

Title: Cosmology 2022

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Collection: Cosmology (2021/2022)

Date: April 11, 2022 - 9:00 AM

URL: <https://pirsa.org/22040117>

INTRODUCE SPATIAL CURVATURE $K = (\text{CURVATURE LENGTH})^{-2}$

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2 \right)$$

$$d\Omega^2 = (d\theta)^2 + (\sin\theta)^2 (d\phi)^2$$

$$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho$$

$$\frac{\dot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$$

$$\dot{\rho} = -3H(\rho + p)$$

$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a(t)^2 p(t) \bar{g}_{ij} \end{pmatrix}$$

$\bar{g}_{ij}$ SPATIAL METRIC

$$H(a) = H_0 \left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda \right]^{1/2}$$

$$\Omega_k = -\frac{K}{H_0^2}$$

$$K = \{0, \pm 1\}$$

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$$K = \{0, \pm 1\}$$

$$x' = cx$$

$$k' = \frac{k}{c^2}$$

$$a(t) = \frac{a}{c}$$

$$a=1$$

$$\dot{\rho} = -3H(\rho + p)$$

$$k = \{0, \pm 1\}$$

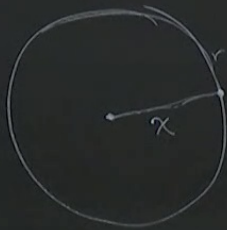
$$\frac{a'(t)}{a(t)} = \frac{\dot{a}}{a}$$

$$a=1$$

$$ds^2 = \frac{(dr)^2}{1 - kr^2} + r^2 d\Omega^2$$

$$\chi = \int_0^r \frac{dr'}{\sqrt{1 - kr'^2}}$$

$$ds^2 = d\chi^2$$



CIRCUMFERENCE $2\pi r$

$$= \begin{cases} \frac{1}{\sqrt{k}} \sin(\sqrt{k}r) & k > 0 \\ r & k = 0 \\ \frac{1}{\sqrt{-k}} \sinh(\sqrt{-k}r) & k < 0 \end{cases}$$

$$r = S_k(\chi) = \begin{cases} \frac{1}{\sqrt{k}} \sin(\sqrt{k}\chi) & k > 0 \\ \chi & k = 0 \\ \frac{1}{\sqrt{-k}} \sinh(\sqrt{-k}\chi) & k < 0 \end{cases}$$

$$\dot{p} = -3H(p+p)$$

$$K = \{0, \pm 1\}$$

$$\frac{a(t)}{a} = \frac{a}{c}$$

$a=1$

$$dl^2 = \frac{(dr)^2}{1-Kr^2} + r^2 d\Omega^2$$

$$x = \int_0^r \frac{dr'}{\sqrt{1-Kr'^2}}$$

$$= \begin{cases} \frac{1}{\sqrt{K}} \sin(\sqrt{K}r) & K > 0 \\ r & K = 0 \\ \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}r) & K < 0 \end{cases}$$

$$dl^2 = (dx)^2 + S_K(x)^2 d\Omega^2$$

$$r = S_K(x) = \begin{cases} \frac{1}{\sqrt{K}} \sin(\sqrt{K}x) & K > 0 \\ x & K = 0 \\ \frac{1}{\sqrt{-K}} \sinh(\sqrt{-K}x) & K < 0 \end{cases}$$

