Title: Cosmology (2021/2022)

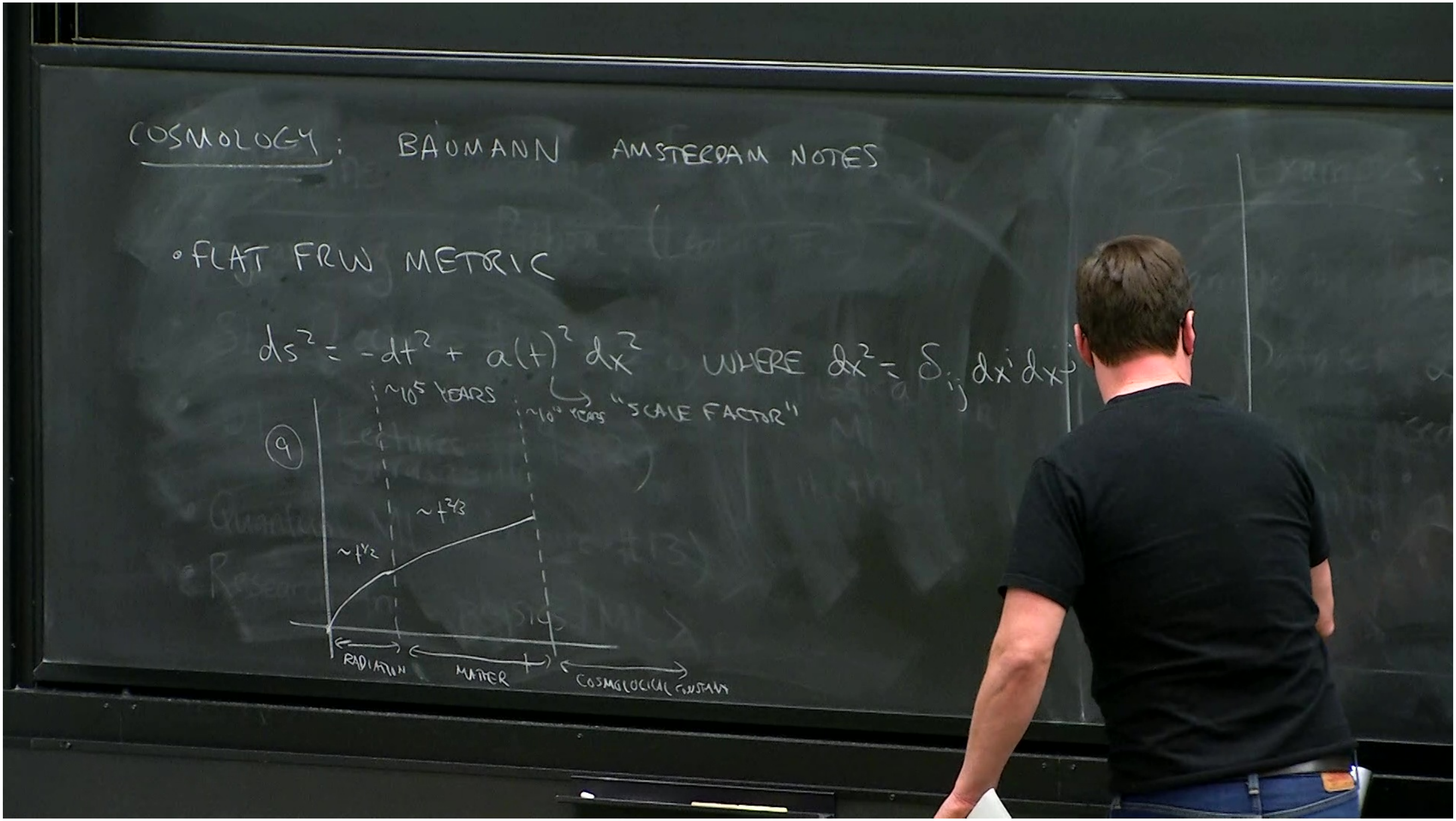
Speakers: Kendrick Smith, Gang Xu

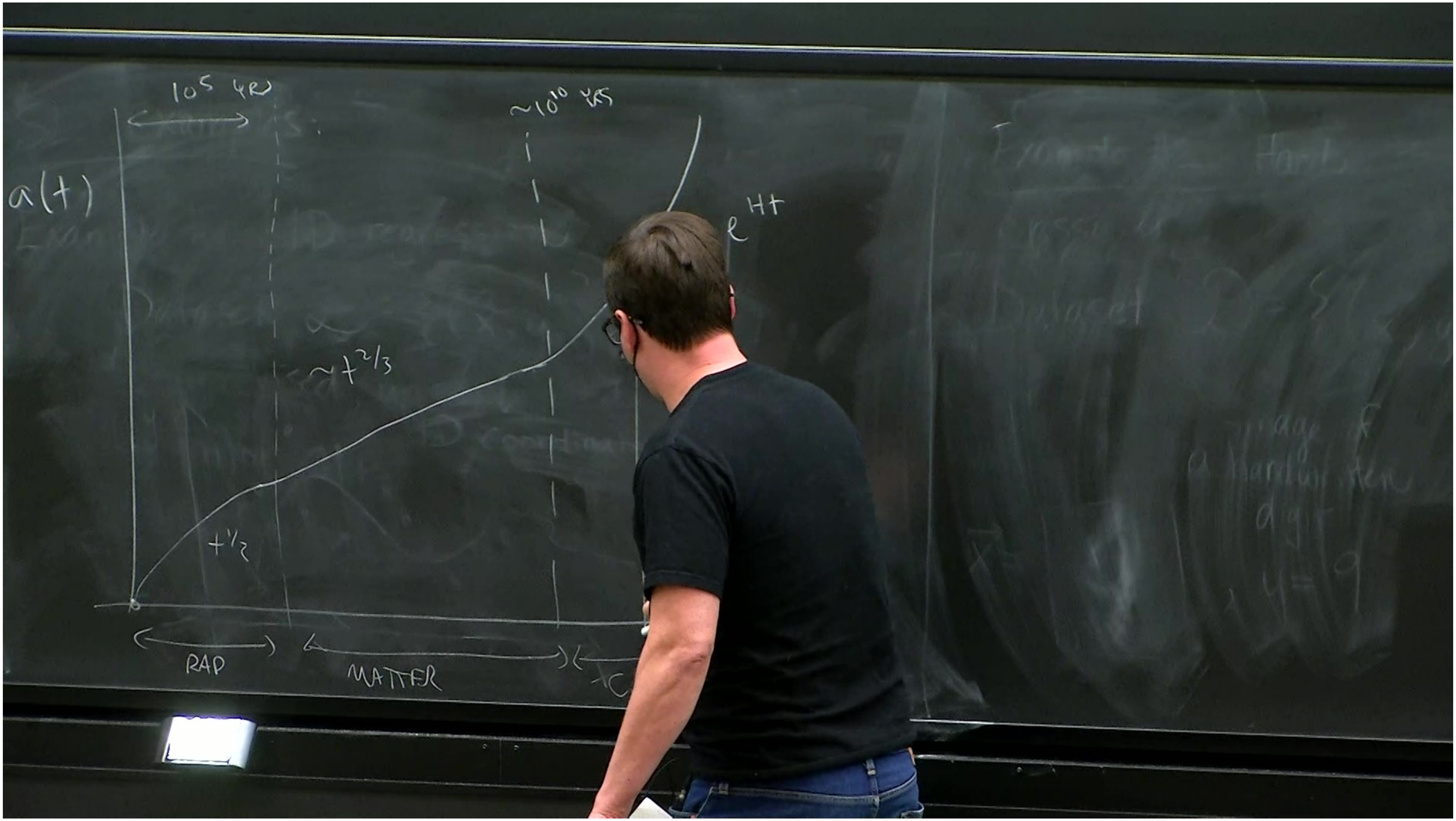
Collection: Cosmology (2021/2022)

Date: April 04, 2022 - 2:00 PM

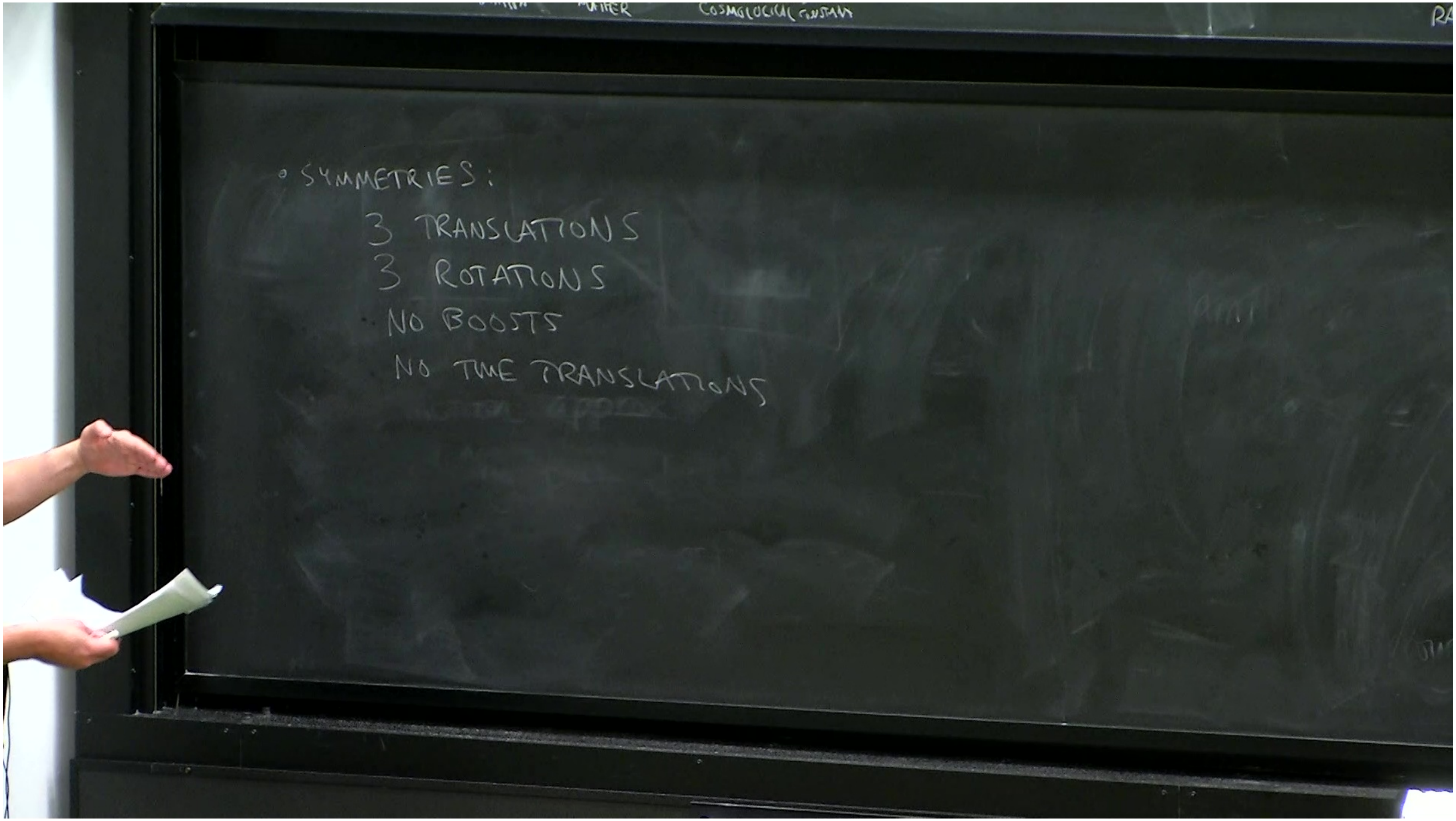
URL: https://pirsa.org/22040076

Abstract: This class is an introduction to cosmology. We'll cover expansion history of the universe, thermal history, dark matter models, and as much cosmological perturbation theory as time permits.


