Title: Quantum Gravity

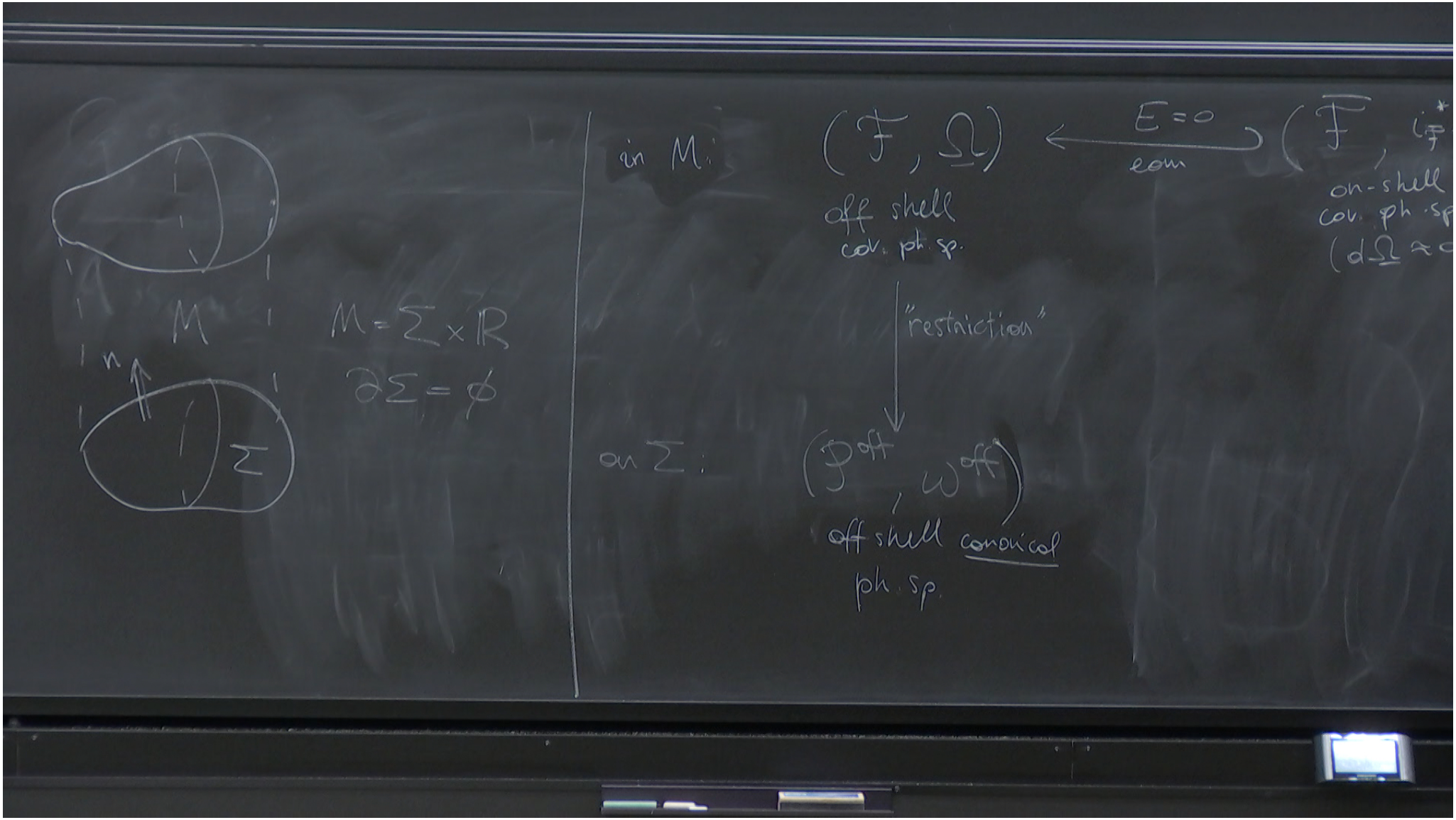
Speakers: Aldo Riello

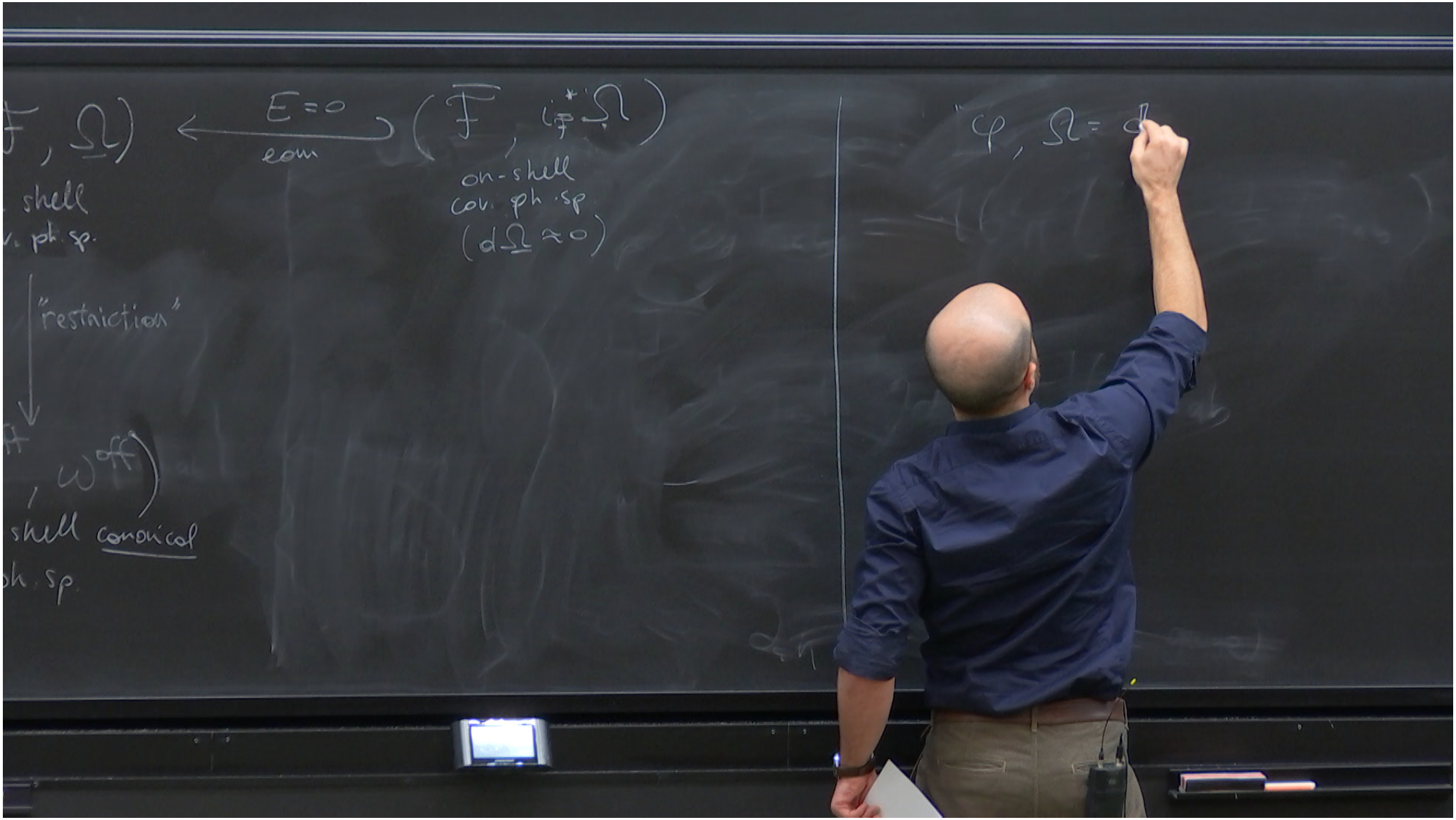
Collection: Quantum Gravity (2021-2022)

