Title: Gravitational entropy and the large scale geometry of spacetime

Speakers: Neil Turok

Series: Quantum Gravity

Date: April 14, 2022 - 2:30 PM

URL: https://pirsa.org/22040112

Abstract: I'll review a new, simpler explanation for the large scale geometry of spacetime, presented recently by Latham Boyle and me in arXiv:2201.07279. The basic ingredients are elementary and well-known, namely Einstein's theory of gravity and Hawking's method of computing gravitational entropy. The new twist is provided by the boundary conditions we proposed for big bang-type singularities, respecting CPT symmetry and analyticity at the bang with finite Weyl curvature. These boundary conditions allow gravitational instantons for universes with positive Lambda, massless (conformal) radiation and positive or negative space curvature. Using these new instantons, we are able to infer the gravitational entropy for a complete set of quasi-realistic, four-dimensional cosmologies. If the total entropy in radiation exceeds that of Einstein's static universe, the gravitational entropy exceeds the de Sitter entropy. As the total entropy in radiation is increased further, the most probable large-scale geometry for the universe becomes increasingly flat, homogeneous and isotropic. I'll also briefly summarize recent progress towards elaborating this picture into a fully predictive cosmological theory.

Zoom Link: https://pitp.zoom.us/j/95474226765?pwd=clN5UHBnSTFiSVAzM0p3dEdDMzV2Zz09

Gravitational entropy and the large scale geometry of spacetime

Neil Turok, University of Edinburgh + PI

with L. Boyle, see recent preprints 2109.06204, 2110.06258, 2201.07279

Boyle, Finn, NT Phys. Rev. Lett. 121 (2018) no.25, 251301
Annals of Physics, 438 (2022) 168767

LCDM: a remarkably economical fit
just 5 parameters

energy content

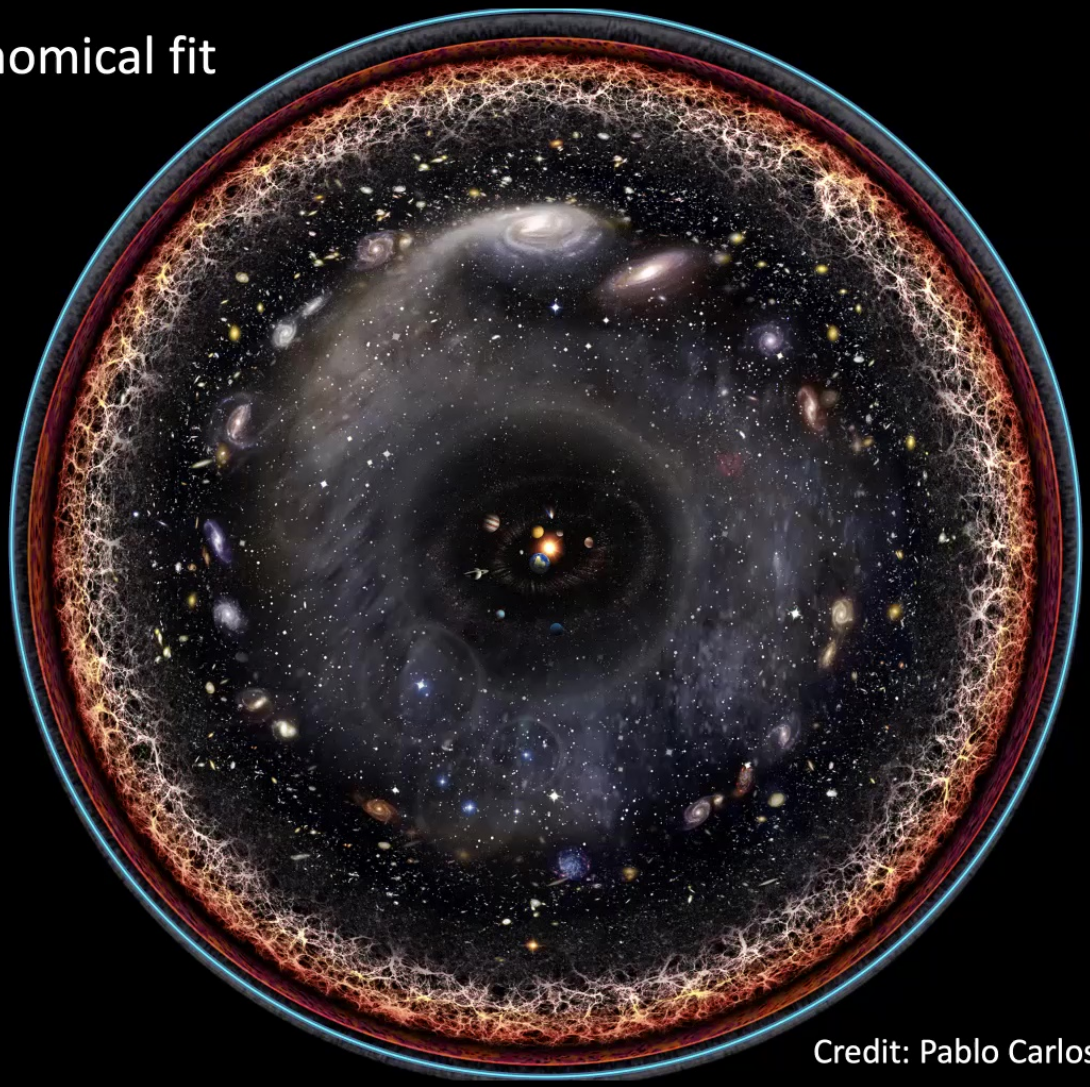
- n_B/n_γ
- ρ_{DM}/ρ_B
- ρ_Λ

comoving curvature

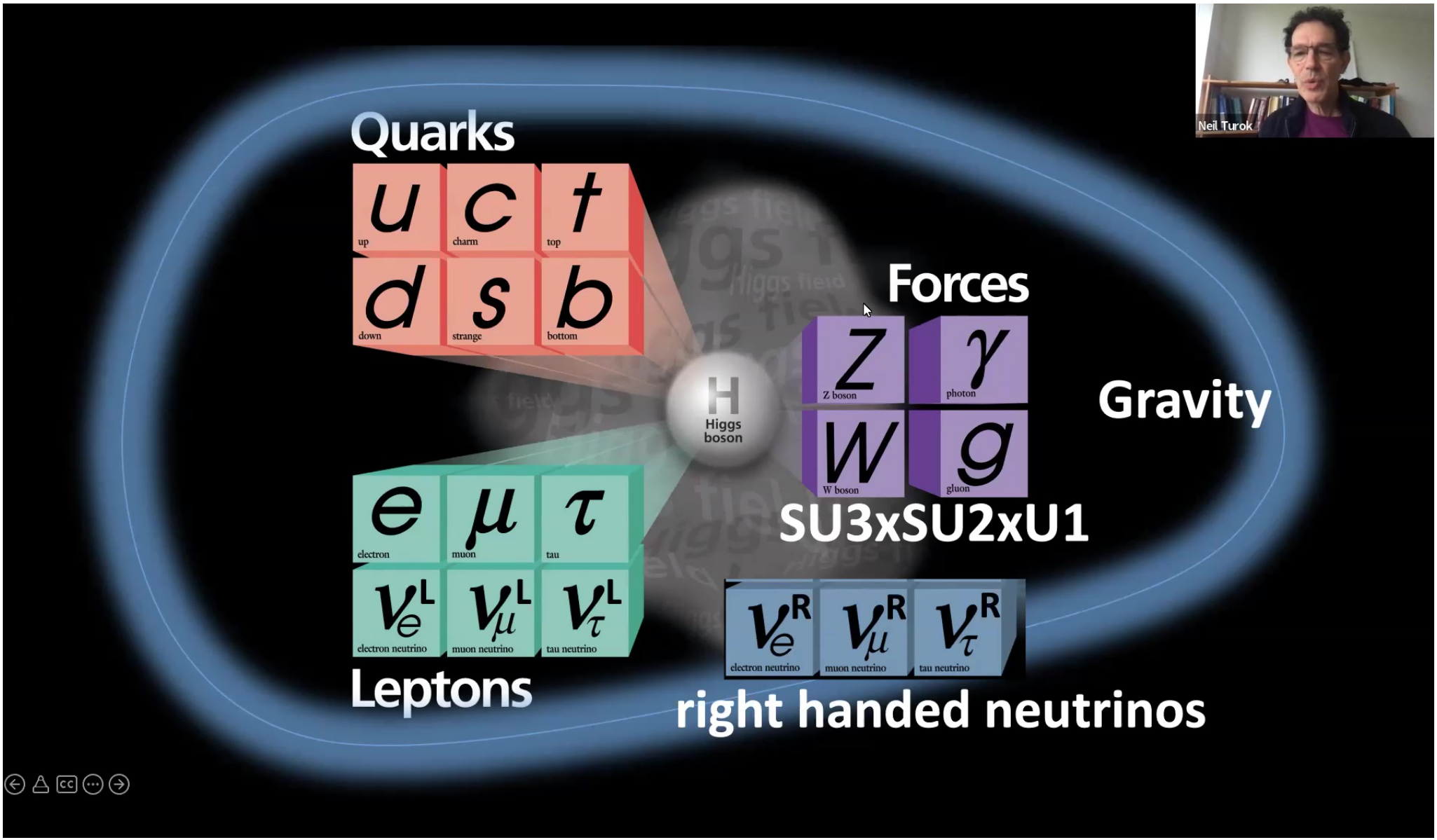
$k^{-2}R_k^{(3)} \sim A_k k^{-\varepsilon}$

