Title: Causal Operations in Quantum Field Theory

Speakers: Ian Jubb

Series: Quantum Foundations

Date: April 08, 2022 - 2:00 PM

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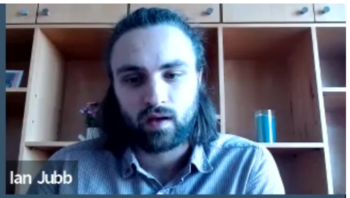
Abstract: While Quantum Field Theory is the most accurate theory we have for predicting the microscopic world, there are still open problems regarding its mathematical description. In particular, the usual quantum mechanical description of measurements, unitary kicks, and other local operations has the potential to produce pathological causality violations. Not all local operations lead to such violations, but any that do cannot be physically realisable. It is an open question whether a given local operation in the theory respects causality, and hence whether a given local operation is physical. In this talk I will work toward a general condition that distinguishes causal and acausal local operations.

Zoom Link: https://pitp.zoom.us/j/98089863001?pwd=K2RWL2lNWFd4VDZYd013eUN3alNmQT09



Causal Operations in Quantum Field Theory

Ian Jubb
DIAS

Borsten, Kells, IJ, *arxiv:1912.06141* , IJ, *arxiv:2106.09027*

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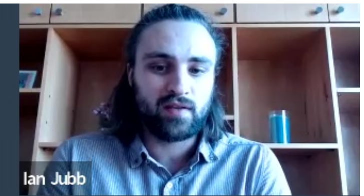
Introduction



- Initial question: I*s the textbook description of measurement in quantum theory, in terms of states, operators, and the projection postulate, consistent with relativity in the setting of QFT?*
- QFT is a relativistic theory, in the sense that it makes accurate predictions in systems where relativity is very relevant.
- These predictions come from scattering probabilities in the theory, which are approximations to the real system, e.g. the state is prepared in the infinite past, and measured in the infinite future, and measured across all of space.
- *What if we ask more of QFT? Can we use the textbook description of measurement to describe multiple (spatially and temporally finite) measurements in the same background spacetime?*
- Can go down the route of measurement models, e.g. Unruh-DeWitt detectors or other probe fields. In any model, tracing out any auxiliary systems gives rise to some *update map* for the main system of interest.
- Here I will be concerned with the physically possible update maps; those that could, at least in principle, arise from some physical operation.

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Plan

- **Setup and background**
  - Quantum Field Theory
  - Local Operations
- **Causal Operations**
  - Sorkin’s Scenario
  - Causality condition
- **Discussion**

Quantum Field Theory

- Real scalar field theory in Minkowski spacetime
- Field operator-valued distribution:

$$\phi(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}} e^{ip.x} + a^\dagger_{\vec{p}} e^{-ip.x} \right)$$

$$p.x = -E_{\vec{p}}t + \vec{p}.\vec{x}$$

$$E_{\vec{p}} = \sqrt{\vec{p}.\vec{p} + m^2}$$

$$\left[a_{\vec{p}}, a^\dagger_{\vec{q}}\right] = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{q})$$

- Vacuum state: $|0\rangle$
- *Non-normalisable* single particle states: $|\vec{p}\rangle = a^\dagger_{\vec{p}}|0\rangle$

$$\langle\vec{p}|\vec{q}\rangle = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})$$

- Single particle Hilbert space, H, consists of states like: $|\varphi\rangle = \int \frac{d^3p}{(2\pi)^3}\varphi(\vec{p})|\vec{p}\rangle \quad , \quad \varphi(\vec{p}) \in L^2(\mathbb{R}^3)$
- Bosonic Fock space: $F = \oplus_{n=0}^{\infty} S\left(H^{\otimes n}\right)$
- Self-adjoint operators, $A^\dagger = A$, on F correspond to observable quantities
- $\langle\Psi|A|\Psi\rangle$ or $\mathrm{tr}(\rho A)$ interpreted as expectation value of observable

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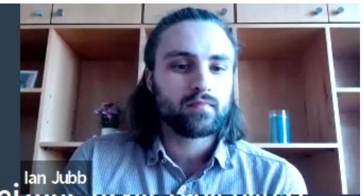
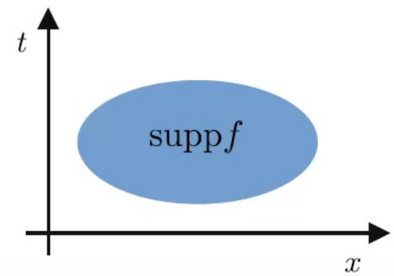
Quantum Field Theory



Fewster, Rejzner, arXiv.1904.04051

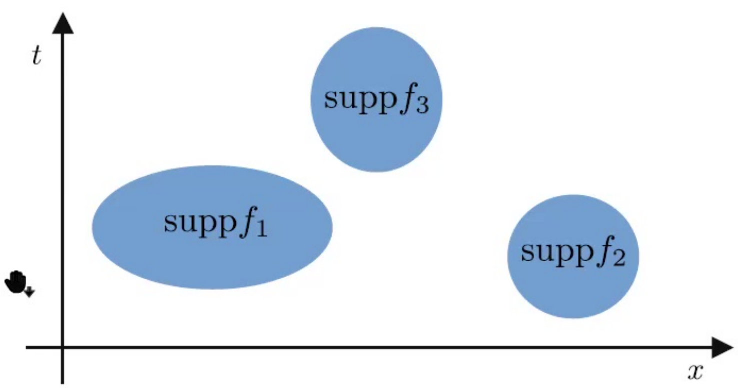
- Integrate, or *smear*, $\phi(t,\vec{x})$ with a test function $f(t,\vec{x})$ to get an operator on F
- *Smeared field operator*:

$$\phi(f) = \int d^4x\, f(t,\vec{x})\phi(t,\vec{x})$$

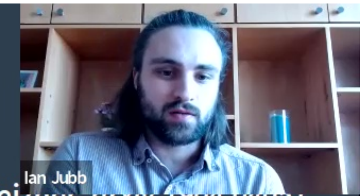


- Example action: $$\phi(f)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\tilde{f}(E_{\vec{p}},\vec{p})^*|\vec{p}\rangle$$
- Algebra, $\mathfrak{A}$, corresponds to complex sums of products of smeared fields and the identity I, e.g.

$$\phi(f_1)\phi(f_2) + i\phi(f_3)^3 + 4I$$



Quantum Field Theory



Fewster, Rejzner, arXiv:1904.04051

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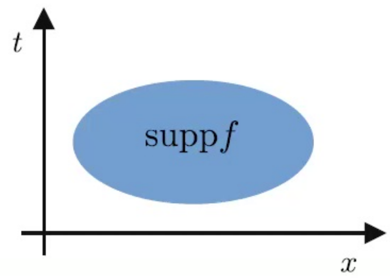
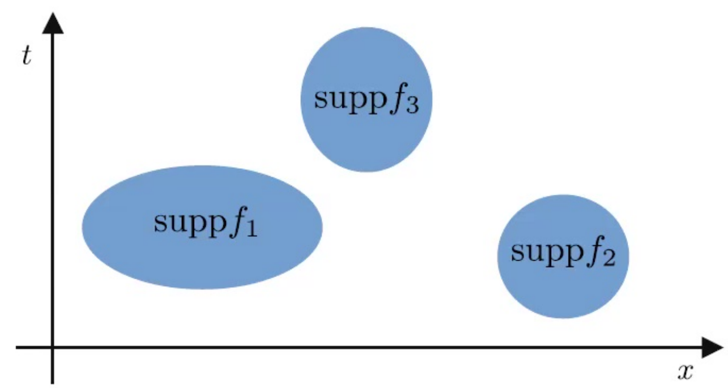


- Example action: $\phi(f)|0\rangle = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\tilde{f}(E_{\vec{p}},\vec{p})^*|\vec{p}\rangle$
- Algebra, $\mathfrak{A}$, corresponds to complex sums of products of smeared fields and the identity I, e.g.

$$\phi(f_1)\phi(f_2) + i\phi(f_3)^3 + 4I$$

- Can also consider functions of such operators, e.g.

$$e^{i\phi(f)}$$



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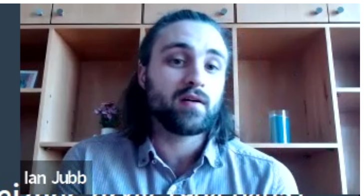
Quantum Field Theory



Fewster, Rejzner, *arXiv:1904.04051*

- **Commutation relations for smeared fields:**

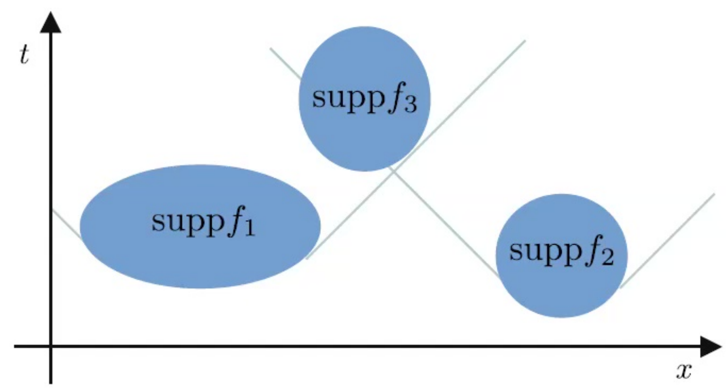
$$[\phi(f),\phi(g)] = \int d^4x d^4x' f(t,\vec{x})g(t',\vec{x}')[\phi(t,\vec{x}),\phi(t',\vec{x}')]$$
$$= i\int d^4x d^4x' f(t,\vec{x})g(t',\vec{x}')\Delta(t,\vec{x},t',\vec{x}')$$
$$= i\Delta(f,g)$$

$$[\phi(t,\vec{x}),\phi(t',\vec{x}')] = i\Delta(t,\vec{x},t',\vec{x}')$$

Pauli-Jordan function (difference of retarded and advanced Green functions) vanishes for *spacelike* points

$[\phi(f_1),\phi(f_2)] = 0$ **as supports spacelike**

$[\phi(f_{1,2}),\phi(f_3)] \neq 0$ **as supports timelike**



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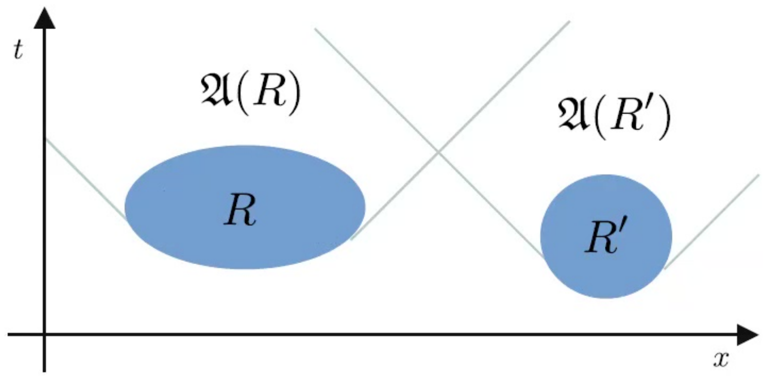
Quantum Field Theory

Fewster, Rejzner, *arxiv:1904.04051*

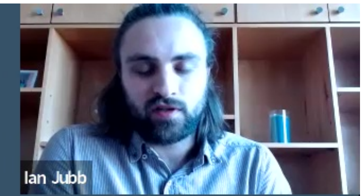
- For each subregion *R*, can form the associated subalgebra:



$$[A, A'] = 0 \ , \ \ \forall\, A \in \mathfrak{A}(R) \ , \ \forall\, A' \in \mathfrak{A}(R')$$

