Title: Machine Learning (2021/2022)

Speakers: Lauren Hayward

Collection: Machine Learning (2021/2022)

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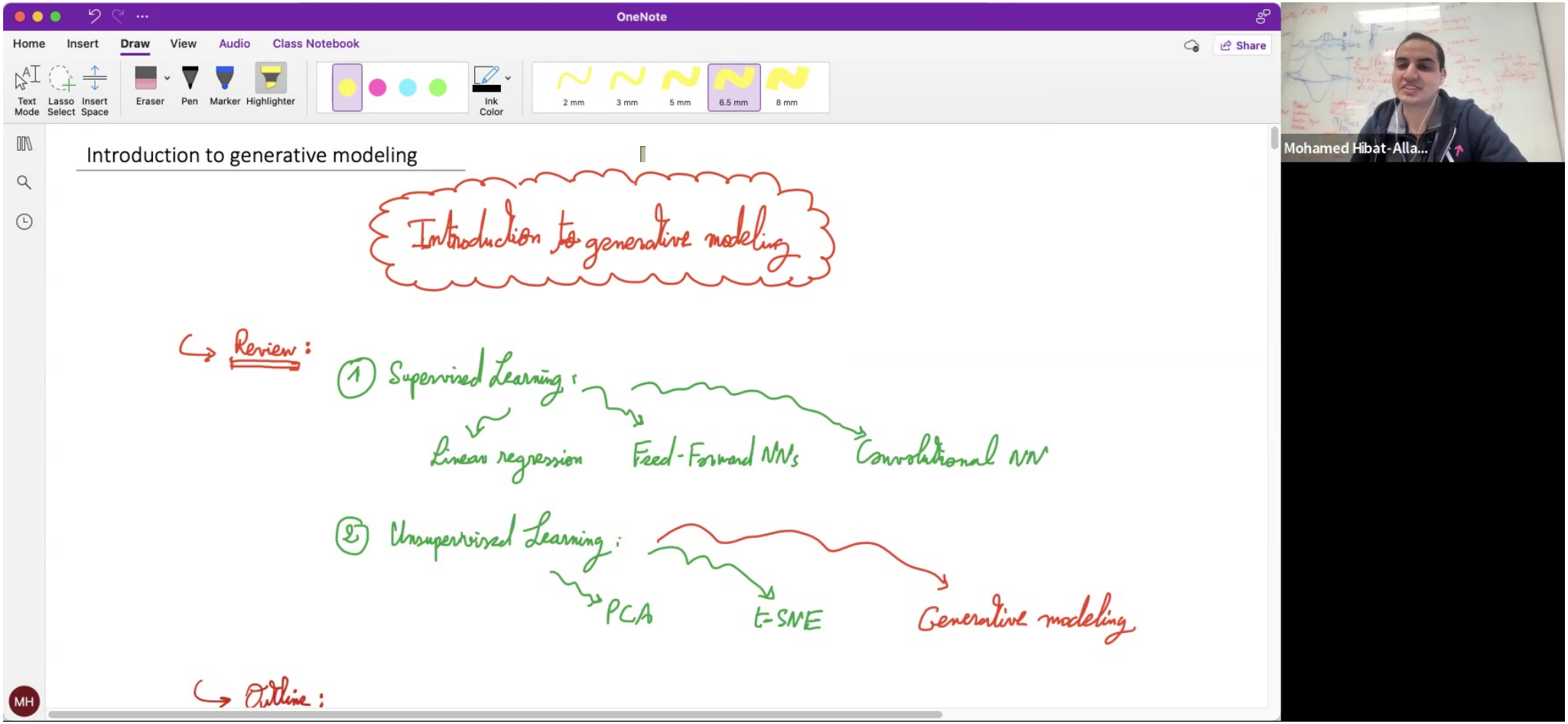
Abstract: This course is designed to introduce modern machine learning techniques for studying classical and quantum many-body problems encountered in condensed matter, quantum information, and related fields of physics. Lectures will focus on introducing machine learning algorithms and discussing how they can be applied to solve problem in statistical physics. Tutorials and homework assignments will concentrate on developing programming skills to study the problems presented in lecture.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Introduction to generative modeling
Introduction to generative modeling
Review:
1 Supervised Learning:
Linear regression
Feed-Forward NNs
Convolutional NN
2 Unsupervised Learning:
PCA
t-SNE
Generative modeling
Outline:
MH
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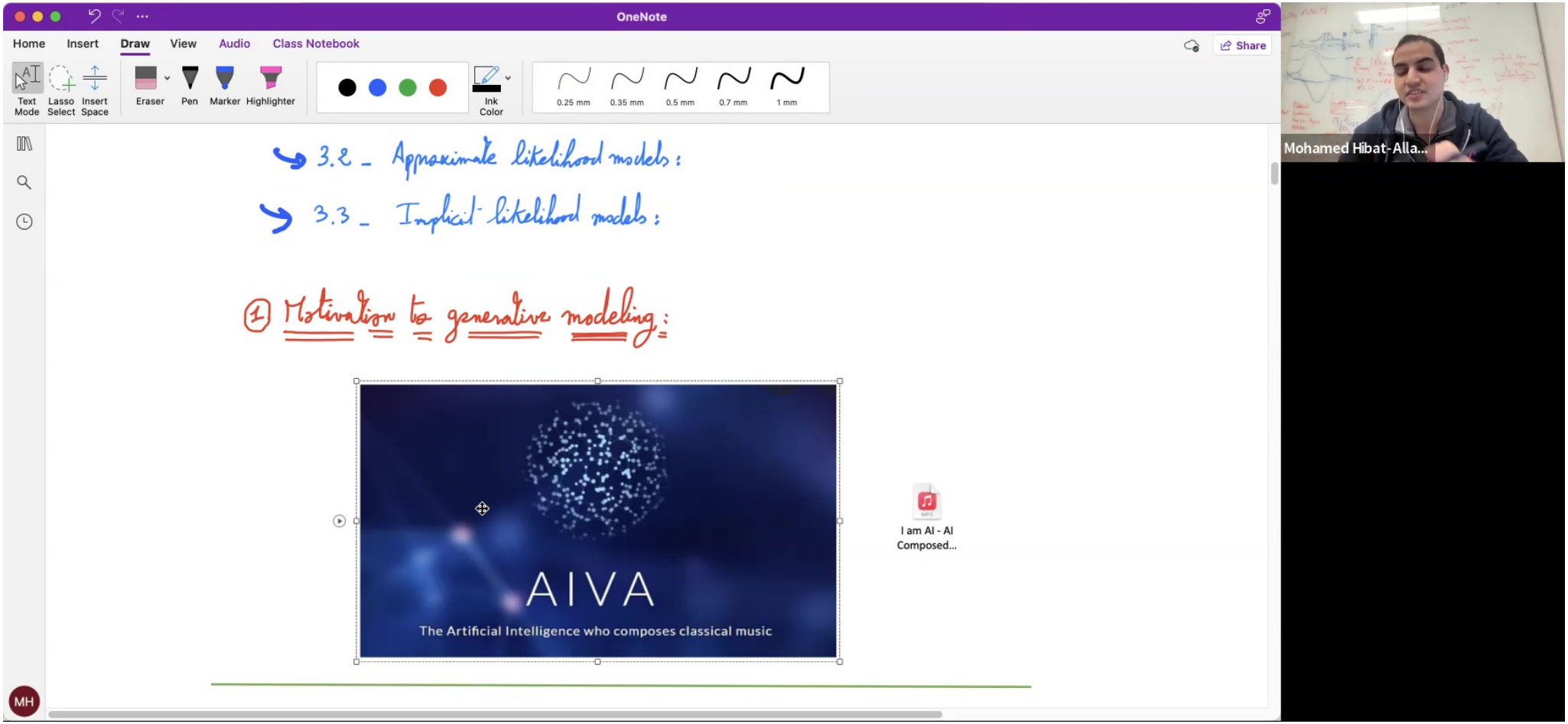
Outline:

1. Motivation for generative modeling:
2. Maximum likelihood Principle & KL divergence:
3. Classification of generative models
   - 3.1 - Exact likelihood models:
   - 3.2 - Approximate likelihood models:
   - 3.3 - Implicit likelihood models:

1) Motivation to generative modeling:

MH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3.2 _ Approximate likelihood models:
3.3 _ Implicit likelihood models:
1 Motivation to generative modeling:
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The Artificial Intelligence who composes classical music
I am AI - AI Composed...
