Title: Cosmology (2021/2022)

Speakers: Kendrick Smith, Gang Xu

Collection: Cosmology (2021/2022)

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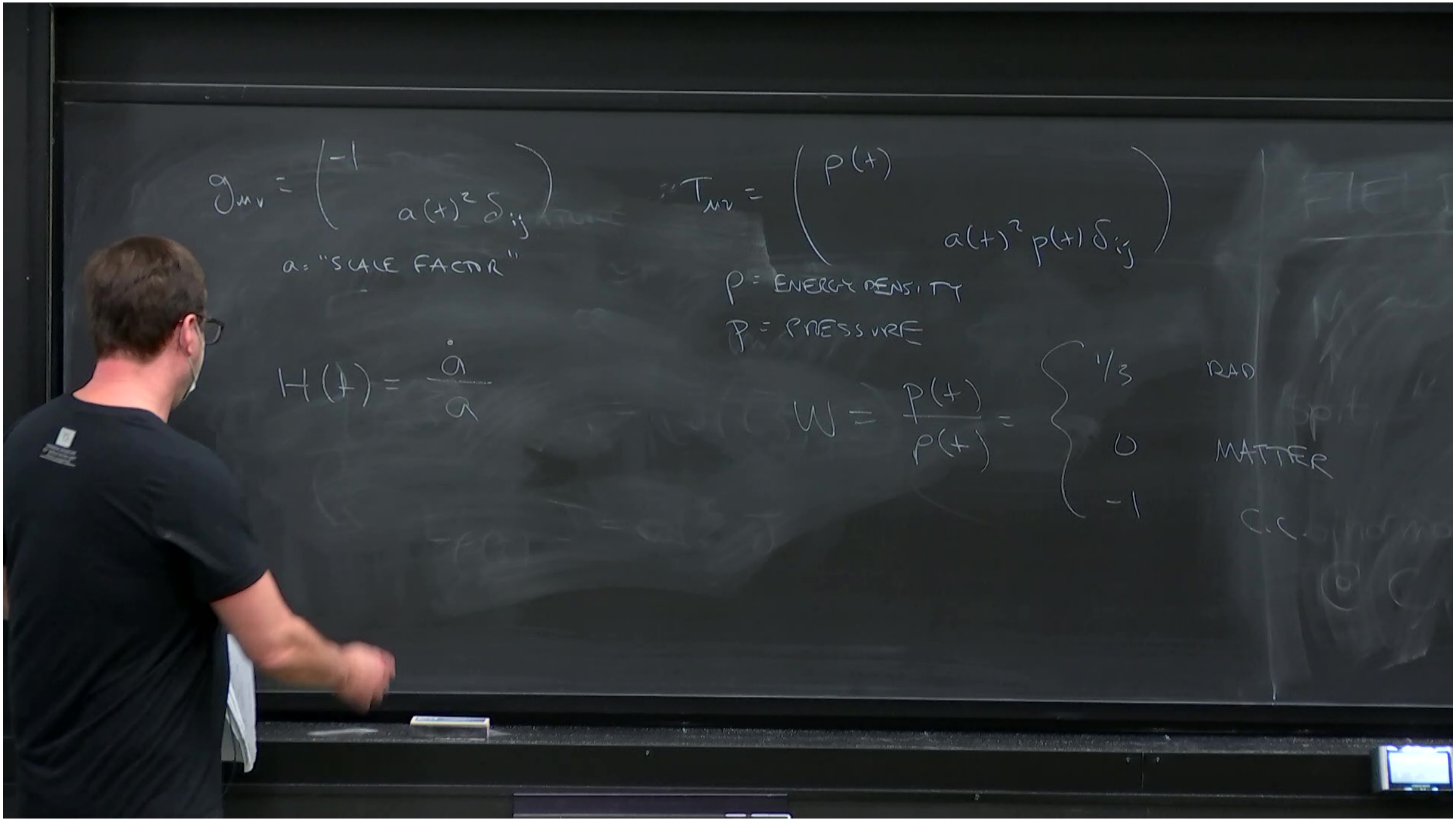
Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.

τ CONF TIME

K SPATIAL CURVATURE


$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a(t)^2 \delta_{ij} \end{pmatrix}$
a: "SCALE FACTOR"
$H(t) = \frac{\dot{a}}{a}$
$T_{\mu\nu} = \begin{pmatrix} \rho(t) & \\ & a(t)^2 p(t) \delta_{ij} \end{pmatrix}$
ρ = ENERGY DENSITY
p = PRESSURE
$w = \frac{p(t)}{\rho(t)} = \begin{cases} 1/3 & \text{RAD} \\ 0 & \text{MATTER} \\ -1 & \text{C.C.} \end{cases}$

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ρ = ENERGY DENSITY

p = PRESSURE

$$H(t) = \frac{\dot{a}}{a}$$

$$a = \frac{1}{1+z}$$

$$w = \frac{p(t)}{\rho(t)} = \begin{cases} 1/3 & \text{RAD} \\ 0 & \text{MATTER} \\ -1 & \text{C.C.} \end{cases}$$

CONTINUITY: $\dot{\rho} = -3H(\rho + p)$

$$\frac{d\log\rho}{d\log a} = -3(1+w) \Rightarrow \rho(a) \propto a^{-3(1+w)}$$

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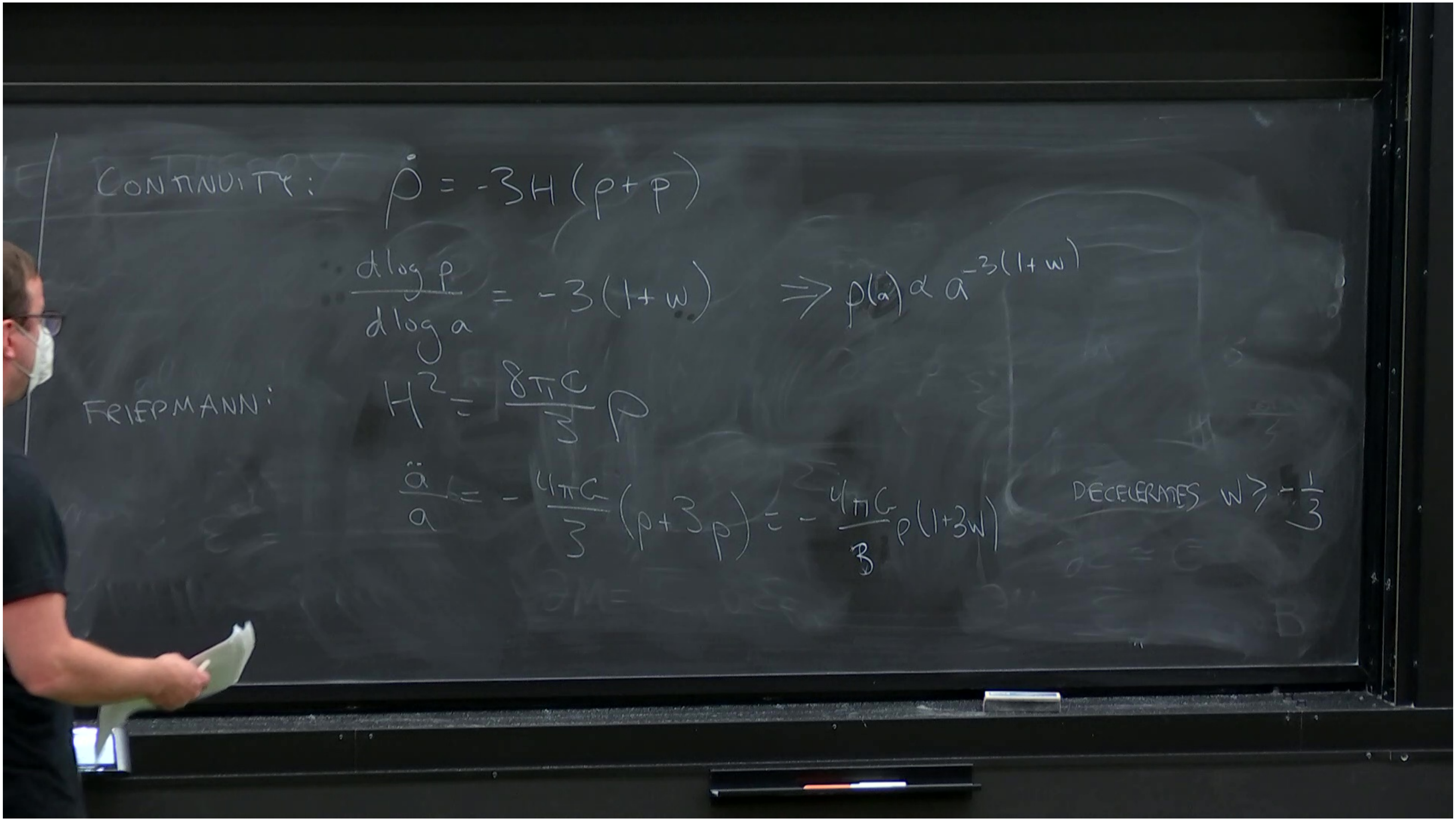
CONTINUITY: $\dot{\rho} = -3H(\rho + p)$
$\frac{d\log\rho}{d\log a} = -3(1+w) \Rightarrow \rho(a) \propto a^{-3(1+w)}$
FRIEDMANN: $H^2 = \frac{8\pi G}{3}\rho$
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) = -\frac{4\pi G}{3}\rho(1+3w)$
DECELERATES $w > -\frac{1}{3}$

