Title: Strong Gravity

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Collection: Strong Gravity (2021/2022)

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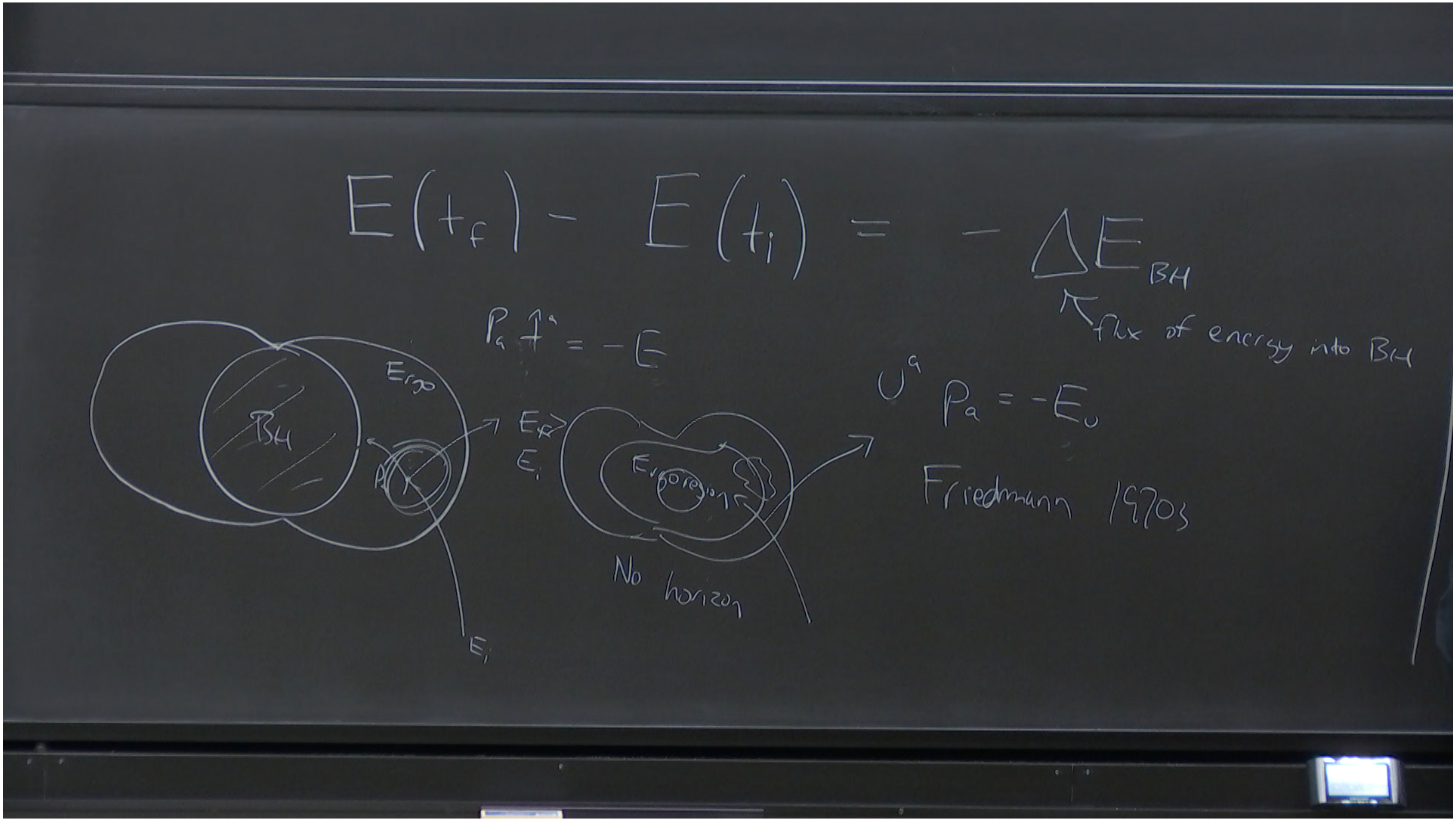
Abstract: This course will introduce some advanced topics in general relativity related to describing gravity in the strong field and dynamical regime. Topics covered include properties of spinning black holes, black hole thermodynamics and energy extraction, how to define horizons in a dynamical setting, formulations of the Einstein equations as constraint and evolution equations, and gravitational waves and how they are sourced.


