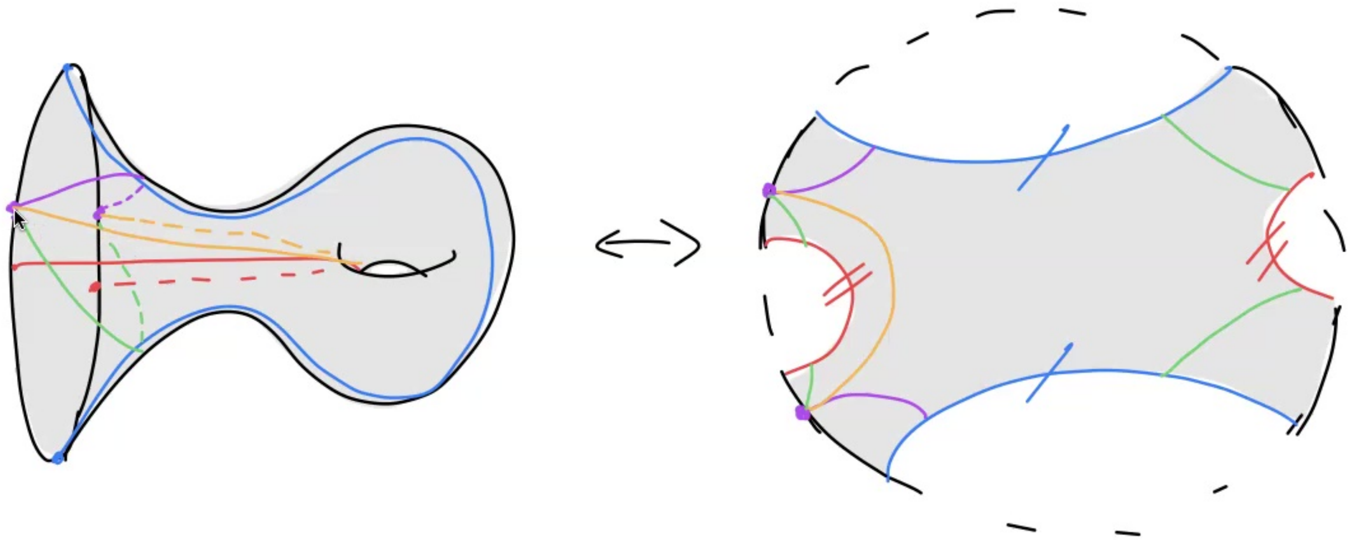
Volume of the interior in JT gravity

Non-perturbative quantum volume:

Challenge: infinite number of extremal geodesics on higher genus surfaces



Taking minimal geodesic on each surface is not an option: we want to continue to Lorentzian! Euclidean minimal geodesic changes abruptly: leads to non-analiticity

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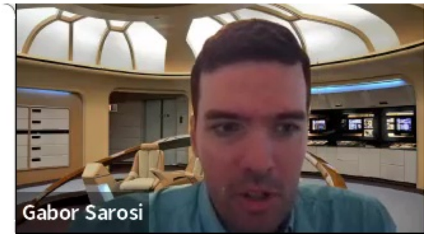


Volume of the interior in JT gravity

Non-perturbative quantum volume:

Challenge: infinite number of extremal geodesics on higher genus surfaces

Prescription: average over a well defined set of extremal geodesics

$$\langle \ell \rangle \equiv \sum_g e^{S_0(1-2g)} \sum_\gamma \langle \ell_\gamma \rangle_{\text{wiggles \& moduli space}}$$

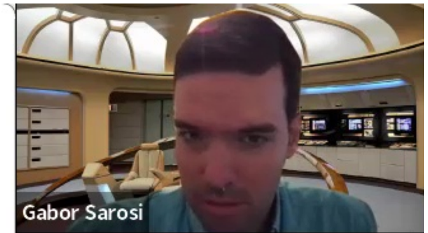
Divergent!