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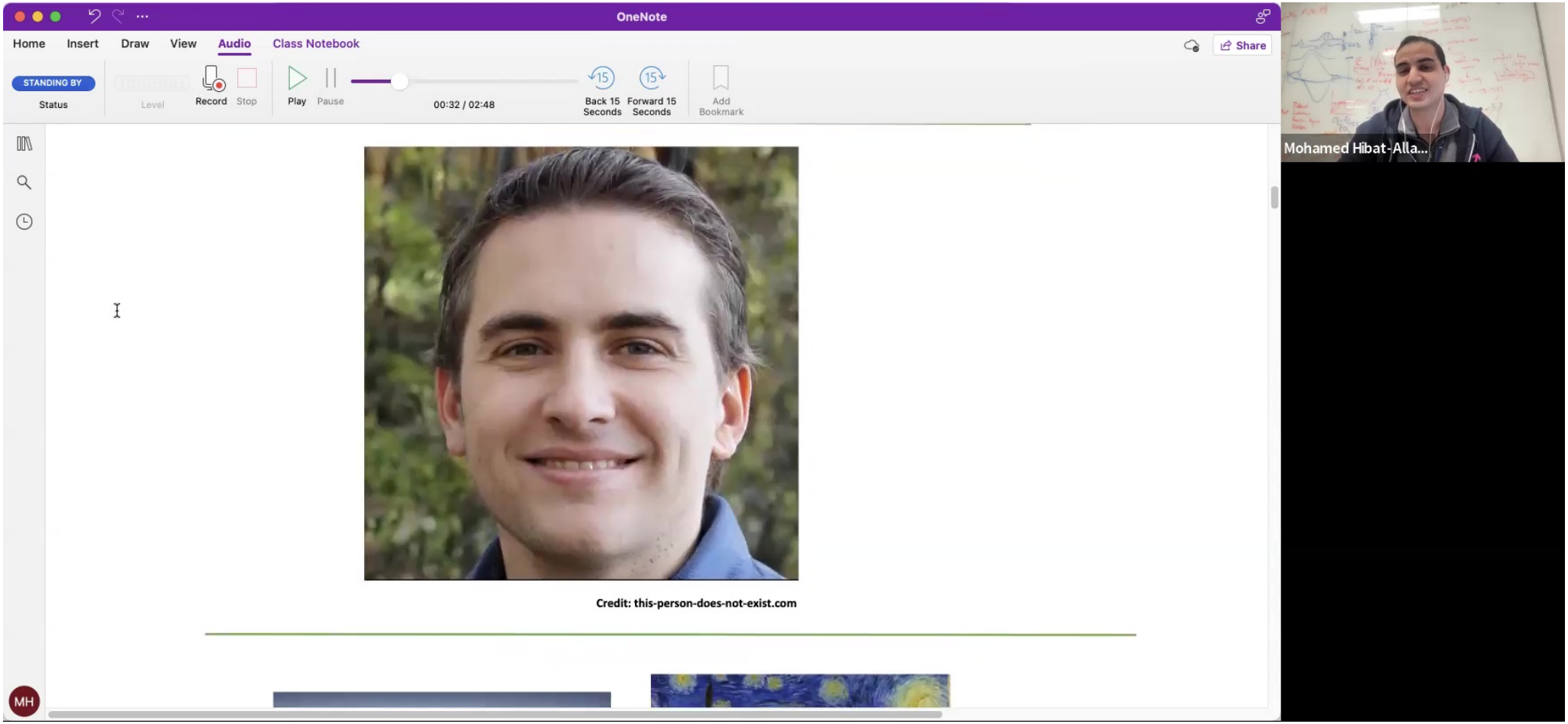
MH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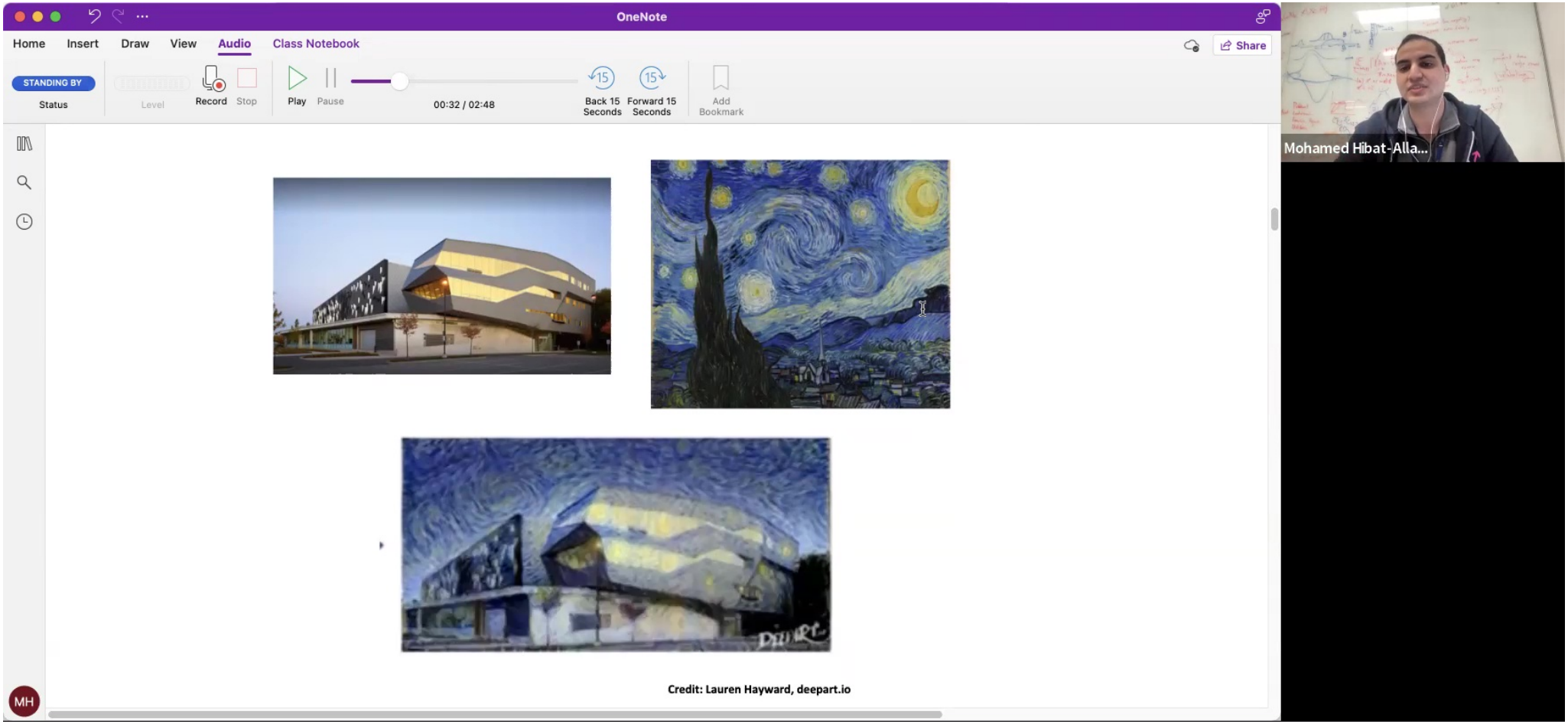
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Credit: Lauren Hayward, deepart.io

Credit: Deep Dream Generator

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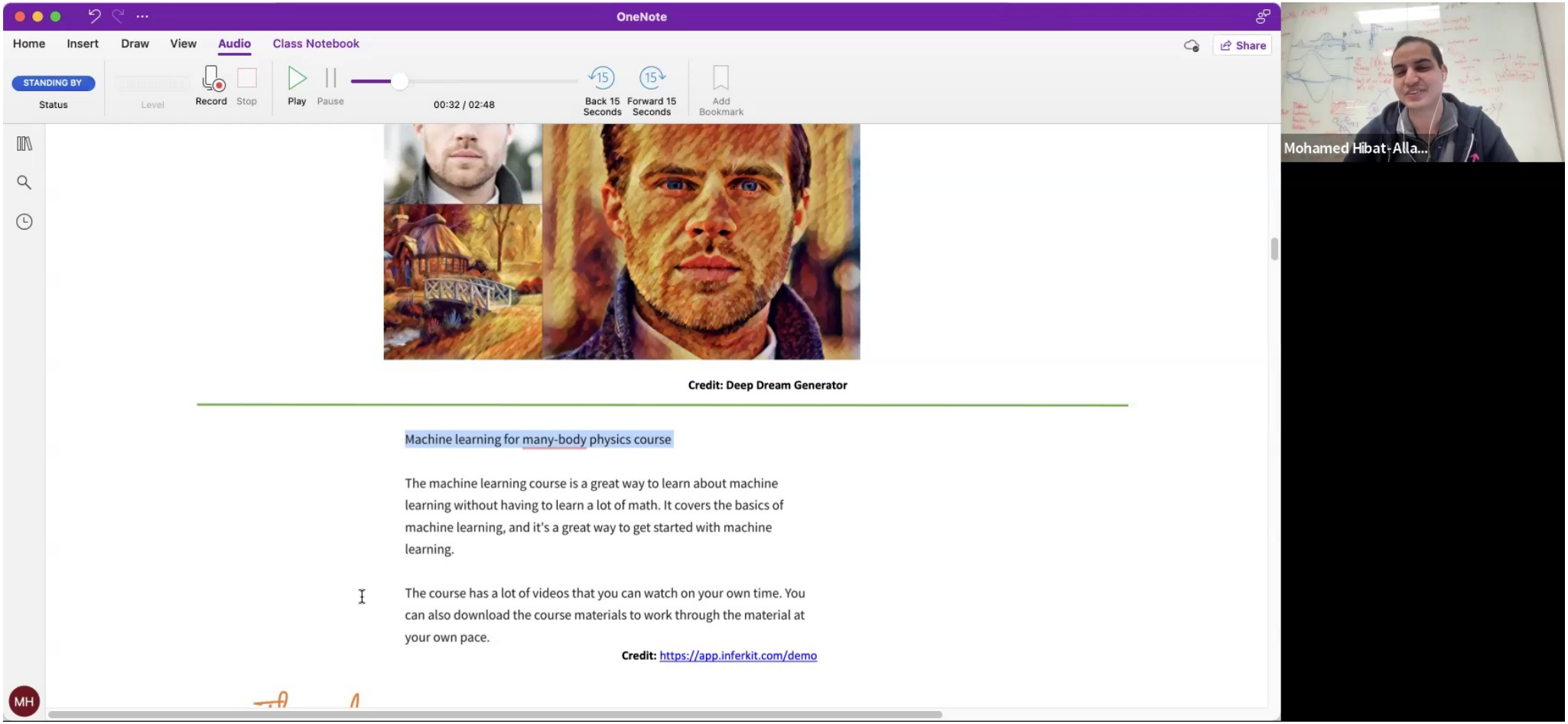