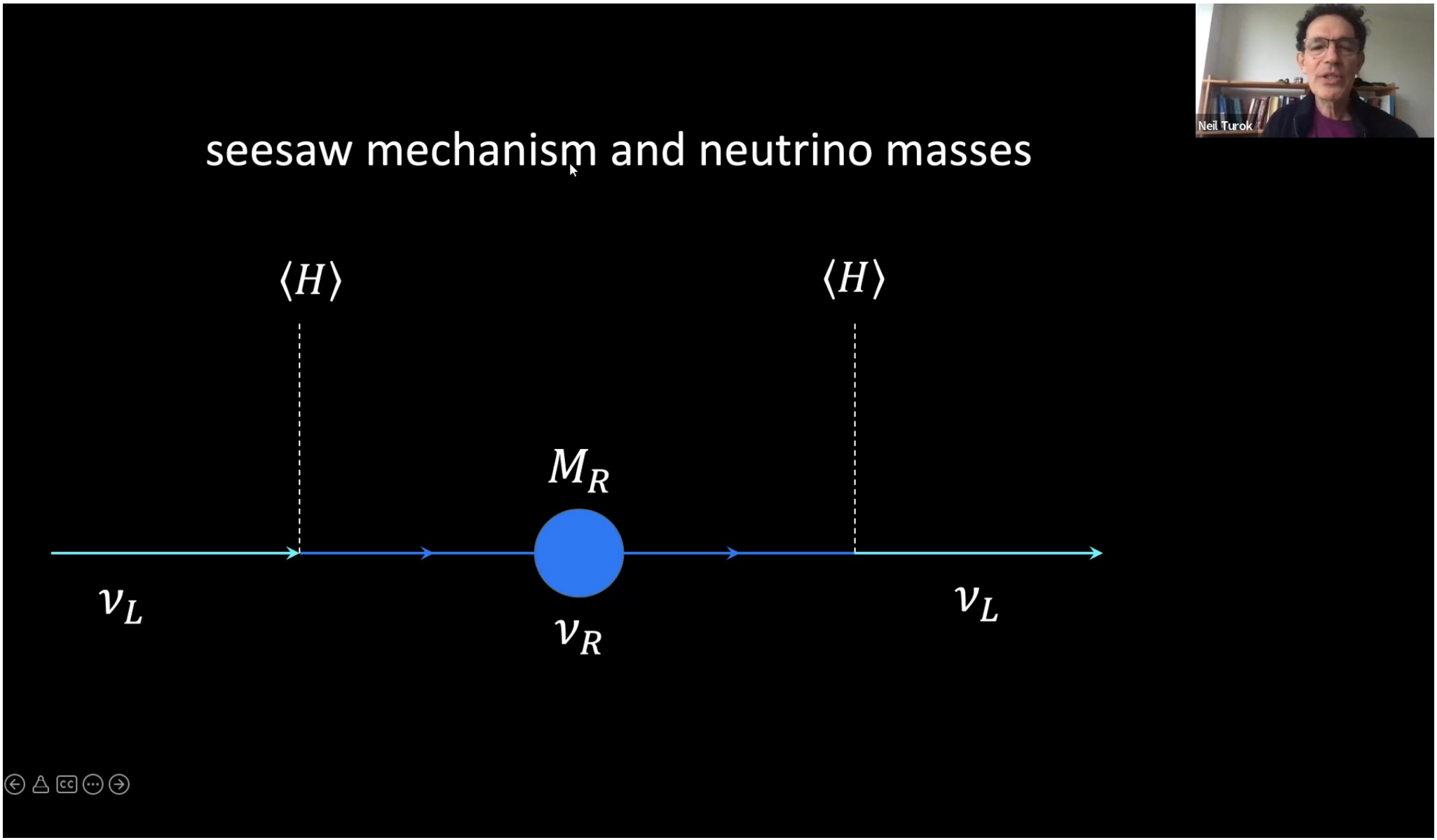
- $A_{k,rms} \sim 7.0\times10^{-5}$
- $\varepsilon \sim 0.017$

many quantities
consistent with zero



Credit: Pablo Carlos Budassi


Neil Turok
Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
H
Higgs boson
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
Gravity
SU3xSU2xU1
e
electron
μ
muon
τ
tau
ν_e^L
electron neutrino
ν_μ^L
muon neutrino
ν_τ^L
tau neutrino
Leptons
ν_e^R
electron neutrino
ν_μ^R
muon neutrino
ν_τ^R
tau neutrino
right handed neutrinos


Neil Turok
seesaw mechanism and neutrino masses
$\langle H \rangle$
$\langle H \rangle$
M_R
ν_L
ν_R
ν_L

our goal: explain nature's observed (and surprisin simplicity with simple and predictive principles

gauge fields

Higgs

$$\Psi = \int e^{\frac{i}{\hbar}\int\left(\frac{R}{16\pi G} - \frac{1}{4}F^2 + \overline{\psi}_i \not{D}\psi - \lambda H\overline{\psi}\psi + |DH|^2 - V(H)\right)}$$

gravity

particles

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gravity

particles

Do (real-time) path integrals exist? Feldbrugge and NT forthcoming preprint

Consider a radiation plus lambda cosmology:

$$\rho = \rho_r + \rho_\Lambda; \quad \rho_r = \frac{r}{a^4 L_{Pl}^2}, \ \rho_\Lambda = \frac{\lambda}{L_{Pl}^2}; \quad L_{Pl}^2 \equiv 8\pi G$$

$$R^{(3)} = 6\kappa$$

$$ds^2 = a(nt)^2\left(-n^2 dt^2 + \gamma_{ij} dx^i dx^j\right)$$

Friedmann equation

$$G_{00} = L_{Pl}^2 T_{00}$$

$$\boxed{\dot{a}^2 = \frac{1}{3}\left(r - 3\kappa a^2 + \lambda a^4\right)}$$

$$\left(\Omega_\kappa \equiv -\frac{3\kappa a^2}{3\dot{a}^2}; \ \Omega_r \equiv \frac{r}{3\dot{a}^2}; \ \Omega_\Lambda \equiv \frac{\lambda a^4}{3\dot{a}^2}\right)$$

dimensions

$$[a] = L, [r] = L^2, [\lambda] = L^{-2}$$

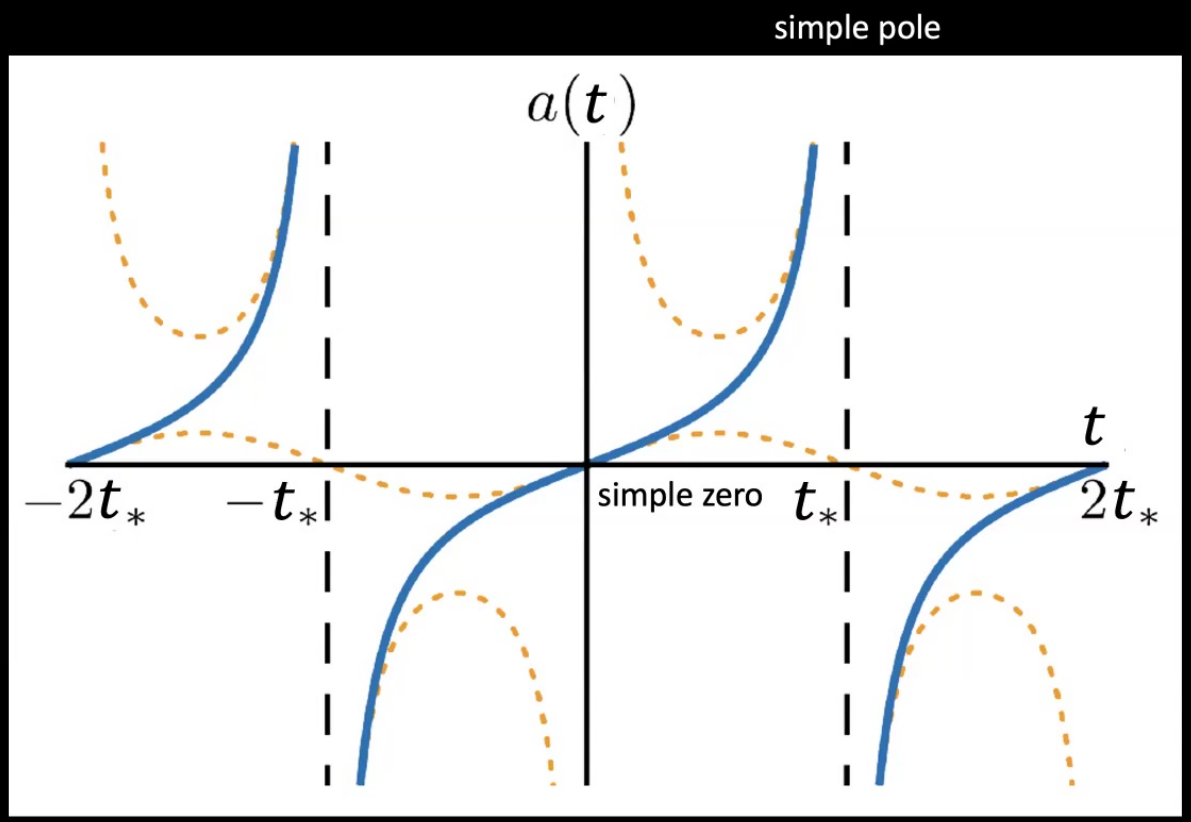
$$\dot{a} \equiv \frac{1}{n}\frac{da}{dt}$$

Jacobi elliptic function

solution $a(nt) = \alpha \, \mathrm{sn}(\beta nt, m)$; $\quad \frac{m}{(1+m)^2} = \frac{\lambda r}{(3\kappa)^2}; \ \alpha = \sqrt{\frac{r(1+m)}{3\kappa}}; \ \beta = \sqrt{\frac{\kappa}{(1+m)}}$

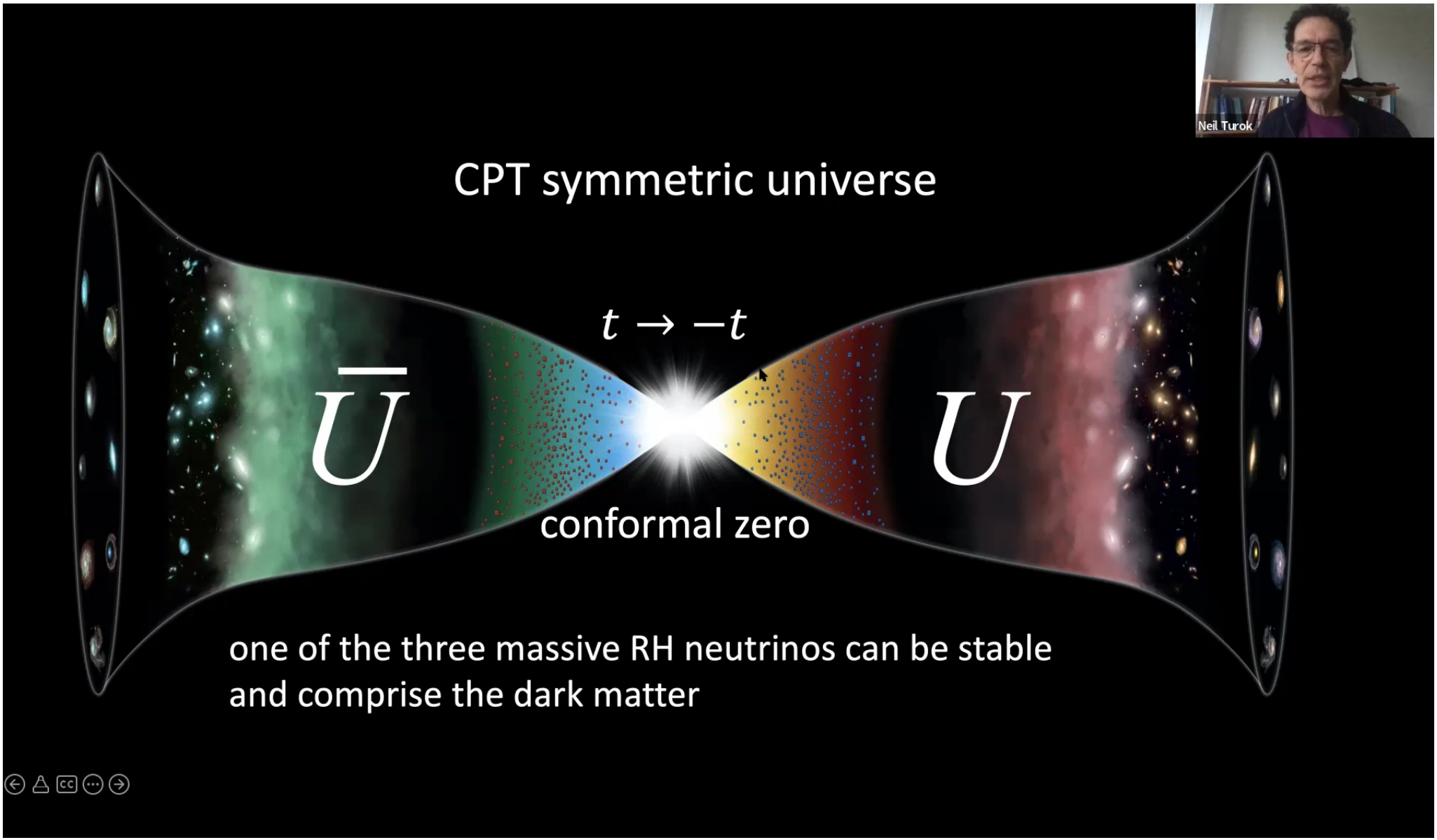
has remarkable analytical properties

(In this talk, I shall generally assume a compact topology: for $\kappa > 0$, a sphere and for $\kappa < 0$, a 3d "tile" of H_3)

