Title: Cosmology (2021/2022)

Speakers: Kendrick Smith, Gang Xu

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Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.

near equilibrium: $\Gamma \gg H$

Out of equilibrium

$$E^2 = p^2 + m^2$$

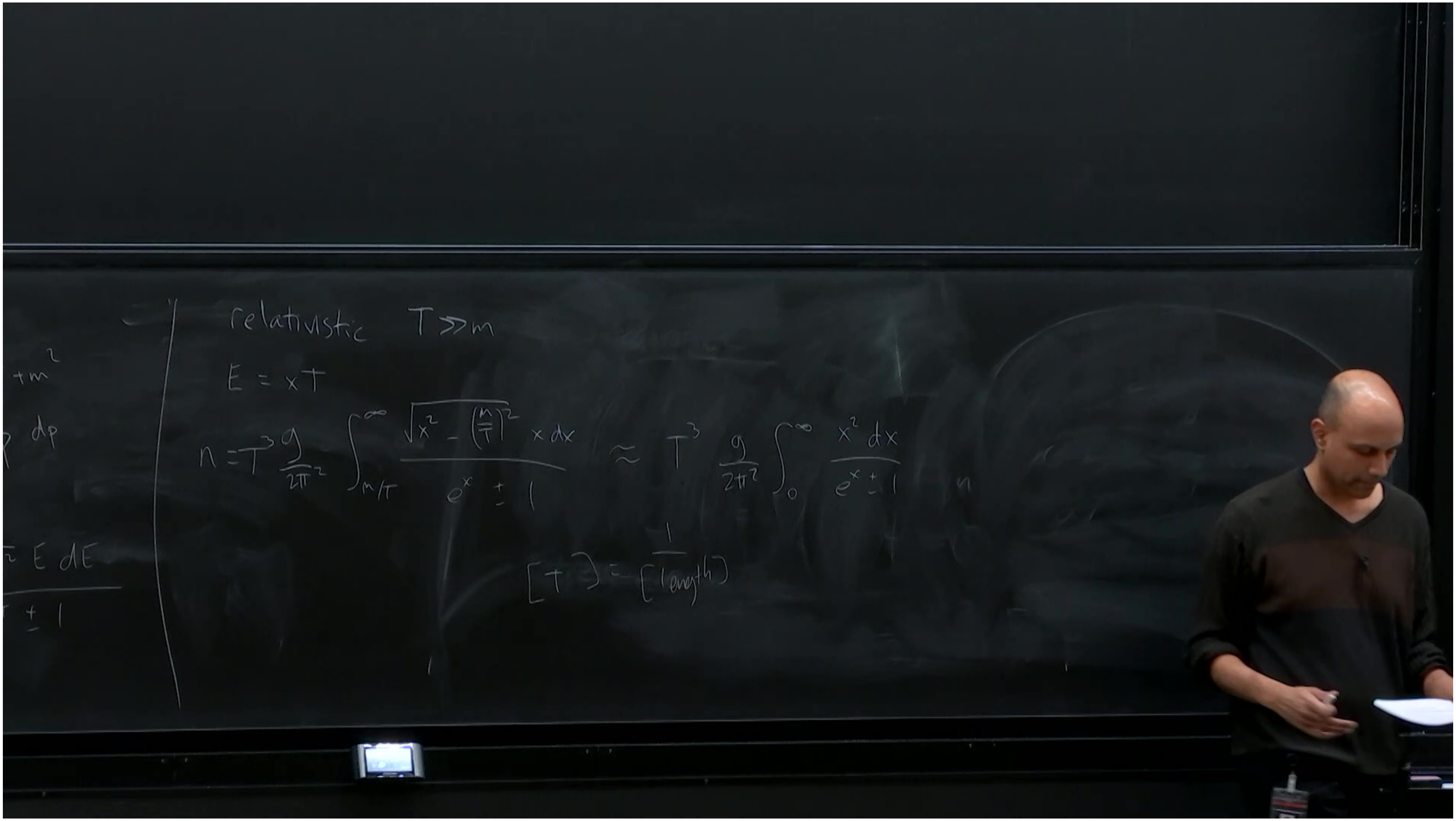
$$E\, dE = p\, dp$$

$$f(E) = \frac{1}{e^{(E-\mu)/T} \pm 1}$$

$$n(x) = g \int \frac{4\pi p^2\, dp}{(2\pi)^3} f(E)$$

$$= \frac{g}{2\pi^2} \int p^2\, dp\, f(E) = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{E/T} \pm 1}$$

for $\mu = 0$


relativistic $T \gg m$
$E = xT$
$n = T^3 \frac{g}{2\pi^2} \int_{m/T}^{\infty} \frac{\sqrt{x^2 - \left(\frac{m}{T}\right)^2}\, x\, dx}{e^x \pm 1} \approx T^3 \frac{g}{2\pi^2} \int_0^{\infty} \frac{x^2\, dx}{e^x \pm 1}$
$[T] = \frac{1}{[\text{length}]}$