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Credit: Deep Dream Generator
Machine learning for many-body physics course
The machine learning course is a great way to learn about machine learning without having to learn a lot of math. It covers the basics of machine learning, and it's a great way to get started with machine learning.
The course has a lot of videos that you can watch on your own time. You can also download the course materials to work through the material at your own pace.
Credit: https://app.inferkit.com/demo
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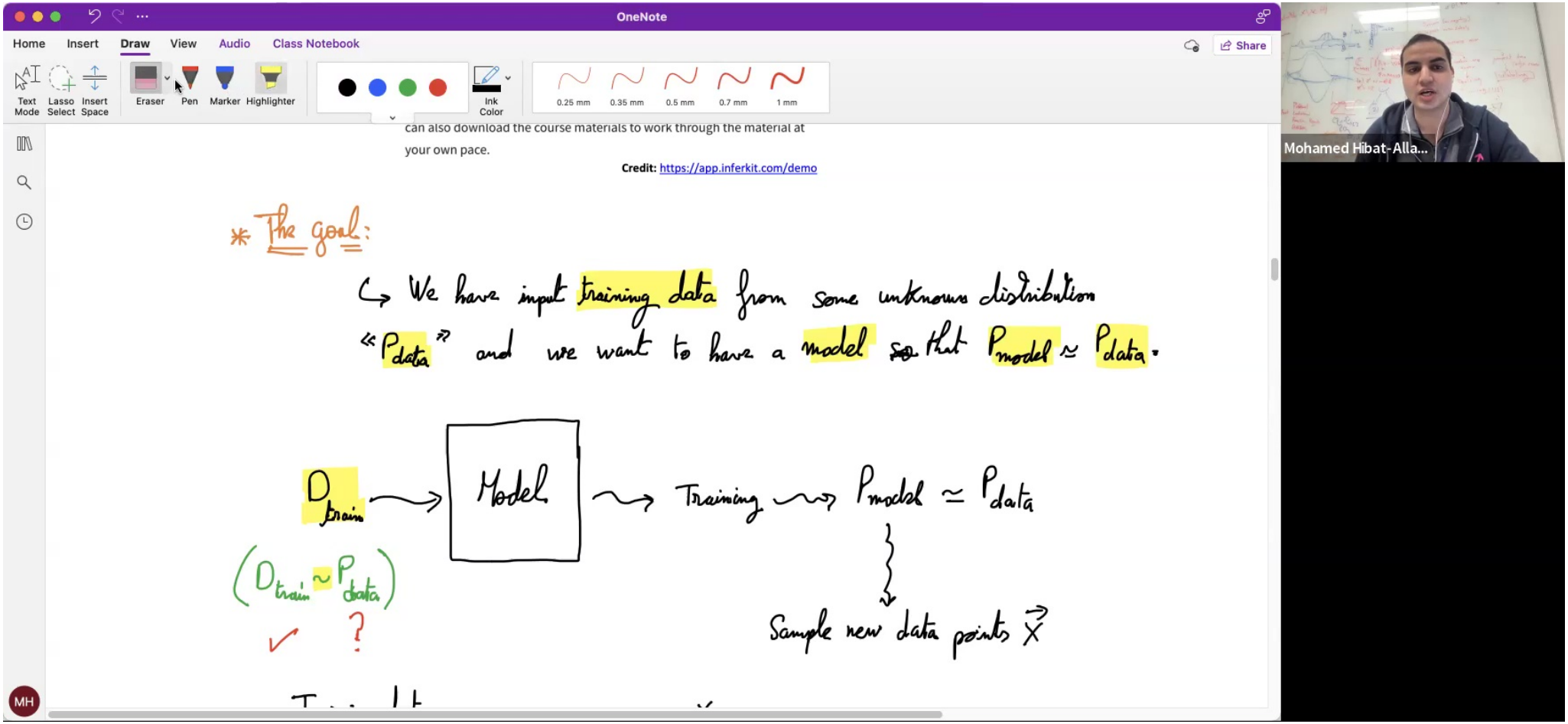
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can also download the course materials to work through the material at your own pace.
Credit: https://app.inferkit.com/demo
* The goal:
↳ We have input training data from some unknown distribution « P_{data} » and we want to have a model so that $P_{model} \simeq P_{data}$.
D_{train}
Model
Training
$P_{model} \simeq P_{data}$
Sample new data points $\vec{x}$
$(D_{train} \simeq P_{data})$
✓ ?
MH
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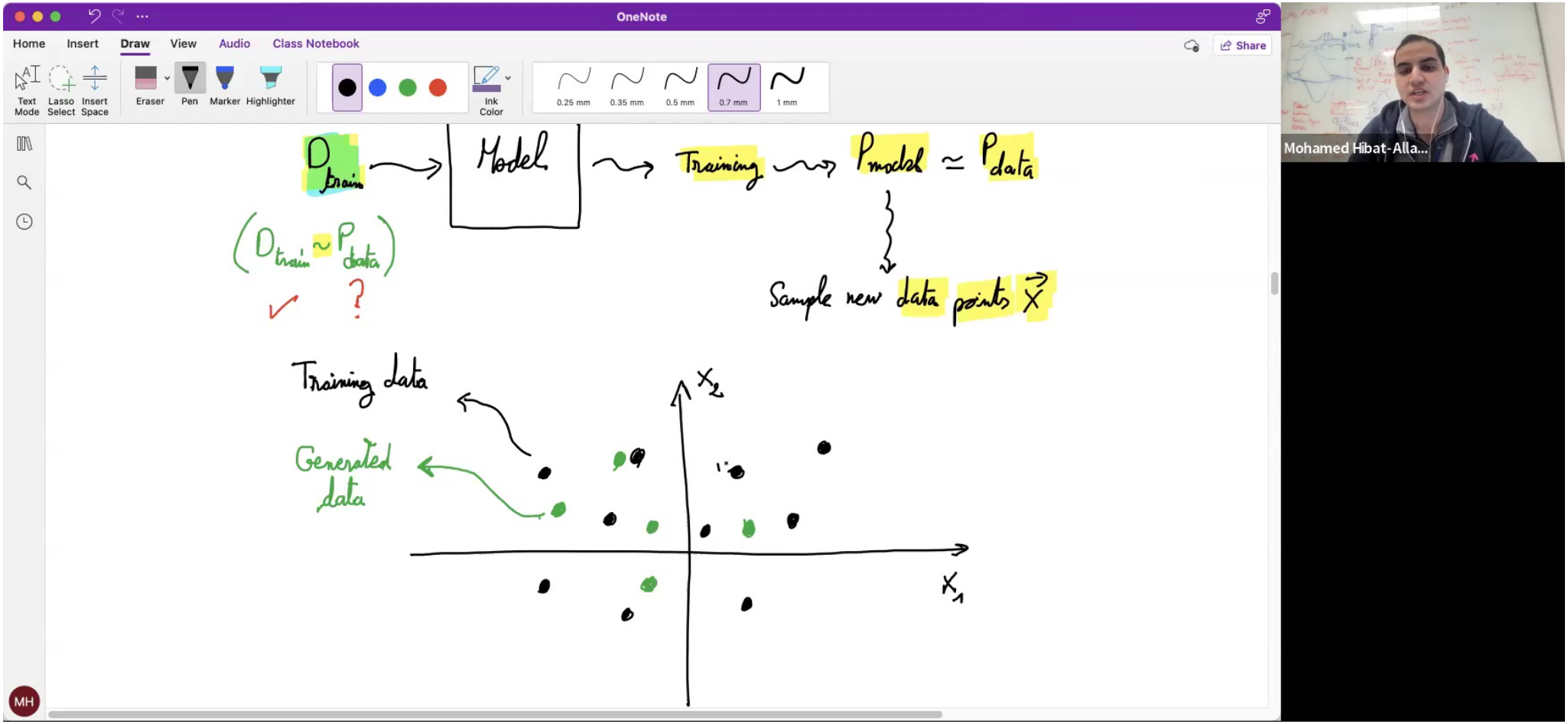
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D_{train}
Model
Training
$P_{model} \simeq P_{data}$
$(D_{train} \sim P_{data})$
?