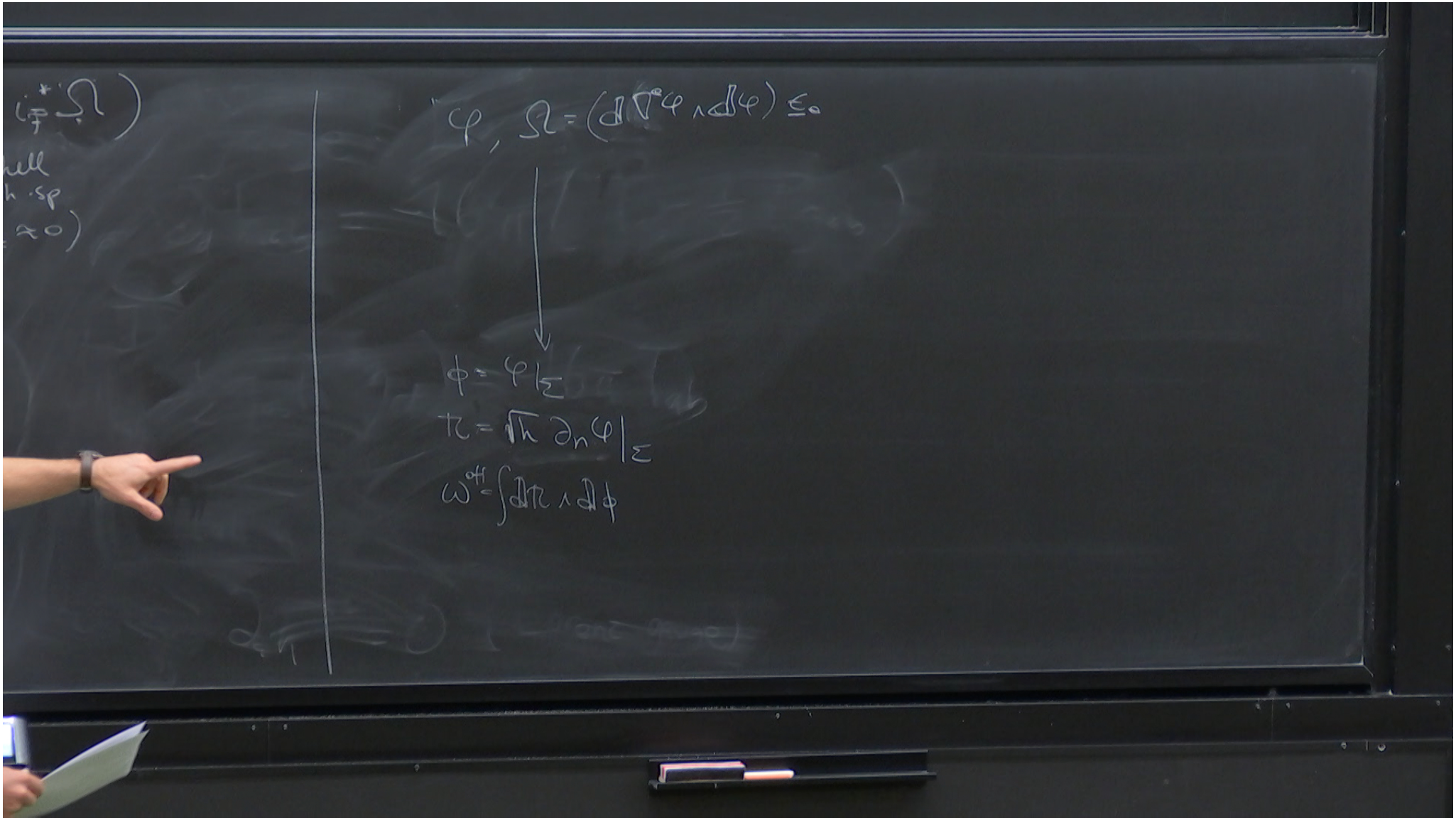
Date: April 29, 2022 - 11:30 AM

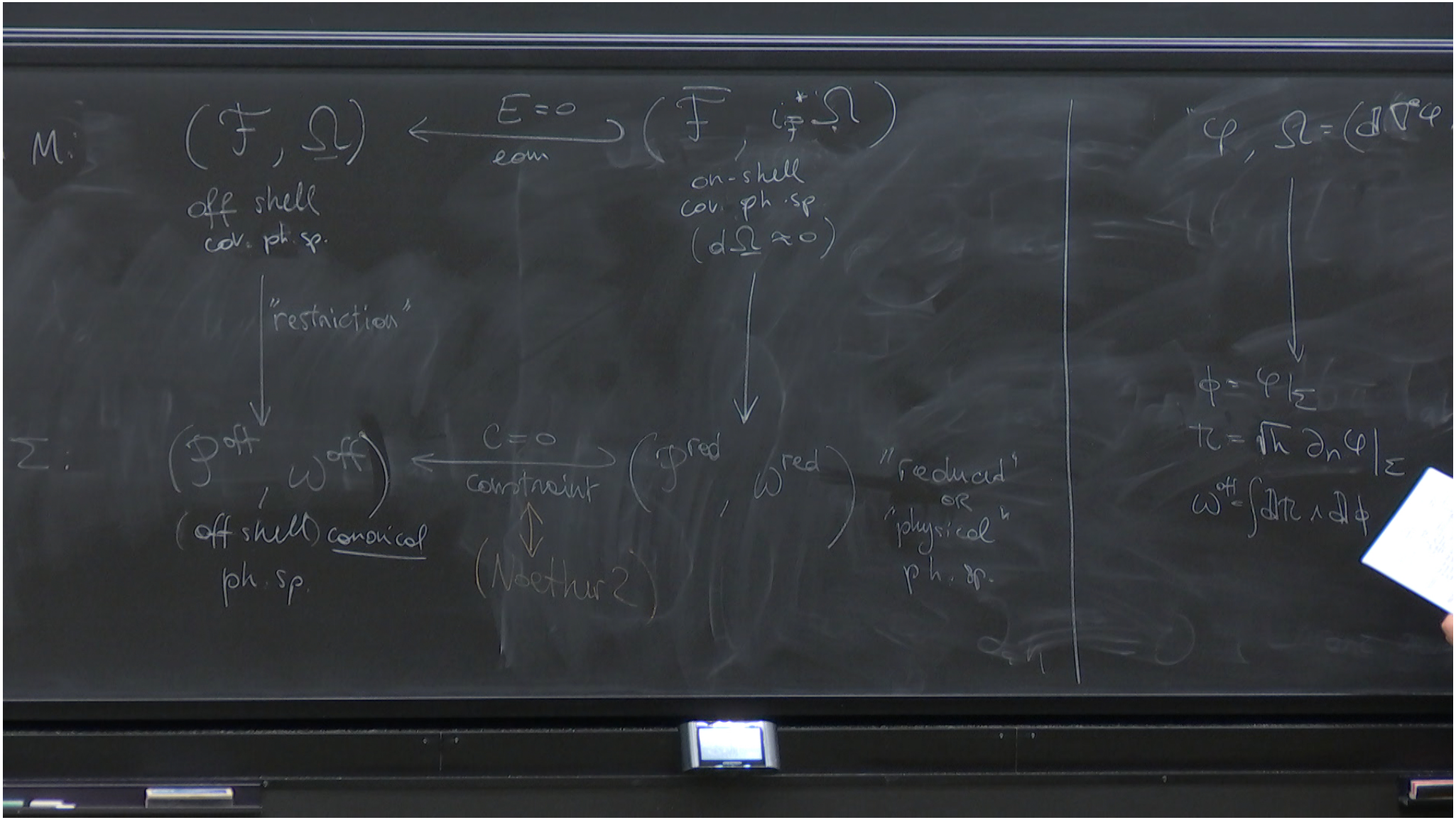
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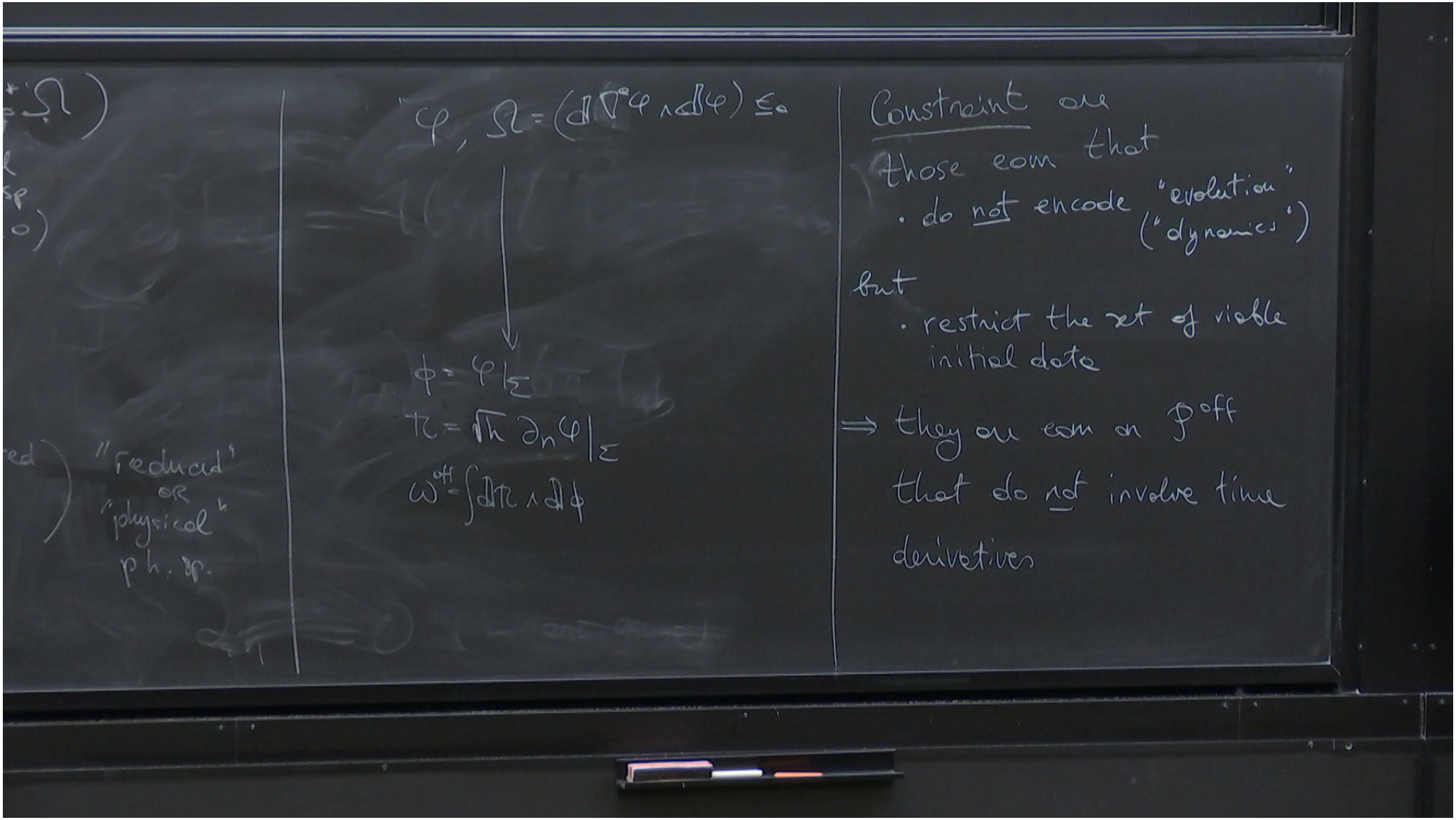
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

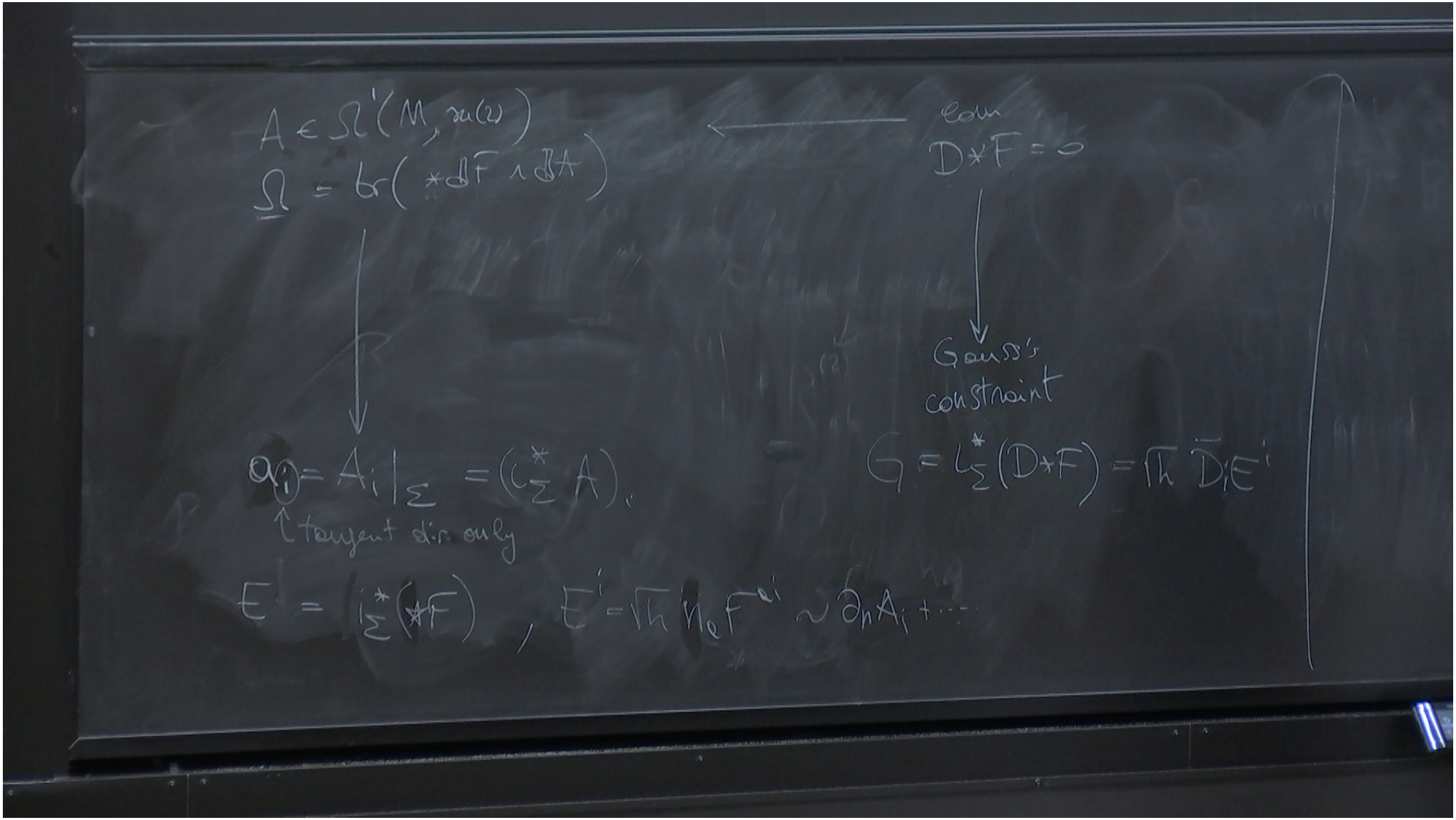

M
n
Σ
M = Σ × R
∂Σ = ∅
in M:
(F, Ω)
E = 0
eom
off shell
cov. ph. sp.
"restriction"
on Σ:
(P^off, ω^off)
off shell canonical
ph. sp.

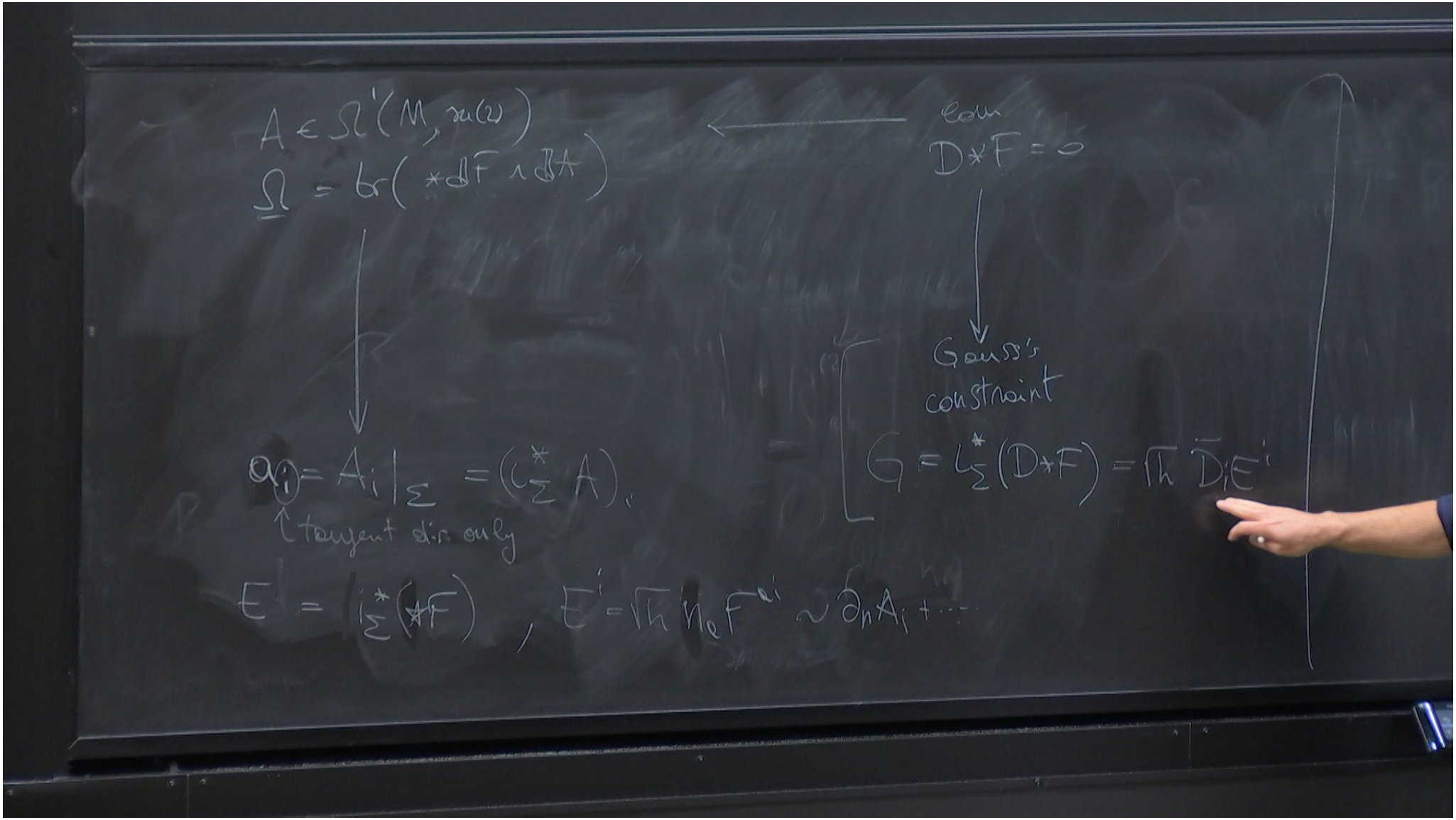

E=0
eom
on-shell
cov. ph. sp
(dΩ ≈ 0)
"restriction"
shell
ph. sp.
shell canonical
sh. sp.