E(t_f) - E(t_i) = - ΔE_BH
flux of energy into BH
P_a t^a = -E
Ergo
BH
E_f >
E_i
E_i
Ergoregion
No horizon
U^a P_a = -E_U
Friedmann 1970s

into BH

Superradiance

Scalar field

Massless Klein-Gordon Eqn.

$$\Box \psi = \nabla_a \nabla^a \psi = 0$$

Stress-energy Tensor

$$T_{ab} = \nabla_a \psi \nabla_b \psi - \frac{1}{2} g_{ab} \nabla_c \psi \nabla^c \psi$$

$$\psi = \Psi(r,\theta)\,\mathrm{Re}\left[e^{-i(\omega t - m\phi)}\right]$$

real part

frequency

azimuthal num.

$$\Delta E_{BH} = \int dt \int_{r=r_+} \chi^a \, T_{ab} \, t^b \, dS_{BH}$$

$$= \int dt \int_{r=r_+} \left[\chi^a (\nabla_a \psi)(\nabla_b \psi) t^b - \frac{1}{2} \cancelto{0}{\chi_a t^a} \nabla_c \psi \nabla^c \psi\right] dS_{BH}$$

$$\Delta E_{BH} = \int dt \int dS_{BH} \left(\partial_t \Psi + \Omega_H \partial_\phi \Psi\right) \partial_t \Psi$$

$$= \underbrace{\left(\omega - m\Omega_H\right)}_{<0} \omega \underbrace{\int dt \int dS_{BH} \left|\bar{\Psi}\right|^2 \sin^2(\omega t - m\phi)}_{\geq 0}$$

$$0 < \omega < m\Omega_H \Rightarrow \Delta E_{BH} < 0$$

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$$= \underbrace{\left(\omega - m\Omega_H\right)}_{<0} \omega \underbrace{\int dt \int dS_{BH} \left|\bar{\Psi}\right|^2 \sin^2(\omega t - m\phi)}_{\geq 0}$$

$$\underline{0 < \omega < m\Omega_H} \Rightarrow \Delta E_{BH} < 0$$

$$\frac{\delta E}{\delta J} = \frac{\omega}{m} \qquad \Updownarrow$$

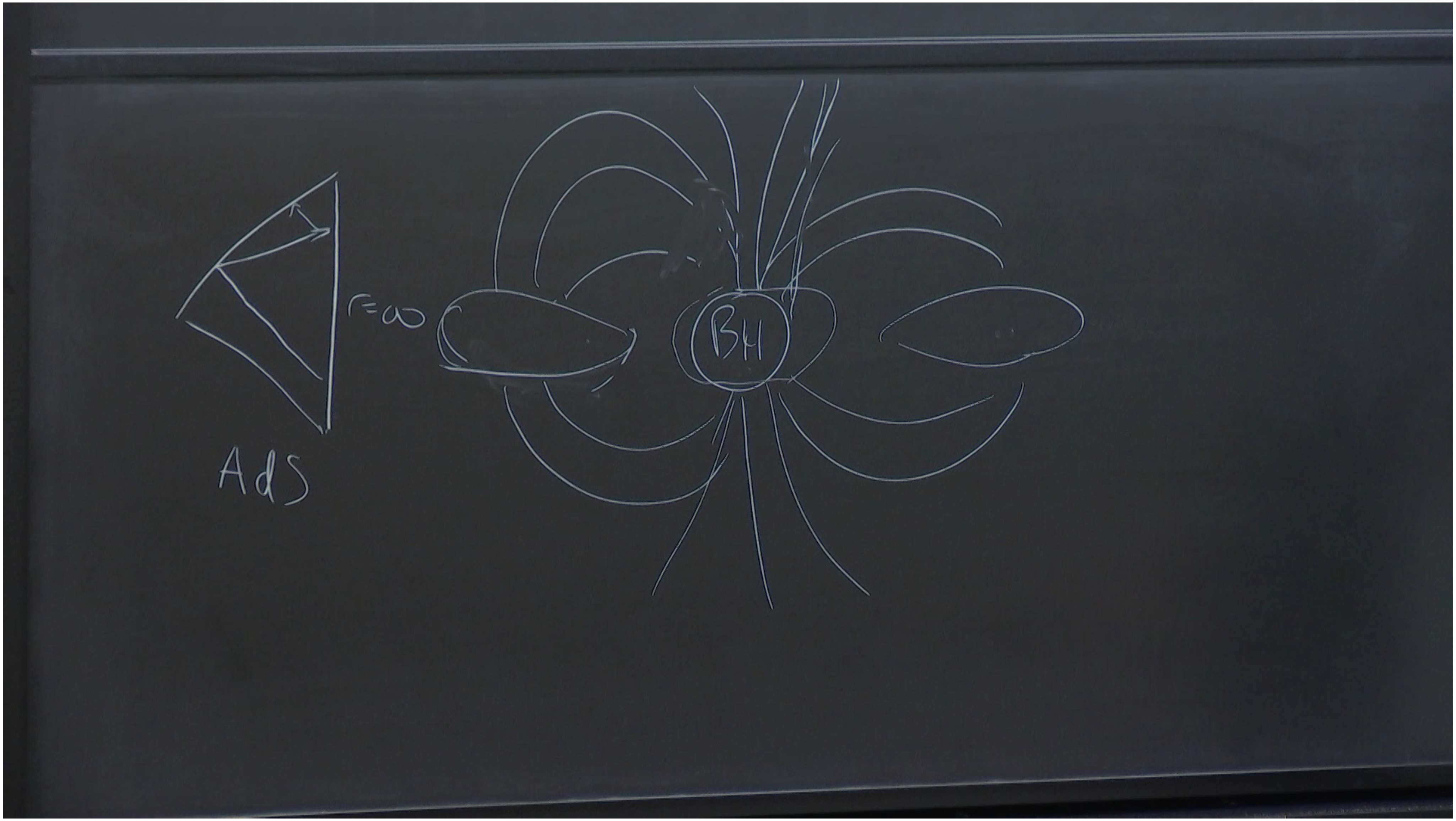
$$\frac{\delta E}{\delta J} < \Omega_H$$