$+m^2$

dp

$E\,dE$

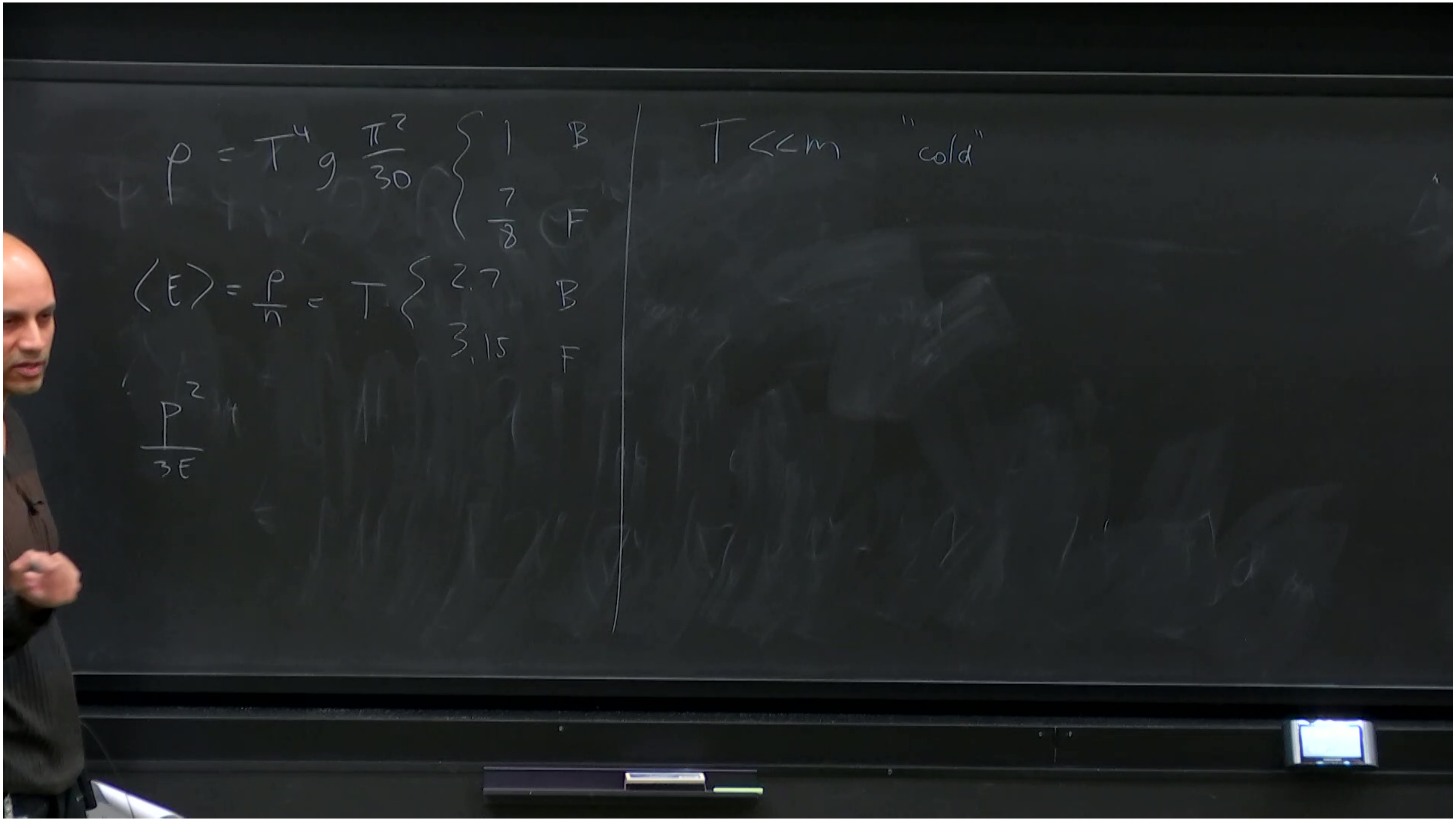
± 1

relativistic $T \gg m$

$E = xT$

$$n = T^3 \frac{g}{2\pi^2} \int_{m/T}^{\infty} \frac{\sqrt{x^2 - \left(\frac{m}{T}\right)^2}\, x\, dx}{e^x \pm 1} \approx T^3 \frac{g}{2\pi^2} \underbrace{\int_0^{\infty} \frac{x^2\, dx}{e^x \pm 1}}_{\substack{2\zeta(3) \quad B \\ \frac{3}{4}\, 2\zeta(3) \quad F}} = T^3 \frac{g}{\pi^2} \zeta(3) \begin{cases} 1 & B \\ \frac{3}{4} & F \end{cases}$$

$$[T] = \frac{1}{[\text{length}]}$$


$\rho = T^4 g \frac{\pi^2}{30} \begin{cases} 1 & B \\ \frac{7}{8} & F \end{cases}$
$\langle E \rangle = \frac{\rho}{n} = T \begin{cases} 2.7 & B \\ 3.15 & F \end{cases}$
$\frac{p^2}{3E}$
$T \ll m$ "cold"

$T \ll m$ "cold" $\qquad \varepsilon = x\,T$

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\; E \; e^{-E/T}}{1 \pm e^{-E/T}}\, dE$$

$$E = m + \varepsilon \Rightarrow n = \frac{g}{2\pi^2} e^{-m/T} \int_0^\infty d\varepsilon \; \sqrt{2m\varepsilon + \varepsilon^2}\,(m+\varepsilon)\, e^{-\varepsilon/T}$$

$$= \frac{g}{2\pi^2} e^{-m/T} \sqrt{2m^3T^3} \underbrace{\int_0^\infty \sqrt{x}\, e^{-x}\, dx}_{\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}}$$

$T \ll m$ "cold" $\qquad \varepsilon = x\,T$

$$n = \frac{g}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\; E\; e^{-E/T}}{1 \pm e^{-E/T}}\, dE$$

$$E = m + \varepsilon \Rightarrow n = \frac{g}{2\pi^2} e^{-m/T} \int_0^\infty d\varepsilon\, \sqrt{2m\varepsilon + \varepsilon^2}\,(m+\varepsilon)\, e^{-\varepsilon/T}$$

$$= \frac{g}{2\pi^2} e^{-m/T} \sqrt{2m^3T^3} \underbrace{\int_0^\infty \sqrt{x}\, e^{-x}\, dx}_{\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}} = e^{-m/T} g \left(\frac{mT}{2\pi}\right)^{3/2}$$

m "cold"

$$\frac{g}{2\pi^2}\int_m^\infty \frac{\sqrt{E^2-m^2}\,E\,e^{-E/T}}{1 \pm e^{-E/T}}\,dE$$