φ, $\Omega = (d\nabla^2\varphi \wedge d\varphi)\,\varepsilon_0$
$\phi = \varphi|_\Sigma$
$\pi = \sqrt{h}\,\partial_n\varphi|_\Sigma$
$\omega^{\text{off}} = \int d\pi \wedge d\phi$


M:
$(\mathcal{F}, \Omega)$
off shell
cov. ph. sp.
$E=0$
eom
$(F, i_F^* \Omega)$
on-shell
cov. ph. sp.
$(d\Omega \approx 0)$
"restriction"
Σ:
$(\rho^{off}, \omega^{off})$
(off shell) canonical
ph. sp.
$C=0$
constraint
(Noether 2)
$(\rho^{red}, \omega^{red})$
"reduced"
or
"physical"
ph. sp.
φ, $\Omega = (d\nabla\varphi$
$\phi = \varphi|_\Sigma$
$\pi = \sqrt{h}\, \partial_n \varphi|_\Sigma$
$\omega^{off} = \int d\pi \wedge d\phi$


"reduced" or "physical" ph. sp.
φ, $\Omega = (d\nabla^a\varphi \wedge d\mathbb{V}(\varphi) \approx 0$
$\phi = \varphi|_\Sigma$
$\pi = \sqrt{h}\,\partial_n\varphi|_\Sigma$
$\omega^{\text{off}} = \int d\pi \wedge d\phi$
Constraint are those eom that
• do not encode "evolution" ("dynamics")
but
• restrict the set of viable initial data
⇒ they are eom on $\mathcal{P}^{\text{off}}$ that do not involve time derivatives

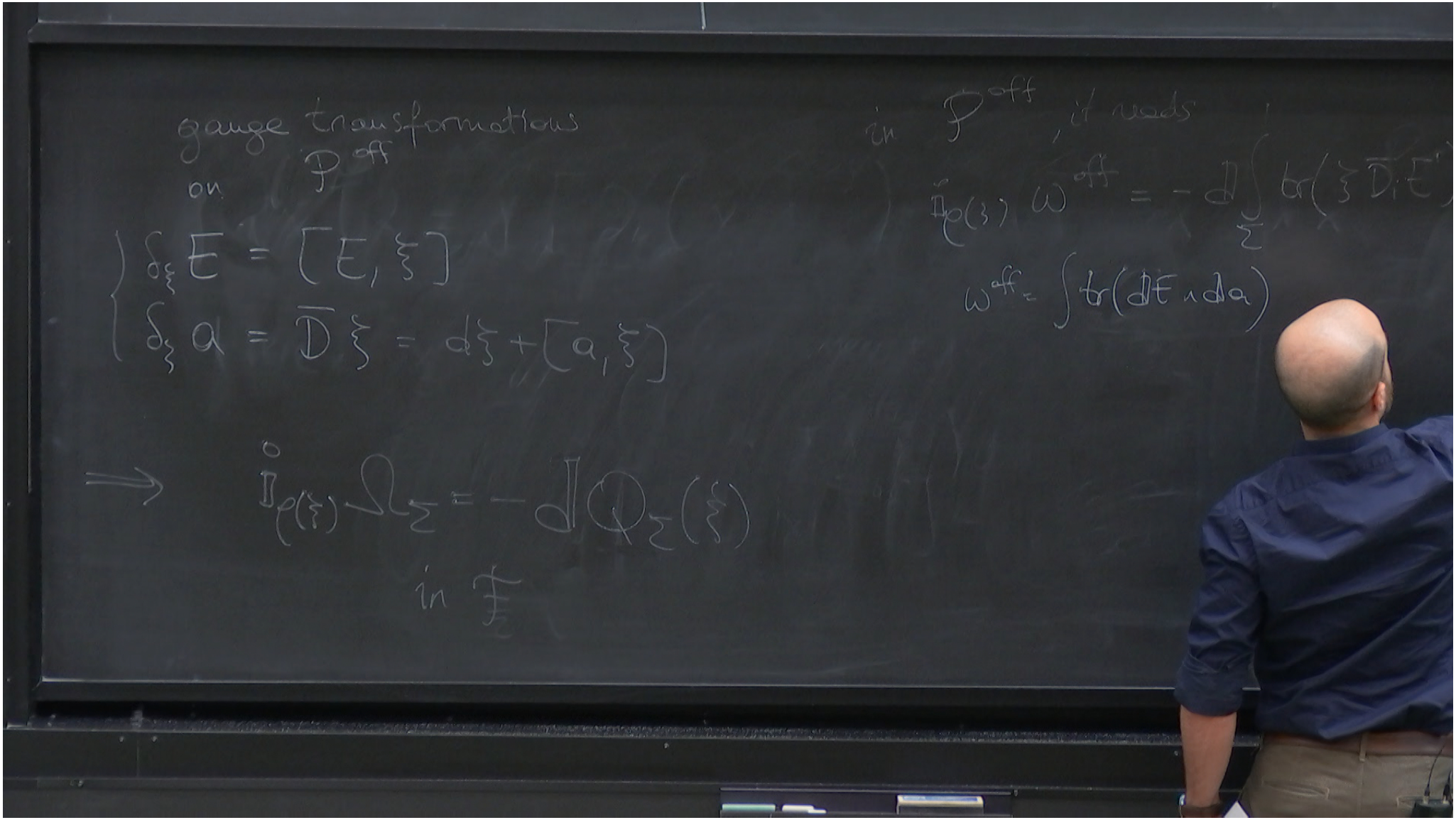

$A \in \Omega^1(M, su(2))$
$\Omega = tr(*dF \wedge dA)$
eom
$D*F = 0$
Gauss's constraint
$q_i = A_i|_\Sigma = (i_\Sigma^* A)_i$
tangent dir. only
$G = i_\Sigma^*(D*F) = \sqrt{h}\, \tilde{D}_i E^i$
$E = i_\Sigma^*(*F)$, $E^i = \sqrt{h}\, n_e F^{ei} \sim \partial_n A_i + \cdots$

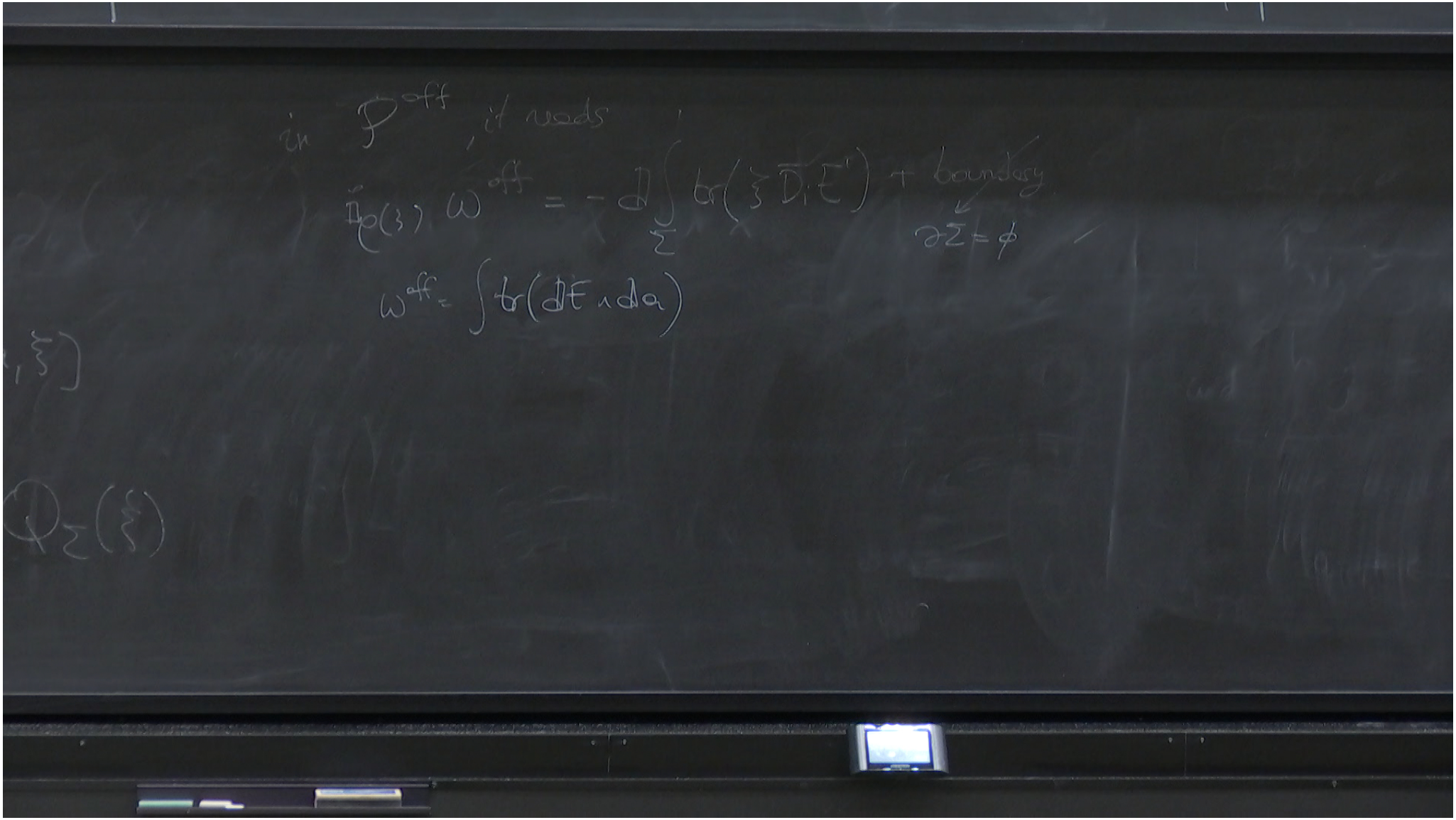

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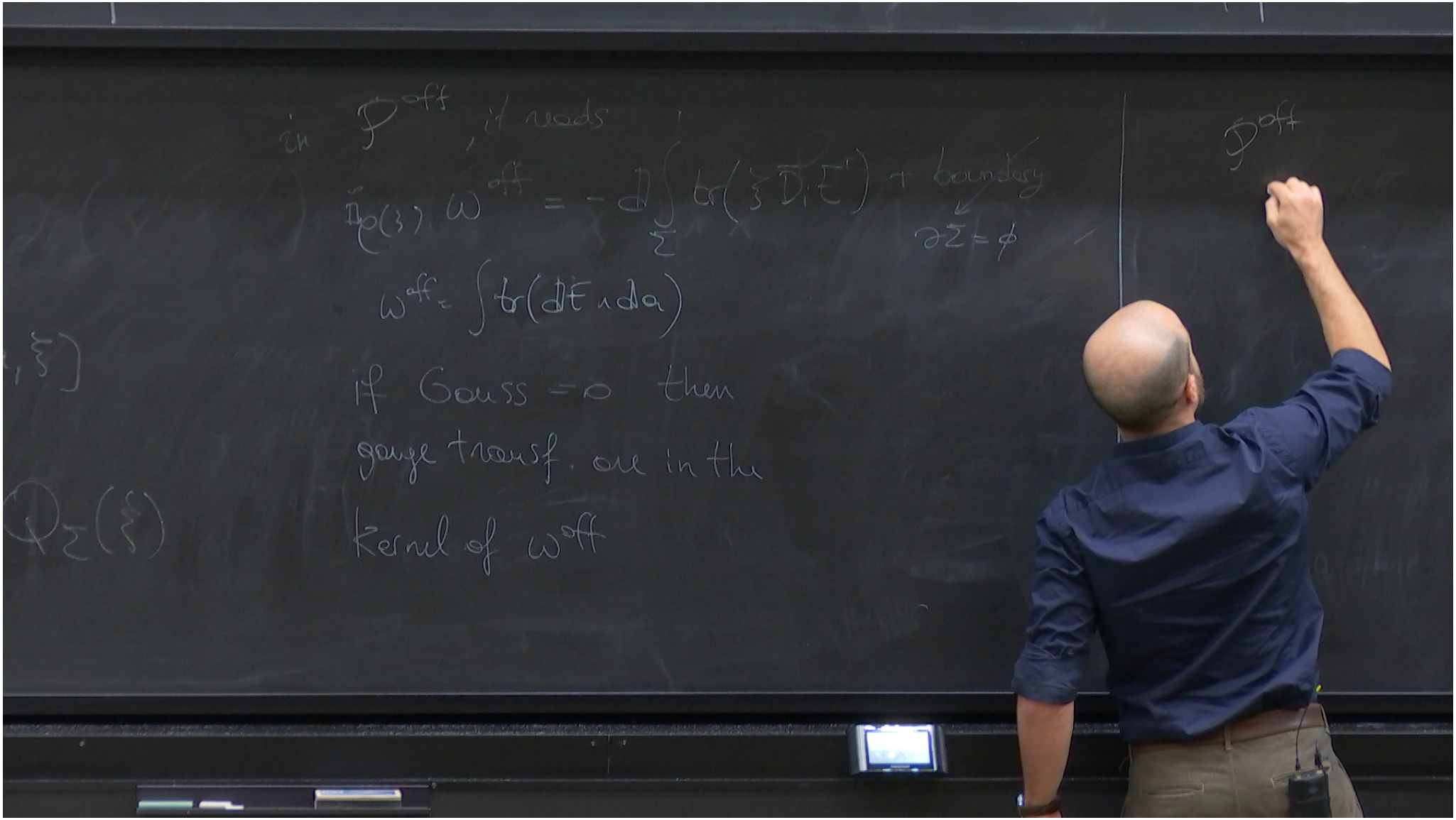
gauge transformations on $\mathcal{P}$ off