CIRCUMFERENCE $2\pi r$

$$H(a), \quad K \neq 0$$

$$a=1 \quad \xleftrightarrow{dx}$$

$$\left(\frac{dx}{dt} = \frac{1}{a(t)} \right)$$

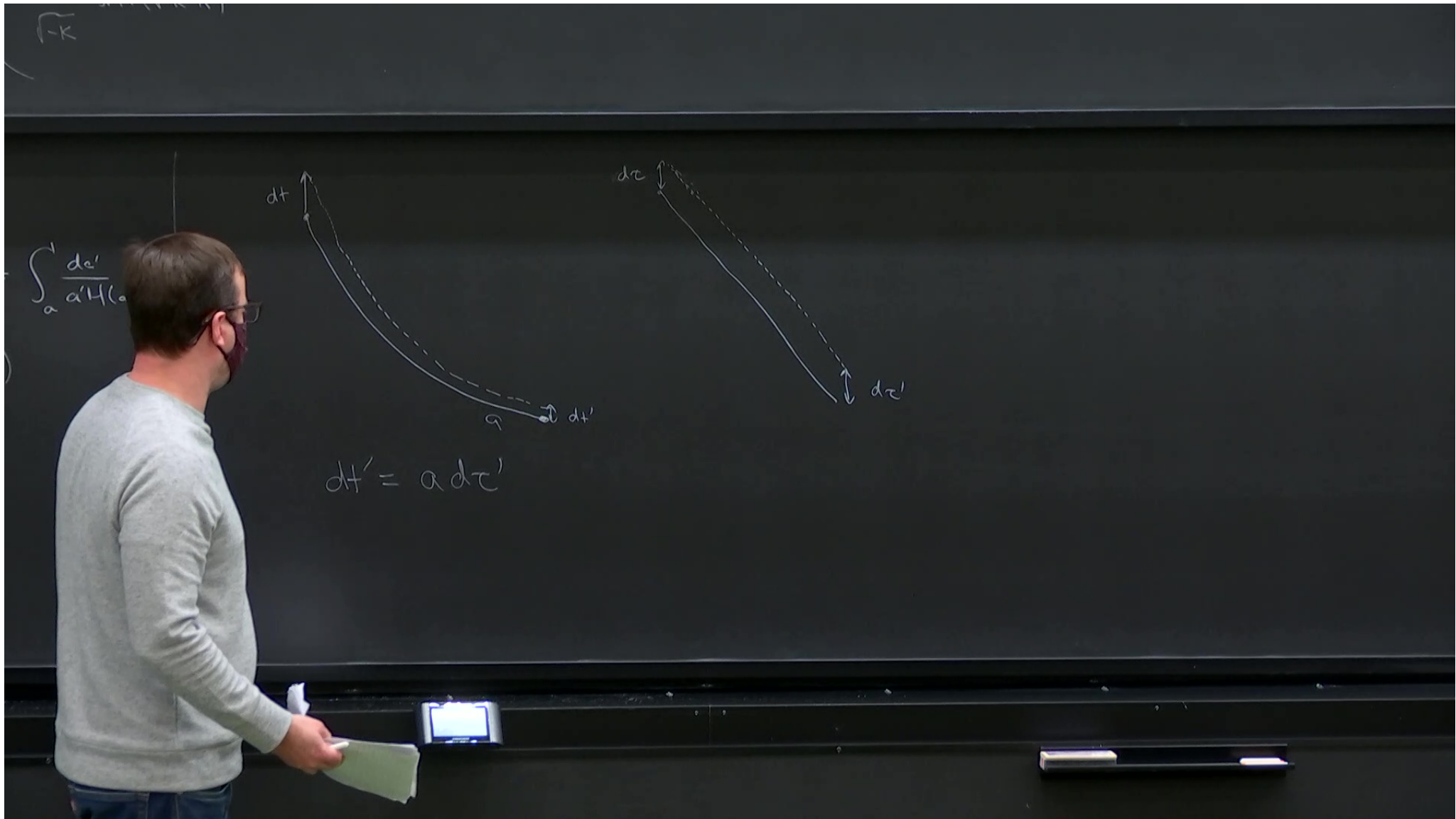
SCALE FACTOR a

$$x = \int \frac{dt}{a} = \int_a^1 \frac{da'}{a' H(a')}$$

$$r = S_K(x)$$

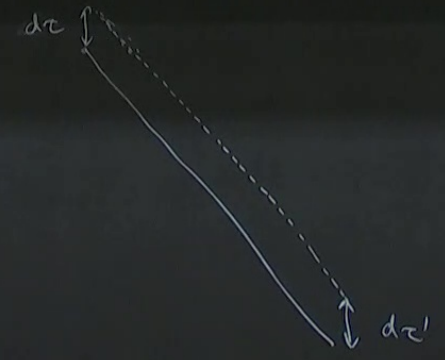
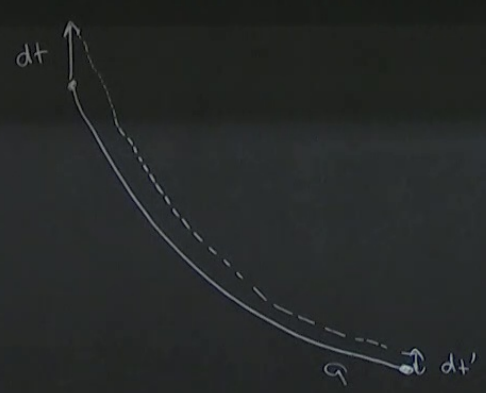
$$\frac{1}{\sqrt{-K}} \sinh(\sqrt{-K} x)$$

$$dt$$



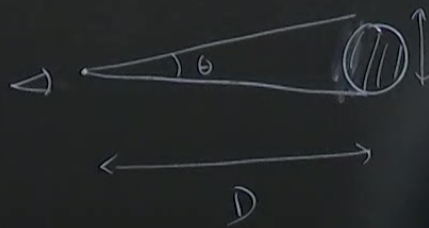
$\sqrt{1-K}$

$$\int_a^1 \frac{da'}{a' H(a')}$$



$$\begin{aligned} dt' &= a d\tau' \\ &= a d\tau \\ &= a dt \end{aligned}$$

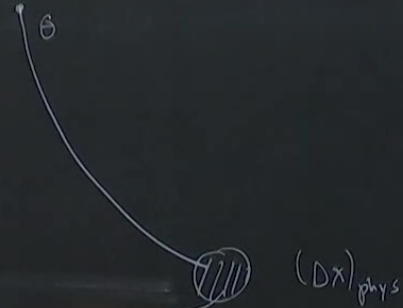
"STANDARD RULER"



