Title: Cosmology (2021/2022)

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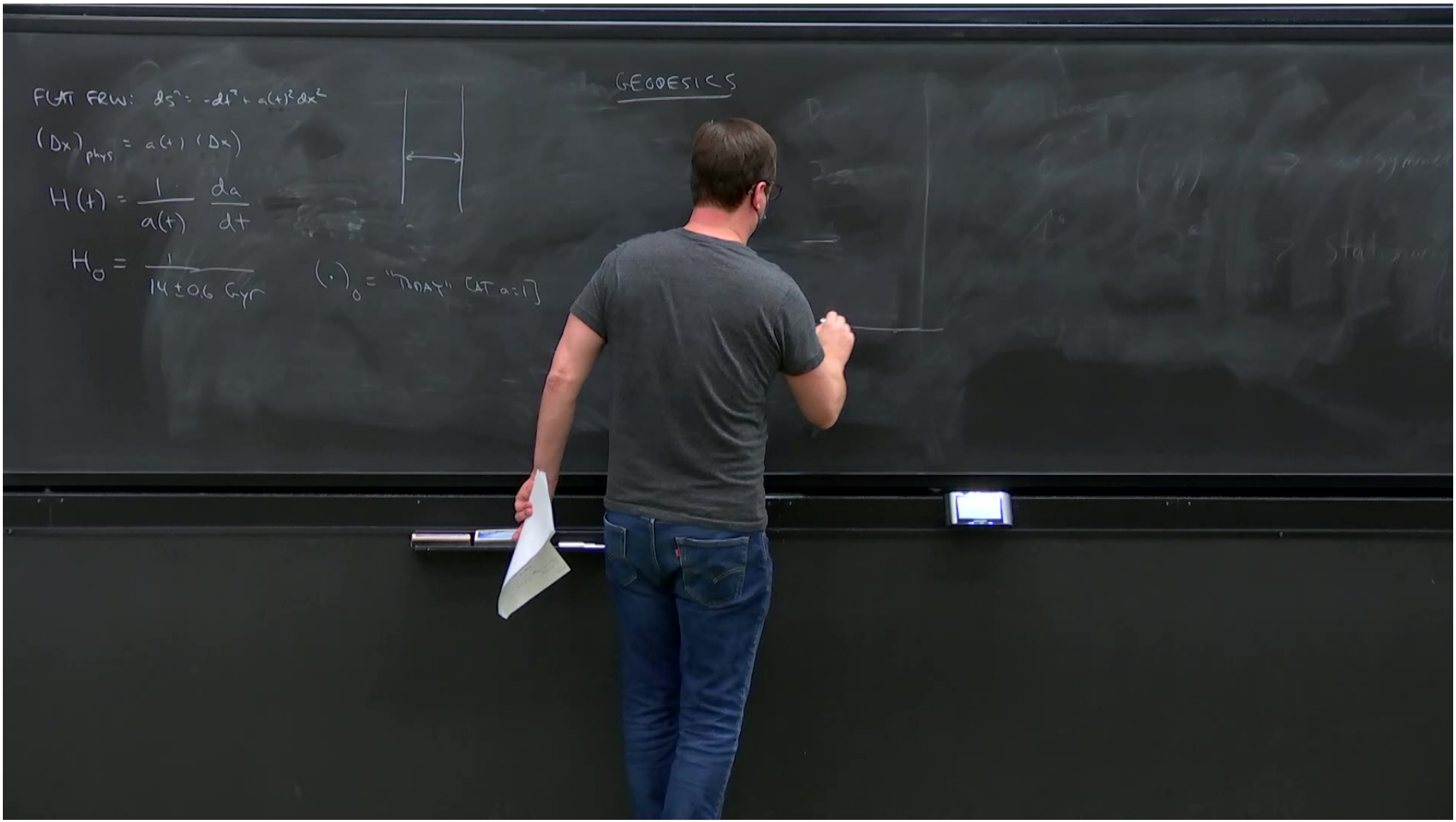
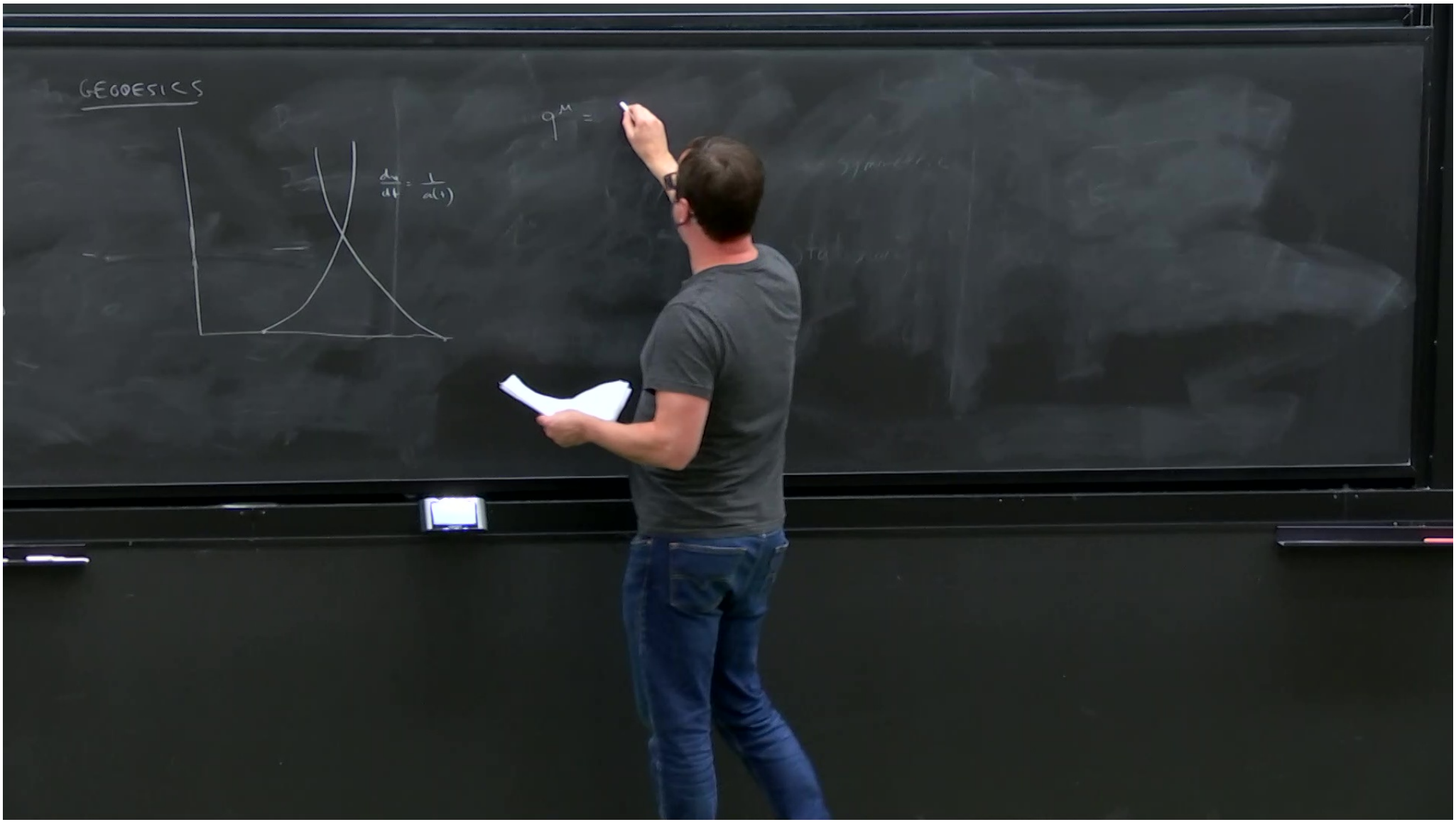
Collection: Cosmology (2021/2022)

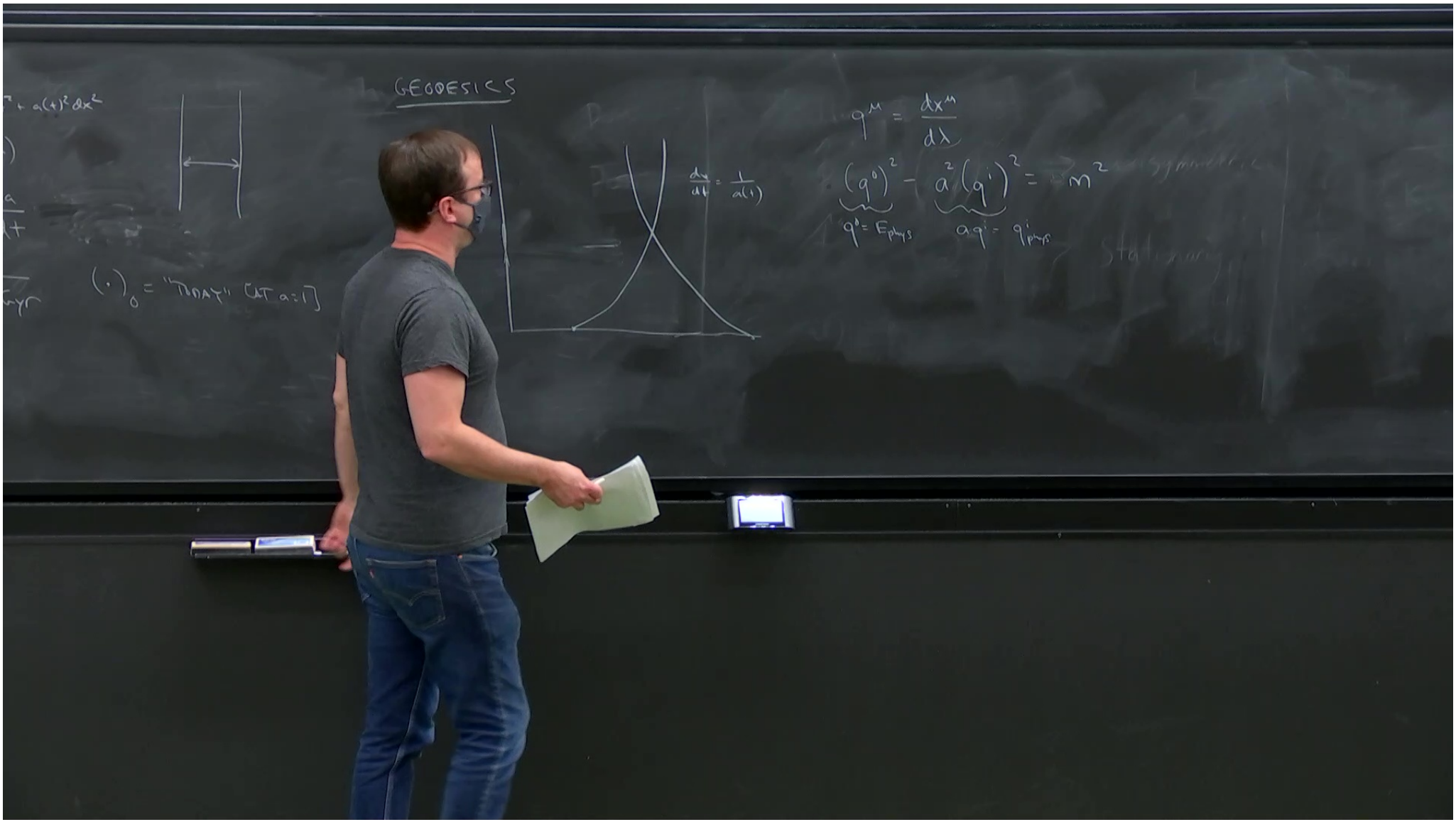
Date: April 05, 2022 - 2:30 PM

URL: https://pirsa.org/22040077

Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.