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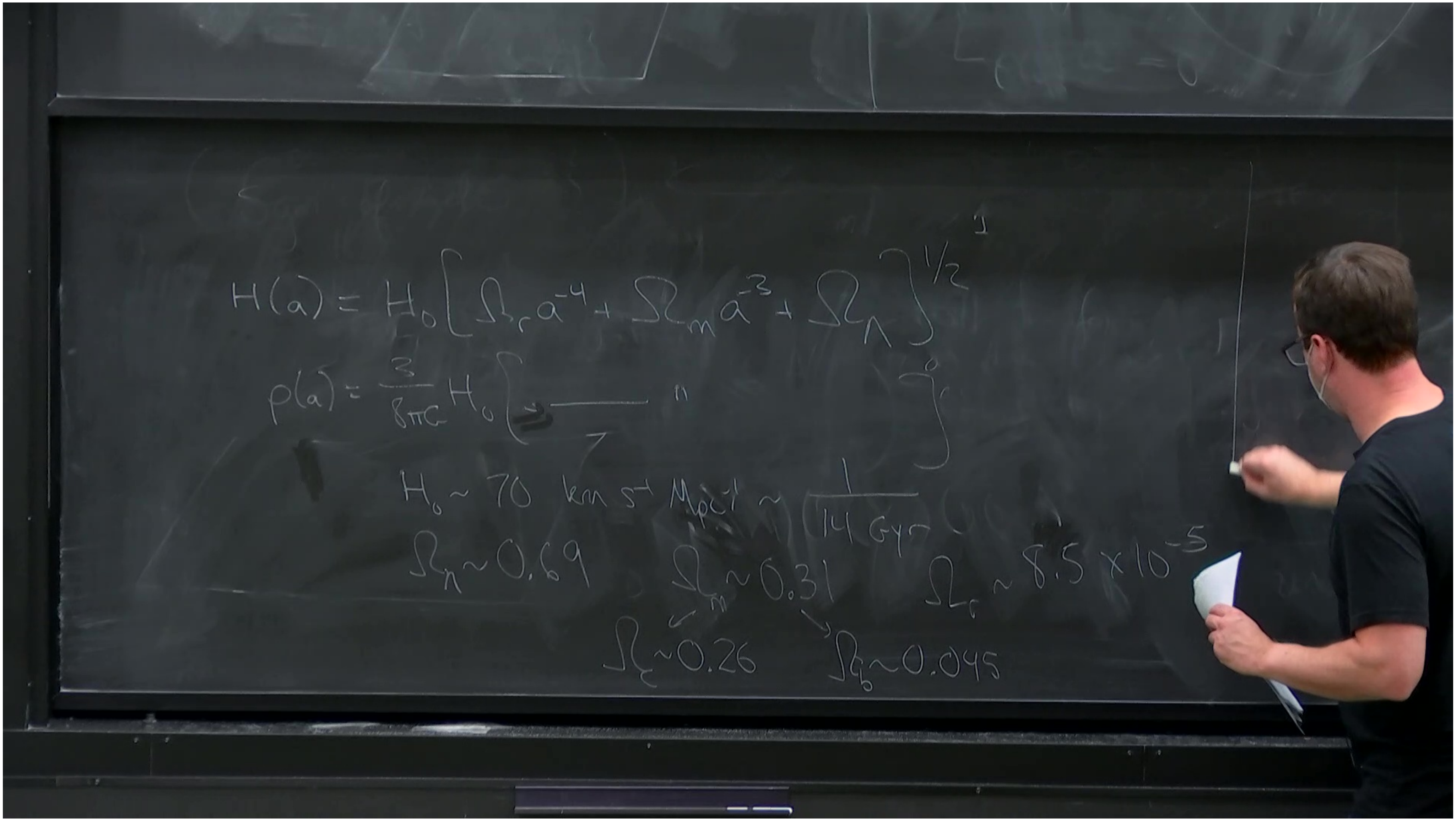
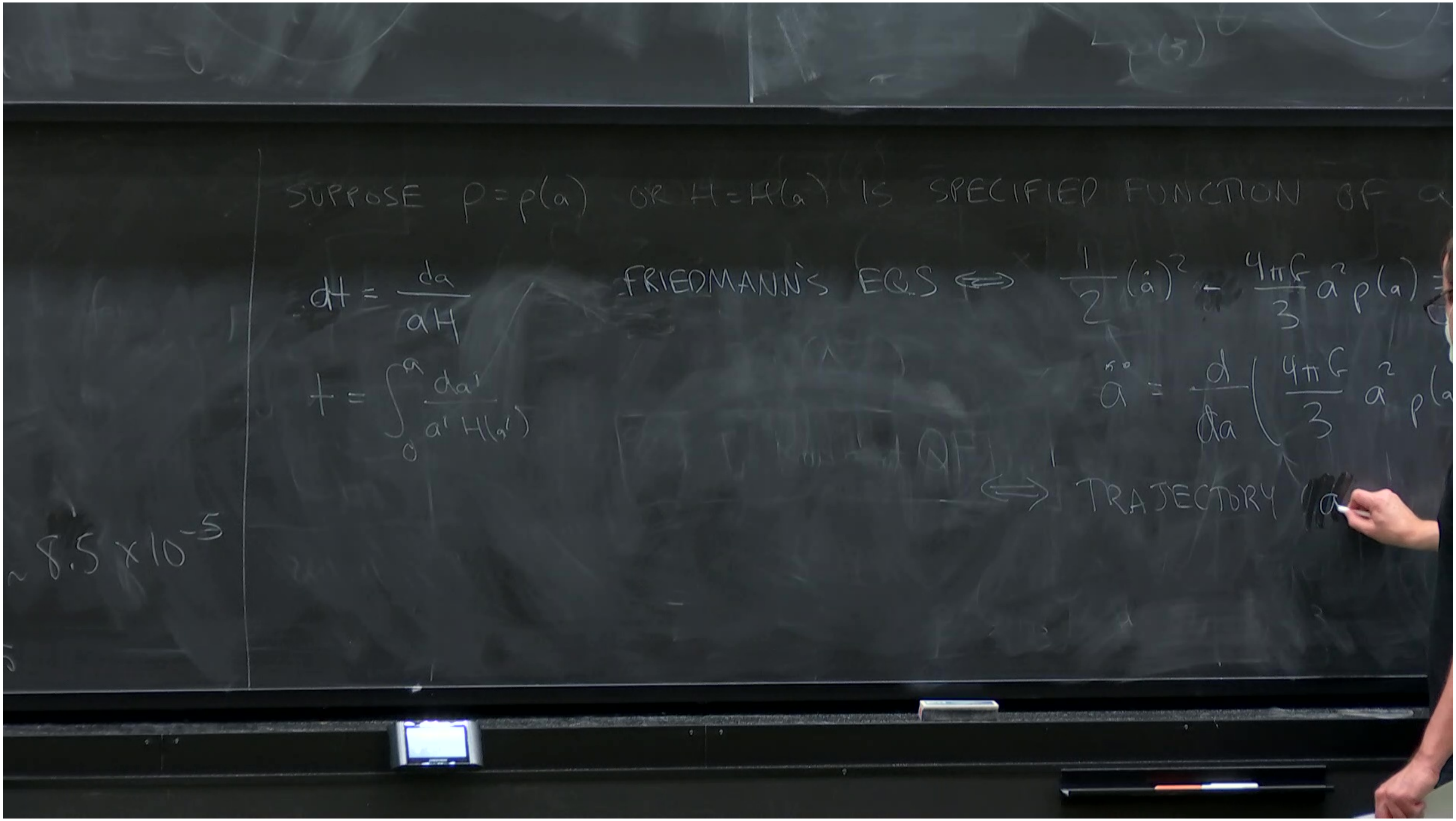
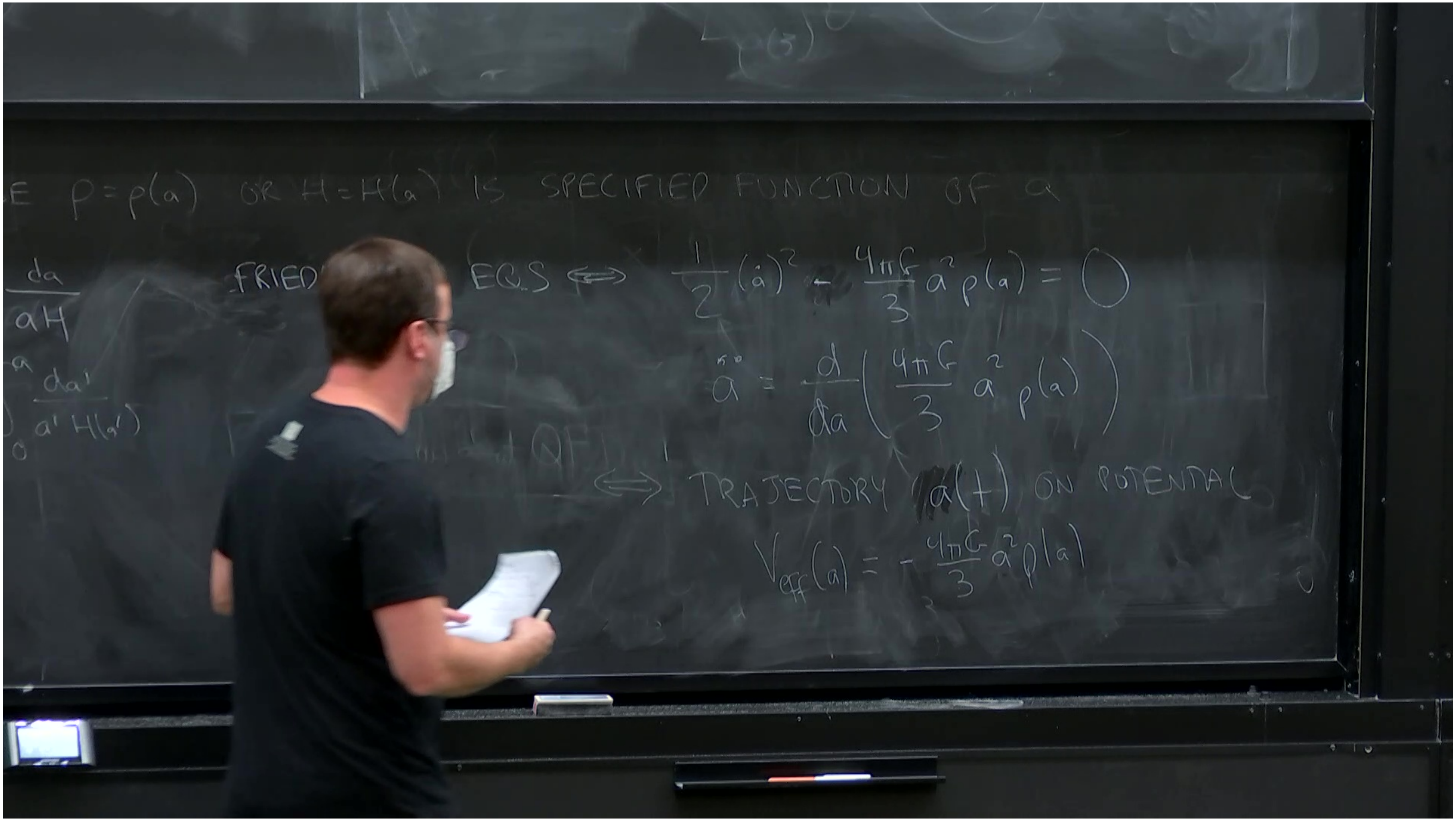
DECELERATES $w > -\frac{1}{3}$