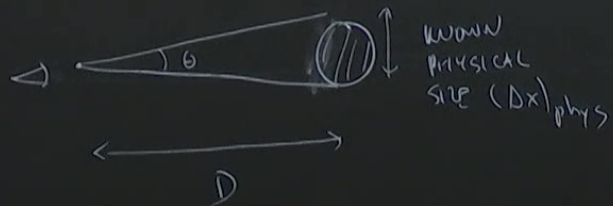
KNOWN
PHYSICAL
SIZE $(\Delta x)_{phys}$

$$\theta = \frac{(\Delta x)_{phys}}{D}$$

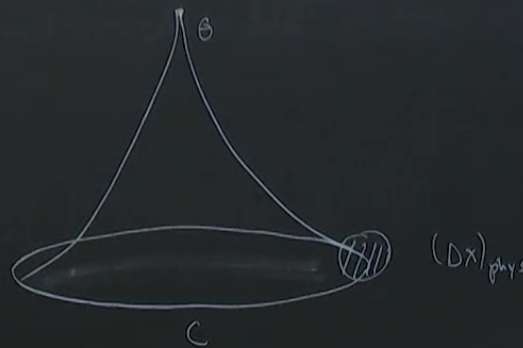
$[\theta] =$



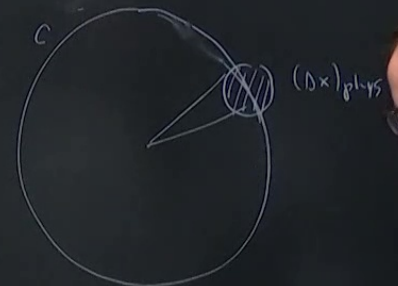
"STANDARD RULER"



$$\theta = \frac{(Dx)_{\text{phys}}}{D} \quad [\theta \ll 1]$$

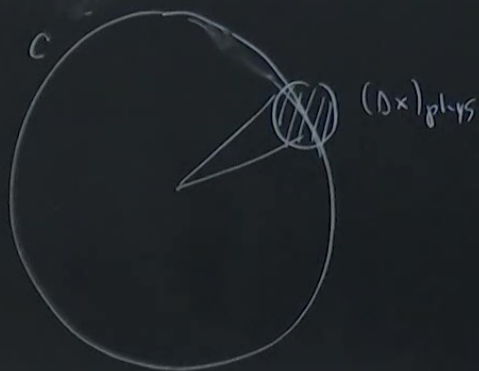


SCALE FACTOR S



$$\theta = \frac{(Dx)_{\text{phys}}}{C}$$

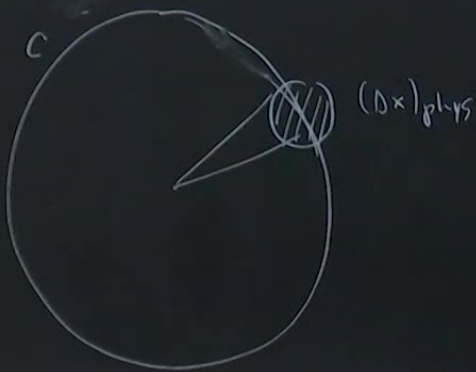
SCALE FACTOR a
TIME SNAPSHOT, TWO SIGNAL



$$\Theta = \frac{2\pi(\Delta x)_{\text{phys}}}{c_{\text{phys}}}$$

$$= \frac{2\pi(\Delta x)_{\text{phys}}}{2\pi a r} \leftarrow \text{NOT } 2\pi a x$$

SCALE FACTOR a
TIME SNAPSHOT, TWO SIGNAL

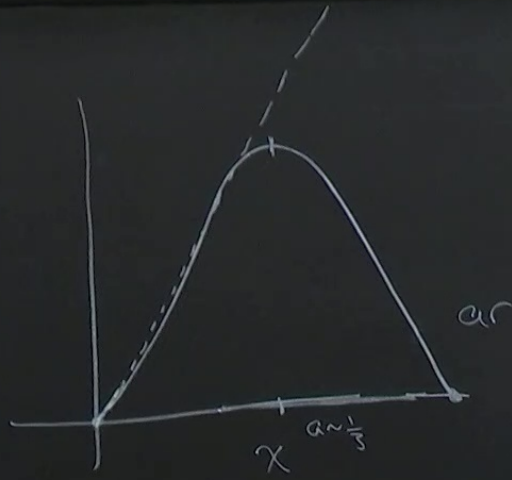


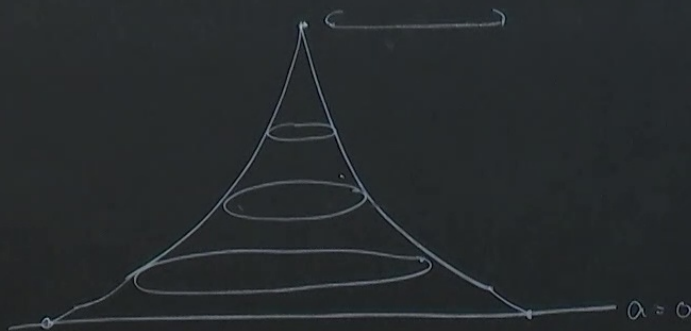
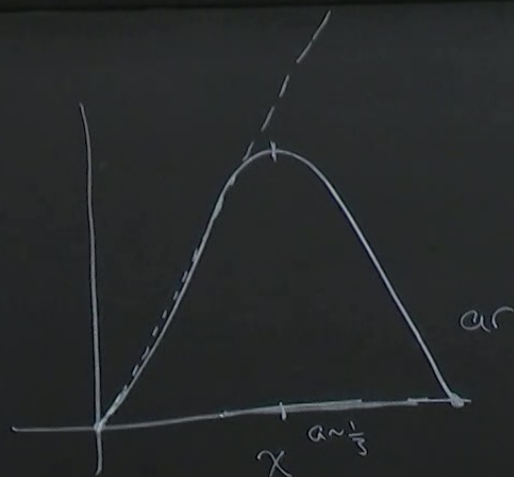
$$\Theta = \frac{2\pi (Dx)_{phys}}{C_{phys}}$$