FLAT FRW: $ds^2 = -dt^2 + a(t)^2 dx^2$
$(\Delta x)_{phys} = a(t)\,(\Delta x)$
$H(t) = \frac{1}{a(t)} \frac{da}{dt}$
$H_0 = \frac{1}{14 \pm 0.6 \text{ Gyr}}$
$(\cdot)_0$ = "TODAY" [AT $a=1$]
GEODESICS


GEODESICS
$q^\mu =$
$\frac{dq}{dt} = \frac{1}{a(t)}$


GEODESICS
$q^\mu = \frac{dx^\mu}{d\lambda}$
$(q^0)^2 - a^2(q^i)^2 = m^2$
$q^0 = E_{phys}$
$a q^i = q^i_{phys}$
$\frac{d\eta}{dt} = \frac{1}{a(t)}$
$(\cdot)_0$ = "TODAY" [AT a=1]

$$q^\mu = \frac{dx^\mu}{d\lambda}$$

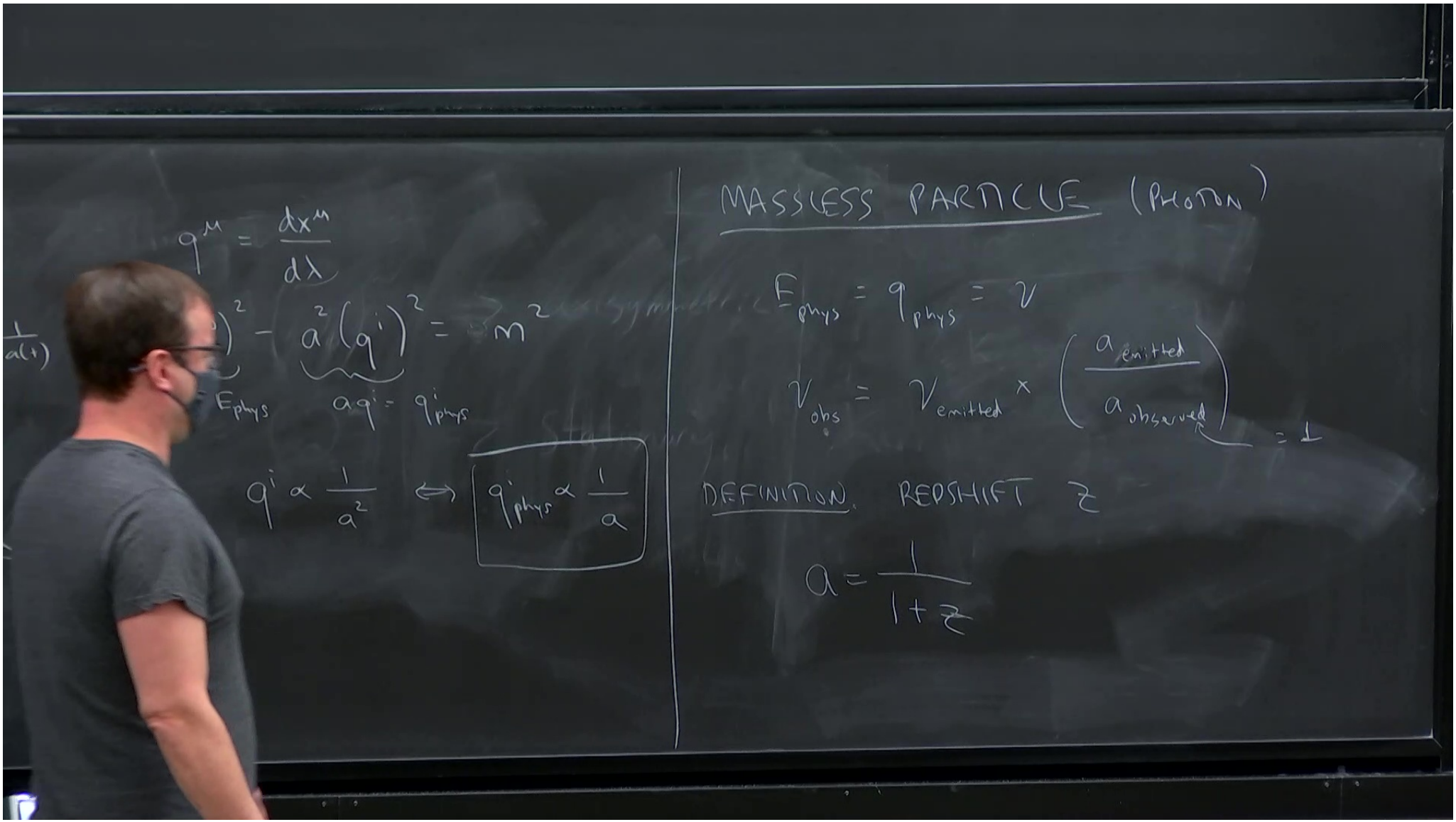
$$\underbrace{(q^0)^2}_{q^0 = E_{phys}} - \underbrace{a^2 (q^i)^2}_{a q^i = q^i_{phys}} = m^2$$

$$q^i \propto \frac{1}{a^2} \iff \boxed{q^i_{phys} \propto \frac{1}{a}}$$

MASSLESS PARTICLE (PHOTON)

$$E_{phys} = q_{phys} = \nu$$

$$\nu_{obs} = \nu_{emitted} \times \underbrace{\left(\frac{a_{emitted}}{a_{observed}}\right)}_{=1}$$


$q^\mu = \frac{dx^\mu}{d\lambda}$
$\frac{1}{a(t)}$
$)^2 - a^2 (q^i)^2 = m^2$
E_{phys}
$a q^i = q^i_{phys}$
$q^i \propto \frac{1}{a^2} \iff q^i_{phys} \propto \frac{1}{a}$
MASSLESS PARTICLE (PHOTON)
$E_{phys} = q_{phys} = \nu$
$\nu_{obs} = \nu_{emitted} \times \left(\frac{a_{emitted}}{a_{observed}}\right)$
$= 1$
DEFINITION: REDSHIFT z
$a = \frac{1}{1+z}$

$$q^\mu = \frac{dx^\mu}{d\lambda}$$

$$\underbrace{(q^0)^2}_{q^0 = E_{phys}} - \underbrace{a^2 (q^i)^2}_{a q^i = q^i_{phys}} = m^2$$

$$q^i \propto \frac{1}{a^2} \iff \boxed{q^i_{phys} \propto \frac{1}{a}}$$

MASSLESS PARTICLE (PHOTON)

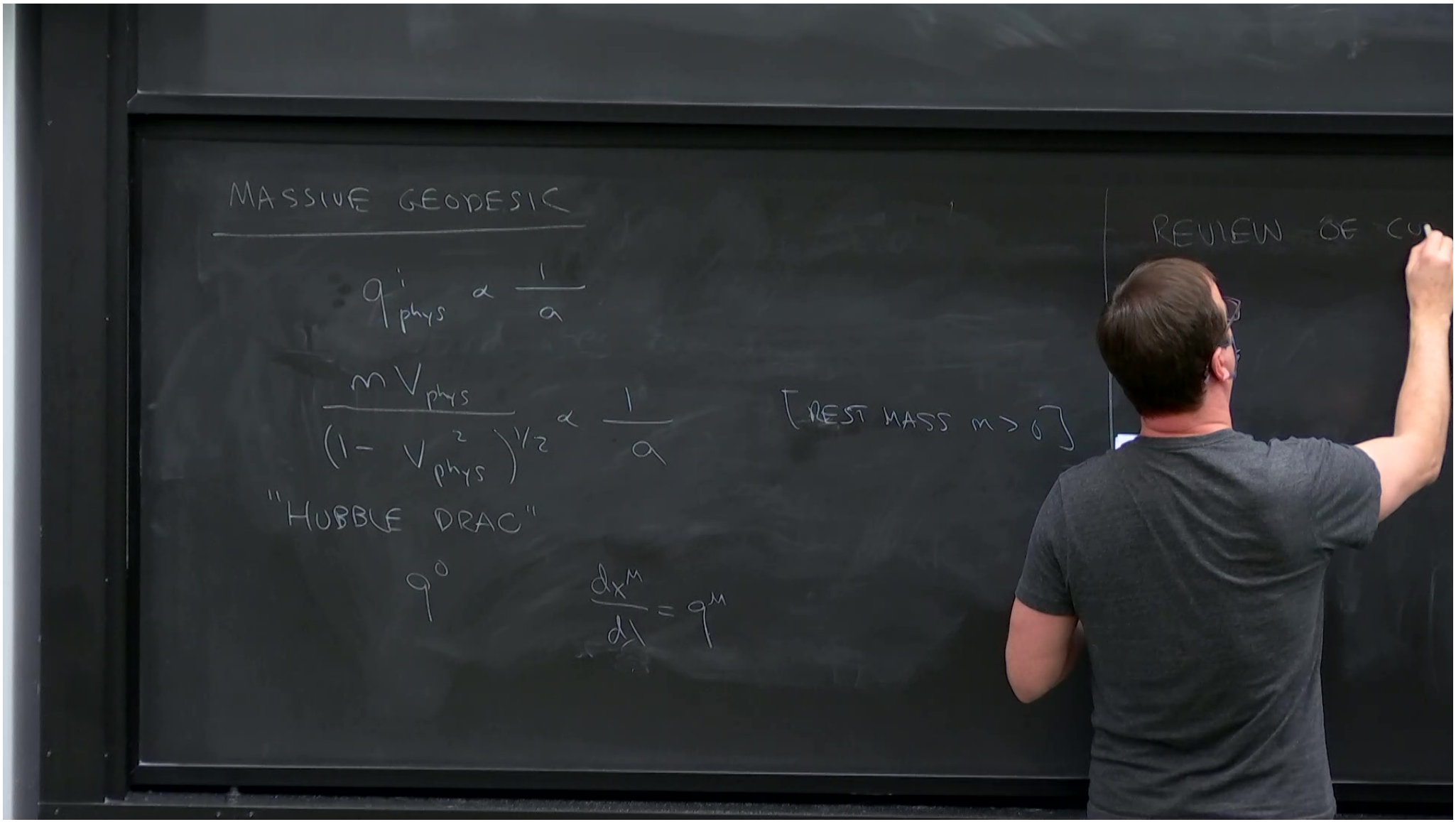
$$E_{phys} = q_{phys} = \nu$$

$$\nu_{obs} = \nu_{emitted} \times \underbrace{\left(\frac{a_{emitted}}{a_{observed}}\right)}_{=1}$$

DEFINITION: REDSHIFT z

$$a = \frac{1}{1+z}$$

$$\boxed{\nu_{obs} = \frac{\nu_{emitted}}{1 + z_{emitted}}}$$

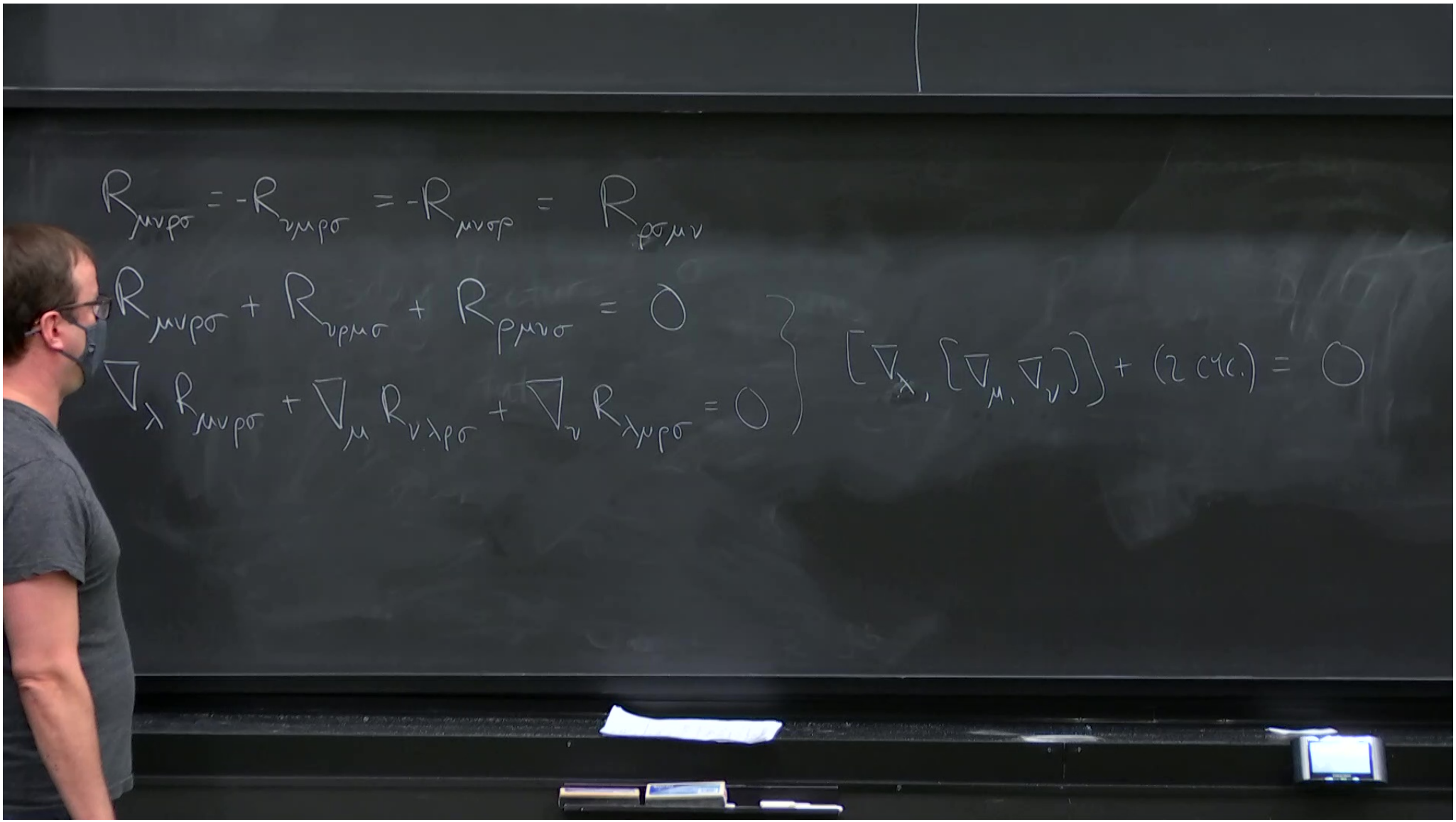

MASSIVE GEODESIC
$q^i_{phys} \propto \frac{1}{a}$
$\frac{m V_{phys}}{(1 - V_{phys}^2)^{1/2}} \propto \frac{1}{a}$
[REST MASS $m > 0$]
"HUBBLE DRAG"
q^0
$\frac{dx^\mu}{d\lambda} = q^\mu$
REVIEW OF CU