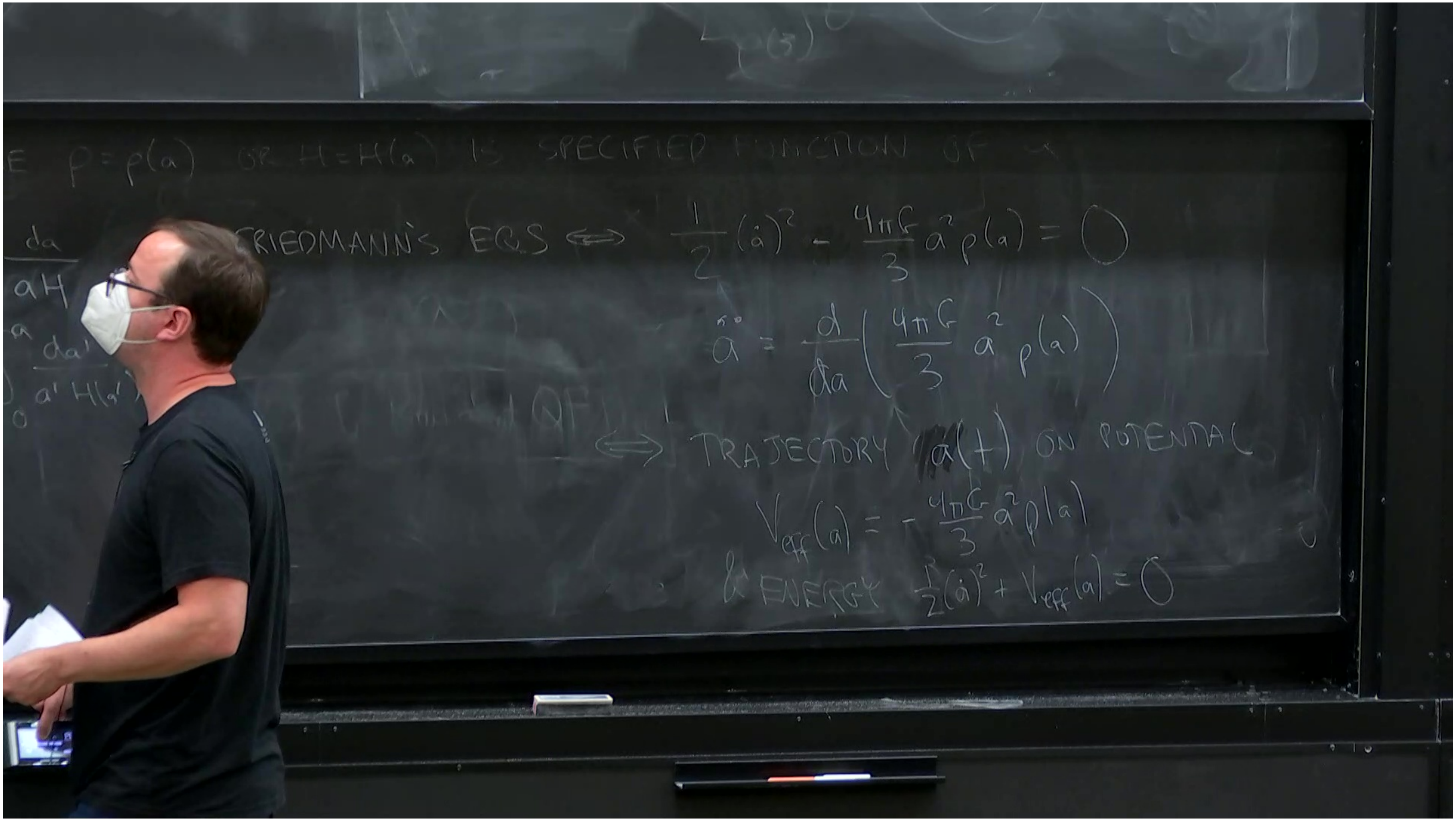
$$\frac{\Lambda}{M_{Pl}^4} \sim 8.3 \times 10^{-121}$$