COSMOLOGY: BAUMANN AMSTERDAM NOTES
• FLAT FRW METRIC
$ds^2 = -dt^2 + a(t)^2 dx^2$ WHERE $dx^2 = \delta_{ij} dx^i dx^j$
~10^5 YEARS
~10^10 YEARS
"SCALE FACTOR"
~t^{1/2}
~t^{2/3}
RADIATION
MATTER
COSMOLOGICAL CONSTANT



10^5 yrs
$\sim 10^{10}$ yrs
$a(t)$
e^{Ht}
$\sim t^{2/3}$
$t^{1/2}$
RAD
MATTER


• SYMMETRIES:
3 TRANSLATIONS
3 ROTATIONS
NO BOOSTS
NO TIME TRANSLATIONS

SYMMETRIES:

- 3 TRANSLATIONS
- 3 ROTATIONS
- NO BOOSTS
- NO TIME TRANSLATIONS

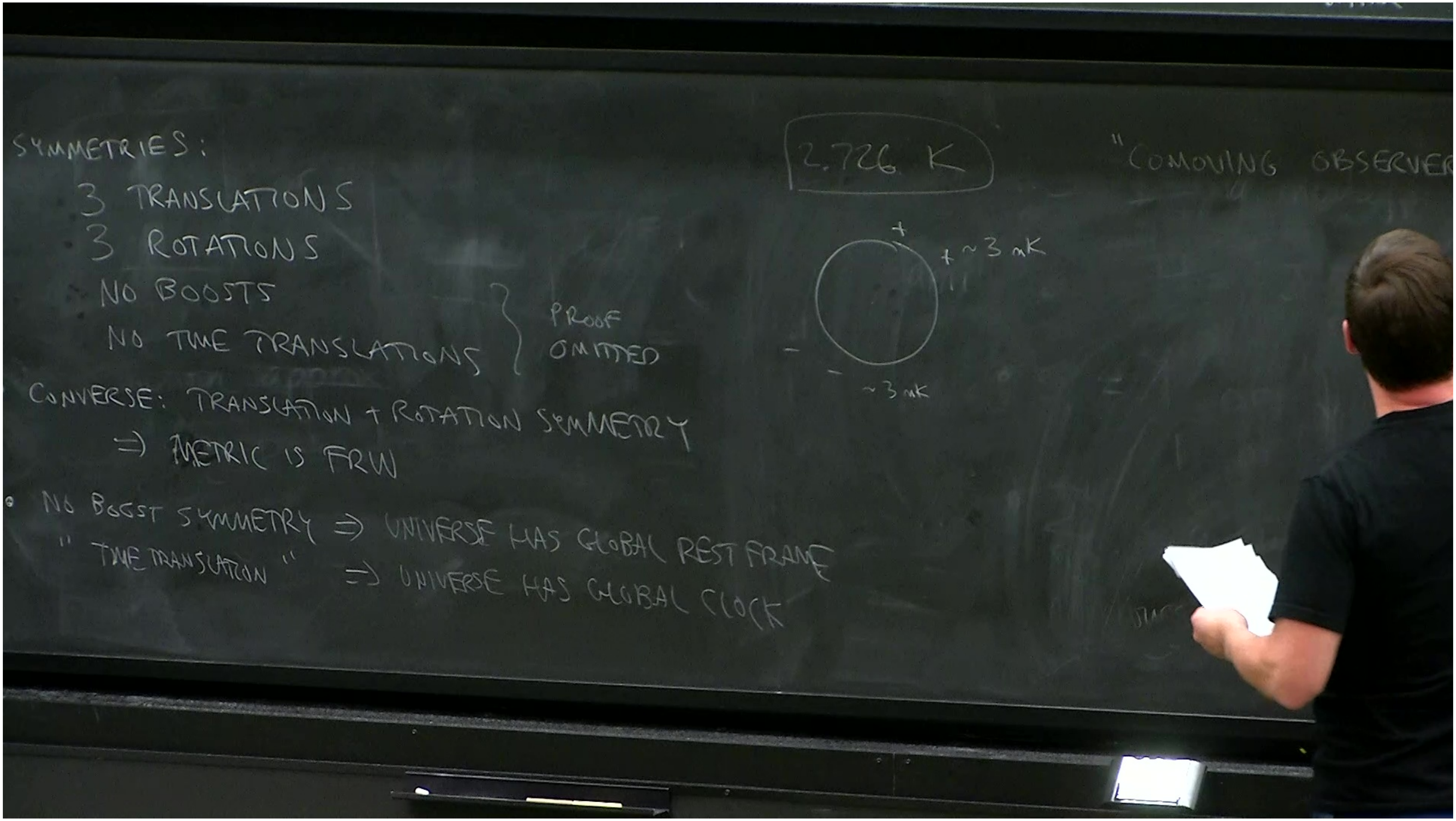
} PROOF OMITTED

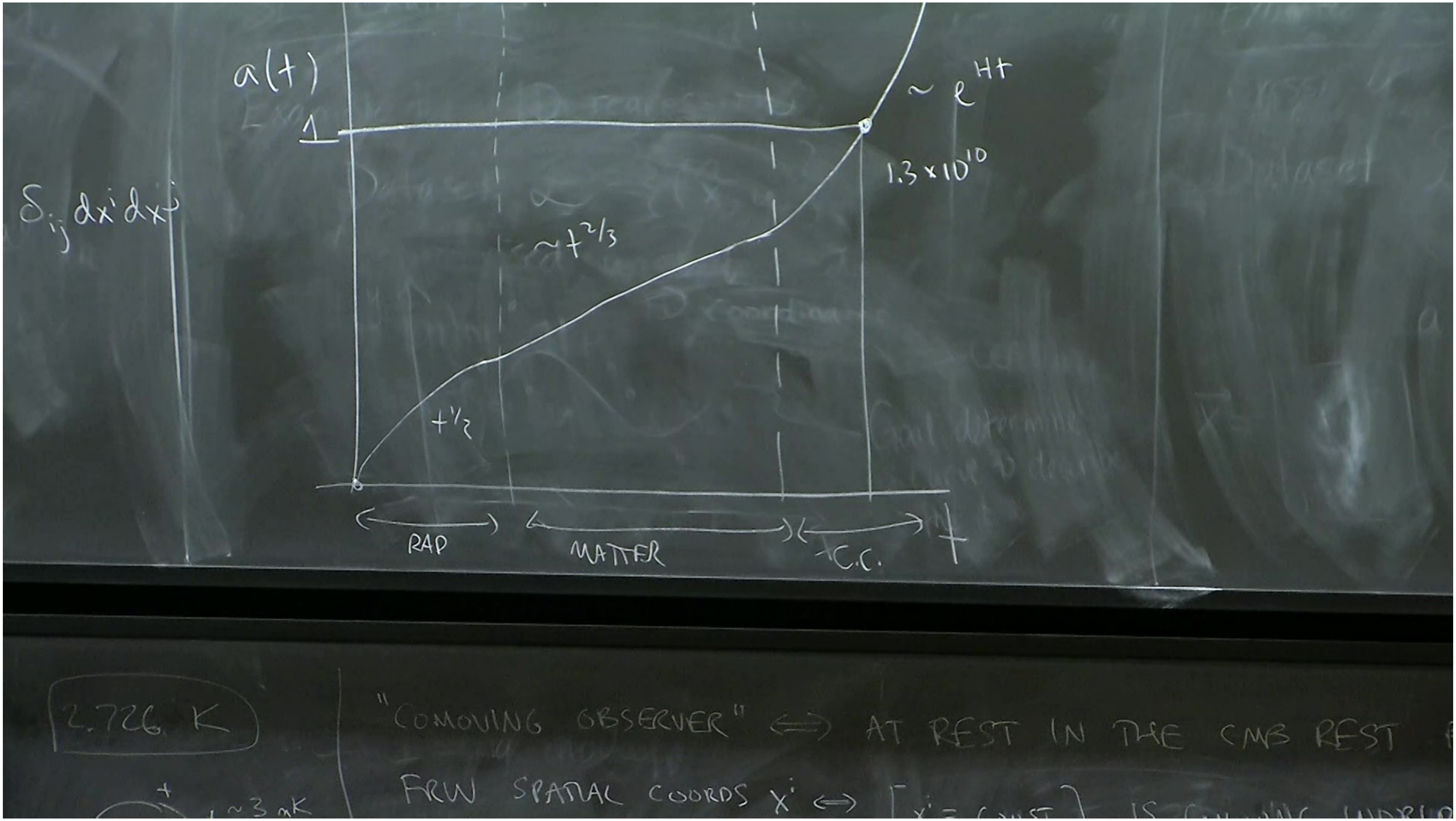
2.726 K

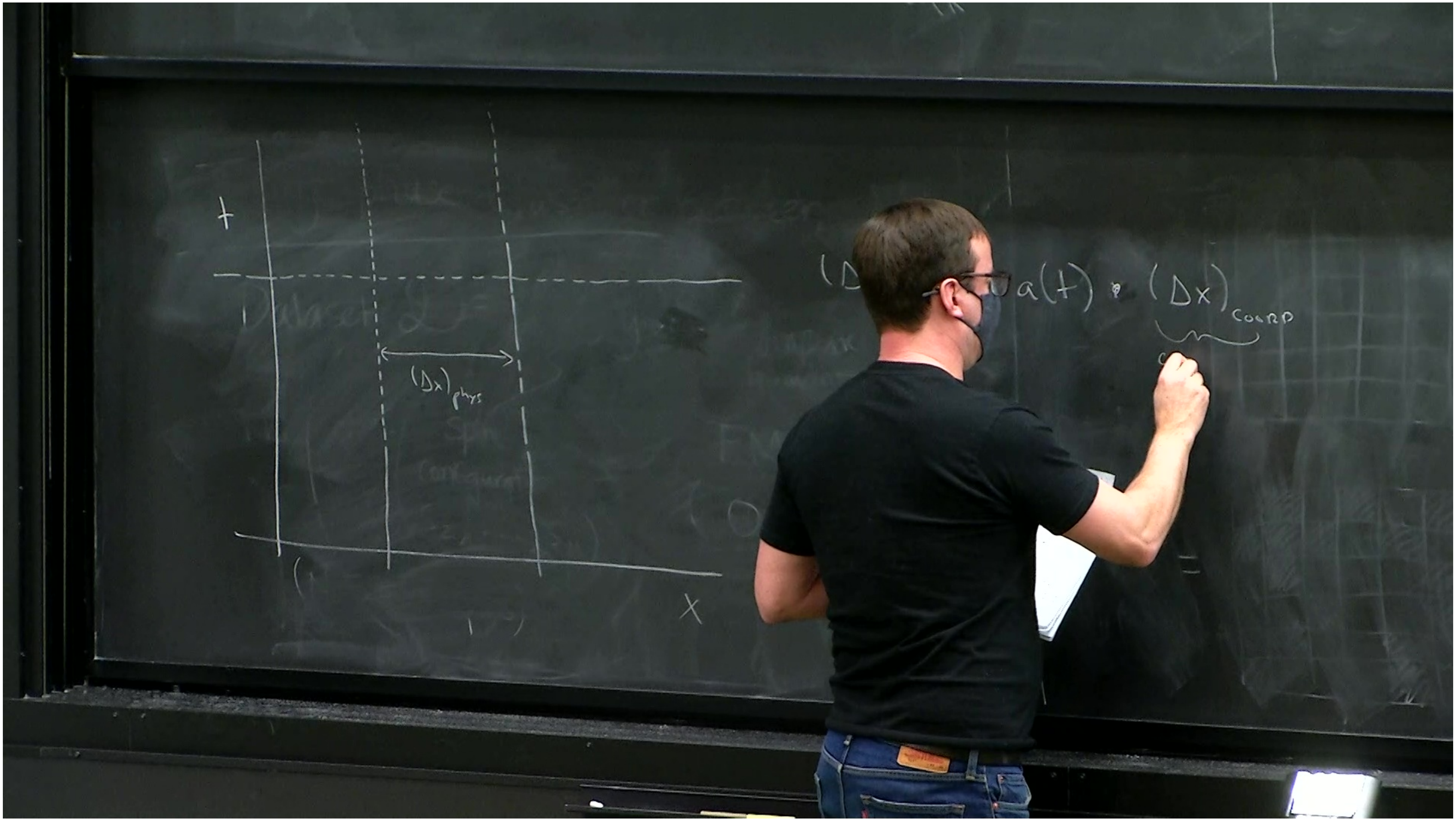
CONVERSE: TRANSLATION + ROTATION SYMMETRY