REVIEW OF CURVATURE

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) X_\rho = R_{\mu\nu\rho}{}^{\sigma} X_\sigma$$

$$R_{\mu\nu\rho}{}^{\sigma} = -\partial_\mu \Gamma^{\sigma}_{\nu\rho} + \partial_\nu \Gamma^{\sigma}_{\mu\rho} - \Gamma^{\sigma}_{\mu\lambda}\Gamma^{\lambda}_{\nu\rho} + \Gamma^{\sigma}_{\nu\lambda}\Gamma^{\lambda}_{\mu\rho}$$

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) X^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \rho_m} = \sum_{i=1}^{m} R_{\mu\nu\rho_i}{}^{\lambda} X^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \lambda \cdots \rho_m} - \sum_{j=1}^{n} R_{\mu\nu\lambda}{}^{\sigma_j} X^{\sigma_1 \cdots \lambda}$$


$R_{\mu\nu\rho\sigma} = -R_{\nu\mu\rho\sigma} = -R_{\mu\nu\sigma\rho} = R_{\rho\sigma\mu\nu}$
$R_{\mu\nu\rho\sigma} + R_{\nu\rho\mu\sigma} + R_{\rho\mu\nu\sigma} = 0$
$\nabla_\lambda R_{\mu\nu\rho\sigma} + \nabla_\mu R_{\nu\lambda\rho\sigma} + \nabla_\nu R_{\lambda\mu\rho\sigma} = 0$
$[\nabla_\lambda, [\nabla_\mu, \nabla_\nu]] + (2 \text{ cyc.}) = 0$

$$\nabla_\nu]] + (2\ \text{cyc.}) = 0$$

$$R_{\mu\nu} = g^{\rho\sigma} R_{\mu\rho\nu\sigma} \quad \text{RICCI}$$

$$R = g^{\mu\nu} R_{\mu\nu} \quad \text{SCALAR}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad \text{EINSTEIN} \quad \Rightarrow \quad \nabla_\mu G^{\mu\nu} = 0 \quad \text{"CONS}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla_\nu]] + (2\ \text{cyc.}) = 0$$

$$R_{\mu\nu} = g^{\rho\sigma} R_{\mu\rho\nu\sigma} \qquad \text{RICCI}$$

$$R = g^{\mu\nu} R_{\mu\nu} \qquad \text{SCALAR}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \qquad \text{EINSTEIN} \quad \Rightarrow \quad \nabla_\mu G^{\mu\nu} = 0 \qquad \text{"CONS}$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \Rightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

$$R_{\mu\nu} = g^{\rho\sigma} R_{\mu\rho\nu\sigma}$$ RICCI

$$R = g^{\mu\nu} R_{\mu\nu}$$ SCALAR

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$ EINSTEIN $\Rightarrow \nabla_\mu G^{\mu\nu} = 0$ "CONSERVED"

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$$

SPECIALIZE TO FRW METRIC

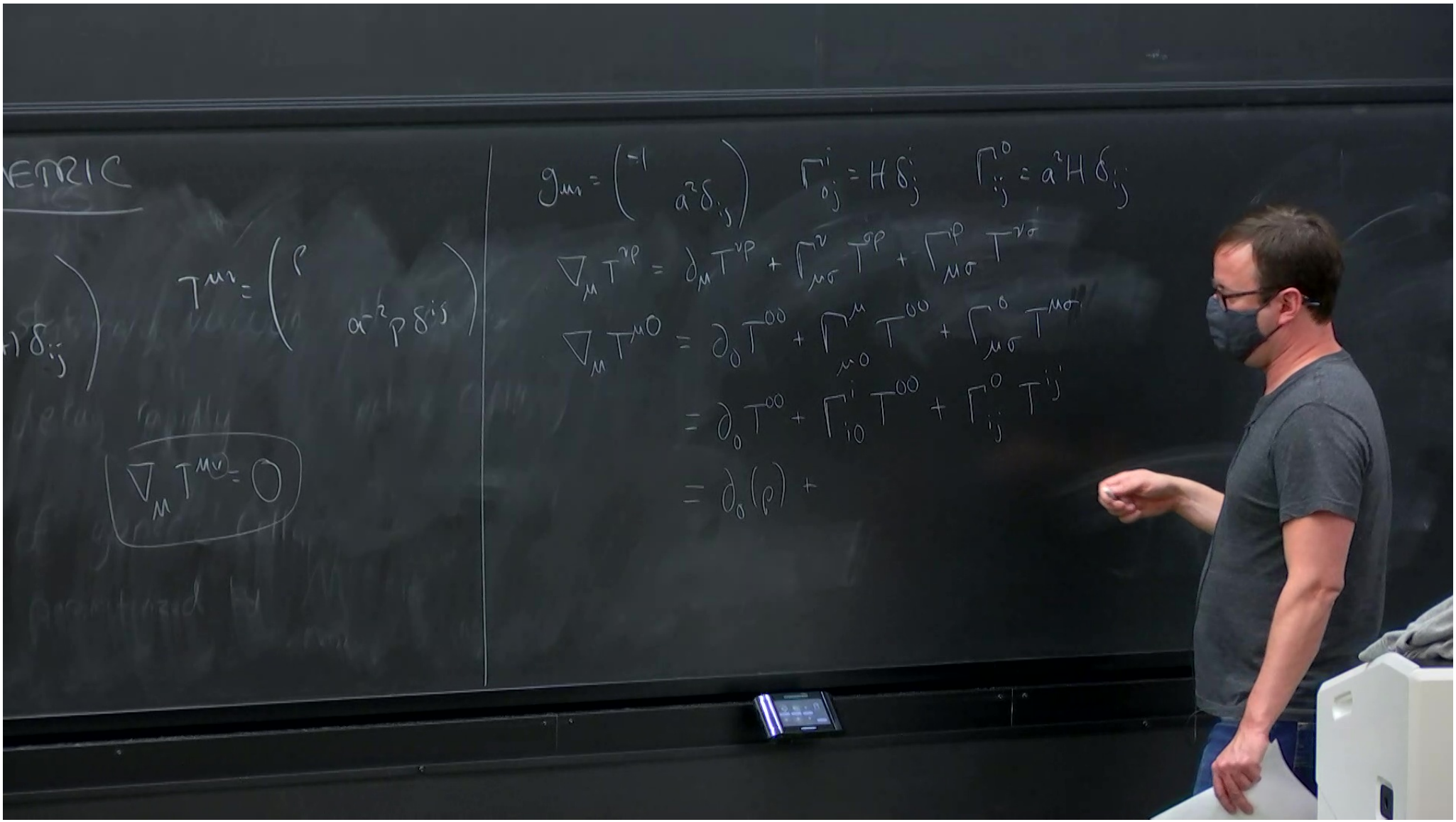
$$T_{\mu\nu} = \begin{pmatrix} \rho(t) & 0 \\ 0 & a(t)^2\, p(t)\, \delta_{ij} \end{pmatrix} \qquad T^{\mu\nu} = \begin{pmatrix} \rho & \\ & a^{-2} p\, \delta^{ij} \end{pmatrix}$$

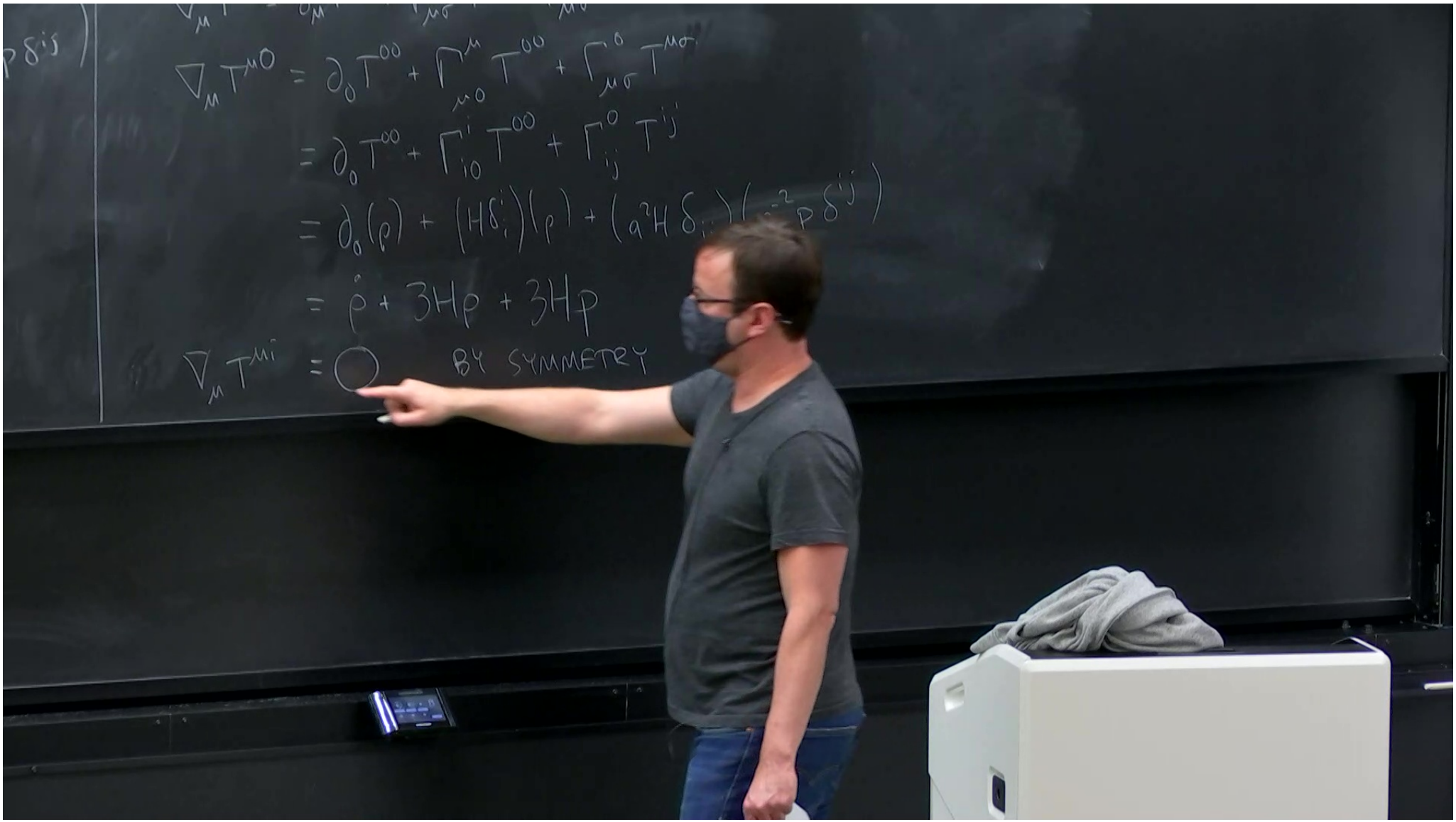
ρ = ENERGY DENSITY

p = PRESSURE

$$\nabla_\mu T^{\mu\nu} = 0$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a^2 \delta_{ij} \end{pmatrix} \qquad \Gamma^{i}$$


NETRIC
$T^{\mu\nu} = \begin{pmatrix} \rho & \\ & a^{-2} p \delta^{ij} \end{pmatrix}$
$\nabla_\mu T^{\mu\nu} = 0$
$g_{\mu\nu} = \begin{pmatrix} -1 & \\ & a^2 \delta_{ij} \end{pmatrix}$
$\Gamma^i_{0j} = H \delta^i_j$
$\Gamma^0_{ij} = a^2 H \delta_{ij}$
$\nabla_\mu T^{\nu\rho} = \partial_\mu T^{\nu\rho} + \Gamma^\nu_{\mu\sigma} T^{\sigma\rho} + \Gamma^\rho_{\mu\sigma} T^{\nu\sigma}$
$\nabla_\mu T^{\mu 0} = \partial_0 T^{00} + \Gamma^\mu_{\mu 0} T^{00} + \Gamma^0_{\mu\sigma} T^{\mu\sigma}$
$= \partial_0 T^{00} + \Gamma^i_{i0} T^{00} + \Gamma^0_{ij} T^{ij}$
$= \partial_0 (\rho) +$