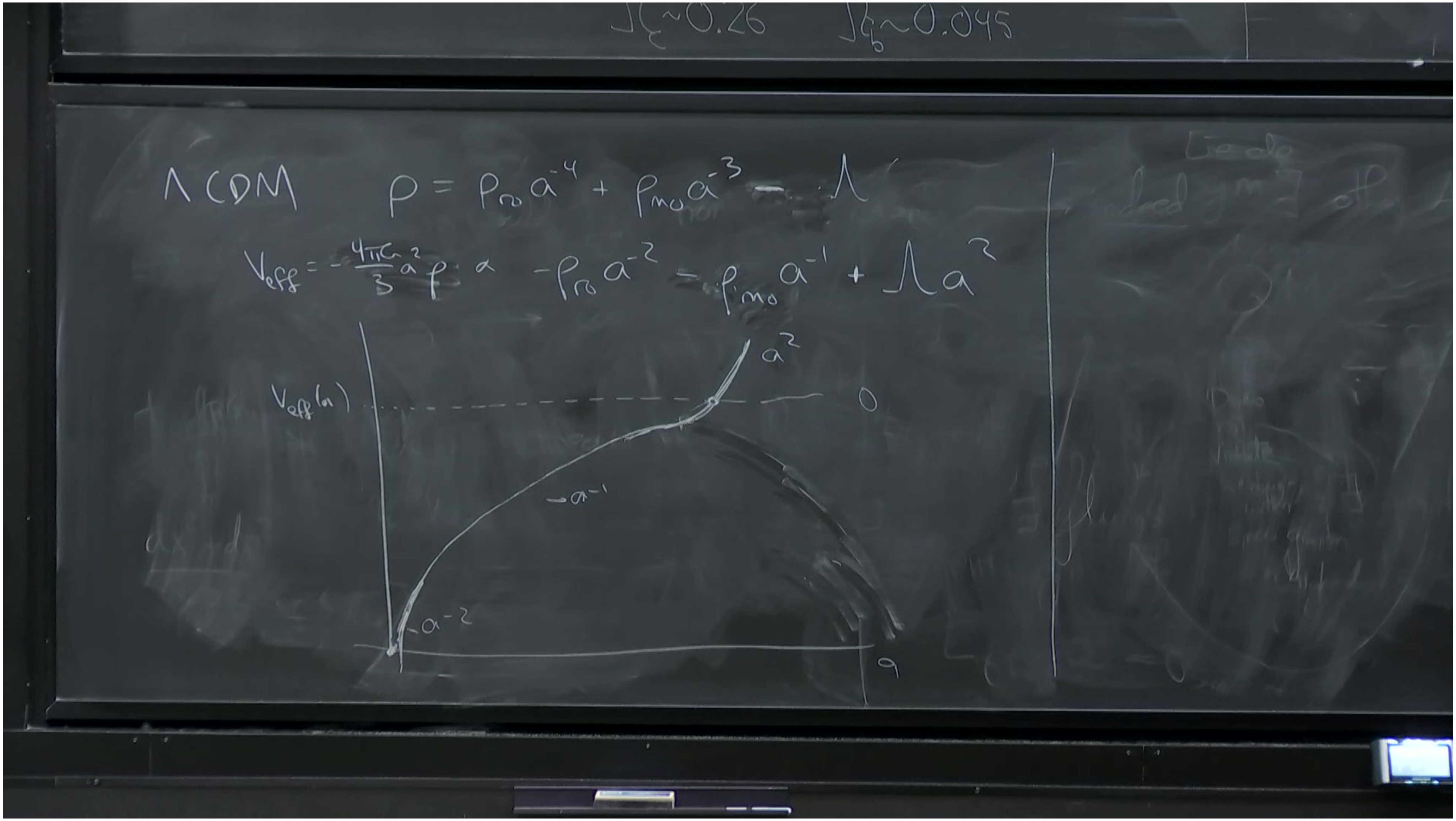


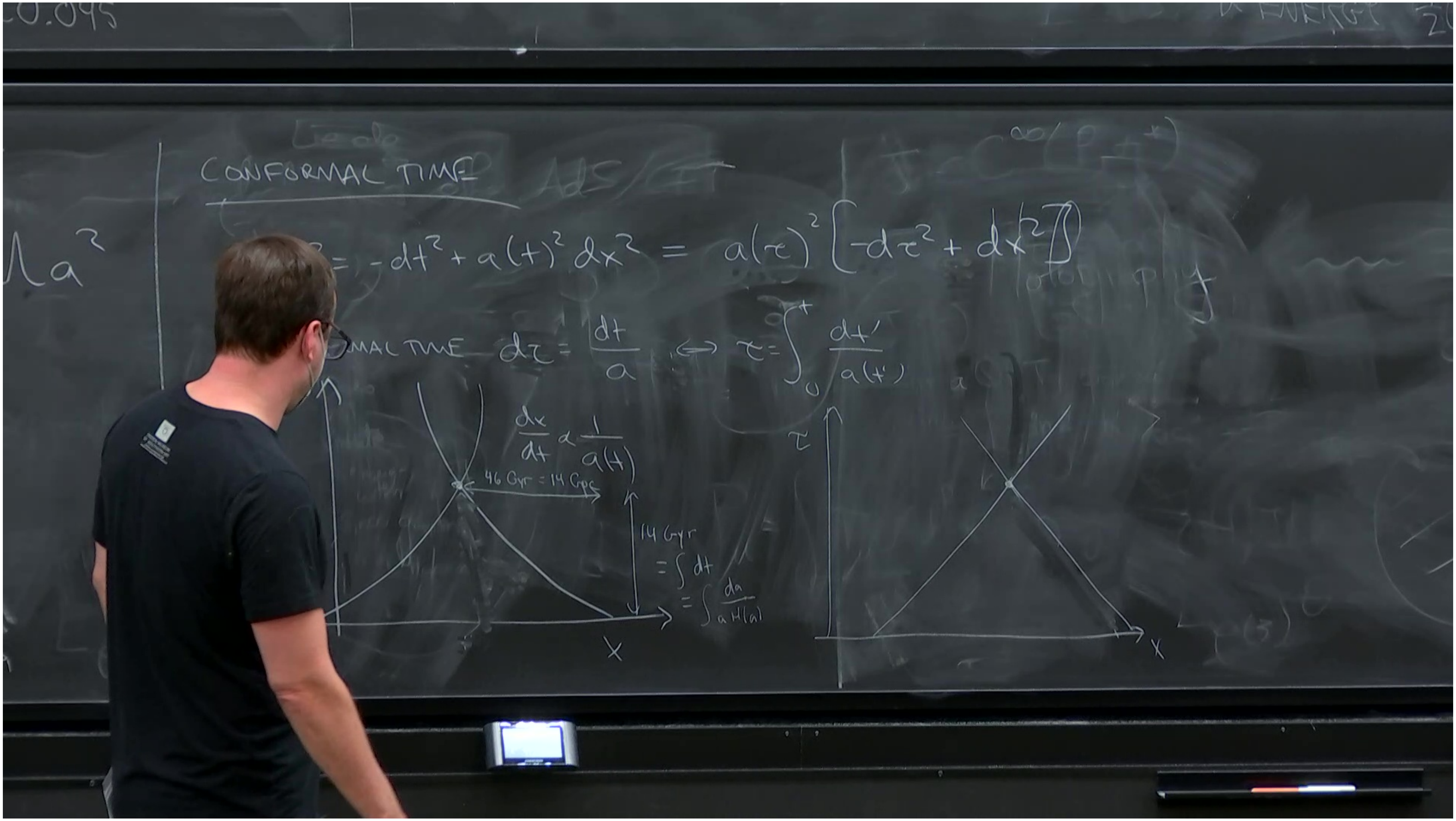
H(a) = H₀[Ωᵣ a⁻⁴ + Ωₘ a⁻³ + Ω_Λ]^1/2
ρ(a) = 3/(8πG) H₀["
H₀ ~ 70 km s⁻¹ Mpc⁻¹ ~ 1/(14 Gyr)
Ω_Λ ~ 0.69
Ωₘ ~ 0.31
Ωᵣ ~ 8.5 × 10⁻⁵
Ω_c ~ 0.26
Ω_b ~ 0.045


SUPPOSE ρ = ρ(a) OR H = H(a) IS SPECIFIED FUNCTION OF a
dt = da / aH
t = ∫_0^a da' / a' H(a')
FRIEDMANN'S EQS ⟺ 1/2 (ȧ)^2 − 4πG/3 a^2 ρ(a) =
ä = d/da (4πG/3 a^2 ρ(a
⟺ TRAJECTORY a
~ 8.5 × 10^-5

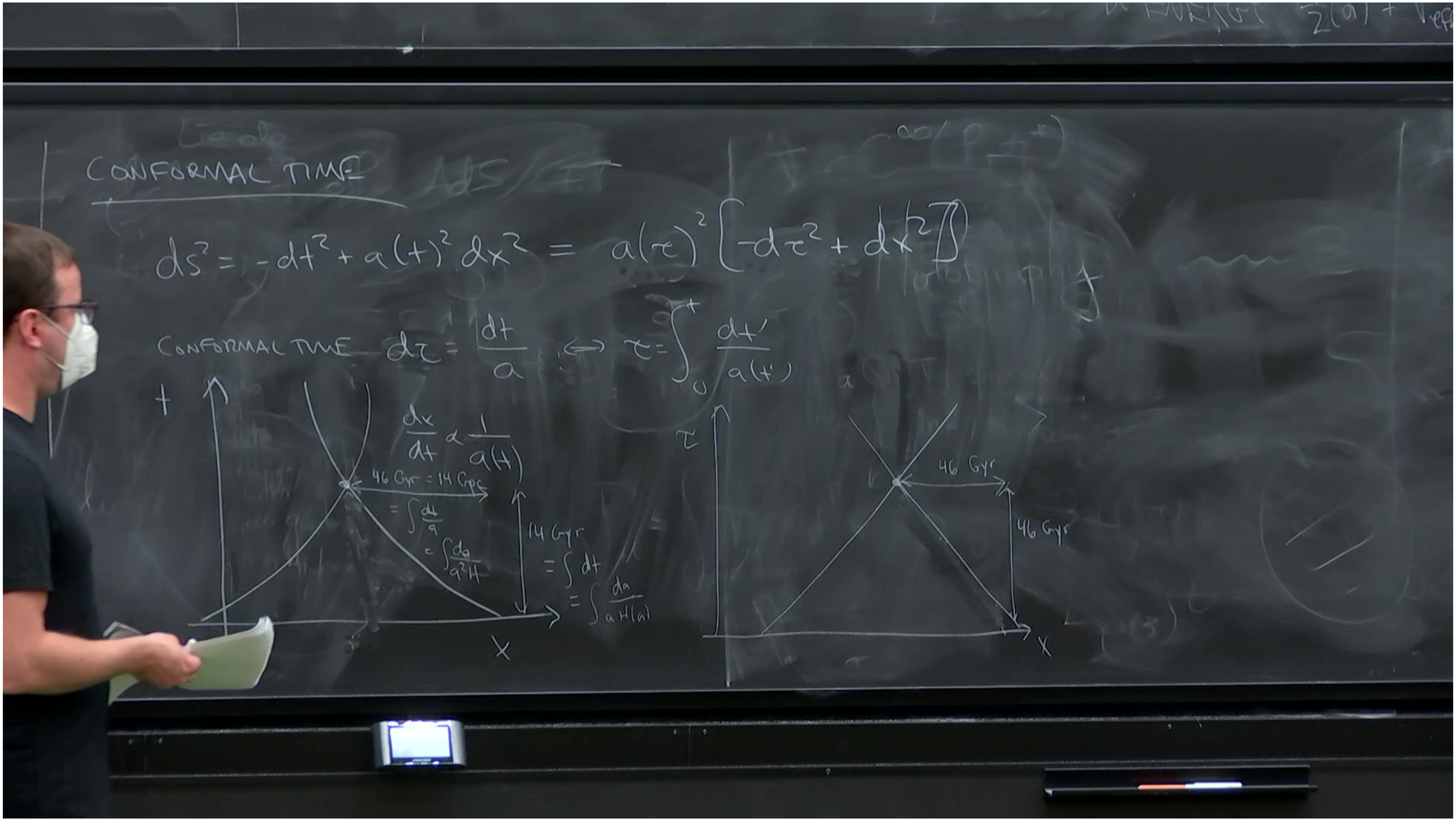

ρ=ρ(a) OR H=H(a) IS SPECIFIED FUNCTION OF a
EQS ⟺ 1/2 (ȧ)² − 4πG/3 a² ρ(a) = 0
ä = d/da (4πG/3 a² ρ(a))
⟺ TRAJECTORY a(t) ON POTENTIAL
V_eff(a) = − 4πG/3 a² ρ(a)

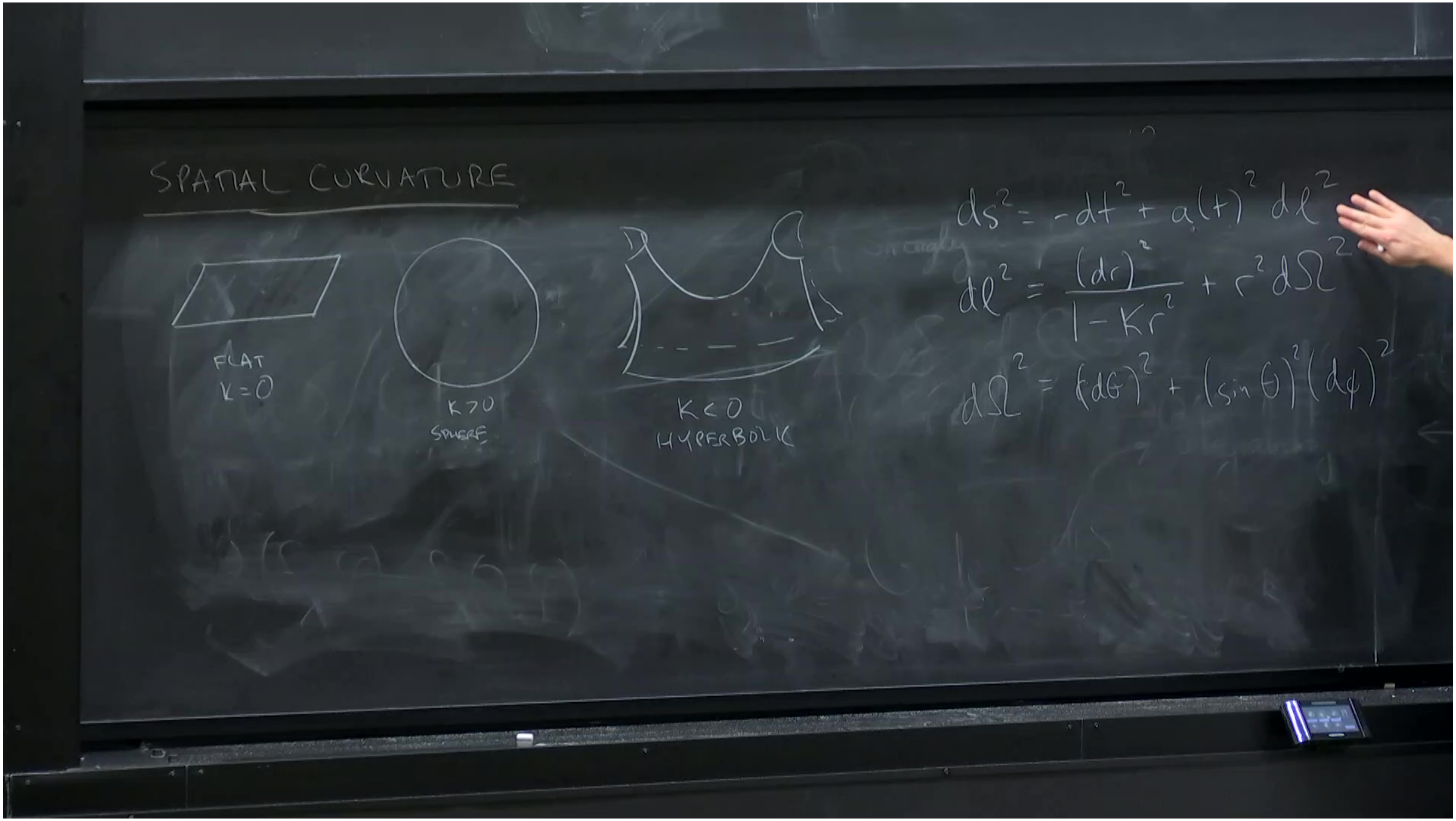

E ρ=ρ(a) OR H=H(a) IS SPECIFIED FUNCTION OF a
FRIEDMANN'S EQS ⟺ 1/2 (ȧ)^2 − 4πG/3 a^2 ρ(a) = 0
ä = d/da (4πG/3 a^2 ρ(a))
⟺ TRAJECTORY a(t) ON POTENTIAL
V_eff(a) = − 4πG/3 a^2 ρ(a)
& ENERGY 1/2 (ȧ)^2 + V_eff(a) = 0