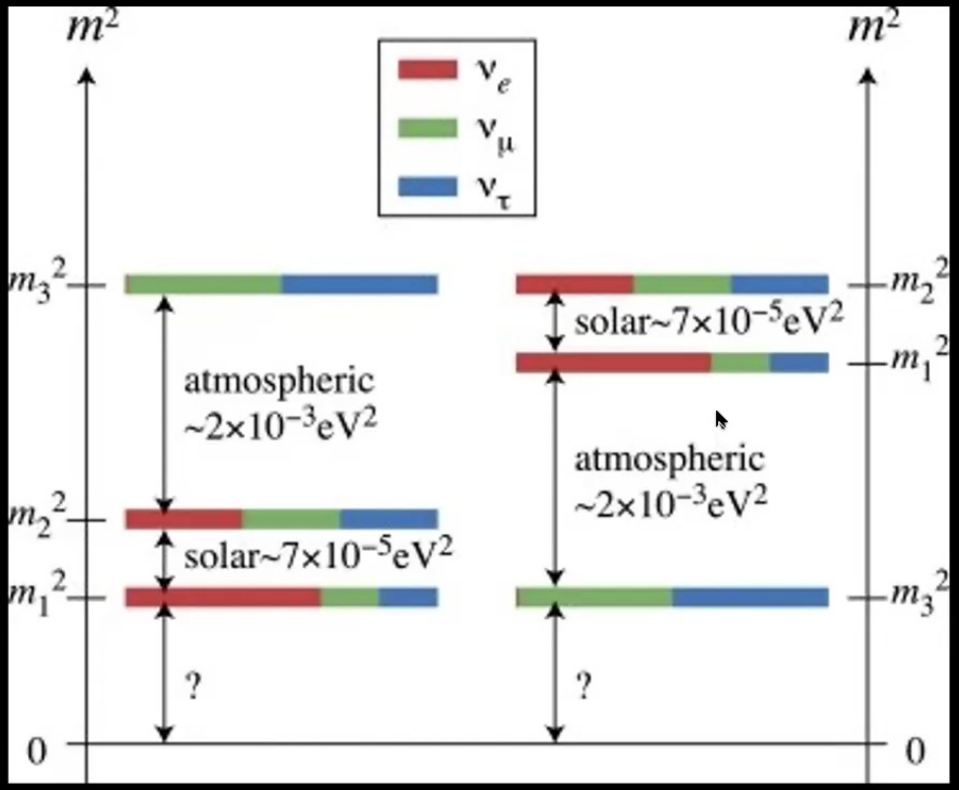


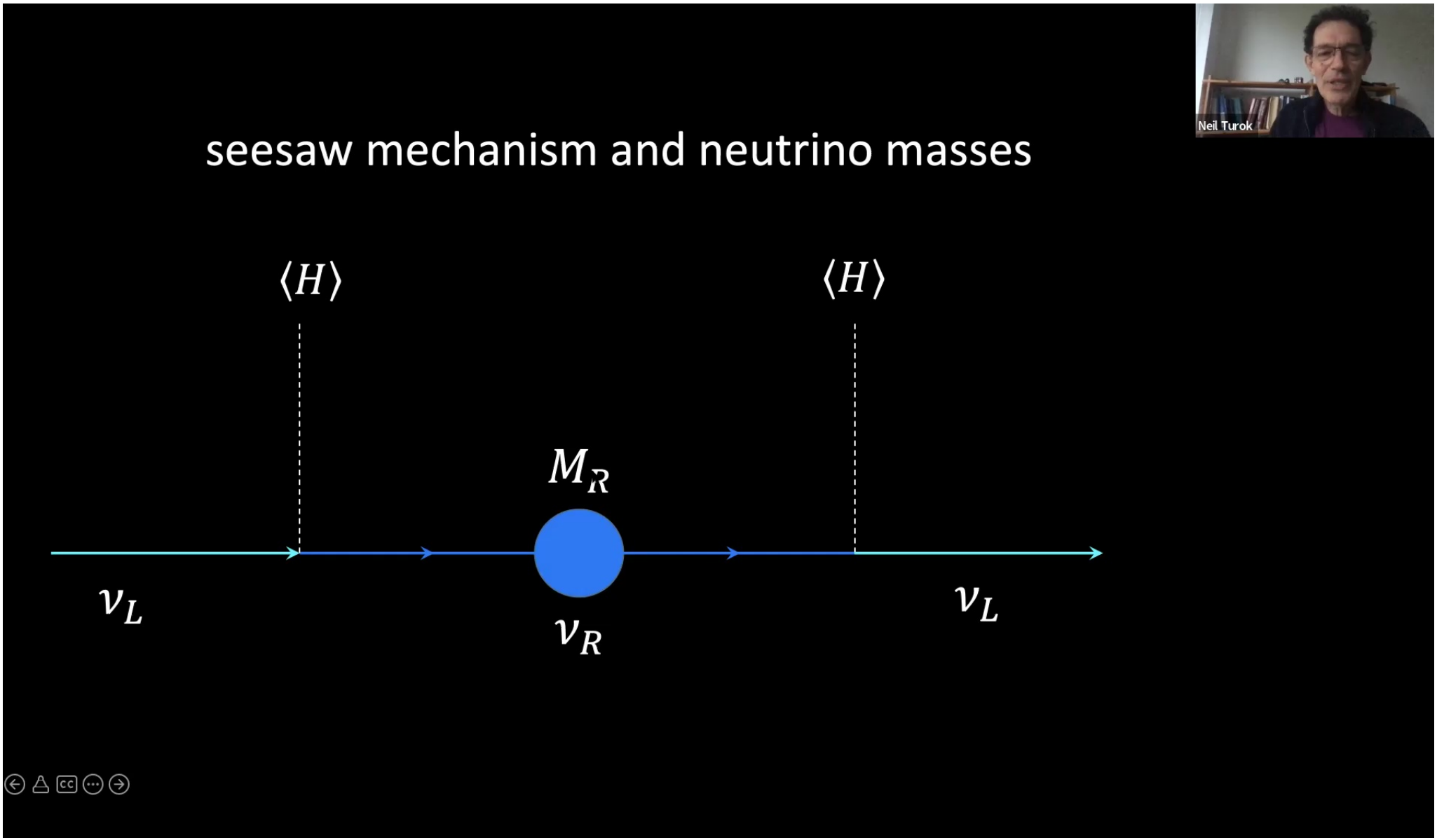
$a(t)$ is single-valued in the complex t-plane (or n-plane since it is a function of nt),
In fact, it is a Jacobi elliptic function: its only singularities are simple poles and it is doubly-periodic in the complex plane of its argument

L. Boyle and NT, hep-th/2109.06204


Neil Turok
CPT symmetric universe
$t \to -t$
$\bar{U}$
U
conformal zero
one of the three massive RH neutrinos can be stable
and comprise the dark matter

Light neutrinos: observed mass differences

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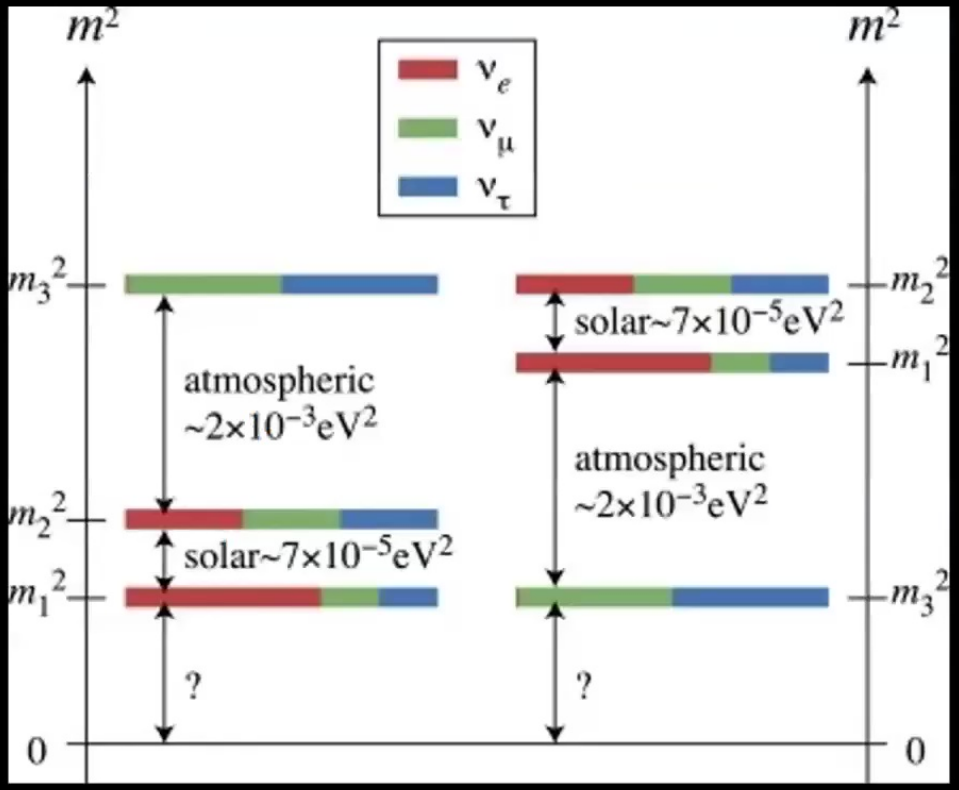
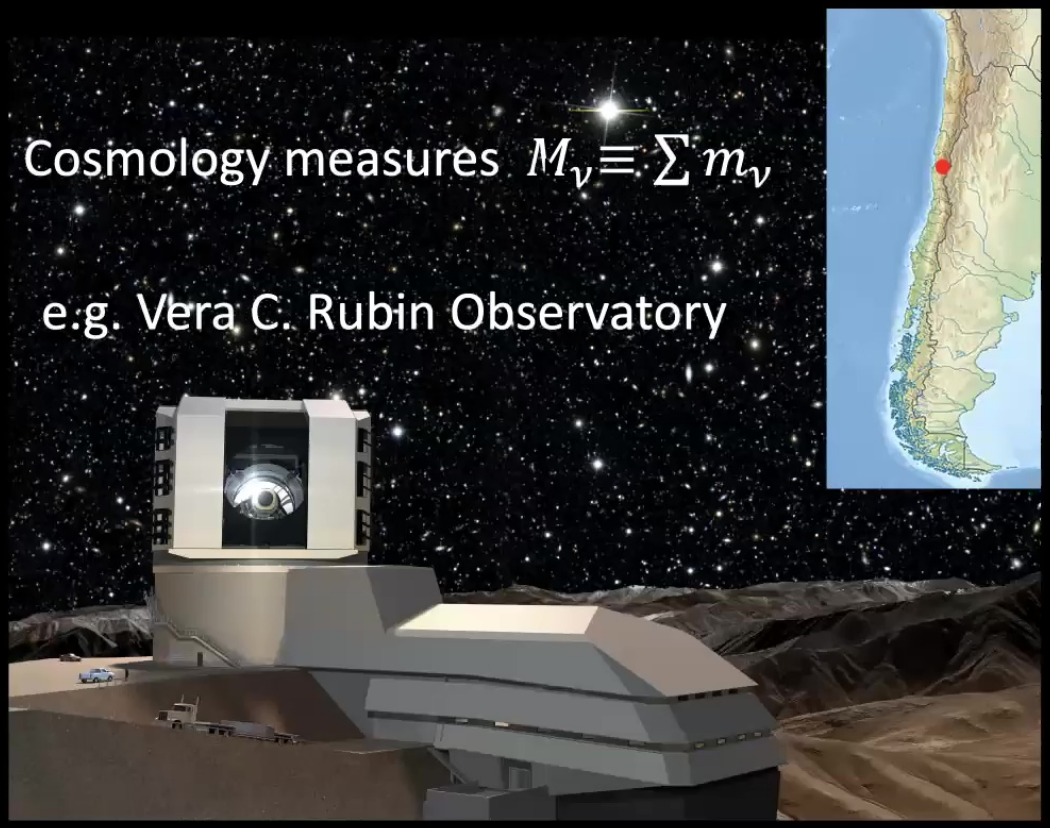
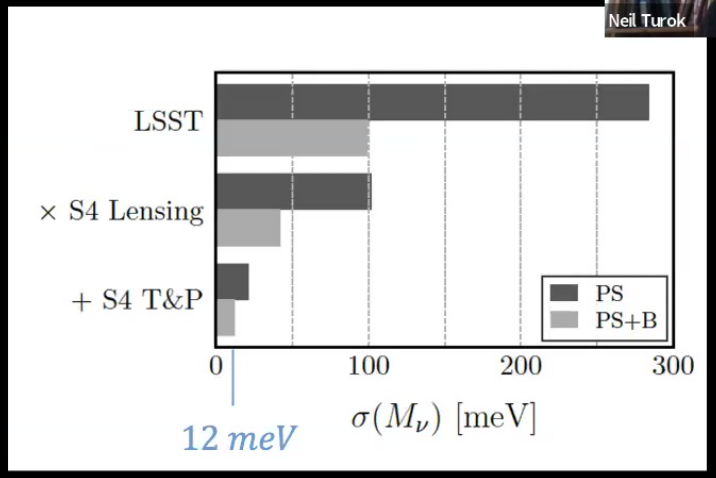