$\varepsilon = xT$

$$m+\varepsilon \Rightarrow n = \frac{g}{2\pi^2} e^{-m/T} \int_0^\infty d\varepsilon \sqrt{2m\varepsilon+\varepsilon^2}\,(m+\varepsilon)\,e^{-\varepsilon/T}$$

"Boltzmann suppression"

$$= \frac{g}{2\pi^2} e^{-m/T} \sqrt{2m^3T^3} \underbrace{\int_0^\infty \sqrt{x}\,e^{-x}\,dx}_{\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}} = e^{-m/T}\, g \left(\frac{mT}{2\pi}\right)^{3/2}$$

$$p = m\,n + \frac{3}{2} n T$$

$$P \approx \frac{g}{2\pi^2} e^{-m/T} \sqrt{2m^3 T^5}$$

$$\frac{p^2}{3E} = \frac{E^2 - m^2}{3E} \approx \frac{2m\varepsilon}{3m} = \frac{2}{3}\varepsilon = \frac{2}{3} T \times$$

$$E = m + \varepsilon$$

$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$

$$P \approx \frac{g}{2\pi^2} e^{-m/T} \sqrt{2m^3 T^5}\, \frac{2}{3} \int_0^\infty x^{3/2} e^{-x}\, dx$$

$$\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right)$$

$$\simeq n T$$

$$= \frac{2}{3}\varepsilon = \frac{2}{3} T x$$

$$T \ll m \Rightarrow P \ll \rho$$

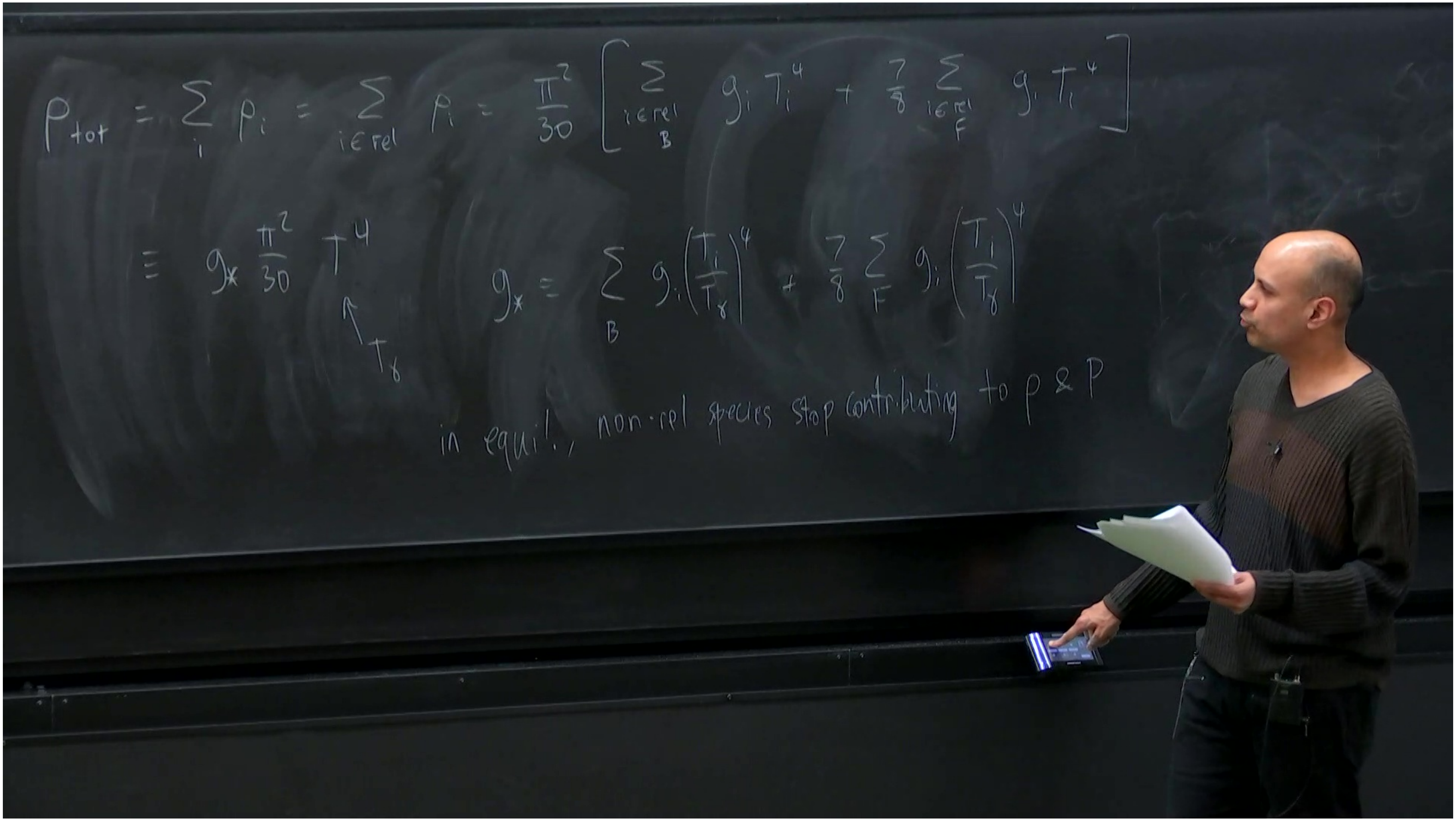
$$w = \frac{P}{\rho} \begin{cases} = \frac{1}{3} & \text{rel} \\ \simeq 0 & \text{nonrel} \end{cases}$$

