Title: Quantum Gravity

Speakers: Aldo Riello

Collection: Quantum Gravity (2021-2022)

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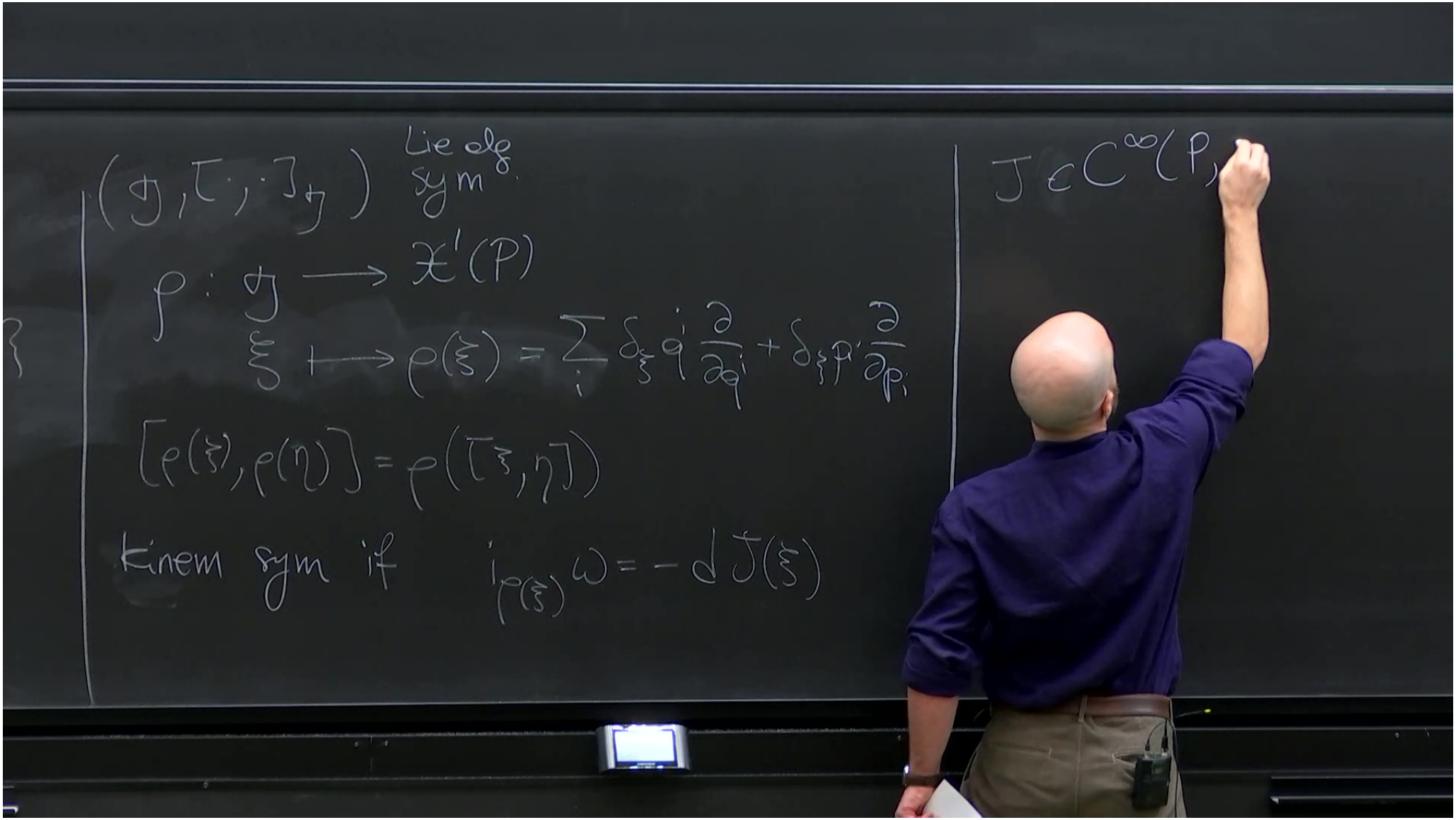
Recap

- $\omega \in \Omega^2(P)$
  - 1) $d\omega = 0$
  - 2) non-deg.
- HVF X : $i_X \omega = -df_X$

  Ham generator
- $[X_f, X_g] = X_{\{f,g\}}$


$(\mathfrak{g}, [\cdot,\cdot]_{\mathfrak{g}})$ Lie alg sym
$\rho : \mathfrak{g} \longrightarrow \mathfrak{X}'(P)$
$\xi \longmapsto \rho(\xi) = \sum \delta_\xi q^i \frac{\partial}{\partial q^i} + \delta_\xi p^i \frac{\partial}{\partial p_i}$
$[\rho(\xi), \rho(\eta)] = \rho([\xi,\eta])$
kinem sym if $i_{\rho(\xi)}\omega = -dJ(\xi)$
$J \in C^\infty(P,$

$J \in C^\infty(P, \mathfrak{g}^*)$

Rot $\quad \vec{\xi} \cdot \vec{\tau} \in so(3)$

$J(\xi) = (\vec{p} \times \vec{q}) \cdot \vec{\xi}$

$\omega = \sum dv_i \wedge dq^i + \frac{1}{2} B_{ij} dq^i \wedge dq^j$

$\delta_\xi q^i \frac{\partial}{\partial q^i} + \delta_\xi p^i \frac{\partial}{\partial p_i}$

$= -d J(\xi)$

$J \in C^\infty(P, \mathfrak{g}^*)$

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Rot $\quad \vec{\xi}.\vec{\tau} \in so(3)$

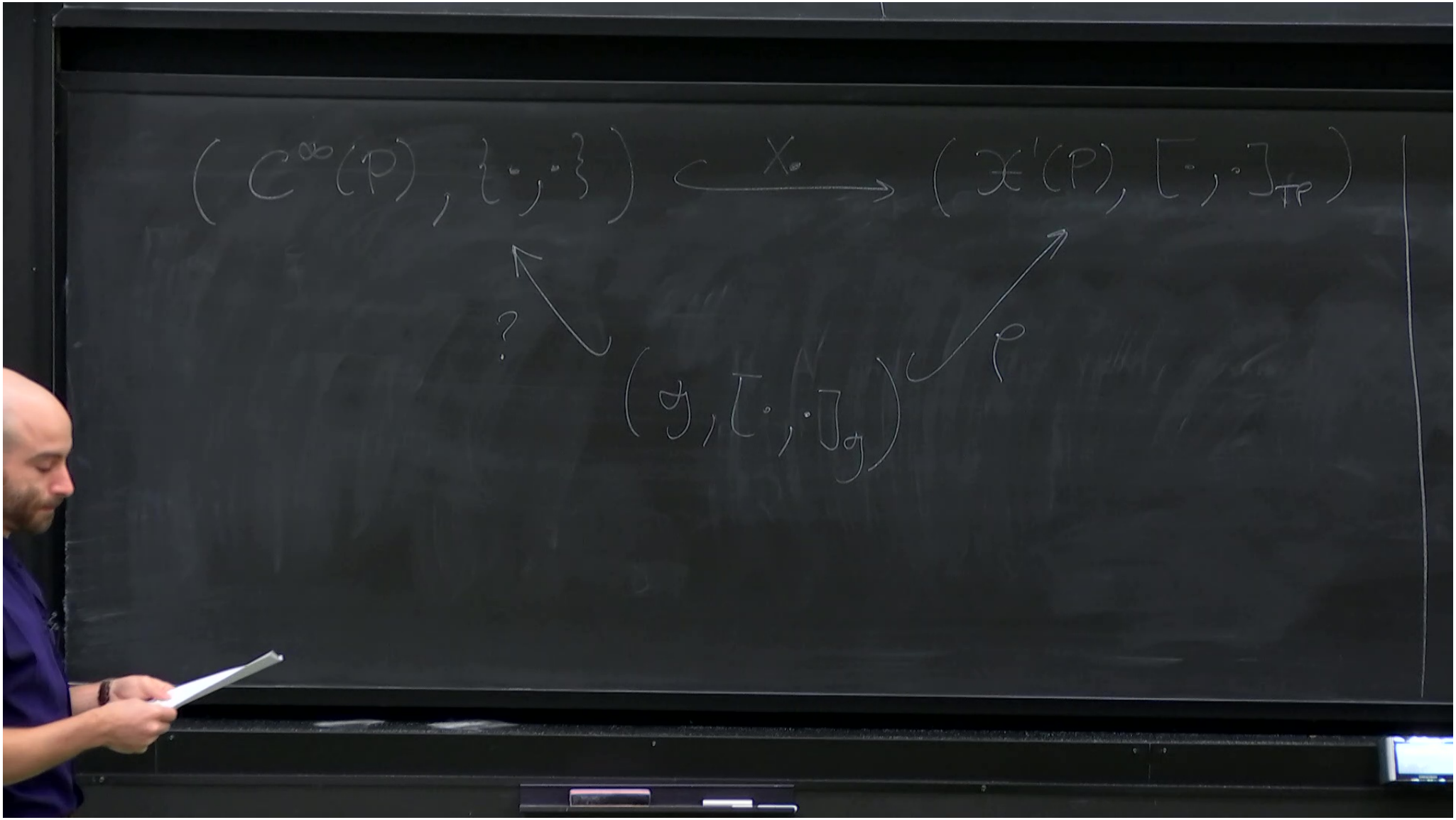
$J(\xi) = (\vec{p} \times \vec{q}) \cdot \vec{\xi}$

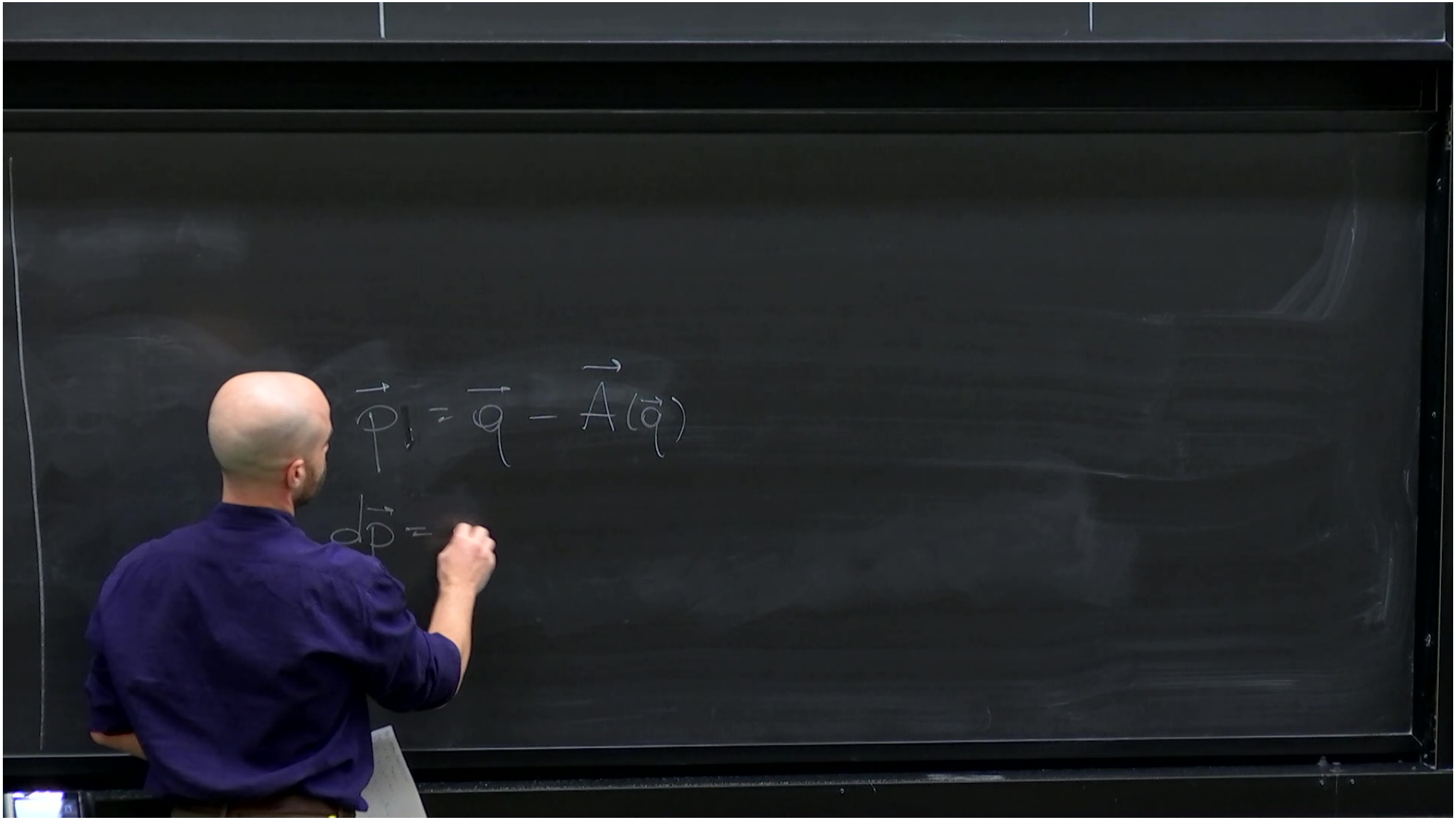
$\omega = \sum d v_i \wedge dq^i + \frac{1}{2} B_{ij} dq^i \wedge dq^j$