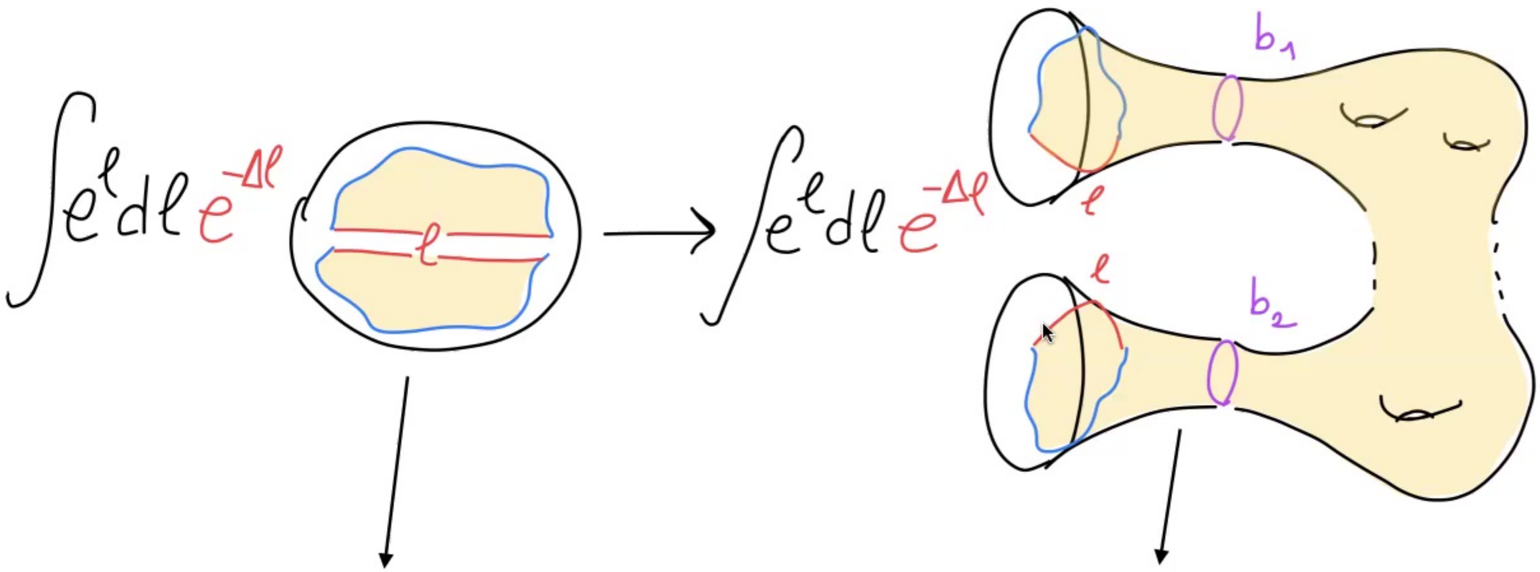
Natural regularization:

$$\langle \ell \rangle \equiv -\lim_{\Delta \to 0} \frac{d}{d\Delta} \sum_g e^{S_0(1-2g)} \sum_\gamma \langle e^{-\Delta \ell_\gamma} \rangle_{\text{wiggles \& moduli space}}$$

Related to two point function [Yang,Saad,...]

Volume of the interior in JT gravity





$$|\psi^{\text{Disk}}_{\frac{\beta}{2}+it}(\ell)|^2$$

$$\int b_1 db_1 b_2 db_2 \psi^{\text{Tr}}_{\frac{\beta}{2}+it}(\ell, b_1)\psi^{\text{Tr}}_{\frac{\beta}{2}-it}(\ell, b_2)[V_{g,2}(b_1, b_2) + \text{disc}]$$

(sum over non-int geodesics turns 1bndy moduli space into 2bndy moduli space [Mirzakhani]**)**

Volume of the interior in JT gravity

Result:

$$\langle \ell(t) \rangle = -\frac{e^{-S_0}}{4\pi^2 Z_{\text{disk}}(\beta)} \int_0^\infty dE_1 dE_2 \langle \rho(E_1)\rho(E_2)\rangle M(E_1, E_2) e^{-\frac{1}{2}\beta(E_1+E_2) - i(E_1-E_2)t}$$

$$M(E_1, E_2) = \frac{16\pi^4}{(E_1 - E_2)[\cosh 2\pi\sqrt{2E_1} - \cosh 2\pi\sqrt{2E_2}]}$$

$$\propto \frac{1}{(E_1 - E_2)^2}$$

In matrix integrals, universally:

$$\langle \rho(E_1)\rho(E_2)\rangle = \langle \rho(E_1)\rangle\langle \rho(E_2)\rangle + \langle \rho(E_1)\rangle \delta(E_1 - E_2) - \frac{\sin^2\left[\pi\langle\rho(E_2)\rangle(E_1 - E_2)\right]}{\pi^2(E_1 - E_2)^2}$$

disk

time indep
divergence
(Δ regulated)

trumpet

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Volume of the interior in JT gravity