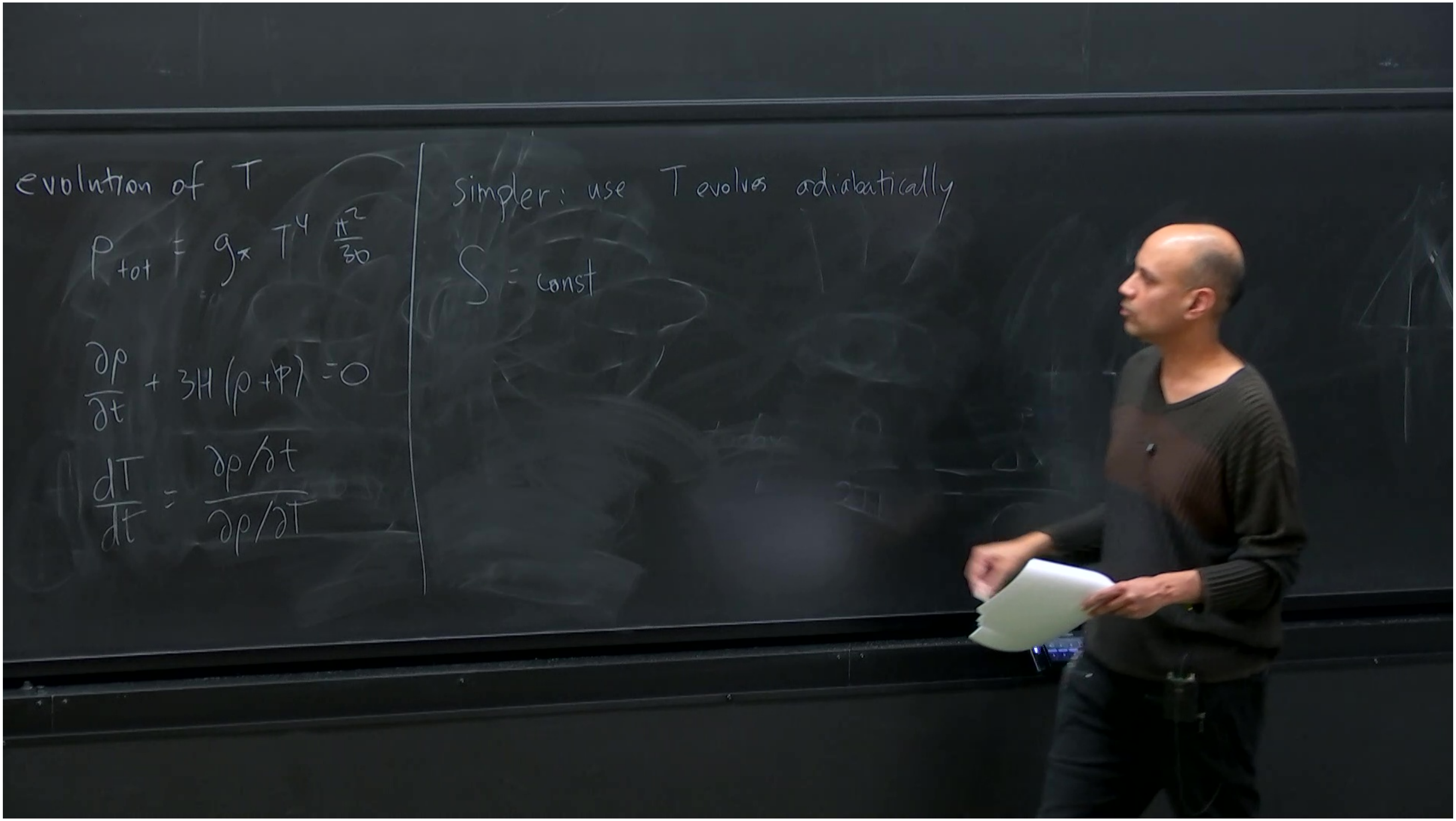
$$\rho_{tot} = \sum_i \rho_i = \sum_{i \in rel} \rho_i = \frac{\pi^2}{30}\left[\sum_{\substack{i \in rel \\ B}} g_i T_i^4 + \frac{7}{8}\sum_{\substack{i \in rel \\ F}} g_i T_i^4\right]$$

$$\rho_{tot} = \sum_i \rho_i = \sum_{i \in rel} \rho_i = \frac{\pi^2}{30}\left[\sum_{\substack{i \in rel \\ B}} g_i T_i^4 + \frac{7}{8}\sum_{\substack{i \in rel \\ F}} g_i T_i^4\right]$$

$$\equiv g_* \frac{\pi^2}{30} \underset{\substack{\uparrow \\ T_\gamma}}{T}{}^4$$

$$g_* = \sum_B g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8}\sum_F g_i \left(\frac{T_i}{T}\right)^4$$


$\rho_{tot} = \sum_i \rho_i = \sum_{i \in rel} \rho_i = \frac{\pi^2}{30} \left[\sum_{\substack{i \in rel \\ B}} g_i T_i^4 + \frac{7}{8} \sum_{\substack{i \in rel \\ F}} g_i T_i^4 \right]$
$\equiv g_* \frac{\pi^2}{30} T^4$
T_γ
$g_* = \sum_B g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8} \sum_F g_i \left(\frac{T_i}{T_\gamma}\right)^4$
in equil., non-rel species stop contributing to ρ & P


evolution of T
$\rho_{tot} = g_* T^4 \frac{\pi^2}{30}$
$\frac{\partial \rho}{\partial t} + 3H(\rho + P) = 0$
$\frac{dT}{dt} = \frac{\partial \rho / \partial t}{\partial \rho / \partial T}$
Simpler: use T evolves adiabatically
$S = $ const

evolution of T

$$\rho_{tot} = g_* T^4 \frac{\pi^2}{30}$$

$$\frac{\partial \rho}{\partial t} + 3H(\rho + P) = 0$$

$$\frac{dT}{dt} = \frac{\partial \rho / \partial t}{\partial \rho / \partial T}$$