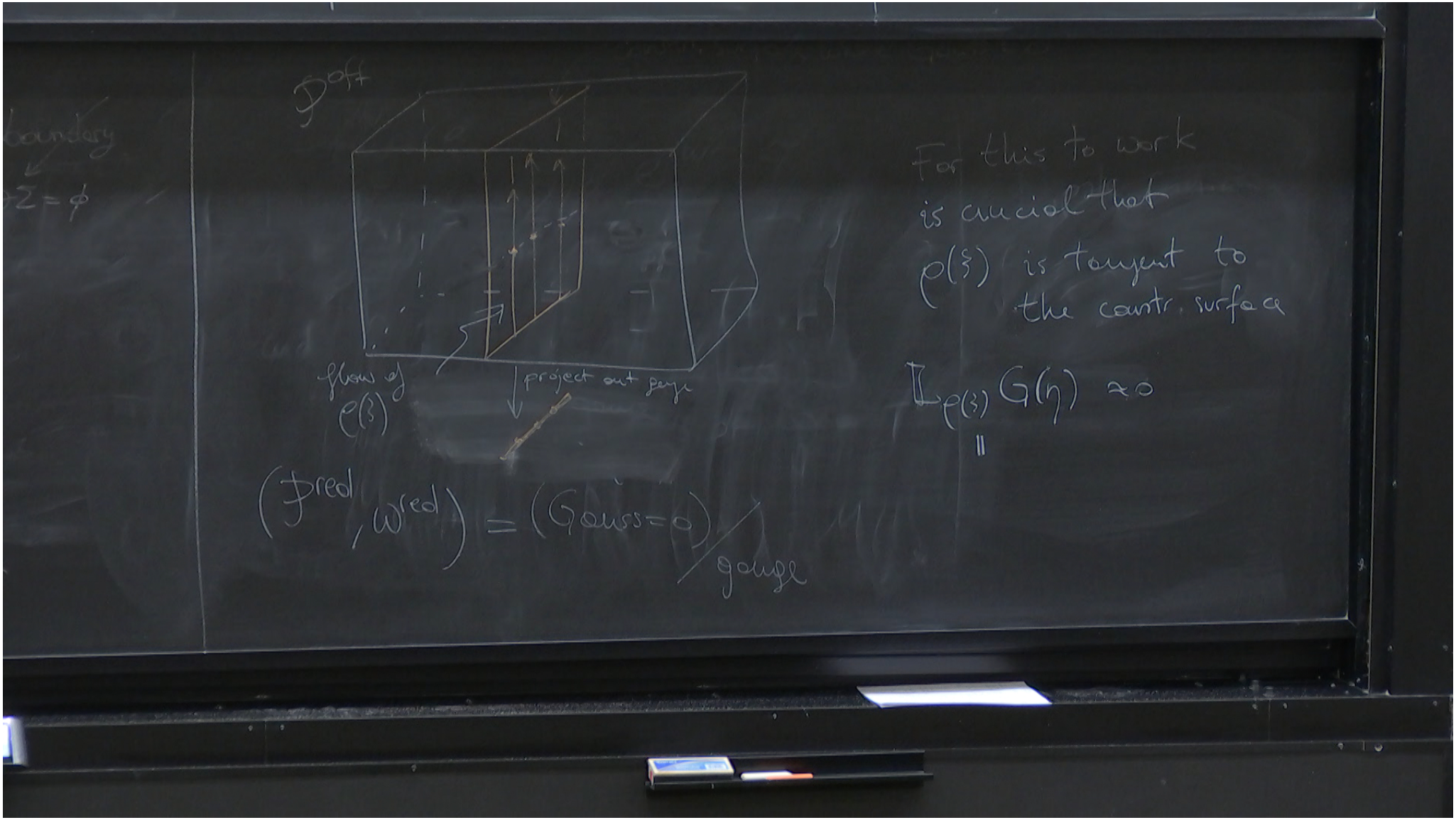
$$\delta_\xi E = [E, \xi]$$

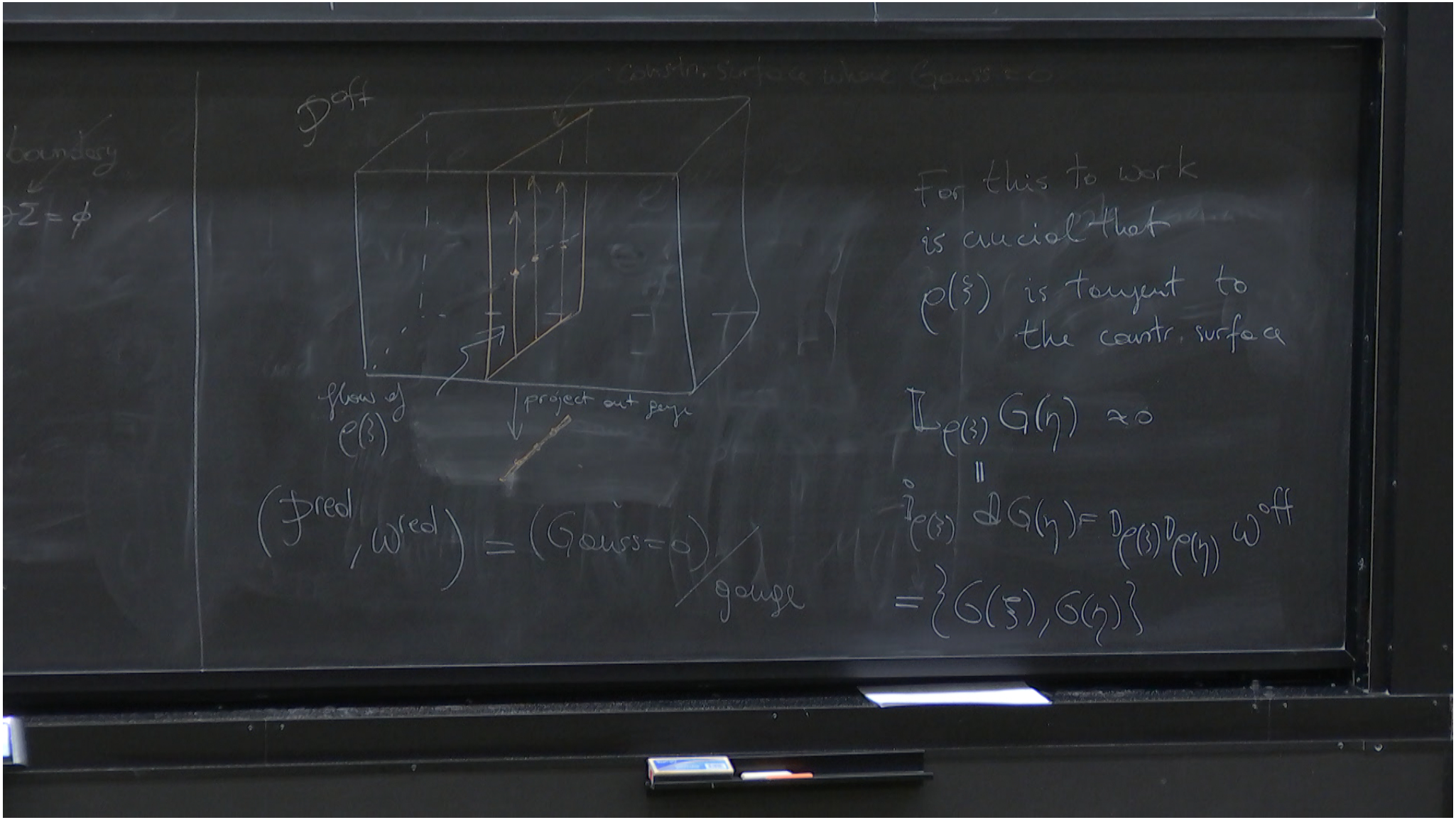
$$\delta_\xi a = \overline{D}\xi = d\xi + [a, \xi]$$

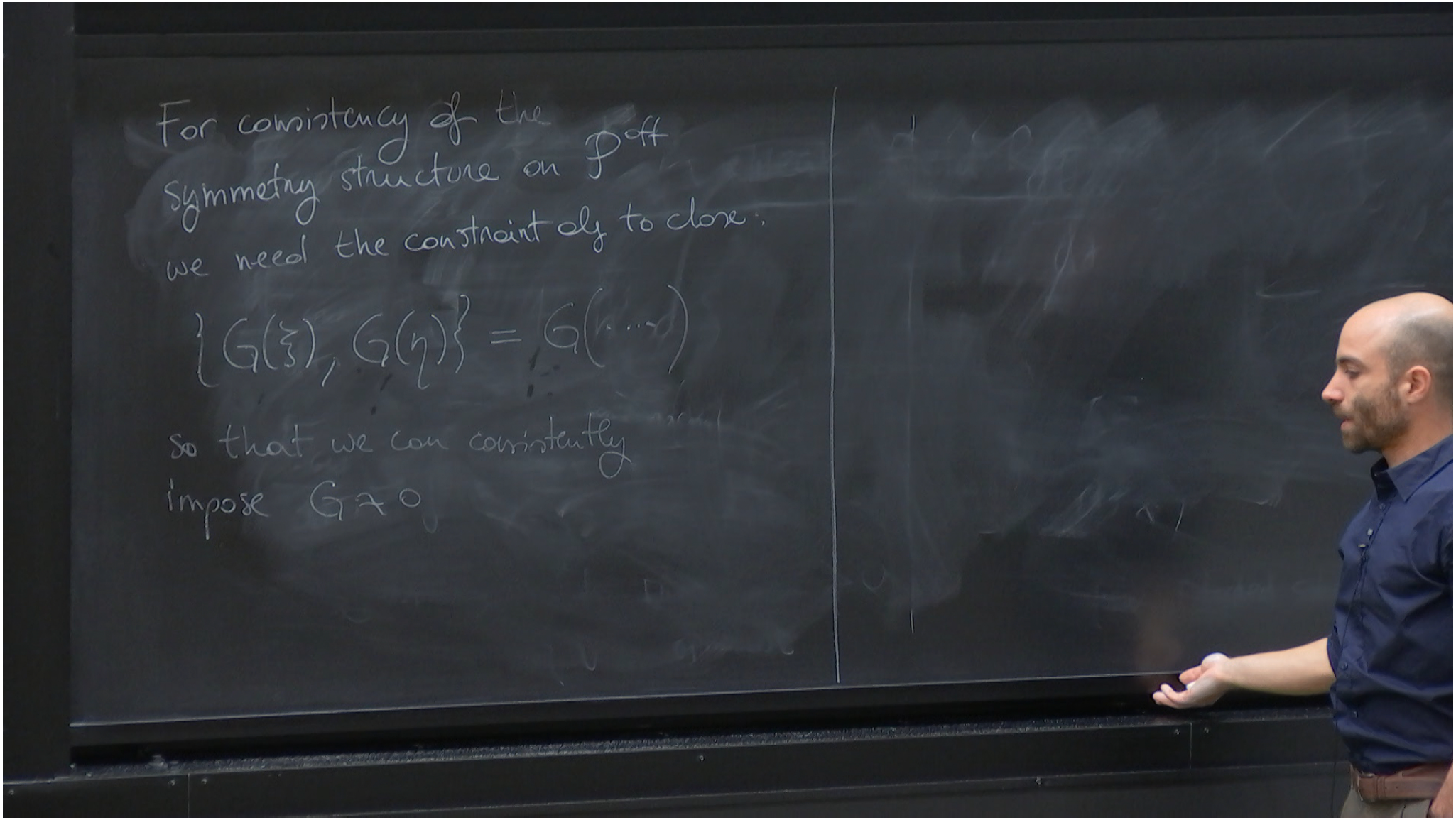

gauge transformations
on $\mathcal{P}^{off}$
$\begin{cases} \delta_\xi E = [E, \xi] \\ \delta_\xi a = \bar{D}\xi = d\xi + [a, \xi] \end{cases}$
$\Rightarrow \quad \mathbb{I}_{\rho(\xi)} \Omega_\Sigma = -d Q_\Sigma(\xi)$
in $\mathcal{F}_\Sigma$
in $\mathcal{P}^{off}$, it reads
$\mathbb{I}_{\rho(\xi)} \omega^{off} = -d \int_\Sigma tr(\xi \bar{D} E)$
$\omega^{off} = \int tr(\delta E \wedge \delta a)$

