Title: JT gravity with matter, generalized ETH, and Random Matrices - Baurzhan Mukhametzhanov

Speakers:

Series: Quantum Fields and Strings

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Abstract: JT gravity in AdS was shown by Saad, Shenker and Stanford to be described by a matrix ensemble of random hamiltonians. We couple JT to a bulk scalar field and extend the matrix ensemble to include a second matrix, dual to the scalar field. We therefore consider a 2-matrix model that can be thought of as a (better defined) generalization of Eigenstate Thermalization Hypotheses: it is a coupled matrix model of a random hamiltonian and a random operator. The 2-matrix model has an interesting integrability structure: correlation functions are expressed via SL(2,R) 6j-symbols that obey unlacing rules and Yang-Baxter equations. We compute the two-sided 2-point function on the double-trumpet geometry from the matrix model and find agreement with the matter loop contribution in the bulk. Based on work in progress with Jafferis, Kolchmeyer and Sonner.

Zoom Link: https://pitp.zoom.us/j/92268935307?pwd=dytCdTJlWVc3QXkweWxzcy83Sk9PZz09

Based on work in progress with
D.Jafferis, D.Kolchmeyer and J.Sonner

Generalized ETH, JT with matter, and Random Matrices

Perimeter Institute,
Quantum Fields and Strings Seminar
12 April, 2022

Baur Mukhametzhanov, IAS

- Random Matrices: Energy Level Statistics in chaotic systems and 2d Quantum Gravity
- Generalized Eigenstate Thermalization Hypotheses as a Matrix Model
- ETH in holographic theories: JT gravity with matter
- Conclusions

Short summary

- JT gravity is equivalent to a random matrix model. The matrix has two interpretations:

  Random Hamiltonian of dual quantum mechanics

  Triangulations of 2d surfaces via t'Hooft diagrams

- We will generalize this to include matter fields ϕ

  Random operator $\mathcal{O}$, generalized ETH

  t'Hooft diagrams of $\mathcal{O}$ have geometric interpretation of a particle propagating on geodesics of a 2d surface created by H

- More generally, conjecture that holographic theories (in 2d) are described by multi-matrix models

$$\int dHd\mathcal{O}\ldots e^{-N\,\mathrm{tr}V(H,\mathcal{O},\ldots)}$$

where we introduce a matrix $\mathcal{O}$ for every "single-trace" light operator. At large c (Generalized Free Fields) it is not gaussian, but integrable. Single-trace potential is due to locality of the bulk. (Tensor models in higher d?)

3

Random Matrices: Wigner vs. t'Hooft

4

Random Matrices
High-energy spectrum of chaotic hamiltonians

- Random matrices were introduced in physics by Wigner to describe energy level statistics of chaotic hamiltonians. Subsequently studied by Dyson, Mehta, Bohigas, Giannoni, Schmit and many others.

- Consider a chaotic hamiltonian and pick a microcanoincal band of energies $(E - \Delta E, E + \Delta E)$. Hamiltonian is a random $e^{S(E)} \times e^{S(E)}$ matrix, assuming the spectrum is dense ($e^{S(E)}$ is large).

- There is no accepted definition of "chaotic". Instead, we recognize chaotic systems by characteristic properties: dense spectra obeying RMT statistics, sensitivity to initial conditions, no integrals of motion etc.

5

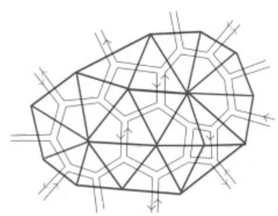
Random Matrices

2d Quantum Gravity

- In the 80's random matrices were used to construct quantum gravity in two dimensions. (David, AmbjØrn, Durhuus, Frohlich, Kazakov, Migdal, Kostov, Brezin, Douglas, Shenker, Gross...)

- E.g. consider matrix integral $\int dH\, e^{-N\,\mathrm{tr}(H^2+gH^3)}$. Perturbative expansion at large N is computed by t'Hooft diagrams.