Cosmology measures $M_\nu \equiv \sum m_\nu$

e.g. Vera C. Rubin Observatory



Chen et al



Power Spec
Bi spec

Prediction: lightest neutrino is massless

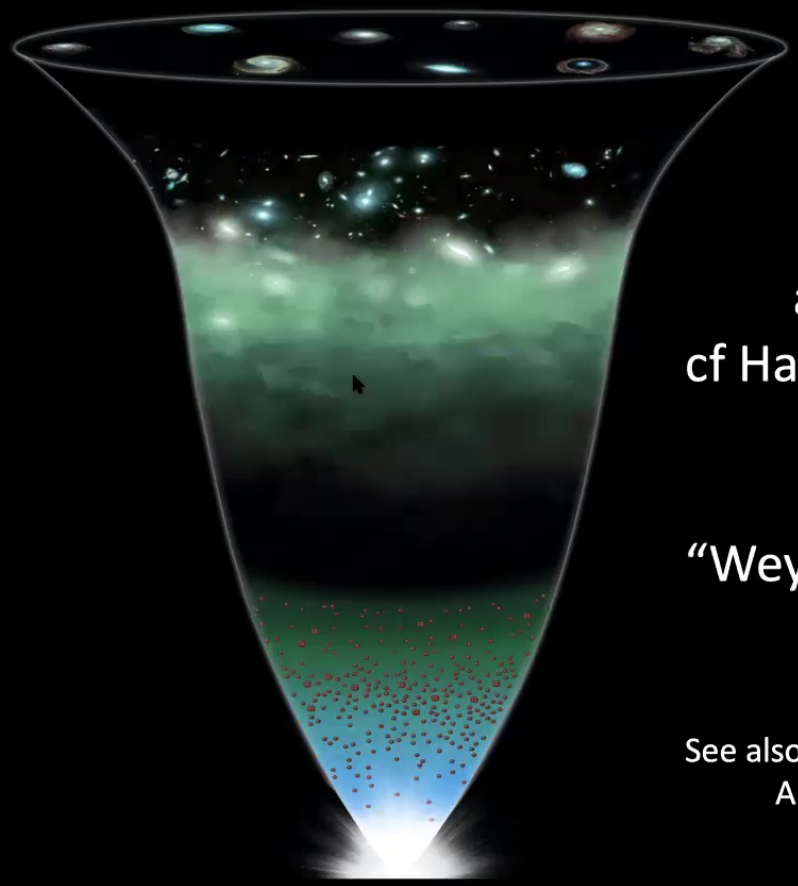
Normal hierarchy: $M_\nu \equiv \sum m_\nu = 60\ meV$

Inverted hierarchy: $M_\nu = 100\ meV$

CPT symmetric Universe is double-sheeted

w/L. Boyle

hep-th/2109.06204



w/L. Boyle

hep-th/2109.06204

analytic mirror
cf Hawking bcs for AdS.

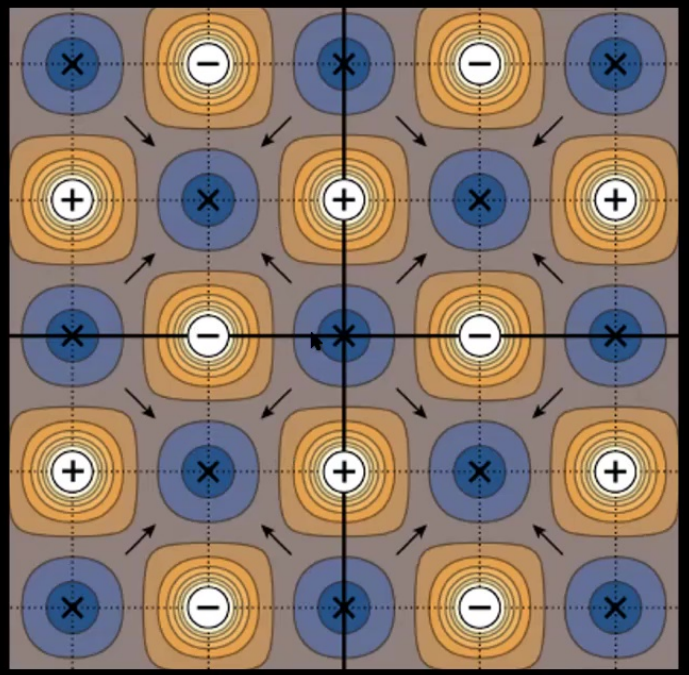
resonates with
“Weyl curvature hypothesis”
Penrose and Tod

See also
A.D. Sakharov, JETP Lett.5, 24 (1967);
Sov. Phys. JETP 52, 349 (1980).

$\mathrm{Im}[nt]$

$|a(nt)|$

fundamental domain is rectangular



$\mathrm{Re}[nt]$

Periodicity in imaginary time implies a Hawking temperature

L. Boyle and NT, hep-th/2109.06204

the puzzling large-scale geometry of spacetime

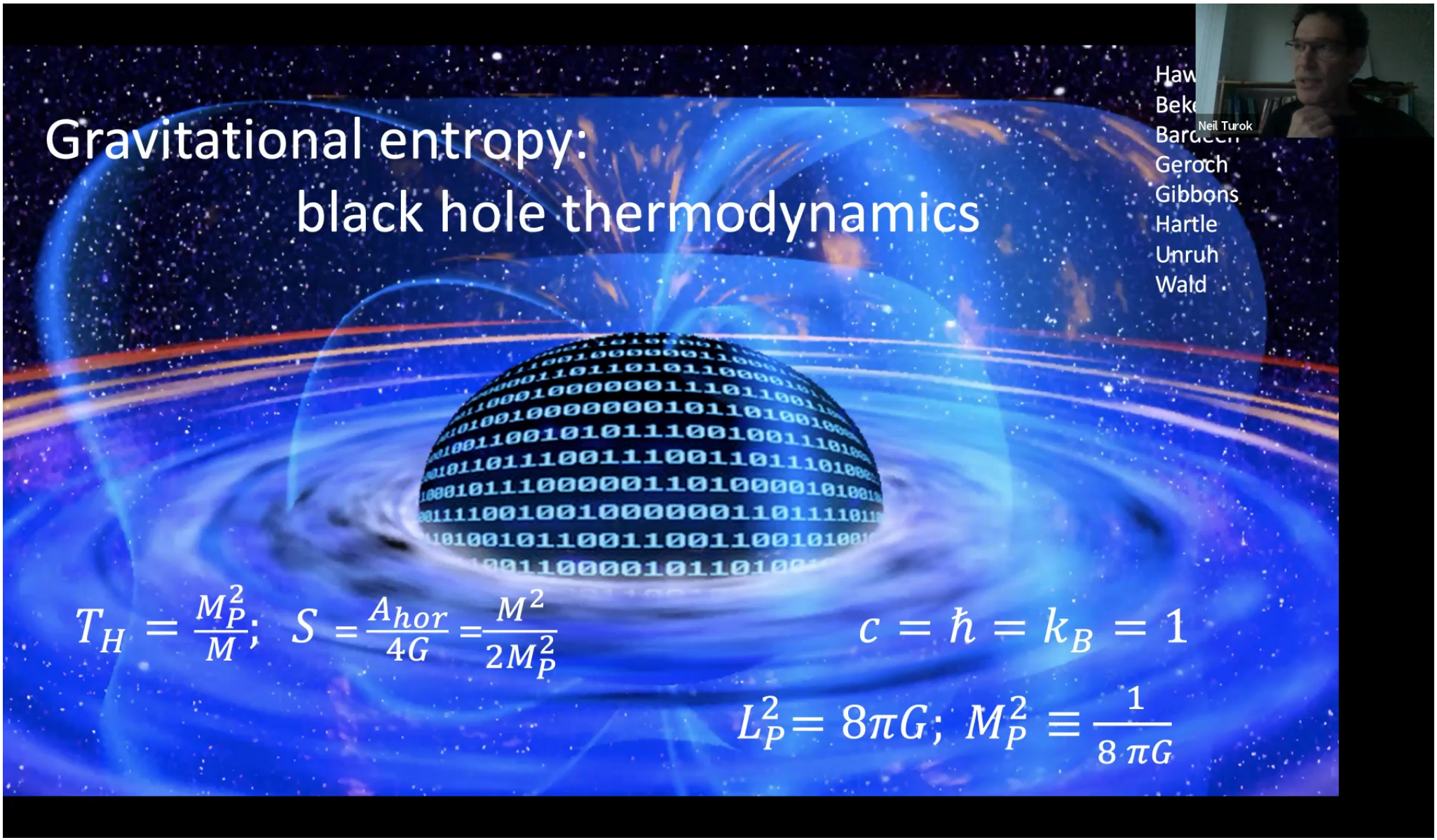
Penrose

flatness closer to home…

one explanation

a better explanation

- the earth is large ($\sim 10^{50}$ atoms)
- gravity, dissipation
- thermodynamics (entropy)

