

Title: Strong Gravity

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Abstract: This course will introduce some advanced topics in general relativity related to describing gravity in the strong field and dynamical regime. Topics covered include properties of spinning black holes, black hole thermodynamics and energy extraction, how to define horizons in a dynamical setting, formulations of the Einstein equations as constraint and evolution equations, and gravitational waves and how they are sourced.

$$D = S_J < \frac{SE}{S_{CH}}$$

Hawking (1971)

BH Area Thm:

$$\delta A_{BH} \geq 0$$

Cosmic Censorship, Null Energy
Condition (NEC)

BH

$$A_{BH} =$$

BH Area

$$A_{\text{BH}} = \int \sqrt{|g_{\theta\theta} g_{\phi\phi}|} \Big|_{r=r_+} d\theta d\phi$$

$$= 4\pi (r_+^2 + a^2)$$

$$= 8\pi \left(M^2 + \sqrt{M^4 - J^2} \right)$$

← BH Angular Mom.

$$g_{\theta\theta} g_{\phi\phi} \Big|_{r=r_+} = (r_+^2 + a^2) \sin^2 \theta$$

$$r_+ = M + \sqrt{M^2 - a^2}$$

$$\begin{aligned}
 \frac{\delta A_{\text{BH}}}{8\pi} &= 2M \delta M + \frac{1}{2} (M^4 - J^2)^{-1/2} (4M^3 \delta M - 2J \delta J) \\
 &= \frac{J}{\sqrt{M^4 - J^2}} \left[\underbrace{\left(\frac{2M \sqrt{M^4 - J^2}}{J} + \frac{2M^3}{J} \right)}_{= \frac{2M}{J} (M^2 + \sqrt{M^4 - J^2})} \delta M - \delta J \right] \\
 &= \frac{2M}{J} (M^2 + \sqrt{M^4 - J^2}) \delta M - \delta J = \frac{2r_+}{a} \delta M - \delta J = \delta \mathcal{E}_+^{-1}
 \end{aligned}$$

Dir Angular Mom.

δJ)

$$\delta J = \frac{\bar{a}}{\sqrt{1-\bar{a}^2}} (\Omega_H \delta M - \delta J)$$

≥ 0

$$\delta A_{BH} \geq 0 \Leftrightarrow \delta J \Omega_H < \delta M$$

$$= \frac{2M}{J} (M^2 + \sqrt{M^4 - J^2}) = \frac{r_g}{a} = \sqrt{2}$$

Define $M_{ir} = \sqrt{\frac{A_{B4}}{16\pi}}$

For $a=0$ $M_{ir} = \left(\frac{4\pi (2M_{B4})^2}{16\pi} \right)^{1/2} = M_{B4}$

For $a=$
 $A_{B4} =$

$$= \frac{c r_+}{a} = \sqrt{2}^{-1}$$

$$c r_{\text{BH}} - 0 \Leftrightarrow \delta) \Omega_{\text{top}} < \delta M$$

For $a = M$

$$A_{\text{BH}} = 4\pi (M_{\text{BH}}^2 + M_{\text{BH}}^2) = 8\pi M_{\text{BH}}^2$$

$$M_{\text{ir}} = \frac{1}{\sqrt{2}} M_{\text{BH}}$$

$$E_{\text{rot}} = M_{\text{BH}} - M_{\text{ir}} = \left(1 - \frac{1}{\sqrt{2}}\right) M_{\text{BH}} \approx 0.292 M_{\text{BH}}$$

$$\frac{1}{4M}$$

$$a = \left(\underbrace{\left(1 - \frac{2M}{r}\right)^{-1/2}}_{V^{-1}} \quad 0 \quad 0 \quad 0 \right)$$

$$= U^b \nabla_b U_a = \underbrace{U^+}_{V^{-1}} \cancel{\partial_+ U_a} - \Gamma_{t_n}^{t+} \underbrace{U^+ U_t}_{=-1} = \Gamma_{t_r}^{t+} \delta_a^r = \frac{M}{r(r-2M)} \delta_a^r$$

$V \rightarrow 0$ as $r \rightarrow r_f$

$$\frac{M}{r^2 \sqrt{1 - \frac{2M}{r}}}$$

$\frac{D}{r} = 0$

$$V_a = \left(1 - \frac{2M}{r}\right)^{1/2} \frac{M}{r^2 \left(1 - \frac{2M}{r}\right)^{1/2}} = \frac{M}{r^2}$$

$$K = V_a \Big|_{r=2M} = \frac{1}{4M}$$

1st Law Area

$$SM = \frac{K}{8\pi} SA_{B4} + \Omega_{B4} SJ$$

Conf. $dE = T ds - p dV$

3rd Law Can't achieve $K=0$ $M^2 S^2 M^2 = 2 J^2$
 $K=0 \Rightarrow \bar{a}=1$, i.e. extremal BH

General Penrose Process

Matter Ten (ignore backreaction)

a | BH

$$J_a = -T_{ab} \uparrow^b$$

$$\nabla_a J^a = -(\nabla_a T_{ab}) \uparrow^b - T_{ab} \nabla_a \uparrow^b = 0$$

Killing's Egn

Apply Stoke's Thm

$$\int_M \nabla_a J^a \sqrt{-g} d^n x = \int_{\partial M} n_a J^a \sqrt{\gamma} d^{n-1} x$$

$\uparrow$
unit normal

Stoke's Thm

$$0 = \int_M \nabla_a J^a \sqrt{-g} d^n x = \int_{\partial M} n_a J^a \sqrt{\gamma} d^{n-1} x$$

$\uparrow$
 unit normal

$$\int_{t=t_f} n_a J^a \sqrt{\gamma} d^3 x - \int_{t=t_i} n_a J^a \sqrt{\gamma} d^3 x$$