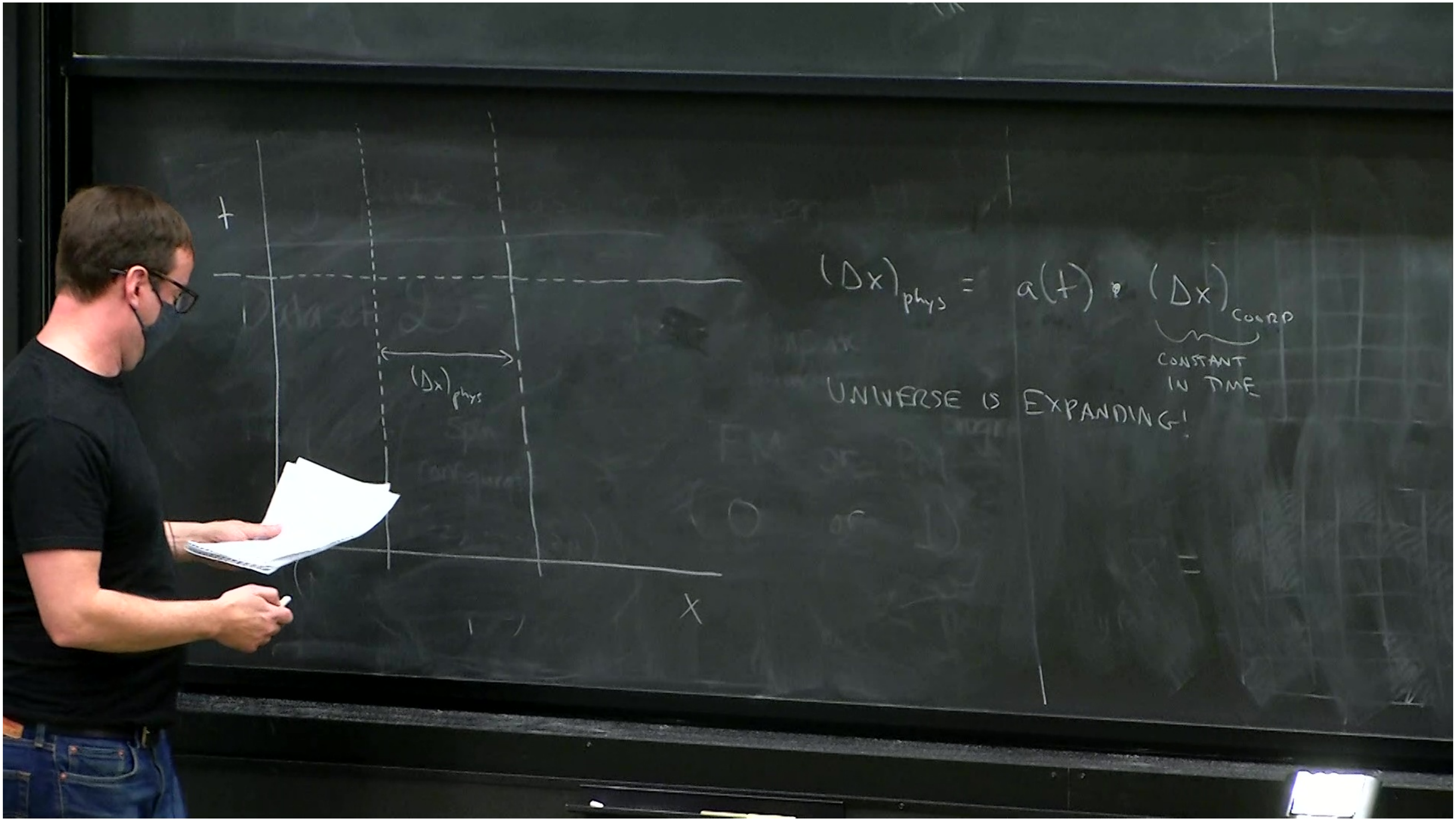
$\Rightarrow$ METRIC IS FRW

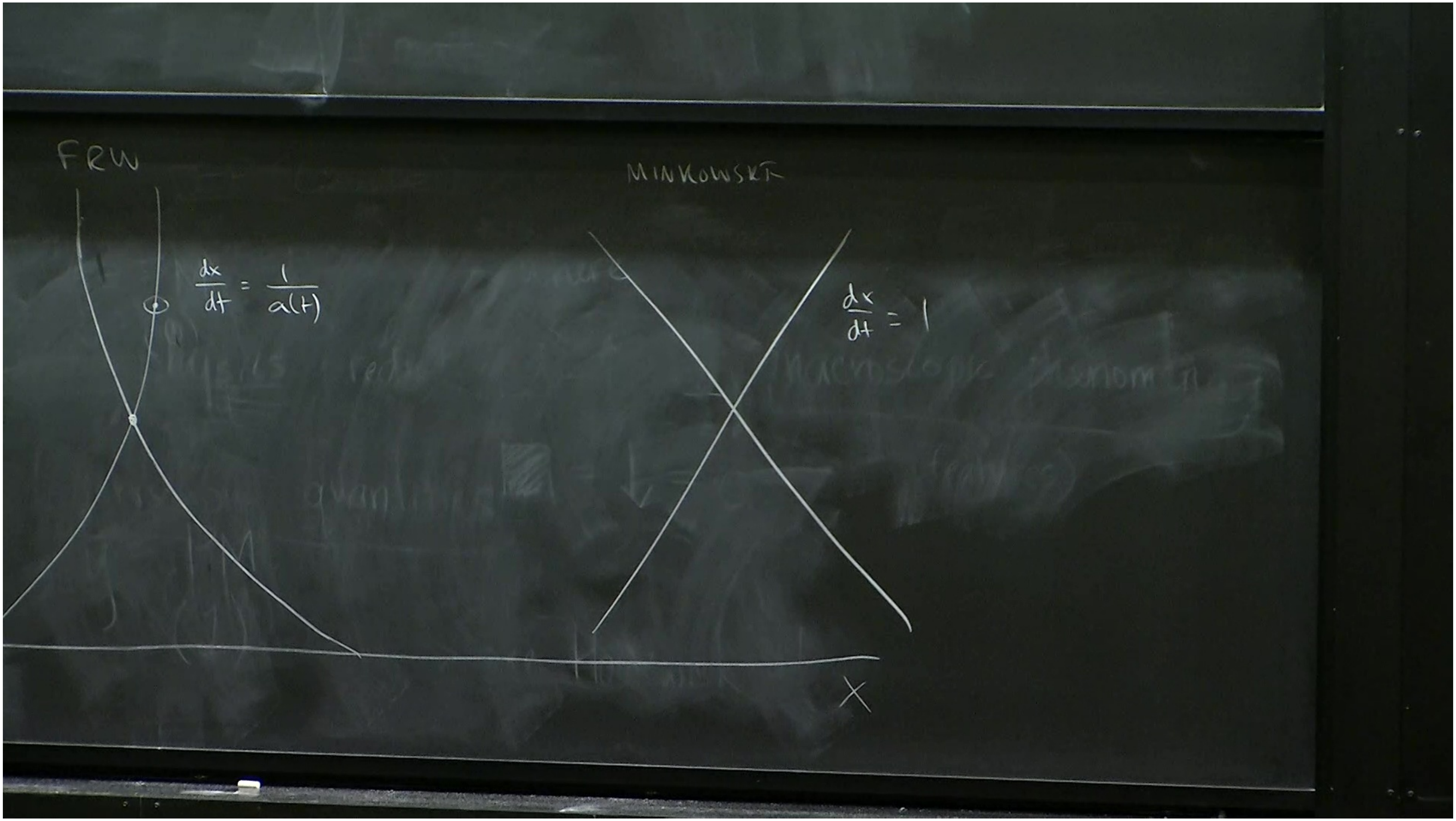
- NO BOOST SYMMETRY $\Rightarrow$ UNIVERSE HAS GLOBAL REST FRAME
- " TIME TRANSLATION " $\Rightarrow$ UNIVERSE HAS GLOBAL CLOCK

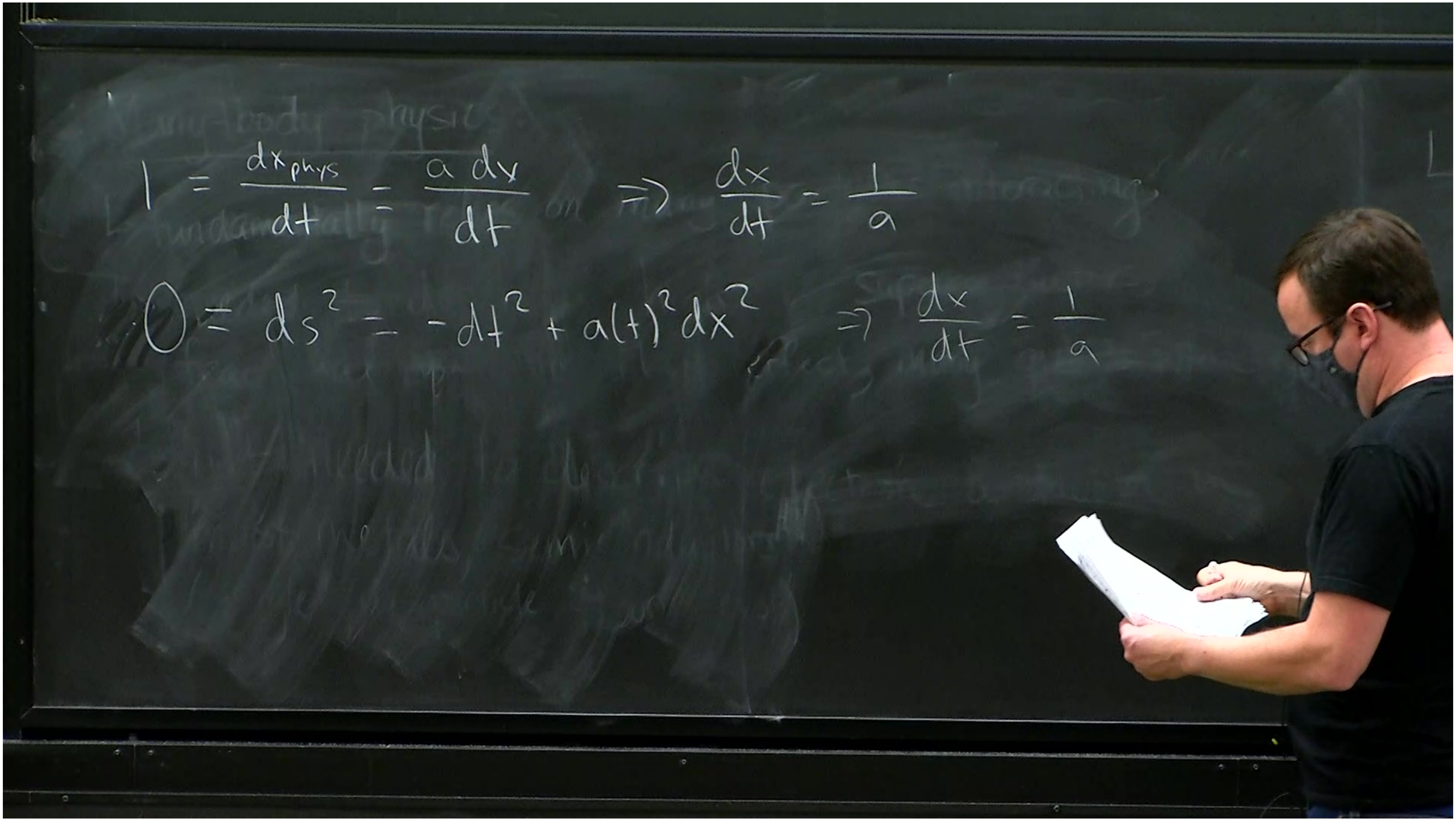

SYMMETRIES:
3 TRANSLATIONS
3 ROTATIONS
NO BOOSTS
NO TIME TRANSLATIONS
PROOF OMITTED
CONVERSE: TRANSLATION + ROTATION SYMMETRY
⇒ METRIC IS FRW
NO BOOST SYMMETRY ⇒ UNIVERSE HAS GLOBAL REST FRAME
" TIME TRANSLATION " ⇒ UNIVERSE HAS GLOBAL CLOCK
2.726 K
+ ~3 mK
~3 mK
"COMOVING OBSERVER

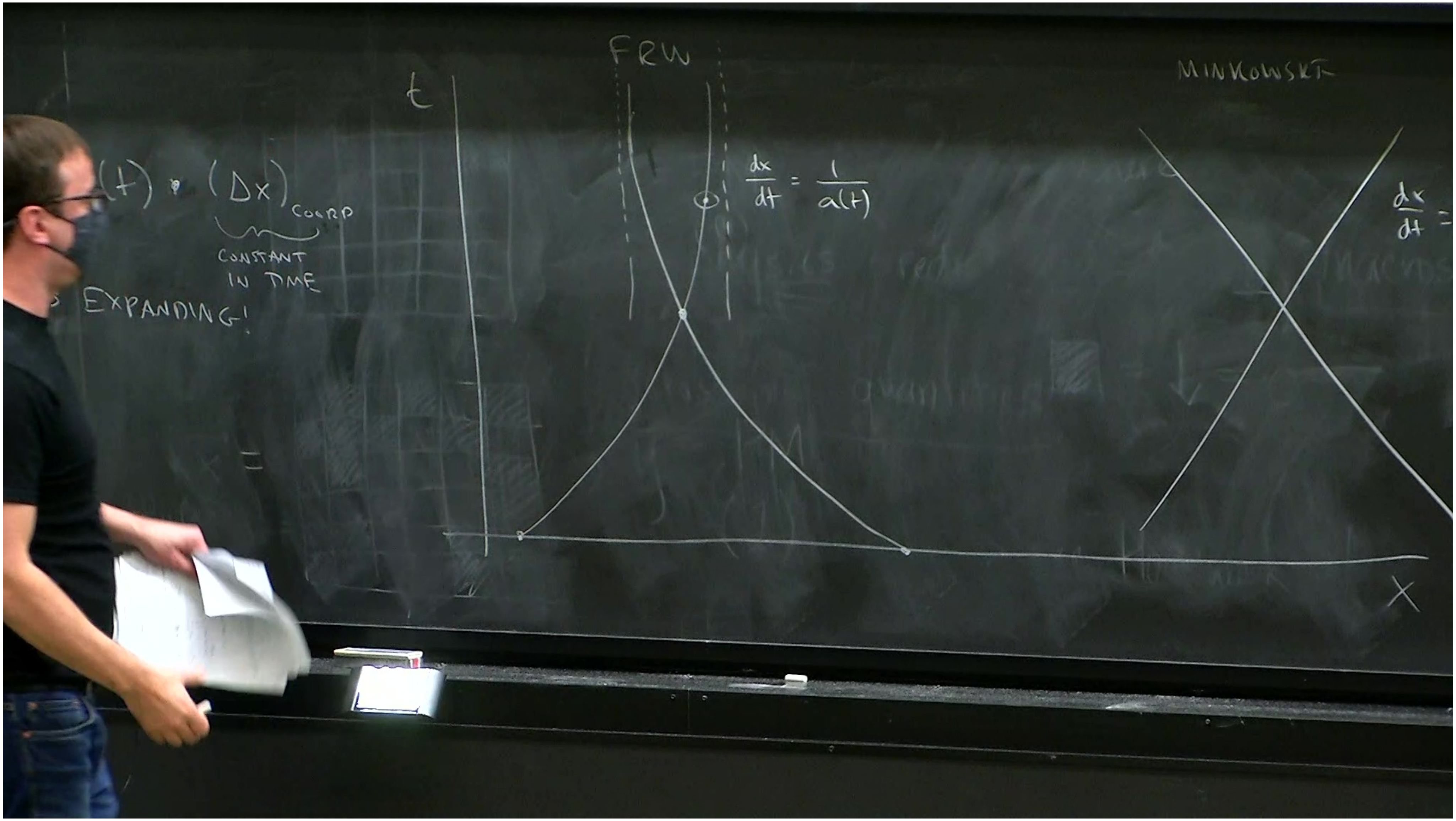

a(t)
1
~ e^Ht
1.3 x 10^10
δij dx^i dx^j
~ t^2/3
t^1/2
RAD
MATTER
C.C.
t
2.726 K
"COMOVING OBSERVER" ⟺ AT REST IN THE CMB REST
FRW SPATIAL COORDS x^i ⟺