  The dual lattice can be thought of as a triangulation of a 2d surface. Parameters of the matrix integral can be tuned such that diagrams with many faces dominate, effectively taking size of triangulation to zero and describing smooth fluctuating surfaces. This realized the idea of t'Hooft in a precise way.



6

Random Matrices

Wigner = t'Hooft

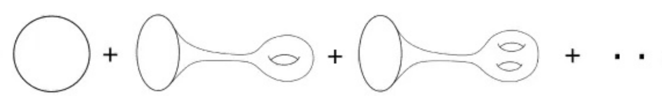
- Remarkably, in 2019 Saad, Shenker and Stanford showed that two applications of RMT could sometimes be equivalent via AdS/CFT.

- They considered Jackiw-Teitelboim (JT) gravity in two dimensions

$$I_{JT} = -S_0\chi - \frac{1}{2}\int_M \phi(R+2) - \int_{\partial M}\phi_b K$$

and showed that it is equivalent to a (double-scaled) matrix integral

$$\int_{\substack{\text{bdries}\\ \beta_1,\ldots,\beta_n}} Dg\ e^{-I_{JT}} = \int dH\ e^{-N\ \mathrm{tr}V(H)}\ \mathrm{tr}e^{-\beta_1 H}\ldots\mathrm{tr}e^{-\beta_n H}$$

- It holds perturbatively in 1/N (genus expansion), e.g. $\langle \mathrm{tr}\, e^{-\beta H}\rangle =$ 

Eigenstate Thermalization Hypothesis

8

Eigenstate Thermalization Hypothesis (ETH)

- Matrix elements of light operator $\mathcal{O}$ in energy eigenstates are random (Deutsch; Srednicki; Rigol, Dunjko, Olshanii)

$$\langle E_n | \mathcal{O} | E_m \rangle = \langle \mathcal{O} \rangle_{\bar{E}} \, \delta_{n,m} + e^{-S/2} f(\bar{E}, \Delta E) R_{nm}, \qquad \overline{|R_{nm}|^2} = 1$$

where $f(\bar{E}, \Delta E)$ is non-zero in a band $\Delta E \lesssim 1/\beta$.

- At very long times: $\Delta E \ll E_T, \quad f(\bar{E}, \Delta E) \approx \text{const}$. Universal RMT predictions.
- At finite times the structure of $f(\bar{E}, \Delta E)$ is important.

- ETH describes 2pt function: $\langle \mathcal{O}(t)\mathcal{O}(0) \rangle_c = e^{-S} \sum_{nm} e^{i(E_n - E_m)t} e^{-S} |f(\bar{E}, E_n - E_m)|^2 |R_{nm}|^2 \approx \int d\omega \; e^{i\omega t} |f(\bar{E}, \omega)|^2$

- To describe 4pt functions and higher need non-gaussianities of R_{nm} .

9

Eigenstate Thermalization Hypothesis

Generalization to a matrix model

- We take ETH ansatz seriously and promote it to a matrix model

$$\int dH d\mathcal{O}\ exp\left(-N\ \mathrm{tr}V(H) - \underbrace{\frac{1}{2}\sum_{nm} e^{S(E_n)/2} e^{S(E_m)/2} \,|f(E_n, E_m)|^{-2}\, \mathcal{O}_{nm}\mathcal{O}_{mn}}_{\mathrm{tr}V(H,\mathcal{O})}\right)$$

- This could apply (with more general $V(H, \mathcal{O})$) to a generic chaotic system, where $\mathcal{O}$ - any simple/few body operator.

- We will focus on holographic systems at large c.

10

Eigenstate Thermalization Hypothesis

Matrix Model description of holographic systems

- In a holographic system we take $\mathcal{O}$ - ''single trace'' operator. (In gauge theory sense; not to be confused with trace in the matrix model). Multi-twist operators are constructed as products of $\mathcal{O}$ and H, e.g. $\mathcal{O}\partial\mathcal{O} = -i\mathcal{O}[H, \mathcal{O}]$.