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disk

time indep divergence (Δ regulated)

trumpet

doubly non-perturbative

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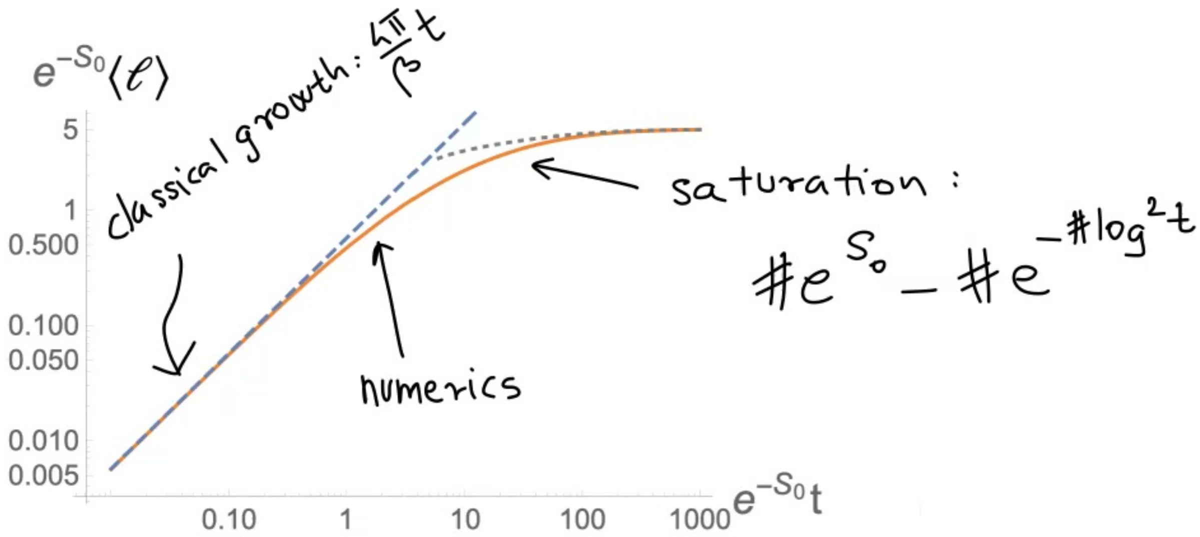
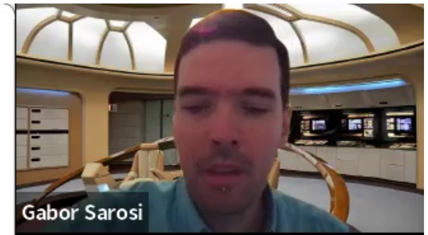
Volume of the interior in JT gravity