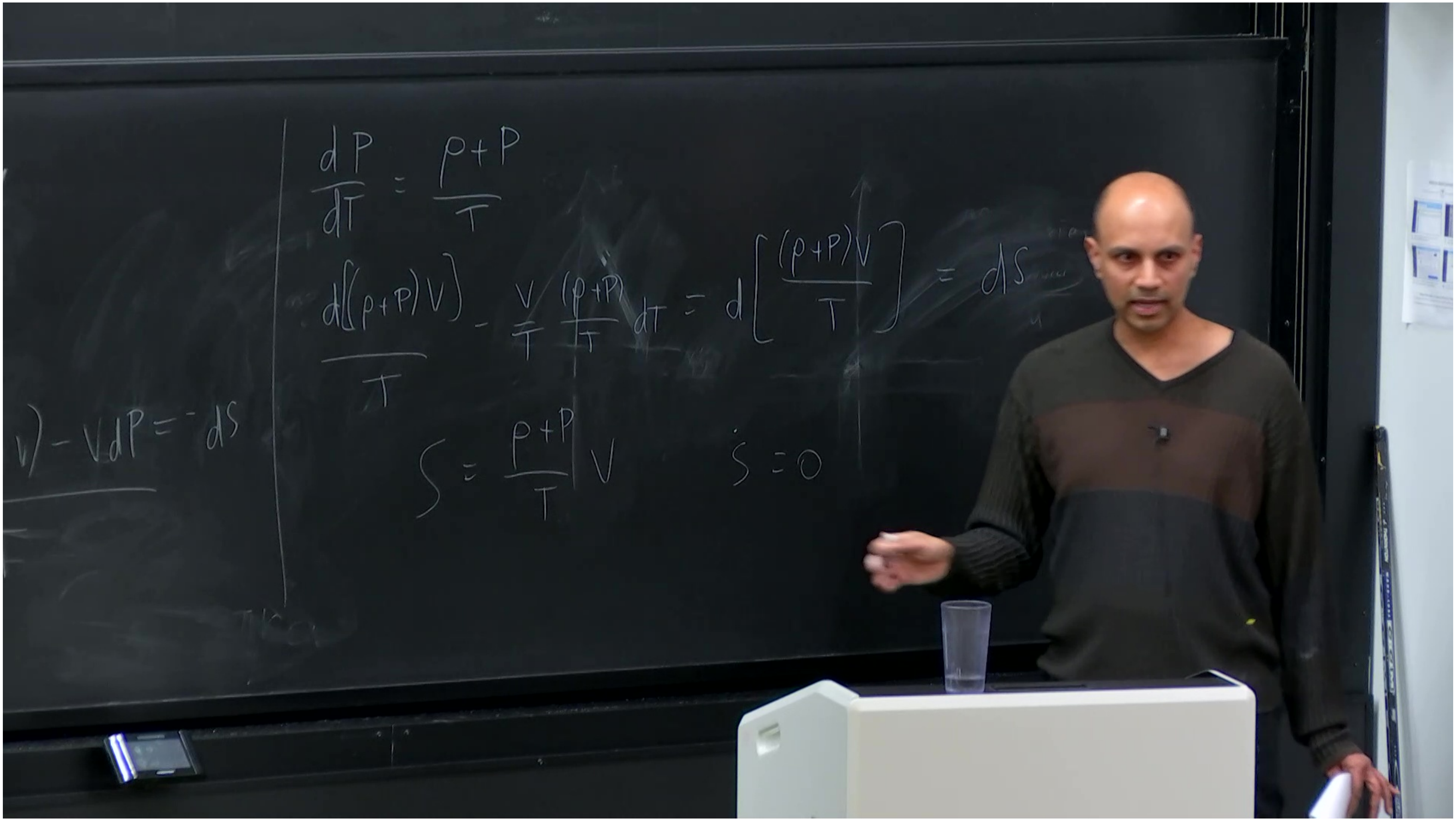
simpler: use T evolves adiabatically

$$S = \text{const} \Rightarrow dS = 0$$

$$dE + PdV - TdS = 0$$

$$d(\rho V) + PdV - TdS = 0 \quad = \frac{d((\rho+P)V) - VdP}{T} = dS$$

$$PdV + VdP = d(PV)$$


$\frac{dP}{dT} = \frac{\rho + P}{T}$
$\frac{d[(\rho+P)V]}{T} - \frac{V}{T}\frac{(\rho+P)}{T}dT = d\left[\frac{(\rho+P)V}{T}\right] = dS$
$V) - VdP = -dS$
$S = \frac{\rho+P}{T}V$
$\dot{S} = 0$

