Title: Cosmology (2021/2022)

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Collection: Cosmology (2021/2022)

Date: April 22, 2022 - 2:30 PM

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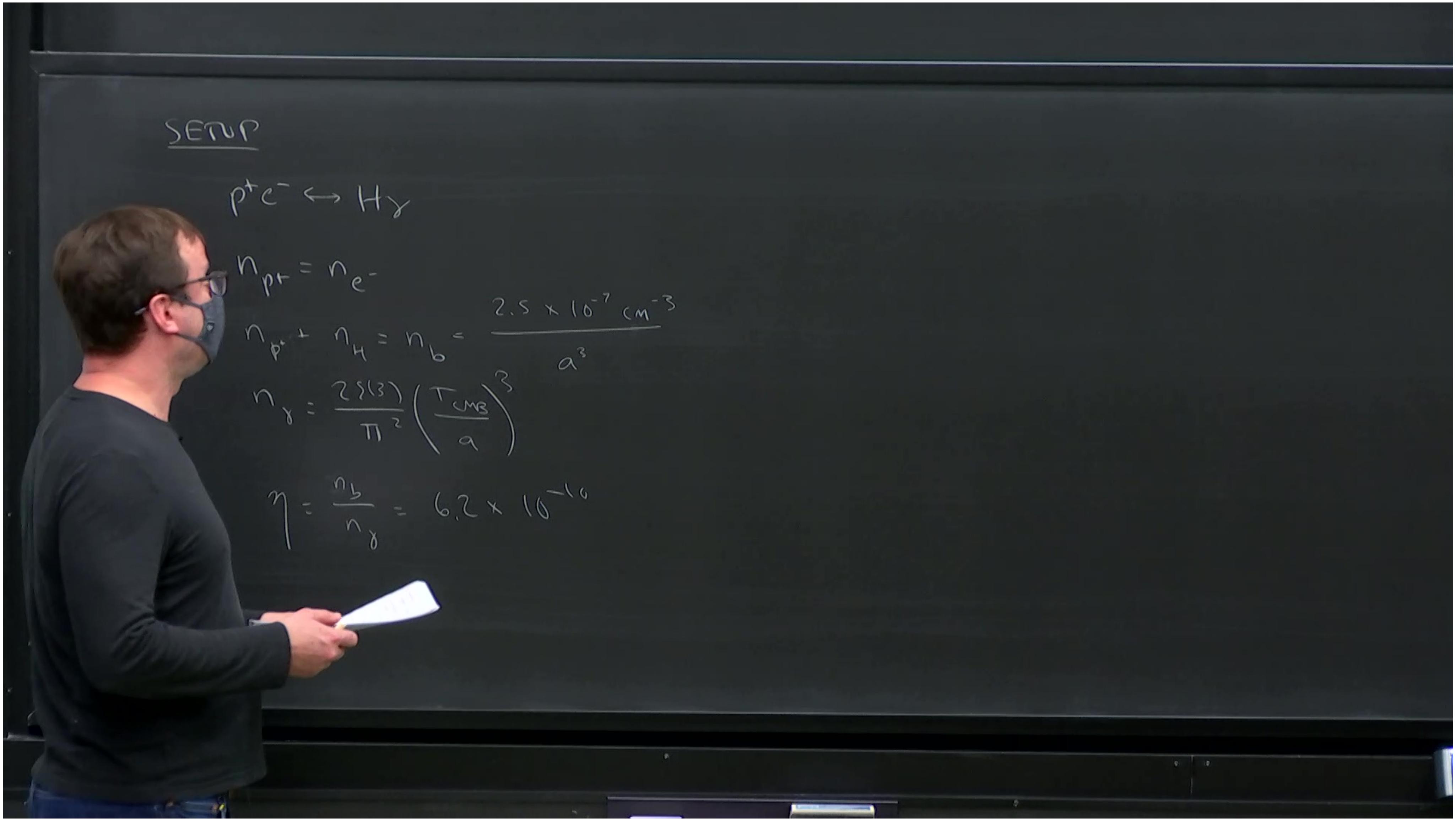
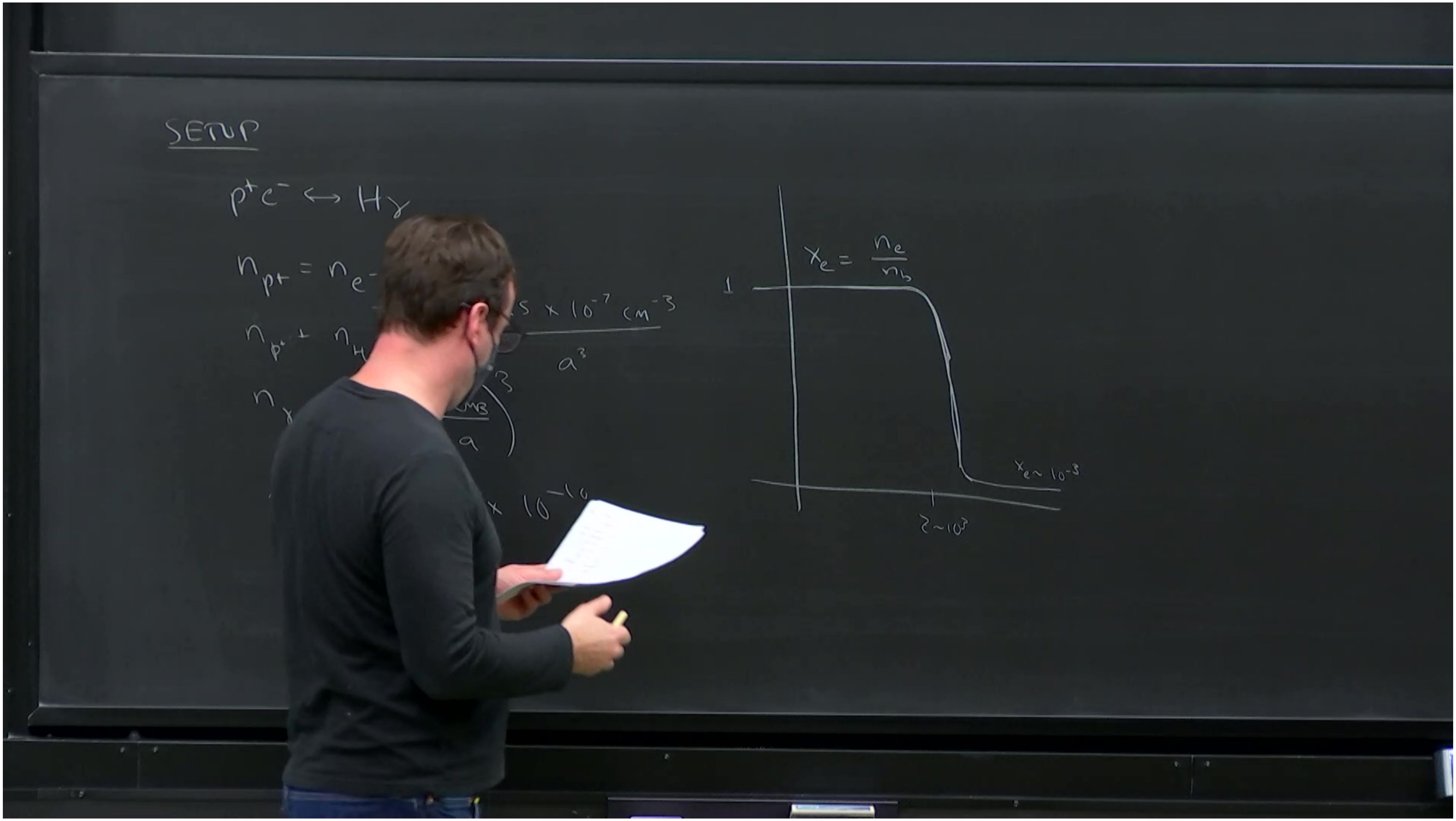
Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.