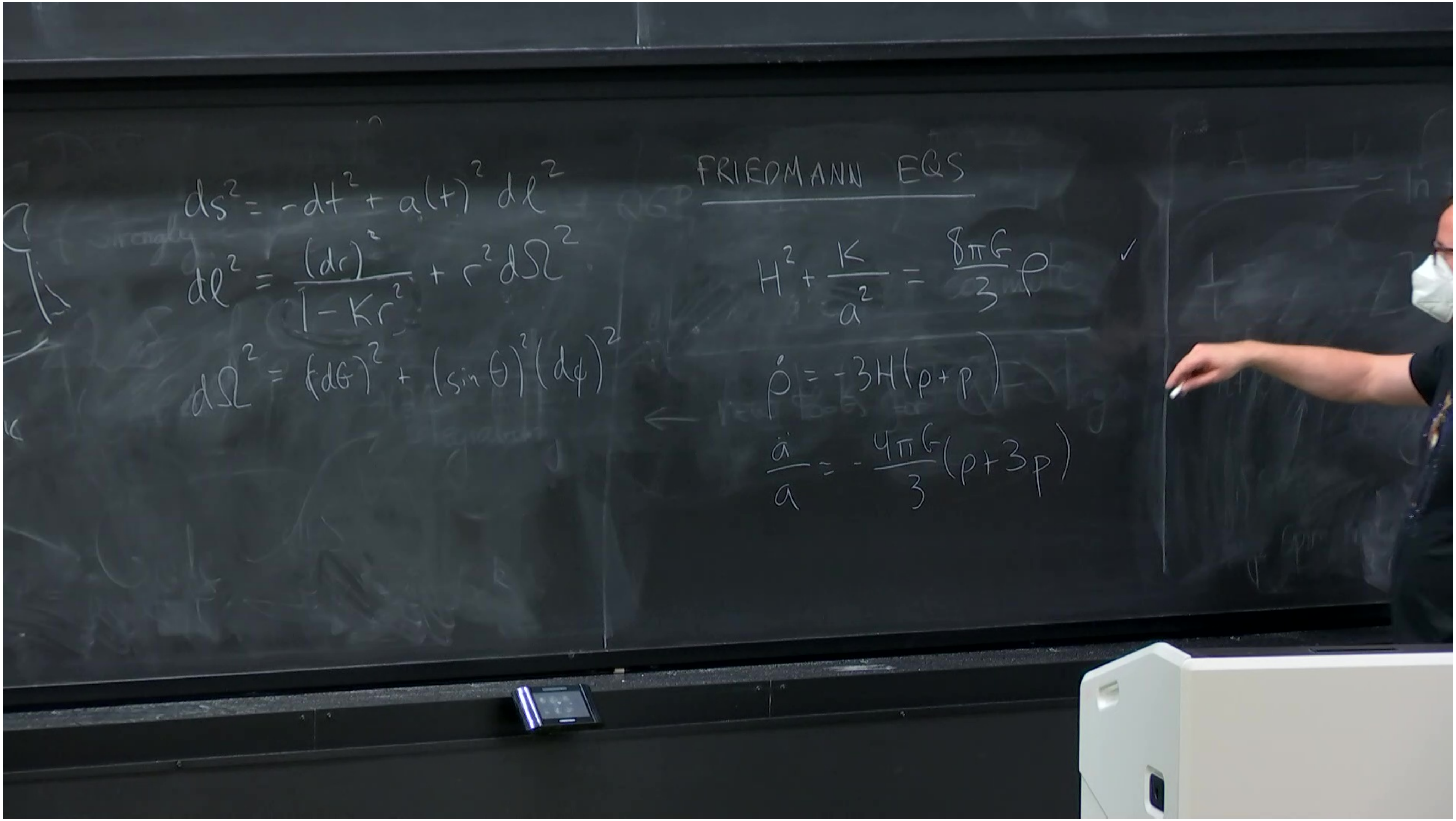

$\Omega_c \sim 0.26$ $\Omega_b \sim 0.045$
ΛCDM $\rho = \rho_{r0} a^{-4} + \rho_{m0} a^{-3} + \Lambda$
$V_{eff} = -\frac{4\pi G}{3} a^2 \rho \propto -\rho_{r0} a^{-2} - \rho_{m0} a^{-1} + \Lambda a^2$
a^2
$V_{eff}(a)$
0
$-a^{-1}$
$-a^{-2}$
a



CONFORMAL TIME
= -dt² + a(t)² dx² = a(τ)² [-dτ² + dx²]
MAL TIME dτ = dt/a ⟺ τ = ∫₀ᵗ dt'/a(t')
dx/dt ∝ 1/a(t)
46 Gyr = 14 Gpc
14 Gyr
= ∫ dt
= ∫ da/(a H(a))
x
τ
x


CONFORMAL TIME
ds² = -dt² + a(t)² dx² = a(τ)² [-dτ² + dx²]
CONFORMAL TIME dτ = dt/a ⟺ τ = ∫₀ᵗ dt'/a(t')
dx/dt ∝ 1/a(t)
46 Gyr = 14 Gpc
= ∫ dt/a
= ∫ da/a²H
14 Gyr
= ∫ dt
= ∫ da/aH(a)
t
x
τ
46 Gyr
46 Gyr
x


SPATIAL CURVATURE
FLAT
K=0
K>0
SPHERE
K<0
HYPERBOLIC
$ds^2 = -dt^2 + a(t)^2 \, d\ell^2$
$d\ell^2 = \frac{(dr)^2}{1-Kr^2} + r^2 d\Omega^2$
$d\Omega^2 = (d\theta)^2 + (\sin\theta)^2 (d\phi)^2$


$ds^2 = -dt^2 + a(t)^2 d\ell^2$
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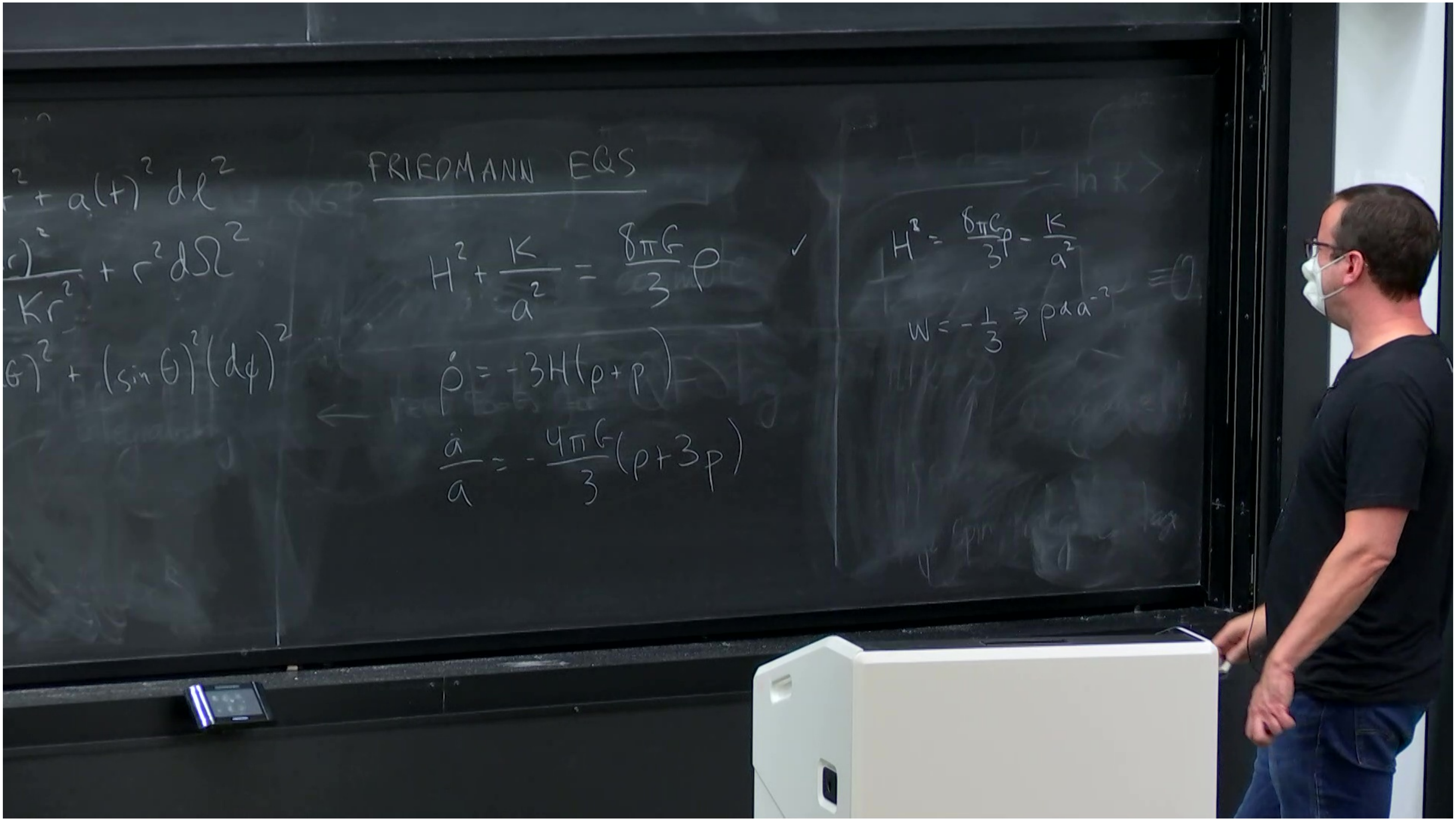
FRIEDMANN EQS
$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$
$\dot{\rho} = -3H(\rho + p)$
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