$$s = \frac{S}{V} = \frac{\rho + P}{T}$$

$$\rho = g \frac{\pi^2}{30} T^4 \begin{cases} 1 & B \\ 7/8 & F \end{cases}$$

$$P = \rho/3$$

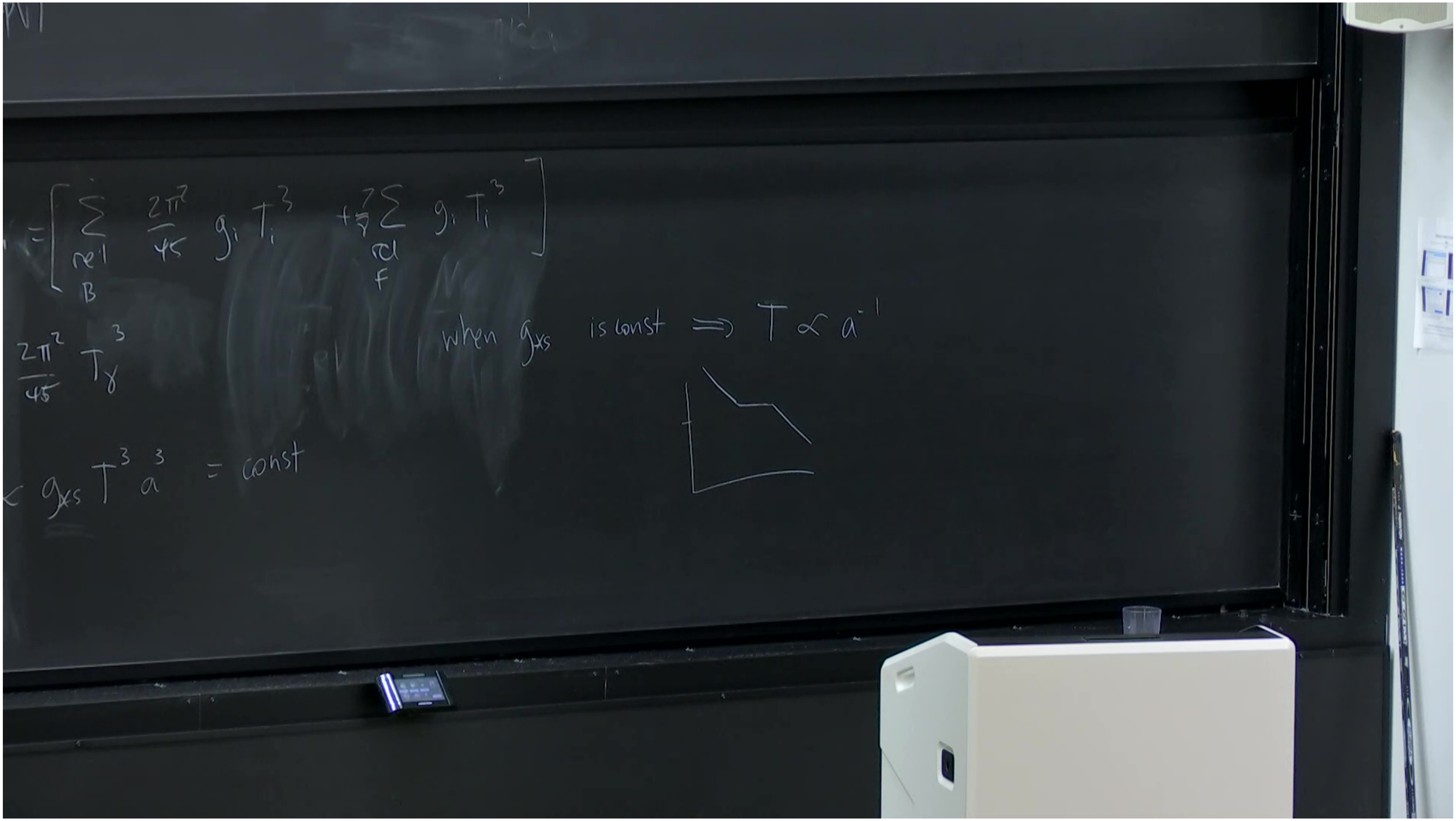
$$s = \frac{2\pi^2}{45} g T^3 \begin{cases} 1 \\ 7/8 \end{cases}$$