- We start with Mean Field Theory. The gaussian matrix model/ETH ansatz can't describe MFT

  Wick contractions and planar limit for $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$ don't match

  Operator $\mathcal{O}$ in MFT obeys constraints

ETH as a Matrix Model

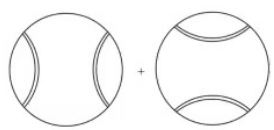
Gaussian ETH $\neq$ MFT

- MFT (GFF) correlators are computed by Wick contractions

$$\mathrm{tr}(e^{-\beta H}\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}) =$$

- On the other hand, in a Matrix Model only two of the diagrams are planar

$$\mathrm{tr}(e^{-\beta H}\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}) =$$



- In particular, OTOC vanishes. Instant scrambling.

12

ETH as a Matrix Model

Gaussian ETH $\neq$ MFT

- Another obstruction is that operators in MFT obey non-trivial constraints

$$[\mathcal{O}(t), \mathcal{O}(0)] = e^{iHt}\mathcal{O}e^{-iHt}\mathcal{O} - \mathcal{O}e^{iHt}\mathcal{O}e^{-iHt} = 2i\,\mathrm{Im}\,G(t)\cdot\mathbf{1}$$

- Generic matrices $H, \mathcal{O}$ do not obey such constraints.

- In the correct Matrix Model we will enforce this by adding (constraint)^2 with a large coefficient in the potential.

13

Review of JT gravity with matter

JT with matter

- To make progress, we focus on JT gravity coupled to a free scalar

$$I_{JT} + \frac{1}{2}\int (\partial\phi)^2 + m^2\phi^2$$

- Dual operator $\mathcal{O}$ has dimension $\Delta = 1/2 + \sqrt{m^2 + 1/4}$.

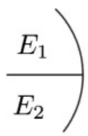
- 1-loop det on the disk renormalizes G_N. On the double-trumpet, gives non-trivial result that we will discuss.

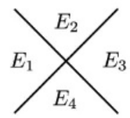
15

JT with matter
Disk correlators

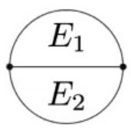
- Correlators on the disk for GFF coupled to Schwarzian are known (Mertens, Turiaci, Verlinde; Yang; Kitaev, Suh)

 = Sum over chord diagrams

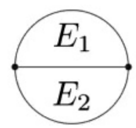
- Feynman rules: 

$= \Gamma(\Delta \pm i\sqrt{E_1} \pm i\sqrt{E_2})^{1/2}$, 

$= \begin{Bmatrix} \Delta & \sqrt{E_1} & \sqrt{E_2} \\ \Delta & \sqrt{E_3} & \sqrt{E_4} \end{Bmatrix}$ ($SL(2,\mathbb{R})$ 6j-symbol)

- For example: 2pt = 

, 4pt = 

- Or in eucl time: $\mathrm{tr}\ e^{-\beta H}\mathcal{O}(\tau)\mathcal{O}(0) = \int_0^\infty dE_1 dE_2\ \sinh 2\pi\sqrt{E_1}\ \sinh 2\pi\sqrt{E_2}\ e^{-\tau E_1 - (\beta-\tau)E_2}$ 

JT with matter

More boundaries/Higher genus

- Selberg trace formula computes the 1-loop determinant on negative curvature Riemann surface as a sum over closed primitive geodesics

$$Z_{1-loop} = \det(\nabla^2 + m^2)^{-1/2} = \exp\left(\#A + \sum_{\gamma}\sum_{n=1}^{\infty}\frac{1}{n}\frac{e^{-n\Delta b_\gamma}}{1-e^{-nb_\gamma}}\right)$$

- Feynman rules still hold with more boundaries or higher genus, where we sum over all possible ways of connecting operators by geodesics.