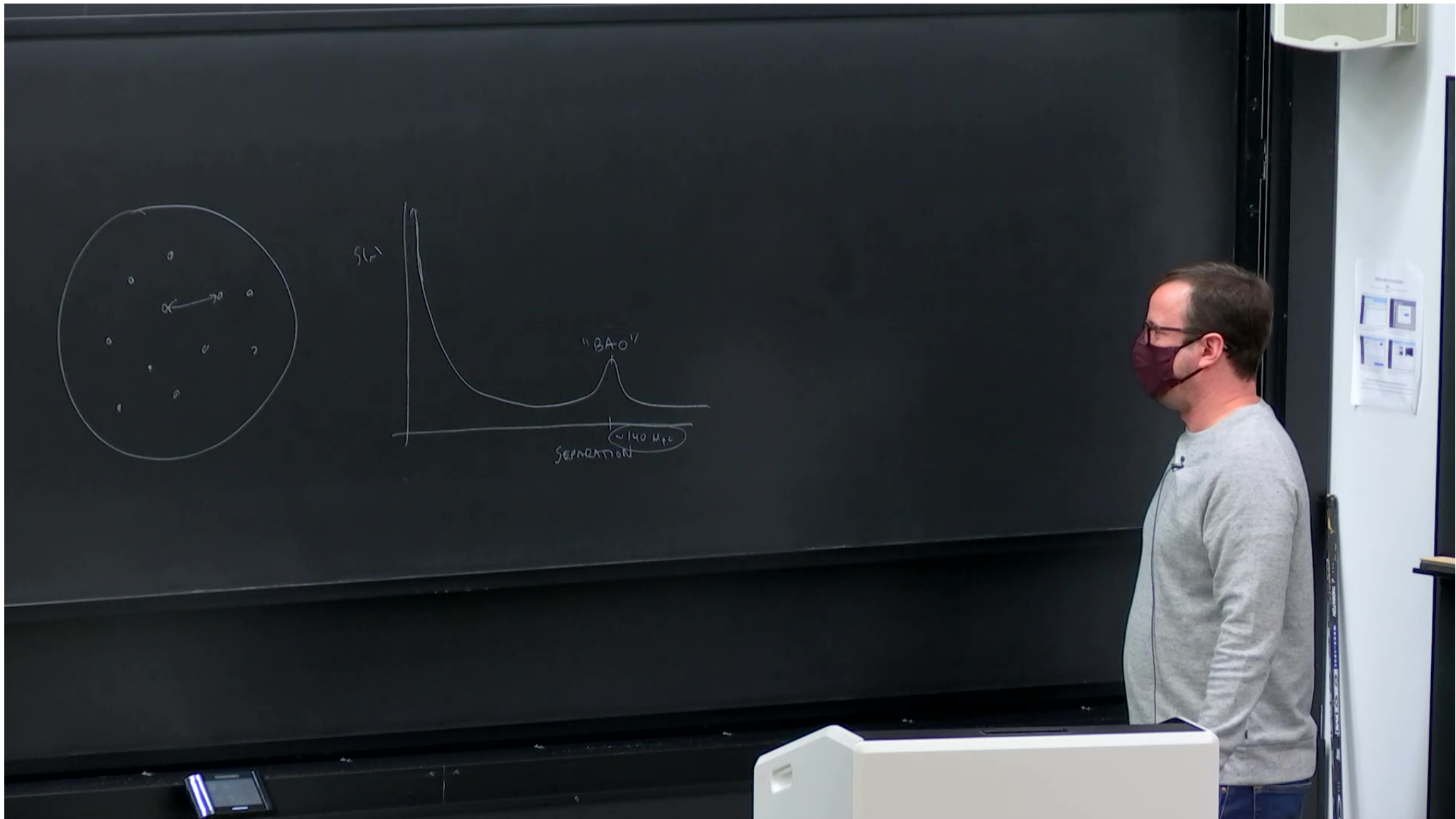
$$= \frac{2\pi (Dx)_{phys}}{2\pi ar}$$

← NOT $2\pi a x$

$$= \frac{(Dx)_{phys}}{(ar)} = \text{"ANGULAR DIAMETER DISTANCE"} D_A$$







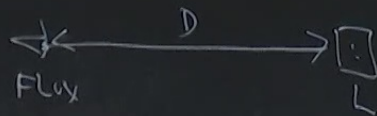
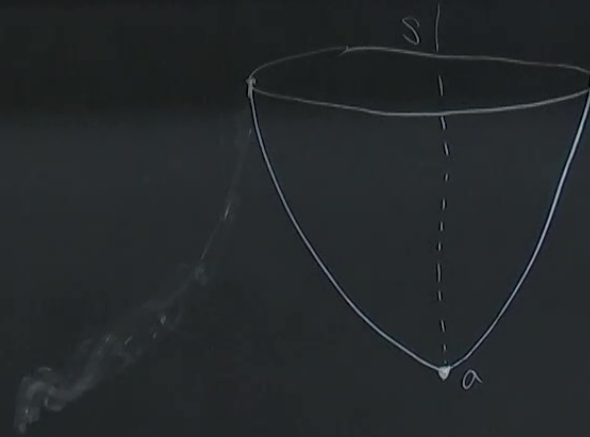