t
(Δx) phys
X
a(t)
(Δx) coord

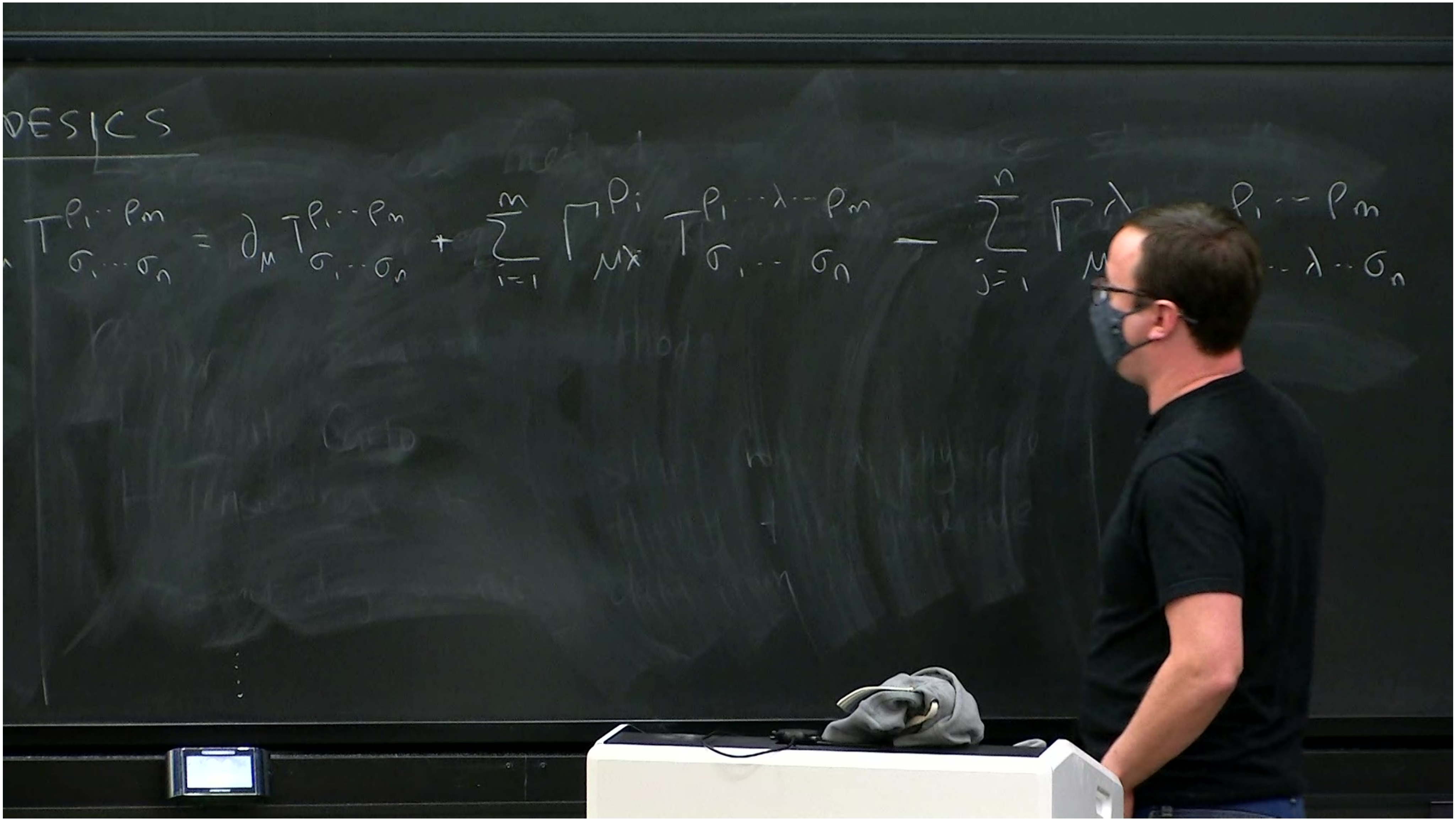

t
(Δx)phys
x
(Δx)phys = a(t) · (Δx)COORD
CONSTANT IN TIME
UNIVERSE IS EXPANDING!

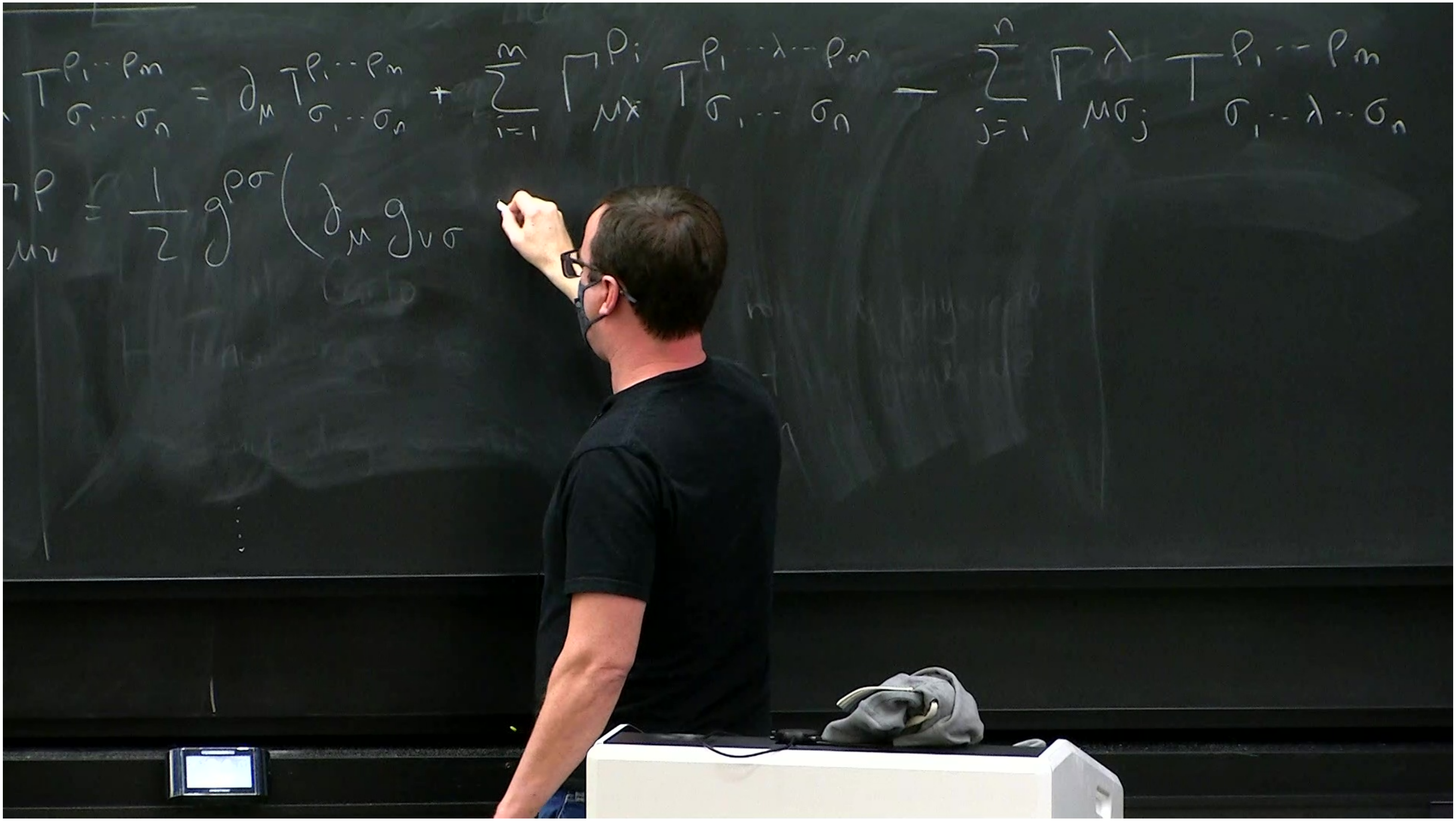

FRW
$\frac{dx}{dt} = \frac{1}{a(t)}$
MINKOWSKI
$\frac{dx}{dt} = 1$
x


$1 = \frac{dx_{phys}}{dt} = \frac{a\,dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{a}$
$0 = ds^2 = -dt^2 + a(t)^2 dx^2 \Rightarrow \frac{dx}{dt} = \frac{1}{a}$

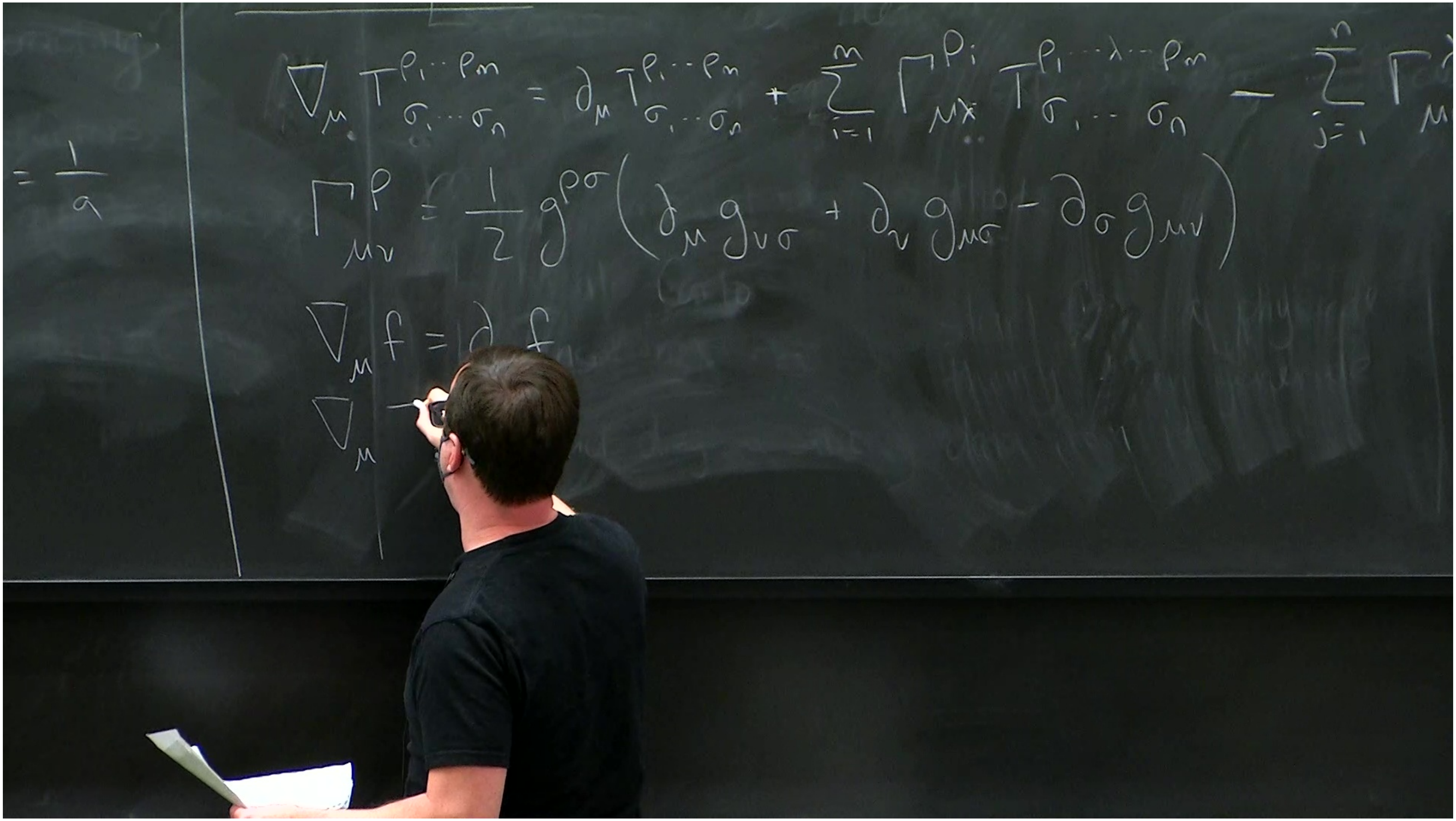


$(t)\cdot(\Delta x)_{COORD}$
CONSTANT IN TIME
EXPANDING!
FRW
t
$\frac{dx}{dt} = \frac{1}{a(t)}$
MINKOWSKI
$\frac{dx}{dt} =$
x