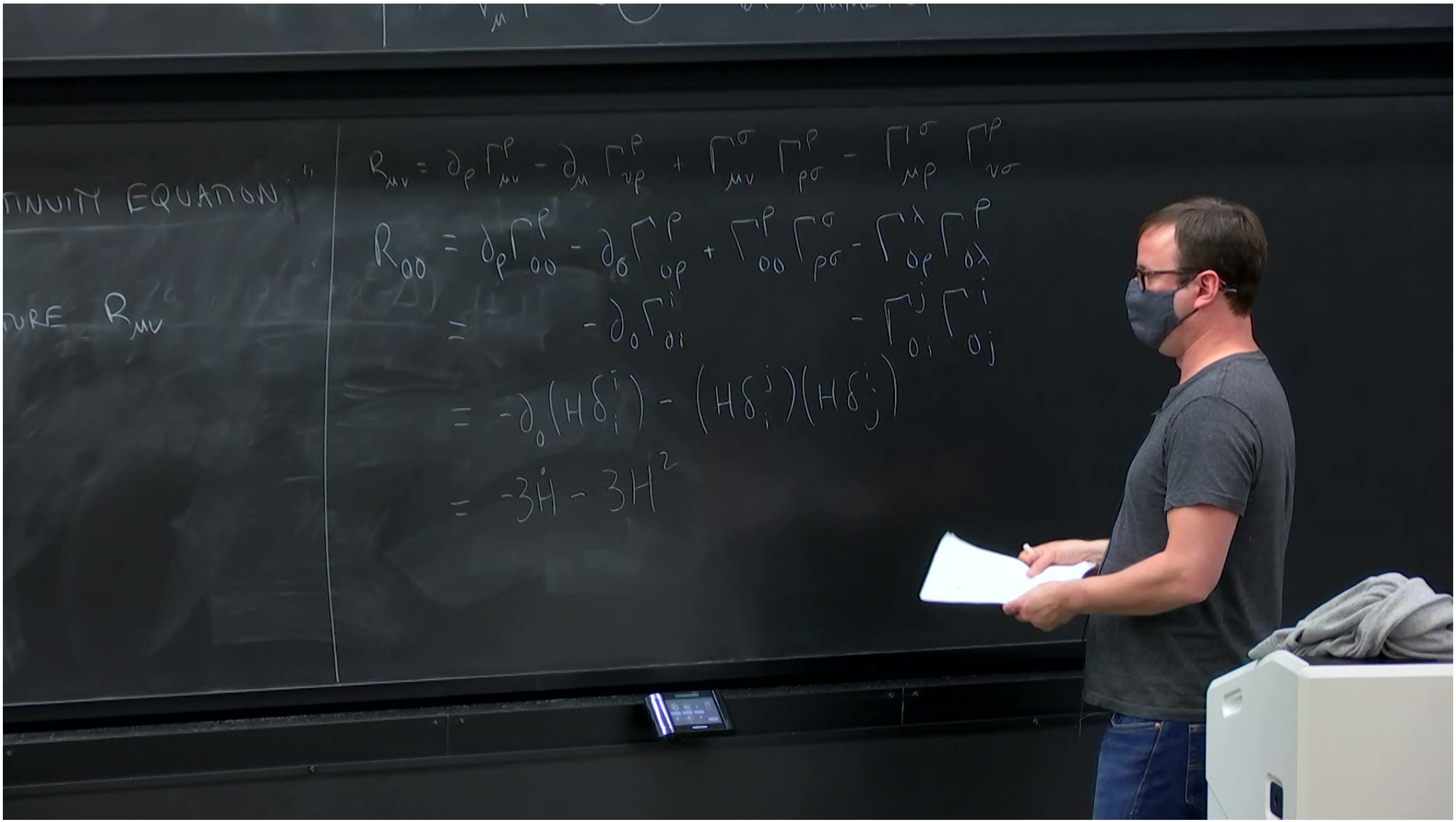

$\nabla_\mu T^{\mu 0} = \partial_0 T^{00} + \Gamma^\mu_{\mu 0} T^{00} + \Gamma^0_{\mu\sigma} T^{\mu\sigma}$
$= \partial_0 T^{00} + \Gamma^i_{i0} T^{00} + \Gamma^0_{ij} T^{ij}$
$= \partial_0(\rho) + (H\delta^i_i)(\rho) + (a^2 H \delta_{ij})(a^{-2} p \delta^{ij})$
$= \dot{\rho} + 3H\rho + 3Hp$
$\nabla_\mu T^{\mu i} = 0$ BY SYMMETRY

$$\boxed{\dot{\rho} = -3H(\rho + p)}$$

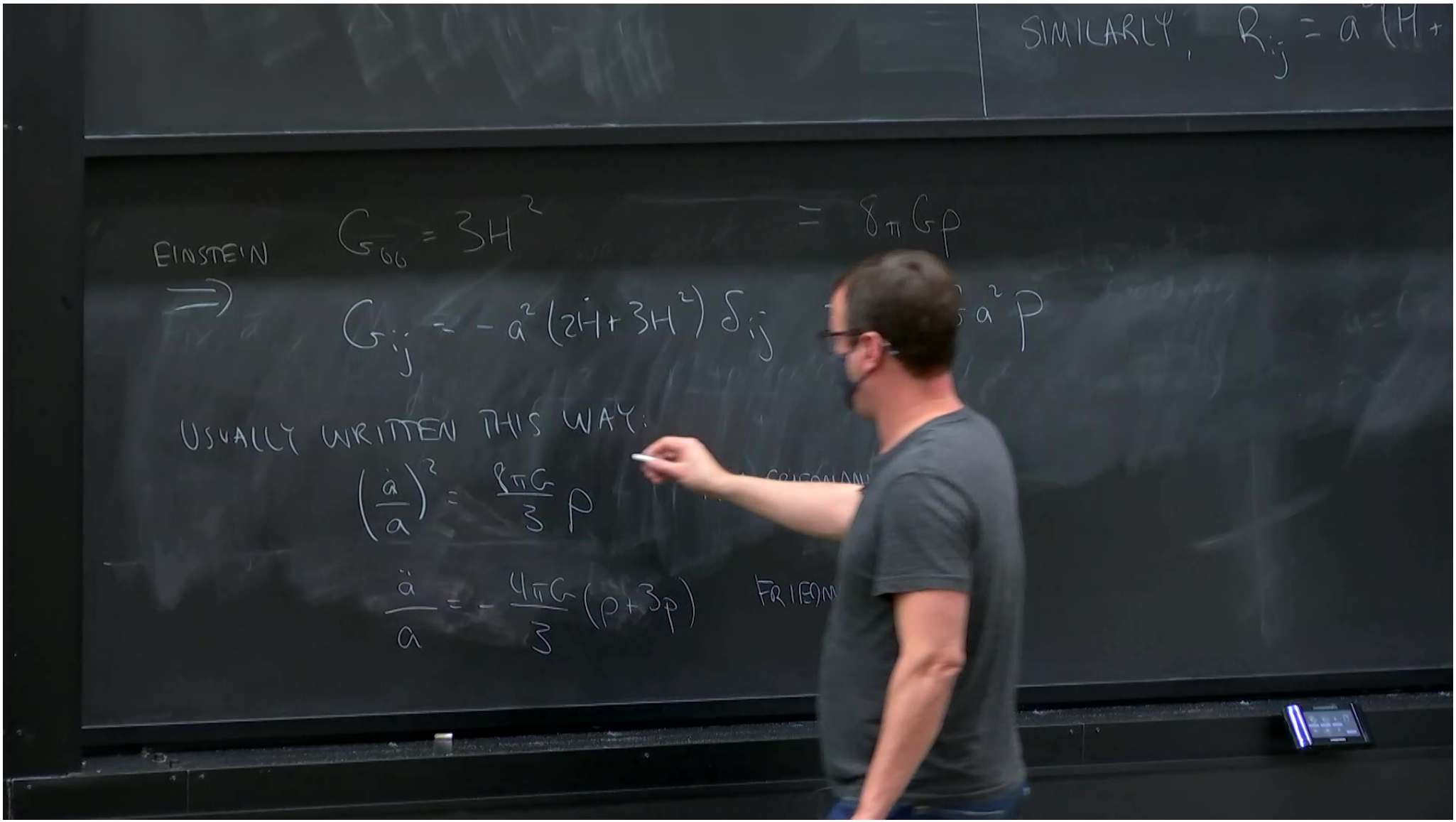
"CONTINUITY EQUATION"

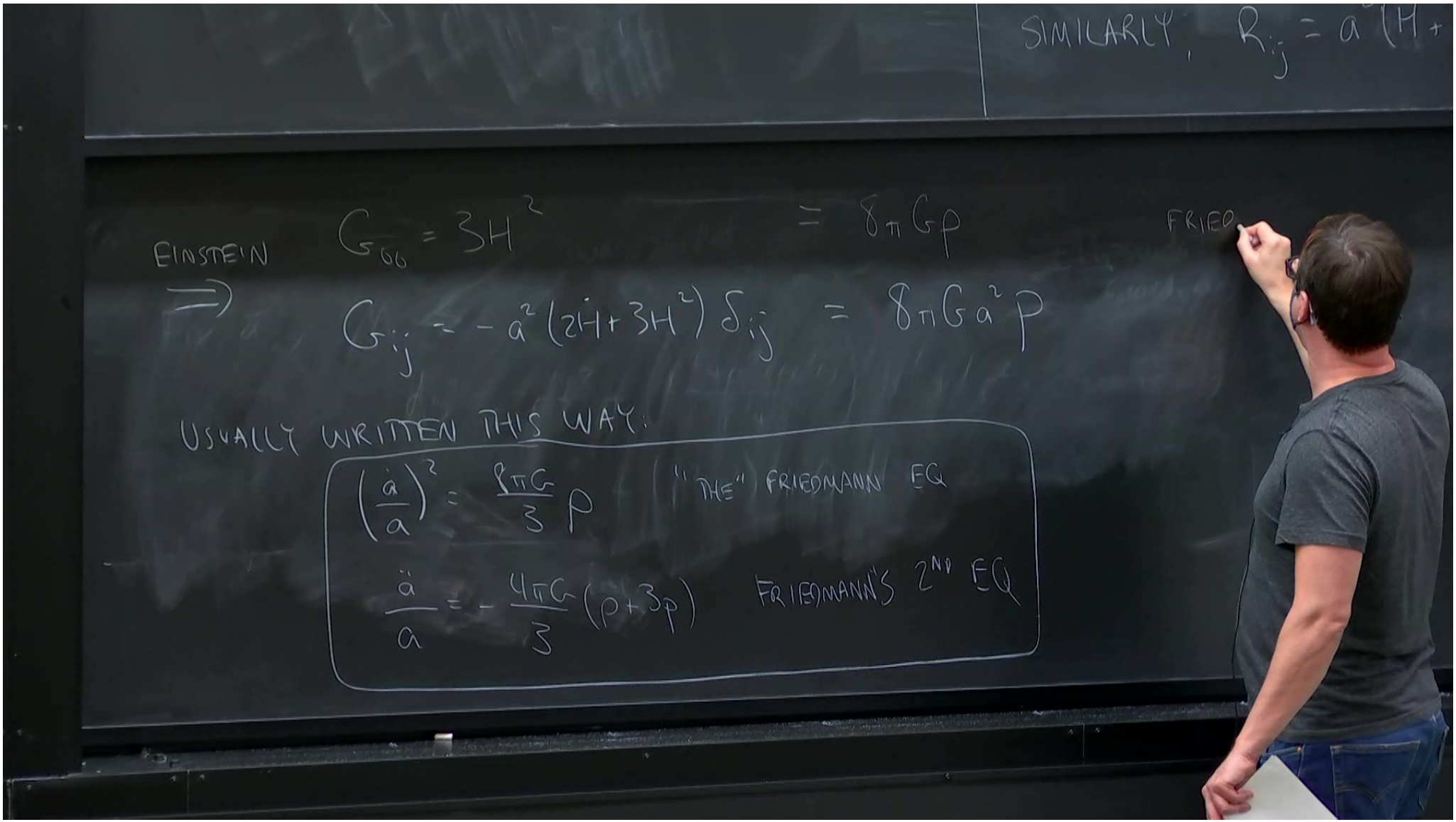
LHS OF EINSTEIN $\Rightarrow$ RICCI CURVATURE $R_{\mu\nu}$