17

2-Matrix Model
for JT + free scalar

18

2-Matrix Models

Chord disk correlators ↔ ''Chord'' Matrix potential

- We would like to write a Matrix Model of $H, \mathcal{O}$ which reproduces JT/matter correlators

$$V_{SSS}(H) + V(H, \mathcal{O})$$

- Disk correlators $\langle \mathcal{O}_{i_1i_2}\mathcal{O}_{i_2j_3}\dots\mathcal{O}_{i_ni_1}\rangle$ determine the potential. This is similar to 1-matrix model, where disk density of states $\rho_0(E)$ determines the potential, while higher genus/more boundaries are determined by loop equations.

- We imagine constructing a class of 2-matrix models, where the potential is constructed out of chord diagrams/6j-symbols. For example

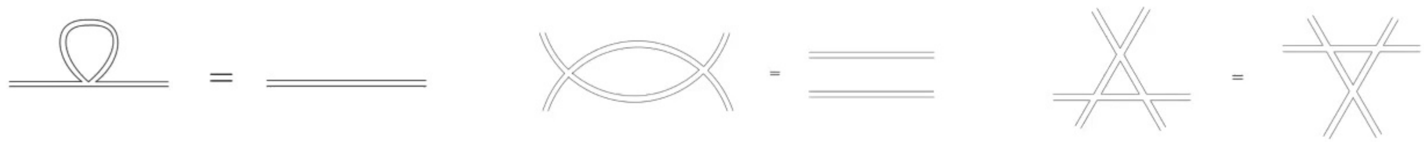
$$\sum_{nm} \Gamma(\Delta \pm i\sqrt{E_n} \pm i\sqrt{E_m})^{-1}\mathcal{O}_{nm}\mathcal{O}_{mn} + \sum (\Gamma\Gamma\Gamma\Gamma)^{-1/2} \begin{Bmatrix} \Delta & \sqrt{E_{n_1}} & \sqrt{E_{n_2}} \\ \Delta & \sqrt{E_{n_3}} & \sqrt{E_{n_4}} \end{Bmatrix} \mathcal{O}_{n_1n_2}\mathcal{O}_{n_2n_3}\mathcal{O}_{n_3n_4}\mathcal{O}_{n_5n_1} + \dots$$

19

2-Matrix Models

Chord disk correlators ↔ ''Chord'' Matrix potential

- ''Chord'' matrix potential gives chord disk correlators. Loop diagrams are all proportional to chord diagrams due to ''unlacing'' relations and Yang-Baxter obeyed by 6j



- A general potential of this type produces disk correlators that are arbitrary linear combinations of chord diagrams

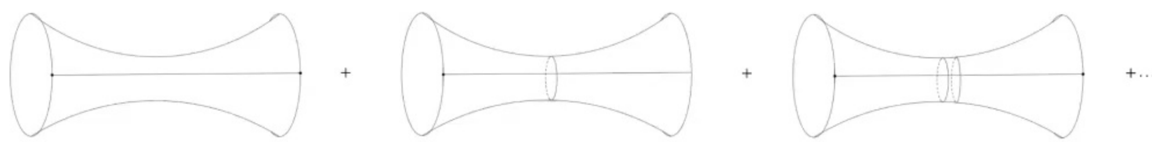
Chord Matrix Potential ≅ Chord disk correlators

- JT/matter correlators are a limit (analogous to double-scaling) in the space of such Matrix Models. We didn't derive this in general, but can construct the potential systematically in ''q-deformed'' JT (q-deformed SYK).
- For now, we will assume that a potential reproducing JT on the disk exists and see the implications for more boundaries.