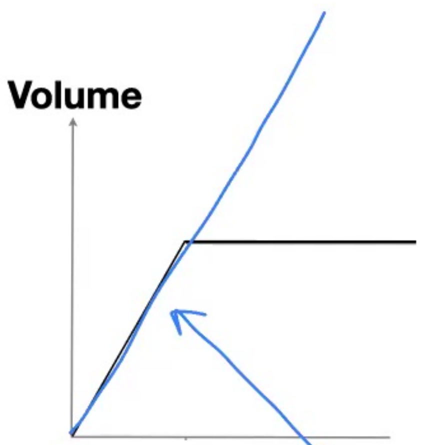


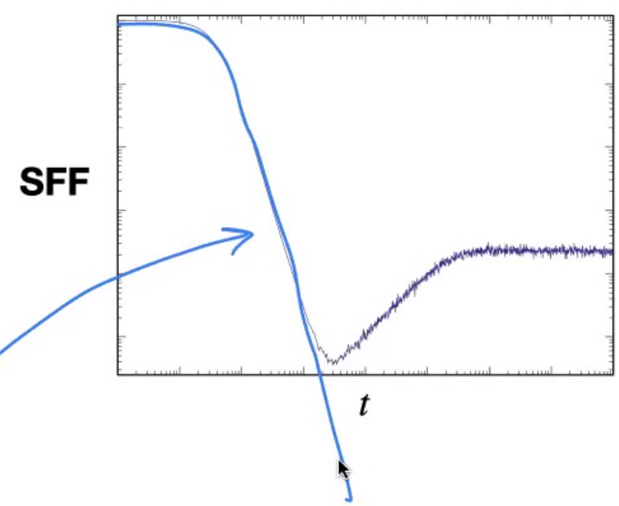


Volume of the interior in JT gravity

Very similar quantity: spectral form factor $\overline{Z(\beta - it)Z(\beta + it)} \equiv \textbf{SFF}(t)$

$$\int_0^\infty dE_1 dE_2 \langle \rho(E_1)\rho(E_2)\rangle e^{-\frac{1}{2}\beta(E_1+E_2) - i(E_1-E_2)t} \times \begin{cases} M(E_1, E_2) & \textbf{for } \langle \ell(t) \rangle \\ 1 & \textbf{for SFF}(t) \end{cases}$$





$$\langle \rho(E_1)\rho(E_2)\rangle = \langle \rho(E_1)\rangle\langle \rho(E_2)\rangle + \langle \rho(E_1)\rangle \delta(E_1 - E_2) - \frac{\sin^2\left[\pi\langle \rho(E_2)\rangle(E_1 - E_2)\right]}{\pi^2(E_1 - E_2)^2}$$

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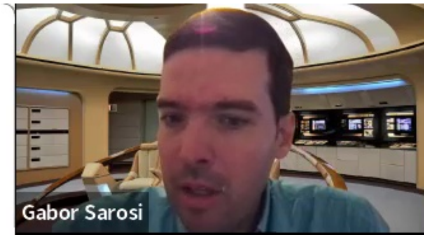

Volume of the interior in JT gravity

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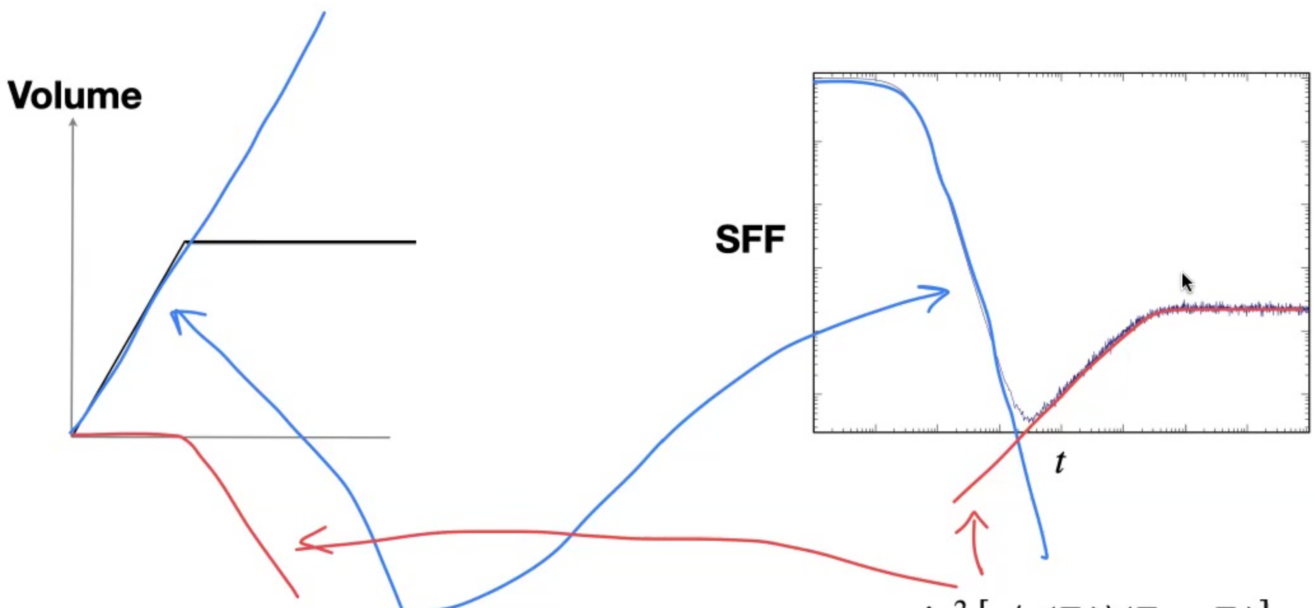