Sample new data points $\vec{x}$
Training data
Generated data
x_2
x_1
MH
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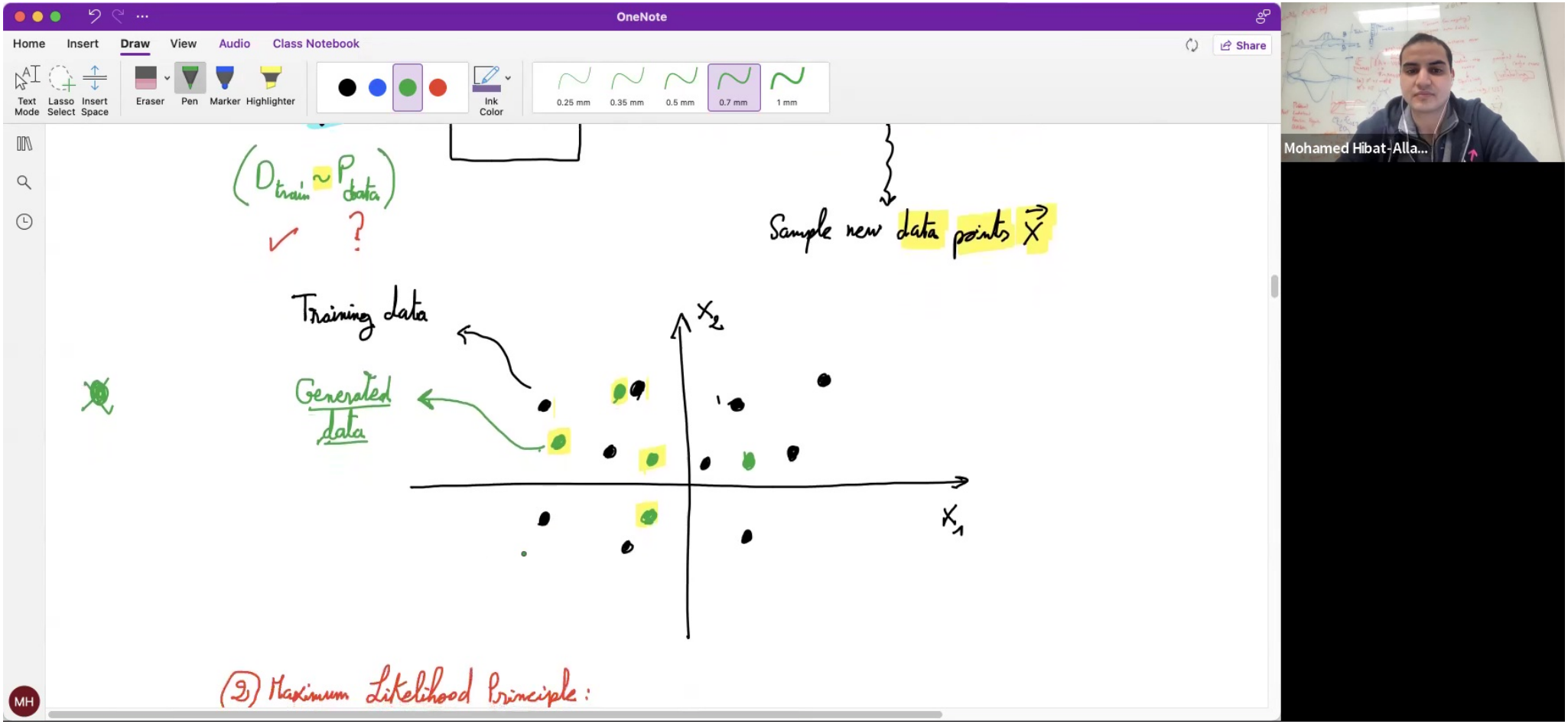
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$(D_{train} \sim P_{data})$
✓ ?