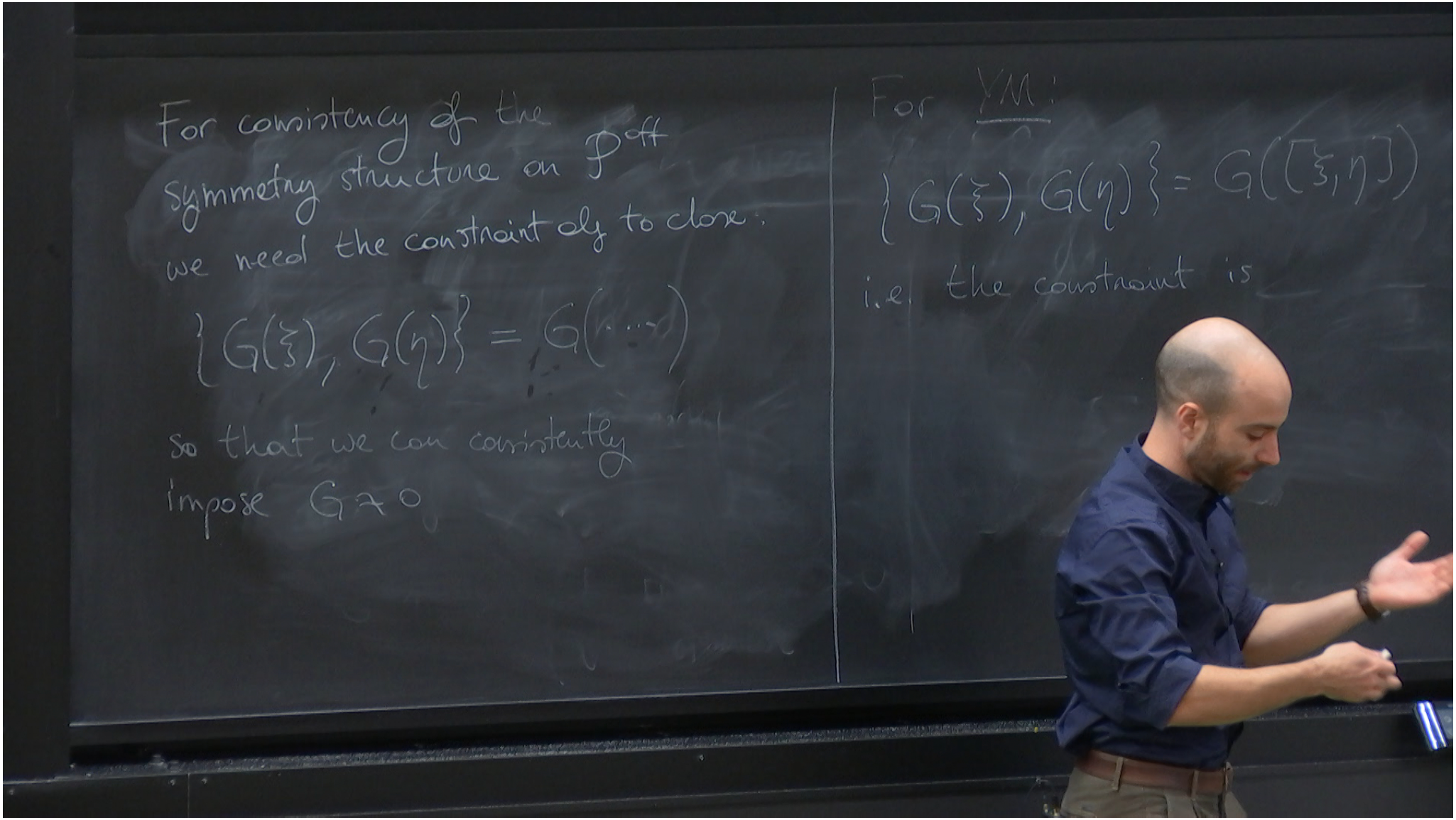

in $\mathcal{P}^{off}$, it reads
$\delta_{p(\xi)}\omega^{off} = -\delta\int_{\Sigma} tr(\xi \bar{D}_i \bar{E}^i)$ + boundary
$\partial\bar{\Sigma} = \phi$
$\omega^{off} = \int tr(\delta E \wedge \delta a)$
$Q_\Sigma(\xi)$

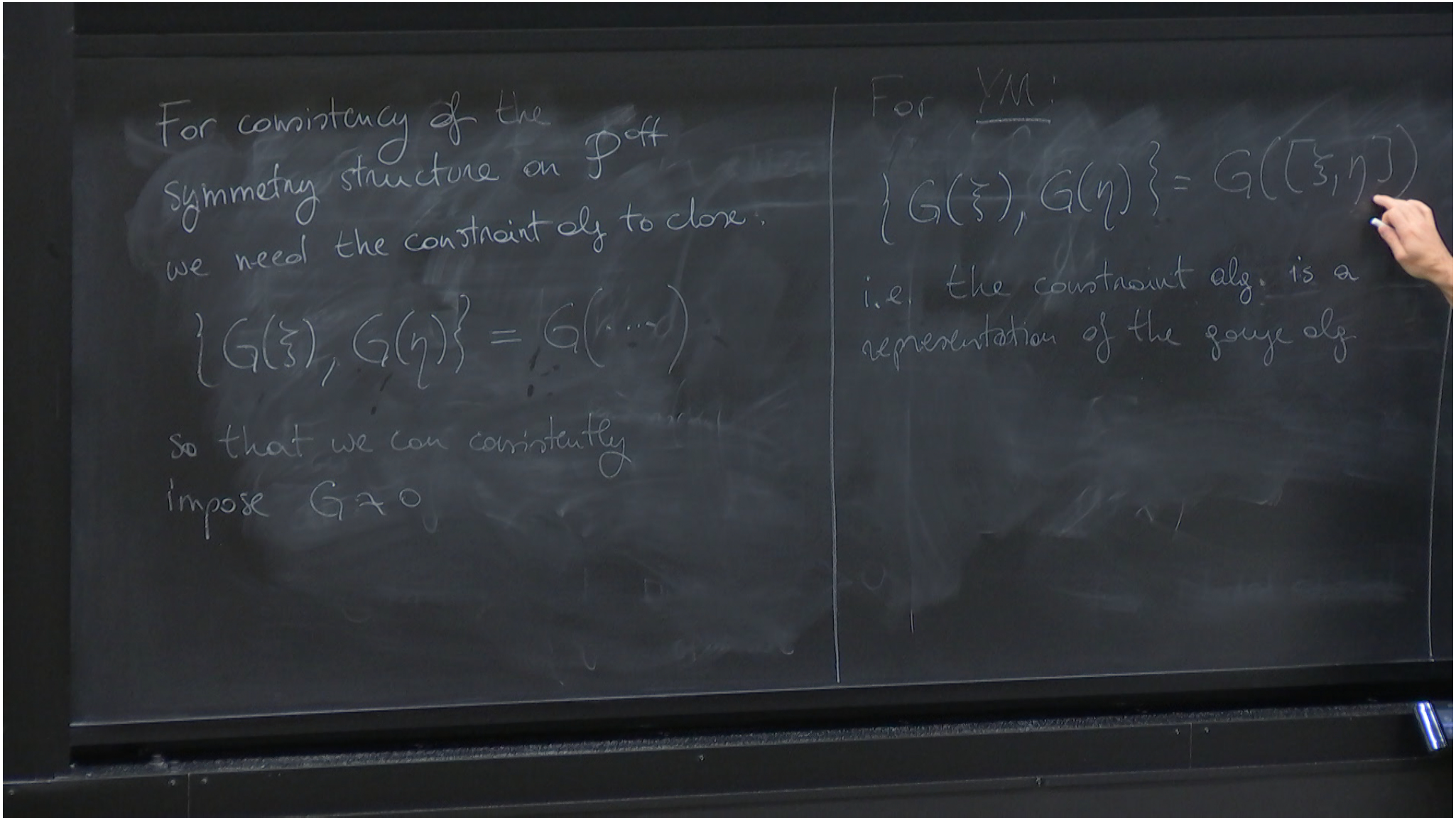

in $\mathcal{P}^{off}$, it reads
$\delta_{\alpha(\xi)} \omega^{off} = -\delta \int_\Sigma tr(\xi \bar{D}_i \bar{E}^i)$ + boundary
$\partial\Sigma = \phi$
$\omega^{off} = \int tr(\delta E \wedge \delta a)$
If Gauss = 0 then
gauge transf. are in the
kernel of ω^{off}
$\mathcal{P}^{off}$

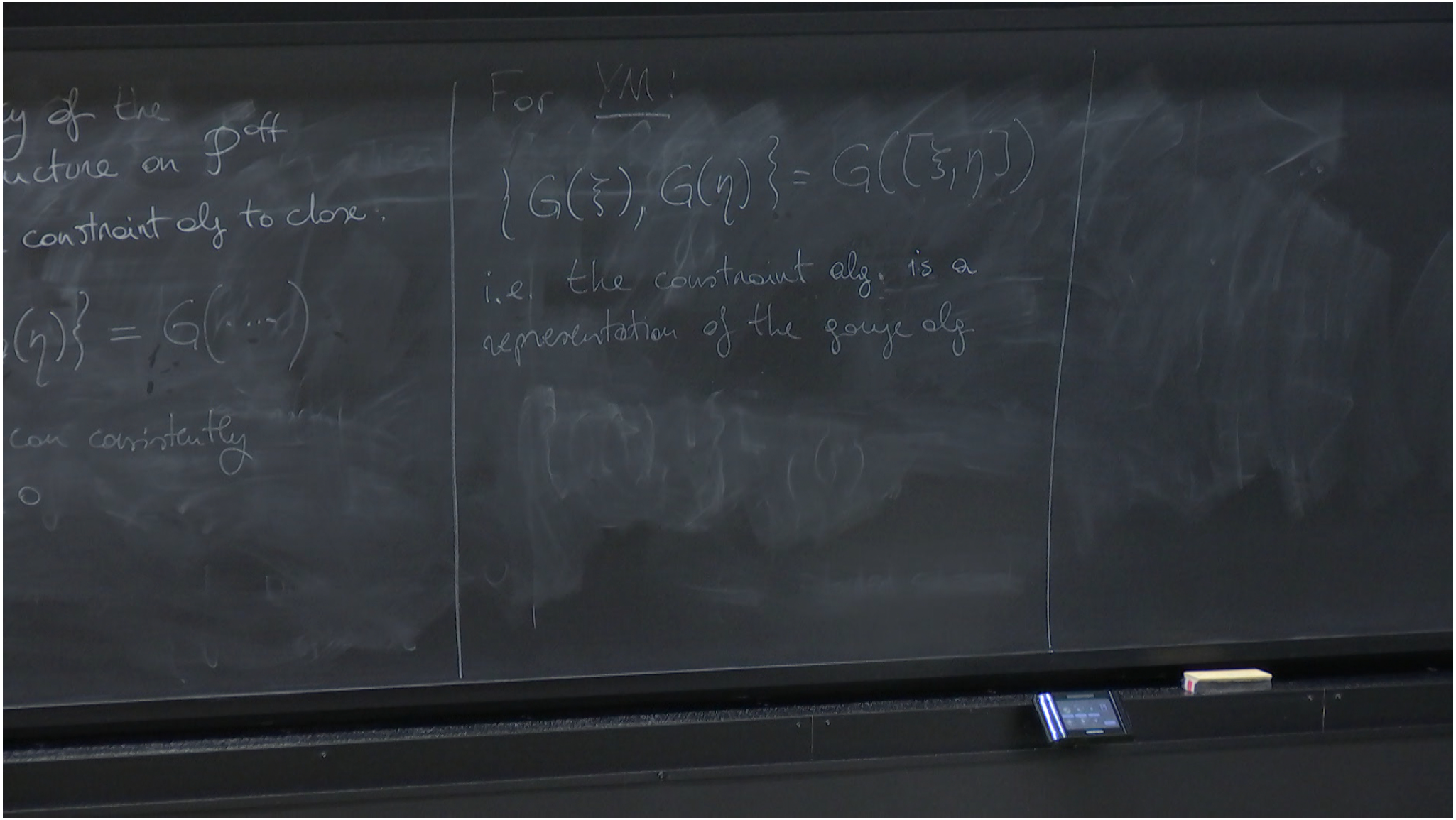

boundary
$\mathcal{D}^{off}$
flow of $\rho(\beta)$
project out gauge
$(\mathcal{F}_{red}, \omega_{red}) = (Gauss=0)/gauge$
For this to work is crucial that $\rho(\beta)$ is tangent to the constr. surface
$\mathbb{L}_{\rho(\beta)} G(\eta) \approx 0$