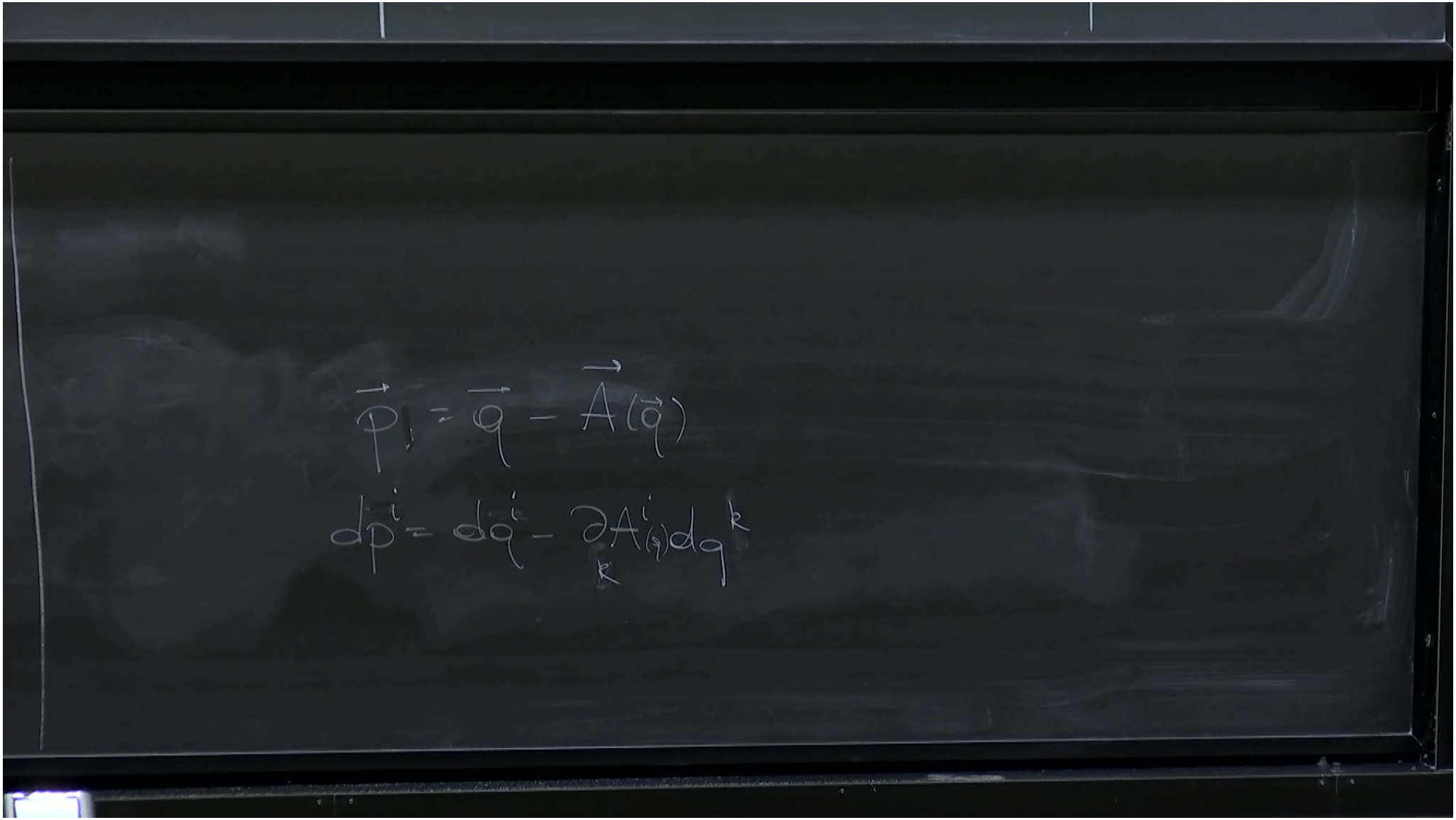
dynamical sym $\{H, J(\xi)\} = 0$

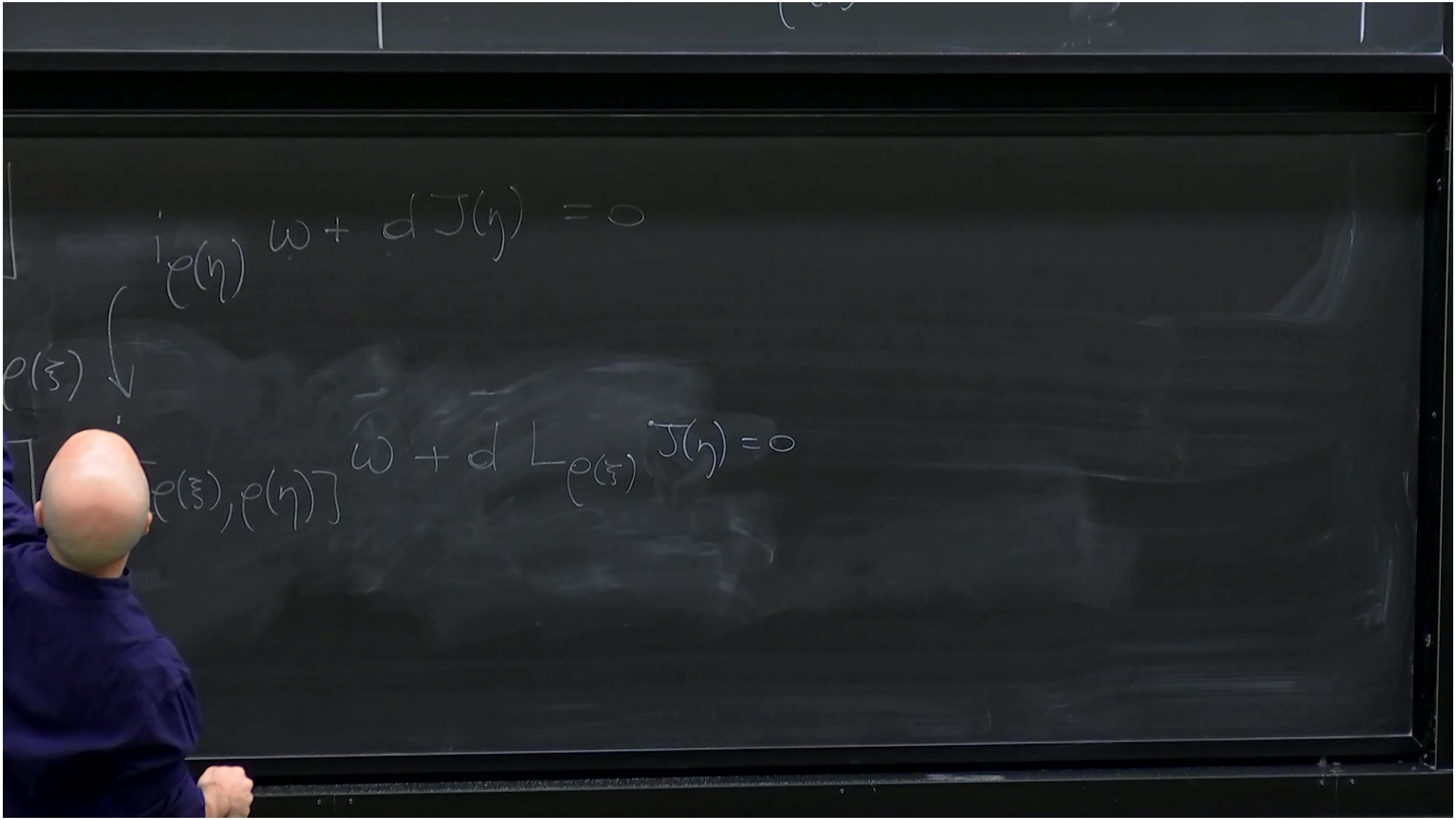
$\delta_\xi q^i \frac{\partial}{\partial q^i} + \delta_\xi p^i \frac{\partial}{\partial p_i}$

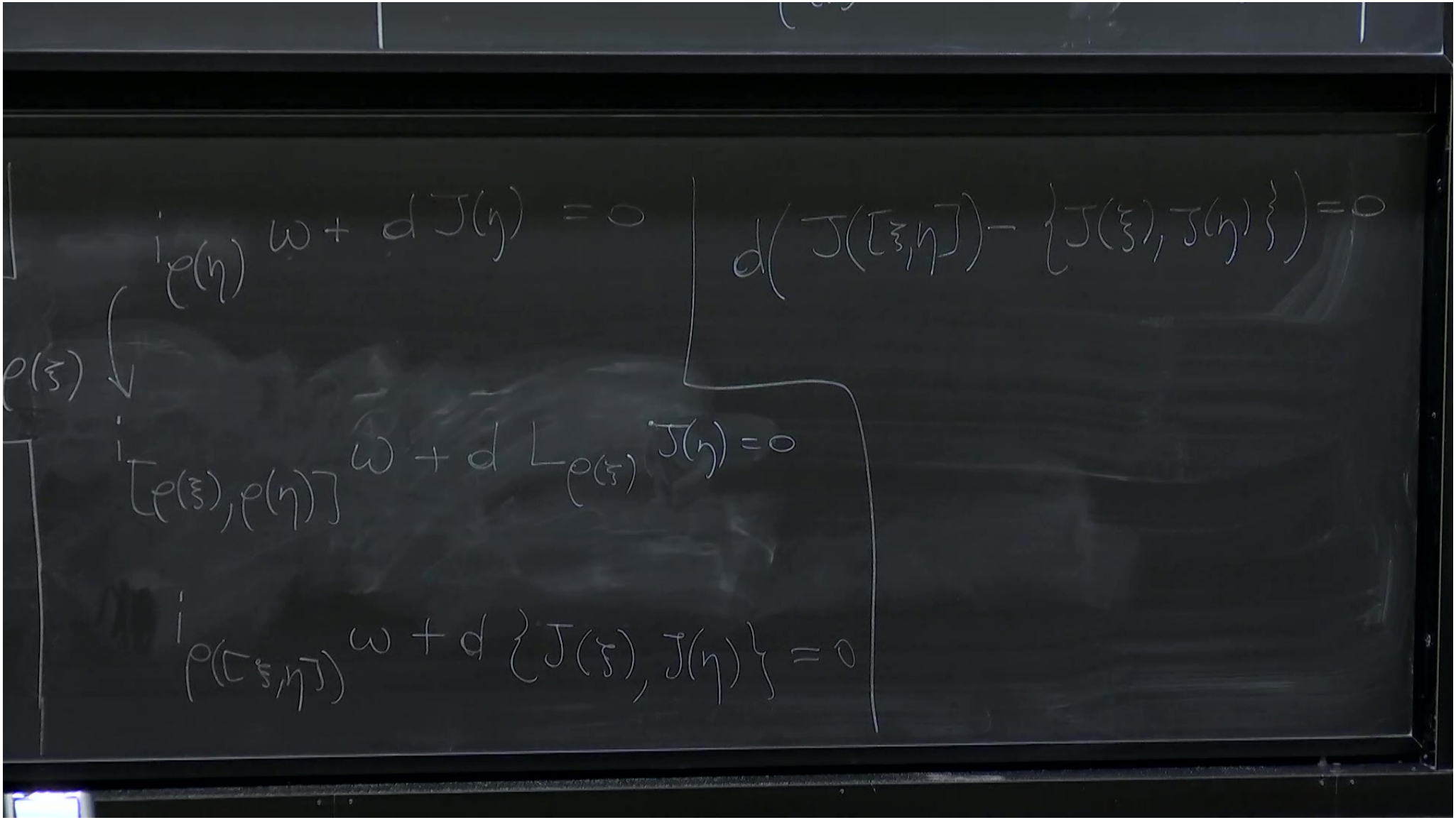
$= -d J(\xi)$


$(C^\infty(P), \{\cdot,\cdot\})$
$X_\bullet$
$(\mathfrak{X}'(P), [\cdot,\cdot]_{TP})$
?
$(\mathfrak{g}, [\cdot,\cdot]_{\mathfrak{g}})$
ρ


$\vec{p} = \vec{q} - \vec{A}(\vec{q})$
$d\vec{p} =$


$\vec{p} = \vec{q} - \vec{A}(\vec{q})$
$dp^i = dq^i - \frac{\partial A^i_{(q)}}{\partial q^k} dq^k$


$i_{\rho(\eta)}\omega + dJ(\eta) = 0$
$\rho(\xi)$
$[\rho(\xi), \rho(\eta)]$
$\omega + d L_{\rho(\xi)} J(\eta) = 0$


$i_{\rho(\eta)}\omega + dJ(\eta) = 0$
$d(J([\xi,\eta]) - \{J(\xi), J(\eta)\}) = 0$
$\rho(\xi)$
$i_{[\rho(\xi),\rho(\eta)]}\omega + d L_{\rho(\xi)} J(\eta) = 0$
$i_{\rho([\xi,\eta])}\omega + d\{J(\xi), J(\eta)\} = 0$

$$i_{\rho(\eta)}\omega + dJ(\eta) = 0$$

$$d\big(J([\xi,\eta]) - \{J(\xi), J(\eta)\}\big) = 0$$

$$k(\xi,\eta) \in \mathbb{R}$$

$\rho(\xi)\downarrow$

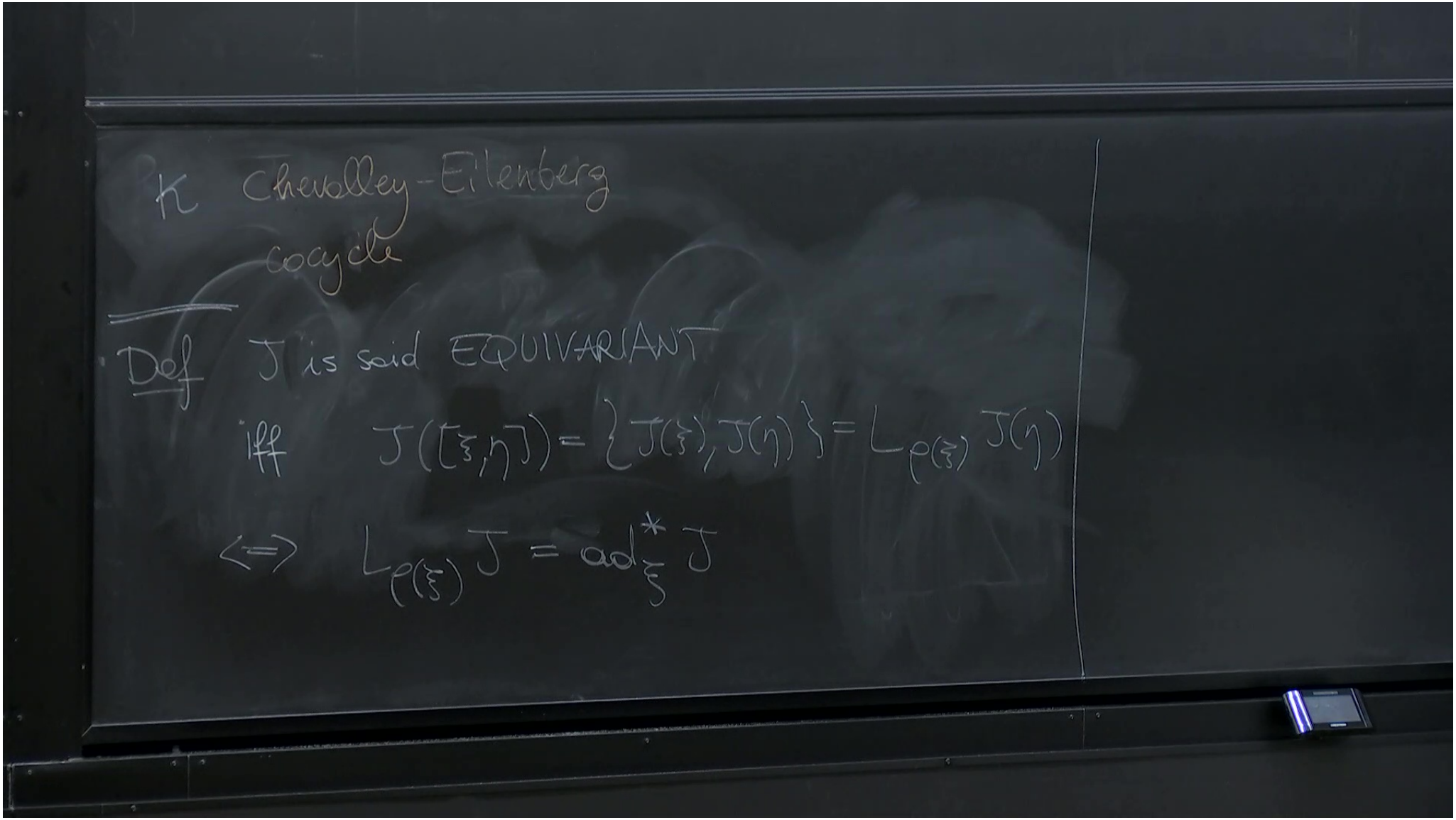
$$i_{[\rho(\xi),\rho(\eta)]}\omega + d\,L_{\rho(\xi)}J(\eta) = 0$$