SETUP
$p^+e^- \leftrightarrow H\gamma$
$n_{p^+} = n_{e^-}$
$n_{p^+} + n_H = n_b = \frac{2.5 \times 10^{-7}\ cm^{-3}}{a^3}$
$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{T_{CMB}}{a}\right)^3$
$\eta = \frac{n_b}{n_\gamma} = 6.2 \times 10^{-10}$

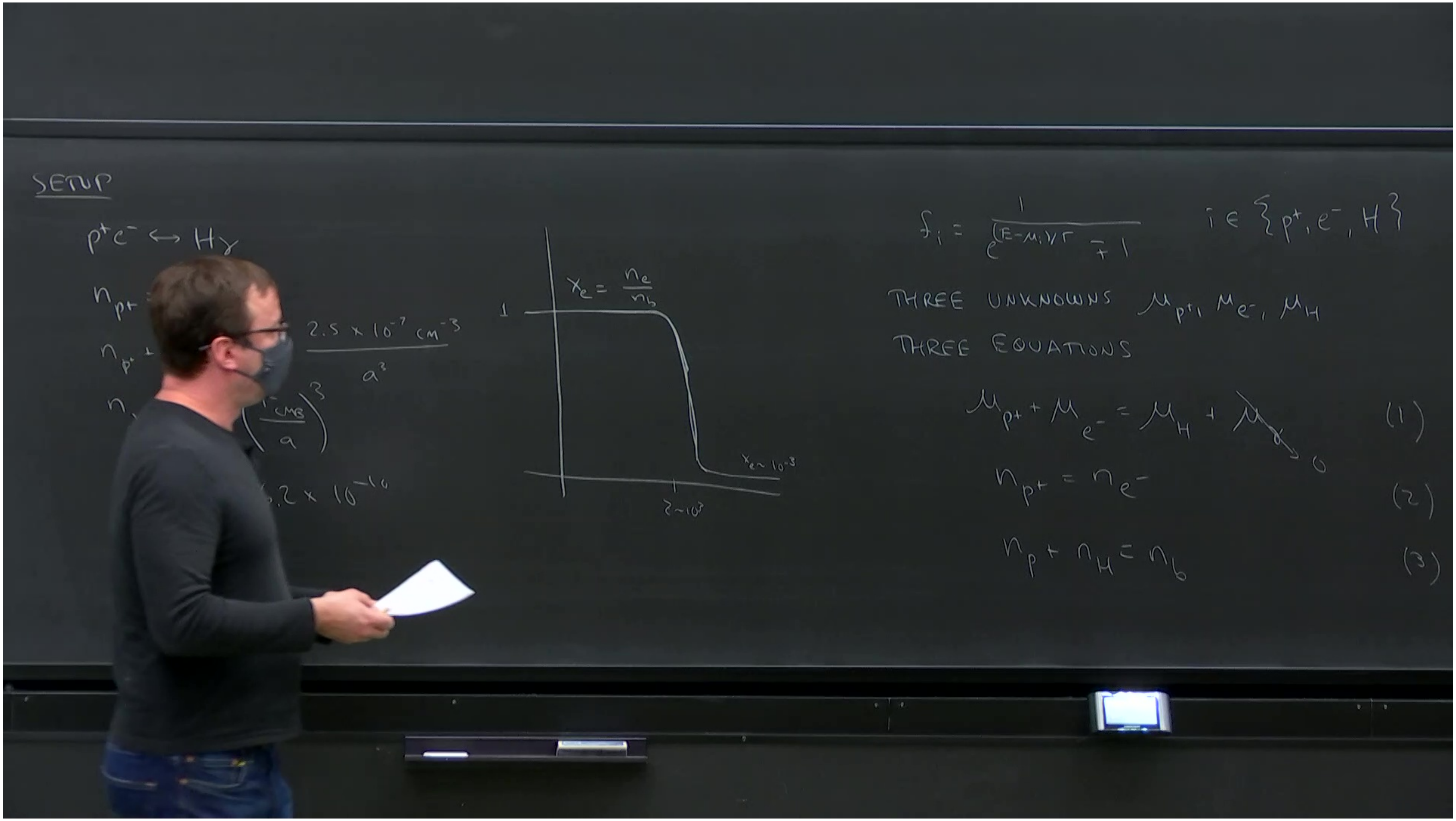

SETUP
p+e- ↔ Hγ
n_p+ = n_e-
5 x 10^-7 cm^-3
a^3
n_p+ + n_H
X_e = n_e / n_b
1
X_e ~ 10^-3
z ~ 10^3

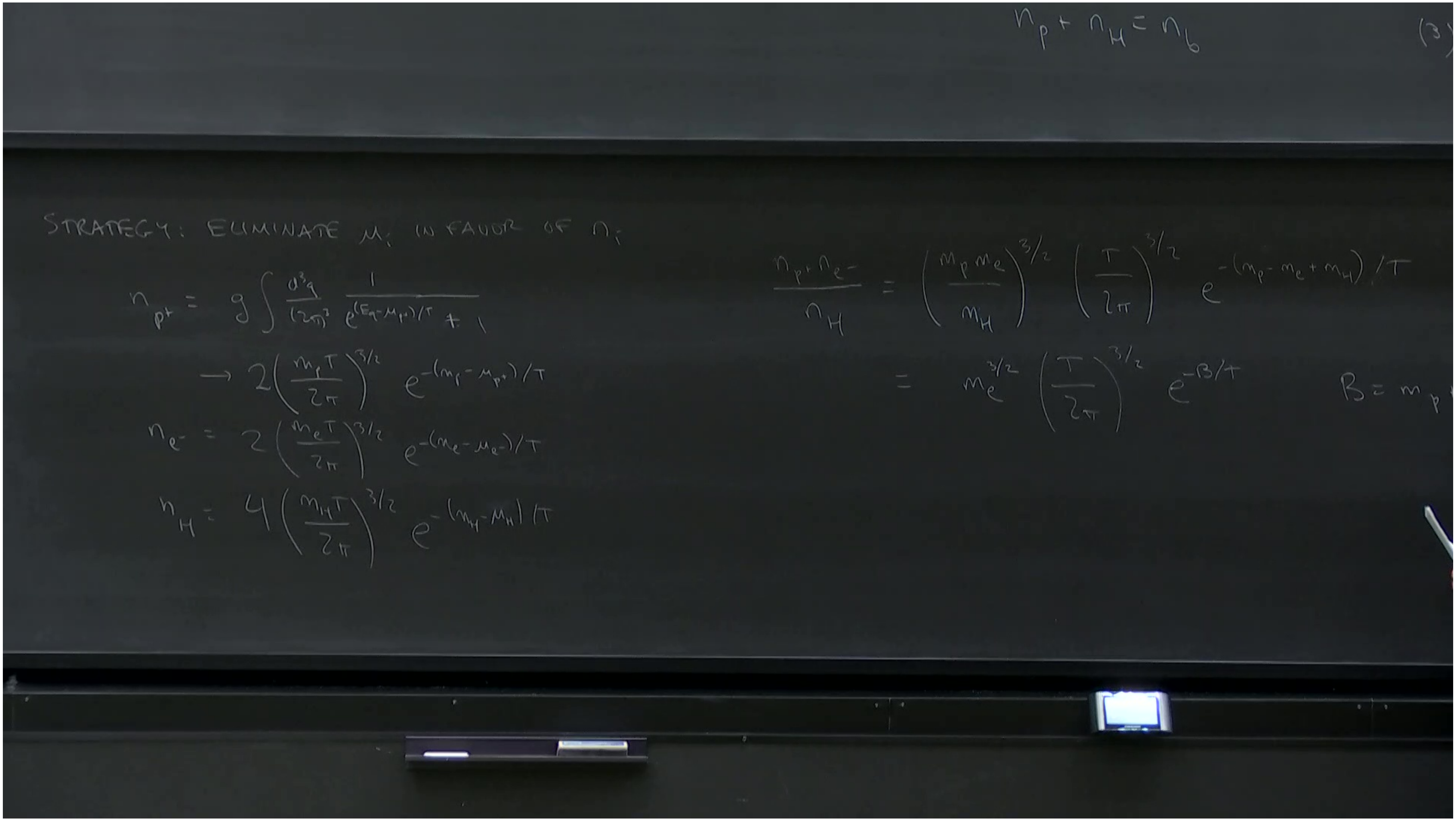
$$f_i = \frac{1}{e^{(E-\mu_i)/T} \mp 1} \qquad i \in \{p^+, e^-, H\}$$