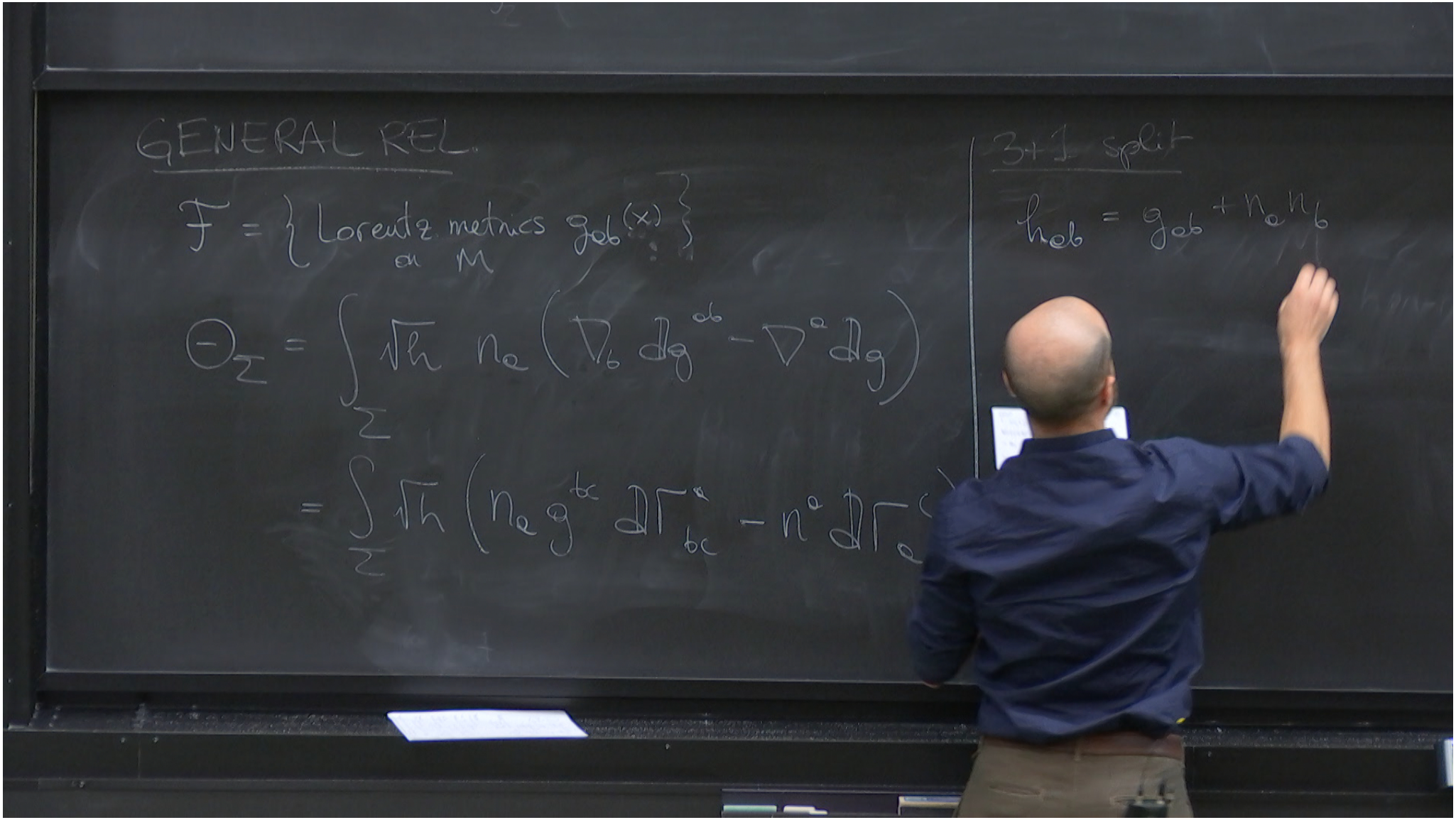

boundary
$\partial Z = \phi$
$\mathcal{F}^{off}$
constr. surface where Gauss = 0
flow of $\rho(\beta)$
project out gauge
$(\mathcal{F}^{red}, \omega^{red}) = (\text{Gauss}=0)/\text{gauge}$
For this to work is crucial that $\rho(\xi)$ is tangent to the constr. surface
$\mathbb{L}_{\rho(\beta)} G(\eta) \approx 0$
$= i_{\rho(\beta)} d G(\eta) = D_{\rho(\beta)} D_{\rho(\eta)} \omega^{off}$
$= \{G(\xi), G(\eta)\}$

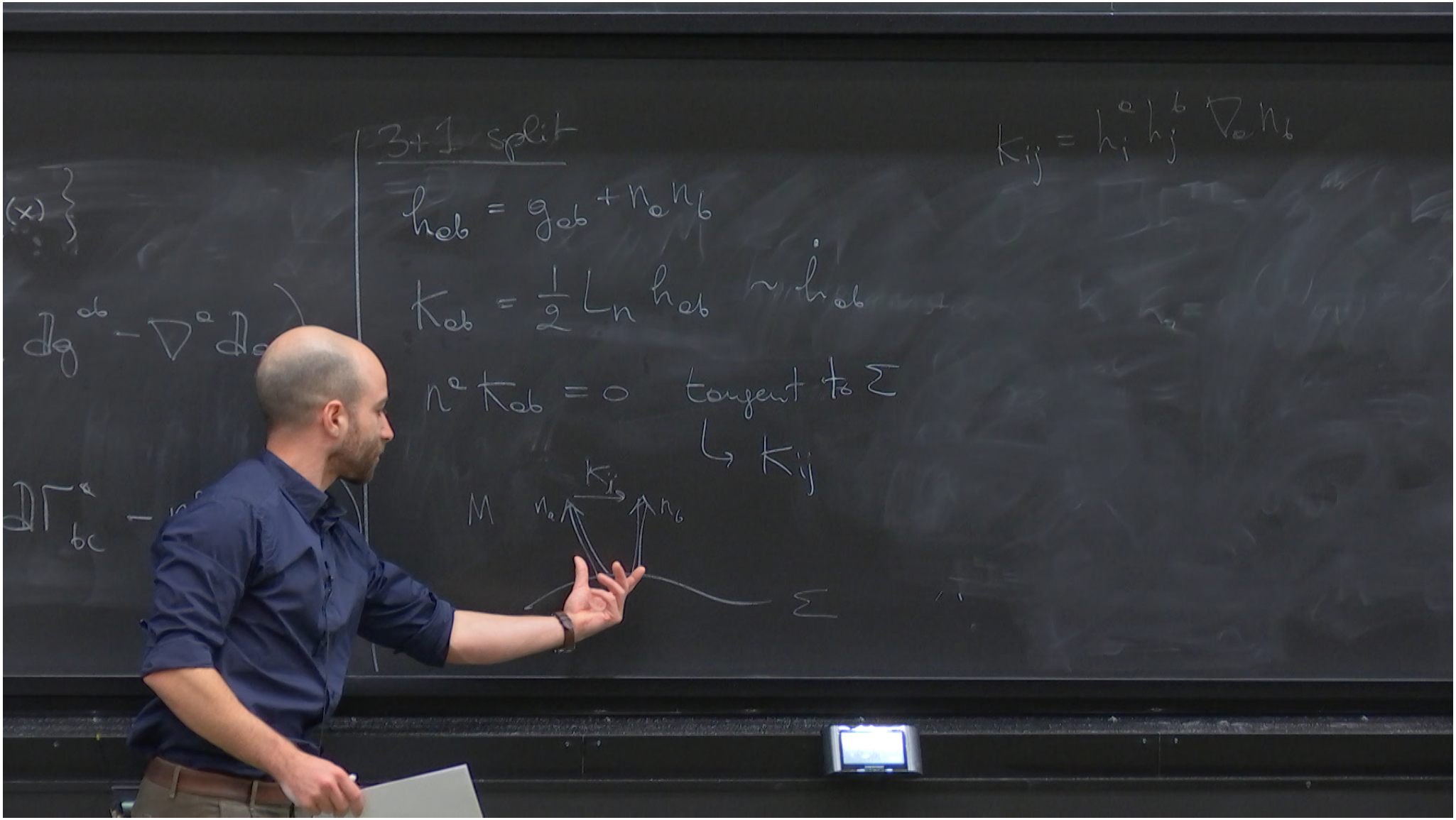

For consistency of the
symmetry structure on $\mathcal{P}^{off}$
we need the constraint alg to close:
$\{G(\xi), G(\eta)\} = G(\dots)$
so that we can consistently
impose $G \approx 0$


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For YM:
$\{G(\xi), G(\eta)\} = G([\xi, \eta])$
i.e. the constraint is


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representation of the gauge alg


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GENERAL REL.
$\mathcal{F} = \{$ Lorentz metrics $g_{ab}(x)$ on M $\}$
$\Theta_\Sigma = \int_\Sigma \sqrt{h}\, n_a \left(\nabla_b dg^{ab} - \nabla^a dg \right)$
$= \int_\Sigma \sqrt{h} \left(n_a g^{bc} d\Gamma^a_{bc} - n^a d\Gamma_a \right)$
3+1 split
$h_{ab} = g_{ab} + n_a n_b$


3+1 split
$h_{ab} = g_{ab} + n_a n_b$
$K_{ab} = \frac{1}{2} \mathcal{L}_n h_{ab} \sim \dot{h}_{ab}$
$n^a K_{ab} = 0$ tangent to Σ
↳ K_{ij}
$K_{ij} = h_i^a h_j^b \nabla_a n_b$
M
n_a
K_{ij}
n_b
Σ

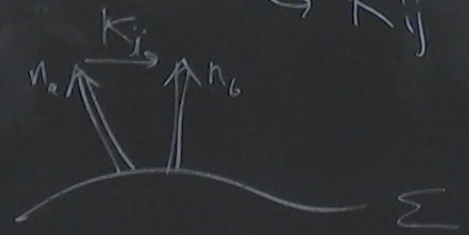
split

$= g_{ab} + n_a n_b$

$= \frac{1}{2} \mathcal{L}_n h_{ab} \sim \dot{h}_{ab}$

$K_{ab} = 0$ tangent to Σ

↳ K_{ij}



$K_{ij} = h_i^a h_j^b \nabla_a n_b$

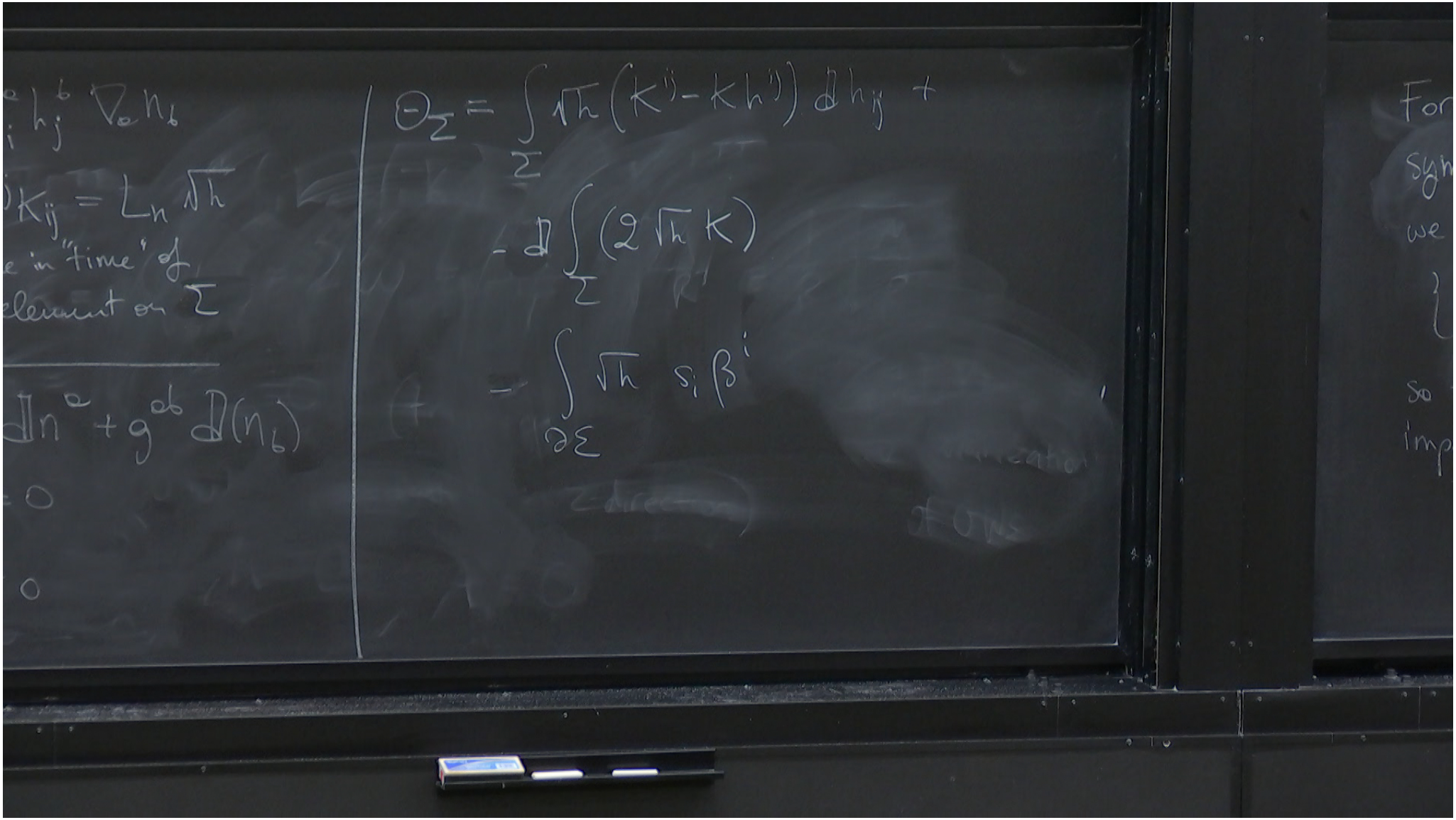
$K \equiv h^{ij} K_{ij} = \mathcal{L}_n \sqrt{h}$

↑ change in "time" of vol. element on Σ

$\beta^a = \mathbb{D} n^a + g^{ab} \mathbb{D}(n_b)$

$n_a \beta^a = 0$

$d\beta \neq 0$


$h_j^b \nabla_a n_b$
$K_{ij} = \mathcal{L}_n \sqrt{h}$
in "time" of
element on Σ
$\delta n^a + g^{ab} \delta(n_b)$
$\Theta_\Sigma = \int_\Sigma \sqrt{h}\,(K^{ij} - K h^{ij})\,\delta h_{ij} +$
$- \delta \int_\Sigma (2\sqrt{h} K)$
$- \int_{\partial\Sigma} \sqrt{h}\, s_i \beta^i$
For
we
so
imp

GENERAL REL.

$$\mathcal{F} = \{ \text{Lorentz metrics } g_{ab}(x) \text{ on } M \}$$

$$\Theta_\Sigma = \int_\Sigma \sqrt{h}\, n_a \left(\nabla_b dg^{ab} - \nabla^a dg \right)$$

$$= \int_\Sigma \sqrt{h} \left(n_a g^{bc} d\Gamma^a_{bc} - n^a d\Gamma^c_{ac} \right)$$