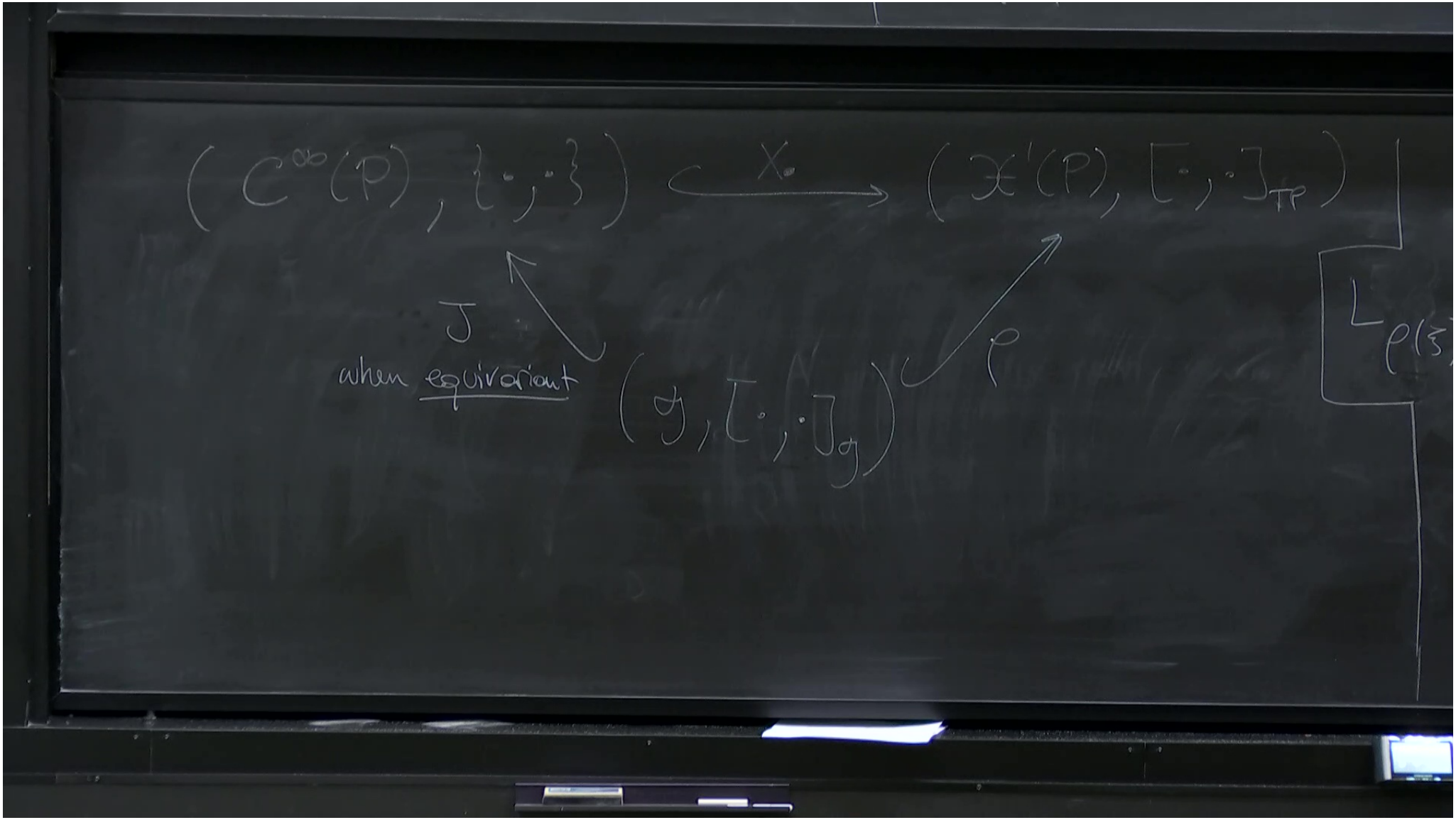
Ex $\quad dk(\xi,\eta) = 0$

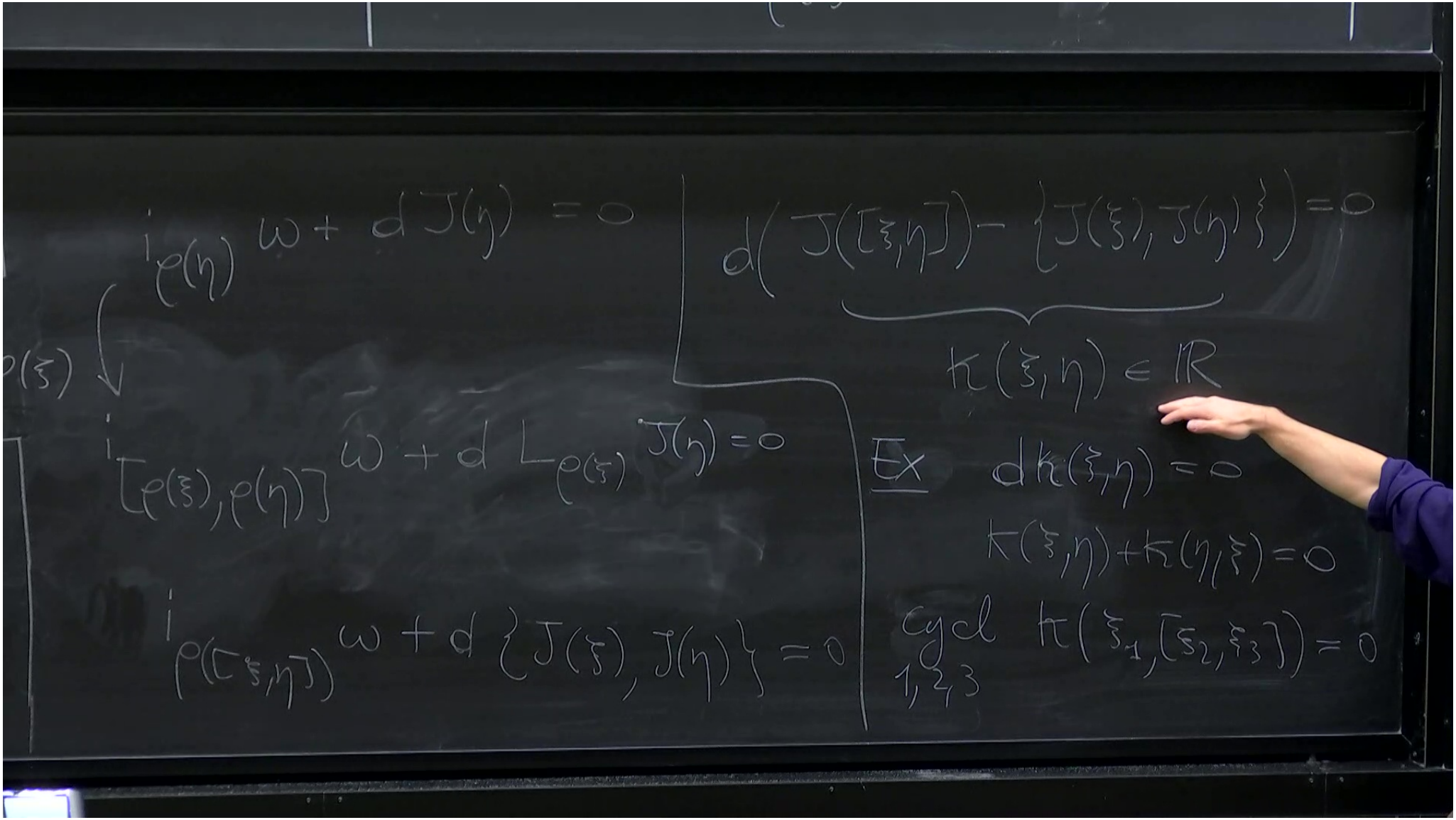
$$k(\xi,\eta) + k(\eta,\xi) = 0$$

$$i_{\rho([\xi,\eta])}\omega + d\{J(\xi), J(\eta)\} = 0$$

$$\underset{1,2,3}{\text{cycl}}\; k(\xi_1,[\xi_2,\xi_3]) = 0$$


K Chevalley-Eilenberg cocycle
Def J is said EQUIVARIANT
iff $J([\xi,\eta]) = \{J(\xi), J(\eta)\} = L_{\rho(\xi)} J(\eta)$
$\Leftrightarrow$ $L_{\rho(\xi)} J = ad^*_\xi J$


$(C^\infty(P), \{\cdot,\cdot\})$
$X_\bullet$
$(\mathfrak{X}(P), [\cdot,\cdot]_{TP})$
J
when equivariant
$(\mathfrak{g}, [\cdot,\cdot]_{\mathfrak{g}})$
ρ
$L_{\rho(\xi)}$

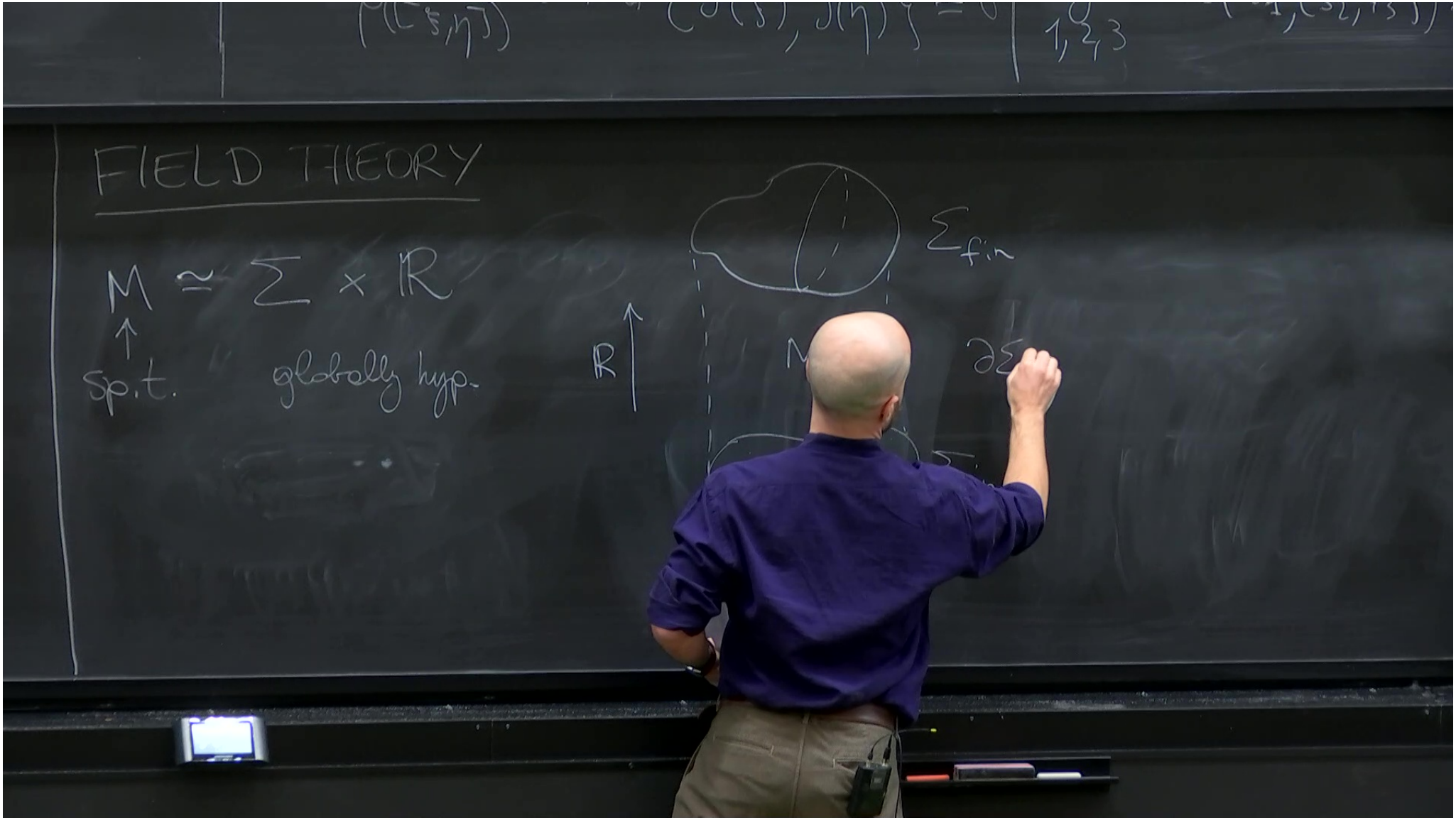

$i_{\rho(\eta)}\omega + d J(\eta) = 0$
$d(J([\xi,\eta]) - \{J(\xi), J(\eta)\}) = 0$
$k(\xi,\eta) \in \mathbb{R}$
$i_{[\rho(\xi),\rho(\eta)]}\omega + d L_{\rho(\xi)} J(\eta) = 0$
Ex $dk(\xi,\eta) = 0$
$k(\xi,\eta) + k(\eta,\xi) = 0$
$i_{\rho([\xi,\eta])}\omega + d\{J(\xi), J(\eta)\} = 0$
$\underset{1,2,3}{cycl} k(\xi_1,[\xi_2,\xi_3]) = 0$