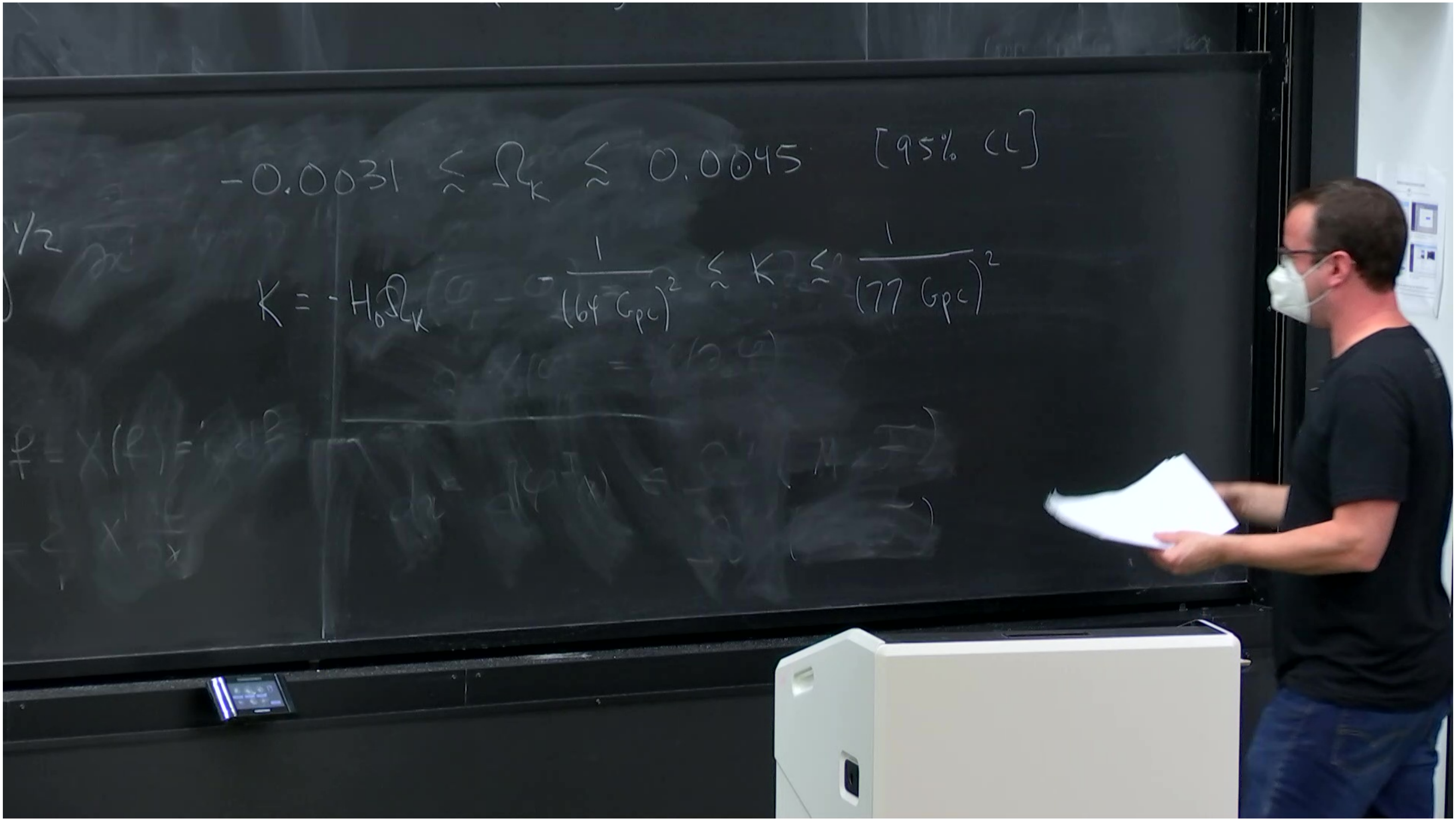

FRIEDMANN EQS
$H^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho$
$\dot\rho = -3H(\rho + p)$
$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$
$w = -\frac{1}{3} \Rightarrow \rho \propto a^{-2}$
$+ r^2 d\Omega^2$
$(\sin\theta)^2(d\phi)^2$

"ASTRO" PARAMETERIZATION:

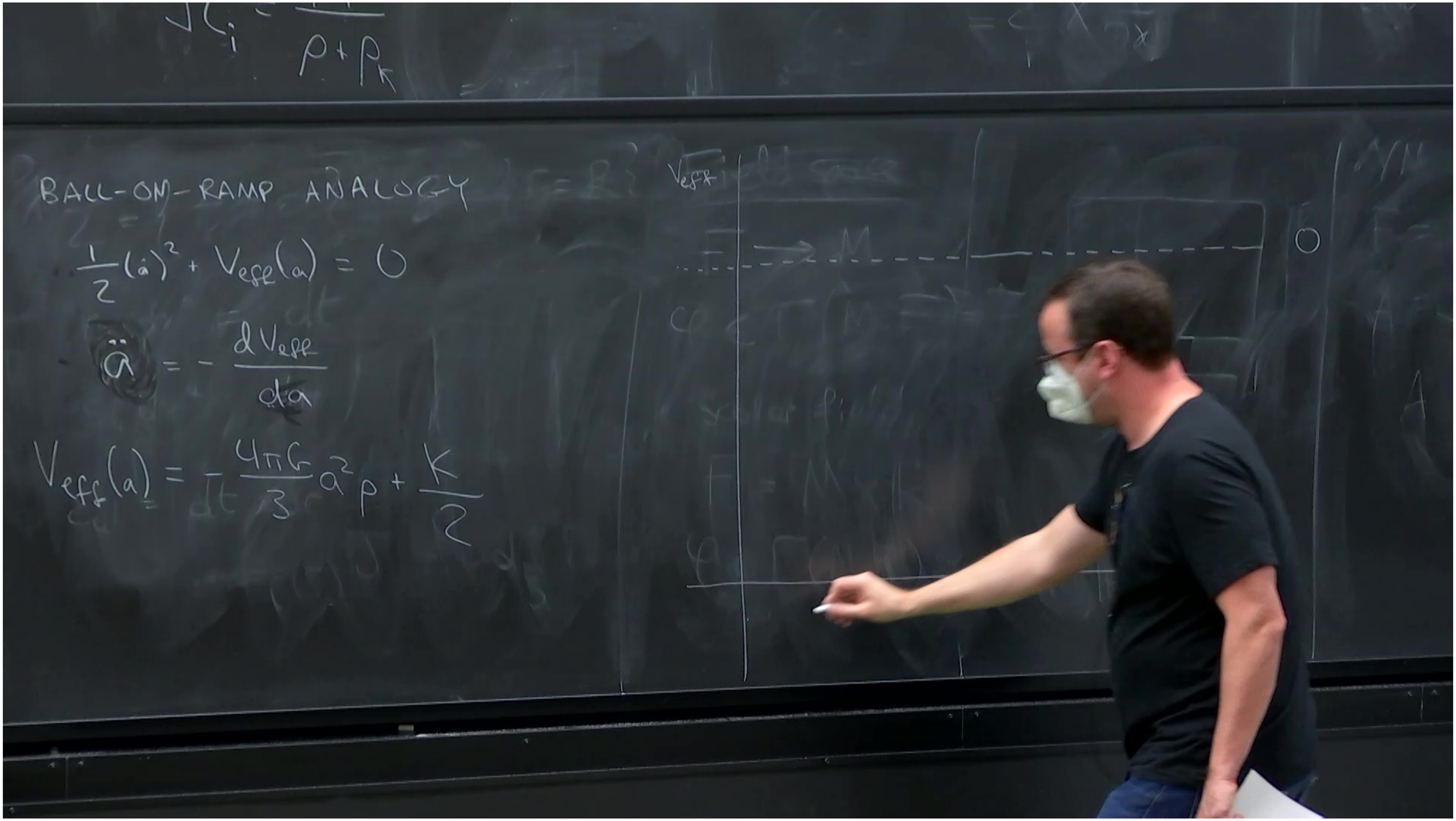
$$H(a) = H_0 \left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_K a^{-2} + \Omega_\Lambda \right]^{1/2}$$

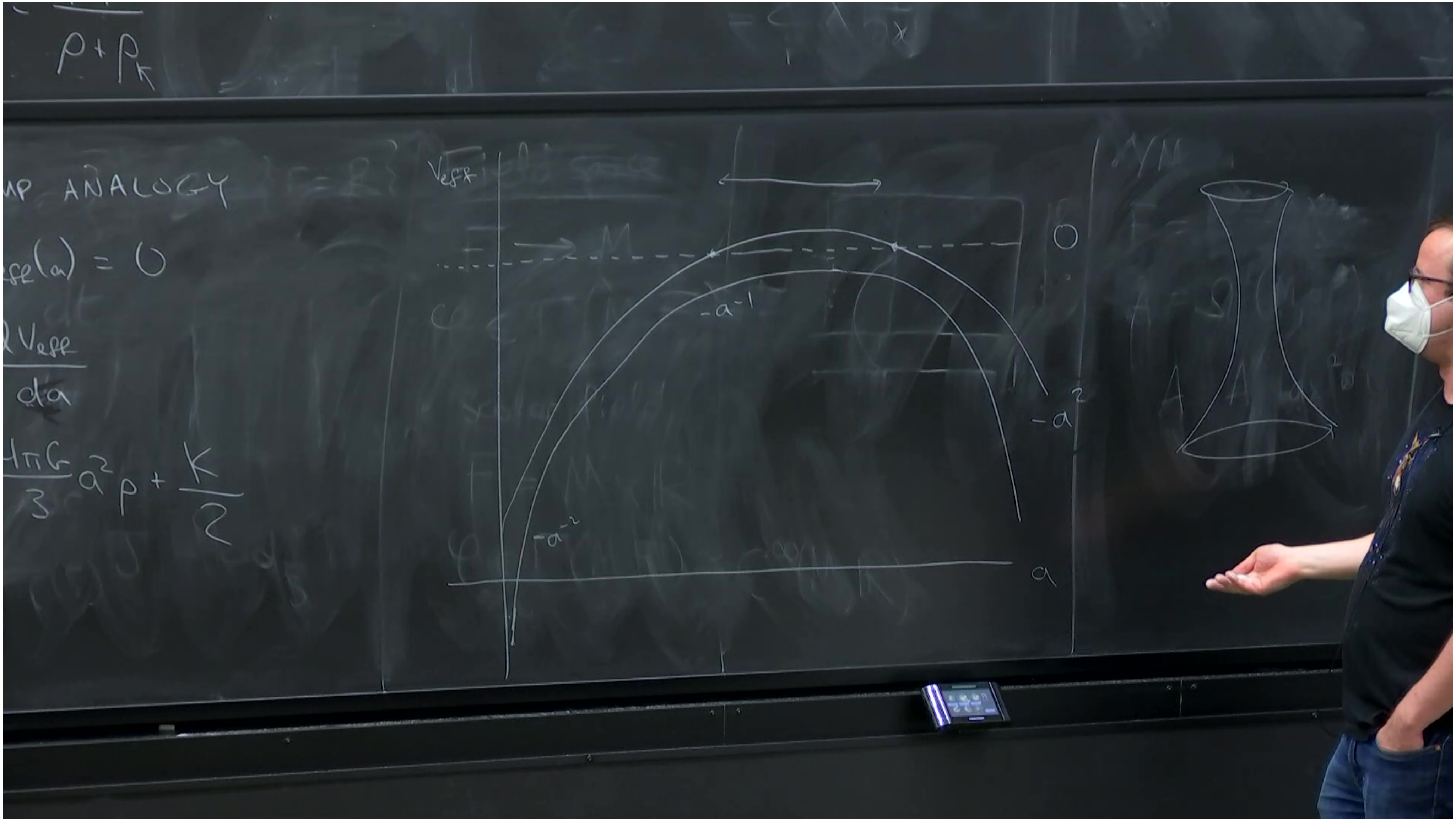
$$H^2 = \frac{8\pi G}{3} \left(\rho + \rho_K \right) \qquad \rho_K \equiv -\frac{3}{8\pi G} \frac{K}{a^2}$$