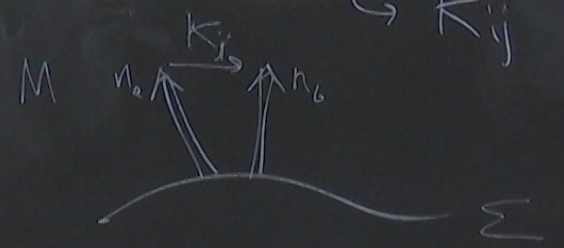
3+1 split

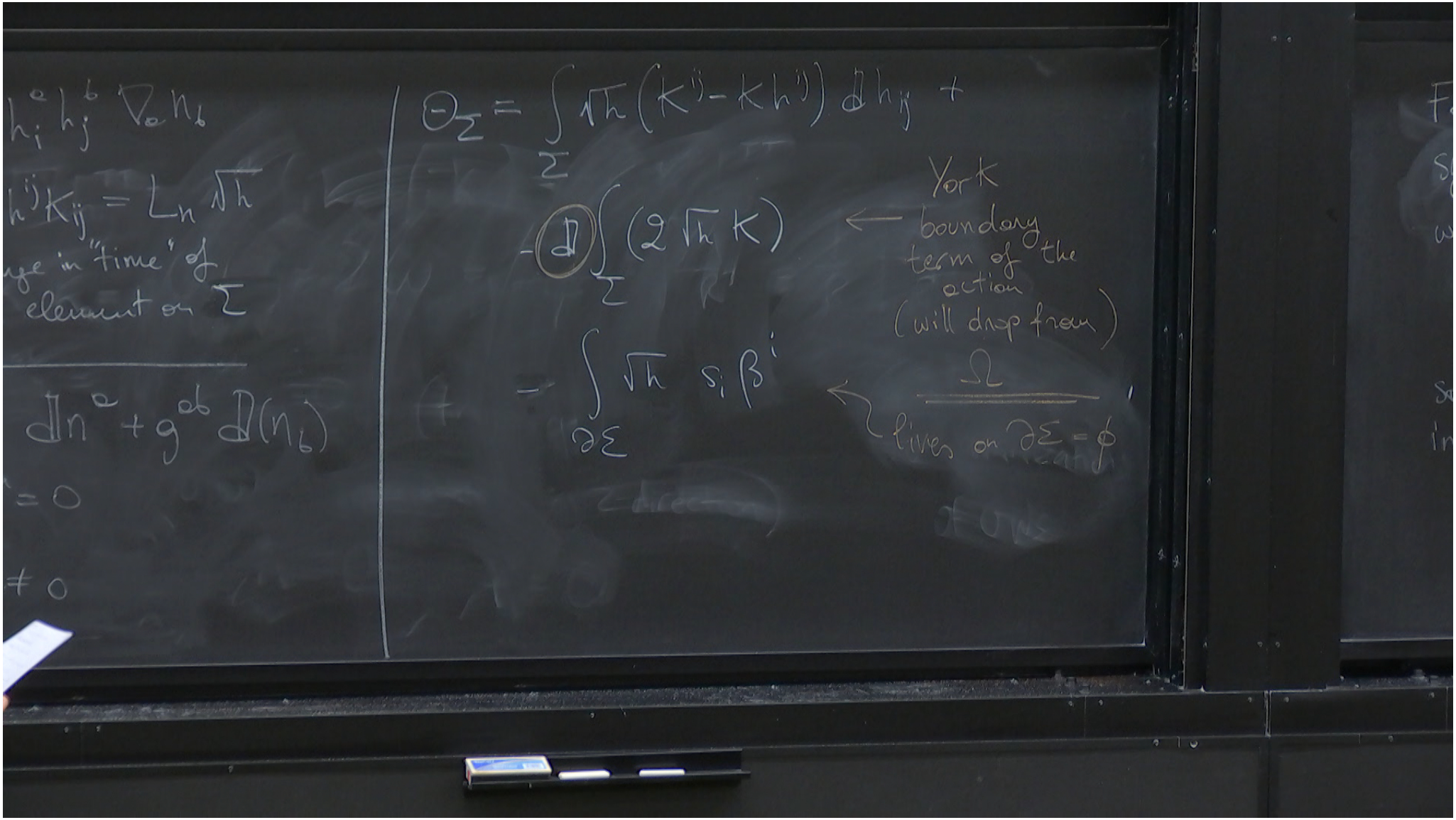
$$h_{ab} = g_{ab} + n_a n_b$$

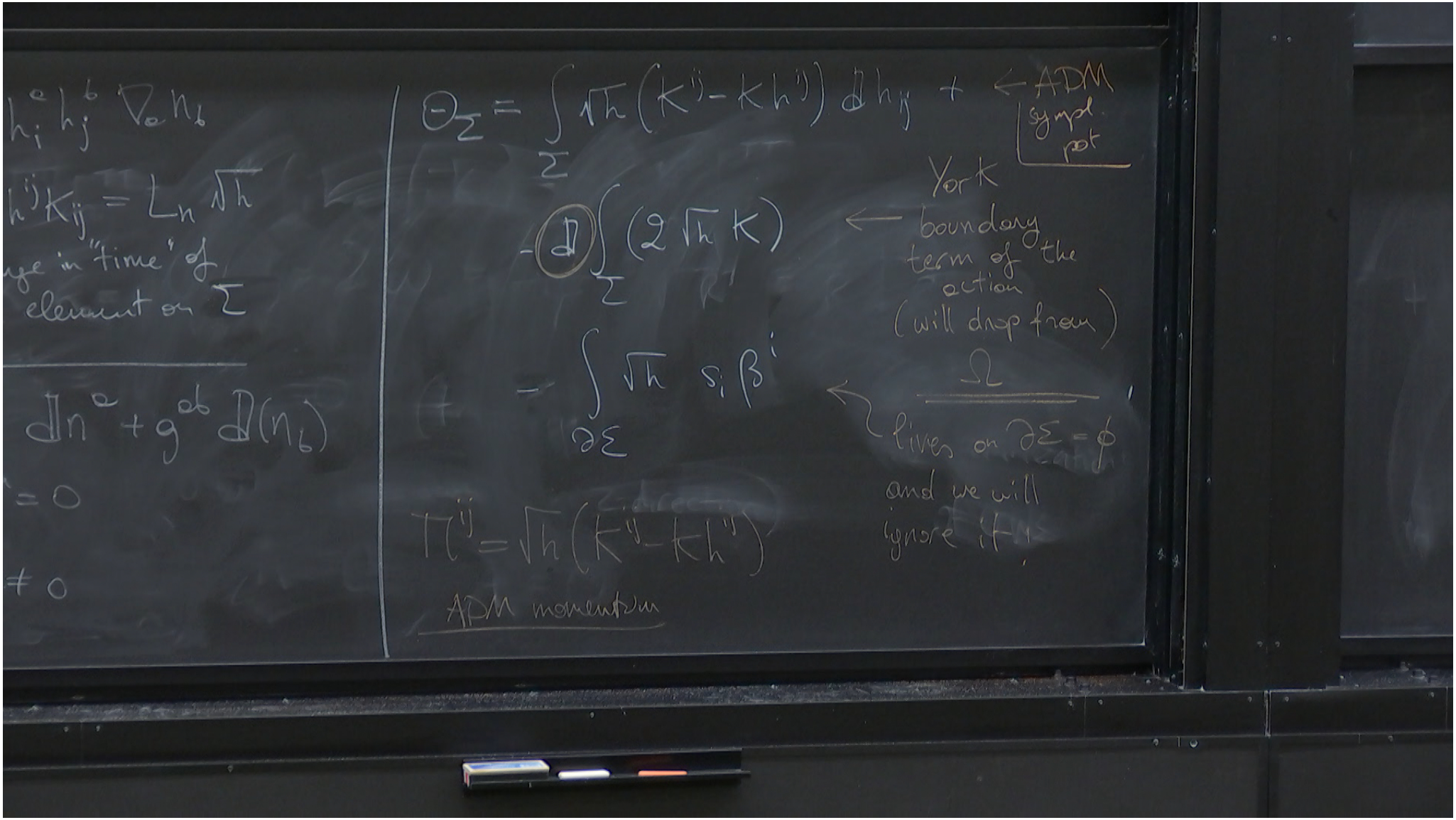
$$K_{ab} = \frac{1}{2} \mathcal{L}_n h_{ab} \sim \dot{h}_{ab}$$

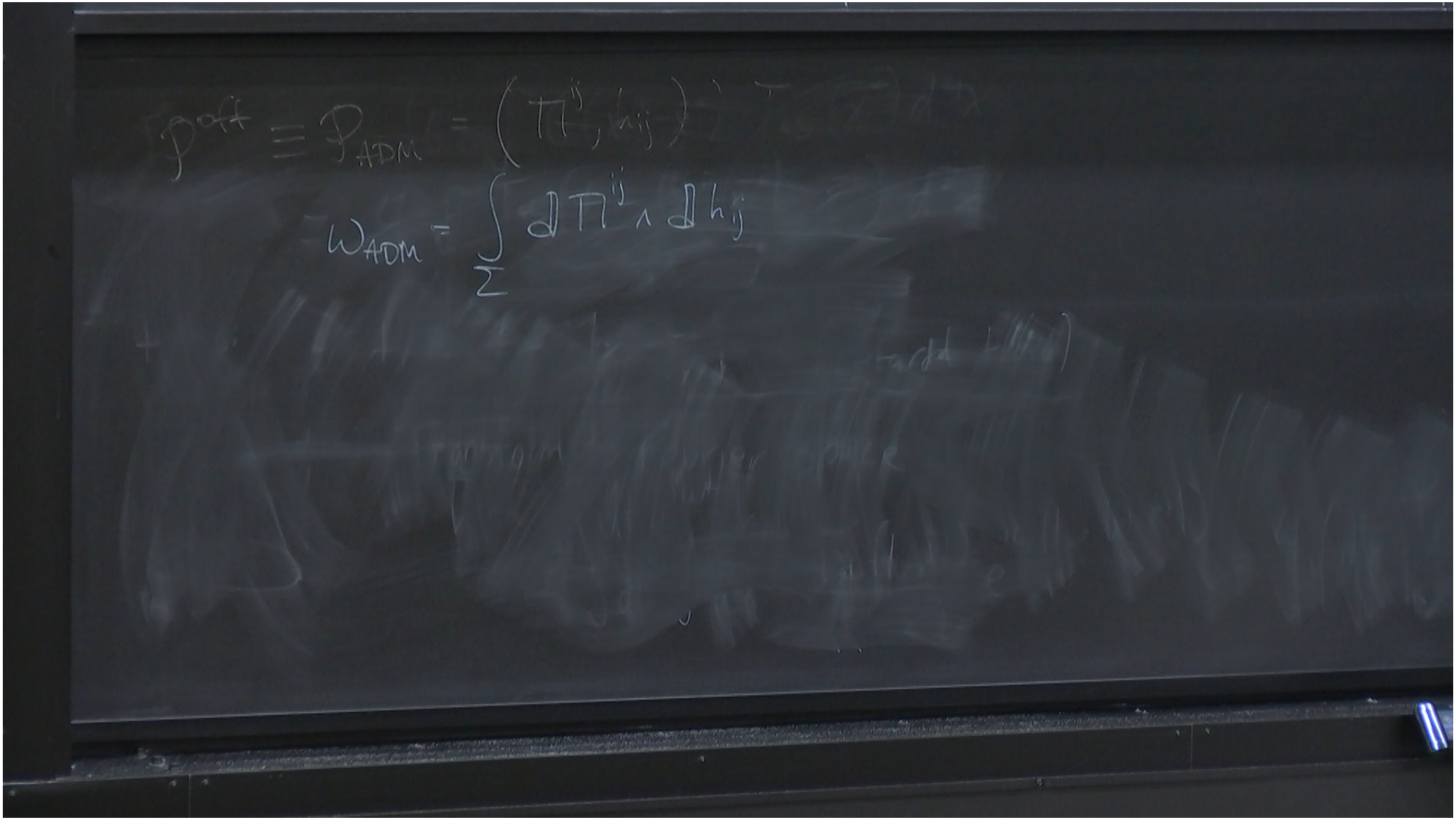
$n^a K_{ab} = 0$ tangent to Σ

$\hookrightarrow K_{ij}$




$h_i^a h_j^b \nabla_a n_b$
$h^{ij} K_{ij} = \mathcal{L}_n \sqrt{h}$
ge in "time" of
element on Σ
$dn^a + g^{ab} \mathcal{D}(n_b)$
$= 0$
$\neq 0$
$\Theta_\Sigma = \int_\Sigma \sqrt{h} (K^{ij} - K h^{ij}) \mathcal{D} h_{ij} +$
$-\mathcal{D} \int_\Sigma (2\sqrt{h} K)$
York boundary term of the action (will drop from Ω)
$-\int_{\partial\Sigma} \sqrt{\sigma}\, s_i \beta^i$
lives on $\partial\Sigma = \phi$


$h_i^a h_j^b \nabla_a n_b$
$h^{ij} K_{ij} = \mathcal{L}_n \sqrt{h}$
ge in "time" of
element on Σ
$dn^a + g^{ab} \mathcal{D}(n_b)$
$= 0$
$\neq 0$
$\Theta_\Sigma = \int_\Sigma \sqrt{h} (K^{ij} - K h^{ij}) \, d h_{ij} +$
ADM sympl pot
$-\frac{1}{\kappa} \int_\Sigma (2 \sqrt{h} K)$
York boundary term of the action (will drop from Ω)
$- \int_{\partial\Sigma} \sqrt{\sigma} \, s_i \beta^i$
lives on $\partial\Sigma = \phi$ and we will ignore it.
$\pi^{ij} = \sqrt{h} (K^{ij} - K h^{ij})$
ADM momentum


$\mathcal{P}^{\text{off}} \equiv \mathcal{P}_{\text{ADM}} = (\pi^{ij}, h_{ij})$
$\omega_{\text{ADM}} = \int_\Sigma d\pi^{ij} \wedge dh_{ij}$