"STANDARD CANDLE"

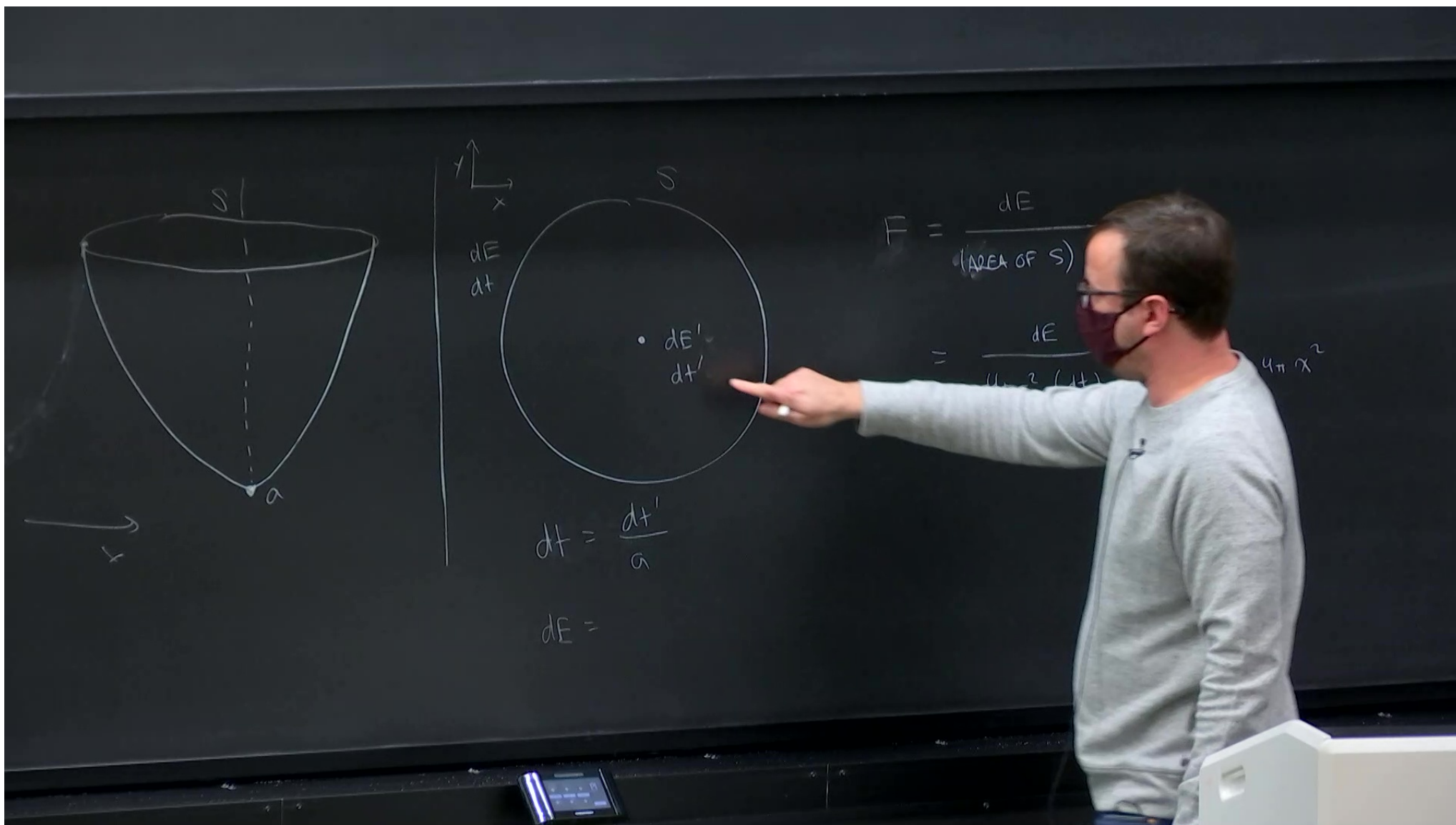
$$L = \frac{\text{ENERGY EMITTED}}{\text{TIME}}$$