- ***Einstein Causality***: subalgebras associated to spacelike regions commute:

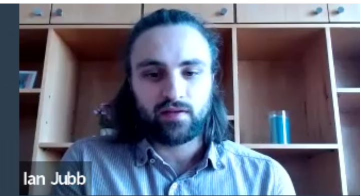
$$\boxed{[\mathfrak{A}(R), \mathfrak{A}(R')] = 0}$$



Setup and Background

Local Operations

Local Operations



- Example: consider the ideal measurement associated with some diagonalisable self-adjoint operator:

$$A \in \mathfrak{A}(R) \ , \ A^\dagger = A \ , \ A = \sum_n \lambda_n P_n$$

- *Non-selective ideal measurement* amounts to the state-update:

$$\rho \mapsto \tilde{\mathcal{E}}^0_A(\rho) = \sum_n P_n \rho P_n$$

- Updated state is useful for calculating expectation values of other operators: $\mathrm{tr}(\tilde{\mathcal{E}}^0_A(\rho)B) = \mathrm{tr}(\rho \mathcal{E}^0_A(B))$
- We can instead focus on the dual map which updates operators:

$$B \mapsto \mathcal{E}^0_A(B) = \sum_n P_n B P_n$$

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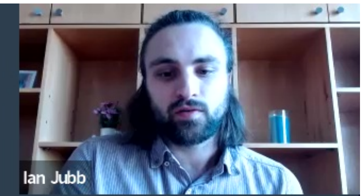
Local Operations

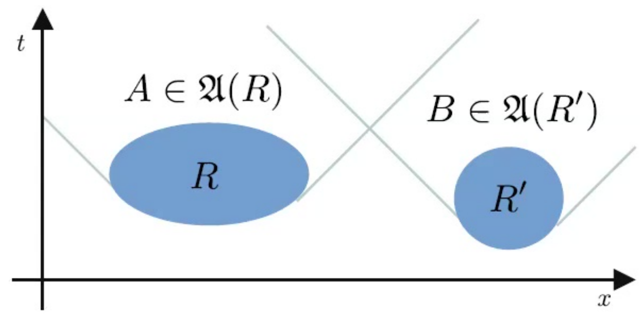


- **Example: consider the *ideal measurement* associated with some diagonalisable self-adjoint operator:**

$$A \in \mathfrak{A}(R) \ , \ \ A^\dagger = A \ , \ \ A = \sum_n \lambda_n P_n \ , \ \ B \mapsto \mathcal{E}^0_A(B) = \sum_n P_n B P_n$$

- **Spacelike case:** $[A, B] = 0 \ , \ \ [P_n, B] = 0$

$$\mathcal{E}^0_A(B) = \sum_n P_n B P_n = \sum_n P_n B = B$$



- **In general, for any spacelike region *R'*,**

$$\boxed{\mathcal{E}^0_A(\cdot)|_{\mathfrak{A}(R')} = 1}$$

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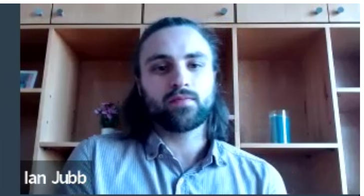
Local Operations

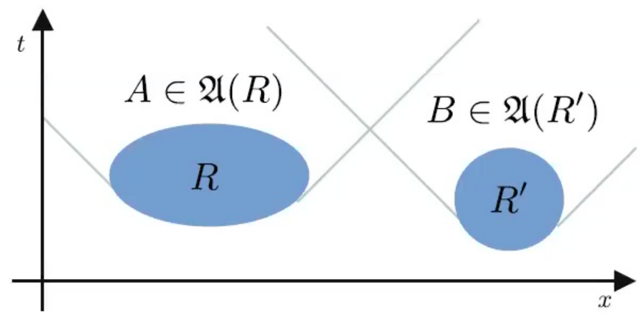


- Example: *unitary kick* with some self-adjoint operator $A \in \mathfrak{A}(R)$, $A^\dagger = A$

$$B \mapsto \mathcal{U}_A(B) = e^{iA} B e^{-iA}$$

- If operators are localisable in spacelike regions, then $[A, B] = 0$

  and $\mathcal{U}_A(B) = e^{iA} B e^{-iA} = B$



- In general, for any spacelike region *R'*,

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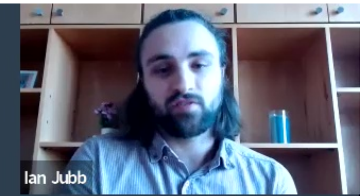
Local Operations

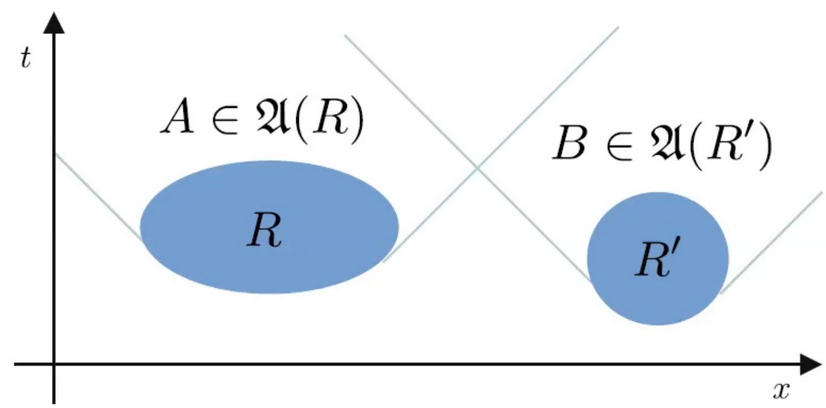


- Example: *weak measurement* of some self-adjoint operator $A \in \mathfrak{A}(R)$, $A^\dagger = A$

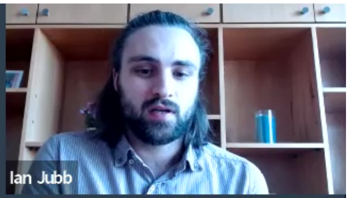
$$B \mapsto \mathcal{W}^\sigma_A(B) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}} d\alpha \, e^{-\frac{(A-\alpha)^2}{4\sigma^2}} B e^{-\frac{(A-\alpha)^2}{4\sigma^2}}$$

- Again,

$$\mathcal{W}^\sigma_A(\cdot)|_{\mathfrak{A}(R')} = 1$$



Local Operations



Definition. (local)

$\mathcal{E}(I) = I$

A completely positive trace-preserving map is *local* to a region *R* if, for any spacelike region *R'*,

$$\mathcal{E}_R(\cdot)|_{\mathfrak{A}(R')} = 1$$

- *Einstein causality* ensures that any map, $\mathcal{E}_A(\cdot)$, constructed from a local operator, $A \in \mathfrak{A}(R)$, in some functional way, e.g. $\mathcal{E}_A^0(\cdot)$, $\mathcal{U}_A(\cdot)$, $\mathcal{W}_A^\sigma(\cdot)$, is local to *R*
- This locality condition ensures that expectation values, and probability distributions, associated with spacelike operators are unchanged, e.g.

$$\mathrm{tr}(\rho \mathcal{E}_R(B)) = \mathrm{tr}(\rho B) \ , \ \ \forall\, B \in \mathfrak{A}(R')$$

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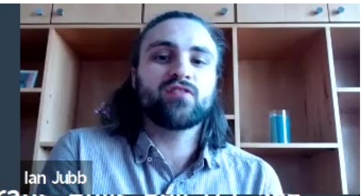
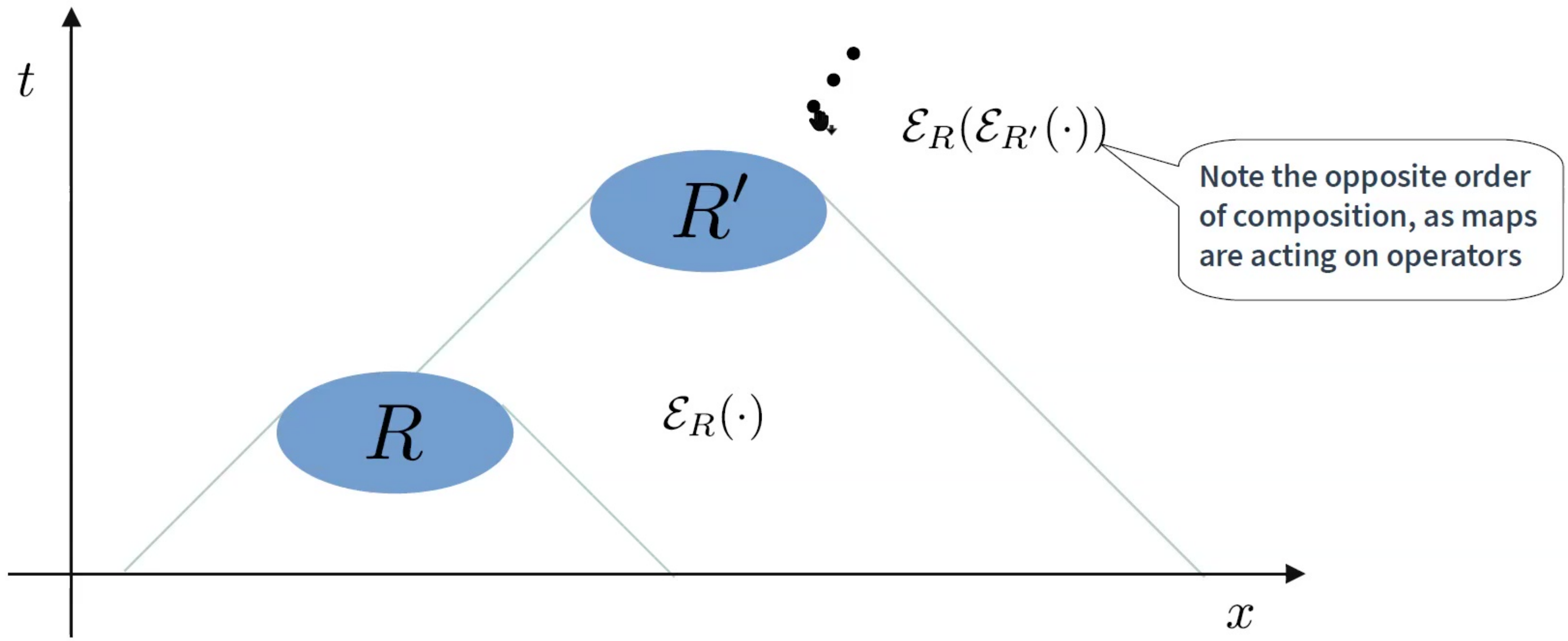
Local Operations

Hellwig, Kraus, *Phys. Rev. D* 1, 566



- Multiple local operations:



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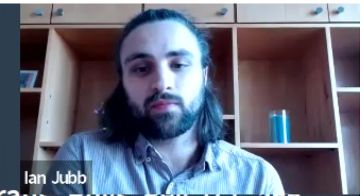
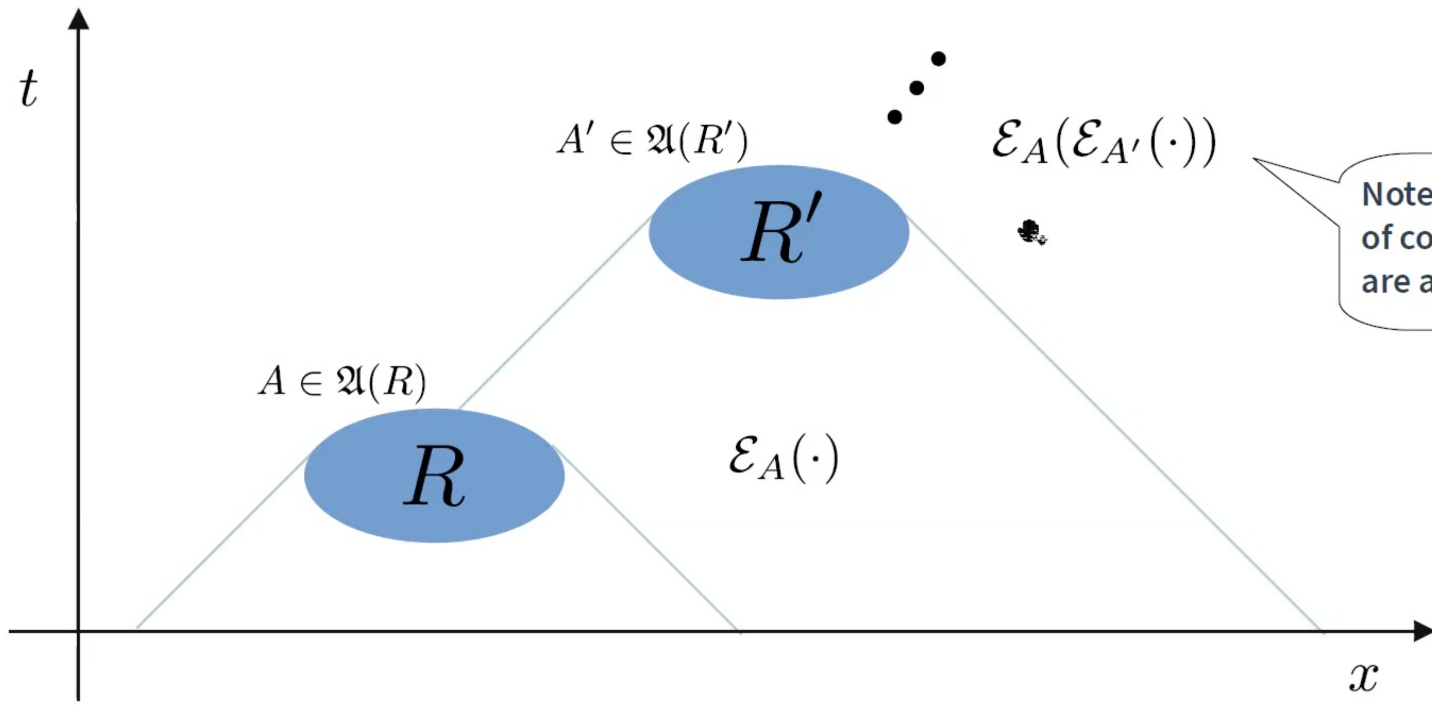


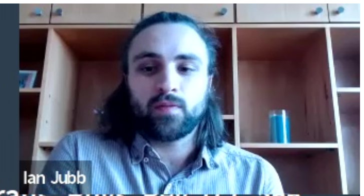
Hellwig, Kraus, *Phys. Rev. D* 1, 566

- Multiple local operations associated to local operators:



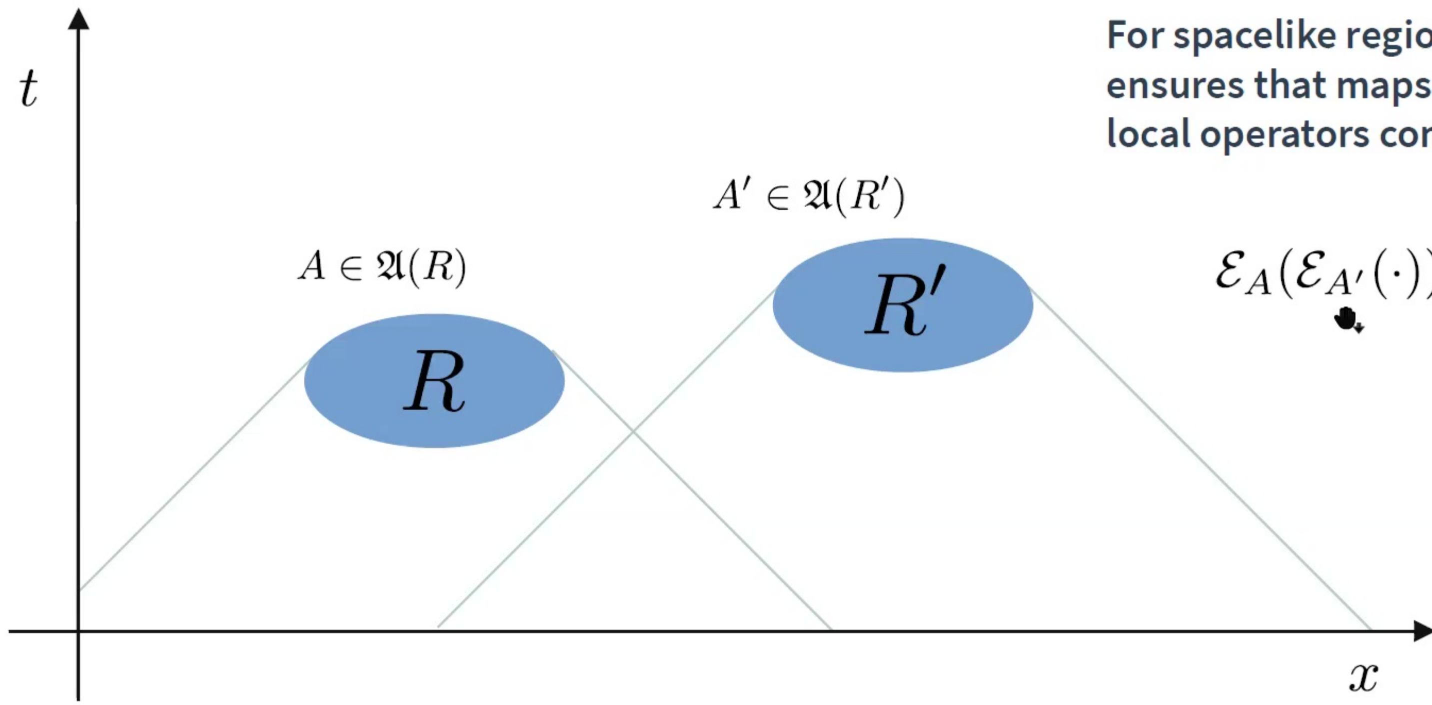
Note the opposite order of composition, as maps are acting on operators

Local Operations



Hellwig, Kraus, *Phys. Rev. D* 1, 566

- Multiple local operations associated to local operators:



For spacelike regions, *Einstein causality* ensures that maps constructed from local operators commute, i.e.

$$\mathcal{E}_A(\mathcal{E}_{A'}(\cdot)) = \mathcal{E}_{A'}(\mathcal{E}_A(\cdot))$$

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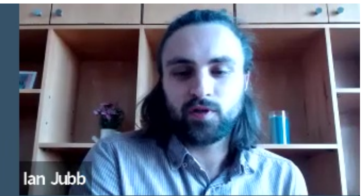
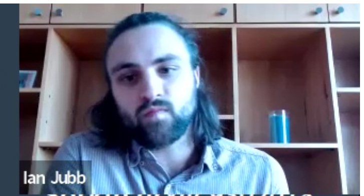


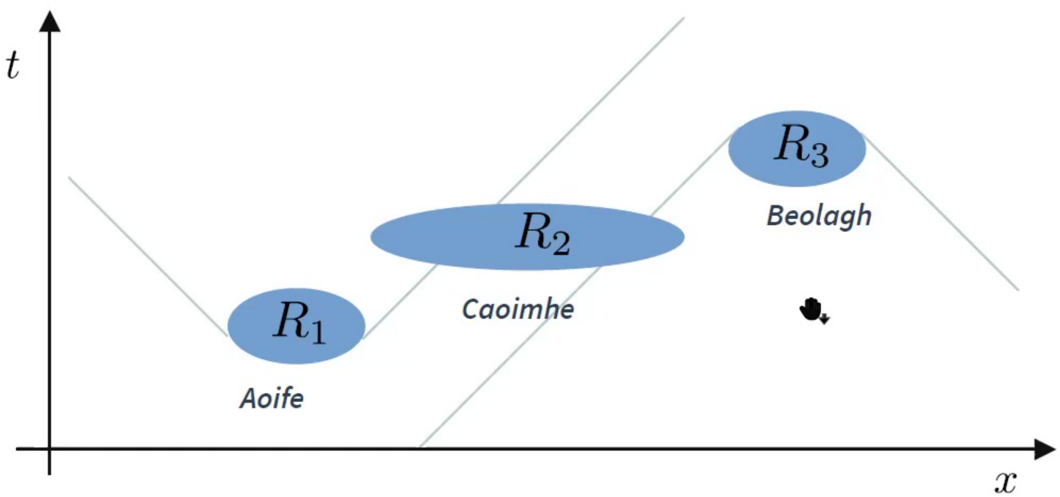
Causal Operations

Sorkin's Scenario



Sorkin's Scenario

- Consider 3 agents, Aoife, Caoimhe, and Beolagh, acting in their respective regions:



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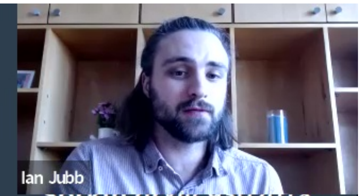
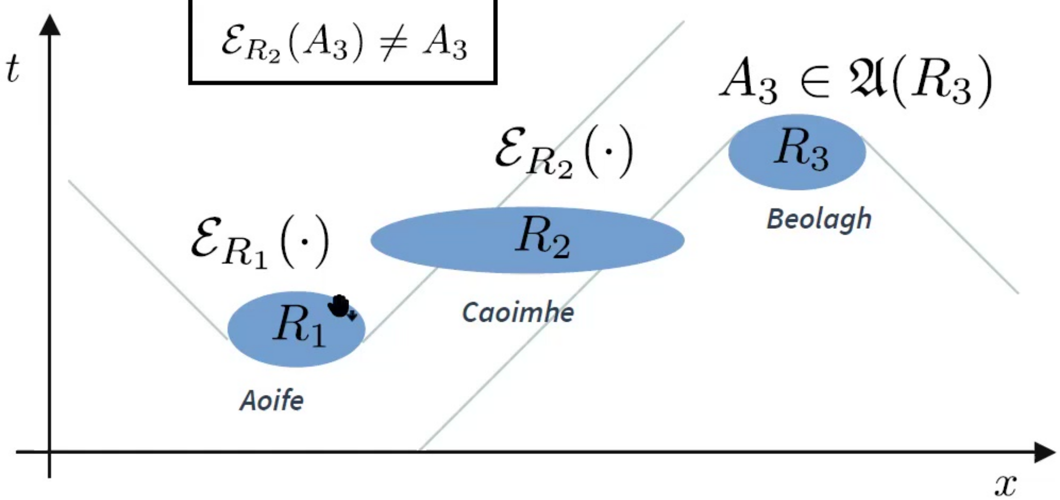
Sorkin's Scenario



Sorkin arXiv:9302018

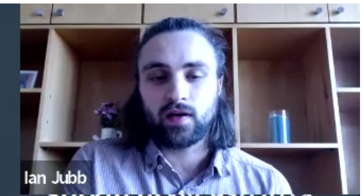
- Consider 3 agents, Aoife, Caoimhe, and Beolagh, acting in their respective regions:

$$\mathcal{E}_{R_1}(A_3) = A_3$$
$$\mathcal{E}_{R_2}(A_3) \neq A_3$$



1) Aoife performs local operation described by the update map:
$$\mathcal{E}_{R_1}(\cdot)$$
2) Caoimhe performs local operation described by the update map:
$$\mathcal{E}_{R_2}(\cdot)$$
3) Beolagh measures the expectation value of some local self-adjoint operator.