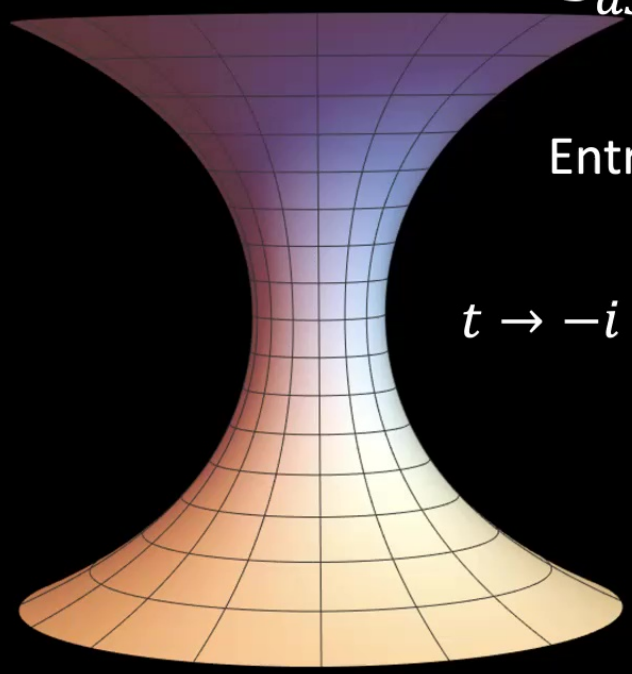

Gravitational entropy:
black hole thermodynamics
Haw
Beke
Bardeen
Geroch
Gibbons
Hartle
Unruh
Wald
Neil Turok
$T_H = \frac{M_P^2}{M}; \; S = \frac{A_{hor}}{4G} = \frac{M^2}{2M_P^2}$
$c = \hbar = k_B = 1$
$L_P^2 = 8\pi G; \; M_P^2 \equiv \frac{1}{8\,\pi G}$

de Sitter

$$T_{\text{Hawking}} = \frac{H}{2\pi}; \;\; H^2 = \frac{8\pi G}{3}\rho_\Lambda \; ; \; M_P^2 \equiv \frac{1}{8\,\pi G}$$

$$S_{dS} = \frac{A_{hor}}{4G} = \frac{24\pi^2 M_P^4}{\rho_\Lambda} \quad \left(= S_{BH}\left(M = \frac{4\pi}{3}\rho_\Lambda H^{-3}\right)\right)$$

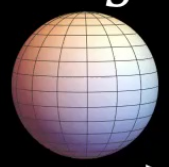
($\approx 3.26\times10^{122}$ for measured ρ_Λ)



Entropy calculated using Euclidean path integral

$t \to -i\, t_E$

S^4



$a = 1/H$

Ricci scalar trace of Einstein

$$R = \frac{12}{a^2} = \frac{4\rho_\Lambda}{M_P^2}$$

$$\text{Vol} = \frac{8\pi^2}{3}a^4 = \frac{24\pi^2 M_P^4}{\rho_\Lambda^2}$$

$$iS = \int\left(\tfrac{1}{2}M_P^2 R - \rho_\Lambda\right) = \rho_\Lambda\,\text{Vol} = \frac{24\pi^2 M_P^4}{\rho_\Lambda}$$

(this derivation hides a VERY important sign!)

1) The real cosmos is far from equilibrium: radiation temp $T_r \gg T_{dS} \approx 10^{-39} T_{0,CMB}$

2) For spatially compact spacetimes, the Hamiltonian is zero on all states

3) Nevertheless, one can still define a statistical ensemble

4) The expansion rate $H \sim T_r^2 / M_P \ll T_r$; for $T_r \ll M_P$ the expansion is adiabatic

5) The radiation is in local thermal equilibrium. It is an excellent approximation to integrate out the microscopic degrees of freedom (fermions, gauge bosons…) and treat the radiation as a perfect fluid.

Path Integrals: amplitudes and partition functions

1) Transition amplitudes: pure states to pure states: $N \int dx \; e^{iS}$

Picard-Lefschetz theorem:

for any relevant saddle, $|e^{iS}| \leq 1 \;\Rightarrow\; \mathrm{Re}[iS] \leq 0$. Semiclassical exponents *cannot* be positive

(series of papers by Feldbrugge+Lehners+NT, disproving Hartle-Hawking no boundary proposal and Vilenkin/Linde tunneling proposal)

2) Statistical physics: $Z = Tr\left(e^{-\beta H}\right) = e^{S-E/T}$

For geometrical ensembles (polymers, strings, membranes) S is exponentially large

For cosmology, H annihilates physical states so $Z =$ number of physical states

$Z = e^{S} > 1 \Longrightarrow S > 0.$ Semiclassical exponents *must* be positive

Trace over radiation $Z = \mathrm{Tr}(\int dn e^{-iHn})$; $H = H_g + H_r$

Treat radiation as conformal-invariant matter at temperature $T = \beta^{-1}$: H_r independent of a

Hence $\mathrm{Tr}_r\left(e^{-\beta H_r}\right) = e^{S_r - \beta U_r}$; S_r is the (adiabatically conserved) total entropy in radiation, and $U_r(S_r)$ is the associated internal energy in the conformally static frame.
Continue via $\beta = in$, to obtain

$$Z = e^{S_r}\,\mathrm{Tr}_g\left(\int dn\, e^{-i(H_g+U_r)n}\right)$$

This yields a path integral for cosmology with

$$iS = i\frac{V}{L_{Pl}^2}\int dt\, n\left(-3\frac{\dot{a}^2}{n^2} + 3\kappa a^2 - \lambda a^4 - r\right)$$

Note: odd in n

where we have set $U_r = r\,V/L_{Pl}^2$; $L_{Pl}^2 \equiv 8\pi G$

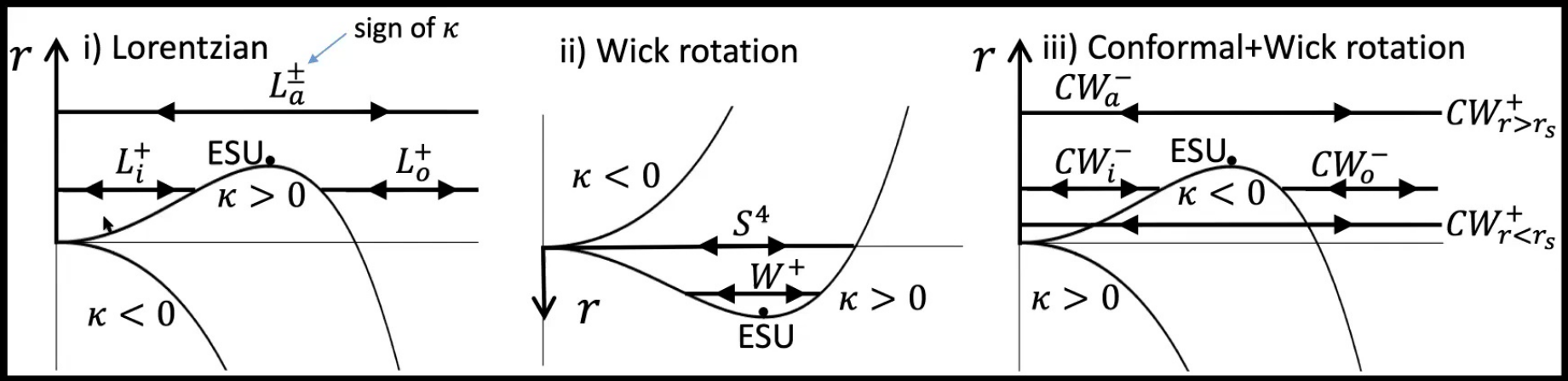
Cosmology with Lambda, radiation, space curvature

Conformal time $ds^2 = a^2(nt)(-n^2dt^2 + \gamma_{ij}dx^i dx^j)$ $\quad R^{(3)} = 6\kappa$

Einstein static universe

Friedmann $3\,\dot{a}^2 + \underbrace{3\kappa a^2 - \lambda a^4}_{V_{eff}(a)} = r;$ $\quad \rho_r = \frac{r}{a^4 L_{Pl}^2};\ \rho_\Lambda = \frac{\lambda}{L_{Pl}^2};$ for $\kappa > 0$, $r_{ESU} = \frac{9\,\kappa^2}{4\lambda}$

(no horizon, zero gravitational entropy)



Two ways to go Euclidean

$$ds^2 = a^2(nt)(-n^2 dt^2 + \gamma_{ij} dx^i dx^j)$$

W: $n = -iN, N$ real

CW: $n = -iN, N$ real

$a(nt) = -ib(Nt), b(t)$ real

("conformal rotation" C; Gibbons+Hawking+Perry)

Whereas W only yields instantons for $\kappa > 0, r \leq r_{ESU}$

CW yields instantons for all values of $\kappa, \lambda > 0, r > 0$

For CPT symmetric cosmology, since $a(nt)$ is odd, use CW

Cosmology with Lambda, radiation, space curvature

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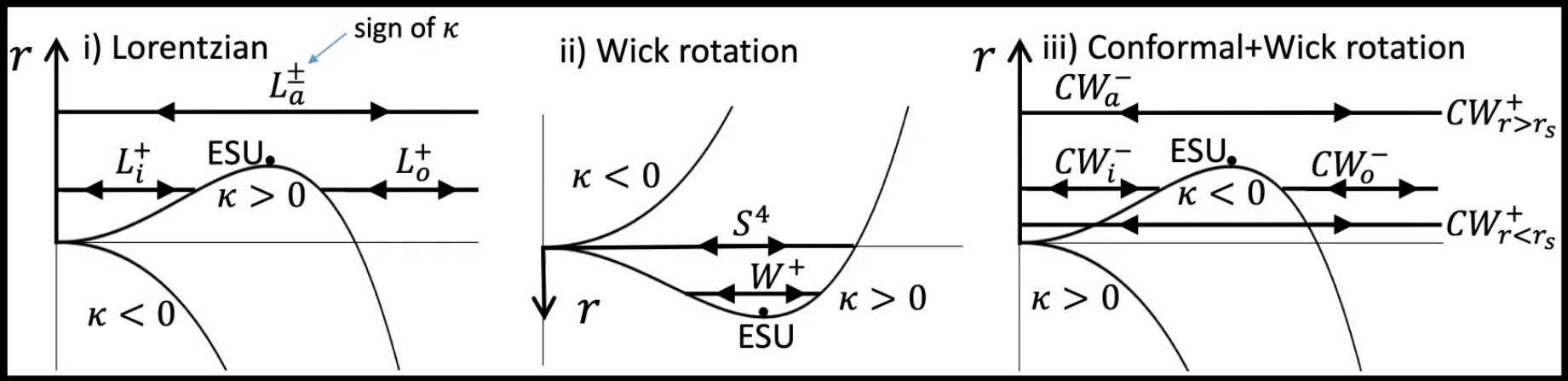