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Gabor Sarosi

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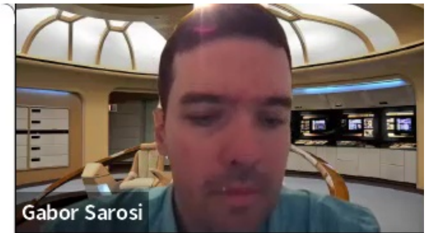

Spectral complexity

The final formula for the volume makes sense in any matrix integral, can also be derived for more general dilaton potential
(with methods of [Maxfield-Turiaci,Witten])

Call it spectral complexity

$$\mathscr{C}(t) \equiv \frac{1}{Z(\beta)^2} \sum_{i \neq j} \frac{1}{[E_i - E_j]^2} e^{-\beta(E_i + E_j)} [1 - \cos(E_i - E_j)t]$$

Can we interpret this quantity?

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Time-averaged density matrix

An extremely simple proxy for complexity:

How much of the Hilbert space is spanned by time evolved thermofield double states up to a given time?

$$\{e^{-iHt}|TFD\rangle \,|\, t < T\}$$

Consider the density matrix:

$$\rho(T) = \frac{1}{T}\int_0^T dt e^{-iHt}|TFD\rangle\langle TFD|e^{iHt}$$

First guess: rankρ

However, instantly full rank due to extremely small eigenvalues

Replace rank with number of eigenvalues bigger than some threshold:

$$\textbf{rank}_\epsilon \rho \equiv \sum_{\lambda_i > \epsilon} 1 = \textbf{Tr}\Theta(\rho - \epsilon)$$

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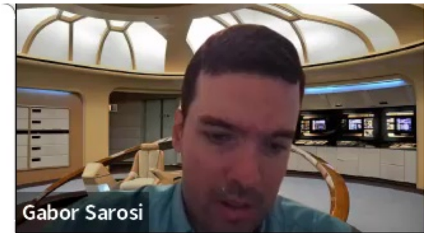

Effective dimension

$$\textbf{rank}_\epsilon \rho \equiv \sum_{\lambda_i > \epsilon} 1 = \textbf{Tr}\Theta(\rho - \epsilon)$$

How is it a proxy for complexity?

Consider an ensemble of states $\mathscr{E}$, $C(\mathscr{E}) \equiv$ **min. num. of gates to prep.** $\mathscr{E}$

$$\textbf{then } C(\mathscr{E}) \geq \frac{\log |\mathscr{E}|}{\log \textbf{choices}}$$

[Roberts-Yoshida]

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Effective dimension

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then $C(\mathscr{E}) \geq \dfrac{\log |\mathscr{E}|}{\log \textbf{choices}}$ [Roberts-Yoshida]

$C(\mathscr{E})$ **is also (roughly) the complexity of the most complex state in** $\mathscr{E}$

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Effective dimension

Consider an ensemble of states $\mathscr{E}$, $C(\mathscr{E}) \equiv$ **min. num. of gates to prep.** $\mathscr{E}$

then $C(\mathscr{E}) \geq \dfrac{\log|\mathscr{E}|}{\log \textbf{choices}}$ [Roberts-Yoshida]

Take $\mathscr{E}_T =$ **subspace with** $\rho(T)$ **eigenvalues** $> \epsilon$

$$|\mathscr{E}_T| \propto \textbf{vol}[\mathbb{C}P^{\mathrm{rank}_\epsilon \rho(T)-1}] \propto \pi^{\mathrm{rank}_\epsilon \rho(T)-1}$$

$$C(\mathscr{E}_T) \geq \#(\mathrm{rank}_\epsilon \rho(T) - 1)$$

Tempting to think that the most complex state in the set is $U_T|TFD\rangle$

Superposing $U_{\delta t}|\psi\rangle, U_{\delta t}^2|\psi\rangle, U_{\delta t}^3|\psi\rangle$ **is cheap but does have an overhead**
[Childs-Kothari-Somma]

