Title: Cosmology (2021/2022)

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Collection: Cosmology (2021/2022)

Date: April 21, 2022 - 9:00 AM

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Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.

DARK MATTER

OBSERVATIONS REQUIRE TWO DARK COMPONENTS:

1) "LIGHT" SECTOR
   - RELATIVISTIC $T \sim 1$ eV
   - $N_{eff} = 2.99 \pm 0.17$
   - $\Omega_{today} \lesssim 0.003 \Leftrightarrow \sum m_\nu \lesssim 120$ meV

   } SM NEUTRINO

2) "HEAVY" SECTOR ("COLD DARK MATTER")
   - NONRELATIVISTIC $T \sim 5$ keV $\Leftrightarrow m \gtrsim 5$ keV
   - $\Omega_{today} \sim 0.27$

$= 2.99 \pm 0.17$

NEUTRINO

$\Omega_{\nu,\text{today}} \lesssim 0.003 \Leftrightarrow \sum m_\nu \lesssim 120$ meV

"χ" SECTOR ("COLD DARK MATTER")

NONRELATIVISTIC $T \sim 5$ keV $\Leftrightarrow m_\chi \gtrsim 5$ keV

$\Omega_{\chi,\text{today}} \sim 0.27$

- UPPER BOUNDS ON INTERACTIONS w/ SM

CAN WE EXPLAIN DM AS A THERMAL RELIC?

CHALLENGE: RECALL FOR NEUTRINOS [1 SPECIES]

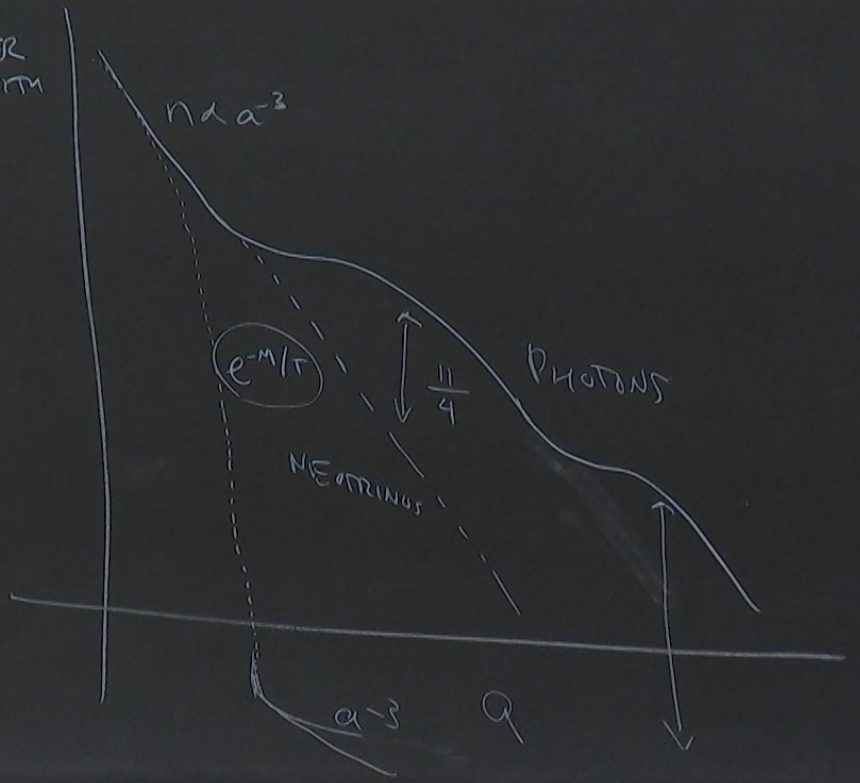
$$\Omega = \frac{m_\nu}{14\ \text{eV}}$$

NUMBER DENSITY n

HARD WITH

"HOT" RELIC ⇔ STILL RELATIVISTIC AT DECOUPLING

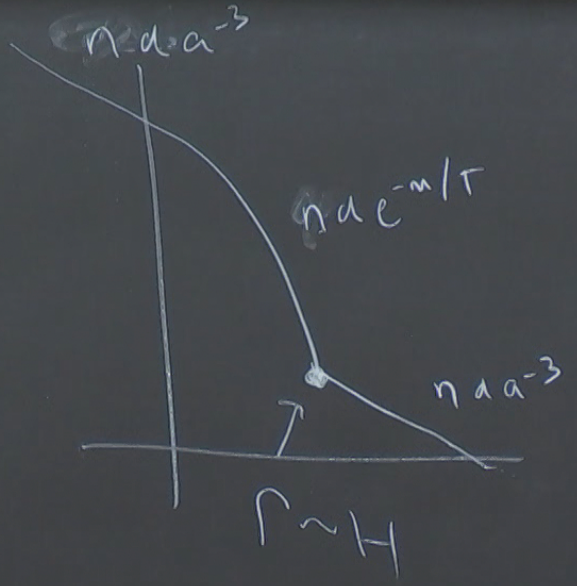
"COLD" RELIC ⇔ NON-REL AT DECOUPLING



CROSS SECTION σ $X\bar{X} \leftrightarrow SM$

PARTICLE MASS m

INSTANTANEOUS FREEZE-OUT



CALCULATE DECOUPLING TEMPERATURE T_* $\Gamma \sim H$

$$\Gamma \sim n_x \sigma v$$

$$\sim \left[(mT)^{3/2} e^{-m/T}\right]\left[\sigma\right]\left[\frac{T}{m}\right]^{1/2}$$