DESICS
$T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} - \sum_{j=1}^{n} \Gamma^{\lambda} \ \ {}^{\rho_1 \cdots \rho_m}_{\cdots \lambda \cdots \sigma_n}$



$T^{\rho_1\cdots\rho_m}_{\sigma_1\cdots\sigma_n} = \partial_\mu T^{\rho_1\cdots\rho_m}_{\sigma_1\cdots\sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1\cdots\lambda\cdots\rho_m}_{\sigma_1\cdots\sigma_n} - \sum_{j=1}^{n} \Gamma^{\lambda}_{\mu\sigma_j} T^{\rho_1\cdots\rho_m}_{\sigma_1\cdots\lambda\cdots\sigma_n}$
$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}(\partial_\mu g_{\nu\sigma}$


$\nabla_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} - \sum_{j=1}^{n} \Gamma$
$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$
$\nabla_\mu f = \partial_\mu f$
∇_μ
$= \frac{1}{a}$

$$\nabla_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}_{\sigma_1 \cdots \sigma_n} - \sum_{j=1}^{n} \Gamma^{\lambda}_{\mu\sigma_j} T$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$$

$$\nabla_\mu f = \partial_\mu f$$

$$\nabla_\mu \nabla_\nu f = \nabla_\nu \nabla_\mu f$$ "TORSION FREE"

GEODESICS

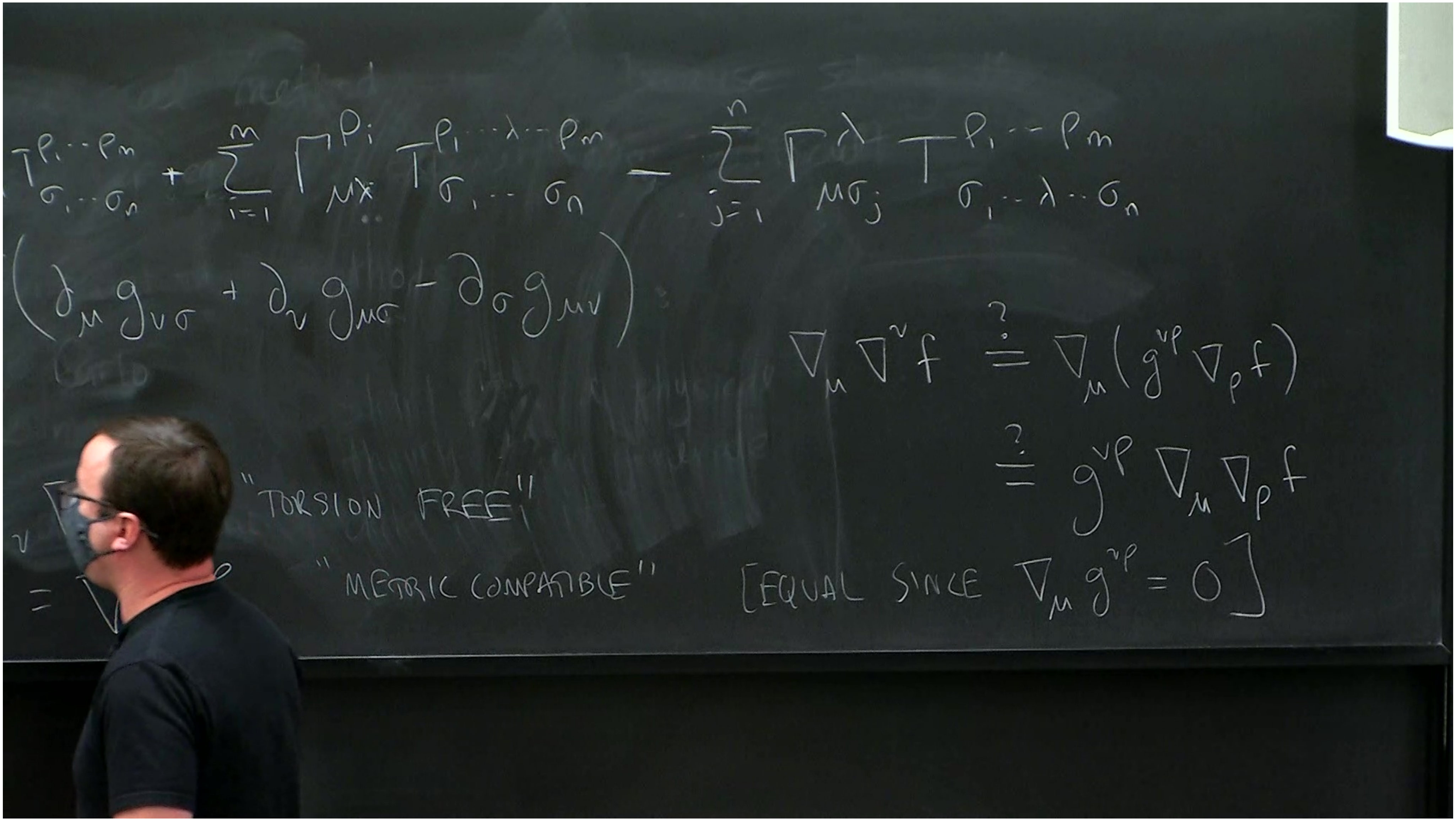
$$\nabla_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} = \partial_\mu T^{\rho_1 \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} + \sum_{i=1}^{m} \Gamma^{\rho_i}_{\mu \lambda} T^{\rho_1 \cdots \lambda \cdots \rho_m}{}_{\sigma_1 \cdots \sigma_n} - \sum_{j=1}^{n} \Gamma^{\lambda}_{\mu \sigma_j} T^{\rho_1 \cdots}{}_{\sigma_1 \cdots \lambda}$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right)$$

$$\nabla_\mu f = \partial_\mu f$$

$$\nabla_\mu \nabla_\nu f = \nabla_\nu \nabla_\mu f$$ "TORSION FREE"

$$\nabla_\mu g_{\nu\rho} = 0 = \nabla_\mu g^{\nu\rho}$$ "METRIC COMPATIBLE"



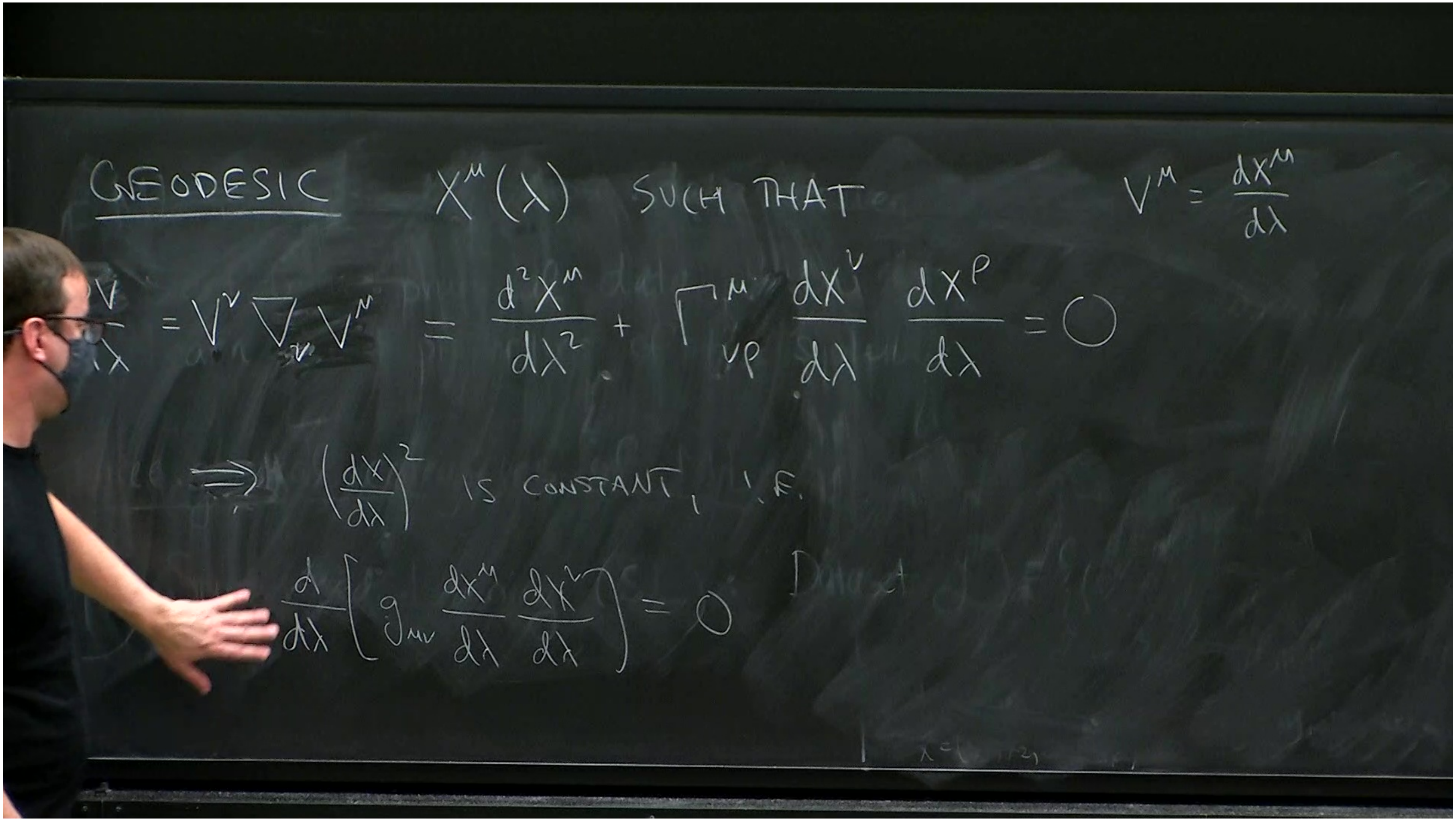
$T^{\rho_1\cdots\rho_n}_{\sigma_1\cdots\sigma_n} + \sum_{i=1}^{n} \Gamma^{\rho_i}_{\mu\lambda} T^{\rho_1\cdots\lambda\cdots\rho_n}_{\sigma_1\cdots\sigma_n} - \sum_{j=1}^{n} \Gamma^{\lambda}_{\mu\sigma_j} T^{\rho_1\cdots\rho_n}_{\sigma_1\cdots\lambda\cdots\sigma_n}$
$(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})$
$\nabla_\mu \nabla^\nu f \stackrel{?}{=} \nabla_\mu (g^{\nu\rho} \nabla_\rho f)$
$\stackrel{?}{=} g^{\nu\rho} \nabla_\mu \nabla_\rho f$
"TORSION FREE"
"METRIC COMPATIBLE"
[EQUAL SINCE $\nabla_\mu g^{\nu\rho} = 0$]