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Effective dimension

$$C(\mathscr{E}_T) \geq \#(\mathrm{rank}_\epsilon \rho(T) - 1)$$

$$\textbf{rank}_\epsilon \rho \equiv \sum_{\lambda_i > \epsilon} 1 = \textbf{Tr}\Theta(\rho - \epsilon)$$

Difficult to compute, but there is a simple lower bound

Cauchy-Schwarz: $\textbf{Tr}[\rho\sqrt{\Theta(\rho-\epsilon)}] \leq \sqrt{\textbf{Tr}\Theta(\rho-\epsilon)\textbf{Tr}\rho^2}$

↓

Minimized for $\rho = \frac{1}{d}I$

(use regularized Θ to show)

$$\textbf{rank}_\epsilon \rho \geq \frac{1}{\textbf{Tr}\rho^2} \equiv d_{\text{eff}} \quad \textbf{provided } \epsilon < d^{-1}$$

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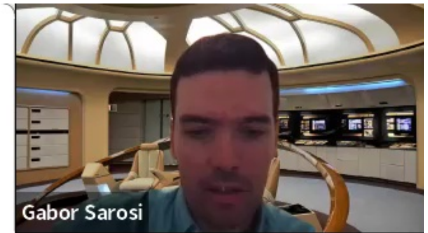
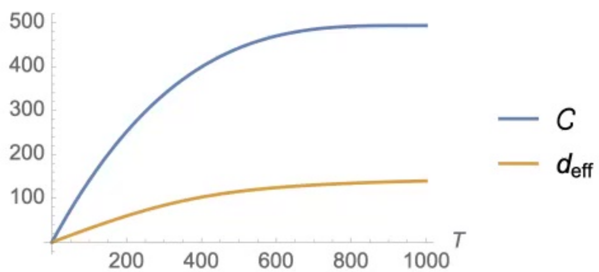


Connection to the volume

$$\rho(T) = \frac{1}{T}\int_0^T dt e^{-iHt}|TFD\rangle\langle TFD|e^{iHt}$$

$$d_{\text{eff}} = \frac{1}{\mathbf{Tr}\rho(T)^2} = \frac{1}{\frac{Z(2\beta)}{Z(\beta)^2} + \frac{\mathscr{C}(2T)}{T^2}}$$



$$\frac{1}{Z(\beta)^2}\sum_{i\neq j}\frac{1}{[E_i - E_j]^2}e^{-\beta(E_i+E_j)}[1 - \cos(E_i - E_j)T]$$

$\propto \ell(2T)$ **for large** T

Pro:

- **Proxy for complexity**
- **Direct function of the late time volume in gravity**

Con:

- **Early time slope different**

$$d_{\text{eff}} \approx \frac{T^2}{\mathscr{C}(2T)} \approx \frac{\beta T}{\gamma^2}$$

- **Saturation independent of volume saturating**

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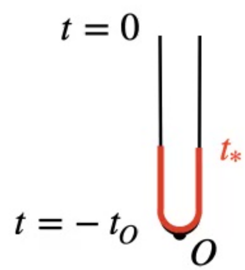
More cons: Switchback states

We should understand the dual of the volume in other states than the TFD

$$|\psi\rangle = O(-t_O)\,|TFD\rangle$$



Both for complexity and volume:

$$2(t_O - t_*)$$

Effective dimension

$$\rho = \int_0^{-t_O} U_t\,|TFD\rangle\langle TFD|\,U_{-t}$$
$$+ \int_0^{t_O} U_t O\Big[U_{-t_O}\,|TFD\rangle\langle TFD|\,U_{t_O}\Big]O^\dagger U_{-t}$$

Claim: Trρ^2 depends only on 2pt func.

Does not know about scrambling

Volume in JT

One can derive a microscopic formula involving limits of 6j symbol

Stay tuned…

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Summary

Defined regularized non-perturbative volume in JT gravity

Showed that it saturates at $t \propto e^{S_0}$, which is expected from complexity

Result is similar to SFF, but the origin of "ramp" and plateau are different

Volume is dual to a simply calculable quantity, dubbed spectral complexity

Questions

Can spectral complexity match some definition of complexity?
Or give a bound?

Is the time-averaged density matrix relevant?

What can we say about other states than the TFD?

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