THREE UNKNOWNS $\mu_{p^+}, \mu_{e^-}, \mu_H$

THREE EQUATIONS

$$\mu_{p^+} + \mu_{e^-} = \mu_H + \cancel{\mu_\gamma} \to 0$$

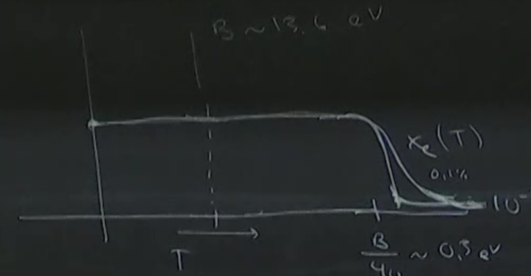

SETUP
p+e- ↔ Hγ
2.5 x 10^-7 cm^-3 / a^3
x_e = n_e / n_b
1
x_e ~ 10^-3
z ~ 10^3
f_i = 1 / (e^((E-μ_i)/T) ∓ 1)
i ∈ {p+, e-, H}
THREE UNKNOWNS μ_p+, μ_e-, μ_H
THREE EQUATIONS
μ_p+ + μ_e- = μ_H + μ_γ → 0 (1)
n_p+ = n_e- (2)
n_p + n_H = n_b (3)


$n_p + n_H = n_b$ (3)
STRATEGY: ELIMINATE μ_i IN FAVOR OF n_i
$n_{p^+} = g \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{(E_q - \mu_{p^+})/T} + 1}$
$\rightarrow 2\left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-(m_p - \mu_{p^+})/T}$
$n_{e^-} = 2\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-(m_e - \mu_{e^-})/T}$
$n_H = 4\left(\frac{m_H T}{2\pi}\right)^{3/2} e^{-(m_H - \mu_H)/T}$
$\frac{n_{p^+} n_{e^-}}{n_H} = \left(\frac{m_p m_e}{m_H}\right)^{3/2} \left(\frac{T}{2\pi}\right)^{3/2} e^{-(m_p + m_e - m_H)/T}$
$= m_e^{3/2} \left(\frac{T}{2\pi}\right)^{3/2} e^{-B/T}$
$B = m_p$

$$n_p + n_H = n_b \qquad (3)$$

$$\frac{n_p n_e}{n_H} = \left(\frac{m_p m_e}{m_H}\right)^{3/2} \left(\frac{T}{2\pi}\right)^{3/2} e^{-(m_p + m_e - m_H)/T}$$

$$= m_e^{3/2} \left(\frac{T}{2\pi}\right)^{3/2} e^{-B/T} \qquad B = m_p + m_e - m_H = 13.6\ \mathrm{eV}$$