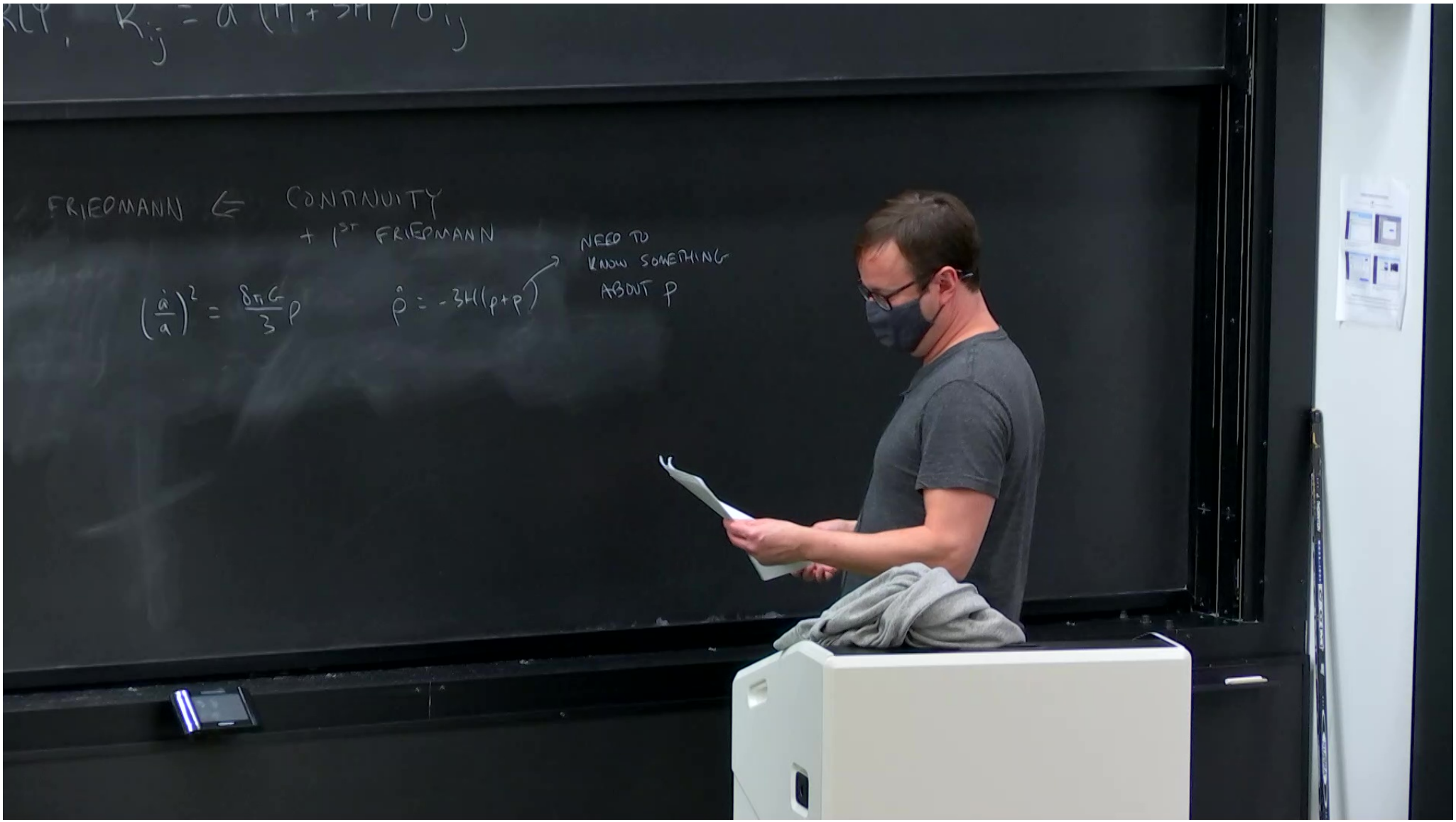
$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\mu \Gamma_\nu$

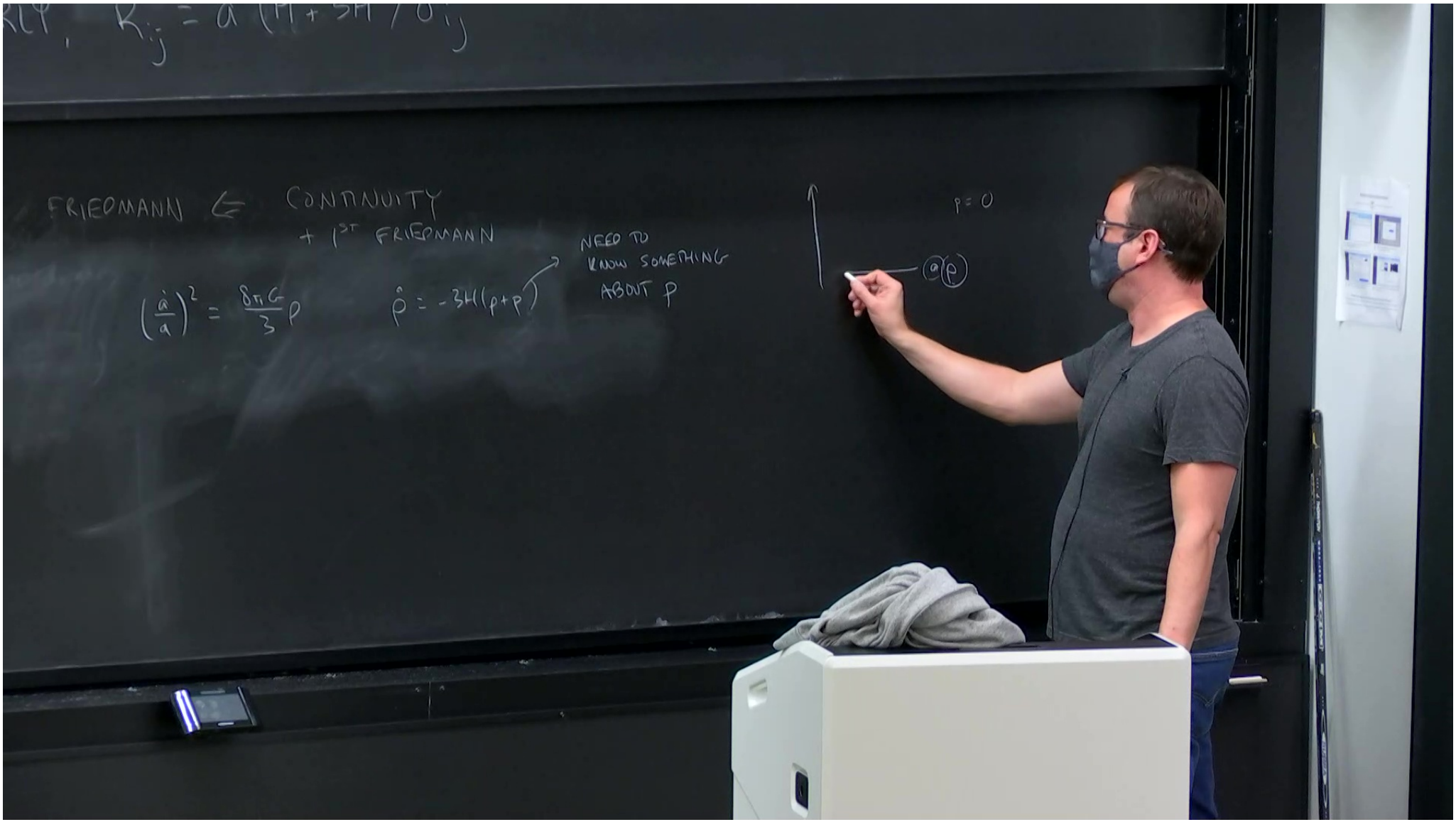

TINUITY EQUATION
URE $R_{\mu\nu}$
$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\mu \Gamma^\rho_{\nu\rho} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\rho\sigma} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\nu\sigma}$
$R_{00} = \partial_\rho \Gamma^\rho_{00} - \partial_0 \Gamma^\rho_{0\rho} + \Gamma^\rho_{00}\Gamma^\sigma_{\rho\sigma} - \Gamma^\lambda_{0\rho}\Gamma^\rho_{0\lambda}$
$= -\partial_0 \Gamma^i_{0i} - \Gamma^j_{0i}\Gamma^i_{0j}$
$= -\partial_0 (H\delta^i_i) - (H\delta^j_i)(H\delta^i_j)$
$= -3\dot{H} - 3H^2$

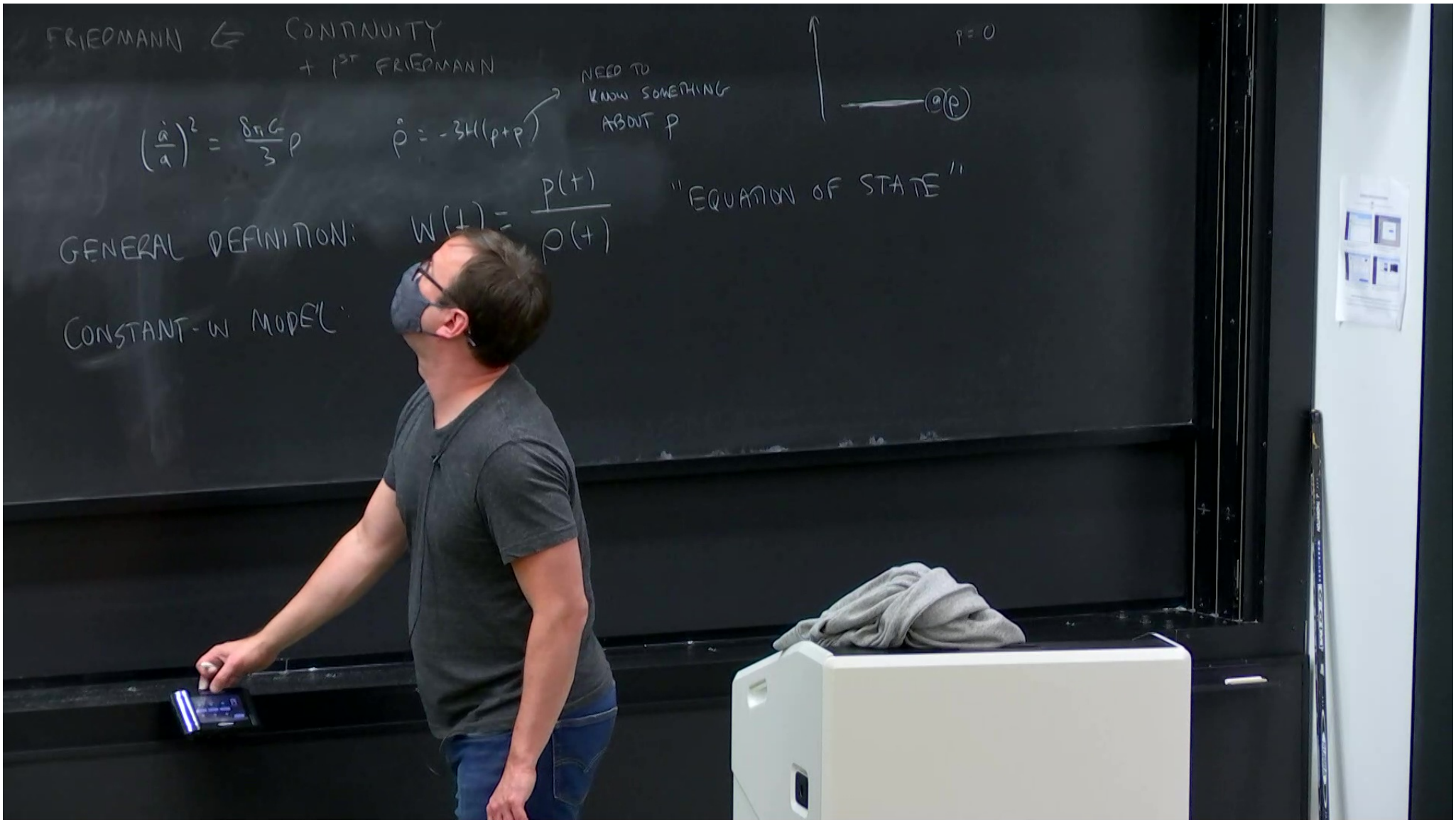
SIMILARLY, $R_{ij} = a^2(\dot{H} + 3H^2)\delta_{ij}$