Sorkin's Scenario



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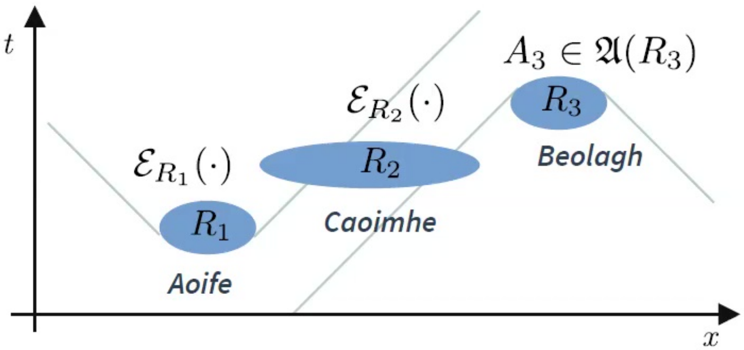

1) Aoife performs local operation described by the update map: $\mathcal{E}_{R_1}(\cdot)$
2) Caoimhe performs local operation described by the update map: $\mathcal{E}_{R_2}(\cdot)$
3) Beolagh measures the expectation value of some local self-adjoint operator.

- Composition rule says this expectation value given by

$$\mathrm{tr}(\rho\,\mathcal{E}_{R_1}(\mathcal{E}_{R_2}(A_3)))$$

- BUT, Aoife is causally disconnected from Beolagh, and hence Beolagh's expectation value should be the same as if Aoife was not there, i.e.

$$\mathrm{tr}(\rho\,\mathcal{E}_{R_2}(A_3))$$

- As an operator equation we want:

$$\mathcal{E}_{R_1}(\mathcal{E}_{R_2}(A_3)) = \mathcal{E}_{R_2}(A_3)$$

whenever Aoife's region is spacelike to Beolagh's

- Einstein Causality *does not* ensure this

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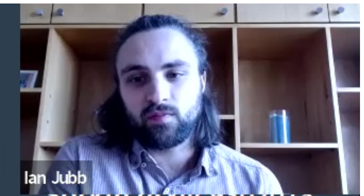
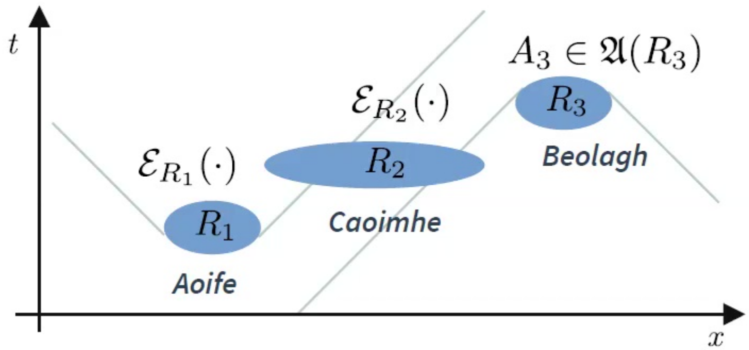
Sorkin's Scenario



Sorkin arXiv:9302018

- Consider 3 agents, Aoife, Caoimhe, and Beolagh, acting in their respective regions:



1) Aoife performs local operation described by the update map: $\mathcal{E}_{R_1}(\cdot)$
2) Caoimhe performs local operation described by the update map: $\mathcal{E}_{R_2}(\cdot)$
3) Beolagh measures the expectation value of some local self-adjoint operator.

- Note, if there is some operator that Beolagh can measure such that

$$\mathcal{E}_{R_1}(\mathcal{E}_{R_2}(A_3)) \neq \mathcal{E}_{R_2}(A_3)$$

then for some state we get

$$\mathrm{tr}(\rho\, \mathcal{E}_{R_1}(\mathcal{E}_{R_2}(A_3))) \neq \mathrm{tr}(\rho\, \mathcal{E}_{R_2}(A_3))$$

- That is, Beolagh can tell (to within some confidence level) whether Aoife has acted or not based on differences in their measured expectation value, i.e. Aoife can signal Beolagh
- This clearly cannot be possible, since such a signal would have to travel faster than light!

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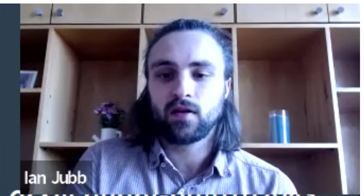
Sorkin's Scenario



Beckman et al *arxiv:quant-ph/0102043*

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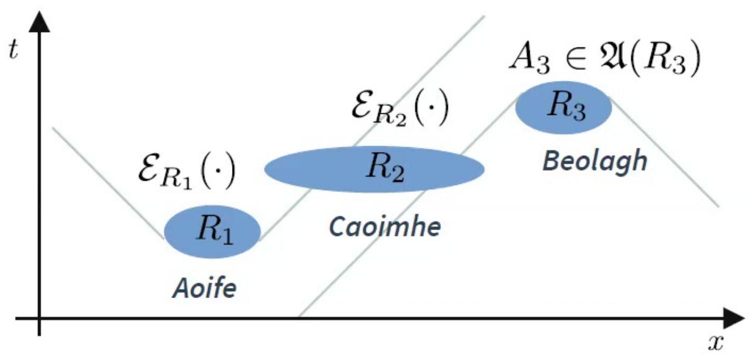

1) Aoife performs local operation described by the update map: $\mathcal{E}_{R_1}(\cdot)$
2) Caoimhe performs local operation described by the update map: $\mathcal{E}_{R_2}(\cdot)$
3) Beolagh measures the expectation value of some local self-adjoint operator.

- Compare to systems in Quantum Information, e.g. some finite dimensional bipartite Hilbert space:

$$H = H_A \otimes H_B$$

- Aoife's region like *part A* of bipartite Hilbert space
- Beolagh's region like *part B* of bipartite Hilbert space
- Caoimhe acts on both parts of the Hilbert space. We're then asking if her actions enable a signal from Aoife to Beolagh
- If they do, they cannot be implemented faster than the light travel time from *part A* to *part B*
- In the relativistic setting of QFT, the spacetime locations of any actions are 'baked in'. Thus, any signal in this setup is superluminal, in which case Caoimhe's map cannot be implemented at all

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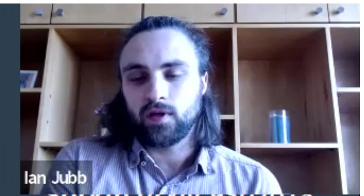
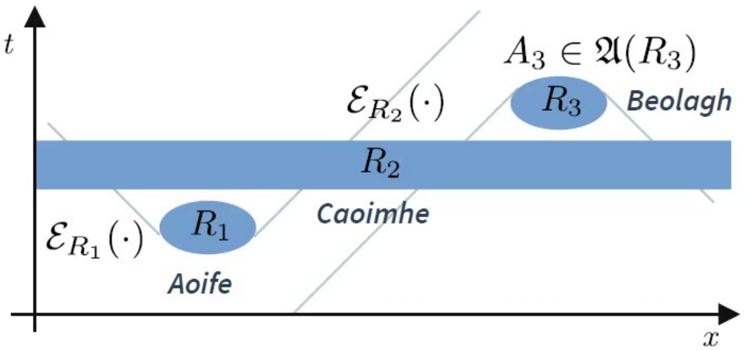
Sorkin's Scenario



Sorkin arXiv:9302018

- Consider 3 agents, Aoife, Caoimhe, and Beolagh, acting in their respective regions:



1) Aoife performs local operation described by the update map: $\mathcal{E}_{R_1}(\cdot)$
2) Caoimhe performs local operation described by the update map: $\mathcal{E}_{R_2}(\cdot)$
3) Beolagh measures the expectation value of some local self-adjoint operator.

- Note, if there is some operator that Beolagh can measure such that

$$\mathcal{E}_{R_?}(\mathcal{E}_{R_?}(A_3)) \neq \mathcal{E}_{R_?}(A_3)$$

We can also ask whether Caoimhe's map enables a signal in the case where it is not even local

$$\mathcal{E}_{R_1}(\mathcal{E}_{R_2}(A_3)) \overset{?}{=} \mathcal{E}_{R_2}(A_3)$$

- That ... fidence level) ... on differences in their measured expectation value, i.e. Aoife can signal Beolagh

- This clearly cannot be possible, since such a signal would have to travel faster than light!

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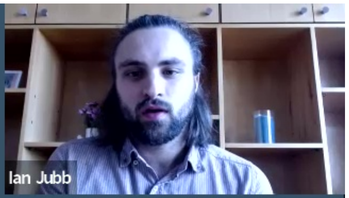

Causal Operations

Examples

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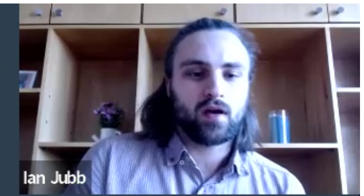
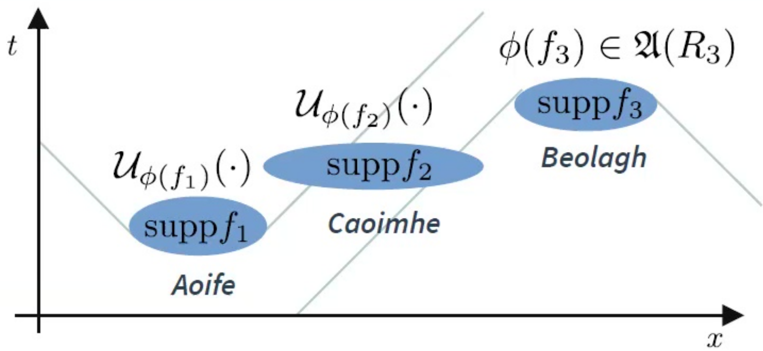
Examples (causal)



- Start in the vacuum state
  1) Aoife performs local operation described by the update map: $\mathcal{U}_{\phi(f_1)}(\cdot) = e^{i\phi(f_1)}(\cdot)e^{-i\phi(f_1)}$
  2) Caoimhe performs local operation described by the update map: $\mathcal{U}_{\phi(f_2)}(\cdot) = e^{i\phi(f_2)}(\cdot)e^{-i\phi(f_2)}$
  3) Beolagh measures the expectation value of some local smeared field operator.