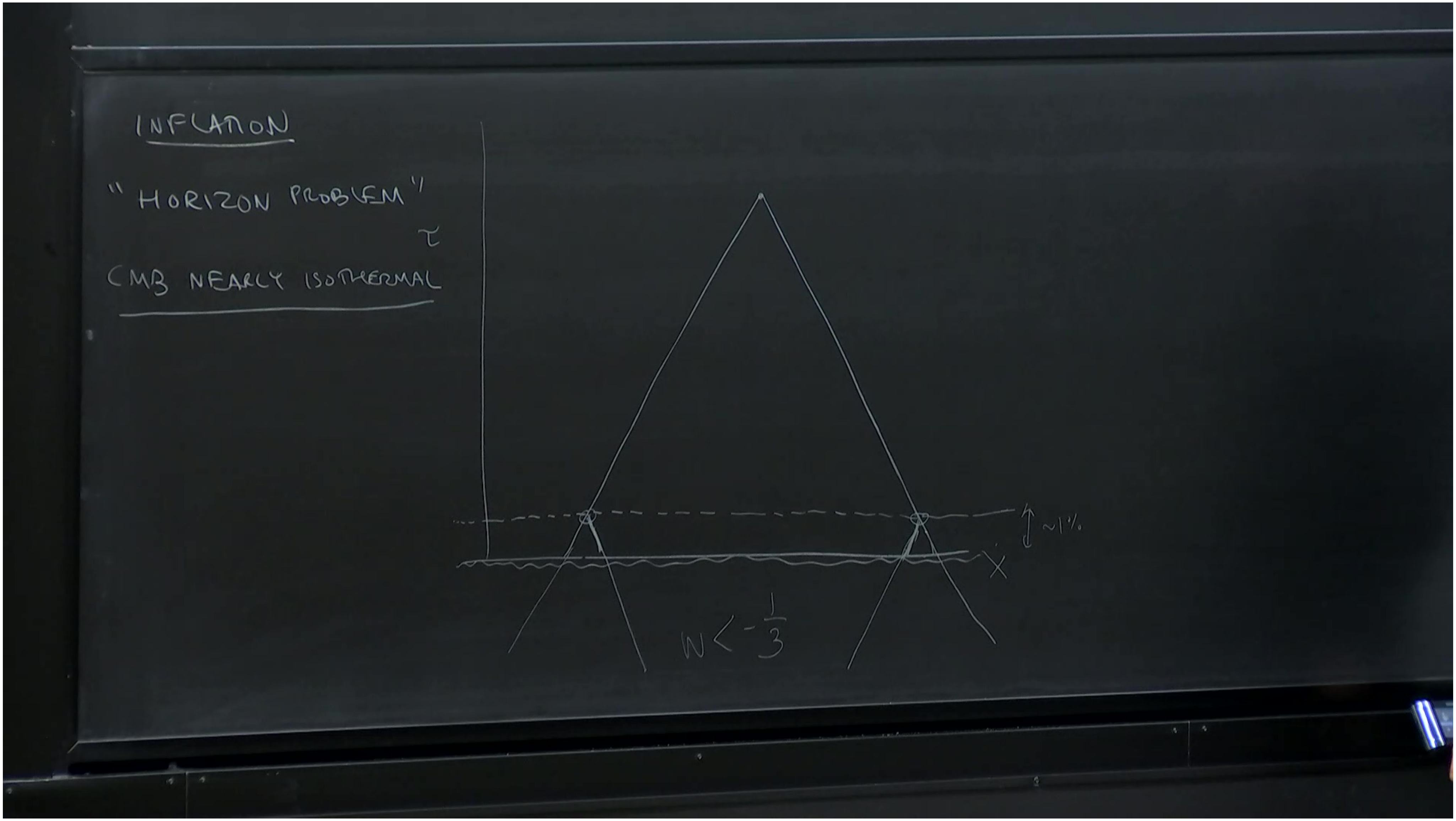
$$n_p = n_e = x_e n_b$$

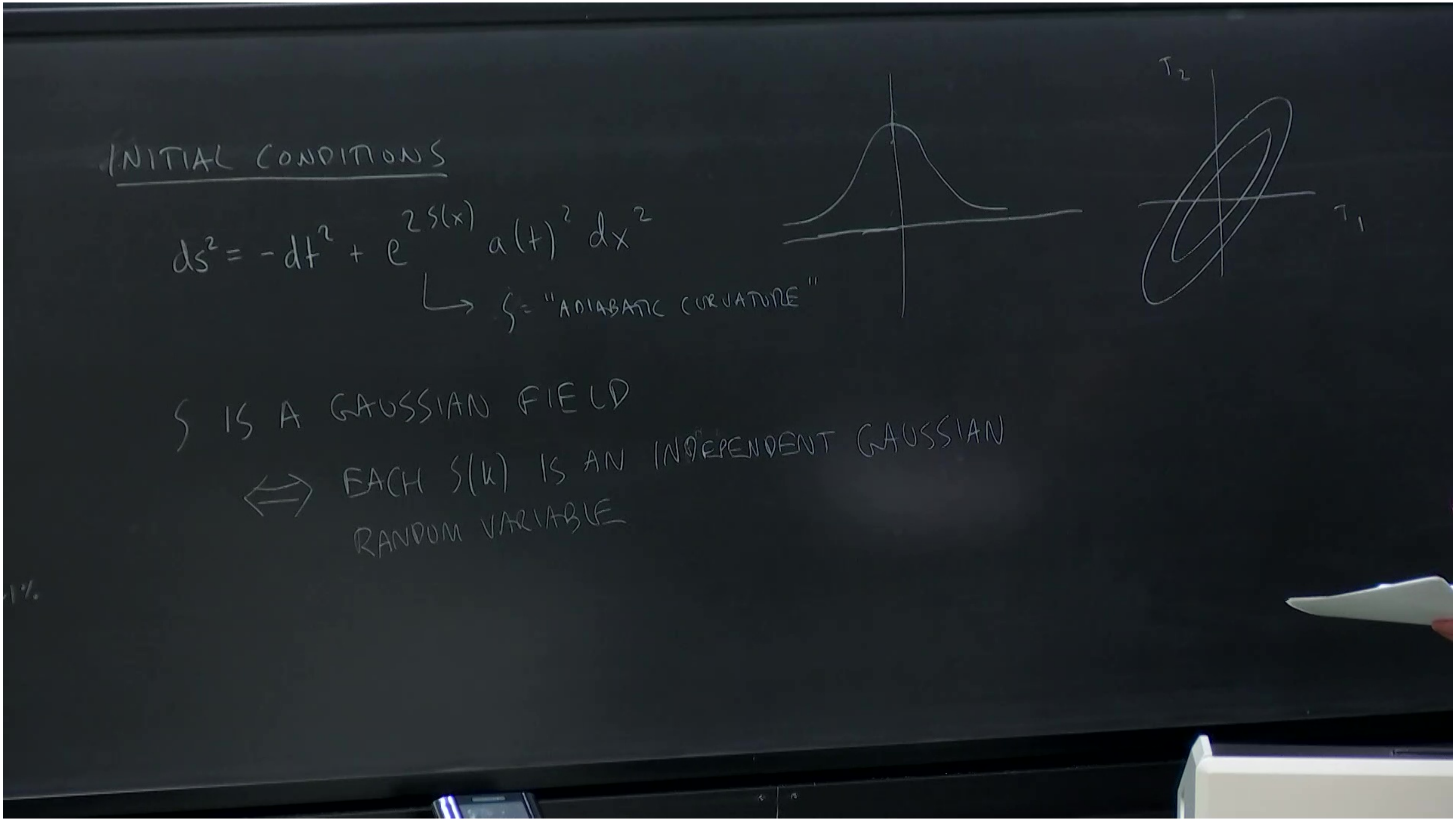
$$n_H = (1 - x_e) n_b$$

$$\Rightarrow \frac{x_e^2}{1 - x_e} = \frac{1}{n_b}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-B/T}$$

$$= \frac{(2\pi)^{1/2}}{\gamma\, \zeta(3)} \frac{1}{\eta} \left(\frac{m_e}{B}\right)^{3/2} \left(\frac{B}{T}\right)^{3/2} e^{-B/T} = (3 \times 10^{15}) \left(\frac{B}{T}\right)^{3/2} e^{-B/T}$$

"SAHA EQUATION"

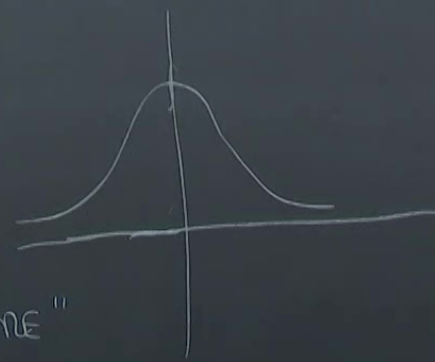

INFLATION
"HORIZON PROBLEM"
τ
CMB NEARLY ISOTHERMAL
~1%
X
W < -1/3

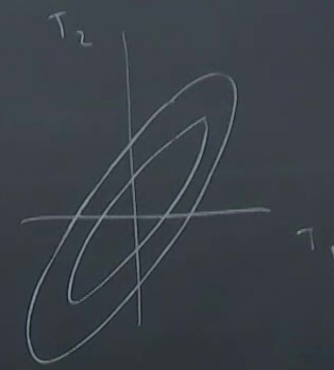

INITIAL CONDITIONS
$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$
ζ = "ADIABATIC CURVATURE"
T_2
T_1
ζ IS A GAUSSIAN FIELD
$\Longleftrightarrow$ EACH $\zeta(k)$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

INITIAL CONDITIONS

$$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$$

$\hookrightarrow$ ζ = "ADIABATIC CURVATURE"





- ζ IS A GAUSSIAN FIELD

  $\iff$ EACH $\zeta(k)$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

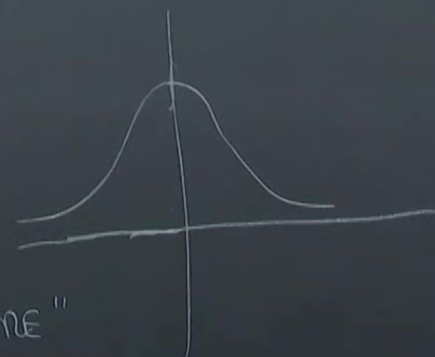
1%

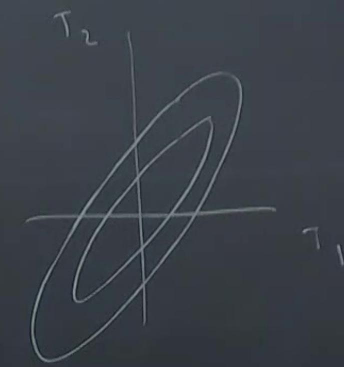
- ζ IS NEARLY SCALE INVARIANT

INITIAL CONDITIONS

$$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$$

$\hookrightarrow$ ζ = "ADIABATIC CURVATURE"