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Note: odd in n

where we have set $U_r = r\,V/L_{Pl}^2$; $L_{Pl}^2 \equiv 8\pi G$

CW rotation follows from boundary conditions which allow

$a(nt)$ odd, analytic at $t = 0$

(Conformal matter does not feel the conformal rotation)

For $\kappa > 0, r \leq r_{ESU}$, a positive semiclassical exponent requires $N > 0$

BUT for $\kappa > 0, r > r_{ESU}$ or $\kappa < 0$, a positive exponent requires $N < 0$

Hence, we require *two* Euclidean descriptions:
usual Wick rotation to integrate out the matter but $N < 0$ for gravity

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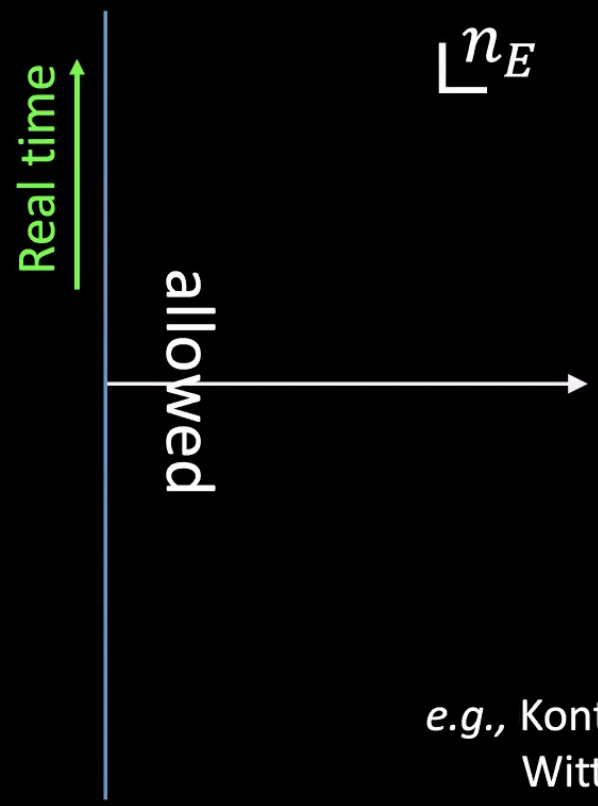
Trad. math physics: Euclidean is fundamental

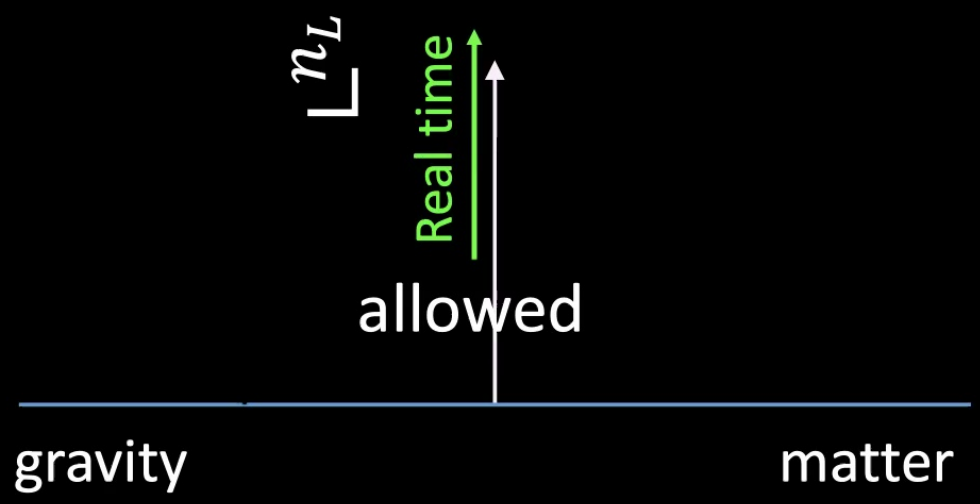
Neil Turok

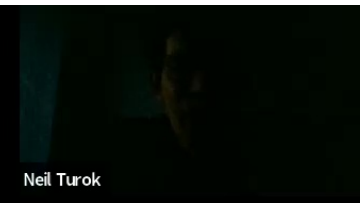


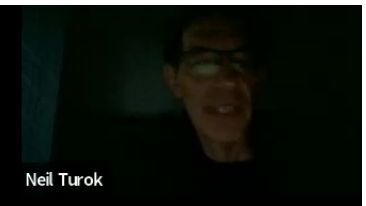
n_E

e.g., Kontsevich-Segal 2105.10161
Witten 2111.06514

cosmological viewpoint:
Lorentzian is fundamental





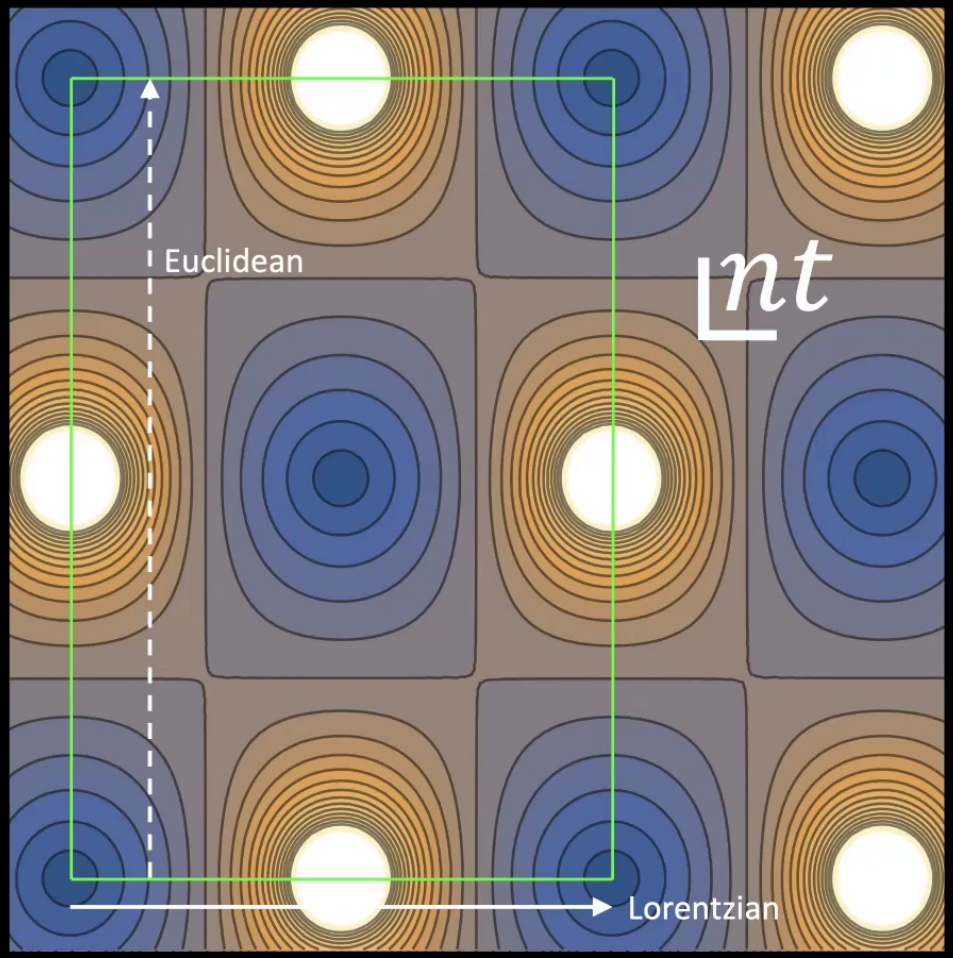


Case	scale factor $a(t)$ or $b(t)$	N	S_g/S_Λ	m	$z_\pm$
W^+	$\sqrt{\frac{3\kappa}{\lambda(1+m)}}\mathrm{dn}(\sqrt{\frac{\kappa}{1+m}}Nt, 1-m)$	$+\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$+\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$e^{\pm\alpha/4}$
$CW^+_{r<r_s}$	$i\sqrt{\frac{3\kappa m}{\lambda(1+m)}}\mathrm{sn}(i\sqrt{\frac{\kappa}{1+m}}Nt, m)$	$+\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$+\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$e^{\pm\alpha/4}$
$CW^+_{r>r_s}$	$i\sqrt{\frac{3\kappa m}{\lambda(1+m)}}\mathrm{sn}(i\sqrt{\frac{\kappa}{1+m}}Nt, m)$	$-\sqrt{\frac{1+m}{\kappa}}2K(1-m)$	$-\frac{(1+m)E(1-m)-2mK(1-m)}{(1+m)^{3/2}}$	$e^{i\theta}$	$e^{\pm i\theta/4}$
CW^-_i	$\sqrt{\frac{3\lvert\kappa\rvert m}{\lambda(1+m)}}\mathrm{sn}(\sqrt{\frac{\lvert\kappa\rvert}{1+m}}Nt, m)$	$-\sqrt{\frac{1+m}{\lvert\kappa\rvert}}2K(m)$	$2\frac{(1+m)E(m)-(1-m)K(m)}{(1+m)^{3/2}}$	$e^{-\alpha}$	$\pm ie^{-\alpha/4}$
CW^-_a	$\sqrt{\frac{3\lvert\kappa\rvert m}{\lambda(1+m)}}\mathrm{sn}(\sqrt{\frac{\lvert\kappa\rvert}{1+m}}Nt, m)$	$-\mathrm{Re}\left(\sqrt{\frac{1+m}{\lvert\kappa\rvert}}2K(m)\right)$	$\mathrm{Re}\left(2\frac{(1+m)E(m)-(1-m)K(m)}{(1+m)^{3/2}}\right)$	$e^{i\theta}$	$\pm ie^{\mp i\theta/4}$

Table I: Analytic forms for Euclidean instantons (Fig. 1, ii) and iii)), periodic in t with period unity, from which the Lorentzian solutions (Fig. 1, i)) are obtained by analytic continuation. Here, $\mathrm{sn}(z,m)$, $\mathrm{dn}(z,m)$ are Jacobi elliptic functions and $K(m)$, $E(m)$ are complete elliptic integrals [23]. W denotes Wick rotation and CW combined conformal and Wick rotations. Superscript $\pm$ indicates $\kappa \lesseqgtr 0$; m satisfies $m/(1+m)^2 = \lambda r/(3\kappa)^2$. For $0 < r \leq r_s$, $m = e^{-\alpha}$ with $0 \leq \alpha < \infty$, and for $r \geq r_s$, $m = e^{i\theta}$ with $0 \leq \theta < \pi$