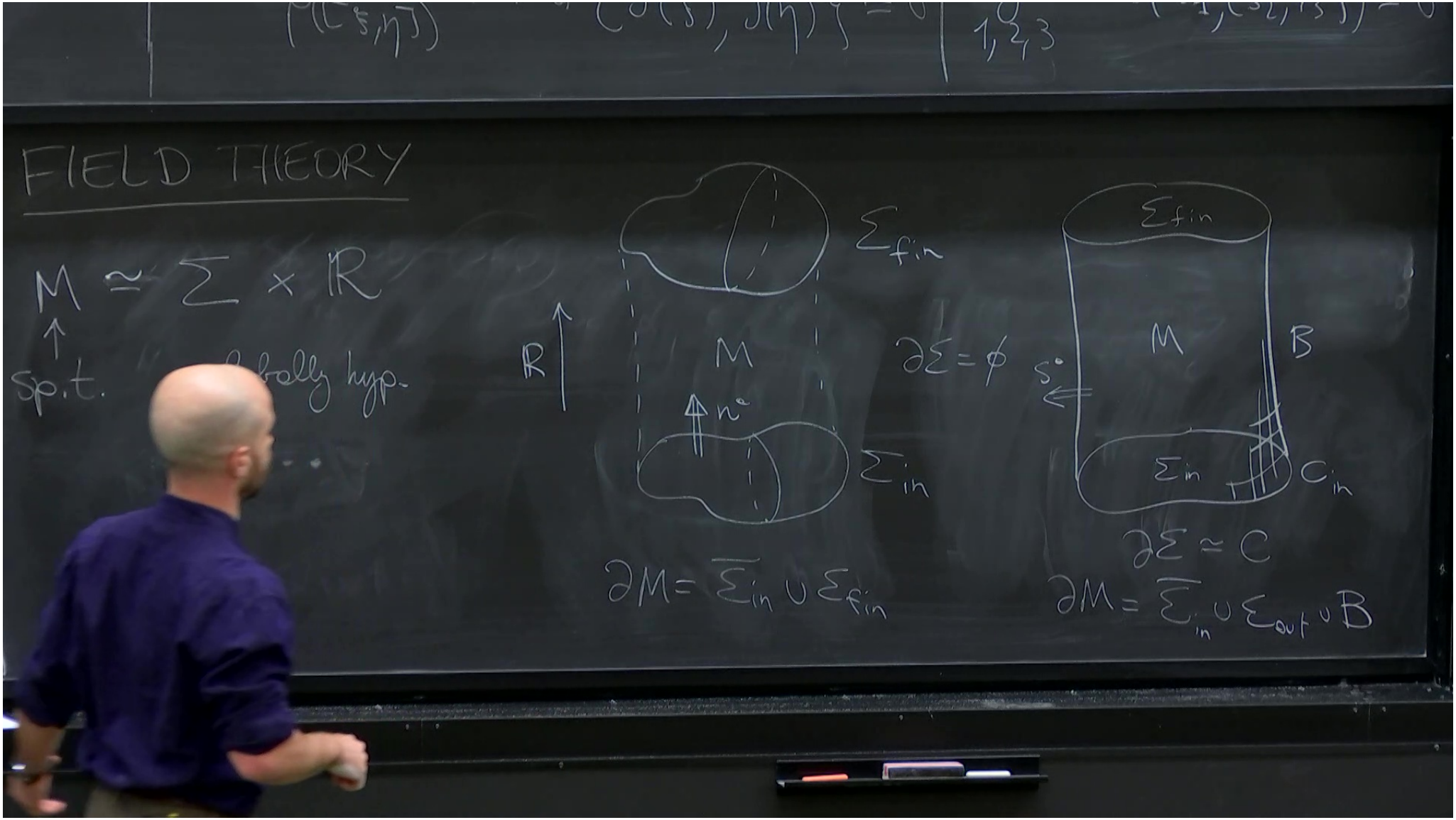
k CE cocycle

DEF Equivariance

$$\{J(\xi), J(\eta)\} = J([\xi, \eta])$$

$$\hookrightarrow \quad L_{\rho(\xi)} J = \mathrm{ad}^*_\xi J$$


FIELD THEORY
M ≃ Σ × ℝ
↑
Sp.t.
globally hyp.
ℝ
Σ fin
M
∂Σ

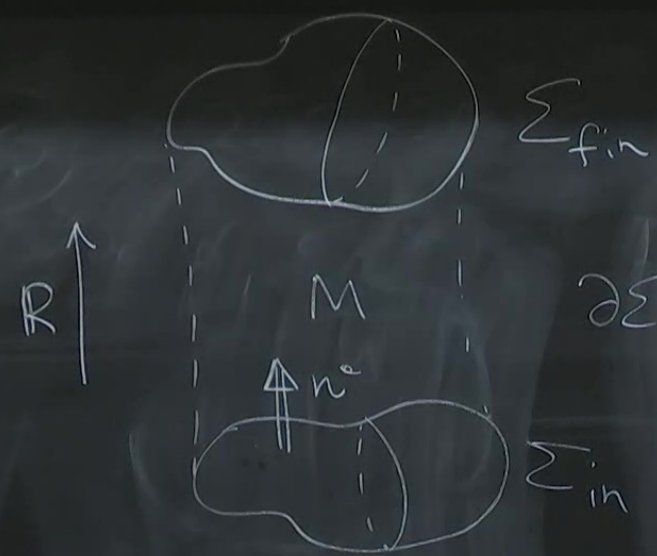

FIELD THEORY
M ≃ Σ × ℝ
sp.t.
ally hyp.
ℝ
M
Σ_fin
Σ_in
n^a
∂Σ = ∅
s^a
M
B
Σ_fin
Σ_in
C_in
∂Σ ≃ C
∂M = Σ̄_in ∪ Σ_fin
∂M = Σ̄_in ∪ Σ_out ∪ B

$\rho([\xi,\eta])$ $\quad \{J(\xi), J(\eta)\} = 0$ $\quad$ $1,2,3$ $\quad \mathcal{H}(\xi_1,(\xi_2,\xi_3)) = 0$

LD THEORY

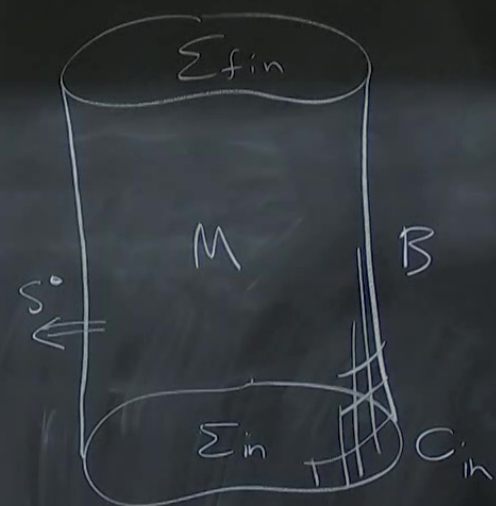
$\simeq \Sigma \times \mathbb{R}$

globally hyp.



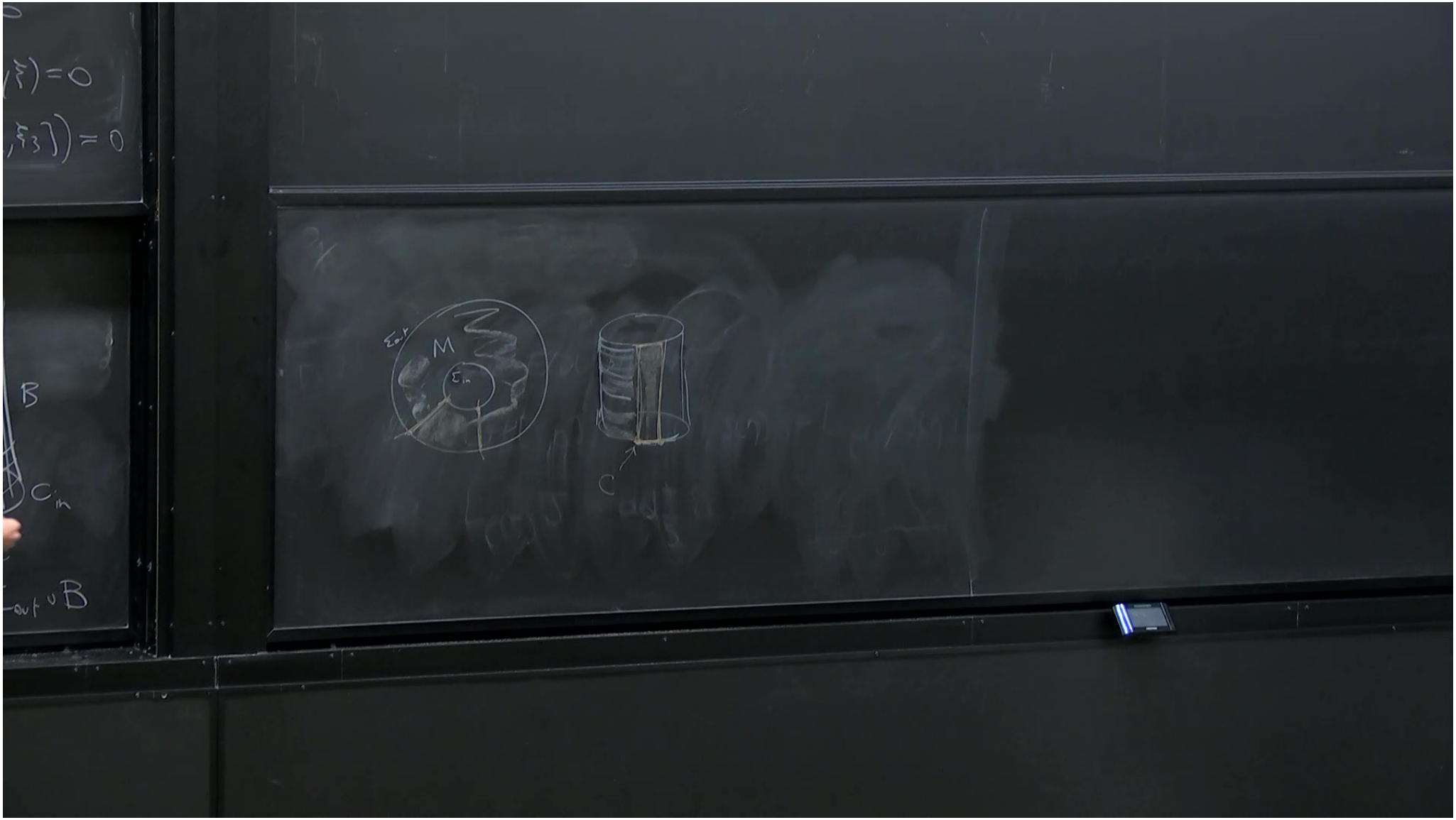
$\partial\Sigma = \phi$

$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{fin}$



$\partial\Sigma \simeq C$

$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{out} \cup B$

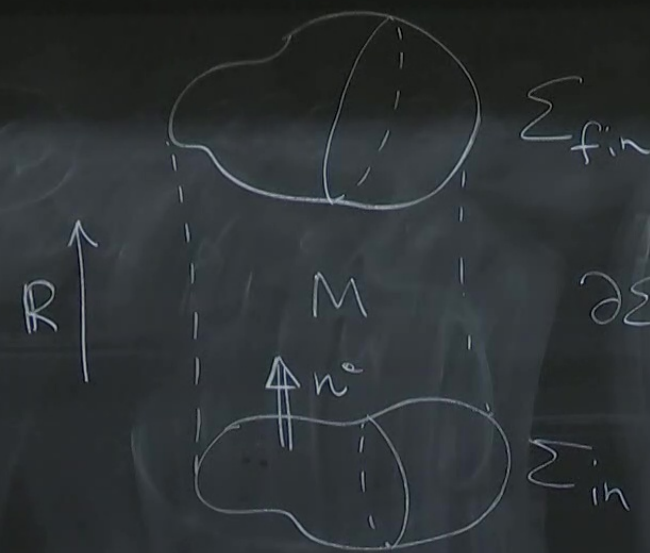

M
C

$\rho([\xi,\eta])\,\omega + d\{J(\xi), J(\eta)\} = 0$ | $\underset{1,2,3}{\text{cycl}}\ \pi(\xi_1,[\xi_2,\xi_3]) = 0$

LD THEORY

$\simeq \Sigma \times \mathbb{R}$

globally hyp.



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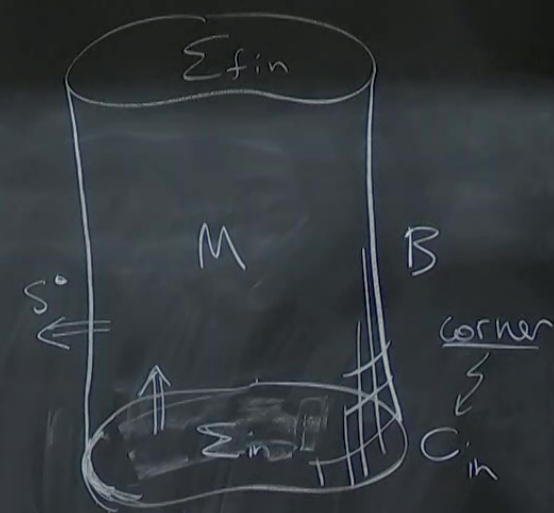


$\partial\Sigma \simeq C$

$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{out} \cup B$

$\rho([\xi,\eta])\,\omega + d\{J(\xi), J(\eta)\} = 0$ | $\underset{1,2,3}{\text{cyc}}\ \pi(\xi_1,[\xi_2,\xi_3]) = 0$

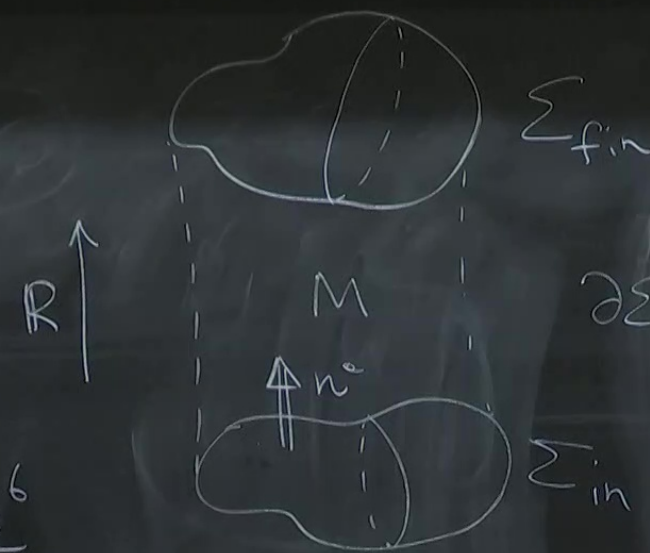
LD THEORY

$\simeq \Sigma \times \mathbb{R}$

globally hyp.