2-Matrix Model

2pt function on double-trumpet

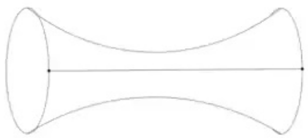
- Given disk correlators, we would like to compute 2pt including 1-loop det (closed geodesics) from the matrix model



$$= \int_0^\infty dE_L dE_R \, \sinh(2\pi\sqrt{E_L})\sinh(2\pi\sqrt{E_R} e^{-\beta_L E_L - \beta_R E_R}\Gamma(\Delta \pm 2i\sqrt{E_L})^{1/2}\Gamma(\Delta \pm 2i\sqrt{E_R})^{1/2}$$

$$\left(\frac{\delta(E_L - E_R)}{\sinh(2\pi\sqrt{E_L})} + \begin{Bmatrix} \Delta & \sqrt{E_L} & \sqrt{E_R} \\ \Delta & \sqrt{E_R} & \sqrt{E_L} \end{Bmatrix} + \sum_{n=0}^{\infty} \begin{Bmatrix} 2\Delta + 2n & \sqrt{E_L} & \sqrt{E_R} \\ \Delta & \sqrt{E_R} & \sqrt{E_L} \end{Bmatrix} + \ldots \right)$$

- We emphasize that this is obtained using Selberg trace formula.



2-Matrix Model

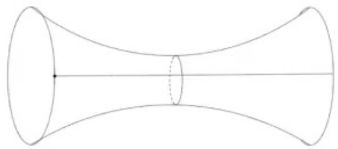
2pt, winding 0

- Winding 0 almost trivially follows from SSS and disk correlators

$$\langle \mathrm{tr} e^{-\beta_L H}\mathcal{O}\ \mathrm{tr} e^{-\beta_R H}\mathcal{O}\rangle = \int dE_L dE_R\ e^{-\beta_L E_L - \beta_R E_R}\langle\rho(E_L)\rangle\langle\rho(E_R)\rangle\langle\mathcal{O}_{E_L E_L}\mathcal{O}_{E_R E_R}\rangle$$

- This reproduces winding 0.

22

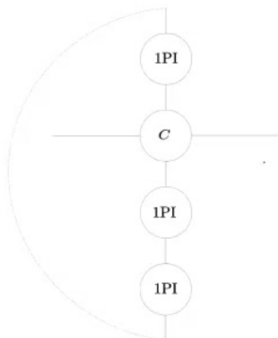


2-Matrix Model

2pt, winding 1

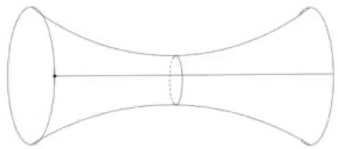
- Need to sum diagrams where we cross one propagator from left to right. Using disk correlators

$$\sum$$



$$= \Gamma(\Delta \pm 2i\sqrt{E_L})^{1/2}\Gamma(\Delta \pm 2i\sqrt{E_R})^{1/2}\begin{Bmatrix} \Delta & \sqrt{E_L} & \sqrt{E_R} \\ \Delta & \sqrt{E_R} & \sqrt{E_L} \end{Bmatrix}$$

- We compute exact connected 4pt function from the disk, than remove a propagator on one leg, and finally glue top and bottom lines. This will become more subtle for winding ≥ 2.



2-Matrix Model

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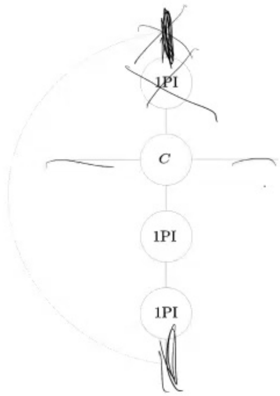

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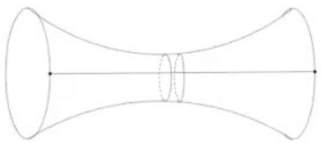
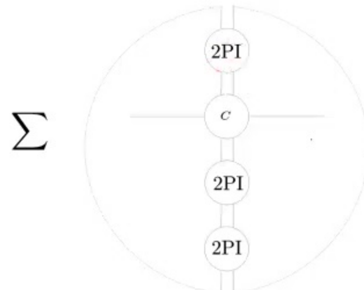