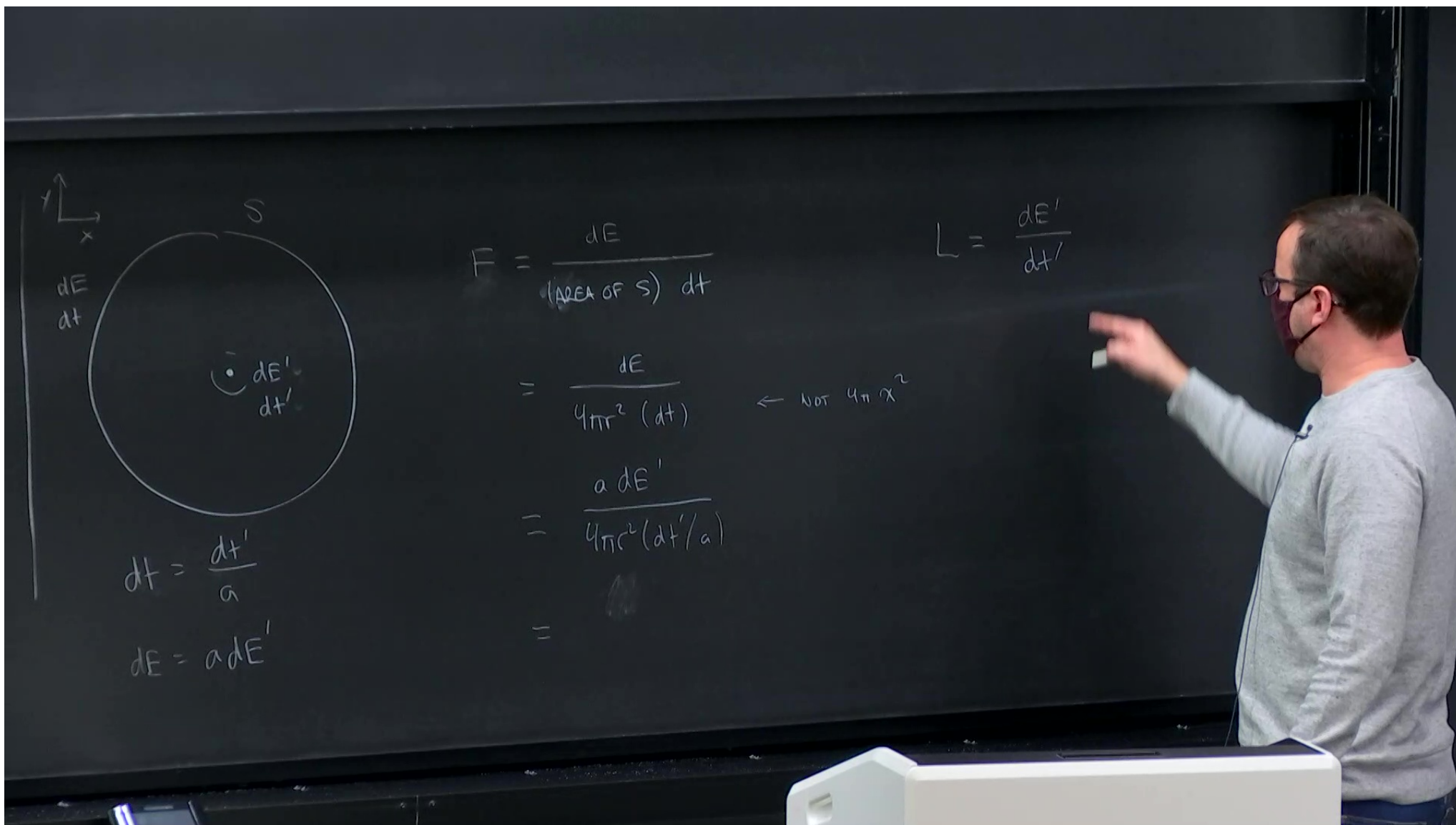
$$F = \frac{\text{ENERGY RECEIVED}}{(\text{COLLECTING AREA})(\text{TIME})}$$