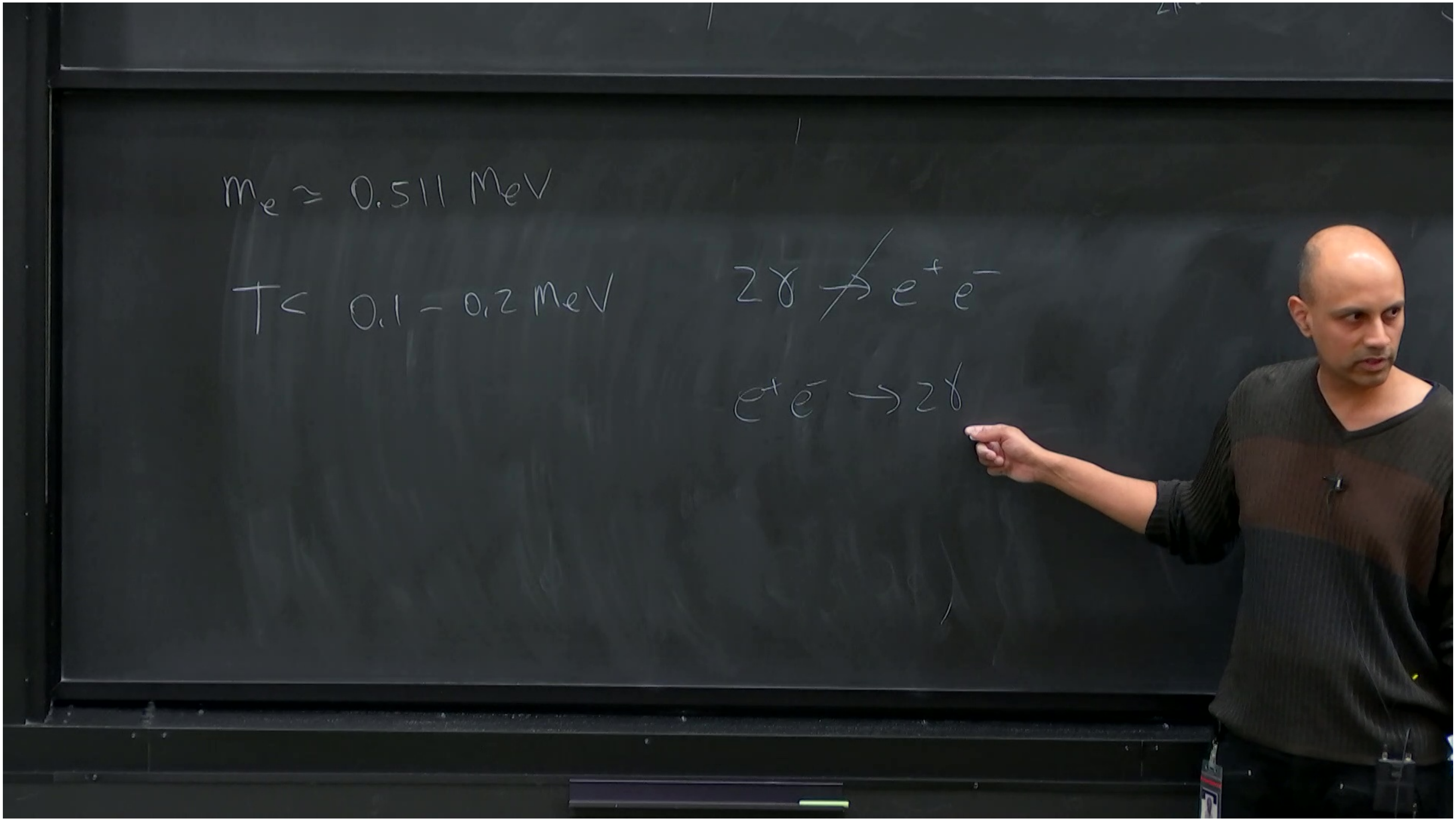
$$s_{tot} = \sum_{i \in rel} s_i = \left[\sum_{\substack{i \in rel \\ B}} \frac{2\pi^2}{45} g_i T_i^3 + \frac{7}{8} \sum_{\substack{rel \\ F}} g_i T_i^3 \right]$$

$$\equiv g_{*s} \frac{2\pi^2}{45} T_\gamma^3$$

$$S_{tot} = s_{tot} V \propto g_{*s} T^3 a^3 = \text{const}$$

when g_{*s} is const


when g*s is const ⟹ T ∝ a^-1
= const


$m_e \simeq 0.511$ MeV
$T < 0.1 - 0.2$ MeV
$2\gamma \not\to e^+ e^-$
$e^+ e^- \to 2\gamma$

$2\pi^2$

$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$

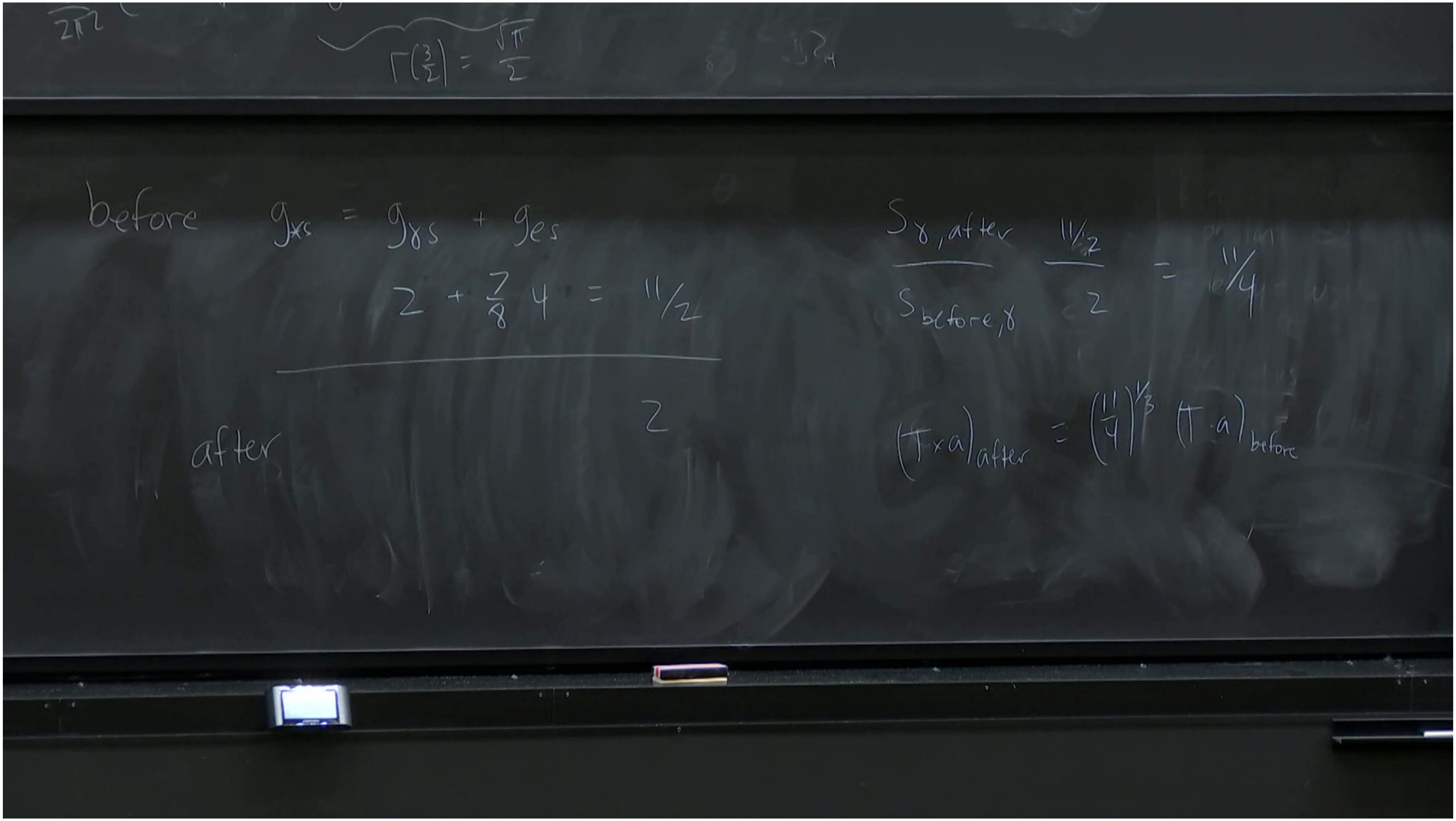
before $\quad g_{*s} = g_{\gamma s} + g_{es}$