$$\sim \sigma m T^2 e^{-m/T}$$

$$H \sim \left[\frac{8\pi G}{3}\rho\right]^{1/2}$$

$$\sim \left[\frac{8\pi G}{3}\frac{\pi^4}{30} g_* T^4\right]^{1/2}$$

$$\sim \left[G T^4\right]^{1/2} \sim \frac{T^2}{M_{Pl}}$$

$$n = g \int \frac{d^3q}{(2\pi)^3} e^{E_q/T} \mp 1$$

$$\to g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \qquad T \ll m$$

$$\sim (mT)^{3/2} e^{-m/T}$$

$$\frac{1}{2} m v^2 \sim T$$

$$v \sim \left(\frac{T}{m}\right)^{1/2}$$

DECOUPLING OCCURS WHEN $\Gamma \sim H$

$$e^{-m/T_*} \sim \frac{1}{\sigma m M_{pl}} \qquad x \equiv \frac{m}{T_*}$$

ASSUMING

$m \sim 1$

$T \ll m$

CALCULATE RELIC ABUNDANCE

$$\rho_0 = m n_0 = m \eta_* g_{*}^3$$

ρ_0: TODAY

g_{*S}

DECOUPLING OCCURS WHEN $\Gamma \sim H$

$$e^{-m/T_*} \sim \frac{1}{\sigma m M_{pl}} \qquad x \equiv \frac{m}{T_*}$$

ASSUMI

$n \sim 1$

CALCULATE RELIC ABUNDANCE

$$\rho_{0} = m n_0 = m n_* a_*^3$$

$\uparrow$ TODAY

$$\sim m n_* \frac{T_{CMB}^3}{a_*^3}$$

$$g_{*S}$$

$\sim H$

$x \equiv \frac{m}{T_\star}$

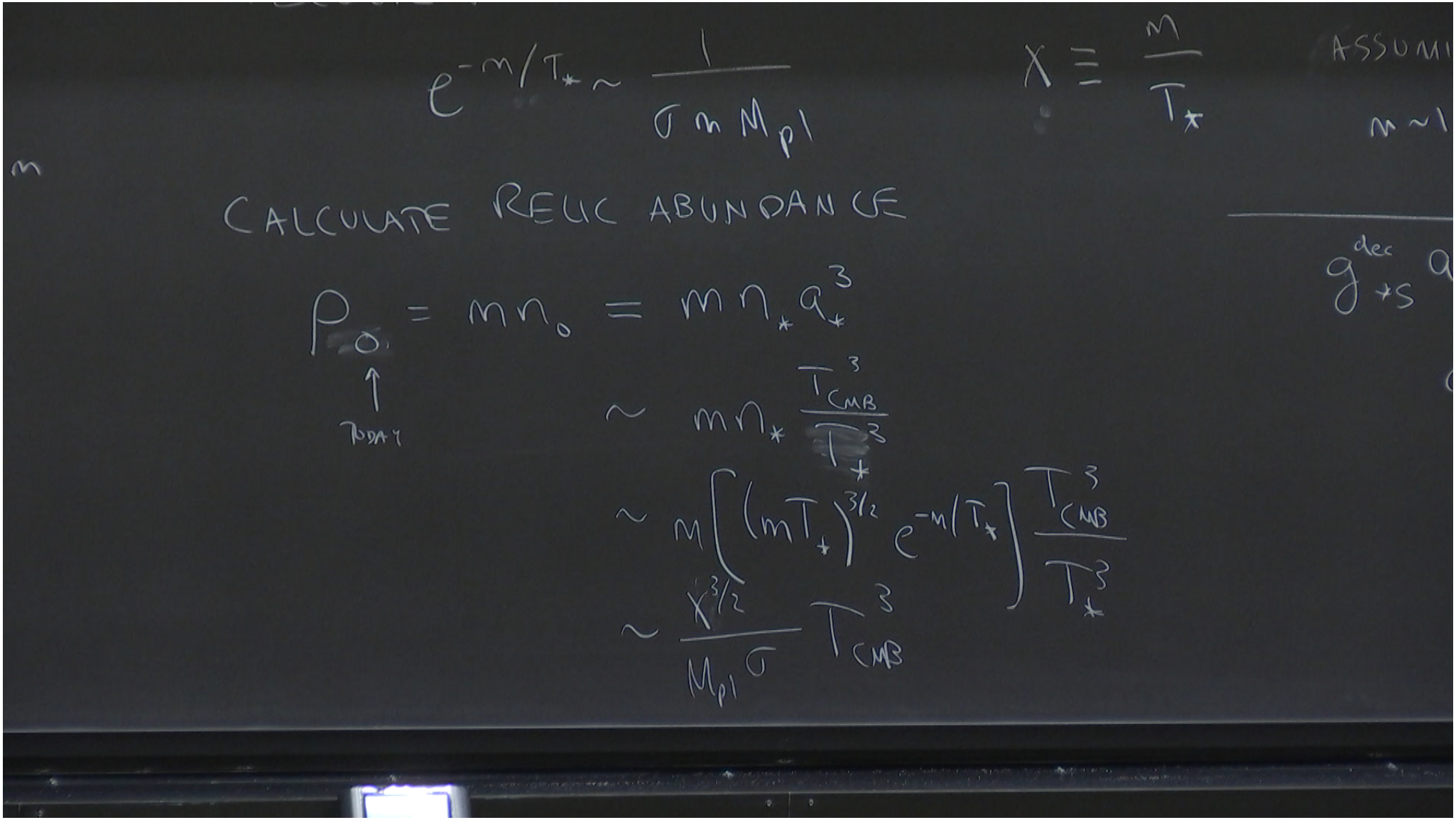
ASSUMING $x \gg 1$

$m \sim 1$ TeV $\quad \sigma \sim 10^{-8}$ GeV^{-2}

$x \sim 30 \qquad 20 \lesssim x \lesssim 50$

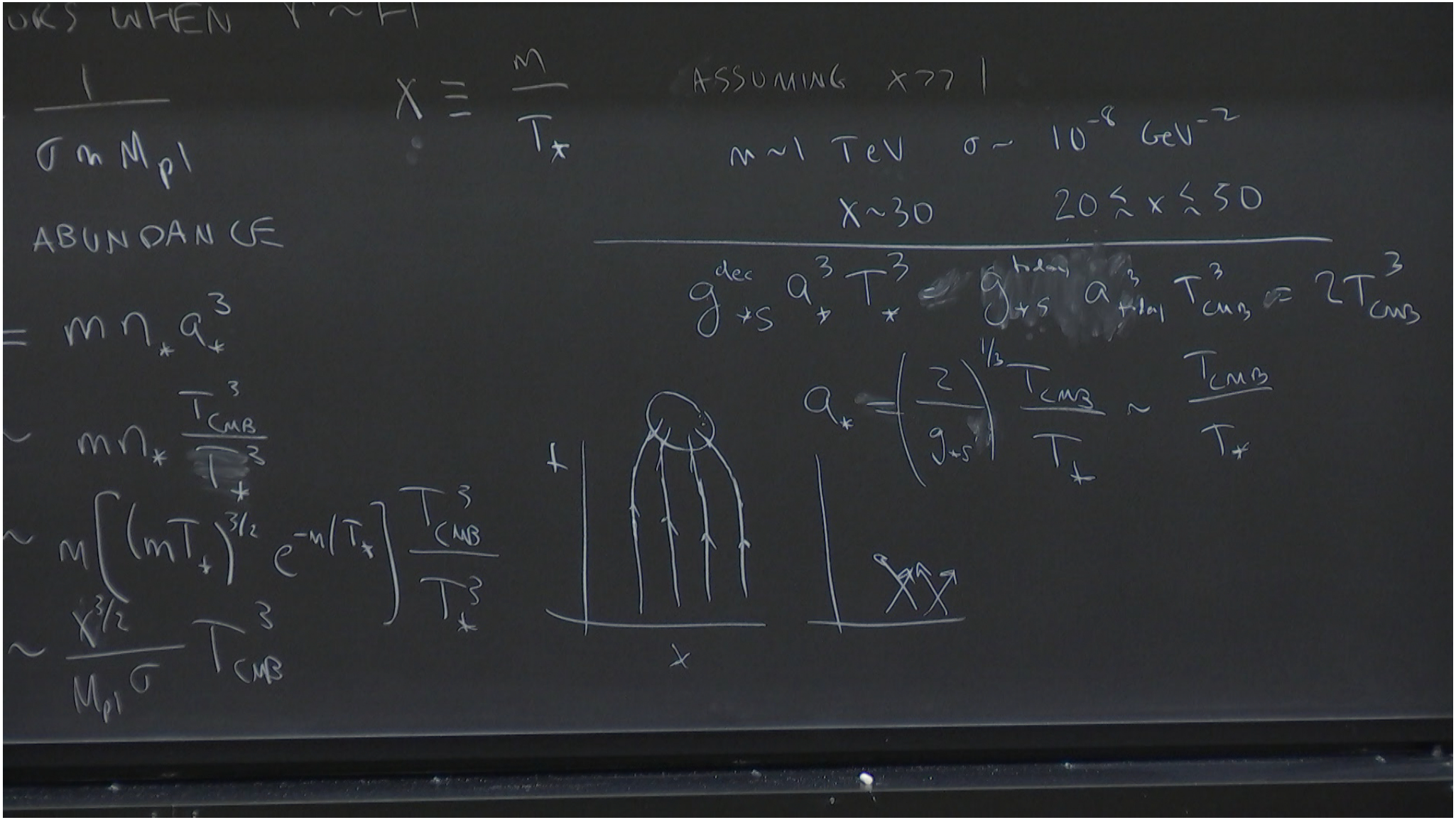
$$g_{\star s}^{dec} a_\star^3 T_\star^3 = g_{\star s}^{today} a_{today}^3 T_{CMB}^3 = 2 T_{CMB}^3$$

$$a_\star = \left(\frac{2}{g_{\star s}}\right)^{1/3} \frac{T_{CMB}}{T_\star} \sim \frac{T_{CMB}}{T_\star}$$