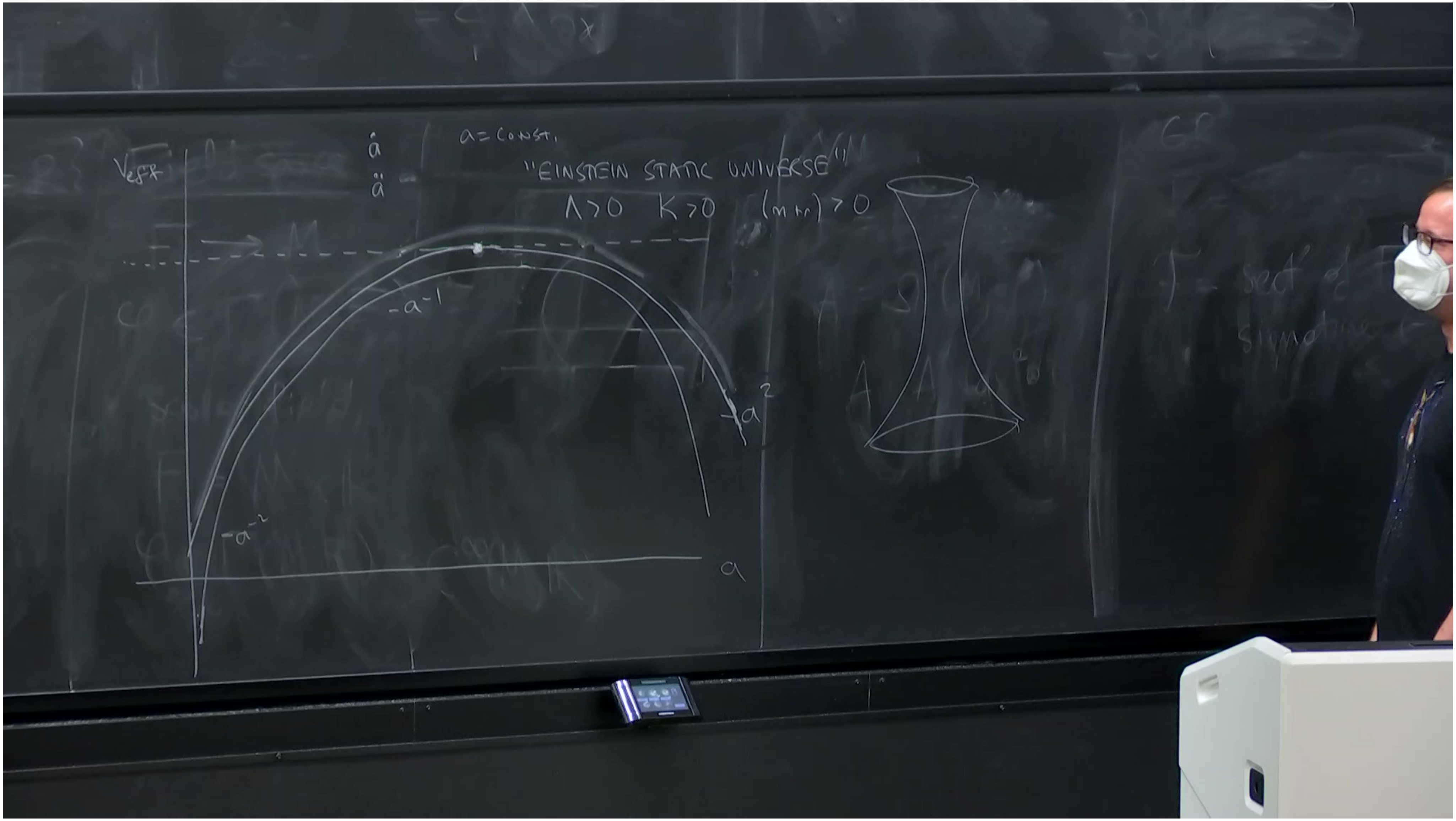
$$\Omega_i = \frac{\rho_i}{\rho + \rho_K}$$



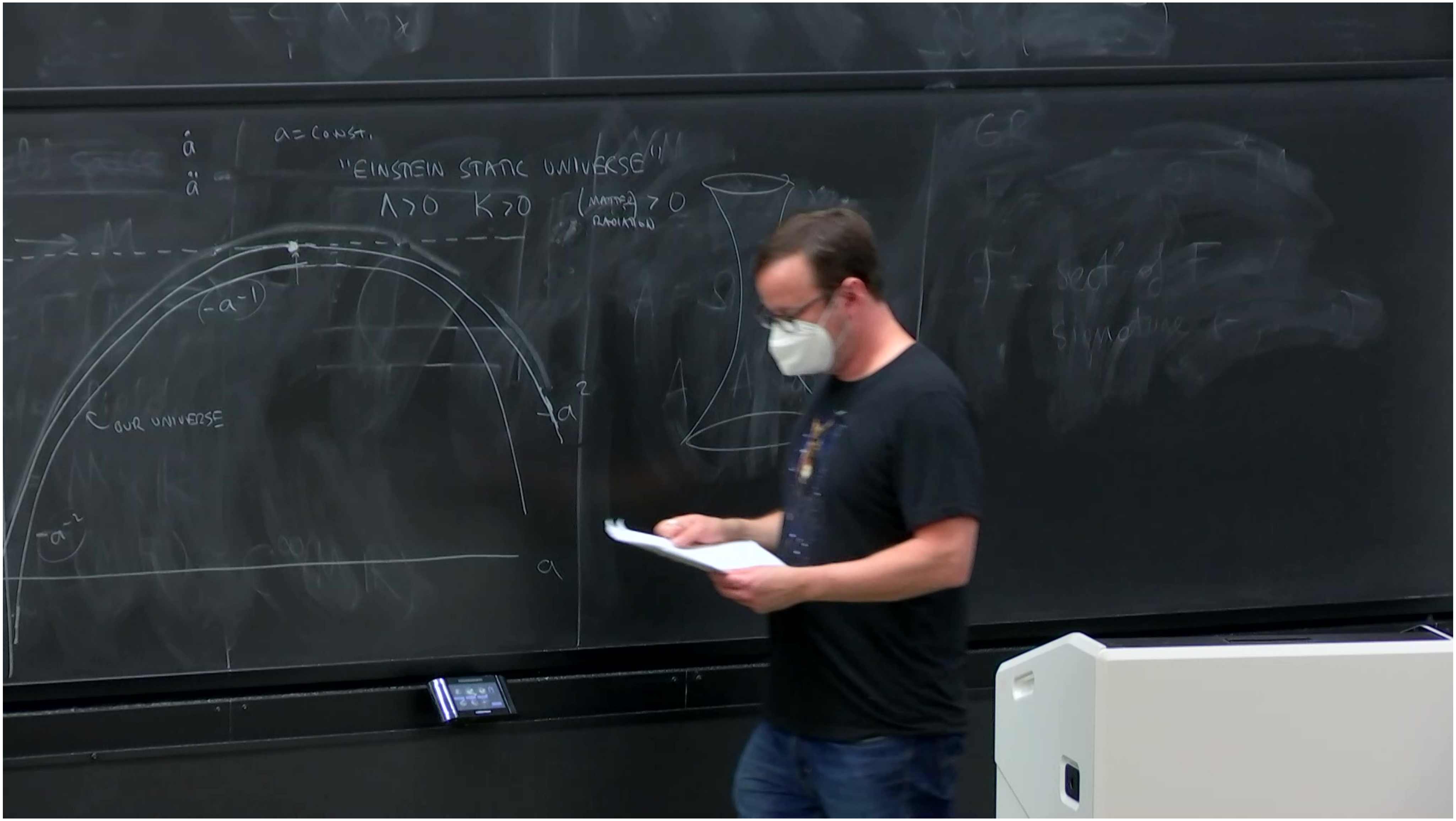
$-0.0031 \lesssim \Omega_k \lesssim 0.0045$ [95% CL]
$-\frac{1}{(64\ \mathrm{Gpc})^2} \lesssim K \lesssim \frac{1}{(77\ \mathrm{Gpc})^2}$


BALL-ON-RAMP ANALOGY
$\frac{1}{2}(\dot{a})^2 + V_{eff}(a) = 0$
$\ddot{a} = -\frac{dV_{eff}}{da}$
$V_{eff}(a) = -\frac{4\pi G}{3}a^2\rho + \frac{K}{2}$
V_{eff}
$\rho + \rho_K$
0