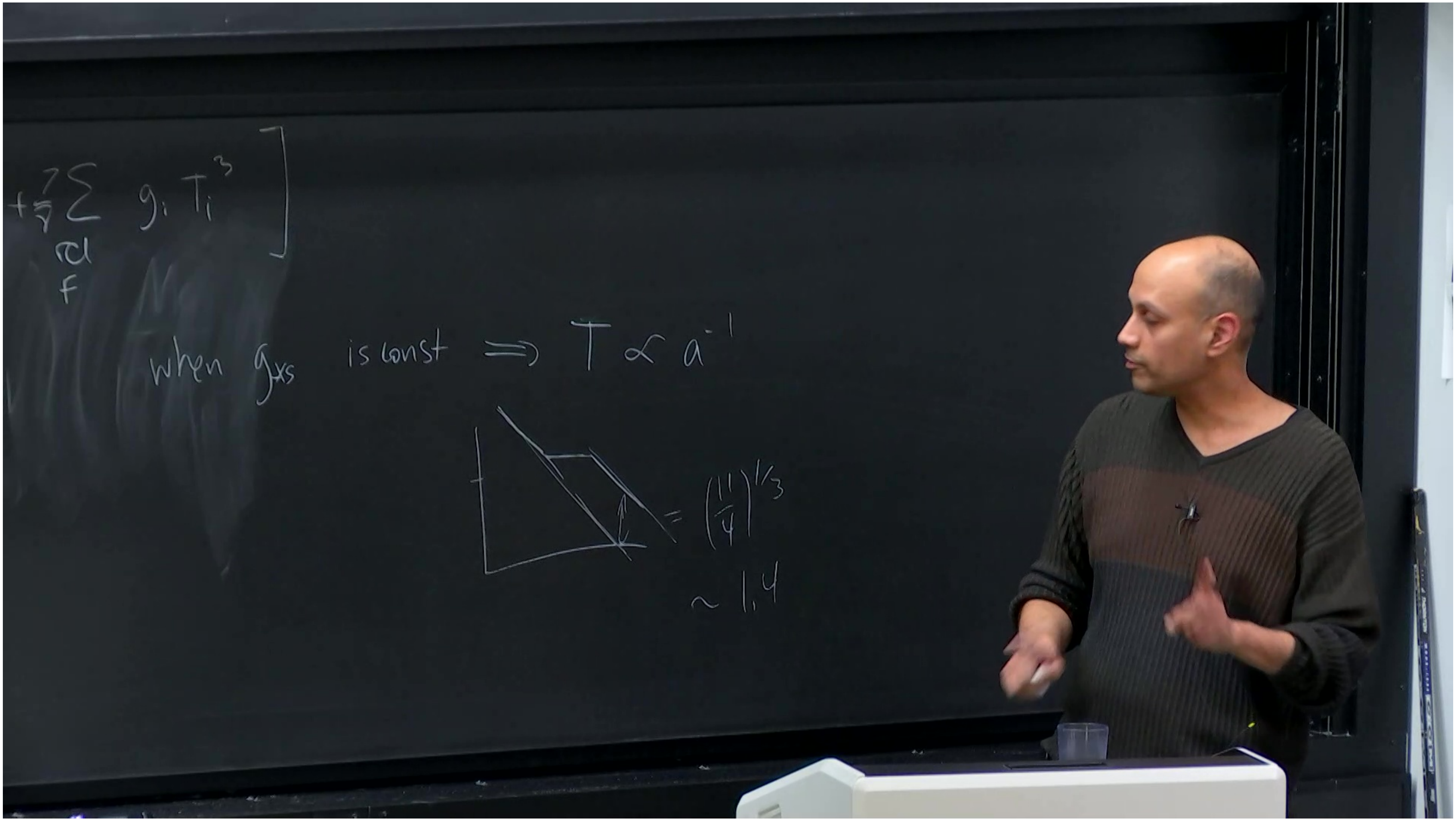
$$2 + \frac{7}{8}\,4 = 11/2$$

$$\frac{}{2}$$

after

$$\frac{S_{\gamma,\text{after}}}{S_{\text{before},\gamma}} \quad \frac{11/2}{2} = 11/4$$


Γ(3/2) = √π/2
before g*s = gγs + ges
2 + 7/8 4 = 11/2
2
after
Sγ,after / Sbefore,γ 11/2 / 2 = 11/4
(T×a)after = (11/4)^1/3 (T·a)before


when g*s is const ⟹ T ∝ a^-1
= (11/4)^1/3
~ 1.4