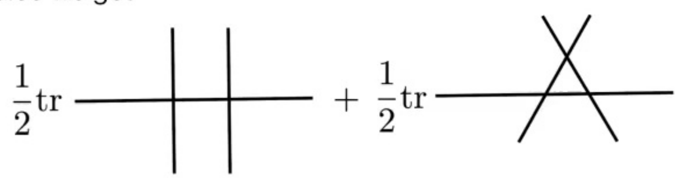
2-Matrix Model

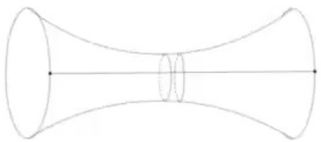
2pt, winding 2

- If we sum over 2PI insertions on both sides, we know the result from disk 6pt function



- To compute on double-trumpet, we remove 2PI graphs on one side and glue. Turns out depends on regularization of the matrix model. With proper choice we get

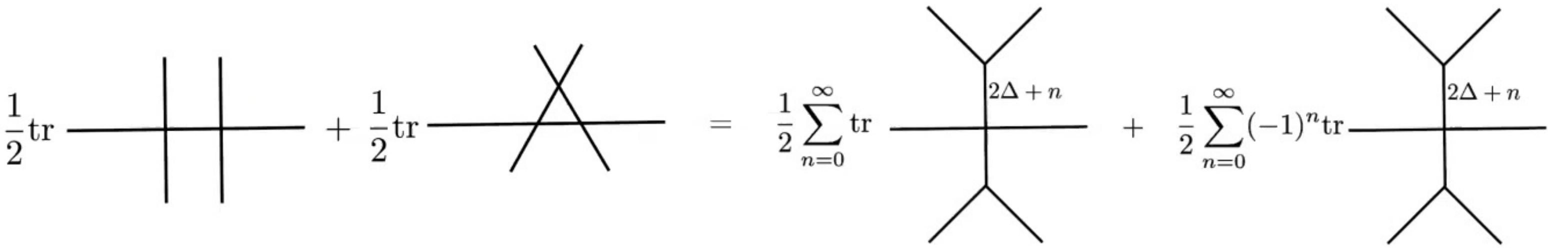


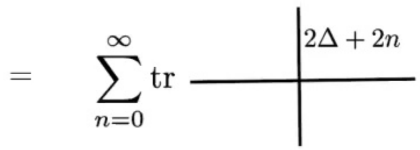


2-Matrix Model

2pt, winding 2

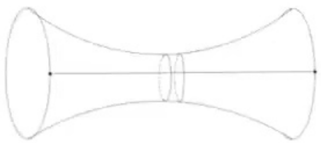
- Finally, use ''pentagon identity'' for 6j-symbol to compute





$$= \sum_{n=0}^{\infty}\begin{Bmatrix} 2\Delta + 2n & \sqrt{E_L} & \sqrt{E_R} \\ \Delta & \sqrt{E_R} & \sqrt{E_L} \end{Bmatrix}$$

25



2-Matrix Model

2pt, winding n>2

- Higher windings work similarly. We get a symmetrized sum and use 6j-symbol (Wilson function), Wilson and Racah polynomial identities to show it reproduces Selberg formula.

26

Conclusions

- Generalized ETH, 2-matrix model of $H, \mathcal{O}$ describing effective field theory of JT with massive scalar.
- t'Hooft diagrams of $\mathcal{O}$ correspond to particle propagating on geodesics of the Riemann surface.
- Integrable matrix model with interactions constructed out of $SL(2,\mathbb{R})$ 6j-symbol. Other groups?
- Full solution of such 2-matrix models? Semi-classical limit, matrix model description of GFF? Low-energy limit? Interactions? Higher d?

27