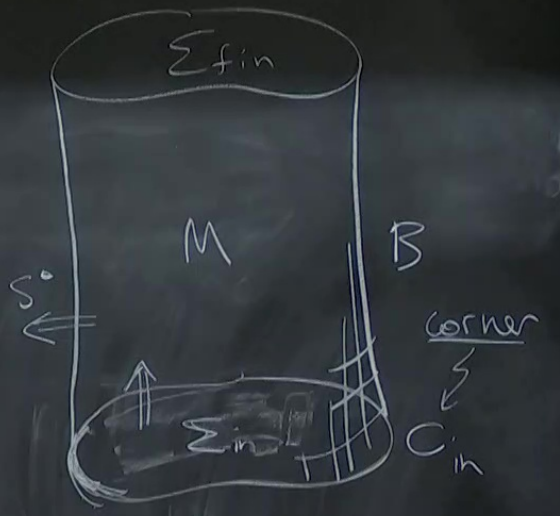
rmal : $\varepsilon^{ab} = \dfrac{n^a s^b - s^a n^b}{\sqrt{1 - n\cdot s}}$

C



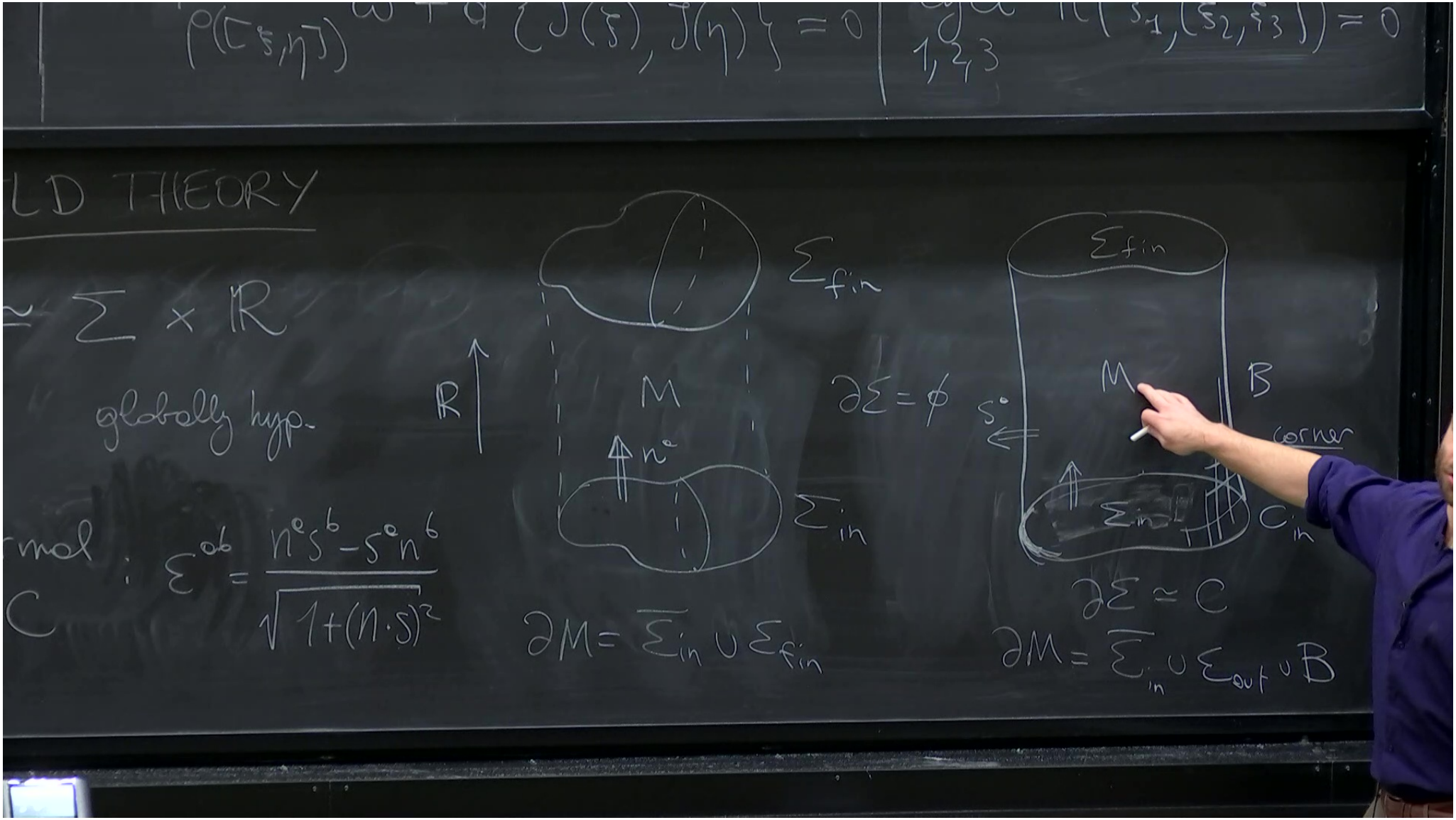
$\partial\Sigma = \phi$

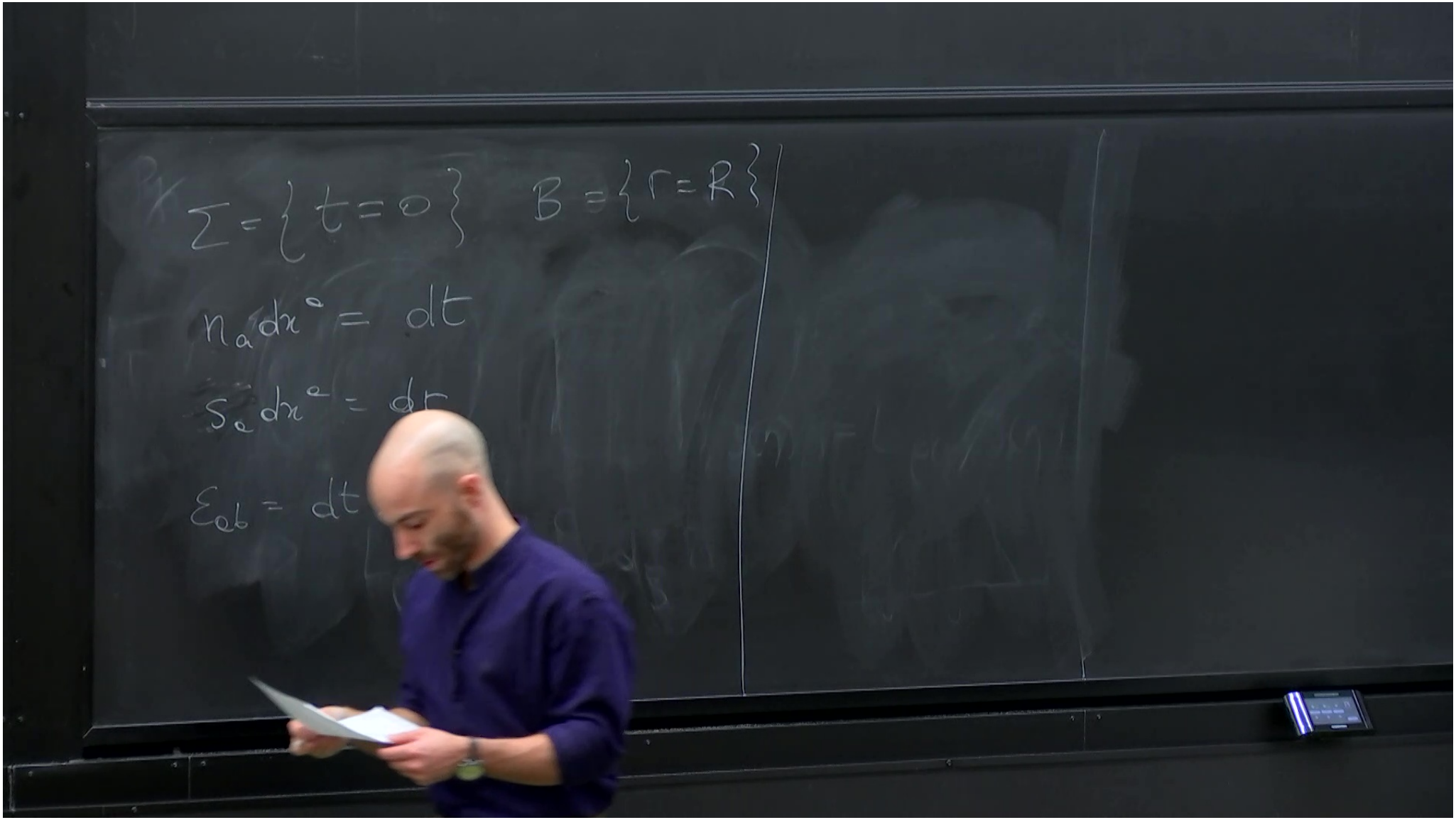
$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{fin}$