- Beolagh's expectation value:

$$\langle 0|\mathcal{U}_{\phi(f_1)}(\mathcal{U}_{\phi(f_2)}(\phi(f_3)))|0\rangle = \langle 0|e^{i\phi(f_1)}e^{i\phi(f_2)}\phi(f_3)e^{-i\phi(f_2)}e^{-i\phi(f_1)}|0\rangle$$

$$= \langle 0|e^{i\phi(f_1)}\left(\phi(f_3) + \Delta(f_3, f_2)\right)e^{-i\phi(f_1)}|0\rangle$$

$$= \langle 0|\phi(f_3) + \Delta(f_3, f_2)|0\rangle$$

$$= \Delta(f_3, f_2)$$

$[\phi(f_3), \phi(f_2)] = i\Delta(f_3, f_2)$

$[\phi(f_3), \phi(f_1)] = 0$

$\langle 0|\phi(f)|0\rangle = 0$

No dependence on Aoife's field

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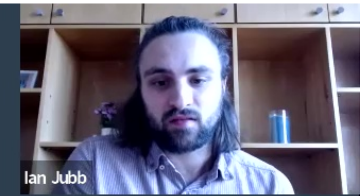
Examples (acausal)



- Start in the vacuum state
  1) Aoife performs local operation described by the update map: $\mathcal{U}_{\phi(f_1)}(\cdot) = e^{i\phi(f_1)}(\cdot)e^{-i\phi(f_1)}$
  2) Caoimhe performs local operation described by the update map: $\mathcal{U}_{\phi(f_2)^2}(\cdot) = e^{i\phi(f_2)^2}(\cdot)e^{-i\phi(f_2)^2}$
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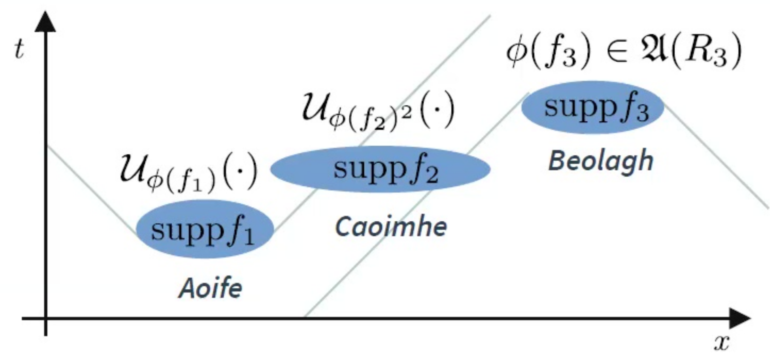

- Beolagh's expectation value:

$$\begin{aligned}\langle 0|\mathcal{U}_{\phi(f_1)}(\mathcal{U}_{\phi(f_2)^2}(\phi(f_3)))|0\rangle &= \langle 0|e^{i\phi(f_1)}e^{i\phi(f_2)^2}\phi(f_3)e^{-i\phi(f_2)^2}e^{-i\phi(f_1)}|0\rangle \\ &= \langle 0|e^{i\phi(f_1)}\left(\phi(f_3) - 2\Delta(f_2,f_3)\phi(f_2)\right)e^{-i\phi(f_1)}|0\rangle \\ &= -2\Delta(f_2,f_3)\langle 0|\phi(f_2) - \Delta(f_1,f_2)|0\rangle\end{aligned}$$

$[\phi(f_3), \phi(f_2)] = i\Delta(f_3, f_2)$

$[\phi(f_3), \phi(f_1)] = 0$

$[\phi(f_1), \phi(f_2)] \neq 0$

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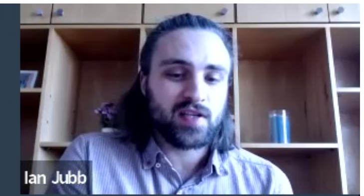
Examples (acausal)



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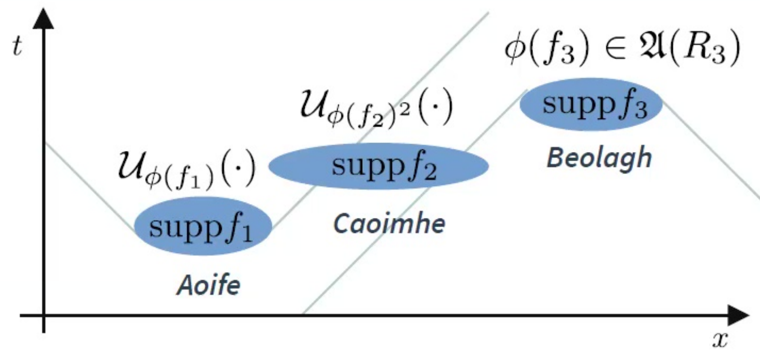

- Beolagh's expectation value:

$$
\begin{aligned}
\langle 0|\mathcal{U}_{\phi(f_1)}(\mathcal{U}_{\phi(f_2)^2}(\phi(f_3)))|0\rangle &= \langle 0|e^{i\phi(f_1)}e^{i\phi(f_2)^2}\phi(f_3)e^{-i\phi(f_2)^2}e^{-i\phi(f_1)}|0\rangle \\
&= \langle 0|e^{i\phi(f_1)}\left(\phi(f_3) - 2\Delta(f_2,f_3)\phi(f_2)\right)e^{-i\phi(f_1)}|0\rangle \\
&= -2\Delta(f_2,f_3)\langle 0|\phi(f_2) - \Delta(f_1,f_2)|0\rangle \\
&= 2\Delta(f_1,f_2)\Delta(f_2,f_3)
\end{aligned}
$$

$[\phi(f_3), \phi(f_2)] = i\Delta(f_3, f_2)$

$[\phi(f_3), \phi(f_1)] = 0$

$[\phi(f_1), \phi(f_2)] \neq 0$

Examples (acausal)

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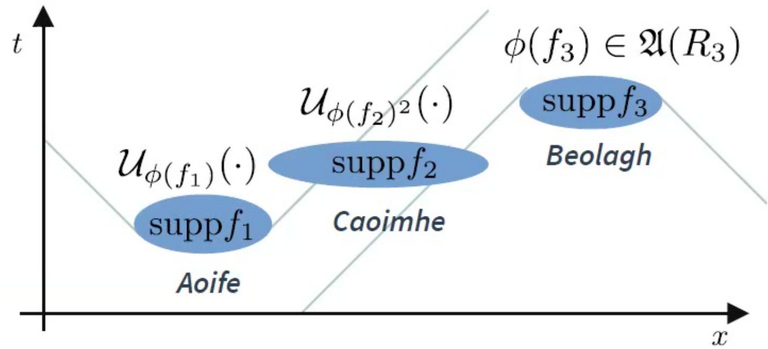

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$[\phi(f_3), \phi(f_2)] = i\Delta(f_3, f_2)$

$[\phi(f_3), \phi(f_1)] = 0$

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Dependence on Aoife's field

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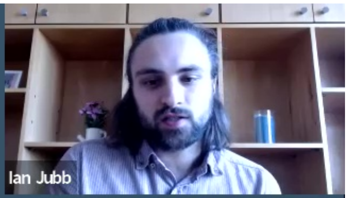
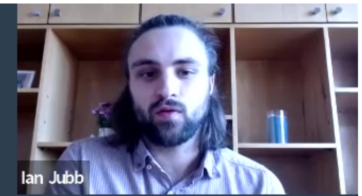


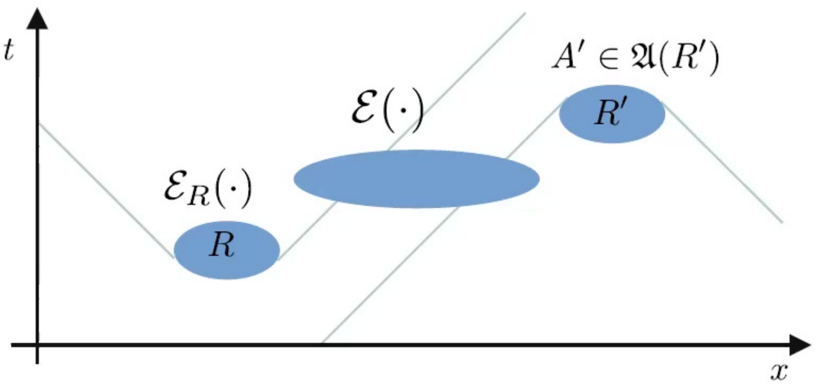
Causal Operations

Causality condition

Causality condition



- Formalise what we have seen:



Definition. (causal w.r.t)

A completely positive trace-preserving map, $\mathcal{E}(\cdot)$, is *causal with respect to* a map $\mathcal{E}_R(\cdot)$ (local to R) if, for all R' spacelike to R, and all $A' \in \mathfrak{A}(R')$, then

$$\mathcal{E}_R(\mathcal{E}(A')) = \mathcal{E}(A')$$

Without reference to any $A' \in \mathfrak{A}(R')$ we could write

$$\mathcal{E}_R(\mathcal{E}(\cdot))|_{\mathfrak{A}(R')} = \mathcal{E}(\cdot)$$

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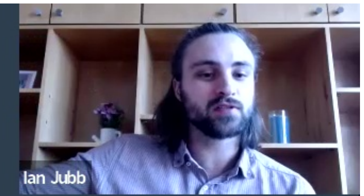
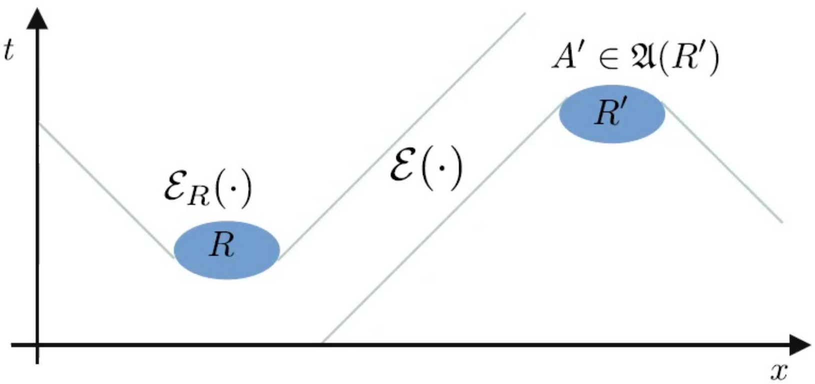
Causality condition



- Remove dependence on specific map local to *R*



Definition. (causal)

A completely positive trace-preserving map, $\mathcal{E}(\cdot)$, is *causal* if, for all regions R, it is causal w.r.t. to all maps local to R, i.e.

$$\mathcal{E}_R(\mathcal{E}(\cdot))|_{\mathfrak{A}(R')} = \mathcal{E}(\cdot)$$

for all R and all completely positive, trace-preserving maps, $\mathcal{E}_R(\cdot)$, local to R.