ρ + ρk
MP ANALOGY
eff(a) = 0
dVeff
da
4πG/3 a²ρ + K/2
Veff
-a⁻¹
-a²
-a⁻²
a
0

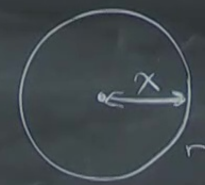


a = CONST.
"EINSTEIN STATIC UNIVERSE"
Λ > 0   K > 0   (m + r) > 0
V_eff
-a^-1
-a^-2
-a^2
a


a = CONST.
"EINSTEIN STATIC UNIVERSE"
$\Lambda > 0$ $K > 0$ (MATTER RADIATION) > 0
$-a^{-1}$
OUR UNIVERSE
$-a^{-2}$
a^2
a

$\Omega_c \sim 0.26 \qquad \Omega_b \sim 0.045$

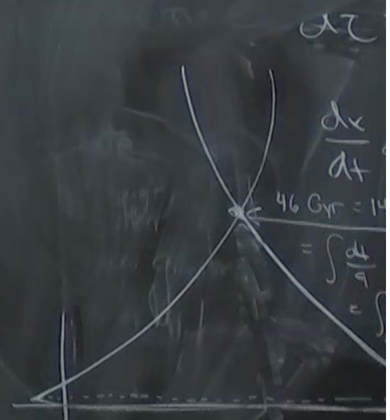
$$d\ell^2 = \frac{(dr)^2}{1-Kr^2} + r^2 d\Omega^2$$

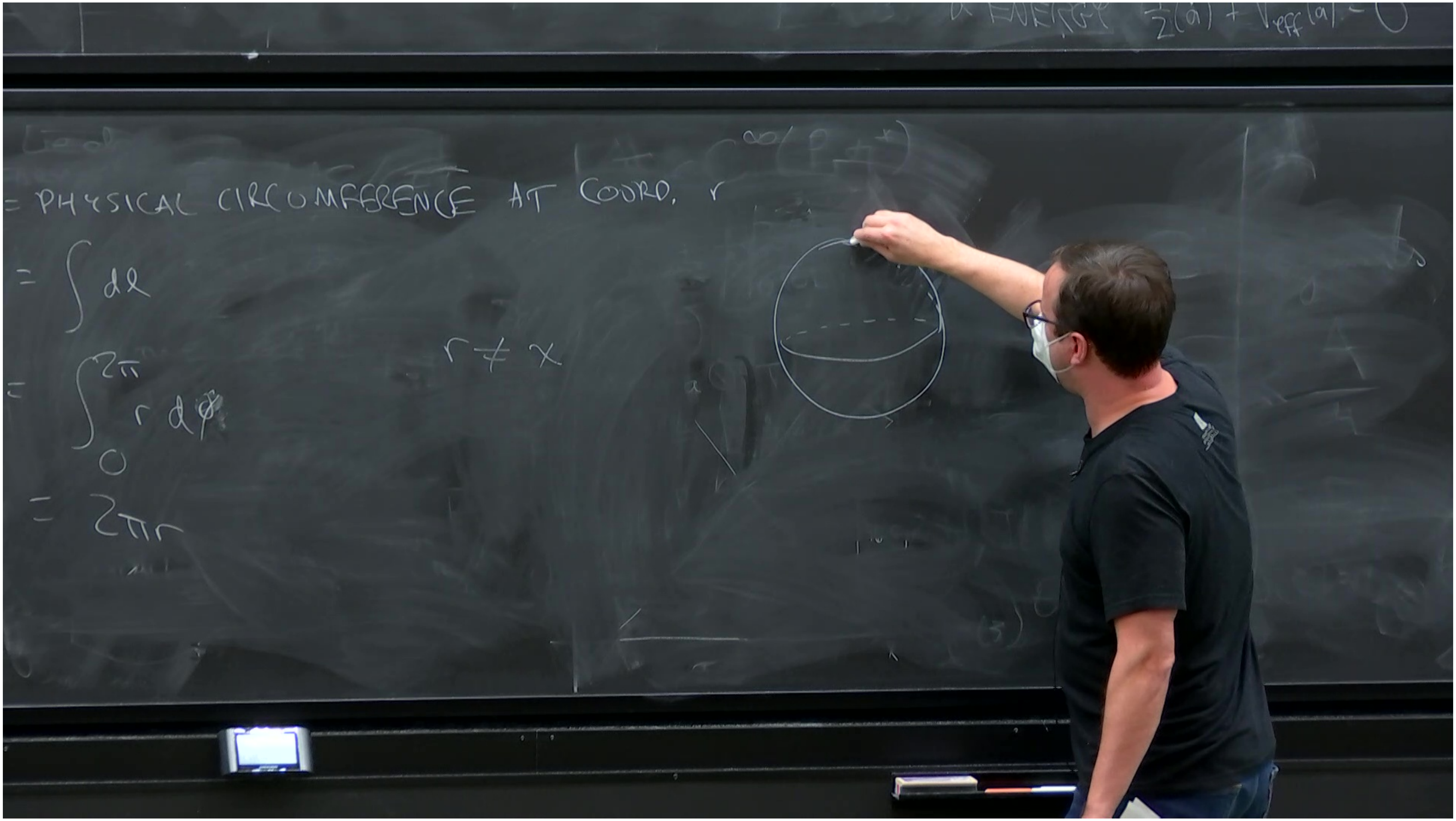


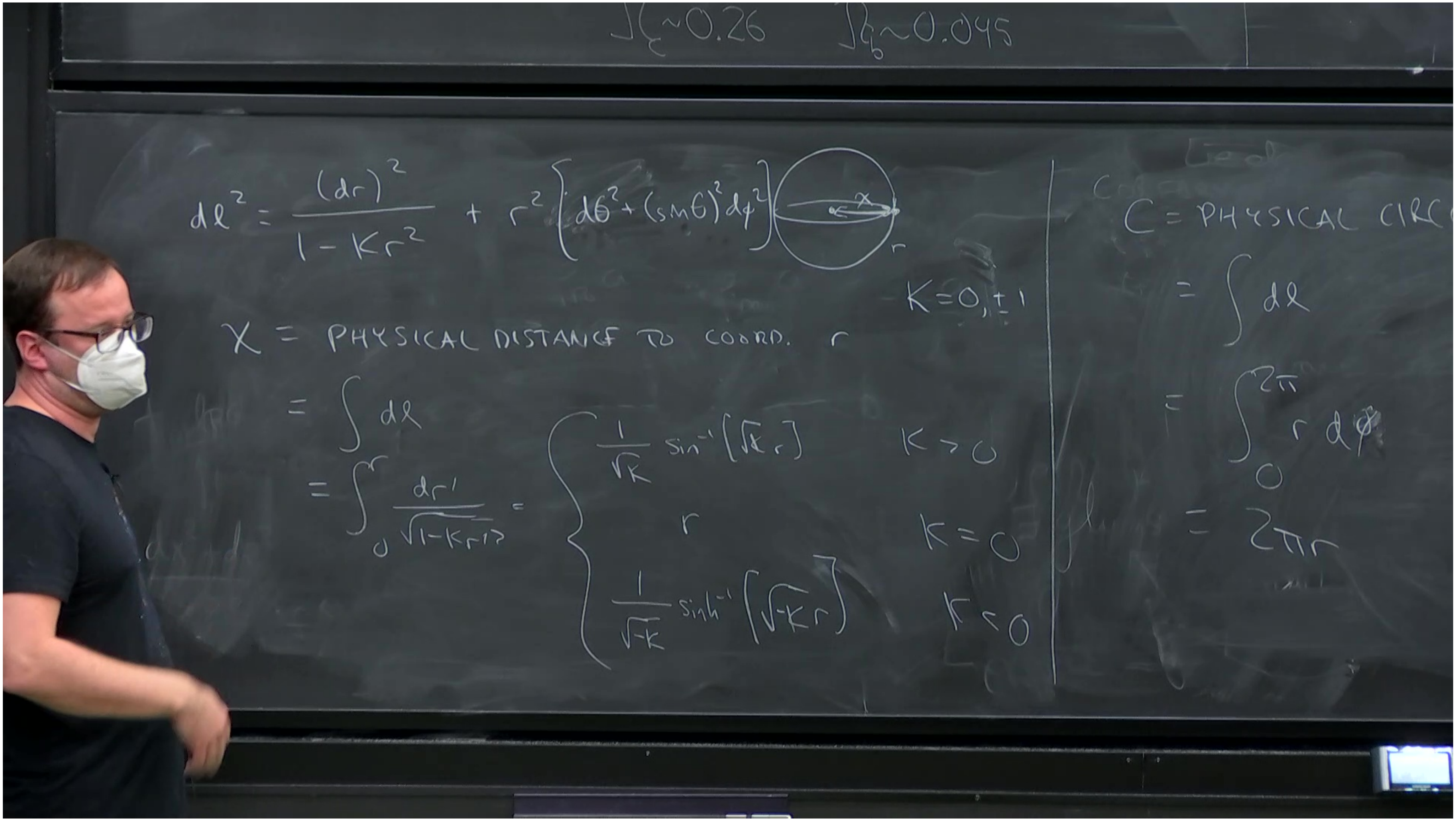
X = PHYSICAL DISTANCE TO COORD. r

$$= \int d\ell$$

$$= \int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = \begin{cases} \frac{1}{\sqrt{K}} \sin^{-1}\left[\sqrt{K}\, r\right] & K>0 \\ r & K=0 \\ \frac{1}{\sqrt{-K}} \sinh^{-1}\left[\sqrt{-K}\, r\right] & K<0 \end{cases}$$




= PHYSICAL CIRCUMFERENCE AT COORD. r
= ∫ dℓ
= ∫_0^{2π} r dφ
= 2πr
r ≠ x



Ω_c ~ 0.26   Ω_b ~ 0.045
dℓ² = (dr)²/(1 − Kr²) + r² [dθ² + (sin θ)² dφ²]
χ
r
K = 0, ±1
χ = PHYSICAL DISTANCE TO COORD. r
= ∫ dℓ
= ∫₀^r dr′/√(1−Kr′²) =
1/√K sin⁻¹[√K r]   K > 0
r   K = 0
1/√−K sinh⁻¹(√−K r)   K < 0
C = PHYSICAL CIRC
= ∫ dℓ
= ∫₀^2π r dφ
= 2πr