$e^{-m/T_*} \sim \frac{1}{\sigma m M_{pl}}$
$x \equiv \frac{m}{T_*}$
ASSUMI
$m \sim 1$
m
CALCULATE RELIC ABUNDANCE
g_{*S}^{dec}
$\rho_0 = m n_0 = m n_* a_*^3$
TODAY
$\sim m n_* \frac{T_{CMB}^3}{T_*^3}$
$\sim m \left[(m T_*)^{3/2} e^{-m/T_*} \right] \frac{T_{CMB}^3}{T_*^3}$
$\sim \frac{x^{3/2}}{M_{pl} \sigma} T_{CMB}^3$

$$\Omega_{cdm} = \frac{\rho_0}{\rho_{tot}} = (0.25)\left(\frac{x}{30}\right)^{3/2}\left(\frac{\sigma}{10^{-7}\ GeV^{-2}}\right)^{-1}$$

$\sigma \sim 10^{-7}\ GeV^{-2}$ "WIMP MIRACLE"


ORS WHEN Γ ~ H
1
σ m M_pl
ABUNDANCE
= m n_* a_*^3
~ m n_* T_CMB^3 / T_*^3
~ m [(m T_*)^{3/2} e^{-m/T_*}] T_CMB^3 / T_*^3
~ x^{3/2} / (M_pl σ) T_CMB^3
x ≡ m / T_*
ASSUMING x >> 1
m ~ 1 TeV σ ~ 10^{-8} GeV^{-2}
X ~ 30 20 ≲ x ≲ 50
g_{*S}^{dec} a_*^3 T_*^3 = g_{*S}^{today} a_{today}^3 T_CMB^3 = 2 T_CMB^3
a_* = (2 / g_{*S})^{1/3} T_CMB / T_* ~ T_CMB / T_*
t
x

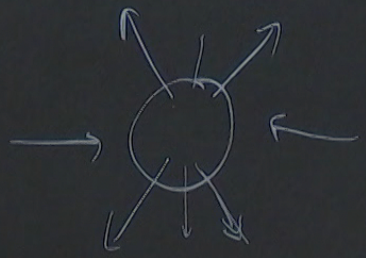
$$\Omega_{cdm} = \frac{\rho_0}{\rho_{tot}} = (0.25)\left(\frac{x}{30}\right)^{3/2}\left(\frac{\sigma}{10^{-7}\ GeV^{-2}}\right)^{-1}$$

$\sigma \sim 10^{-7}\ GeV^{-2}$ "WIMP MIRACLE"

$m \gtrsim 5\ keV$

$m \lesssim 100\ TeV$ "UNITARITY BOUND"

$5\ keV \lesssim m \lesssim 100\ TeV$



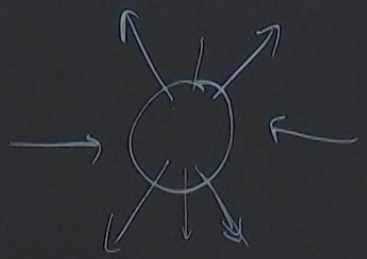
$\sigma < \frac{4\pi}{m}$

$$\Omega_{cdm} = \frac{\rho_0}{\rho_{tot}} = (0.25)\left(\frac{x}{30}\right)^{3/2}\left(\frac{\sigma}{10^{-7}\ GeV^{-2}}\right)^{-1}$$

$\sigma \sim 10^{-7}\ GeV^{-2}$ "WIMP MIRACLE"