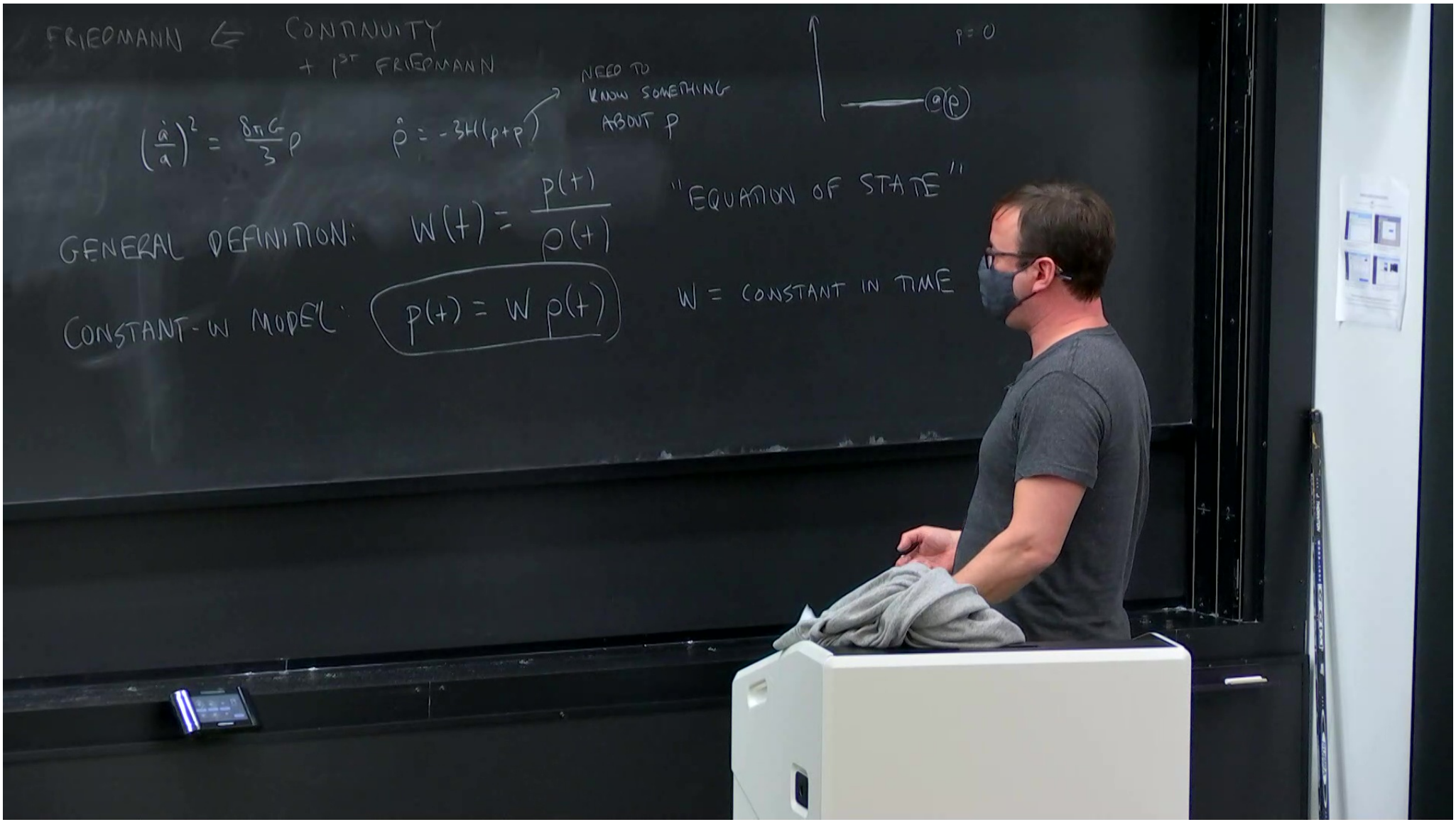

SIMILARLY, $R_{ij} = a^2(H +$
EINSTEIN $G_{00} = 3H^2 = 8\pi G\rho$
$\Rightarrow$
$G_{ij} = -a^2(2\dot{H} + 3H^2)\delta_{ij}$
USUALLY WRITTEN THIS WAY:
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$


SIMILARLY, $R_{ij} = a^2(H +$
EINSTEIN
$G_{00} = 3H^2 = 8\pi G\rho$
FRIED
$\Rightarrow$
$G_{ij} = -a^2(2\dot{H} + 3H^2)\delta_{ij} = 8\pi G a^2 p$
USUALLY WRITTEN THIS WAY:
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
"THE" FRIEDMANN EQ
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$
FRIEDMANN'S 2ND EQ


FRIEDMANN ⇐ CONTINUITY
+ 1ST FRIEDMANN
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$\dot{\rho} = -3H(\rho+p)$
NEED TO
KNOW SOMETHING
ABOUT P


FRIEDMANN ⇐ CONTINUITY
+ 1ST FRIEDMANN
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$\dot{\rho} = -3H(\rho+p)$
NEED TO KNOW SOMETHING ABOUT P
p = 0
@p

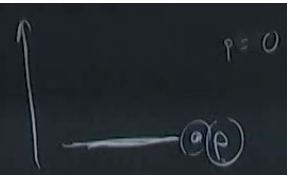

FRIEDMANN ⇐ CONTINUITY
+ 1ST FRIEDMANN
NEED TO KNOW SOMETHING ABOUT P
P = 0
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$\dot{\rho} = -3H(\rho + P)$
"EQUATION OF STATE"
GENERAL DEFINITION: $W(t) = \frac{P(t)}{\rho(t)}$
CONSTANT-W MODEL:


FRIEDMANN ⇐ CONTINUITY
+ 1ST FRIEDMANN
NEED TO KNOW SOMETHING ABOUT p
p = 0
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$
$\dot{\rho} = -3H(\rho + p)$
GENERAL DEFINITION: $w(t) = \frac{p(t)}{\rho(t)}$
"EQUATION OF STATE"
CONSTANT-w MODEL: $p(t) = w\,\rho(t)$
w = CONSTANT IN TIME

FRIEDMANN ⇐ CONTINUITY
+ 1ST FRIEDMANN

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \qquad \dot{\rho} = -3H(\rho + p)$$