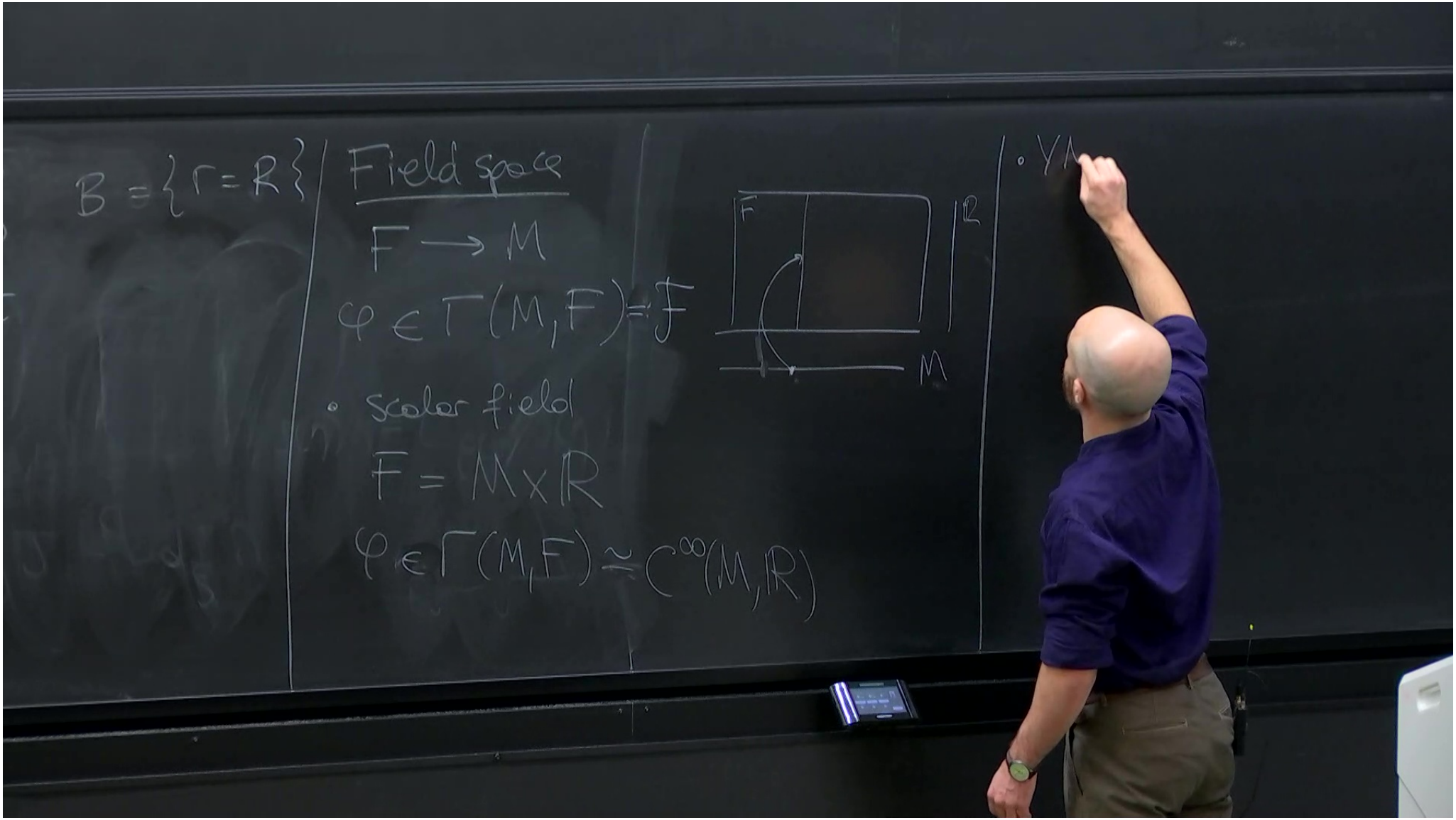


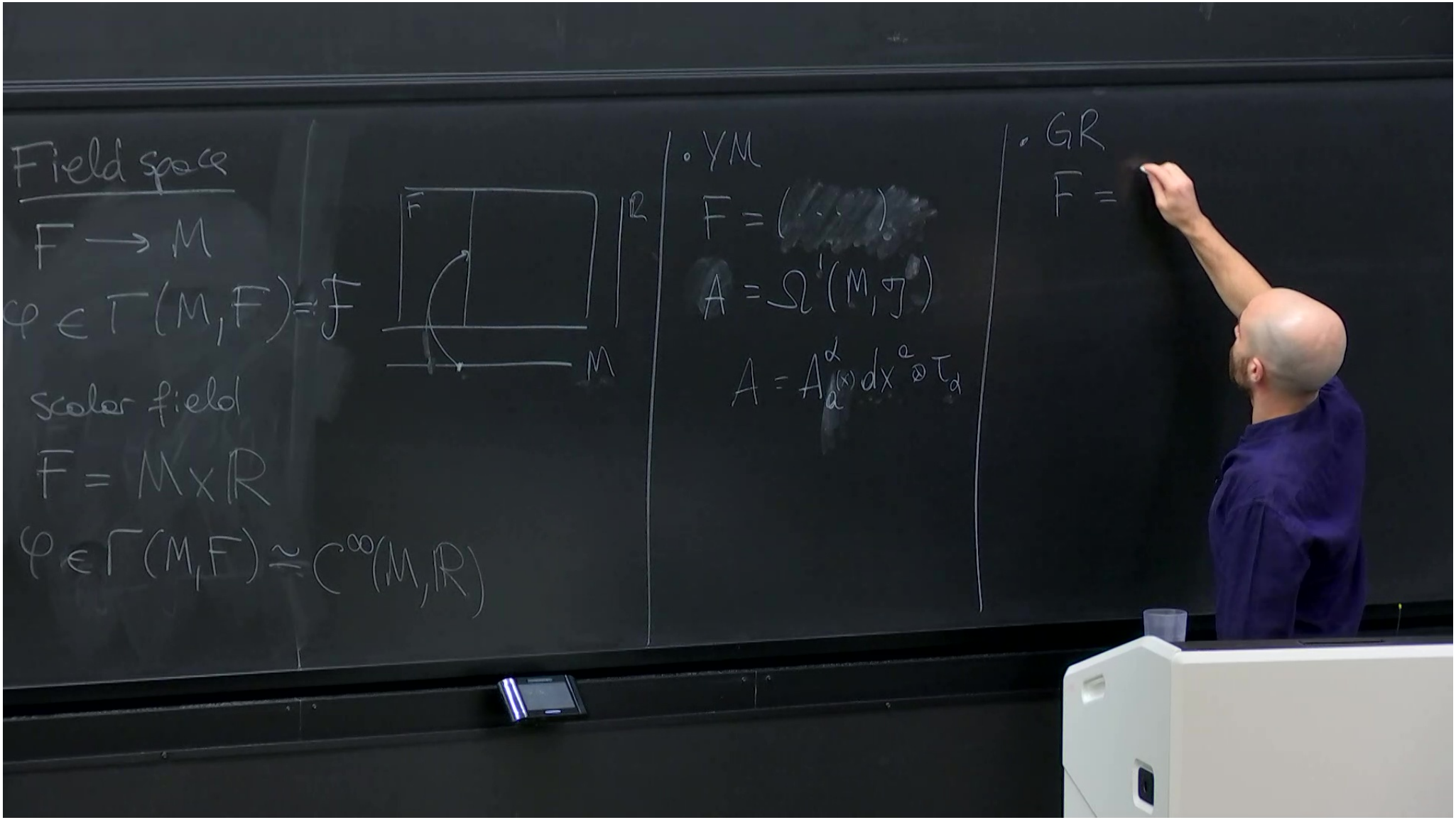
$\partial\Sigma \simeq C$

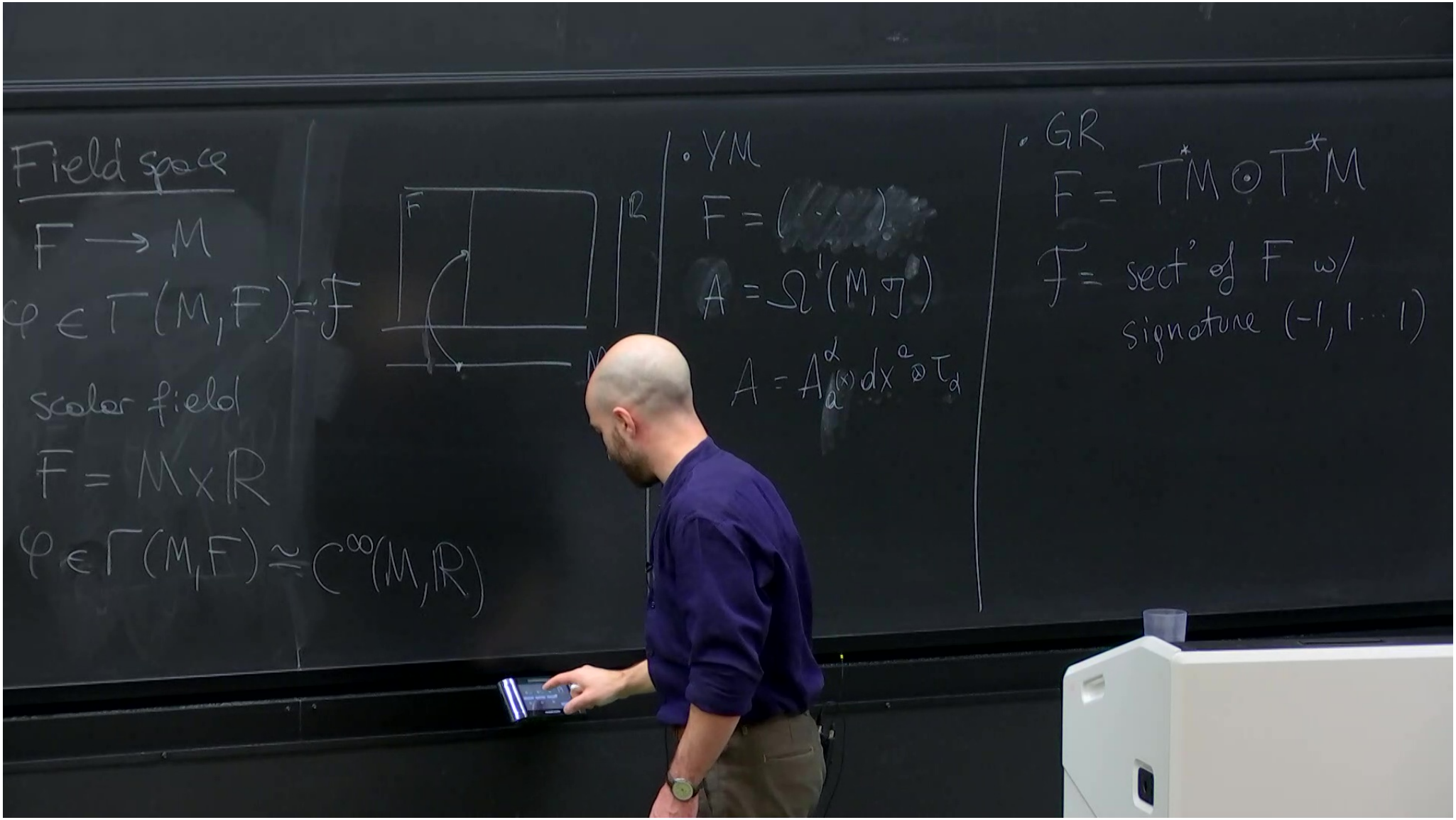
$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{out} \cup B$

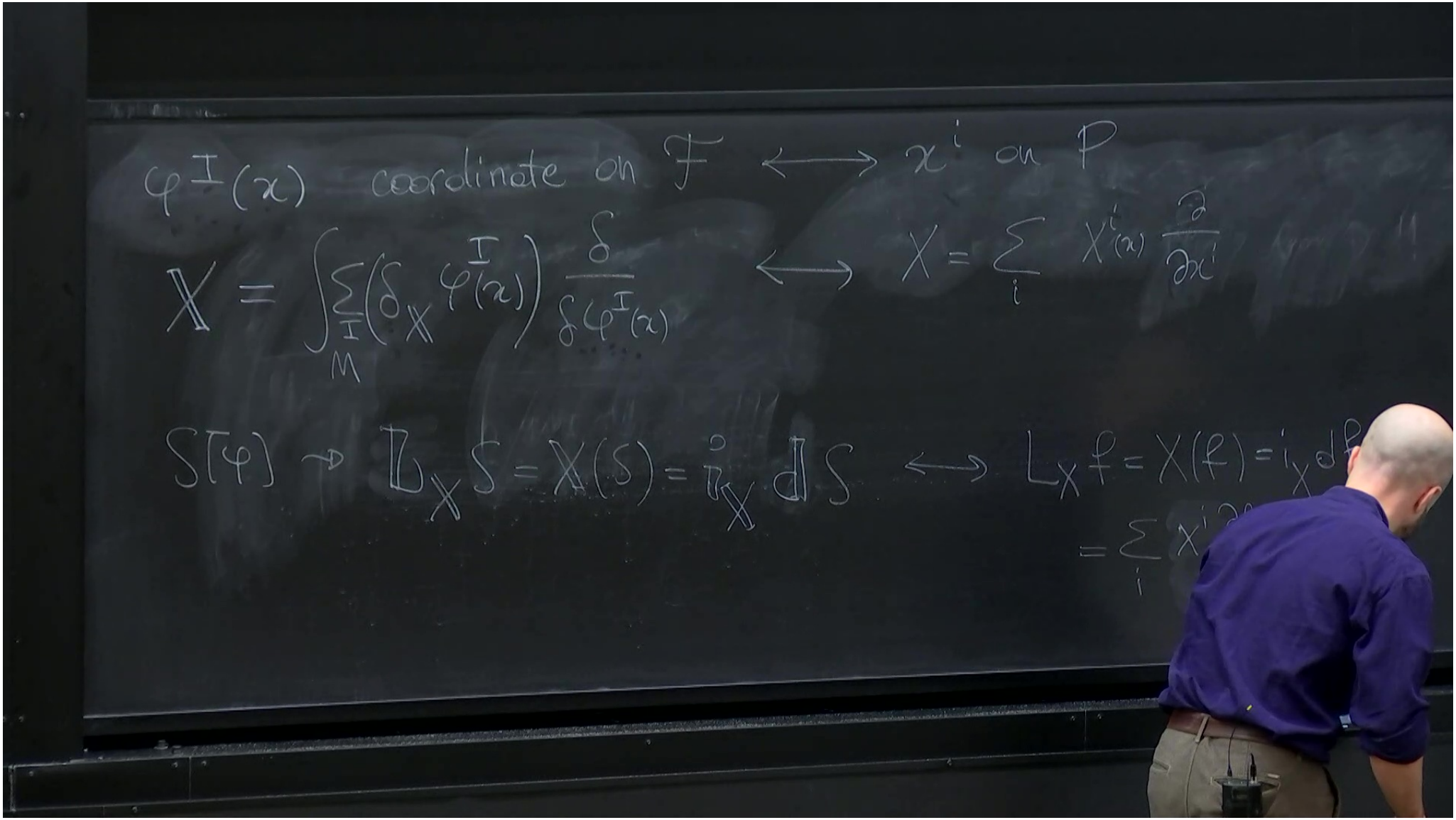

LD THEORY
≃ Σ × ℝ
globally hyp.
R
M
n^a
Σ_{fin}
Σ_{in}
$\partial\Sigma = \phi$
$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{fin}$
Σ_{fin}
M
B
s^a
corner
Σ_{in}
C_{in}
$\partial\Sigma \simeq C$
$\partial M = \overline{\Sigma}_{in} \cup \Sigma_{out} \cup B$
$\varepsilon^{ab} = \frac{n^a s^b - s^a n^b}{\sqrt{1+(n\cdot s)^2}}$

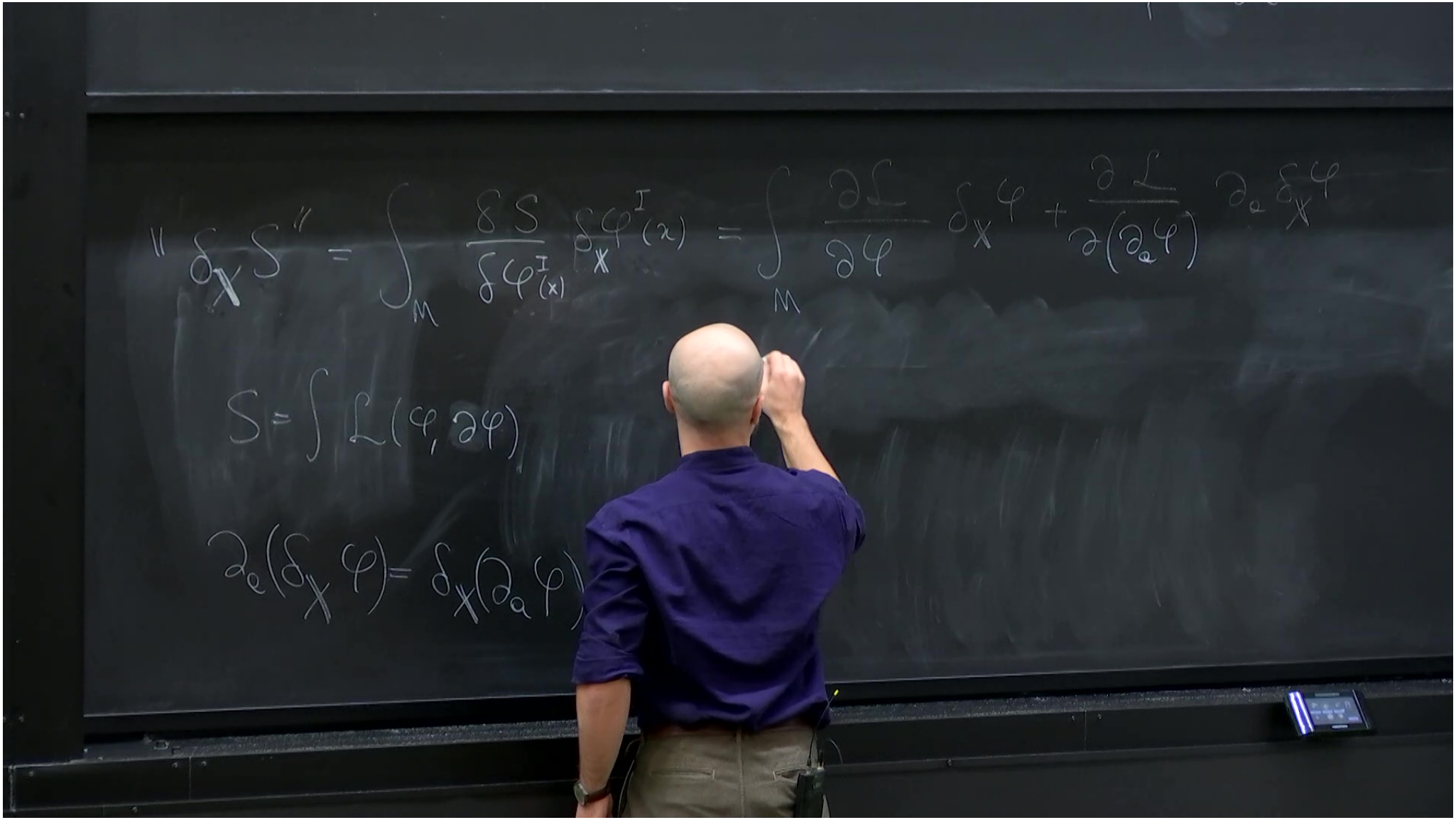

$\Sigma = \{t = 0\}$ $B = \{r = R\}$
$n_a dx^a = dt$
$s_a dx^a = dr$
$\varepsilon_{ab} = dt$