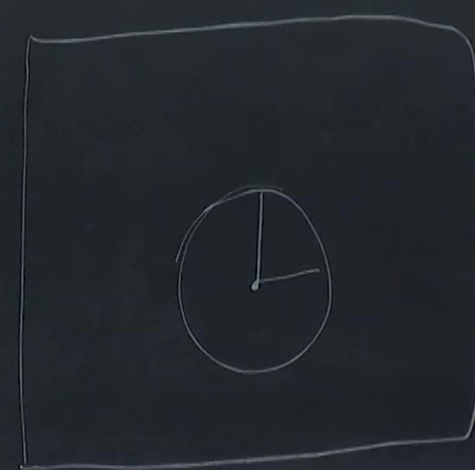
- ζ IS A GAUSSIAN FIELD

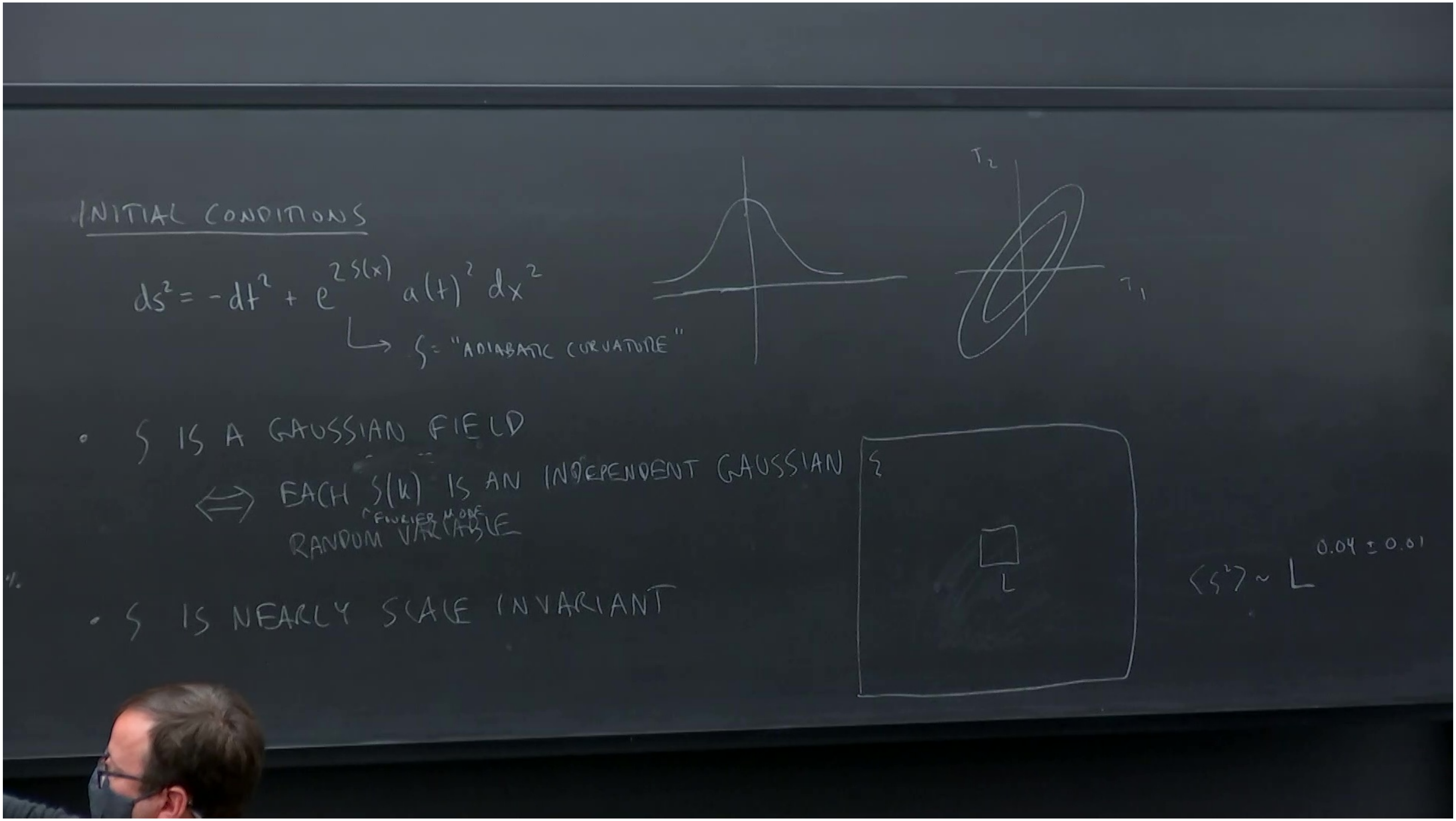
  $\Longleftrightarrow$ EACH $\zeta(k)$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE

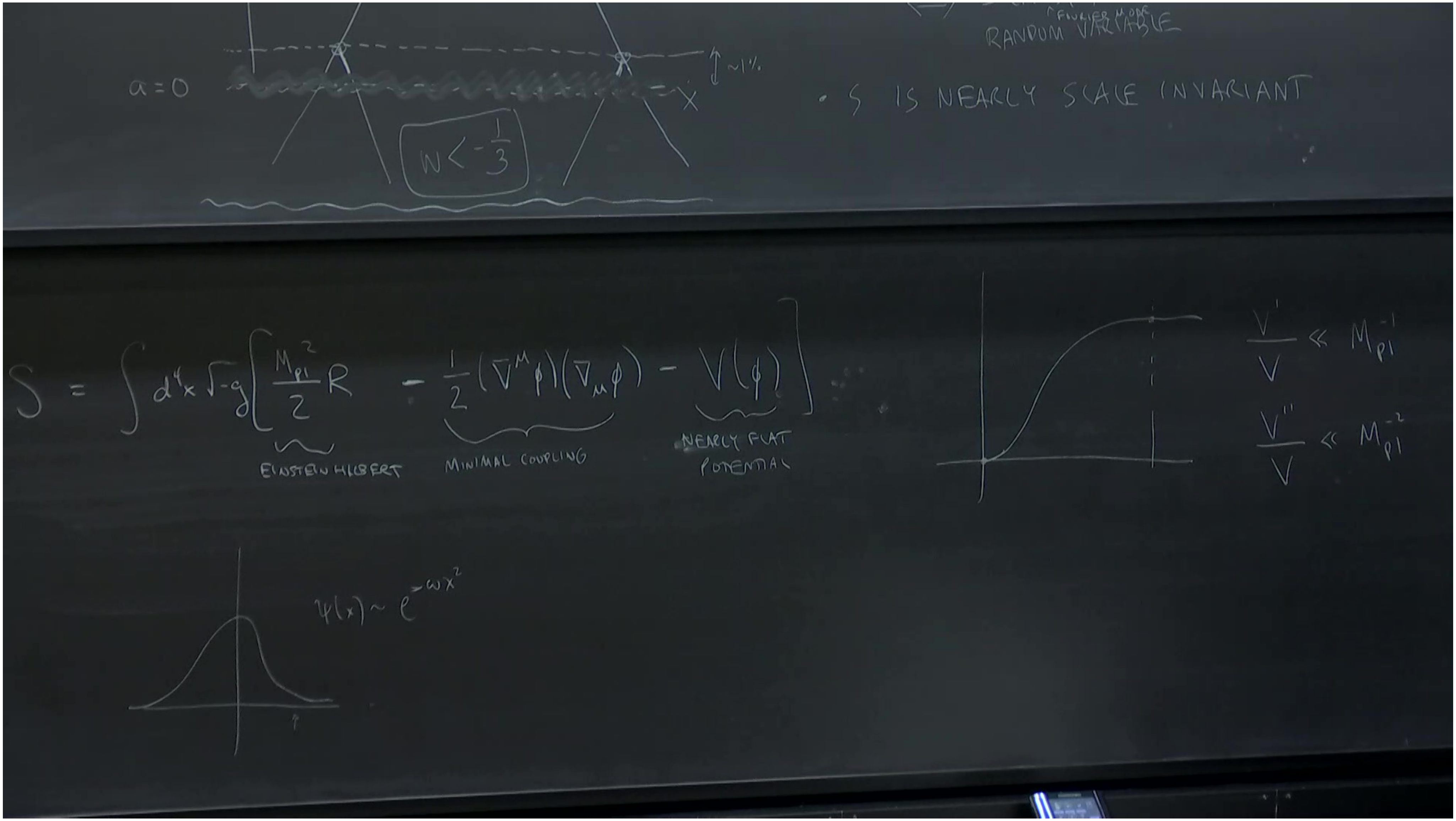
  ^ FOURIER MODE

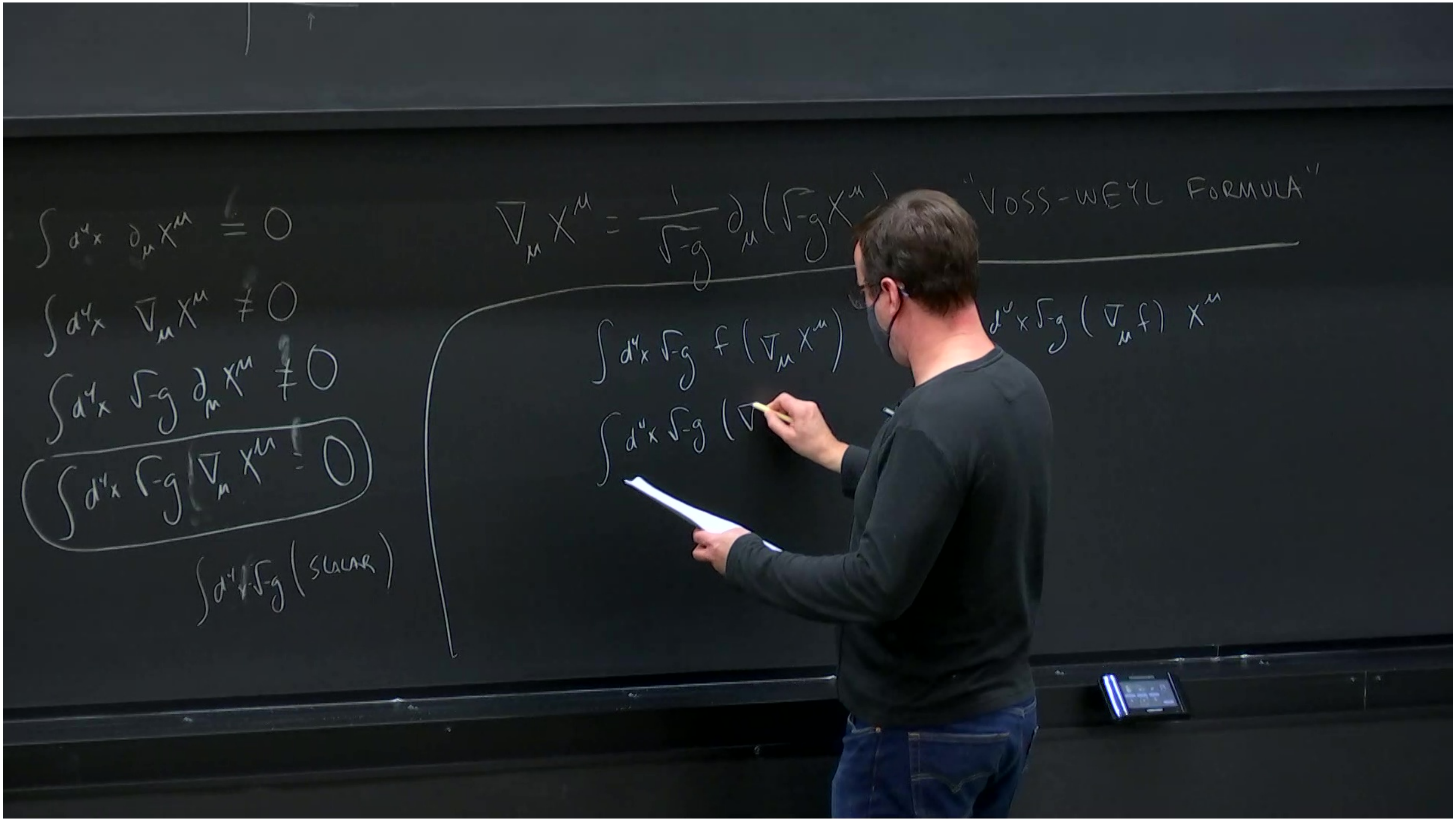
1%

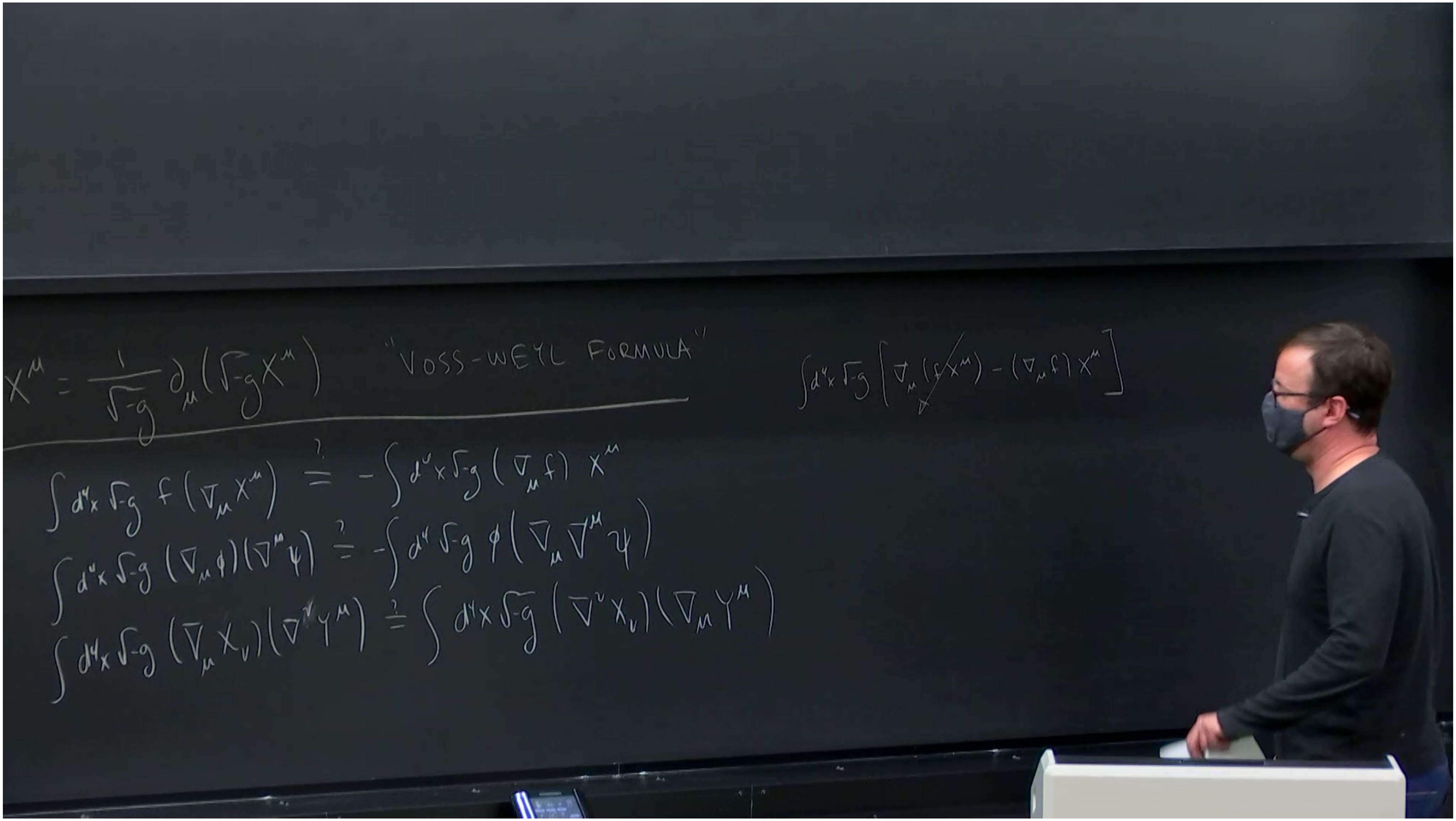
- ζ IS NEARLY SCALE INVARIANT




INITIAL CONDITIONS
$ds^2 = -dt^2 + e^{2\zeta(x)} a(t)^2 dx^2$
ζ = "ADIABATIC CURVATURE"
T_2
T_1
• ζ IS A GAUSSIAN FIELD
⟺ EACH $\zeta(k)$ IS AN INDEPENDENT GAUSSIAN RANDOM VARIABLE
FOURIER MODE
• ζ IS NEARLY SCALE INVARIANT
ζ
L
$\langle \zeta^2 \rangle \sim L^{0.04 \pm 0.01}$


FOURIER MODE
RANDOM VARIABLE
$a=0$
$\sim 1\%$
x
$w < -\frac{1}{3}$
• ζ IS NEARLY SCALE INVARIANT
$S = \int d^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R - \frac{1}{2} (\nabla^\mu \phi)(\nabla_\mu \phi) - V(\phi) \right]$
EINSTEIN HILBERT
MINIMAL COUPLING
NEARLY FLAT POTENTIAL
$\frac{V'}{V} \ll M_{pl}^{-1}$
$\frac{V''}{V} \ll M_{pl}^{-2}$
$\psi(x) \sim e^{-\omega x^2}$


$\int d^4x\, \partial_\mu X^\mu \doteq 0$
$\int d^4x\, \nabla_\mu X^\mu \neq 0$
$\int d^4x \sqrt{-g}\, \partial_\mu X^\mu \neq 0$
$\int d^4x \sqrt{-g}\, \nabla_\mu X^\mu \overset{!}{=} 0$
$\int d^4x \sqrt{-g}\, (\text{SCALAR})$
$\nabla_\mu X^\mu = \frac{1}{\sqrt{-g}}\, \partial_\mu(\sqrt{-g}\, X^\mu)$
"VOSS-WEYL FORMULA"
$\int d^4x \sqrt{-g}\, f\, (\nabla_\mu X^\mu)$