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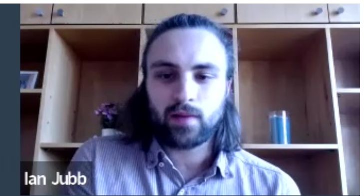
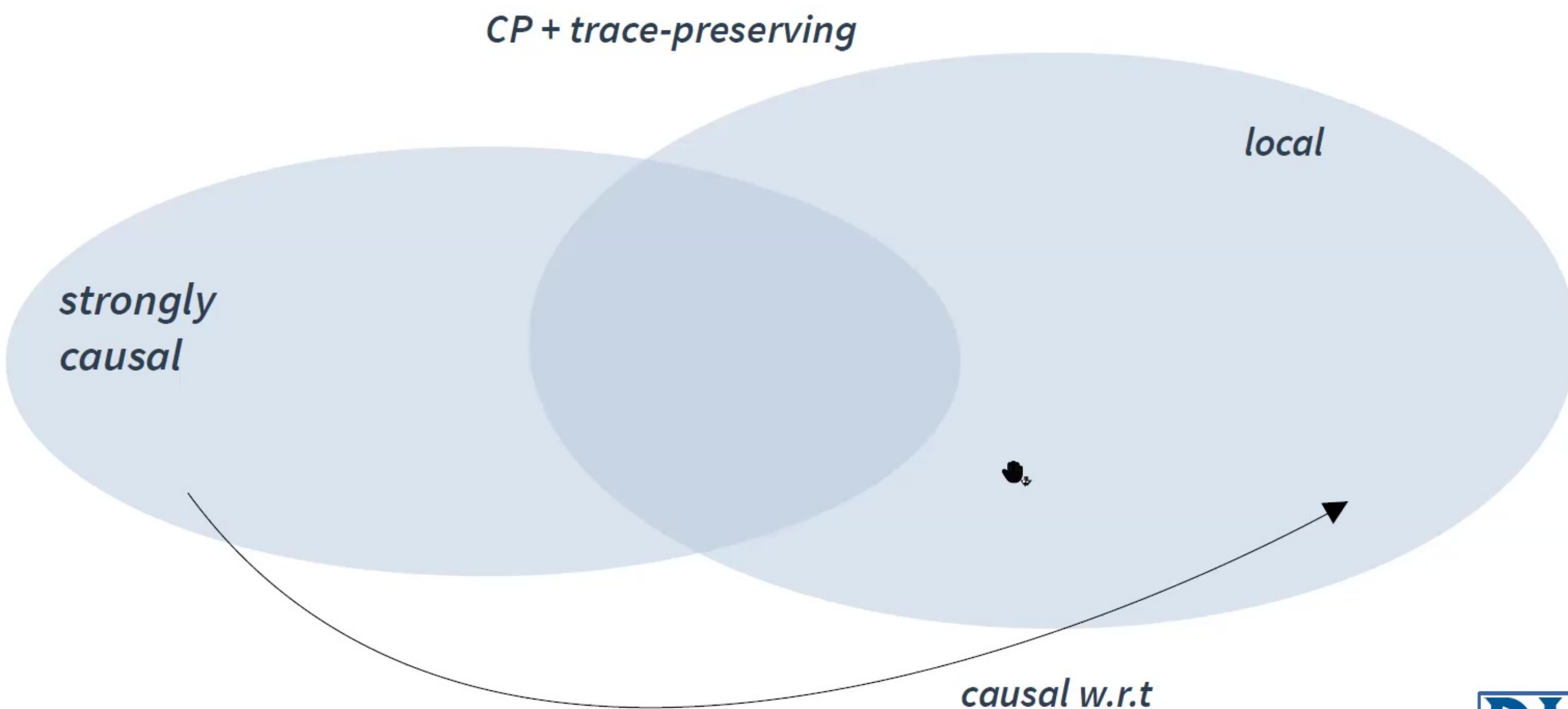
Causality condition



- Venn diagram:



Causality condition

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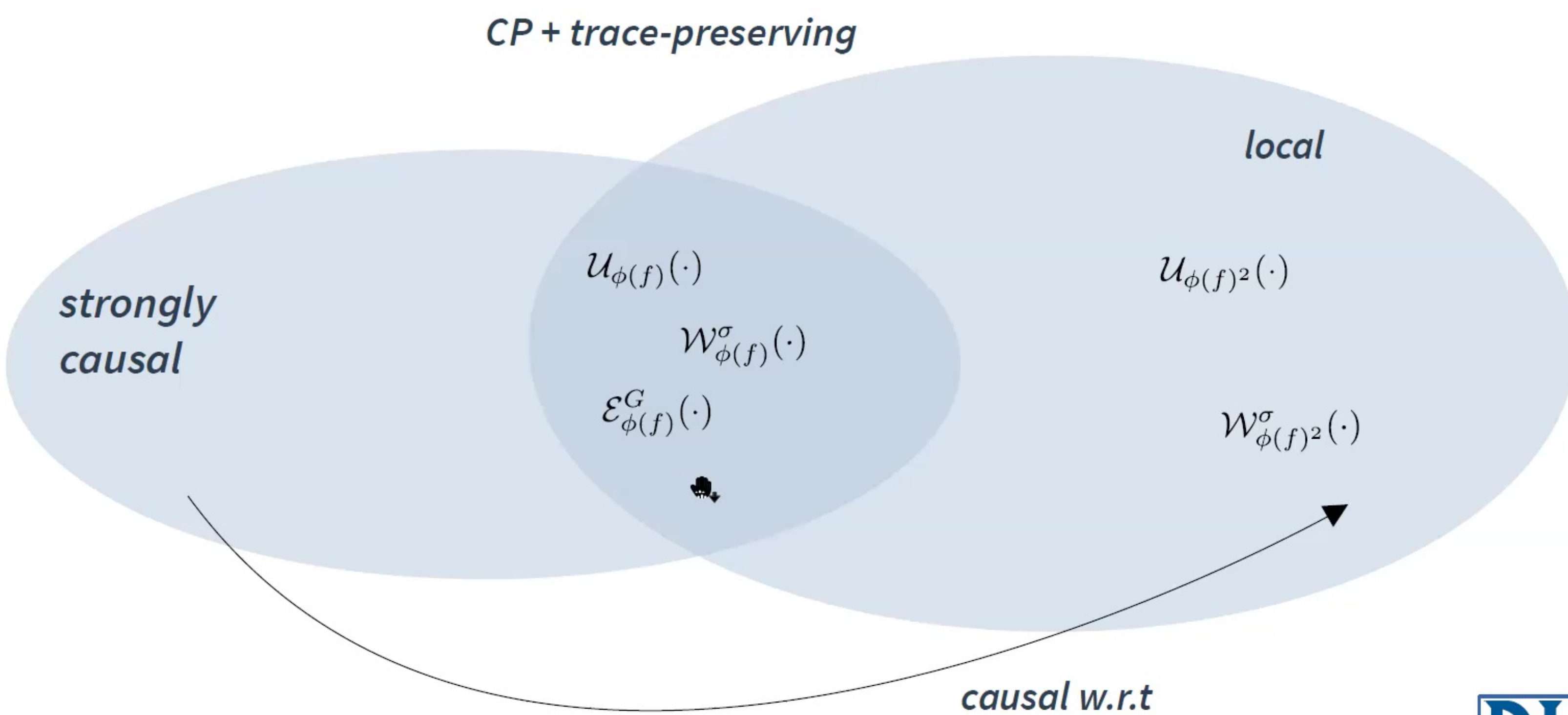
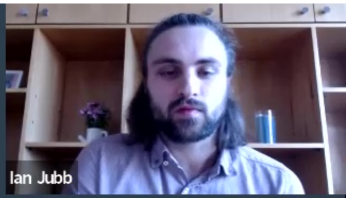


Causality condition



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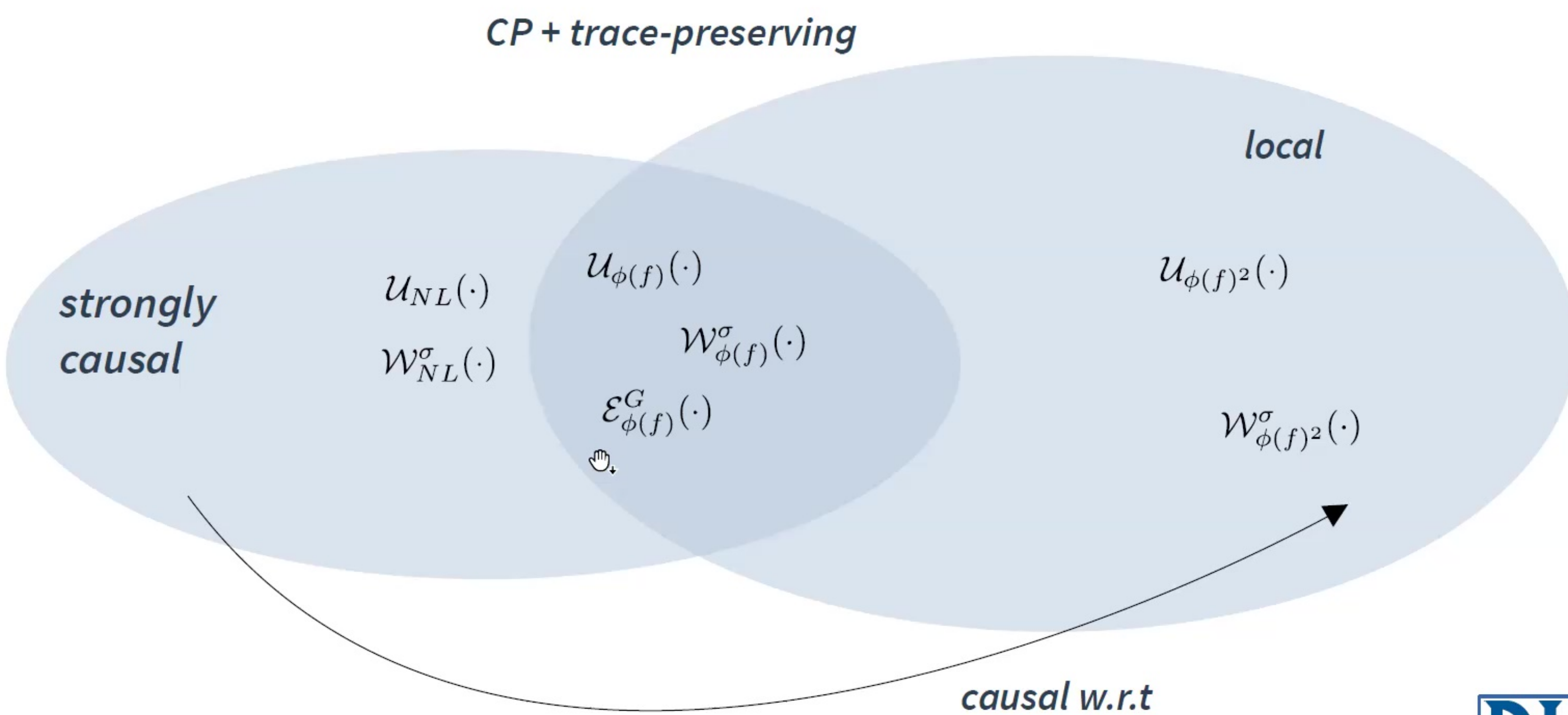

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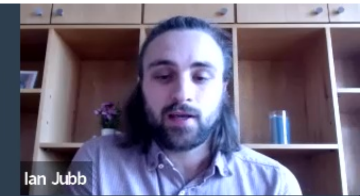
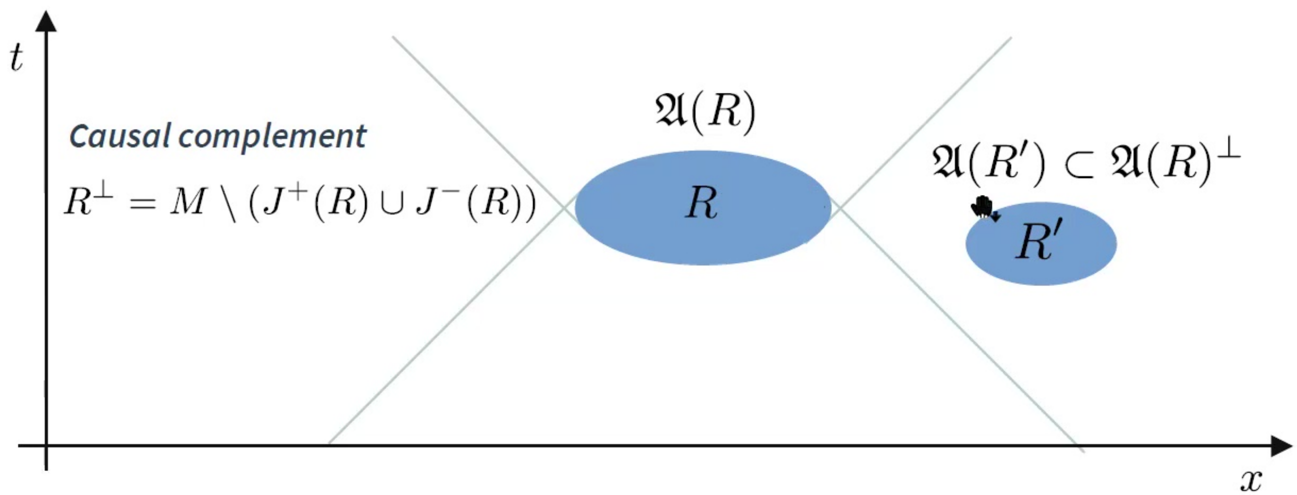
Causality condition