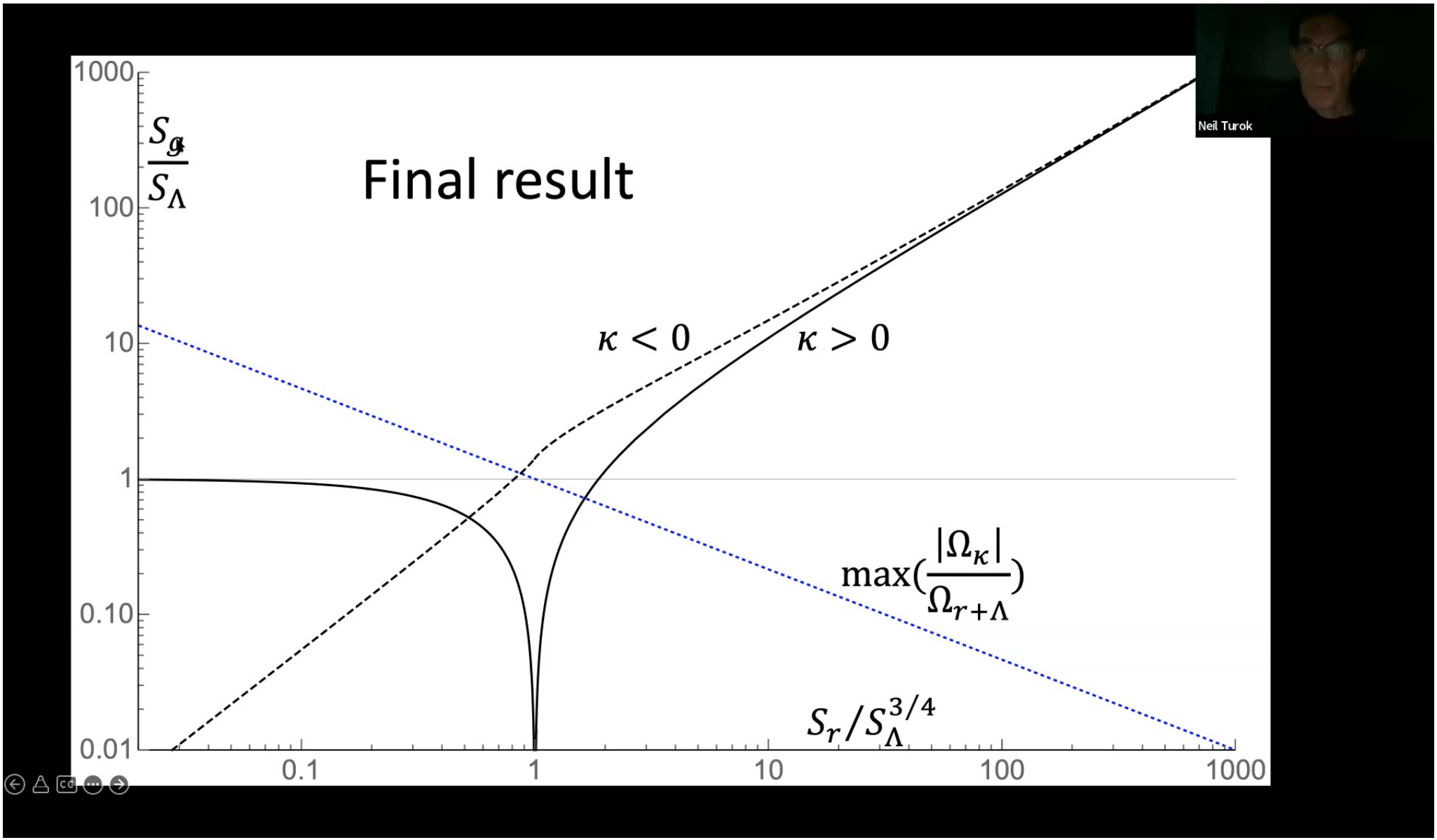
$|a(nt)|$

Fundamental domain is square for flat universe



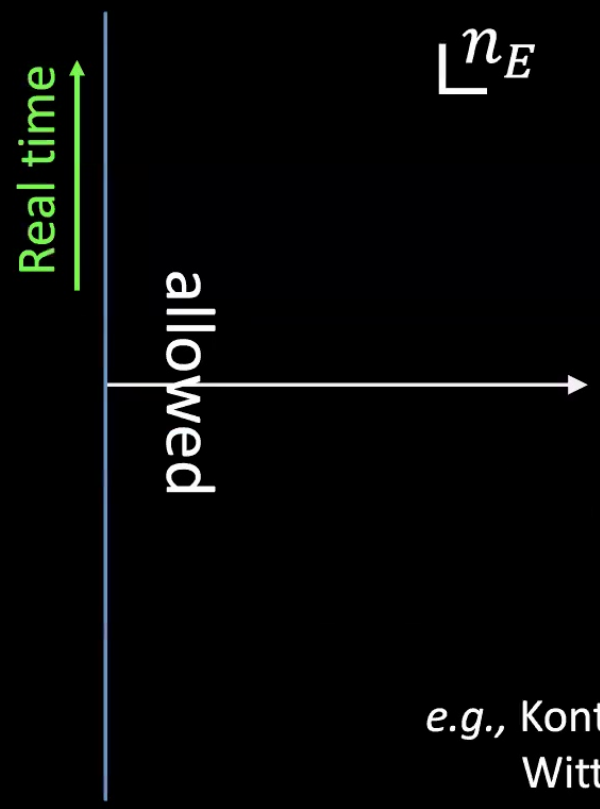
Instanton action is a topological invariant (Cauchy+periodicity)

Neil Turok


Final result
$\frac{S_g}{S_\Lambda}$
1000
100
10
1
0.10
0.01
0.1
1
10
100
1000
$\kappa < 0$
$\kappa > 0$
$\max(\frac{|\Omega_\kappa|}{\Omega_{r+\Lambda}})$
$S_r/S_\Lambda^{3/4}$
Neil Turok

Trad. math physics: Euclidean is fundamental

Neil Turok



n_E

e.g., Kontsevich-Segal 2105.10161
Witten 2111.06514

a better explanation

- the earth is large ($\sim 10^{50}$ atoms)
- gravity, dissipation
- thermodynamics (entropy)

Neil Turok

Consider a radiation plus lambda cosmology:

$$\rho = \rho_r + \rho_\Lambda; \quad \rho_r = \frac{r}{a^4 L_{Pl}^2}, \ \rho_\Lambda = \frac{\lambda}{L_{Pl}^2}; \quad L_{Pl}^2 \equiv 8\pi G$$

Neil Turok

$$R^{(3)} = 6\kappa$$

$$ds^2 = a(nt)^2\left(-n^2 dt^2 + \gamma_{ij} dx^i dx^j\right)$$

Friedmann equation

$$G_{00} = L_{Pl}^2 T_{00}$$

$$\boxed{\dot{a}^2 = \frac{1}{3}\left(r - 3\kappa a^2 + \lambda a^4\right)}$$

$$\left(\Omega_\kappa \equiv -\frac{3\kappa a^2}{3\dot{a}^2}; \ \Omega_r \equiv \frac{r}{3\dot{a}^2}; \ \Omega_\Lambda \equiv \frac{\lambda a^4}{3\dot{a}^2}\right)$$

dimensions

$$[a] = L, [r] = L^2, [\lambda] = L^{-2}$$

$$\dot{a} \equiv \frac{1}{n}\frac{da}{dt}$$

Jacobi elliptic function

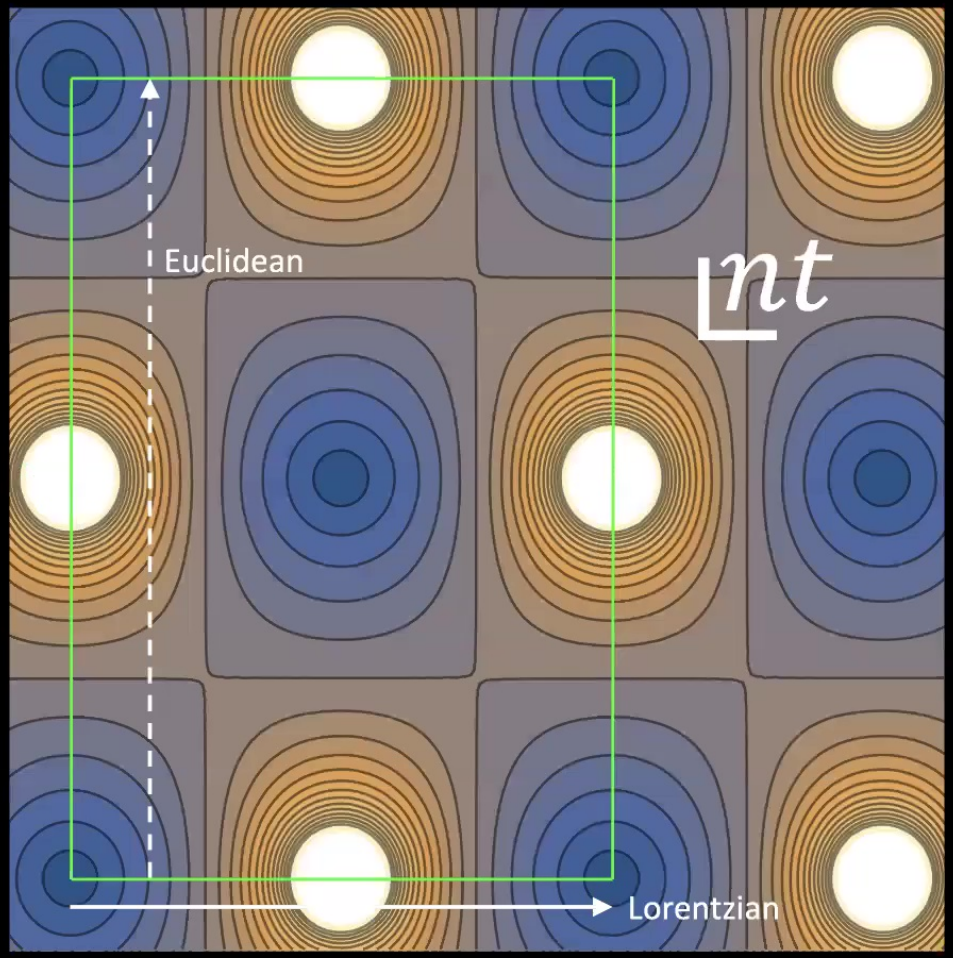
solution $a(nt) = \alpha \, \mathrm{sn}(\beta nt, m)$; $\quad \frac{m}{(1+m)^2} = \frac{\lambda r}{(3\kappa)^2}; \ \alpha = \sqrt{\frac{r(1+m)}{3\kappa}}; \ \beta = \sqrt{\frac{\kappa}{(1+m)}}$

has remarkable analytical properties

(In this talk, I shall generally assume a compact topology: for $\kappa > 0$, a sphere and for $\kappa < 0$, a 3d "tile" of H_3)

$|a(nt)|$

Fundamental domain is square for flat universe



Instanton action is a topological invariant (Cauchy+periodicity)