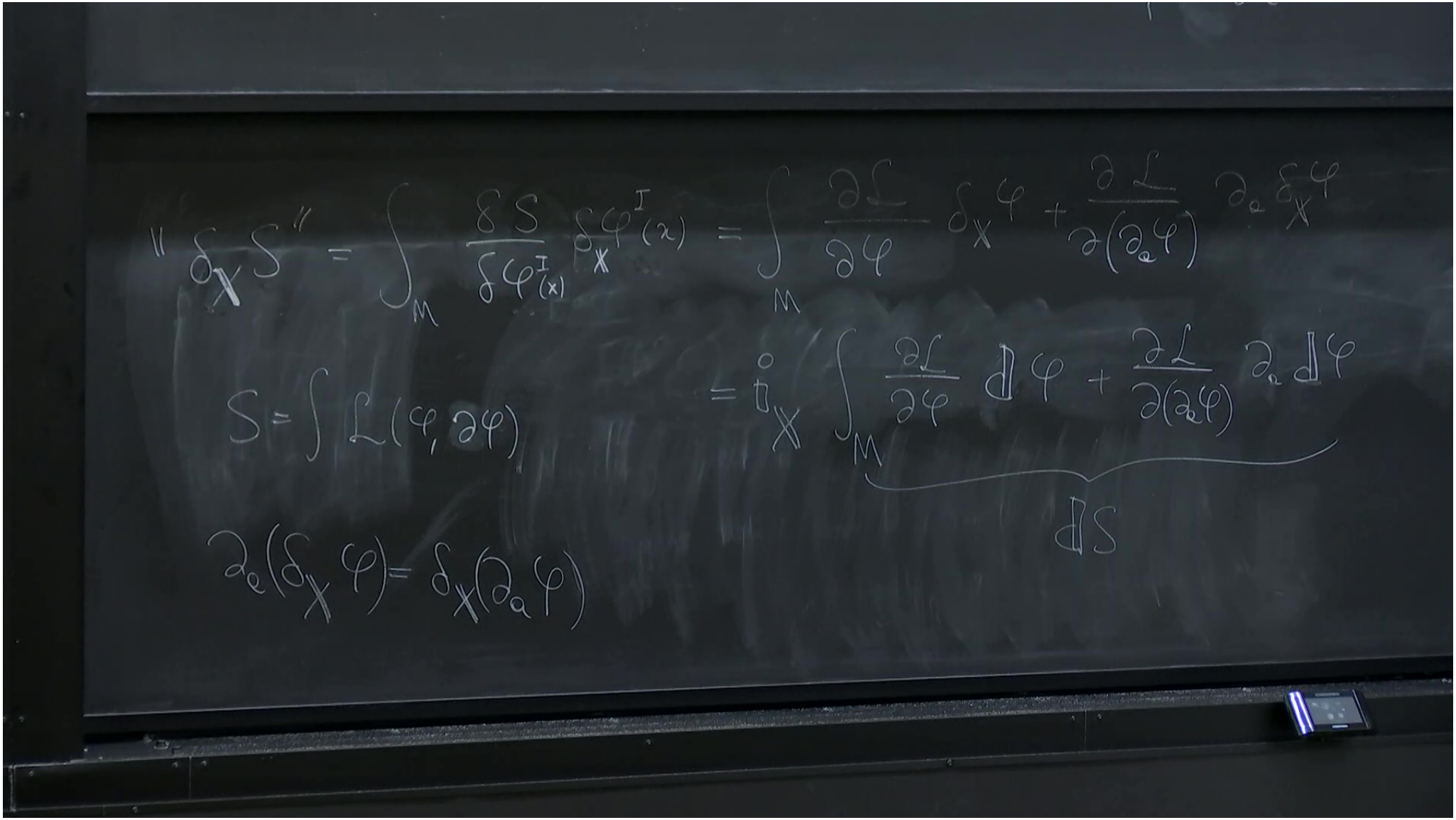

$B = \{r = R\}$
Field space
$F \to M$
$\varphi \in \Gamma(M,F) =: \mathcal{F}$
F
R
M
• scalar field
$F = M \times R$
$\varphi \in \Gamma(M,F) \simeq C^\infty(M,R)$
• YM

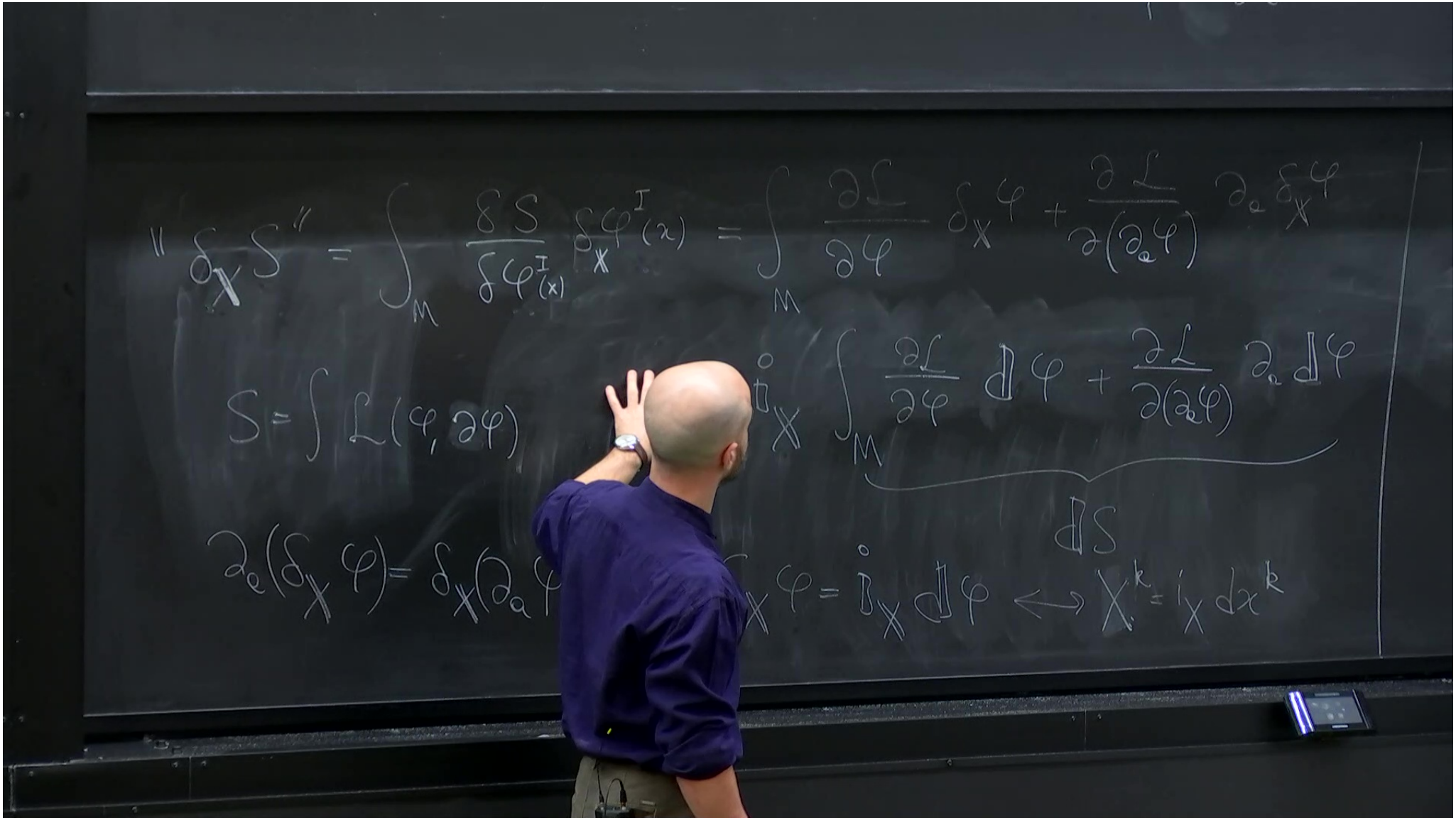

Field space
F → M
φ ∈ Γ(M,F) = 𝓕
F
ℝ
M
scalar field
F = M × ℝ
φ ∈ Γ(M,F) ≃ C^∞(M,ℝ)
• YM
F = (· , ·)
A = Ω¹(M, 𝔤)
A = A^α_a(x) dx^a ⊗ τ_α
• GR
F =

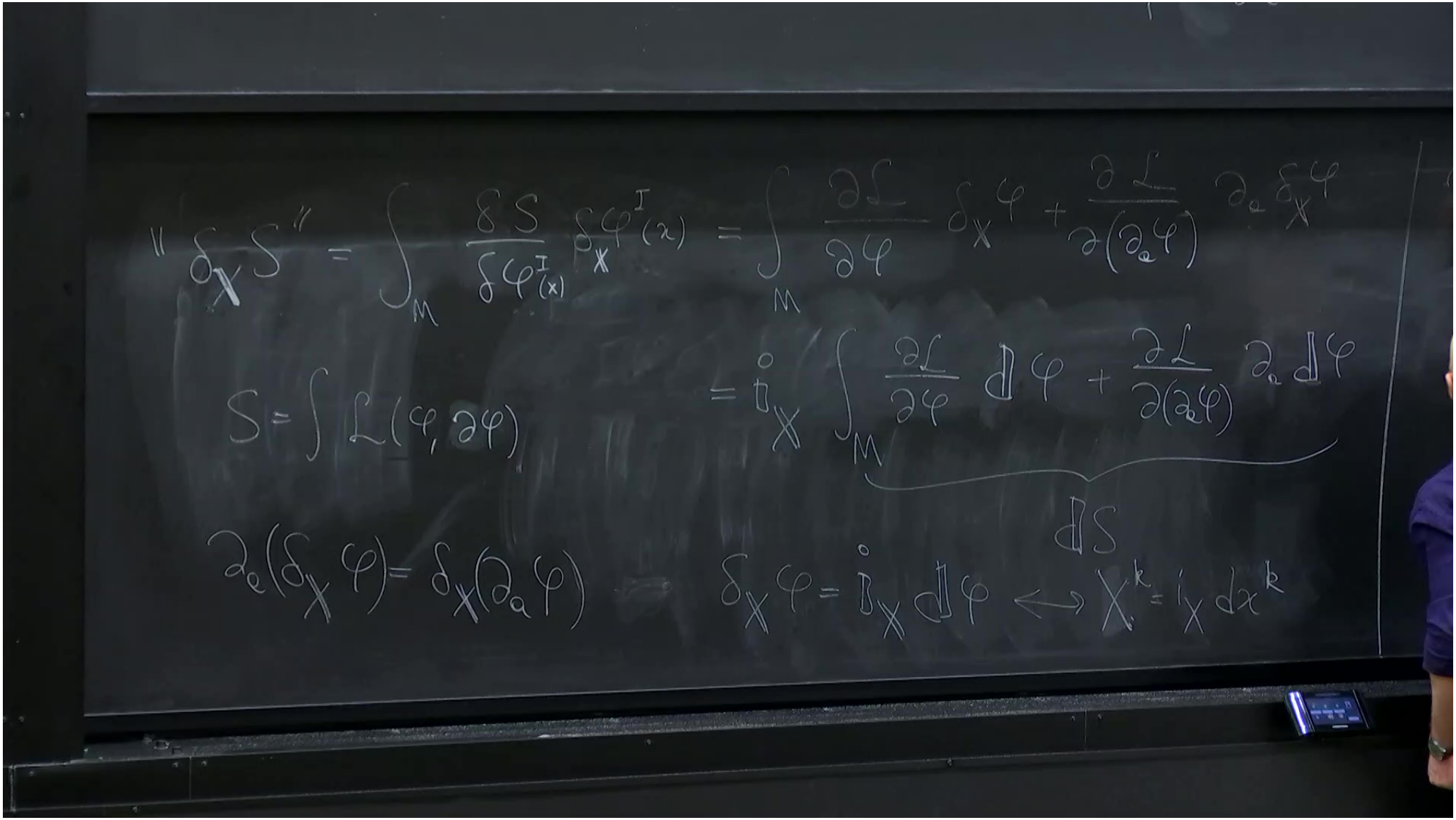

Field space
F → M
φ ∈ Γ(M,F) = 𝓕
scalar field
F = M × ℝ
φ ∈ Γ(M,F) ≃ C^∞(M,ℝ)
F
ℝ
M
• YM
F = (· · ·)
A = Ω^1(M, 𝔤)
A = A^α_a(x) dx^a ⊗ τ_α
• GR
F = T*M ⊙ T*M
𝓕 = sect' of F w/
signature (-1, 1 ··· 1)