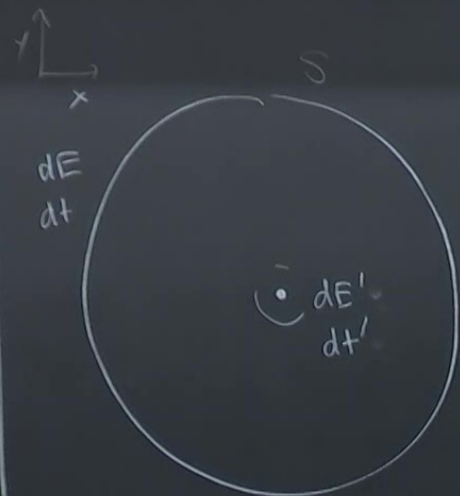
= FUNCTION OF L, a



$$F = \frac{L}{4\pi D^2}$$







$$dt = \frac{dt'}{a}$$

$$dE = a dE'$$

$$F = \frac{dE}{(\text{AREA OF } S) dt}$$

$$= \frac{dE}{4\pi r^2 (dt)} \quad \leftarrow \text{NOT } 4\pi x^2$$

$$= \frac{a dE'}{4\pi r^2 (dt'/a)}$$

$$= \frac{L}{4\pi (r/a)^2}$$

$$L = \frac{dE'}{dt'}$$

$$D_L = \frac{r}{a} \quad \text{"LUMONISITIC DISTANCE"}$$



$$F = \frac{L}{4\pi D^2}$$

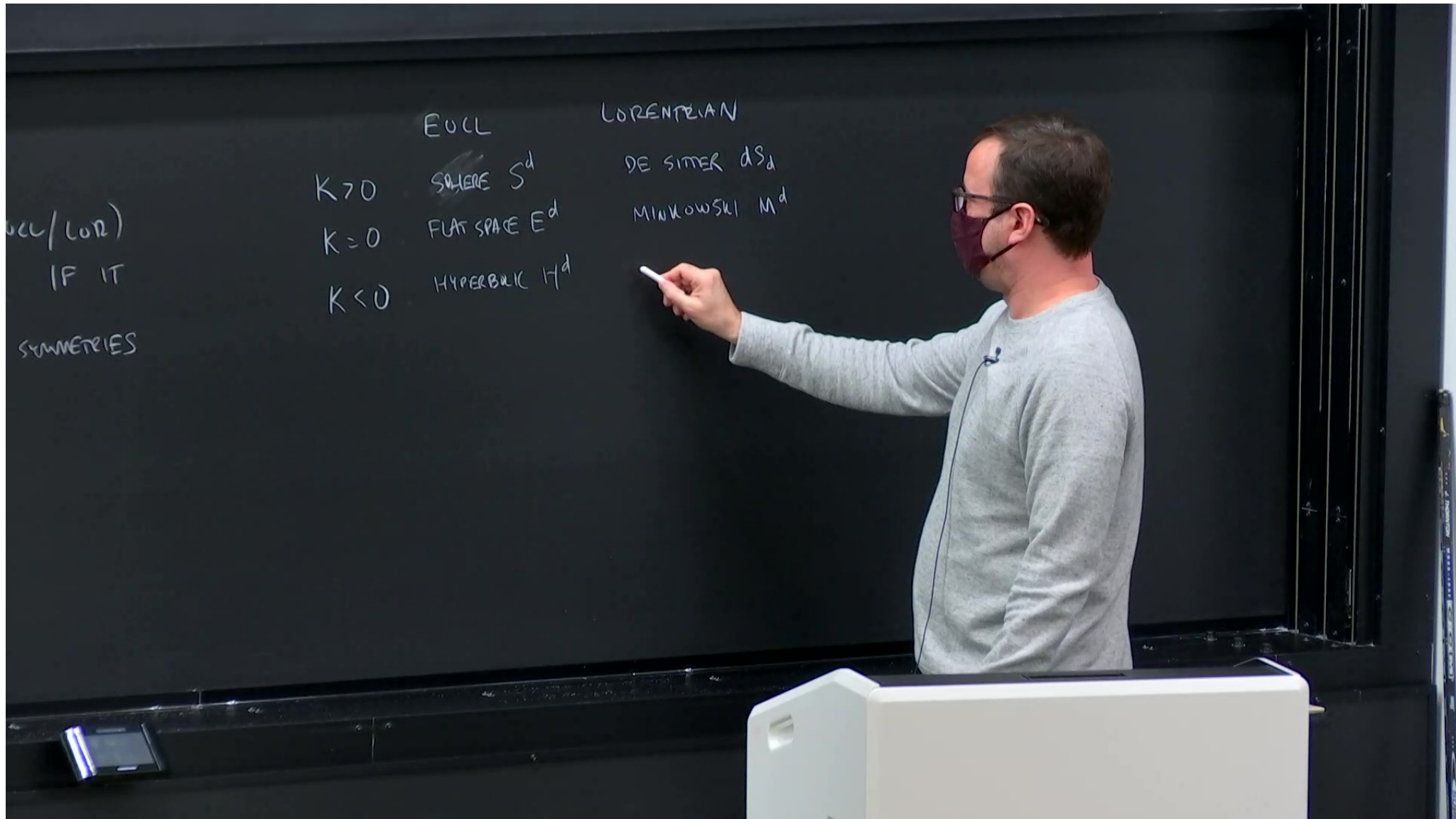
$$dE = a dE'$$

LOOSE ENDS

1) WHY DOES $ds^2 = \frac{(dr)^2}{1 - Kr^2} + r^2 d\Omega^2$

DESCRIBE A SPHERICAL/HYPERBOLIC GEOMETRY?

2) $H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \rho$





$$F = \frac{L}{4\pi D^2}$$

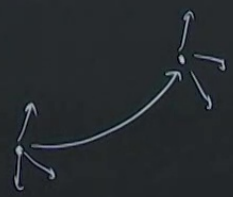
$$dE = a dE'$$

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SPACES OF MAXIMAL SYMMETRY

- A D-DIMENSIONAL SPACE (EULL/LOR) IS MAXIMALLY SYMMETRIC IF IT HAS $\frac{D(D+1)}{2}$ CONTINUOUS SYMMETRIES
- A SPACE HAS CONSTANT CURVATURE



$$F = \frac{L}{4\pi D^2}$$

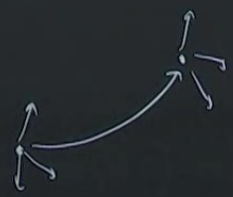
$$dE = a dE'$$

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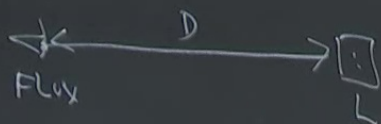