Maximum efficiency ($\bar{a} \to 1$)

$s=0$, scalar wave 0.3%

$s=1$, E-M wave 4.4%

$s=2$, GW wave 138%


r=∞
BH
AdS

Massive Scalar field

$$\nabla_a V^a = \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g}\, V^a \right)$$

Klein-Gordon Eqn. $\Box \phi = \mu^2 \phi$

mass of boson up to $\hbar$

$$\mu^{-1} \gg M_{BH}$$

non-relativistic limit

$$\Box \phi = \nabla_a \left(g^{ab} \partial_b \phi \right) = \frac{1}{\sqrt{-g}} \partial_a \left(\sqrt{-g}\, g^{ab} \partial_b \phi \right)$$

$v^c)$

For Schwarz $\sqrt{-g} = r\sin\theta$

$$g^{tt}\partial_t^2\phi + \partial_i\partial^i\phi - \mu^2\phi = 0 \qquad g^{tt} \simeq -\left(1+\frac{2M}{r}\right)$$

Use ansatz $\phi = \frac{1}{\sqrt{2\mu}}\left[\psi(x)e^{-i\omega t} + \psi^*(x)e^{i\omega t}\right]$

$$\left[-\left(1+\frac{2M}{r}\right)(-i\omega)^2 + \partial_i\partial^i - \mu^2\right]\psi = 0$$

$$(\omega^2-\mu^2)\psi = \left(-\partial_i\partial^i - \frac{2M\omega^2}{r}\right)\psi$$

$\omega \simeq \mu$

$\Rightarrow (\omega-\mu)\psi$

$= r\sin\theta$

$- \mu^2\phi = 0$ $\qquad g^{tt} \simeq -\left(1+\frac{2M}{r}\right)$

$\left[\Psi(x)e^{-i\omega t} + \Psi^*(x)e^{i\omega t}\right]$

$\partial_i\partial^i - \mu^2\big]\Psi = 0$

$\partial_i\partial^i - \frac{2M\omega^2}{r}\big)\psi$

$\omega \simeq \mu$

$\Rightarrow (\omega-\mu)\Psi = \left(-\frac{1}{2\mu}\partial_i\partial^i - \frac{\mu M}{r}\right)\psi$

"fine structure" constant $\alpha = M\mu$

Energy $= \omega - \mu \simeq -\frac{\alpha^2}{2n^2}$

Bohr radius

$r_B = (\mu\alpha)^{-1}$

$l = m = 1$

$$\phi \sim e^{-r/(2r_B)}\, Y_{m=1}^{l=1}(\theta)\, e^{-i(\omega t - m\phi)}$$

$$\mathrm{Re}(\omega) \simeq \mu\left(1 - \frac{\alpha^2}{2n^2}\right)$$

$$|\mathrm{Im}(\omega)| << |\mathrm{Re}(\omega)|$$

$$0 < \omega < m\Omega_H$$

$\alpha = ($