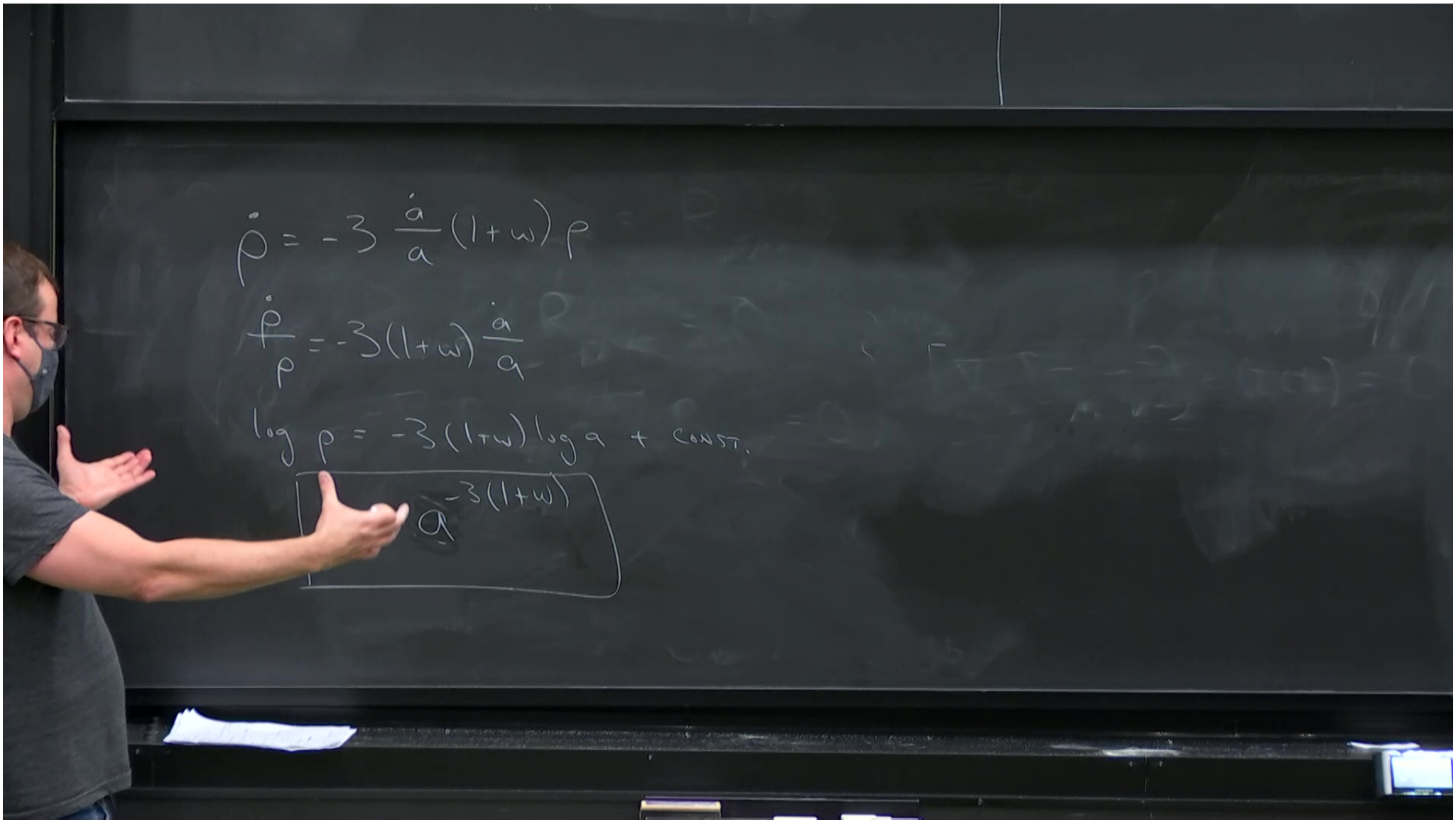
NEED TO KNOW SOMETHING ABOUT p

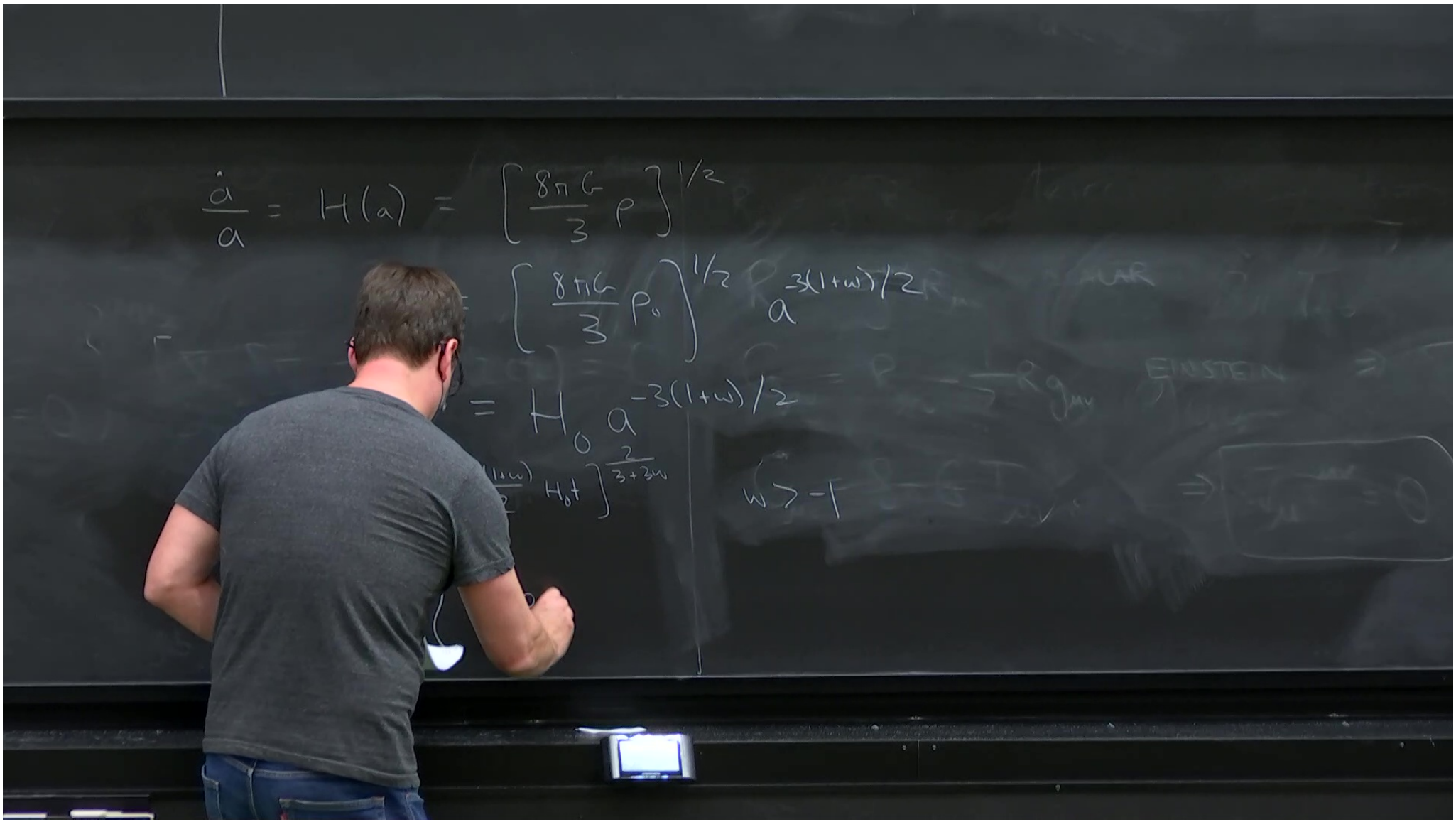


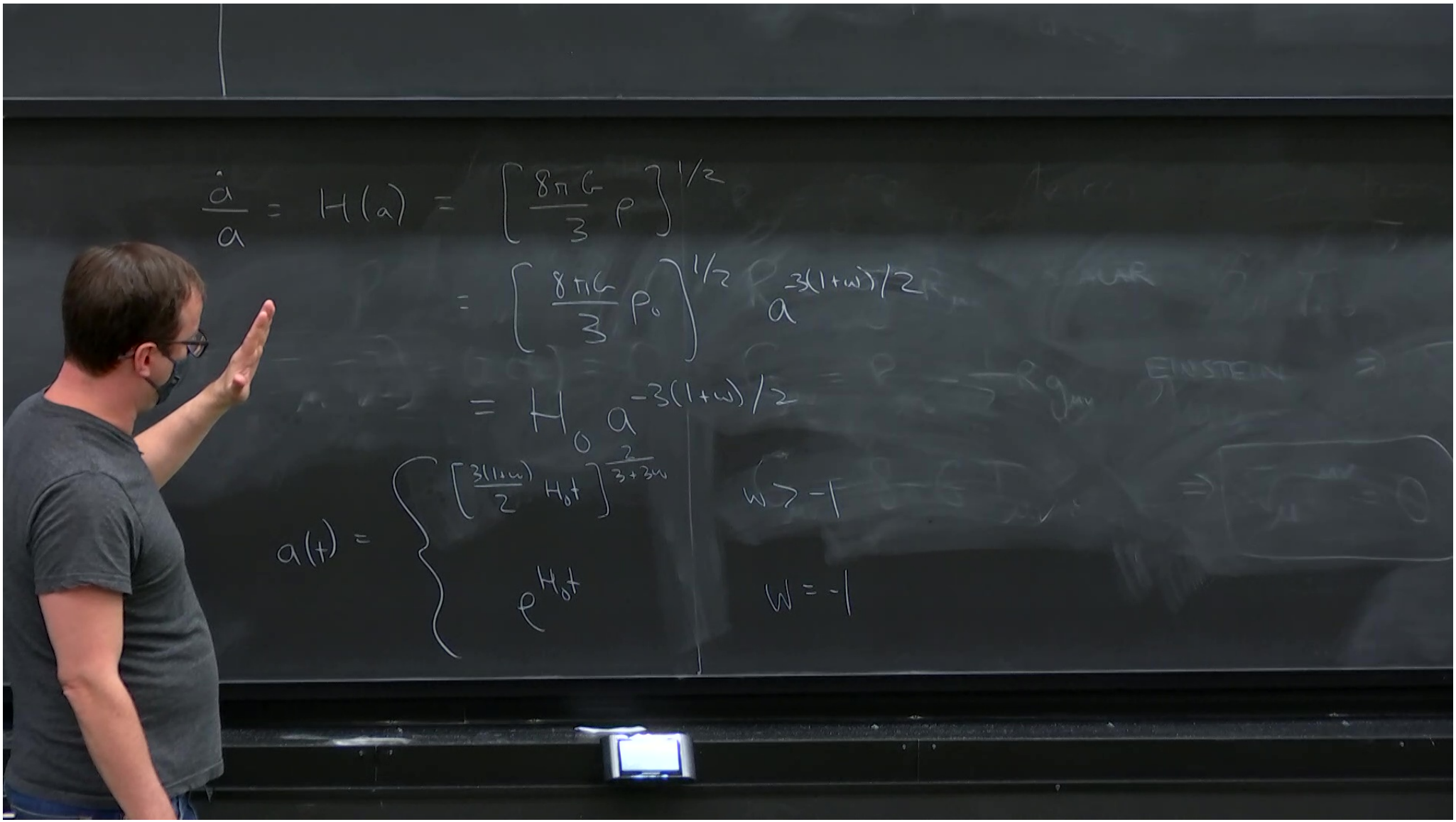
GENERAL DEFINITION: $w(t) = \dfrac{p(t)}{\rho(t)}$ "EQUATION OF STATE"

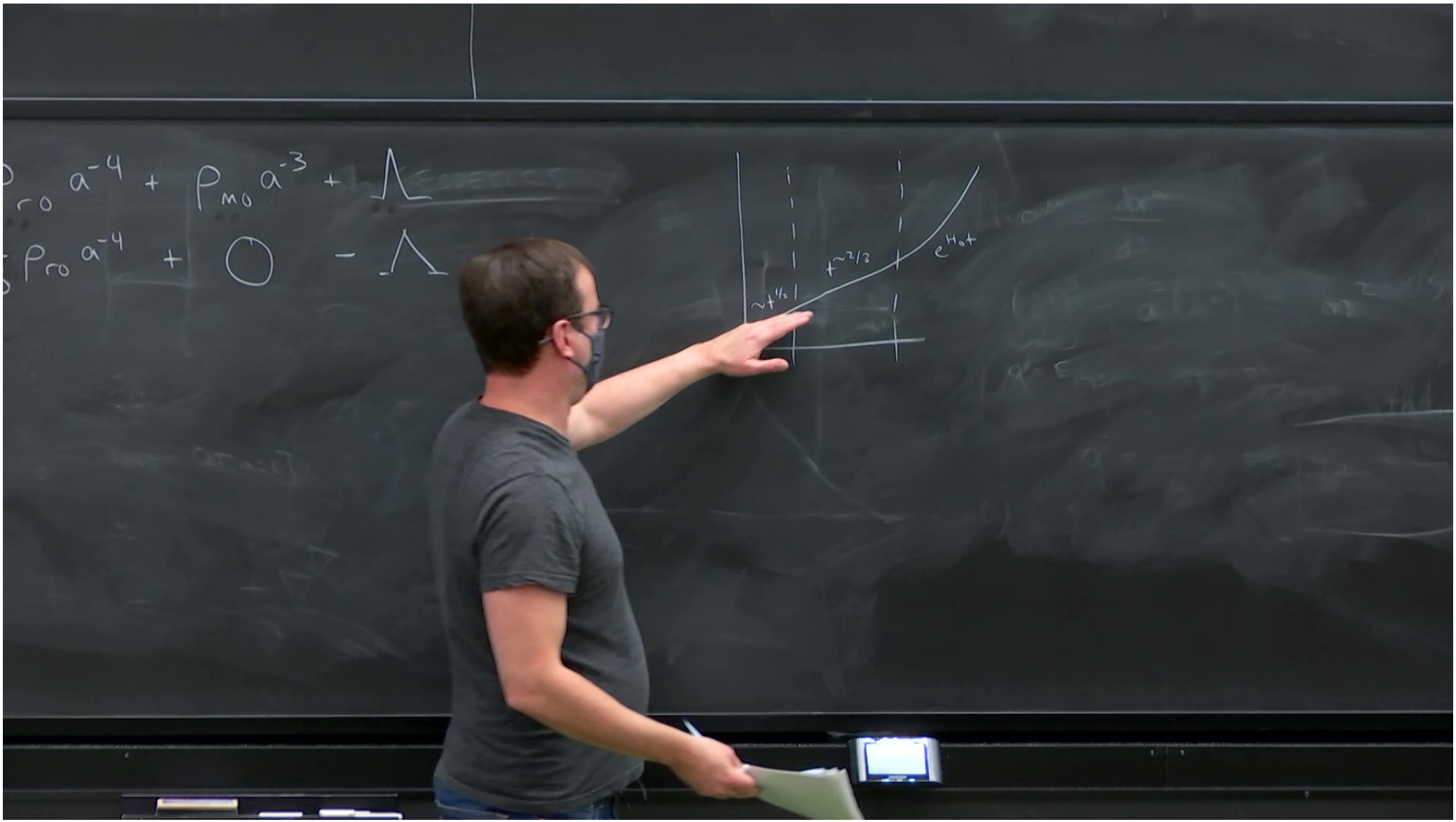
CONSTANT-W MODEL: $p(t) = w\,\rho(t)$ w = CONSTANT IN TIME

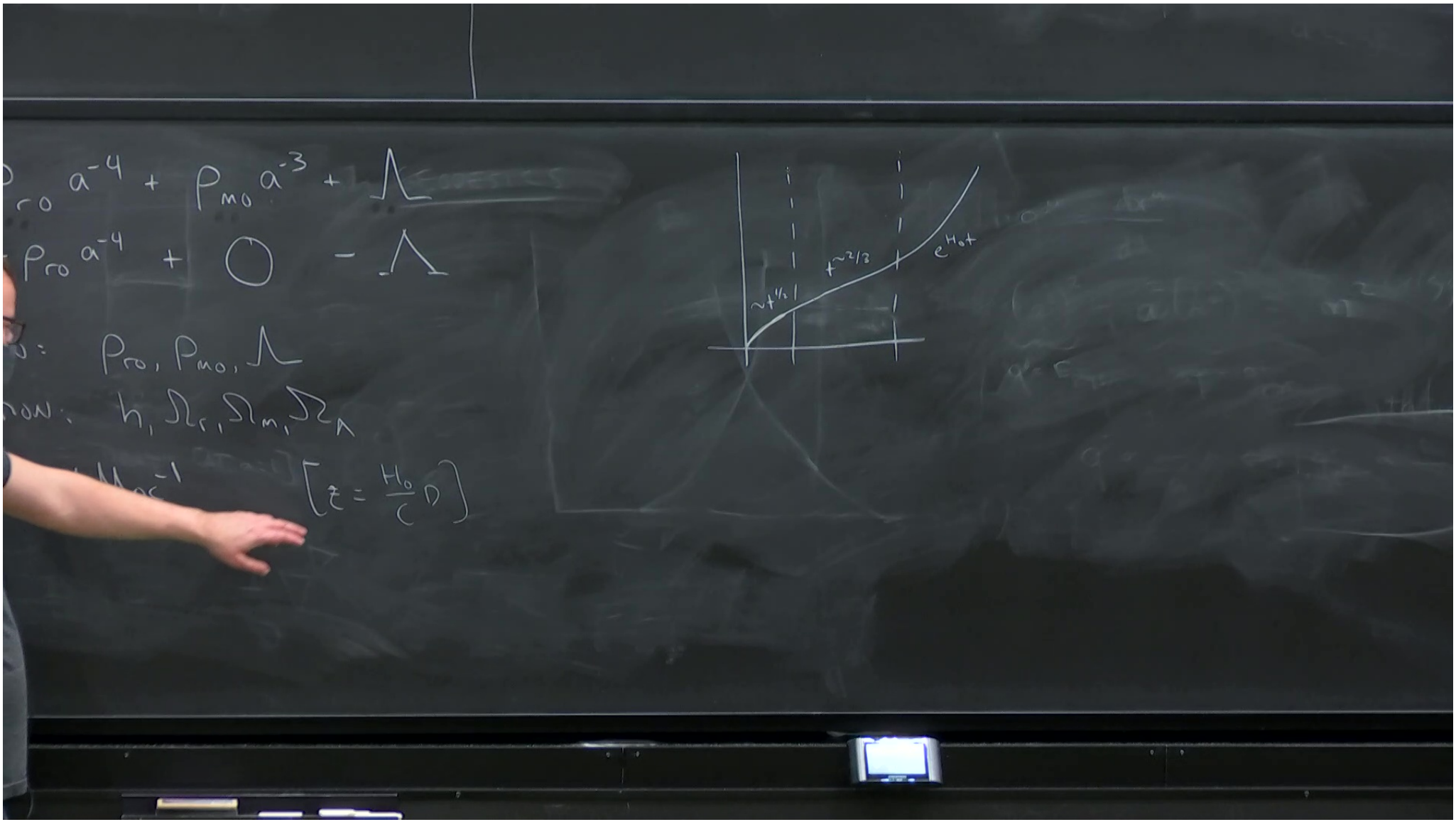
$$w = \begin{cases} 1/3 & \text{RADIATION } [v \sim 1] \\ 0 & \text{MATTER } [v \sim 0] \\ -1 & \text{COSMOLOGICAL CONSTANT} \end{cases}$$

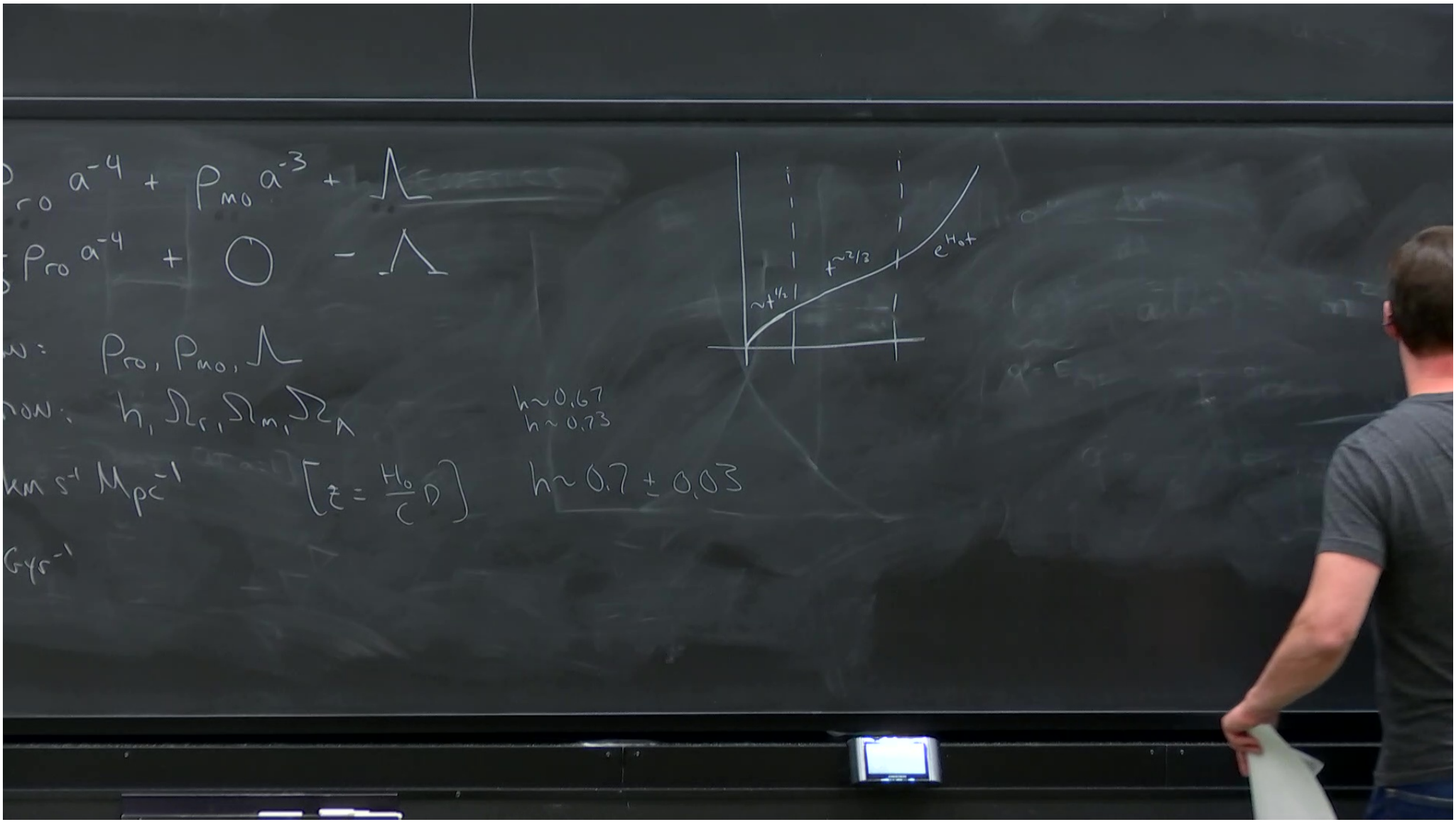

$\dot{\rho} = -3 \frac{\dot{a}}{a}(1+w)\rho$
$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$
$\log \rho = -3(1+w)\log a + \text{CONST.}$
$a^{-3(1+w)}$