Definition. (commutant)

Given some subalgebra, $\mathfrak{A}(R)$, the *commutant* is given by

$$\mathfrak{A}(R)^{\perp} = \{A \in \mathfrak{A} \,|\, [A, B] = 0 \;,\; \forall\, B \in \mathfrak{A}(R)\}$$



Haag duality

$$\mathfrak{A}(R^{\perp}) = \mathfrak{A}(R)^{\perp}$$

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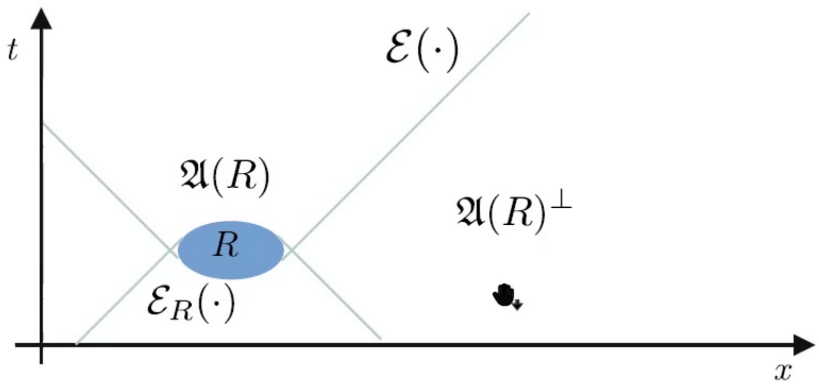
Causality condition

Ian Jubb

- Sufficient condition for strong causality:



Definition. (commutant non-increasing, CNI)

A completely positive trace-preserving map, $\mathcal{E}(\cdot)$, is *commutant non-increasing (CNI)* if, for all regions R, it does not increase the commutant $\mathfrak{A}(R)^{\perp}$, i.e.

$$\mathcal{E}(\mathfrak{A}(R)^{\perp}) \subseteq \mathfrak{A}(R)^{\perp}$$

for all R.

Note, any map local to *R* acts trivially on the commutant, and thus, for any spacelike *R'*,

$$\mathcal{E}_R(\mathcal{E}(\cdot))|_{\mathfrak{A}(R')} = \mathcal{E}(\cdot)$$

This is the criteria for strong causality.

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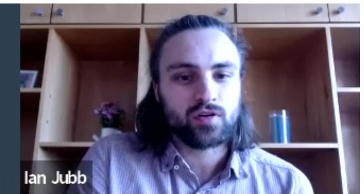
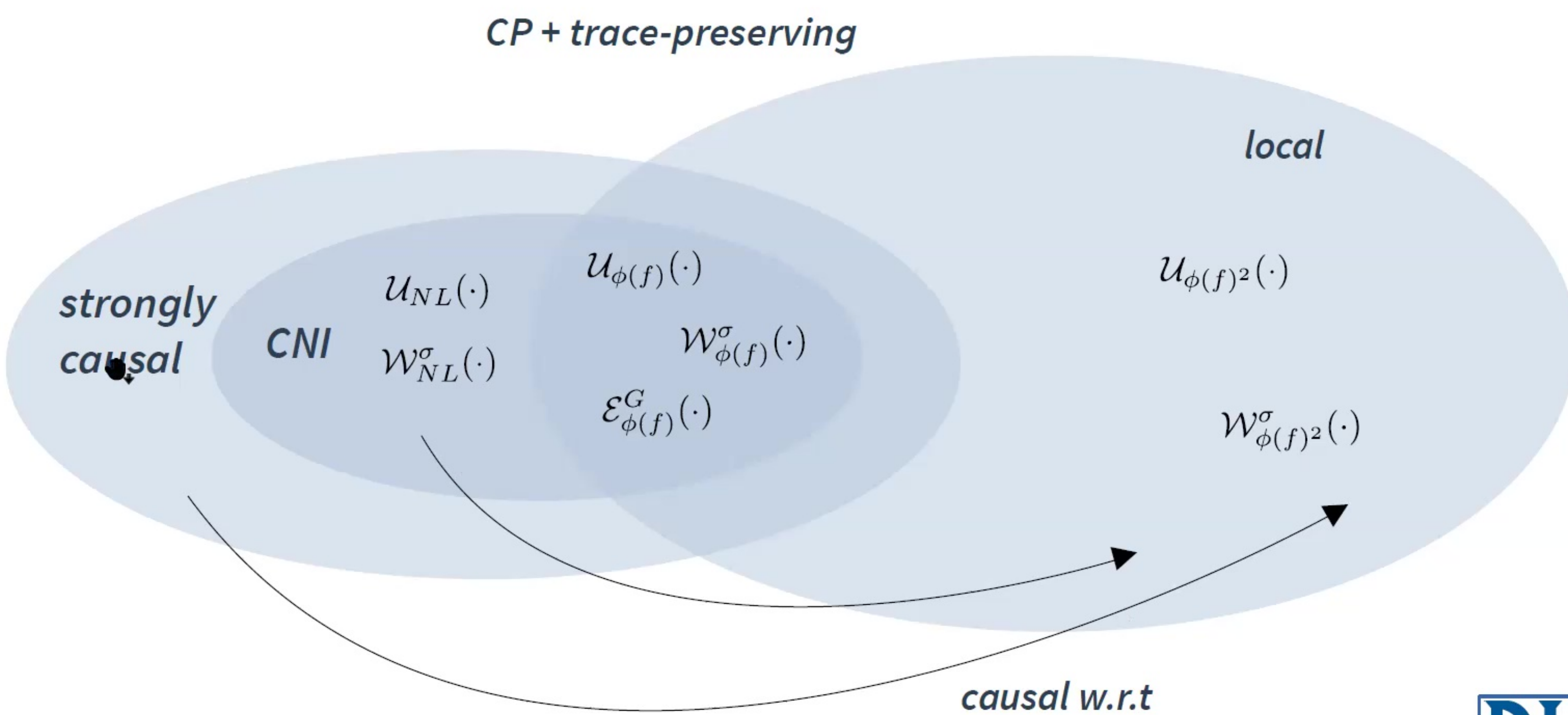
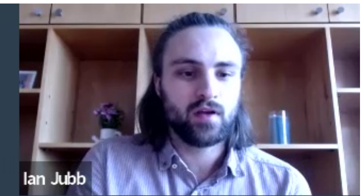
Causality condition



- Venn diagram:



Causality condition



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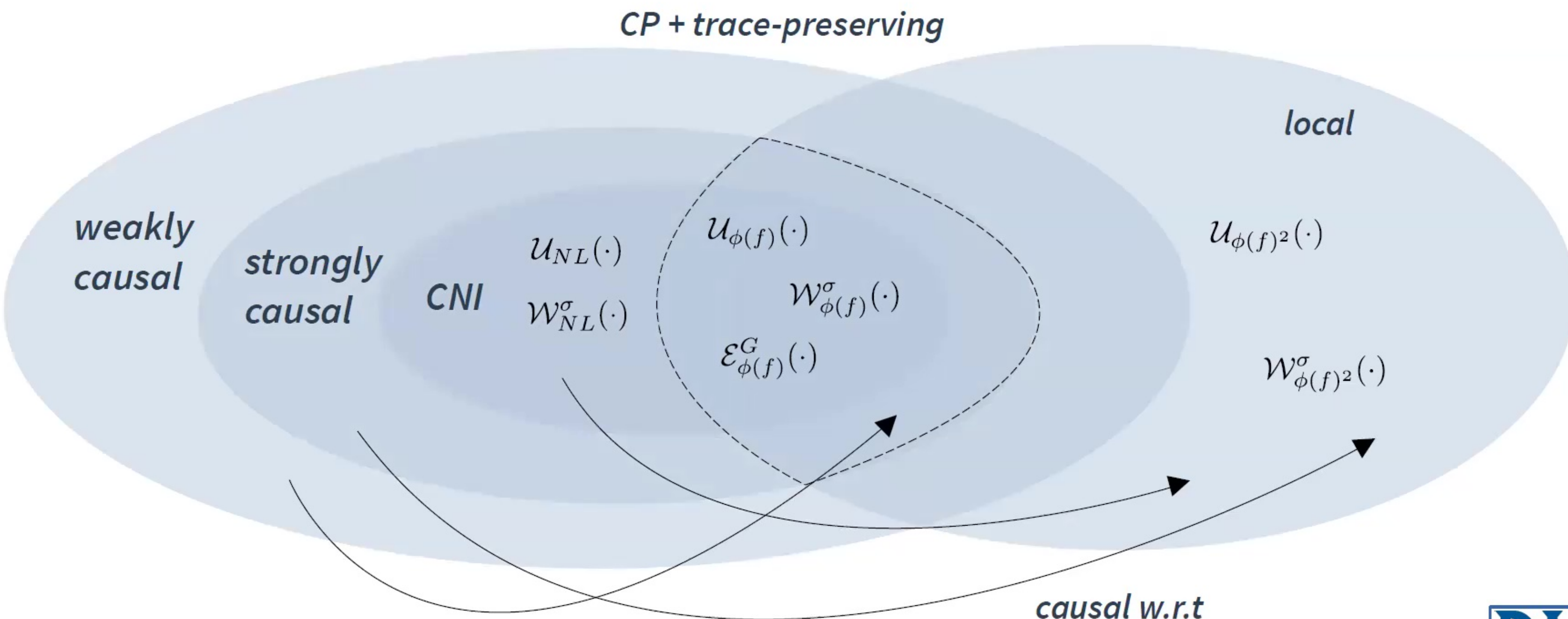

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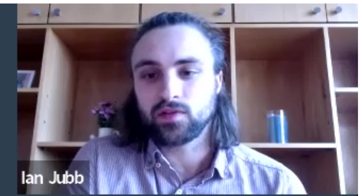
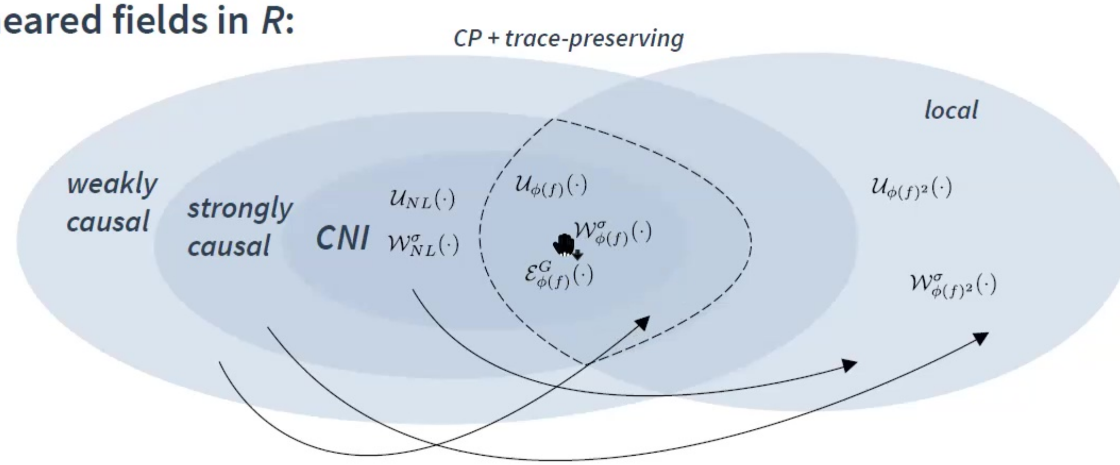
Causality condition