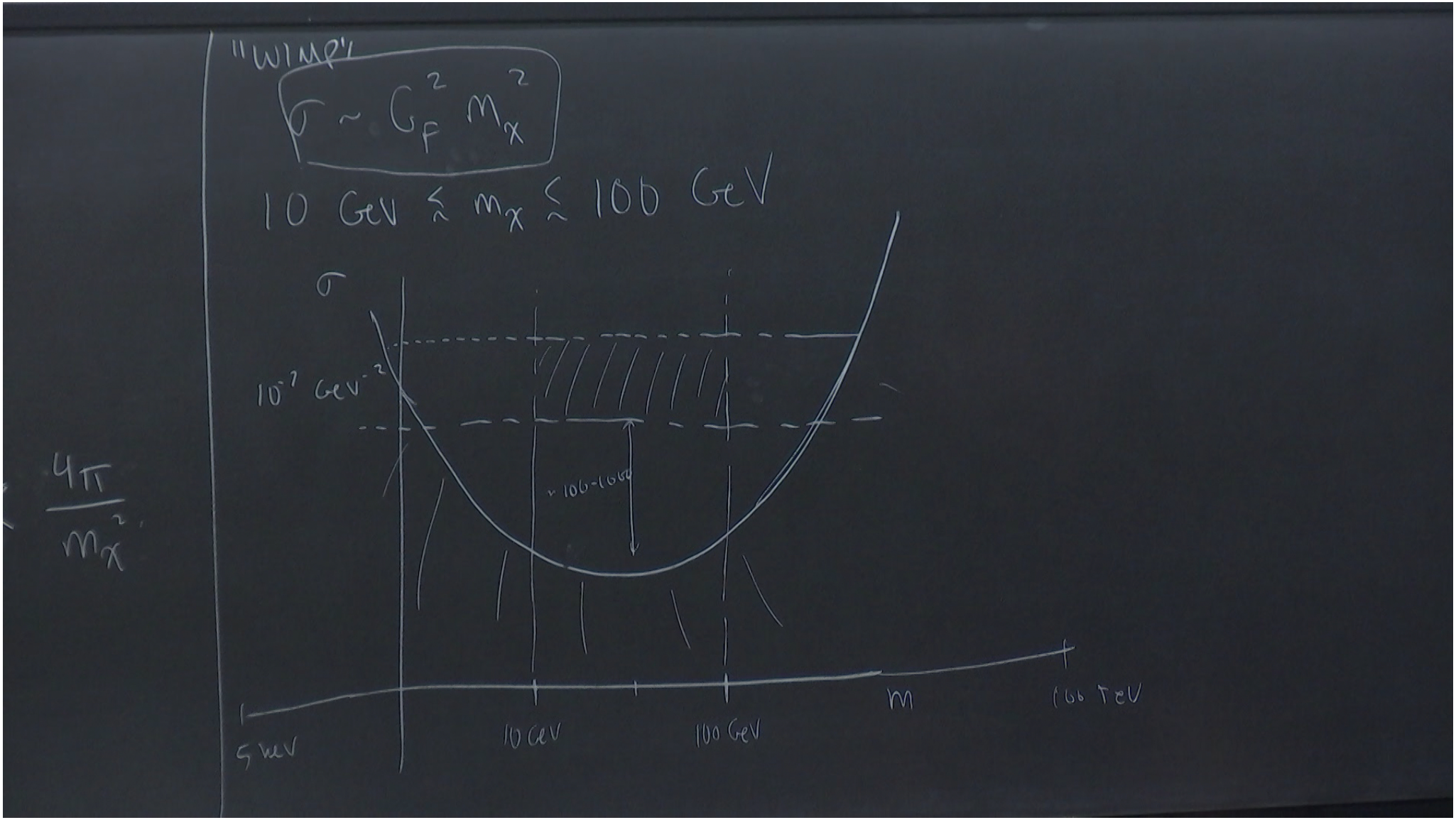
$m \gtrsim 5\ keV$

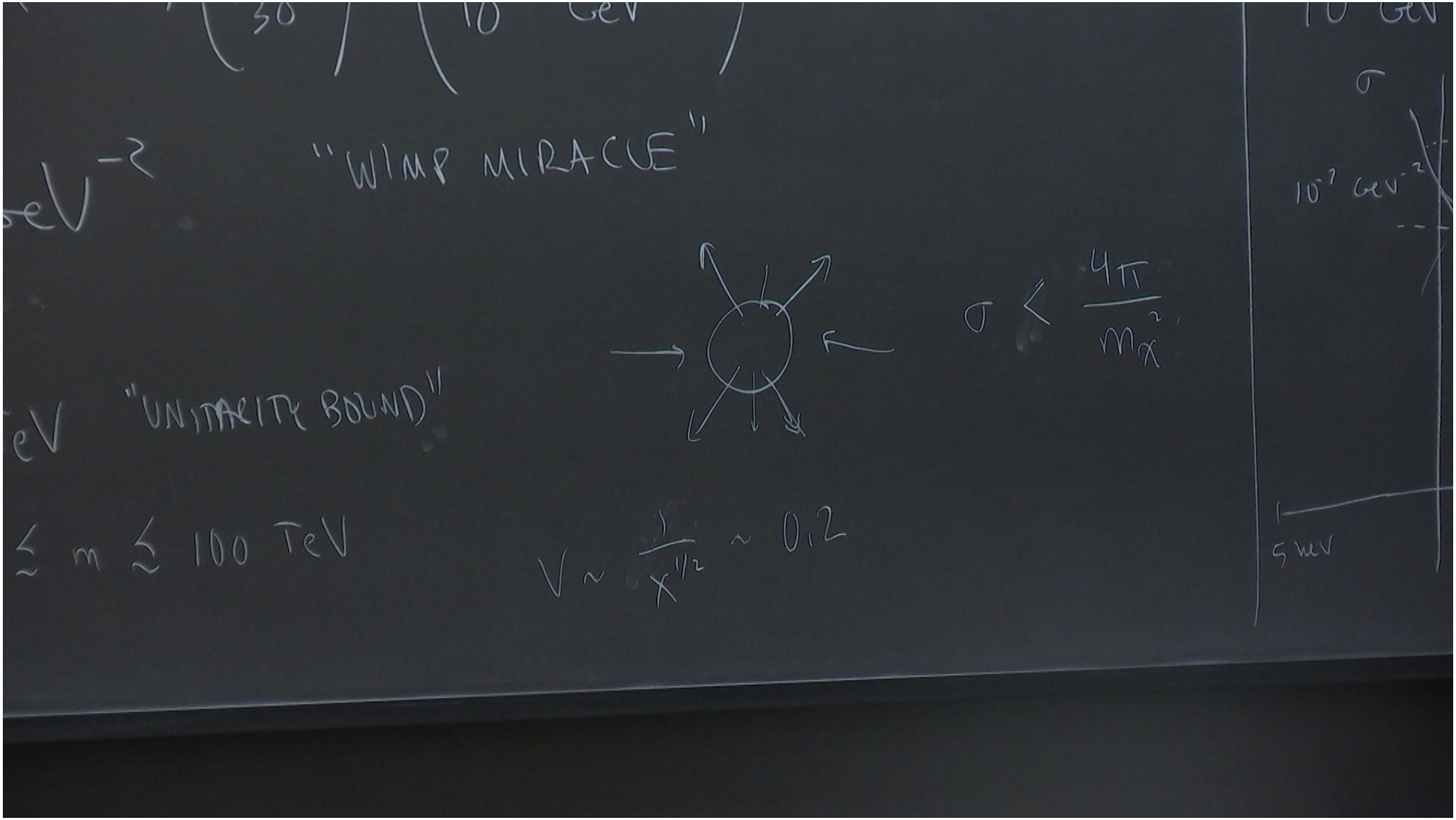
$m \lesssim 100\ TeV$ "UNITARITY BOUND"