$p^{\text{eff}} \equiv p_{ADM} = (\pi^{ij}, h_{ij})$

$\omega_{ADM} = \int_\Sigma \delta\pi^{ij} \wedge \delta h_{ij}$

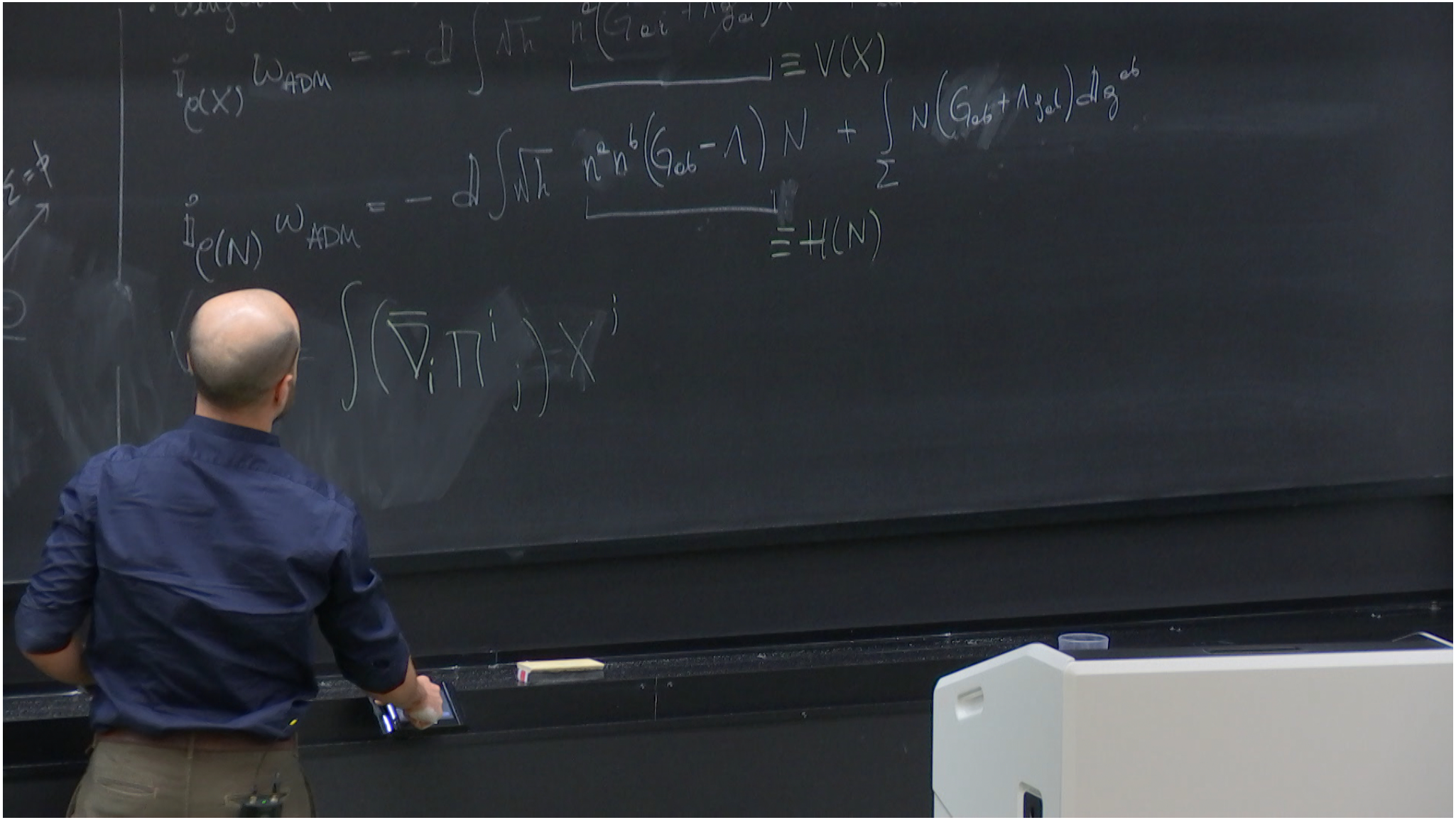
$i_{\rho(\xi)} \Omega_\Sigma = -\delta Q_\Sigma(\xi) + \int_\Sigma i_\xi E - \int_{\partial\Sigma} \xi \cdot \Theta$

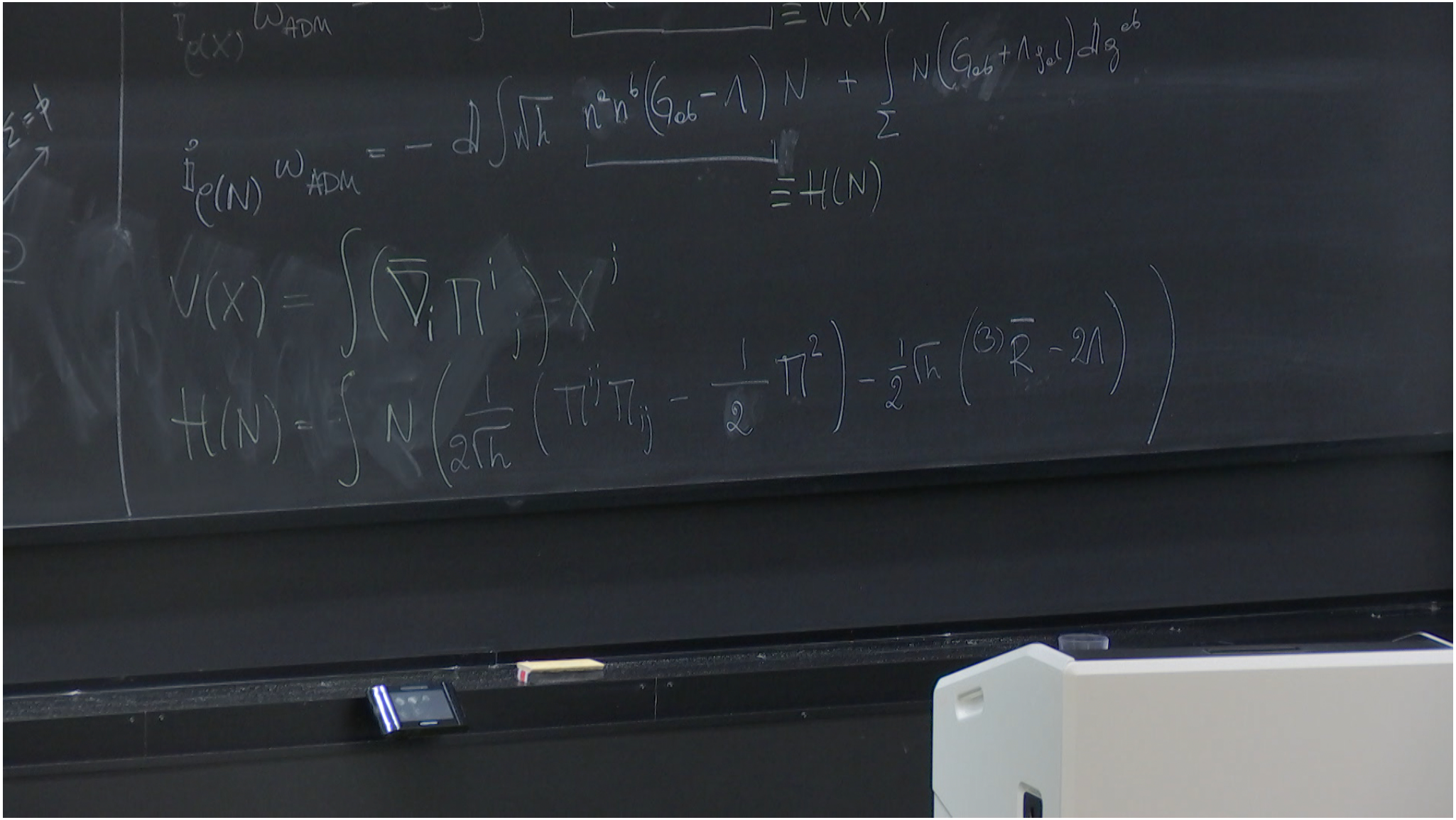
$\partial\Sigma = t$

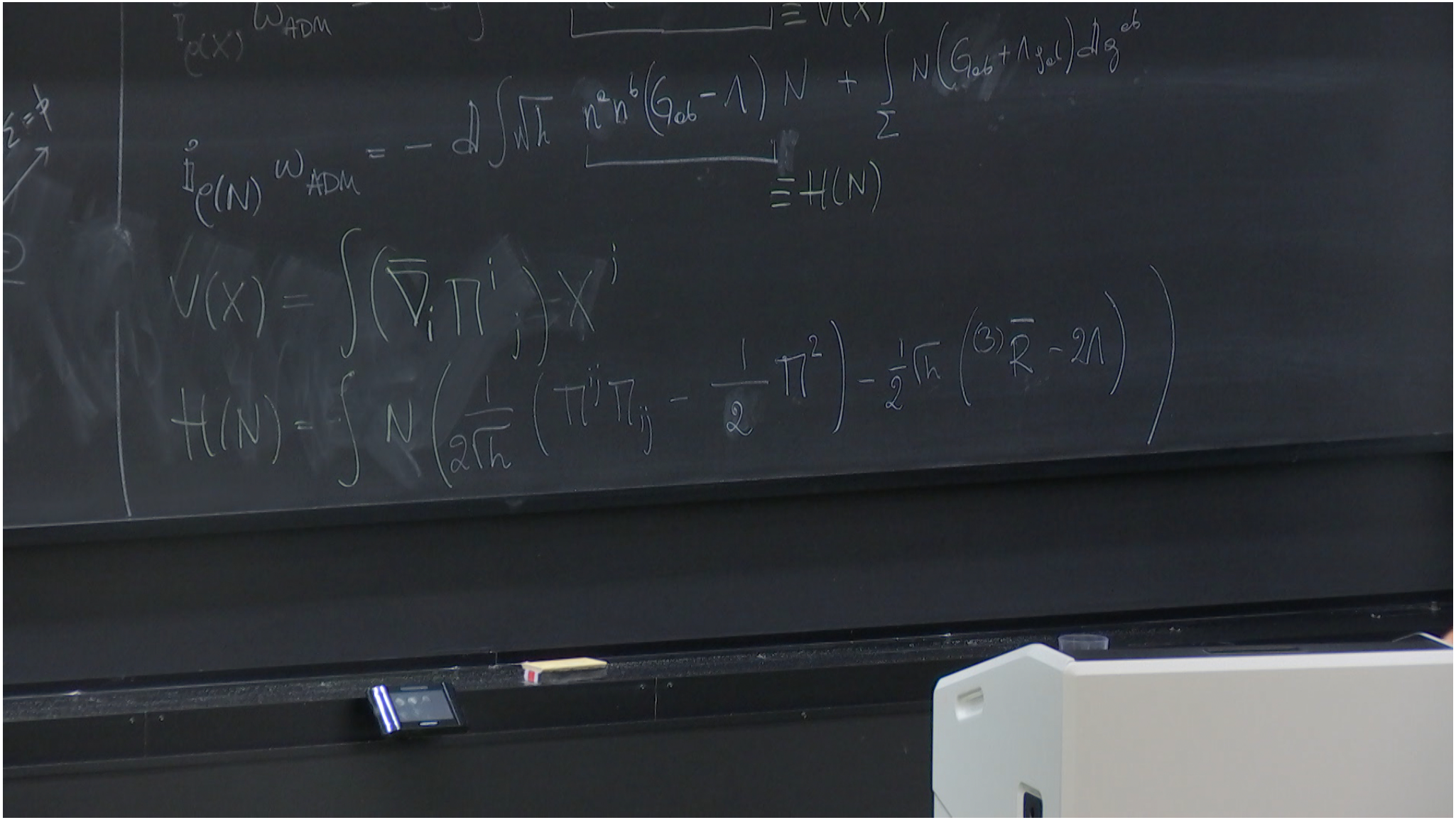
$\xi^a = X^a + N n^a$

↑ tangent vector to Σ

• tangent (spatial


$\equiv V(X)$
$\equiv H(N)$


$\delta_{\xi(X)} \omega_{ADM} = \int \ldots \equiv V(X)$
$\delta_{\xi(N)} \omega_{ADM} = -\delta \int \sqrt{h}\, n^a n^b (G_{ab} - \Lambda) N + \int_\Sigma N (G_{ab} + \Lambda g_{ab}) d^3 g^{ab}$
$\equiv H(N)$
$V(X) = \int (\bar{\nabla}_i \pi^i{}_j) X^j$
$H(N) = \int N \left(\frac{1}{2\sqrt{h}} \left(\pi^{ij} \pi_{ij} - \frac{1}{2} \pi^2 \right) - \frac{1}{2} \sqrt{h} \left({}^{(3)}\bar{R} - 2\Lambda \right) \right)$


ω_{ADM}
$\equiv V(X)$
$\delta_{\varphi(N)}\omega_{ADM} = -\delta\int_\Sigma \sqrt{h}\, n^a n^b (G_{ab} - \Lambda) N + \int_\Sigma N(G_{ab} + \Lambda g_{ab}) d^3x^{ab}$
$\equiv H(N)$
$V(X) = \int (\bar{\nabla}_i \Pi^i{}_j) X^j$
$H(N) = \int N \left(\frac{1}{2\sqrt{h}} \left(\Pi^{ij}\Pi_{ij} - \frac{1}{2}\Pi^2 \right) - \frac{1}{2}\sqrt{h} \left({}^{(3)}\bar{R} - 2\Lambda \right) \right)$