Title: Strong Gravity

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Collection: Strong Gravity (2021/2022)

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Abstract: This course will introduce some advanced topics in general relativity related to describing gravity in the strong field and dynamical regime. Topics covered include properties of spinning black holes, black hole thermodynamics and energy extraction, how to define horizons in a dynamical setting, formulations of the Einstein equations as constraint and evolution equations, and gravitational waves and how they are sourced.

Simple Example of Solving CTT

Vacuum: $\tilde{p}^i = \tilde{E} = 0$

Maximal: $K = \hat{A}^{ij}_{TT} = 0$

maximal "waveless"

Conformally flat: $\tilde{\gamma}_{ij} = f_{ij}$

BC: $\psi \to 1$, X

Eqns: $\partial_i \partial^i \psi + \frac{1}{8}$

$\Delta_L X^i = 0$

Solving CTT

BC: $\psi \to 1$, $\chi^i \to 0$ as $r \to \infty$

Eqns: $\partial_i \partial^i \psi + \frac{1}{8} (\tilde{\mathcal{L}} \chi)^{ij} (\mathcal{L} \chi_{ij}) \psi^{-7} = 0$

$\Delta_L \chi^i = 0$

Σ_0

$r \to \infty$

$(X_{;j})\psi^{-7} = 0$

$\psi = 1$

$\Sigma_0 = \mathbb{R}^3 \Rightarrow \gamma_{ij} = f_{ij}$

$k_{ij} = 0$

$\Sigma_0 = \mathbb{R}^3 / B_R$

At $r=R$ require $D_i s^i = 0 = \tilde{D}_i(\psi^4 s^i)$

$\frac{1}{r^2}\partial_r(\psi^4 r^2) = 0$

$$\partial_r \psi + \frac{\psi}{2r} = 0 \quad \text{at } r = R$$

$$\psi \to 1 \quad \text{as } r \to \infty$$

$$\partial^i \partial_i \psi = 0$$

Unique solution $\psi = 1 + \frac{R}{r}$

$$\gamma_{ij} = \psi^4 \delta_{ij} = \left(1 + \frac{M}{2r}\right)^4 \left(dr^2 + r^2 d\Omega^2\right) \qquad M = 2R$$

Other formulations for solving Const. Eqns.

	Free data	Constrained data
Conformal thin sandwich,	$\tilde{\gamma}_{ij}, K, \tilde{\alpha}, \partial_t \tilde{\gamma}_{ij}$	ψ, β^i
Extended CTS (XCTS) :	$\tilde{\gamma}_{ij}, K, \partial_t K, \partial_t \tilde{\gamma}_{ij}$	ψ, α, β^i