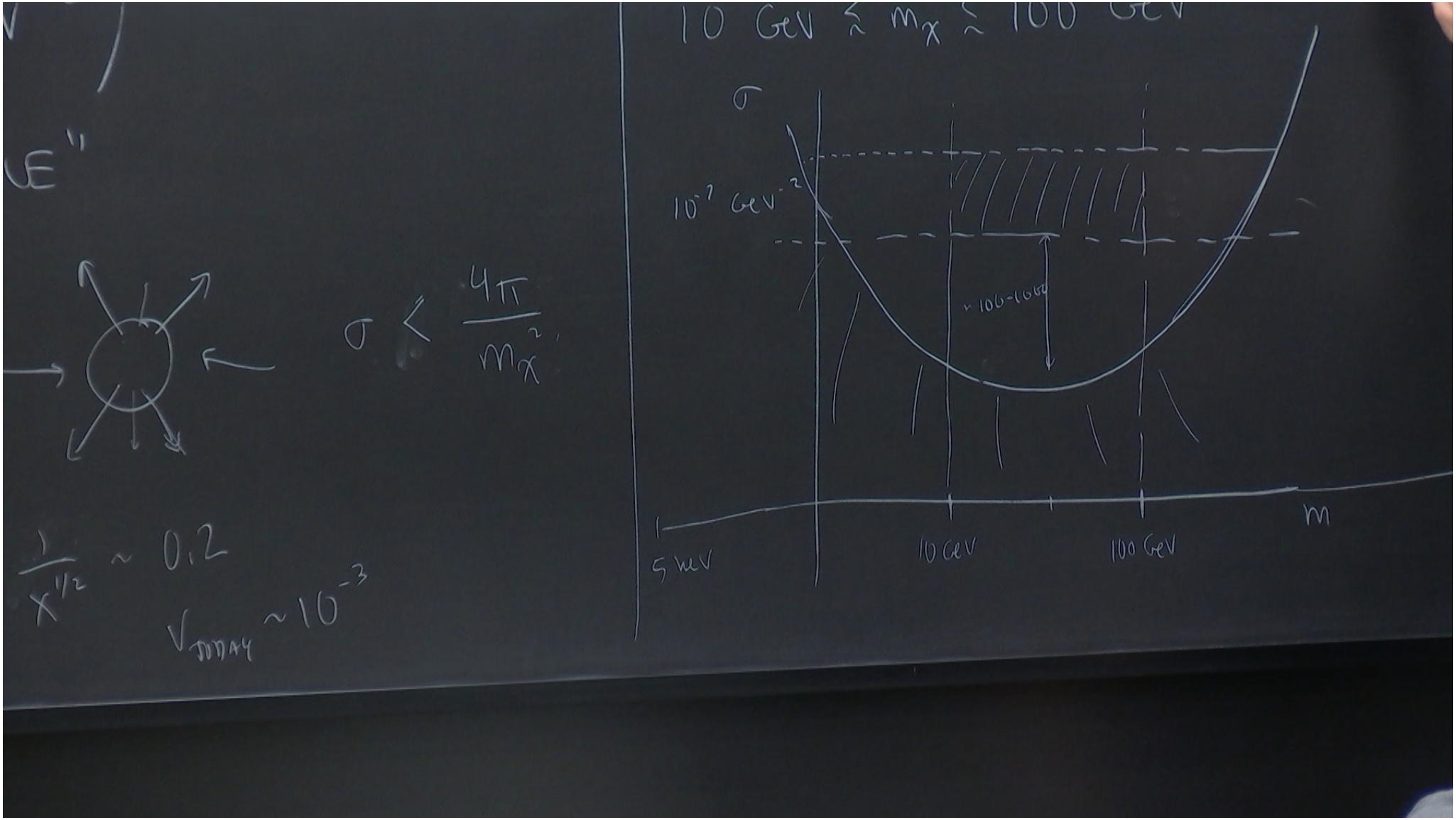


$\sigma < \frac{4\pi}{m_\chi^2}$

$5\ keV \lesssim m \lesssim 100\ TeV$


"WIMP"
$\sigma \sim G_F^2 m_X^2$
10 GeV $\lesssim m_X \lesssim$ 100 GeV
σ
10^{-7} GeV^{-2}
$\sim$ 100-1000
$\frac{4\pi}{m_X^2}$
5 MeV
10 GeV
100 GeV
m
100 TeV


"WIMP MIRACLE"