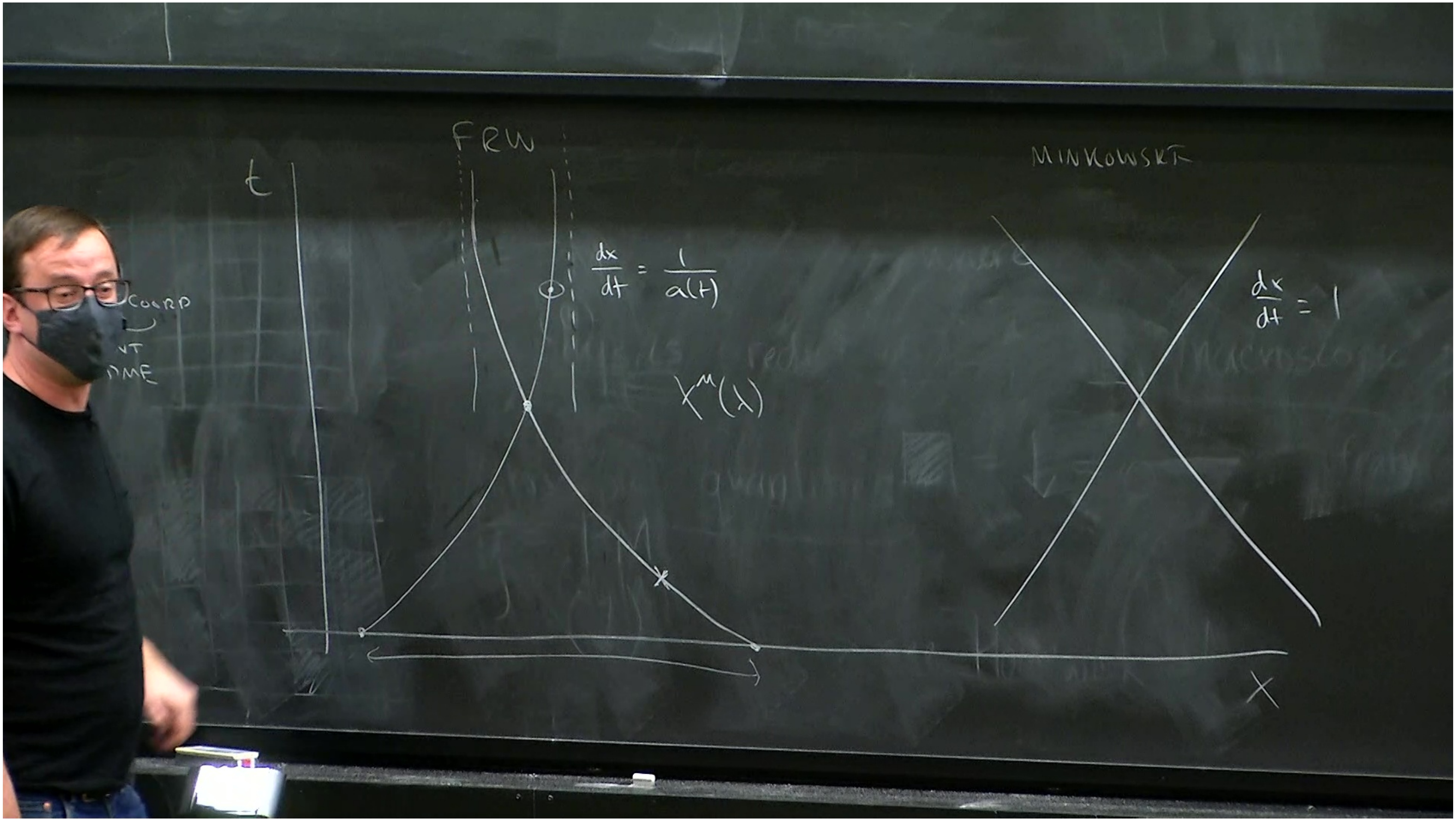
GEODESIC $x^\mu(\lambda)$ SUCH THAT $\qquad V^\mu = \frac{dx^\mu}{d\lambda}$

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0$$

GEODESIC $X^\mu(\lambda)$ SUCH THAT $\qquad V^\mu = \dfrac{dx^\mu}{d\lambda}$

$$\frac{DV}{d\lambda} = V^\nu \nabla_\nu V^\mu = \frac{d^2X^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dX^\nu}{d\lambda}\frac{dX^\rho}{d\lambda} = 0$$


GEODESIC $x^\mu(\lambda)$ SUCH THAT
$V^\mu = \frac{dx^\mu}{d\lambda}$
$= V^\nu \nabla_\nu V^\mu = \frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0$
$\Rightarrow \left(\frac{dx}{d\lambda}\right)^2$ IS CONSTANT, I.E.
$\frac{d}{d\lambda}\left[g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\right] = 0$


FRW
MINKOWSKI
t
dx/dt = 1/a(t)
x^μ(λ)
dx/dt = 1
x

$\frac{dx^\mu}{d\lambda}$

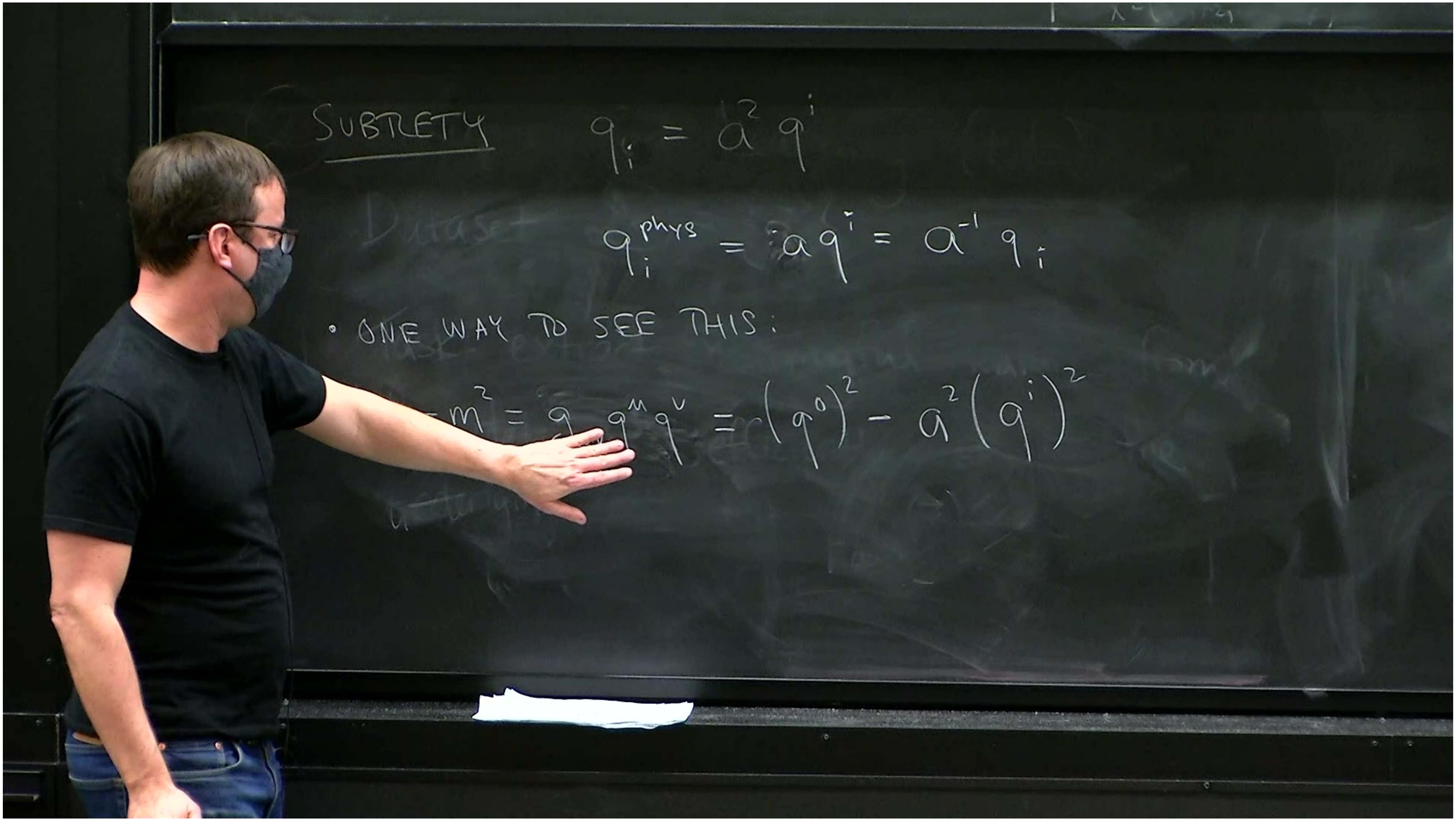
<u>SETUP</u>: PARTICLE WITH MASS $m \geq 0$

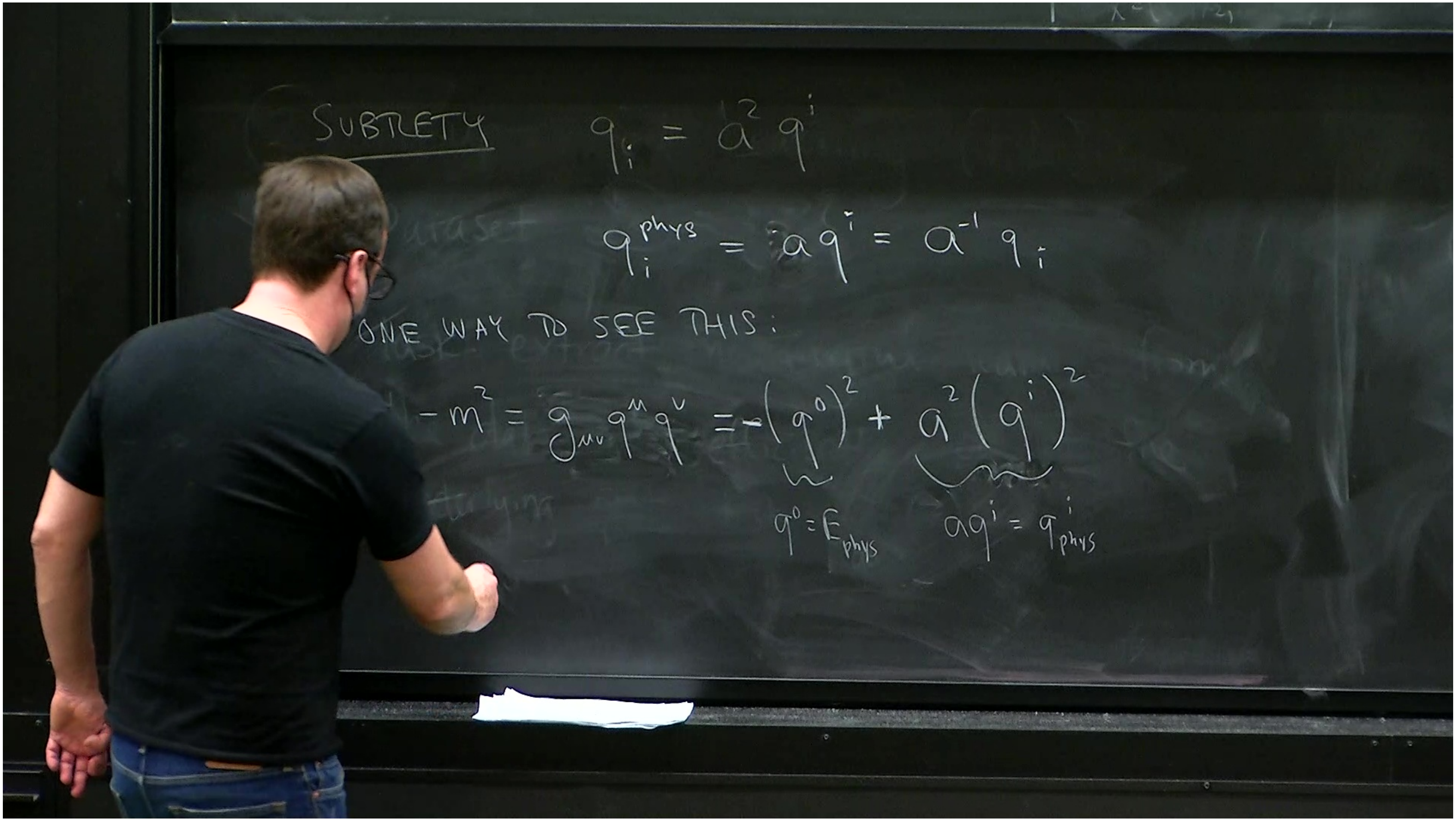
CHOOSE AFFINE PARAMETER λ SO THAT

$$\frac{dx^\mu}{d\lambda} = q^\mu : \left[\begin{array}{c}\text{COVARIANT}\\ \text{4-MOMENTUM OF PARTICLE}\end{array}\right]$$

WHERE $q^\mu q_\mu = -m^2$

NOTE FOR A MASSIVE PARTICLE, $\lambda = \tau/m$.