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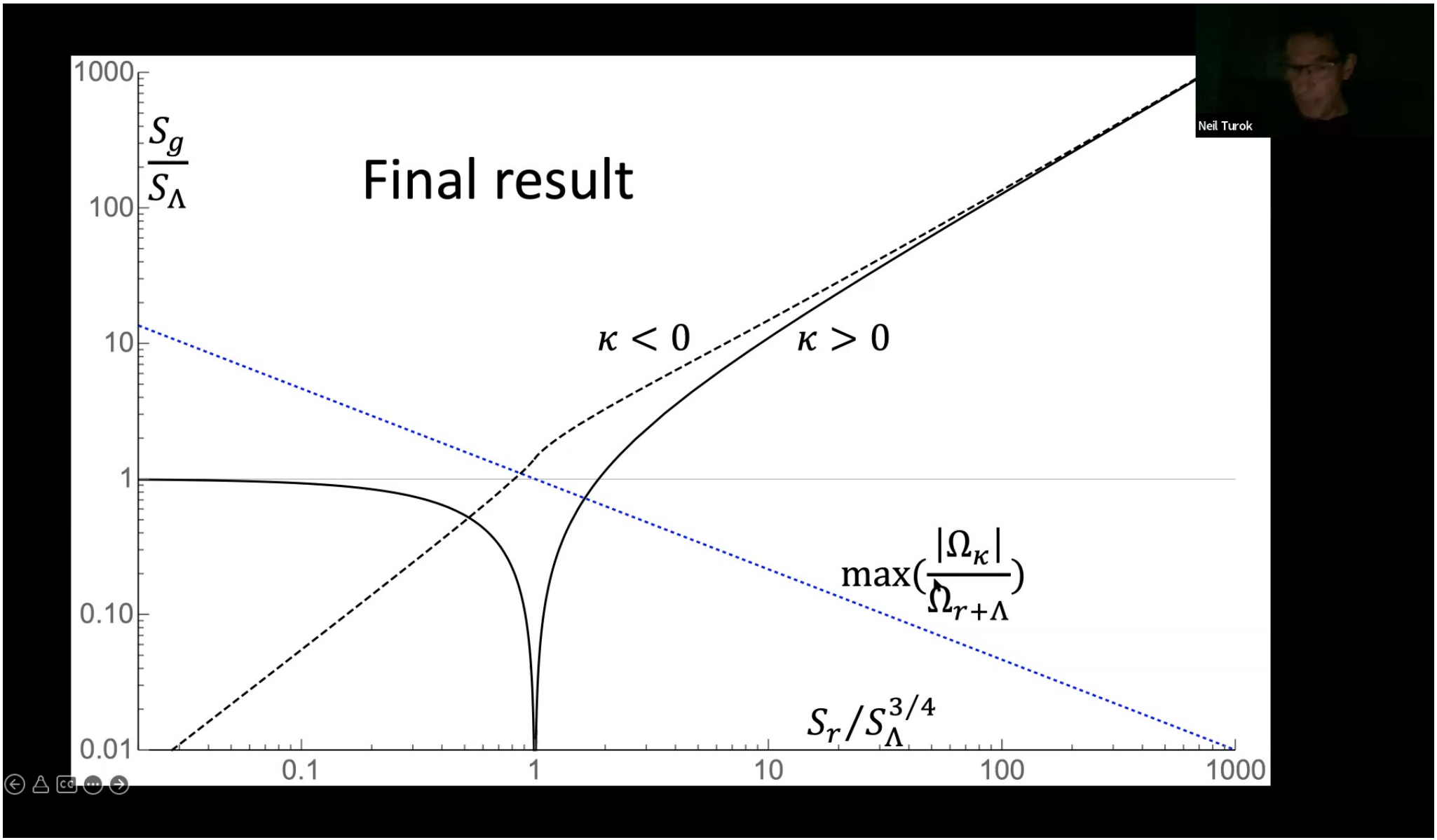
Final result
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$\kappa < 0$
$\kappa > 0$
$\max(\frac{|\Omega_\kappa|}{\Omega_{r+\Lambda}})$
$S_r/S_\Lambda^{3/4}$
Neil Turok

understanding in terms of horizons

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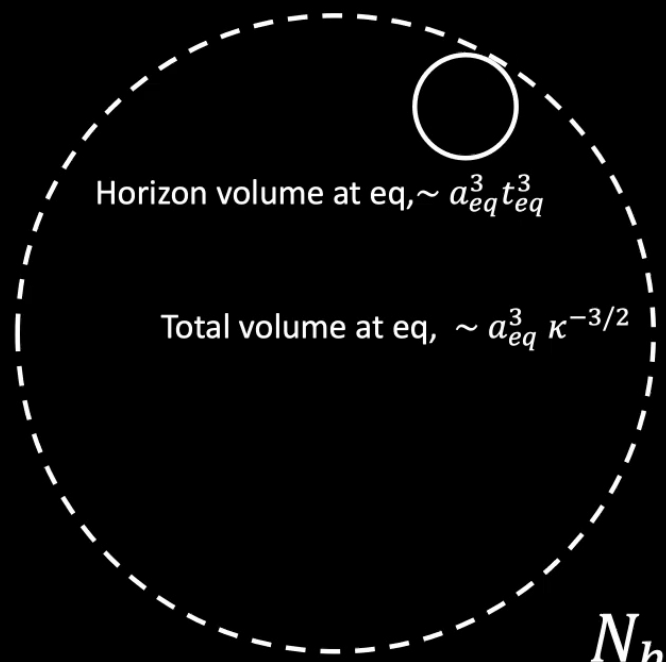
$3\dot{a}^2 = -3\kappa a^2 + r + \lambda a^4$; equal Λ, radiation density at $a_{eq} = (r/\lambda)^{1/4}$

but for $a < a_{eq}$, $a \sim r^{1/2} t$ so $t_{eq} \sim 1/(r\lambda)^{1/4}$



number of horizon volumes at equality

$N_{hor} \sim (\lambda r/\kappa^2)^{3/4}$

multiply by de Sitter entropy λ^{-1}

$$N_{hor}\lambda^{-1} \sim (r/\kappa^2)^{3/4}\, \lambda^{-1/4} \sim S_r S_\Lambda^{1/4}$$

Scalar, tensor and conformal perturbations increase the Euclidean action, decrease the entropy

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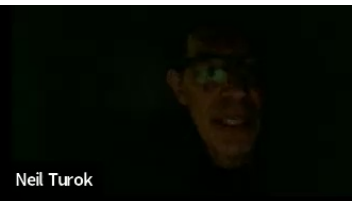
Thus, the most probable universe is flat, homogeneous, isotropic on large scales

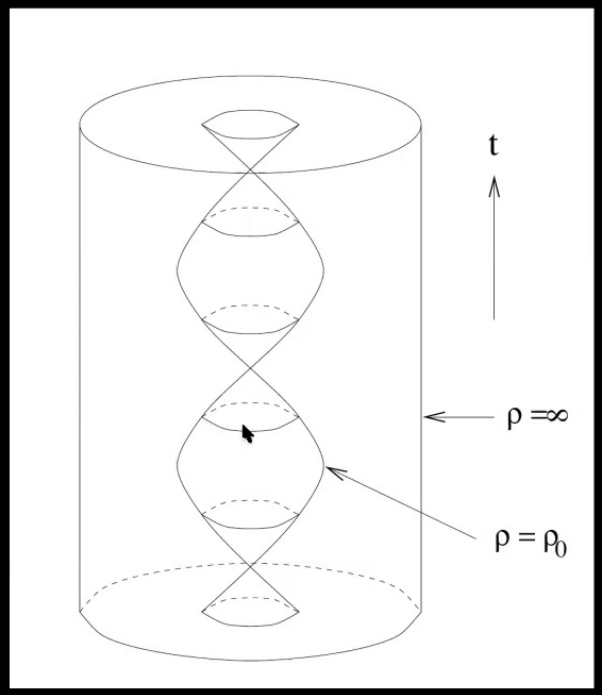
There is no need for any other mechanism to explain these properties

Much remains to be done:

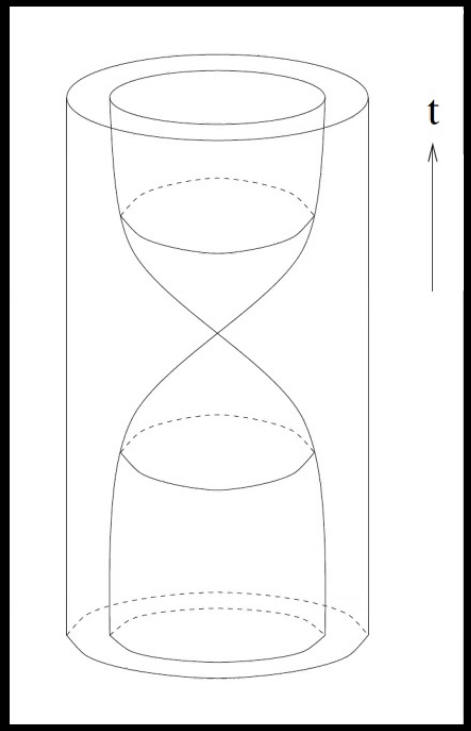
- generalize to non-conformal invariant matter and to include black holes
- explicitly construct the quantum ensemble and show our instantons are the relevant saddles
- use string theory as a 1+1 model cosmology with big bang and a horizon



short strings



long strings



1+1 analog model: string loop in AdS_3 with constant 3-form

w/ Malcolm Perry

See, e.g., Maldacena
hep-th/0001053 [hep-th]
and references therein

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Equations of motion in orthonormal gauge

$$-\ddot{x}^\mu + x''^\mu + \Gamma^\mu_{\nu\lambda}\left(-\dot{x}^\nu\dot{x}^\lambda + x'^\nu x'^\lambda\right) + H^\mu{}_{\nu\lambda}\dot{x}^\nu x'^\lambda = 0;$$

$$H^{\mu\nu\lambda} = H\varepsilon^{\mu\nu\lambda};\ \text{constraints}\ \ \dot{x}^2 + x'^2 = 0,\ \dot{x}x' = 0$$

circular loop

$$x^\mu = (T(\tau), R(\tau)\cos\sigma, R(\tau)\sin\sigma)\,;\quad ds_2^2 = R^2(\tau)(-d\tau^2 + d\sigma^2)$$

$$\Rightarrow\quad -\dot{T}^2(1+R^2) + \dot{R}^2(1+R^2)^{-1} + R^2 = 0\ \ (1)$$

$$-\ddot{T} - \frac{2R}{1+R^2}\dot{R}\dot{T} - \frac{4R}{1+R^2}\dot{R} = 0\ (2)$$

$$(2)\Rightarrow\ \ \dot{T} = -\frac{(2R^2+d)}{1+R^2}\quad (1)\Rightarrow\ \dot{R}^2 - 3R^4 - R^2(4d-1) = d^2$$

$$\boxed{R(\tau) = A\,sn\left(\frac{d}{A}t, m\right);\ A = \frac{d\sqrt{1+m}}{4d-1}}$$

Jacobi elliptic function, again!

Neil Turok
Thank You!