"UNITARITY BOUND"
$\sigma < \frac{4\pi}{m_\chi^2}$
$\lesssim m \lesssim 100$ TeV
$V \sim \frac{1}{x^{1/2}} \sim 0.2$
10^{-7} GeV^{-2}
5 MeV

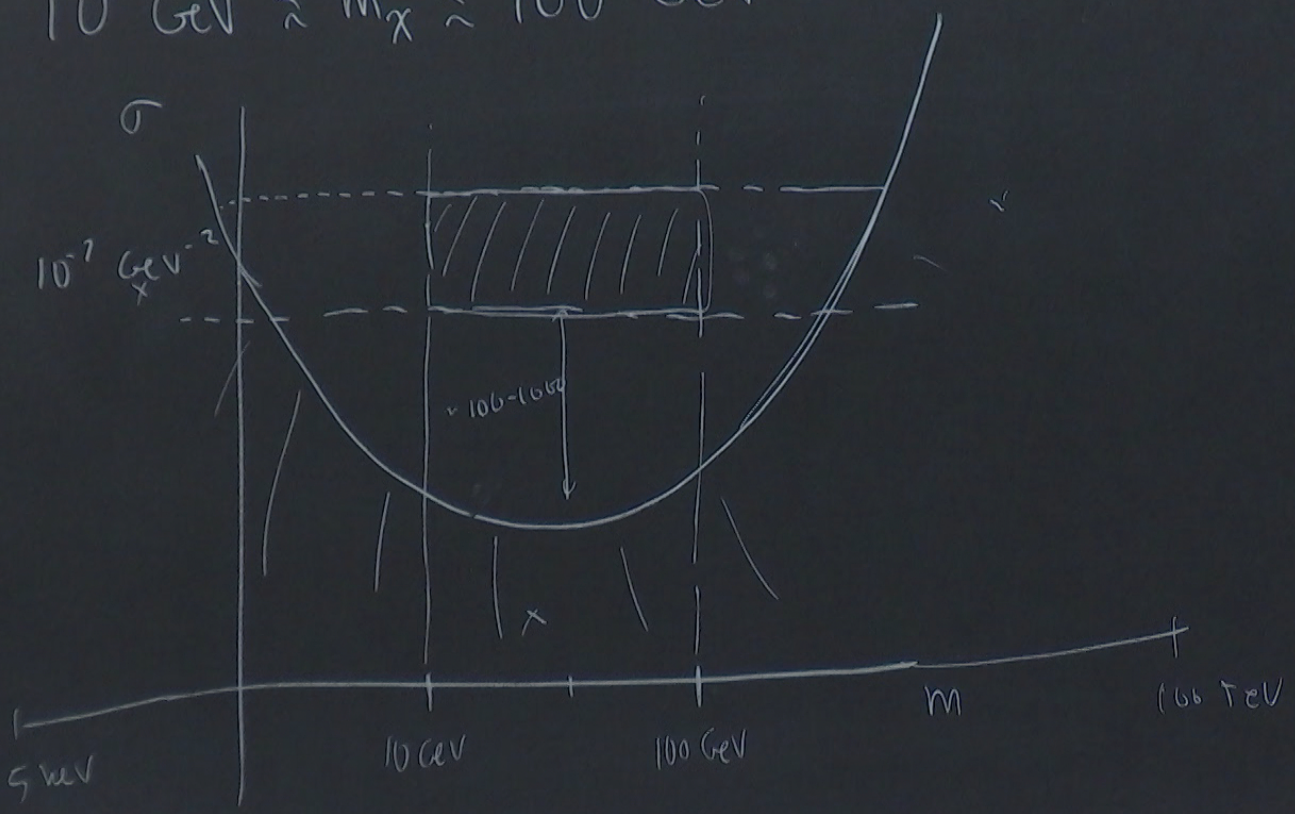

10 GeV ≲ m_χ ≲ 100 GeV
σ
10^-7 GeV^-2
~ 100-1000
m
5 MeV
10 GeV
100 GeV
σ ≤ 4π / m_χ^2
1 / x^1/2 ~ 0.2
v_today ~ 10^-3

"WIMP"