SUBTLETY
$q_i = a^2 q^i$
$q_i^{phys} = a q^i = a^{-1} q_i$
• ONE WAY TO SEE THIS:
$-m^2 = g \; q^\mu q^\nu = (q^0)^2 - a^2 (q^i)^2$


SUBTLETY
$q_i = a^2 q^i$
$q_i^{phys} = a q^i = a^{-1} q_i$
ONE WAY TO SEE THIS:
$-m^2 = g_{\mu\nu} q^\mu q^\nu = -(q^0)^2 + a^2 (q^i)^2$
$q^0 = E_{phys}$
$a q^i = q^i_{phys}$

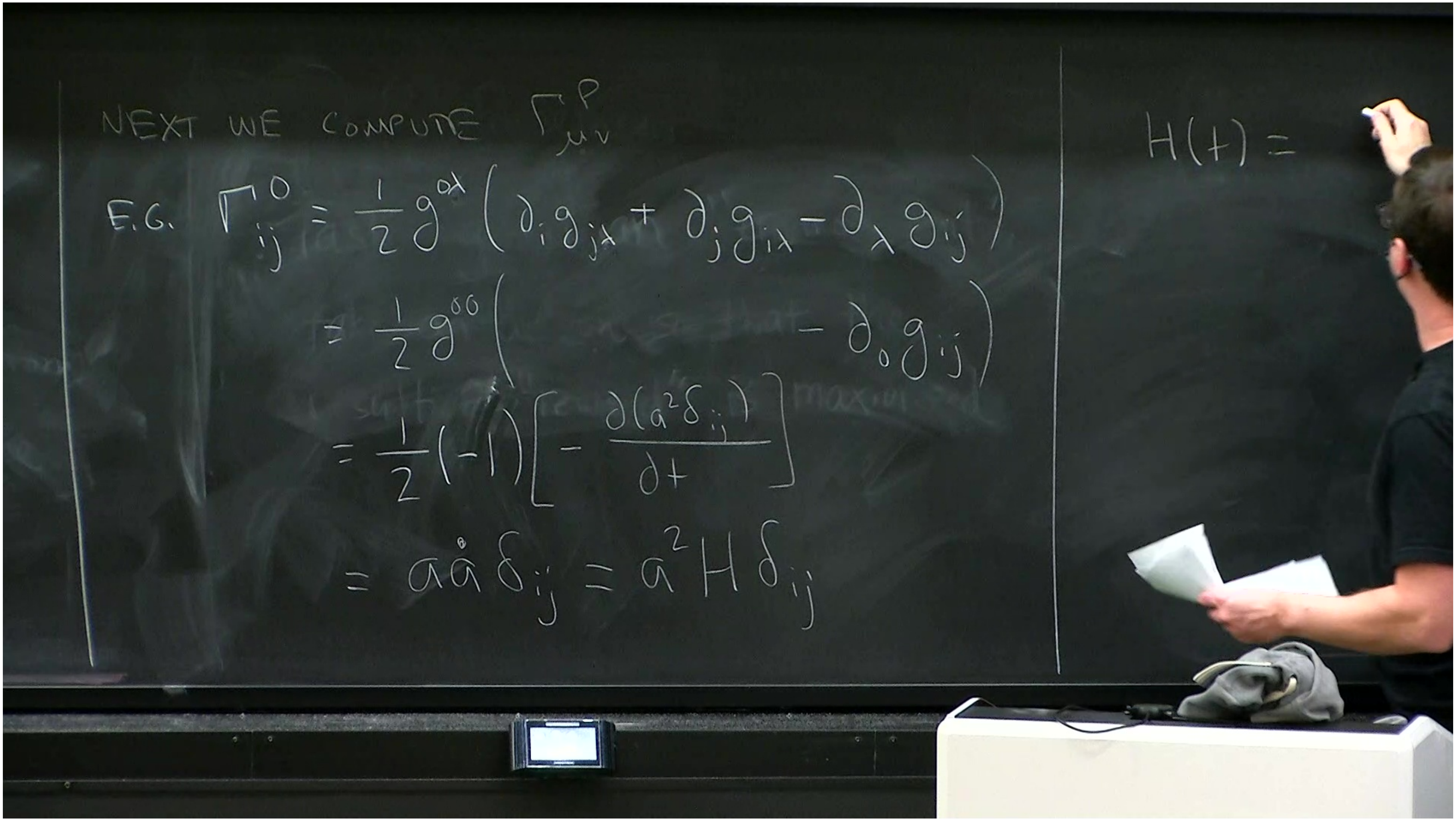
SUBTLETY $\quad q_i = a^2 q^i$

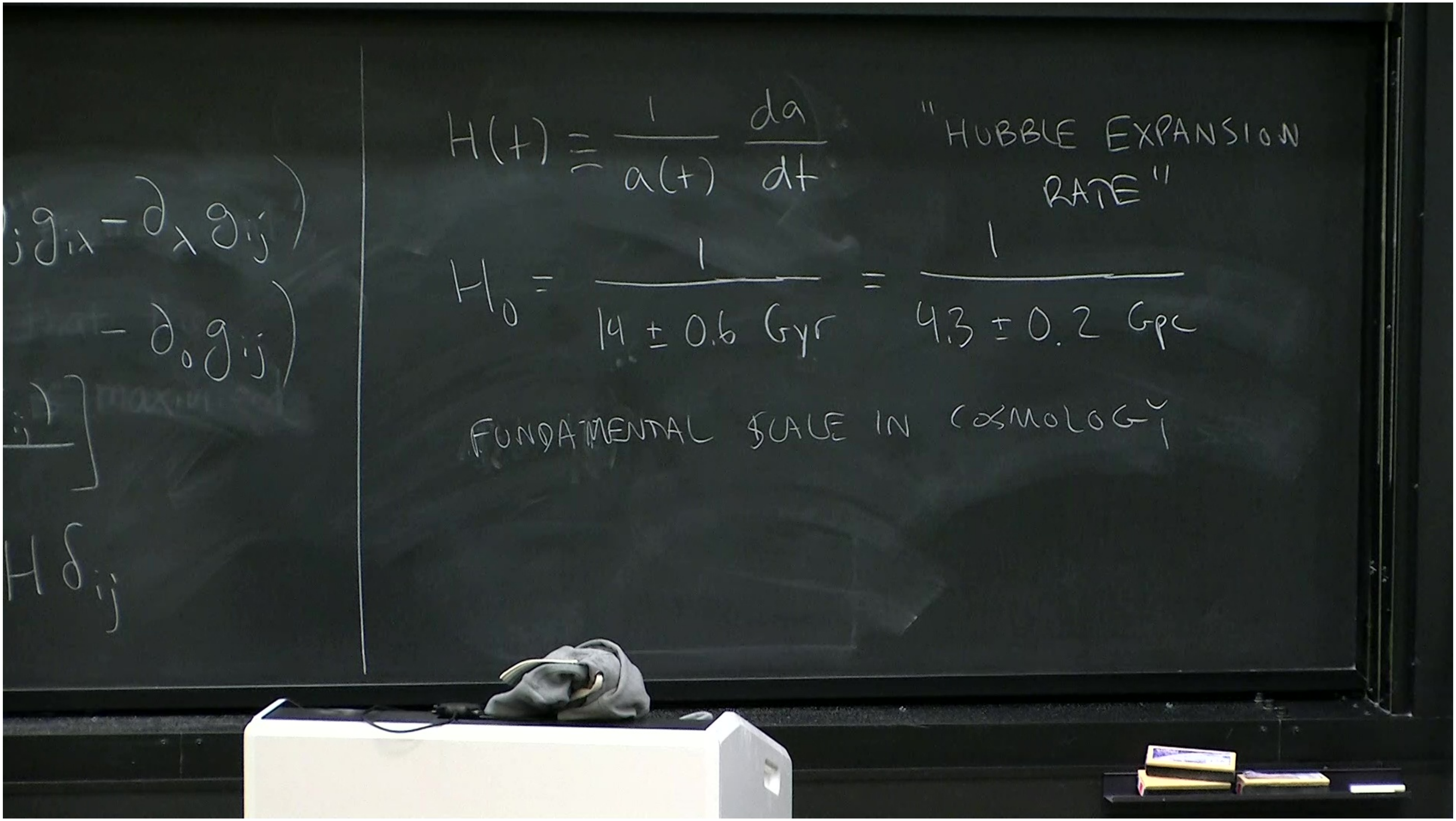
$$q_i^{phys} = a q^i = a^{-1} q_i$$

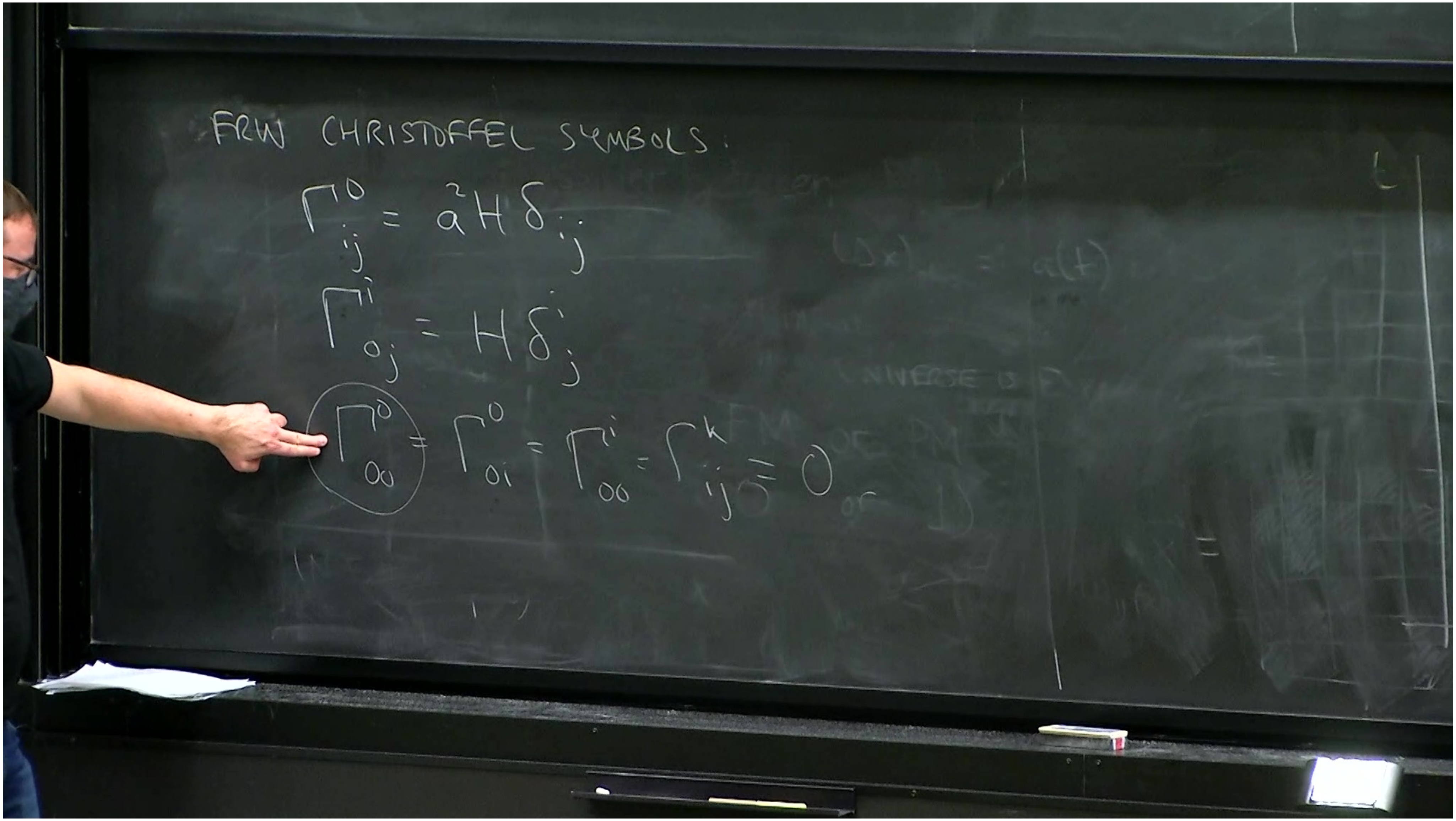
- ONE WAY TO SEE THIS:

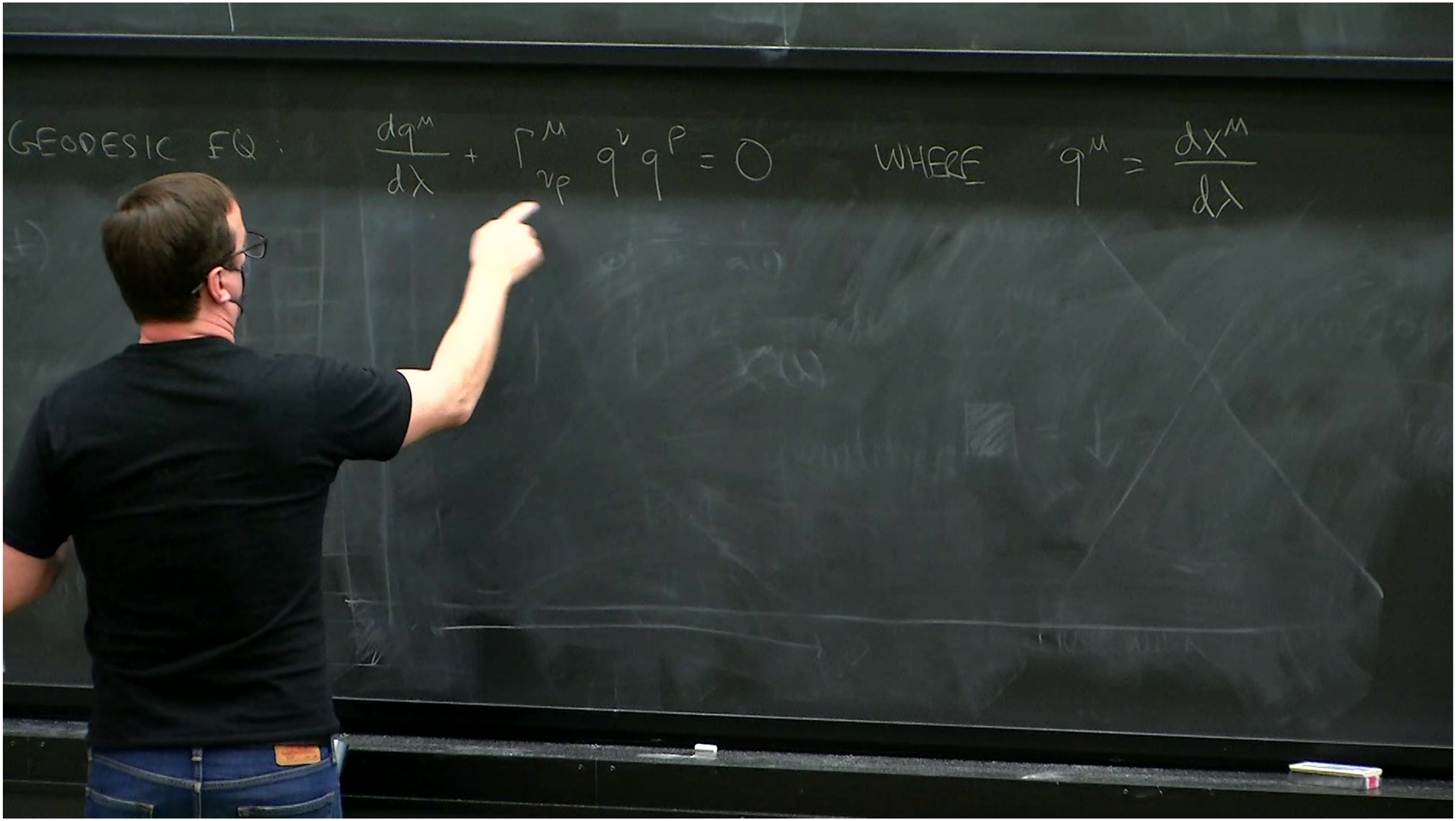
$$-m^2 = g_{\mu\nu} q^\mu q^\nu = -\underbrace{(q^0)^2}_{q^0 = E_{phys}} + \underbrace{a^2 (q^i)^2}_{a q^i = q^i_{phys}}$$

$$\Rightarrow q^i_{phys} = a q^i = a^{-1} q_i$$


NEXT WE COMPUTE $\Gamma^{\rho}_{\mu\nu}$
E.G. $\Gamma^{0}_{ij} = \frac{1}{2} g^{0\lambda} \left(\partial_i g_{j\lambda} + \partial_j g_{i\lambda} - \partial_\lambda g_{ij} \right)$
$= \frac{1}{2} g^{00} \left(- \partial_0 g_{ij} \right)$
$= \frac{1}{2}(-1)\left[- \frac{\partial (a^2 \delta_{ij})}{\partial t} \right]$
$= a \dot{a} \delta_{ij} = a^2 H \delta_{ij}$
$H(t) =$


$H(t) \equiv \frac{1}{a(t)} \frac{da}{dt}$
"HUBBLE EXPANSION RATE"
$H_0 = \frac{1}{14 \pm 0.6 \text{ Gyr}} = \frac{1}{4.3 \pm 0.2 \text{ Gpc}}$
FUNDAMENTAL SCALE IN COSMOLOGY
$\partial_j g_{i\lambda} - \partial_\lambda g_{ij}$
$- \partial_0 g_{ij}$
$H \delta_{ij}$


FRW CHRISTOFFEL SYMBOLS:
$\Gamma^0_{ij} = a^2 H \delta_{ij}$
$\Gamma^i_{0j} = H \delta^i_j$
$\Gamma^0_{00} = \Gamma^0_{0i} = \Gamma^i_{00} = \Gamma^k_{ij} = 0$


GEODESIC EQ: $\frac{dq^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho} q^\nu q^\rho = 0$
WHERE $q^\mu = \frac{dx^\mu}{d\lambda}$

GEODESIC EQ: $\frac{dq^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho} q^\nu q^\rho = 0$ WHERE $q^\mu = \frac{dx^\mu}{d\lambda}$

$$\Rightarrow \frac{dq^0}{d\lambda} + a^2 H \delta_{ij} q^i q^j = 0$$

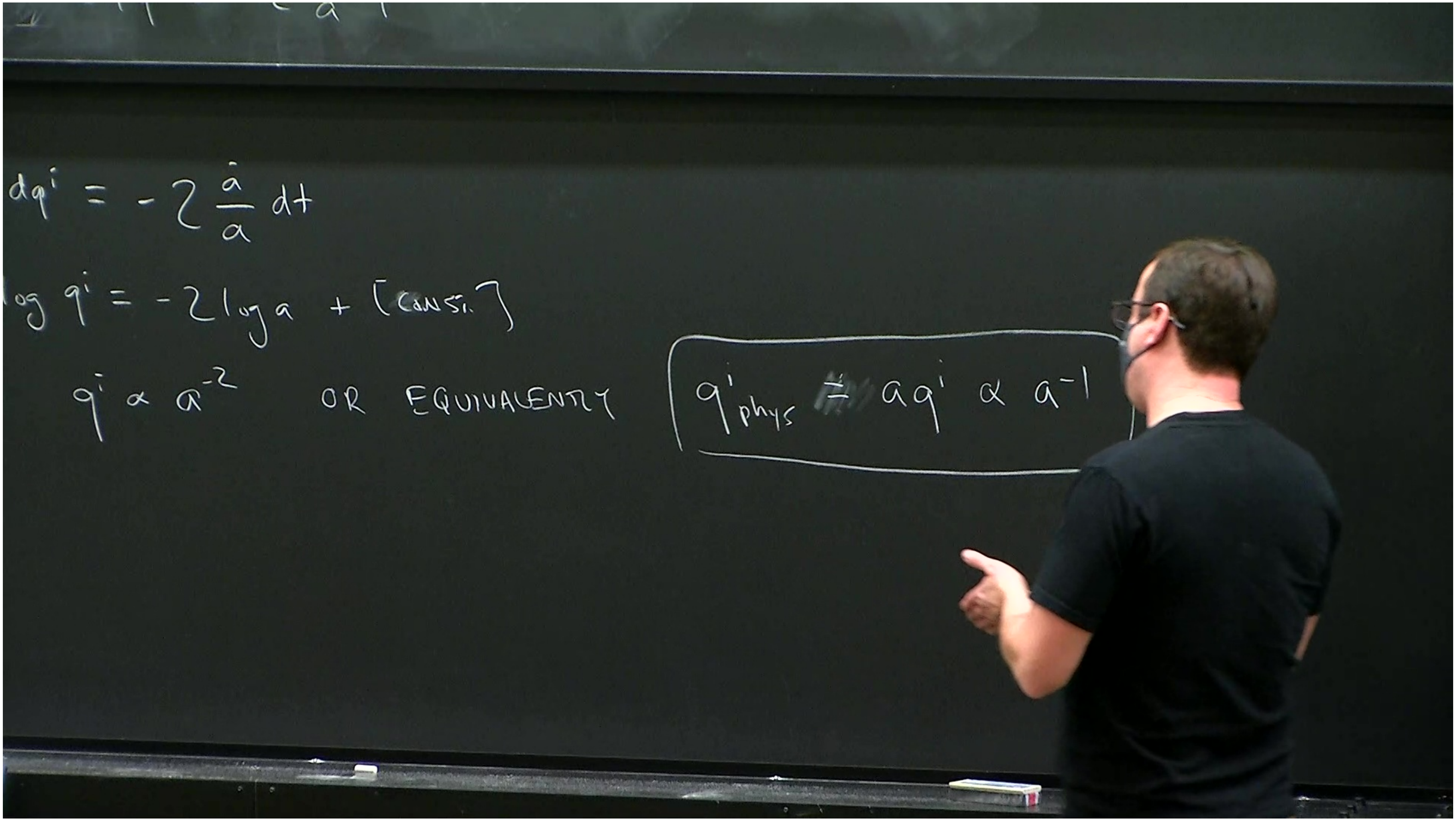
$$\frac{dq^i}{d\lambda} + 2H q^0 q^i = 0$$

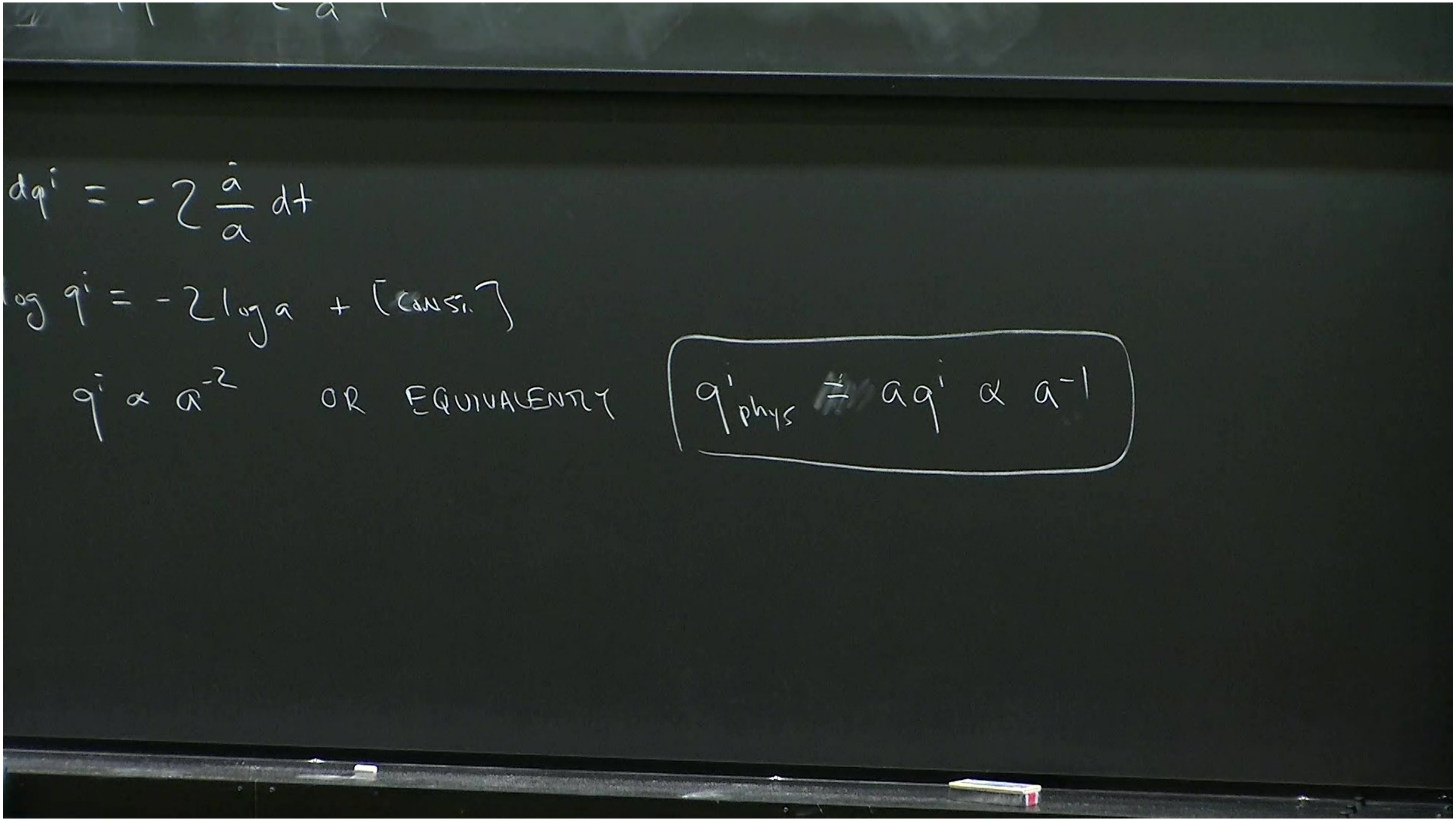
$q^0 = \frac{dt}{d\lambda}$ INTO SECOND EQ:

$$\frac{1}{q^i}\,dq^i = -2\,\frac{\dot a}{a}\,dt$$

$$\log q^i = -2\log a + [\text{CONST.}]$$

$q^i \propto a^{-2}$ OR EQUIVALENTLY $q^i_{phys} = a q^i \propto a^{-1}$


$dq^i = -2\frac{\dot{a}}{a}dt$
$\log q^i = -2\log a + [\text{CONST.}]$
$q^i \propto a^{-2}$ OR EQUIVALENTLY
$\boxed{q^i_{phys} = a q^i \propto a^{-1}}$


$dq^i = -2\frac{\dot{a}}{a}\,dt$
$\log q^i = -2\log a + [\text{CONST.}]$
$q^i \propto a^{-2}$ OR EQUIVALENTLY $\boxed{q^i_{phys} = a q^i \propto a^{-1}}$