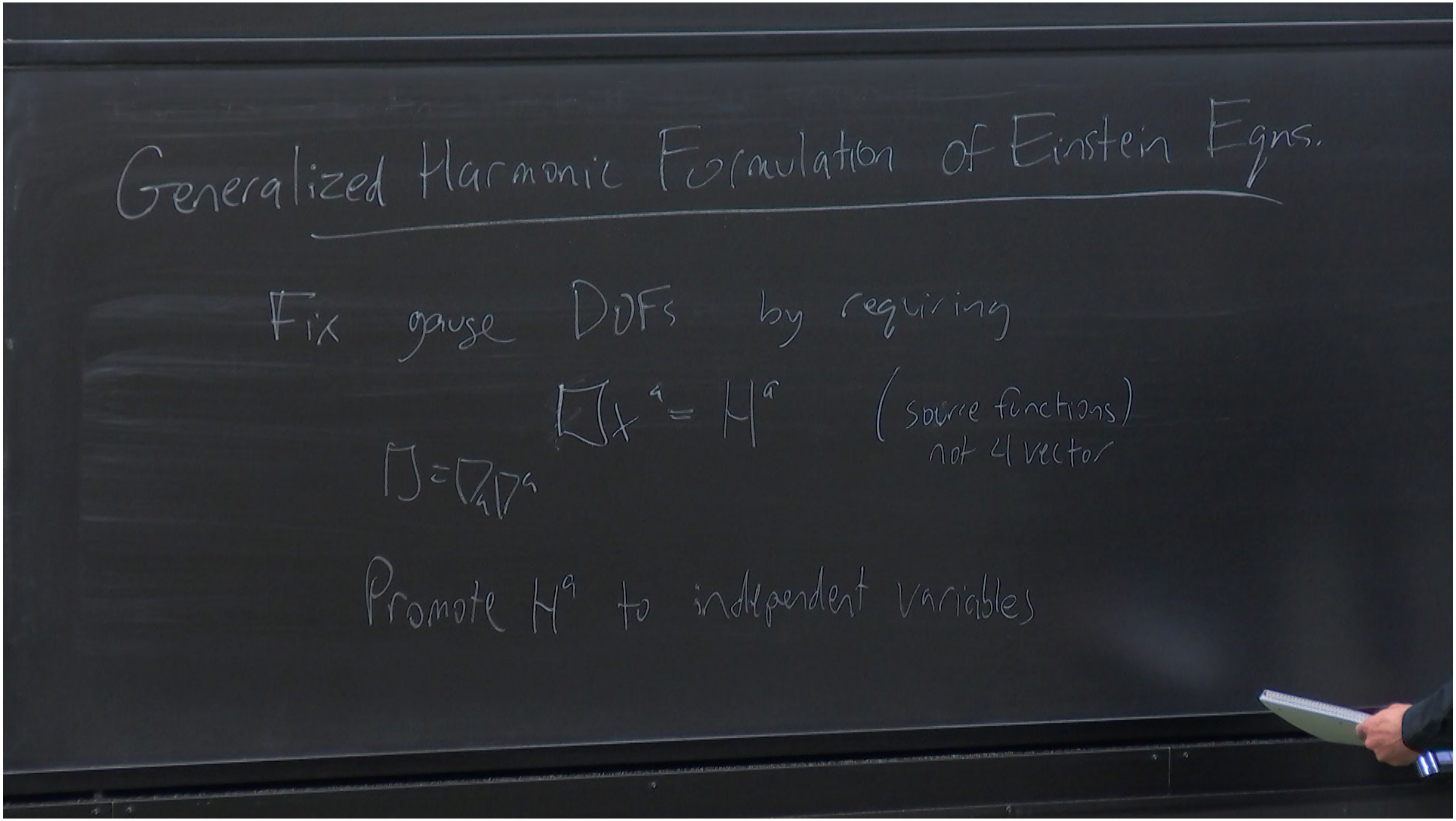
Strategies: Use superposition

$\tilde{\gamma}_{ij} = \gamma^{(1)}_{ij} + \gamma^{(2)}_{ij} - f_{ij}$ to get a binary solution

Quasi-circular, assume approximate helical Killing vector

$$\xi^a = t^a + \Omega_{orb}\,\phi^a$$

$$\mathcal{L}_\xi g_{ab} \approx 0$$


Generalized Harmonic Formulation of Einstein Eqns.
Fix gauge DOFs by requiring
$\Box x^a = H^a$ (source functions)
not 4 vector
$\Box = \nabla_a \nabla^a$
Promote H^a to independent variables

$$R_{ab} = 8\pi \left(T_{ab} - \frac{1}{2} T g_{ab} \right)$$

$$= -\frac{1}{2} g^{cd} \partial_c \partial_d g_{ab} - \partial_c g_{d(a} \partial_{b)} g^{cd} + \Gamma_{(a} \Gamma_{b)} - \Gamma_c \Gamma^c_{\ ab} - \Gamma^c_{\ da} \Gamma^d_{\ cb}$$

$$\Gamma^a = g^{bc} \Gamma^a_{\ bc} = -\Box x^a = \frac{-1}{\sqrt{-g}} \partial_b \left(\sqrt{-g}\, g^{ba} \right)$$

$$H^a + \Gamma^a = 0$$

$$-8\pi \left(2T_{ab} - g_{ab} T \right) = \underbrace{g^{cd} \partial_c \partial_d g_{ab}}_{\text{principal part}} + F\left(g_{ab}, \partial g_{ab}, H_a, \partial_a H_b \right)$$

$$\Gamma^{c}_{ab} - \Gamma^{c}_{da}\Gamma^{d}_{cb}$$

$$\partial_b\left(\sqrt{g}\, g^{ba}\right)$$

$$H_a,\ \partial_a H_b\Big)$$

evolution operator

$$L(H) = 0 \quad (**)$$

Simple Choice

$$H_a = f(g_{ab})$$

Simplest Choice

$$H_a = 0 \quad \text{(harmonic gauge)}$$

Evolve $\{ g_{ab}, \partial_t g_{ab}, H_a, \partial_t H_a \}$ according to (*) and (**)