$\sigma \sim G_F^2 m_x^2$

$\sigma \sim v^2$

$10\ \text{GeV} \lesssim m_x \lesssim 100\ \text{GeV}$



$$\Omega_{cdm} = \frac{\rho_0}{\rho_{tot}} = (0.25)\left(\frac{x}{30}\right)^{3/2}\left(\frac{\sigma}{10^{-7}\ GeV^{-2}}\right)^{-1}$$

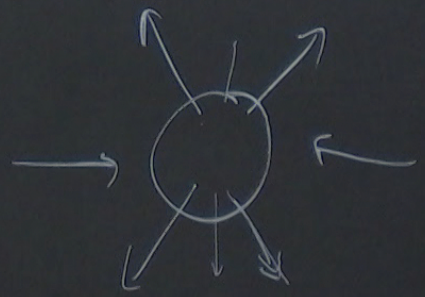
"WIMP MIRACLE"

$\sigma \sim 10^{-7}\ GeV^{-2}$

$m \gtrsim 5\ keV$

$m \lesssim 100\ TeV$

"UNITARITY BOUND"



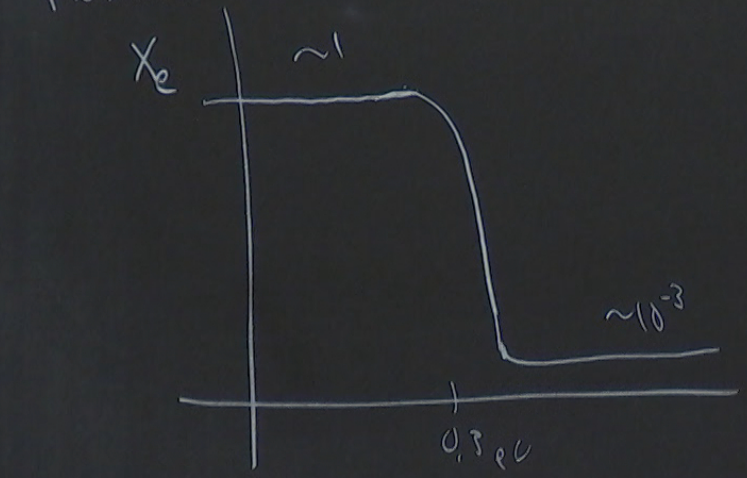
$5\ keV \lesssim m \lesssim 100\ TeV$

$v \sim \frac{1}{x^{1/2}} \sim 0.2$

$v_{today} \sim 1$

- ANNIHILATION
- DECOUPLING
- BINDING
  - HYDROGEN RECOMBINATION $T \sim 0.3$ eV
  - BIG BANG NUCLEOSYNTHESIS $T \sim 10^5$ eV

$p^+ e^- \leftrightarrow H\gamma$



ASSUMPTIONS

- IGNORE He & EXCITED STATES OF H $(g_H = 4)$
- UNIVERSE ELECTRICALLY NEUTRAL $\boxed{n_{e^-} = n_{p^+}}$
- BARYON NUMBER IS CONSERVED

$$n_H + n_{p^+} = n_b = n_{b0} a^{-3} \qquad n_{b0} = \frac{\Omega_b}{m_p} \frac{3H_0^2}{8\pi G} = (2.5 \times 10^{-7})\ cm^{-3}$$

- ASSUME PHOTONS ARE BLACKBODY

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 = n_{\gamma 0} a^{-3} \qquad T = \frac{T_{CMB}}{a} \qquad n_{\gamma 0} = \frac{2\zeta(3)}{\pi^2} T_{CMB} = 410\ cm^{-3}$$

- UNIVERSE ELECTRICALLY NEUTRAL $\boxed{n_{e^-} = n_{p^+}}$

- BARYON NUMBER IS CONSERVED

$$n_H + n_{p^+} = n_b = n_{b0} a^{-3} \qquad n_{b0} = \frac{\Omega_b}{m_p} \frac{3H_0^2}{8\pi G} = (7.5 \times 10^{-7})\ \mathrm{cm}^{-3}$$

- ASSUME PHOTONS ARE BLACKBODY

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 = n_{\gamma 0} a^{-3} \qquad T = \frac{T_{CMB}}{a} \qquad n_{\gamma 0} = \frac{2\zeta(3)}{\pi^2} T_{CMB} = 410\ \mathrm{cm}^{-3}$$

$$\eta = \frac{n_b}{n_\gamma} = \frac{n_{b0}}{n_{\gamma 0}} = 6.2 \times 10^{-10}$$