Rough argument why *weakly causal* implies *CNI*:

- *A* weakly causal map, $\mathcal{E}(\cdot)$, is causal w.r.t all strongly causal local maps, $\mathcal{E}_R(\cdot)$, for all regions *R*. This includes all unitary kicks with smeared fields in *R*:



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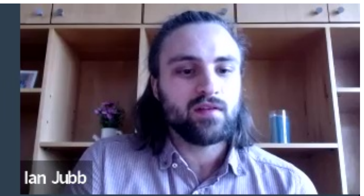
Causality condition



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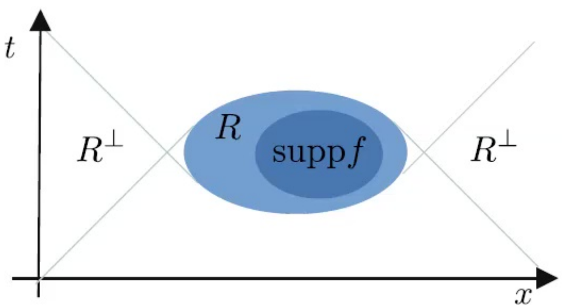

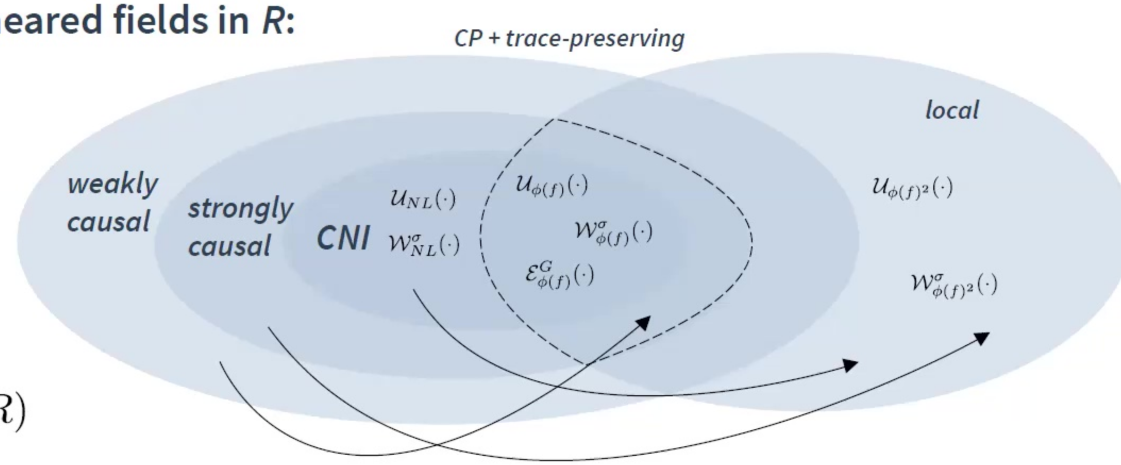

$$\mathcal{U}_{\phi(f)}(\mathcal{E}(\cdot))|_{\mathfrak{A}(R^\perp)} = \mathcal{E}(\cdot) \quad , \quad \forall f \in C_0^\infty(R)$$

- From this we can deduce that, for any $\mathcal{E}(A)$ commutes with $\mathfrak{A}(R)$ for all $A \in \mathfrak{A}(R^\perp) = \mathfrak{A}(R)^\perp$
- Thus, $\mathcal{E}(\mathfrak{A}(R)^\perp) \subseteq \mathfrak{A}(R)^\perp$, i.e. the map is CNI.

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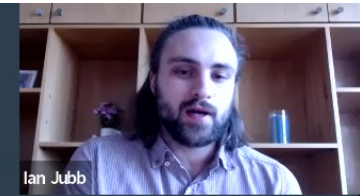
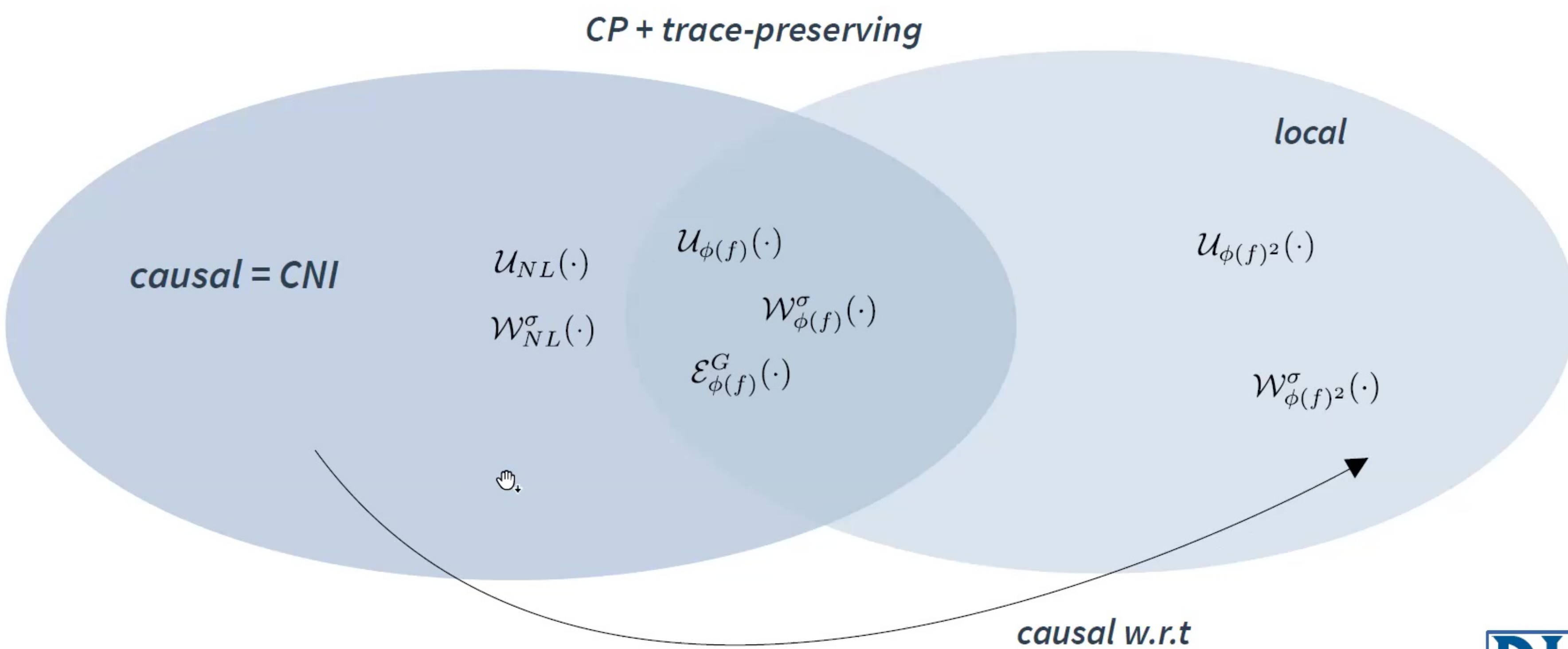
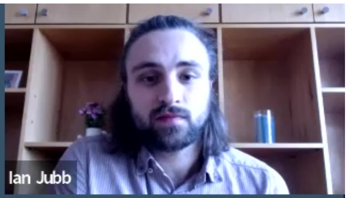
Causality condition



- **Summary:**





Discussion

Open questions/problems

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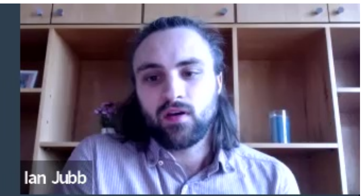
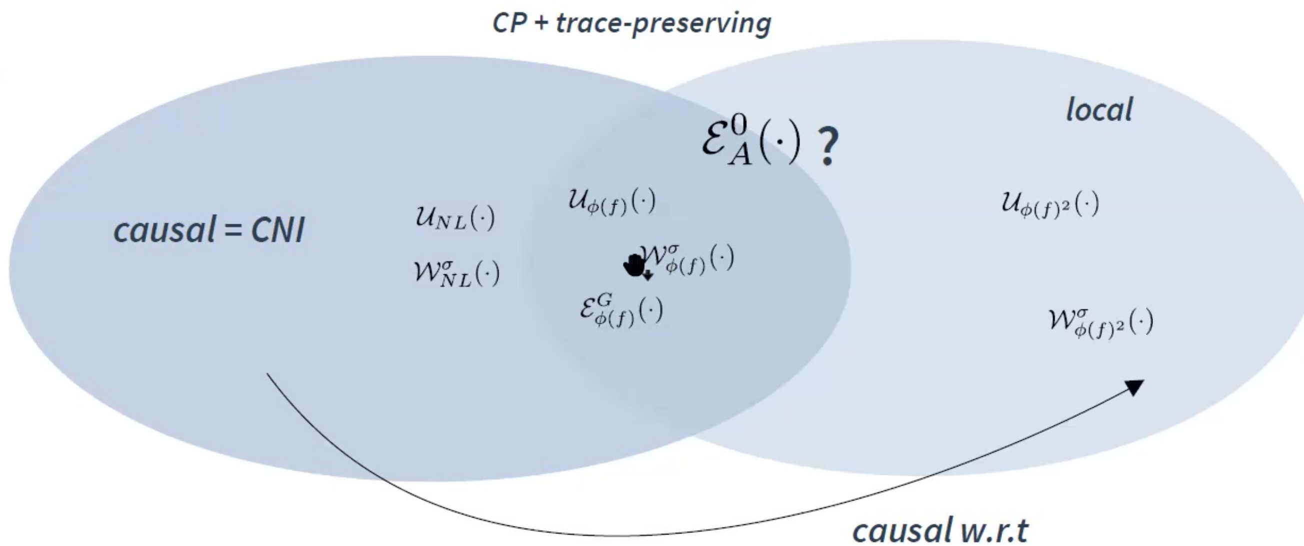
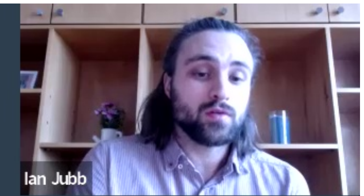


Open questions/problems

- Where do ideal measurements fit in? If they are acausal, is there a 'reasonable' substitute for the projection postulate?



Open questions/problems



- Where do ideal measurements fit in? If they are acausal, is there a 'reasonable' substitute for the projection postulate?

- What does this picture look like for fermionic field theory and gauge theory?

- Can all the causal maps be realised via *local* couplings to auxiliary systems? A *local* Stinespring's Dilation Theorem?