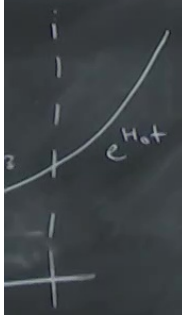

$\frac{\dot{a}}{a} = H(a) = \left[\frac{8\pi G}{3}\rho\right]^{1/2}$
$\left[\frac{8\pi G}{3}\rho_0\right]^{1/2} a^{-3(1+w)/2}$
$= H_0 a^{-3(1+w)/2}$
$w > -1$


$\frac{\dot{a}}{a} = H(a) = \left[\frac{8\pi G}{3}\rho\right]^{1/2}$
$= \left[\frac{8\pi G}{3}\rho_0\right]^{1/2} a^{-3(1+w)/2}$
$= H_0 a^{-3(1+w)/2}$
$a(t) = \begin{cases} \left[\frac{3(1+w)}{2} H_0 t\right]^{\frac{2}{3+3w}} & w > -1 \\ e^{H_0 t} & w = -1 \end{cases}$


$\rho_{r0} a^{-4} + \rho_{M0} a^{-3} + \Lambda$
$\rho_{r0} a^{-4} + 0 - \Lambda$
$\sim t^{1/2}$
$t^{\sim 2/3}$
$e^{H_0 t}$


$\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda$
$\rho_{r0}\, a^{-4} + 0 - \Lambda$
$\rho_{r0}, \rho_{m0}, \Lambda$
$h, \Omega_r, \Omega_m, \Omega_\Lambda$
$\left[\tau = \frac{H_0}{c} D\right]$
$\sim t^{1/2}$
$t^{\sim 2/3}$
$e^{H_0 t}$


$\rho_{r0}\, a^{-4} + \rho_{m0}\, a^{-3} + \Lambda$
$\rho_{r0}\, a^{-4} + 0 - \Lambda$
$\rho_{r0}, \rho_{m0}, \Lambda$
$h, \Omega_r, \Omega_m, \Omega_\Lambda$
$h \sim 0.67$
$h \sim 0.73$
km s^{-1} Mpc^{-1}
$\left[z = \frac{H_0}{c} D\right]$
$h \sim 0.7 \pm 0.03$
Gyr^{-1}
$\sim t^{1/2}$
$t^{\sim 2/3}$
$e^{H_0 t}$



$$\rho_{tot,0} = \frac{3}{8\pi G} H_0^2$$

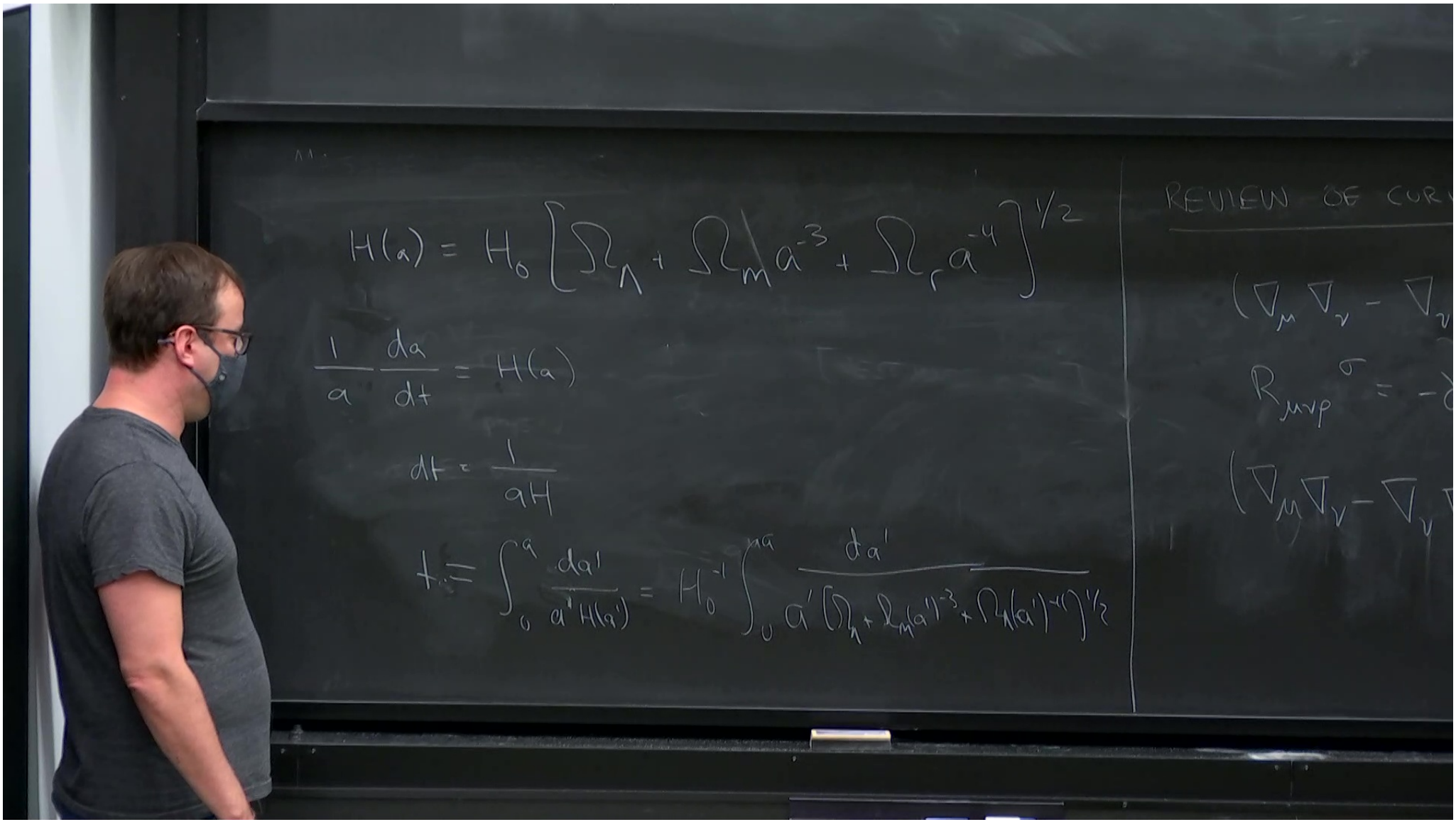
$$\Omega_i = \frac{\rho_{i0}}{\rho_{tot,0}} \qquad [i \in r, m, \Lambda]$$

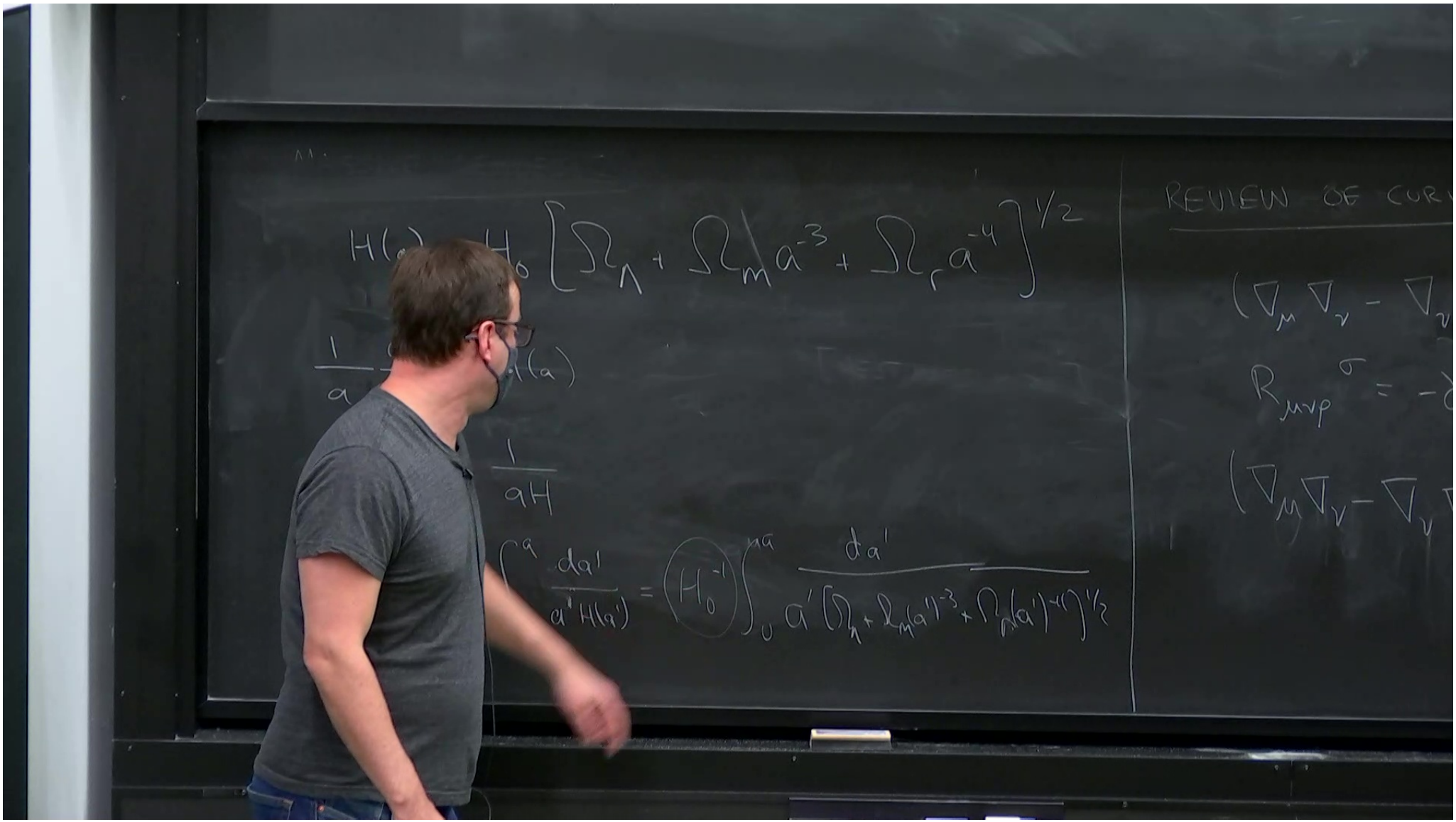
$$h = 0.7 \pm 0.03$$

$$\Omega_m = 0.311 \pm 0.006$$

$$\Omega_\Lambda = 0.689 \pm 0.006$$

$$\Omega_r = (8.5 \pm 0.7) \times 10^{-5}$$


$H(a) = H_0 \left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4} \right]^{1/2}$
$\frac{1}{a} \frac{da}{dt} = H(a)$
$dt = \frac{1}{aH}$
$t = \int_0^a \frac{da'}{a' H(a')} = H_0^{-1} \int_0^a \frac{da'}{a' \left[\Omega_\Lambda + \Omega_m (a')^{-3} + \Omega_r (a')^{-4} \right]^{1/2}}$
REVIEW OF CUR
$(\nabla_\mu \nabla_\nu - \nabla_\nu$
$R_{\mu\nu\rho}{}^{\sigma} = -$
$(\nabla_\mu \nabla_\nu - \nabla_\nu$


REVIEW OF CUR