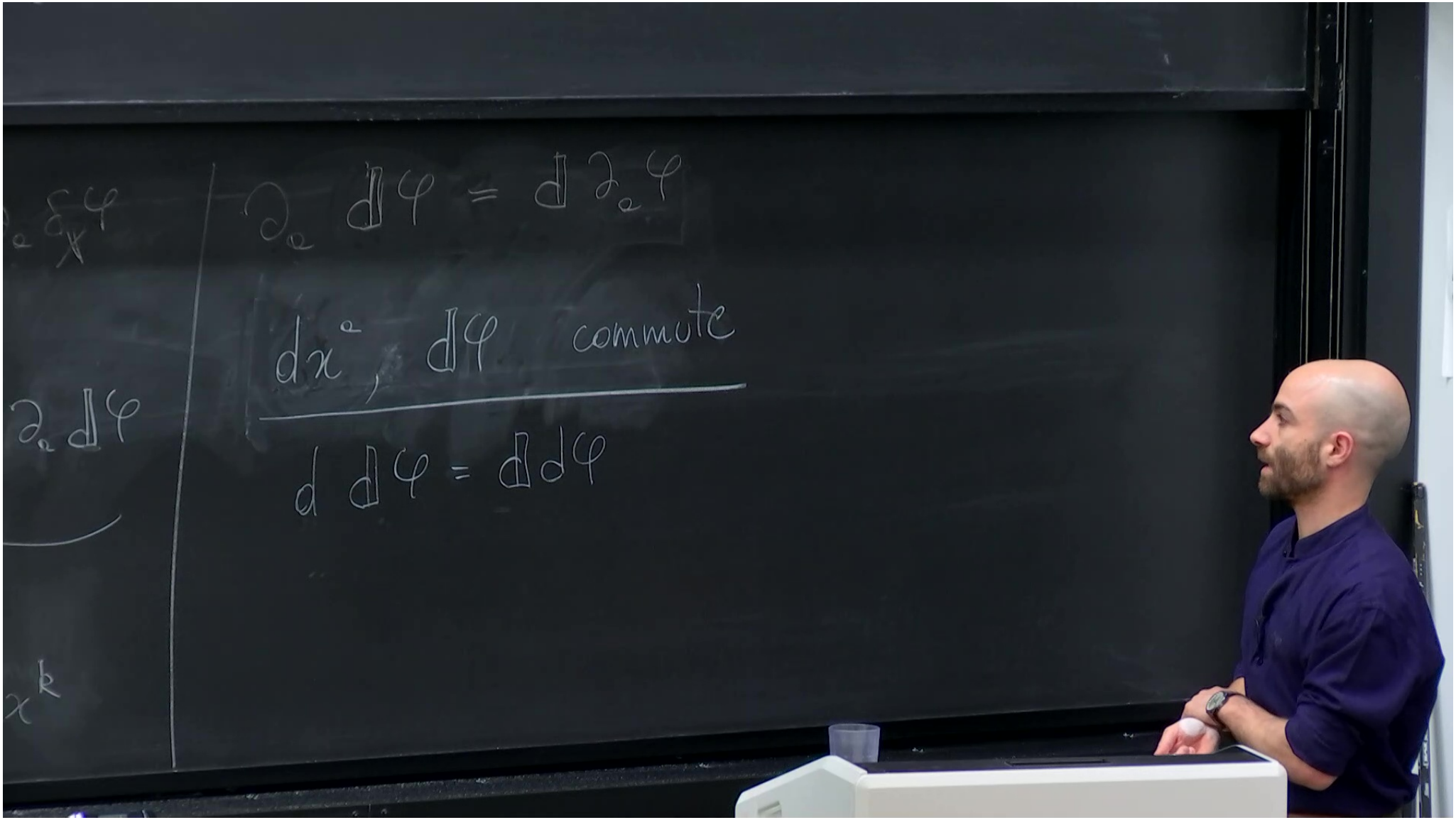

$\varphi^I(x)$ coordinate on $\mathcal{F}$ $\longleftrightarrow$ x^i on P
$X = \int_M \sum_I (\delta_X \varphi^I(x)) \frac{\delta}{\delta \varphi^I(x)}$ $\longleftrightarrow$ $X = \sum_i X^i(x) \frac{\partial}{\partial x^i}$
$S[\varphi] \rightsquigarrow \mathcal{L}_X S = X(S) = i_X dS$ $\longleftrightarrow$ $L_X f = X(f) = i_X df$
$= \sum_i X^i$

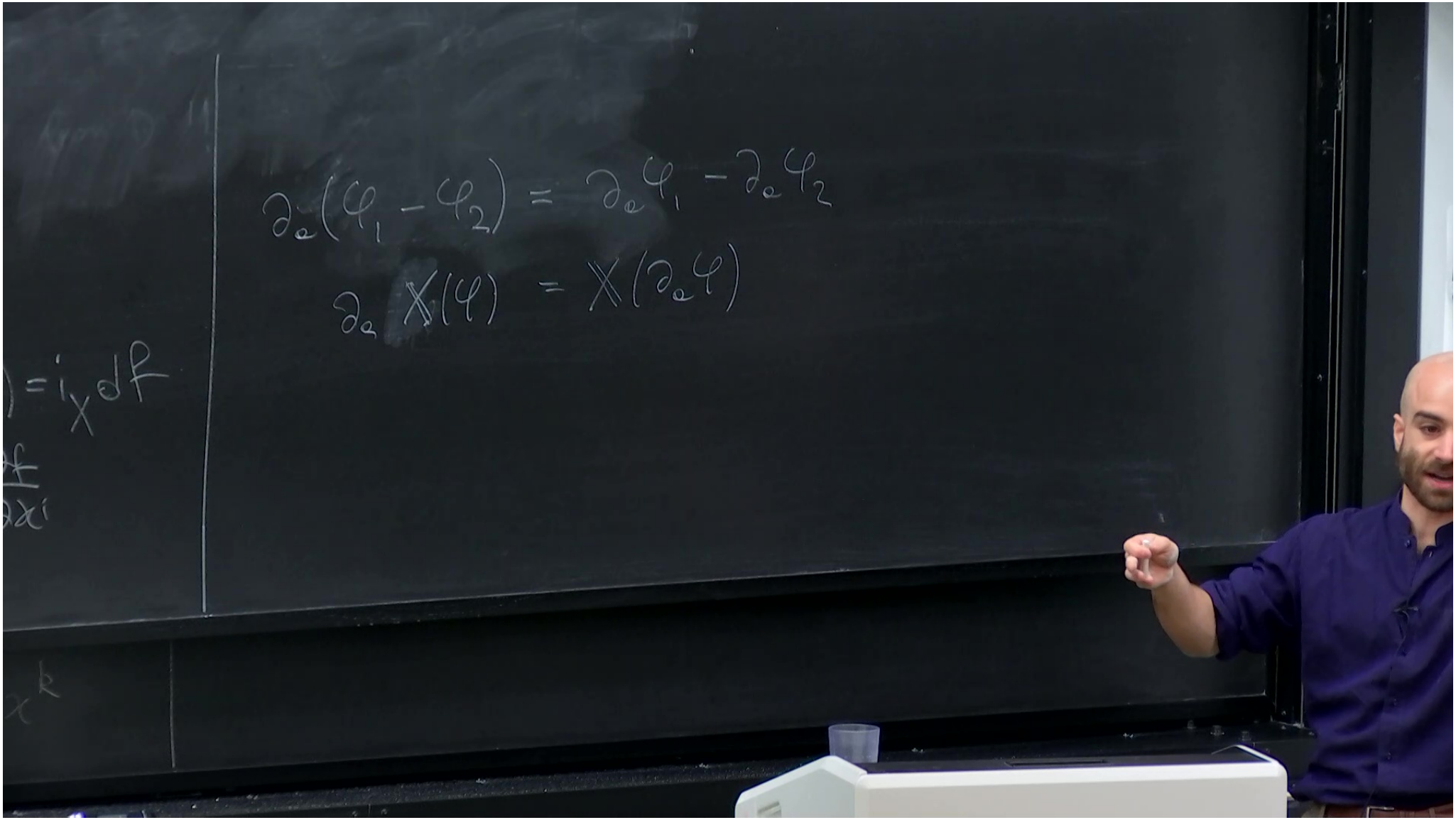

$\delta_X S = \int_M \frac{\delta S}{\delta \varphi^I(x)} \delta_X \varphi^I(x) = \int_M \frac{\partial \mathcal{L}}{\partial \varphi} \delta_X \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a \delta_X \varphi$
$S = \int \mathcal{L}(\varphi, \partial\varphi)$
$\partial_a(\delta_X \varphi) = \delta_X(\partial_a \varphi)$

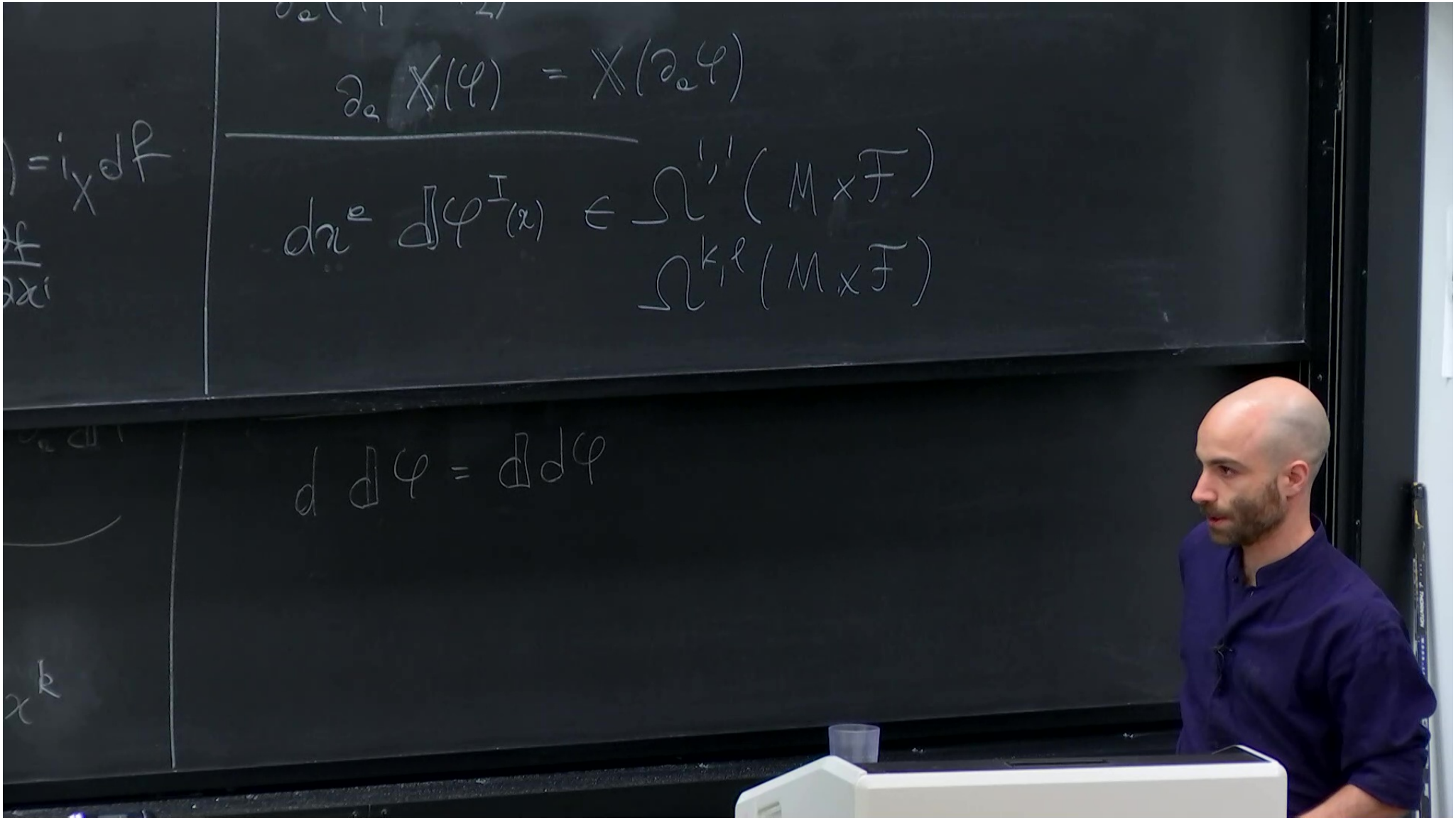

$\delta_\chi S = \int_M \frac{\delta S}{\delta \varphi^I(x)} \delta_\chi \varphi^I(x) = \int_M \frac{\partial \mathcal{L}}{\partial \varphi} \delta_\chi \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a \delta_\chi \varphi$
$S = \int \mathcal{L}(\varphi, \partial\varphi)$
$= i_\chi \int_M \frac{\partial \mathcal{L}}{\partial \varphi} d\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a d\varphi$
dS
$\partial_a(\delta_\chi \varphi) = \delta_\chi(\partial_a \varphi)$


$\delta_X S = \int_M \frac{\delta S}{\delta \varphi^I(x)} \delta_X \varphi^I(x) = \int_M \frac{\partial \mathcal{L}}{\partial \varphi} \delta_X \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a \delta_X \varphi$
$S = \int \mathcal{L}(\varphi, \partial\varphi)$
$\partial_a(\delta_X \varphi) = \delta_X(\partial_a \varphi$
$\int_M \frac{\partial \mathcal{L}}{\partial \varphi} d\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a d\varphi$
dS
$\varphi = i_X d\varphi \leftrightarrow X^k = i_X dx^k$


$\delta_X S = \int_M \frac{\delta S}{\delta \varphi^I(x)} \delta_X \varphi^I(x) = \int_M \frac{\partial \mathcal{L}}{\partial \varphi} \delta_X \varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a \delta_X \varphi$
$S = \int \mathcal{L}(\varphi, \partial\varphi)$
$= i_X \int_M \frac{\partial \mathcal{L}}{\partial \varphi} d\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_a \varphi)} \partial_a d\varphi$
$\partial_a(\delta_X \varphi) = \delta_X(\partial_a \varphi)$
dS
$\delta_X \varphi = i_X d\varphi \leftrightarrow X^k = i_X dx^k$


$\partial_\alpha \mathrm{d}\varphi = \mathrm{d}\partial_\alpha\varphi$
dx^α, $\mathrm{d}\varphi$ commute
$\mathrm{d}\,\mathrm{d}\varphi = \mathrm{d}\mathrm{d}\varphi$
$\partial_\alpha \mathrm{d}\varphi$
x^k


$\partial_a(\varphi_1 - \varphi_2) = \partial_a\varphi_1 - \partial_a\varphi_2$
$\partial_a X(\varphi) = X(\partial_a\varphi)$


$\partial_a X(\varphi) = X(\partial_a \varphi)$
$dx^a \, d\varphi^I(x) \in \Omega^{1,1}(M \times F)$
$\Omega^{k,\ell}(M \times F)$
$d\, d\varphi = d\, d\varphi$