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IF $R_{abcd} = K(g_{ac}g_{bd} - g_{ad}g_{bc})$



$$F = \frac{L}{4\pi D^2}$$

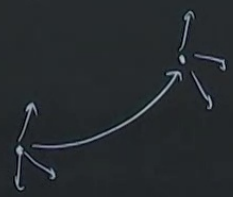
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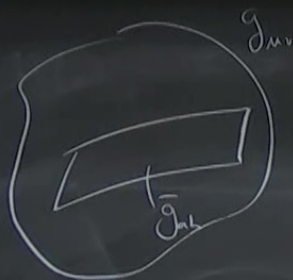


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FOR SOME CONSTANT K



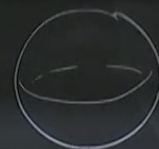
• SPHERE S^d

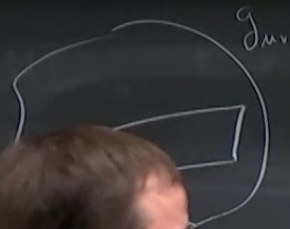
$$X_0^2 + \dots + X_d^2 = R^2$$

W/ METRIC INDUCED BY $ds^2 = \sum_{i=0}^d dx_i^2$

EUCIDEA
• HYPERBOLIC H^d

$$-X_0^2 + X_1^2 + \dots + X_d^2 = R^2$$





• SPHERE S^d

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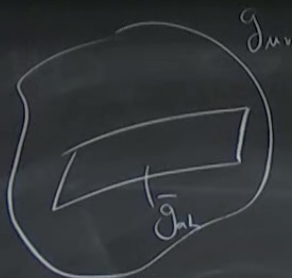
EUCIDEA

• HYPERBOLIC H^d

$$-X_0^2 + X_1^2 + \dots + X_d^2 = R^2$$

W/ METRIC INDUCED BY $ds^2 = -dX_0^2 + \sum_{i=1}^d dX_i^2$





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EUCIDEAN

• HYPERBOLIC H^d

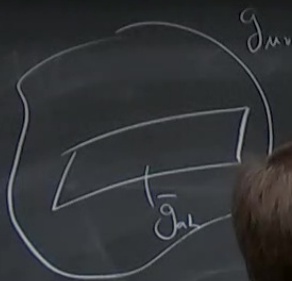
$$-X_0^2 + X_1^2 + \dots + X_d^2 = R^2$$

W/ METRIC INDUCED BY $ds^2 = -dX_0^2 + \sum_{i=1}^d dX_i^2$

$$-X_0^2 + X_1^2 + \dots + X_d^2 = -R^2$$

W/ METRIC INDUCED BY $ds^2 = -dX_0^2 + \sum_{i=1}^d dX_i^2$





• SPHERE S^d

$$X_0^2 + \dots + X_d^2 = R^2$$

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$$-X_0^2 + X_1^2 + \dots + X_d^2 = R^2$$

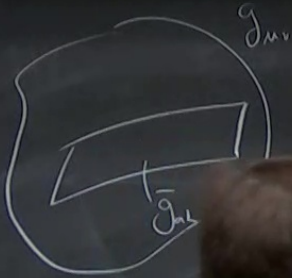
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THE SPHERE dS_d



• SPHERE S^d

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EUCIDEAN
HYPERBOLIC H^d

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• DE SITTER dS_d

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W/ METRIC INDUCED BY $ds^2 = -dX_0^2 + \sum_{i=1}^d dX_i^2$

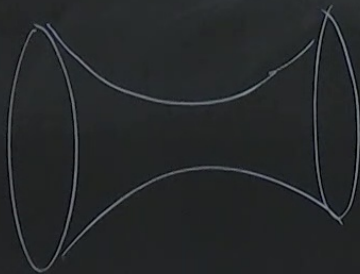


• ANTI DE SITTER AdS_d

$$-X_0^2 - X_1^2 + X_2^2 + \dots + X_d^2 = R^2$$

W/ METRIC INDUCED BY $ds^2 = -dX_0^2 - dX_1^2 + \sum_{i=2}^d dX_i^2$

AdS_2

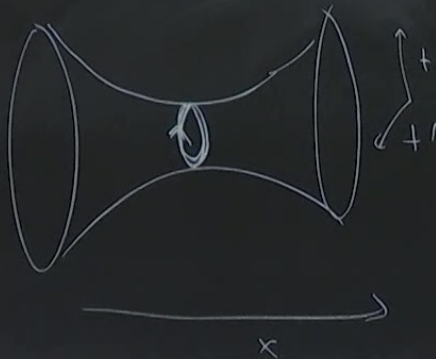


• ANTI DE SITTER AdS_d

$$-X_0^2 - X_1^2 + X_2^2 + \dots + X_d^2 = R^2$$

w/ METRIC INDUCED BY $ds^2 = -dX_0^2 - dX_1^2 + \sum_{i=2}^d dX_i^2$

AdS_2



+ "UNWRAP" PERIODIC COORDINATE

$$0 \leq \theta \leq 2\pi \rightarrow -\infty < x < \infty$$