$d^4x \sqrt{-g}\, (\nabla_\mu f)\, X^\mu$
$\int d^4x \sqrt{-g}\, (\nabla$


$X^\mu = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} X^\mu)$
"VOSS-WEYL FORMULA"
$\int d^4x \sqrt{-g} \left[\nabla_\mu (\cancel{f} X^\mu) - (\nabla_\mu f) X^\mu \right]$
$\int d^4x \sqrt{-g}\, f (\nabla_\mu X^\mu) \stackrel{?}{=} -\int d^4x \sqrt{-g}\, (\nabla_\mu f)\, X^\mu$
$\int d^4x \sqrt{-g}\, (\nabla_\mu \phi)(\nabla^\mu \psi) \stackrel{?}{=} -\int d^4 \sqrt{-g}\, \phi (\nabla_\mu \nabla^\mu \psi)$
$\int d^4x \sqrt{-g}\, (\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) \stackrel{?}{=} \int d^4x \sqrt{-g}\, (\nabla^\nu X_\nu)(\nabla_\mu Y^\mu)$

$$\int d^4x \sqrt{-g}\, (\nabla_\mu X_\nu)(\nabla^\nu Y^\mu) = -\int d^4x \sqrt{-g}\, X^\nu \, (\nabla_\mu \nabla_\nu Y^\mu)$$

$$= -\int d^4x \sqrt{-g}\, X^\nu \left[\nabla_\nu \nabla_\mu Y^\mu - R_{\mu\nu\rho}{}^{\mu} Y^\rho \right]$$

$$= -\int d^4x \sqrt{-g}\, X^\nu \left[\nabla_\nu \nabla_\mu Y^\mu + R_{\nu\rho} Y^\rho \right]$$

$$= \int d^4x \sqrt{-g} \left[(\nabla_\nu X^\nu)(\nabla_\mu Y^\mu) - R_{\nu\rho} X^\nu Y^\rho \right]$$

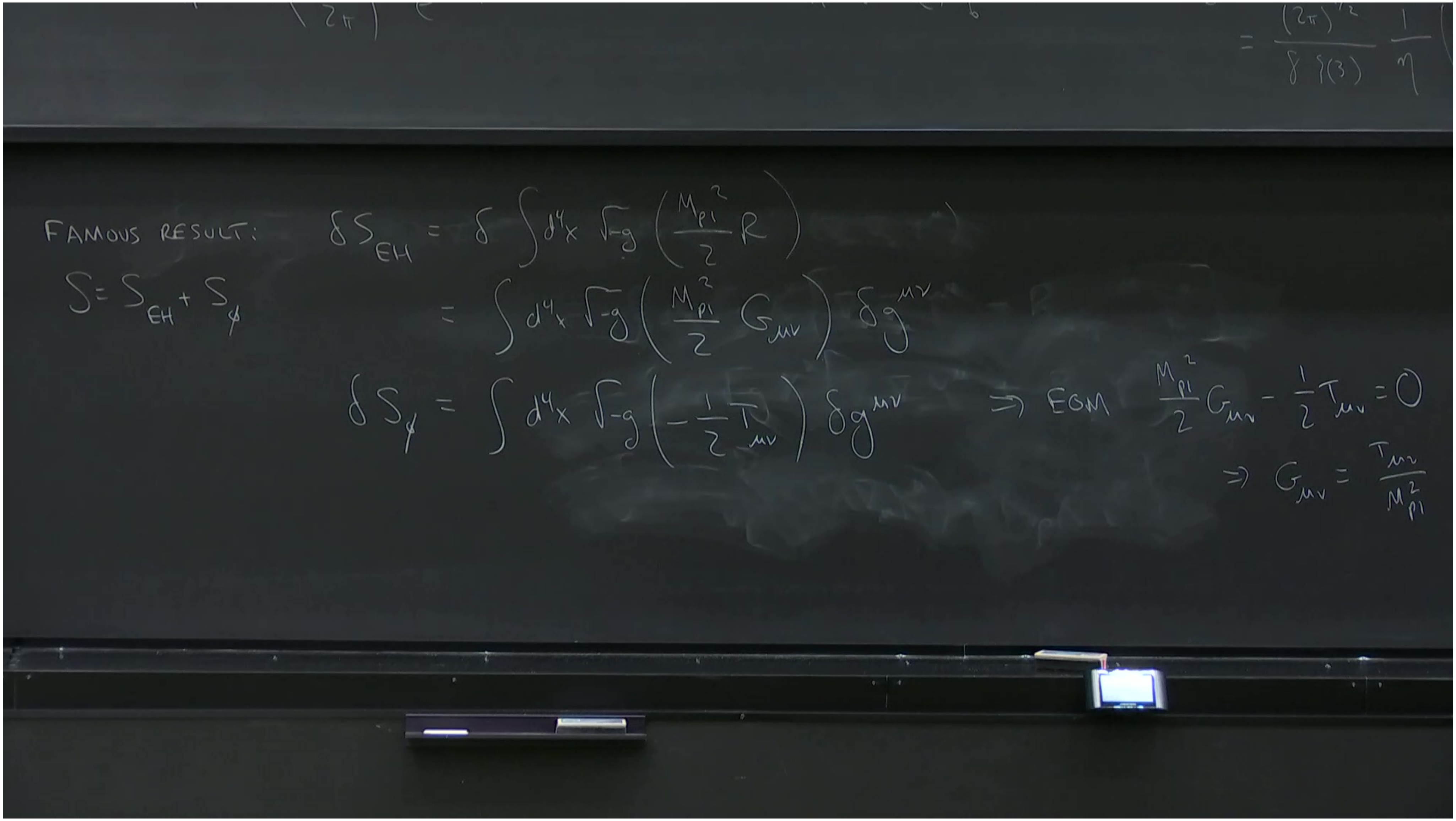
$$\delta S = \int d^4x \sqrt{-g}\left[-(\nabla^\mu \phi)\nabla_\mu(\delta\phi) - V'(\phi)(\delta\phi)\right]$$