$$C^a = H^a - \Box x^a$$

$$R_{ab} - 8\pi (T_{ab} - g_{ab} T) = \nabla_{(a} C_{b)}$$

$n^a n^b$, Hamiltonian Const LHS

$n^a \gamma^b_c$ Mom. Const. LHS

Equivalent to $C_a = \partial_t C_a = 0$ at $t=0$

ccording to (*) ad (**)

$$\nabla^a \nabla_b C_a = g^{cd} \nabla^c \nabla_b C_d$$
$$= g^{cd} (\nabla_b \nabla^c C_d - R^d{}_{cbd} C_d)$$

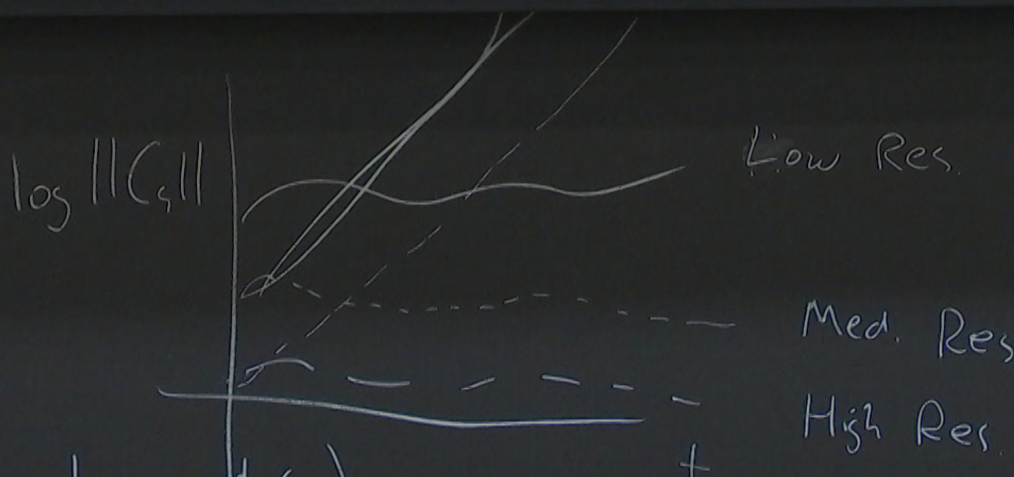
$$(R_{ab} - \tfrac{1}{2} R g_{ab}) - 8\pi T_{ab} = \nabla_{(a} C_{b)} - \tfrac{1}{2} g_{ab} g^{cd} \nabla_{(c} C_{d)}$$

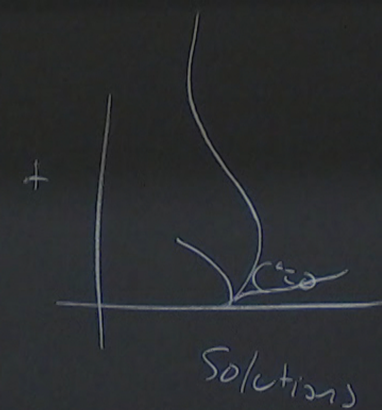
$\nabla^a \to$ $\nabla^a \nabla_{(a} C_{b)} - \tfrac{1}{2} \nabla_b \nabla^{(a} C_{a)} = 0$ by Bianchi Id.

$\nabla^a \nabla_a C_b + \nabla^a \nabla_b C_a - \nabla_b \nabla^a C_a = 0$ Cons. of S-E

$\nabla_a \nabla^a C_b = -R^a{}_b C_a$ (evolution of constraints)

Start $C_a = \partial_t C_a = 0$, C_a will be zero for all time lling vector





Add

$$-K\left(n_{(a}C_{b)} - \frac{1}{2} g_{ab} n^{d} C_{d}\right) \quad \text{to } (*)$$

$$\Box C^{a} = -R^{a}{}_{b} C^{b} - K \nabla_{b}\left[n^{(b} C^{a)}\right]$$

$K \sim \left[\frac{1}{L}\right]$ damps const. violations on time/length scales $1/K$