$$= \int d^4x \sqrt{-g}\left[(\nabla_\mu \nabla^\mu \phi)(\delta\phi) - V'(\phi)(\delta\phi)\right]$$

$$\Rightarrow \boxed{\nabla^2\phi - V'(\phi) = 0}$$

WHERE $\nabla^2\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$

$$\rightarrow \left[-\frac{\partial^2\phi}{\partial t^2} - 3H\frac{\partial\phi}{\partial t} + \frac{1}{a^2}\delta^{ij}(\partial_i\phi)(\partial_j\phi)\right]$$


$= \frac{(2\pi)^{3/2}}{8\,\zeta(3)}\,\frac{1}{\eta}$
FAMOUS RESULT:
$S = S_{EH} + S_\phi$
$\delta S_{EH} = \delta \int d^4x\,\sqrt{-g}\left(\frac{M_{pl}^2}{2} R\right)$
$= \int d^4x\,\sqrt{-g}\left(\frac{M_{pl}^2}{2} G_{\mu\nu}\right)\delta g^{\mu\nu}$
$\delta S_\phi = \int d^4x\,\sqrt{-g}\left(-\frac{1}{2} T_{\mu\nu}\right)\delta g^{\mu\nu}$
$\Rightarrow$ EOM $\frac{M_{pl}^2}{2} G_{\mu\nu} - \frac{1}{2} T_{\mu\nu} = 0$
$\Rightarrow G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{pl}^2}$

$$\delta(\nabla_\mu \phi) = \delta(\partial_\mu \phi) = 0$$

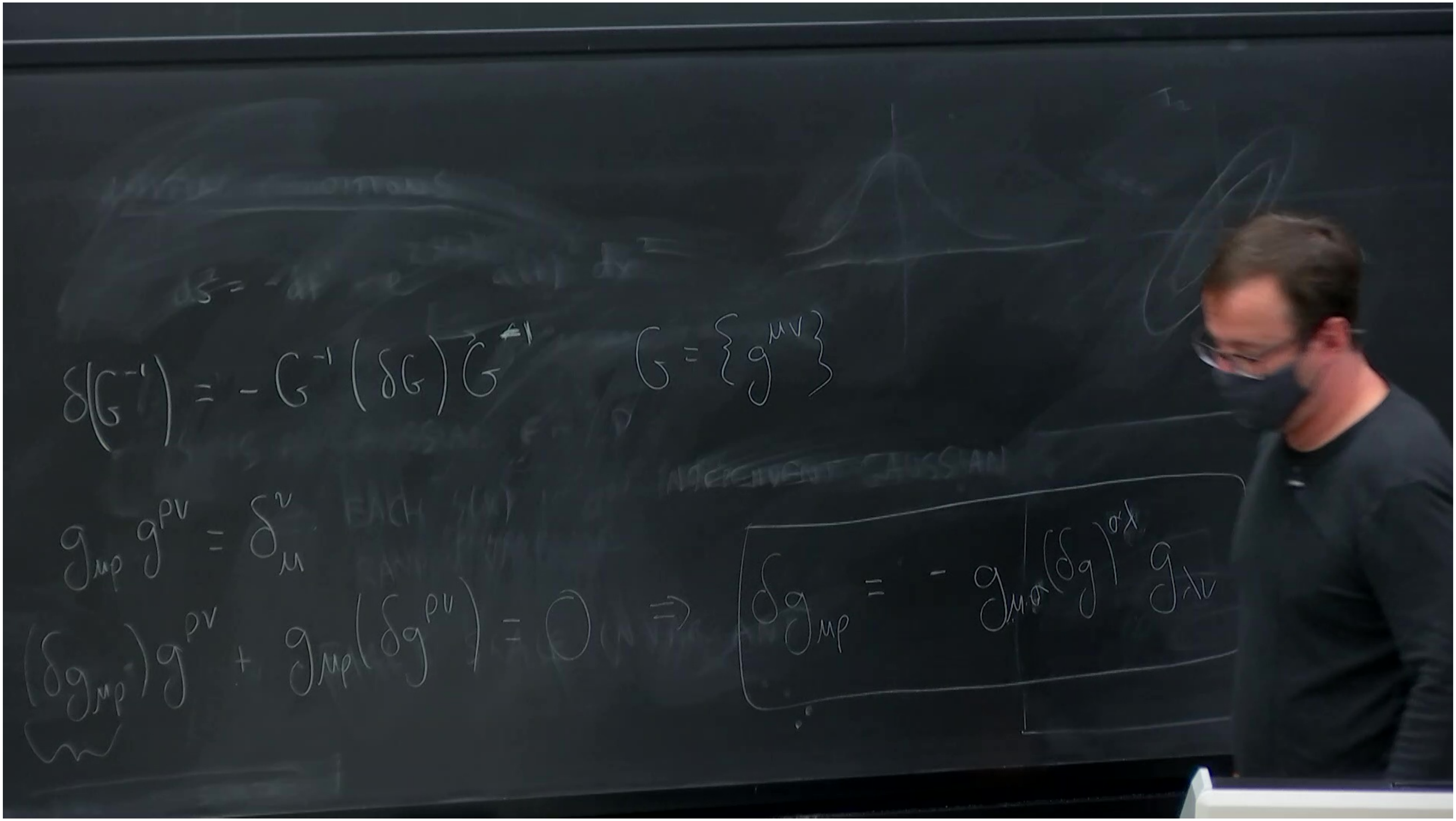
$$\delta(\nabla^\mu \phi) = \delta(g^{\mu\nu} \partial_\nu \phi) = (\partial_\nu \phi)\, \delta g^{\mu\nu}$$

$$\delta(g_{\mu\nu}) = -g_{\mu\rho} (\delta g)^{\rho\sigma} g_{\sigma\nu}$$

MINUS SIGN!

$$\delta(g^{\mu\nu}) = \delta g^{\mu\nu}$$

$$\delta(G^{-1}) = -G^{-1}(\delta$$


$\delta(G^{-1}) = -G^{-1}(\delta G)G^{-1}$
$G = \{g^{\mu\nu}\}$
$g_{\mu\rho}\, g^{\rho\nu} = \delta_\mu^\nu$
$(\delta g_{\mu\rho})\, g^{\rho\nu} + g_{\mu\rho}(\delta g^{\rho\nu}) = 0 \Rightarrow \boxed{\delta g_{\mu\rho} = - g_{\mu\alpha}(\delta g)^{\alpha\lambda} g_{\lambda\nu}}$