Sample new data points $\vec{X}$
Training data
Generated data
X_2
X_1
(2) Maximum Likelihood Principle:
MH
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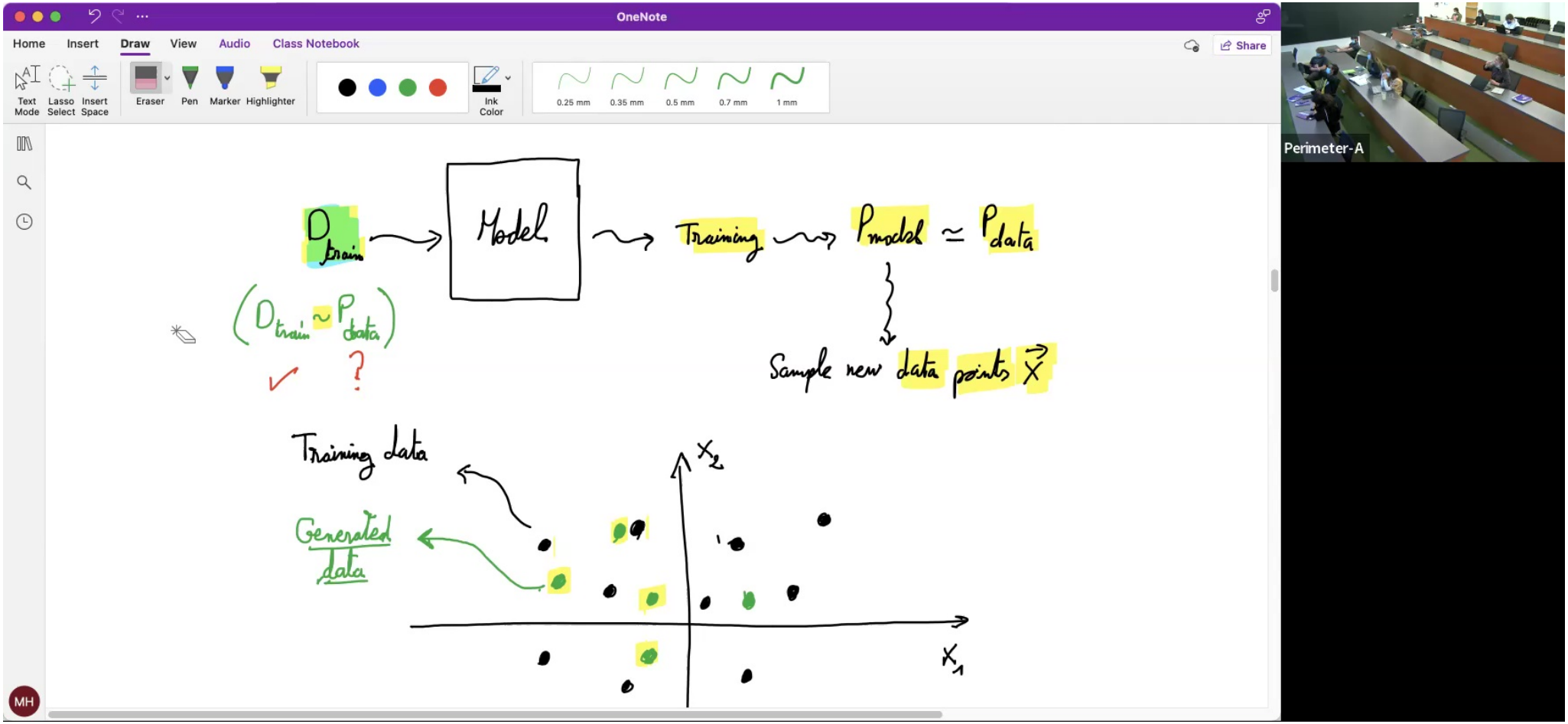
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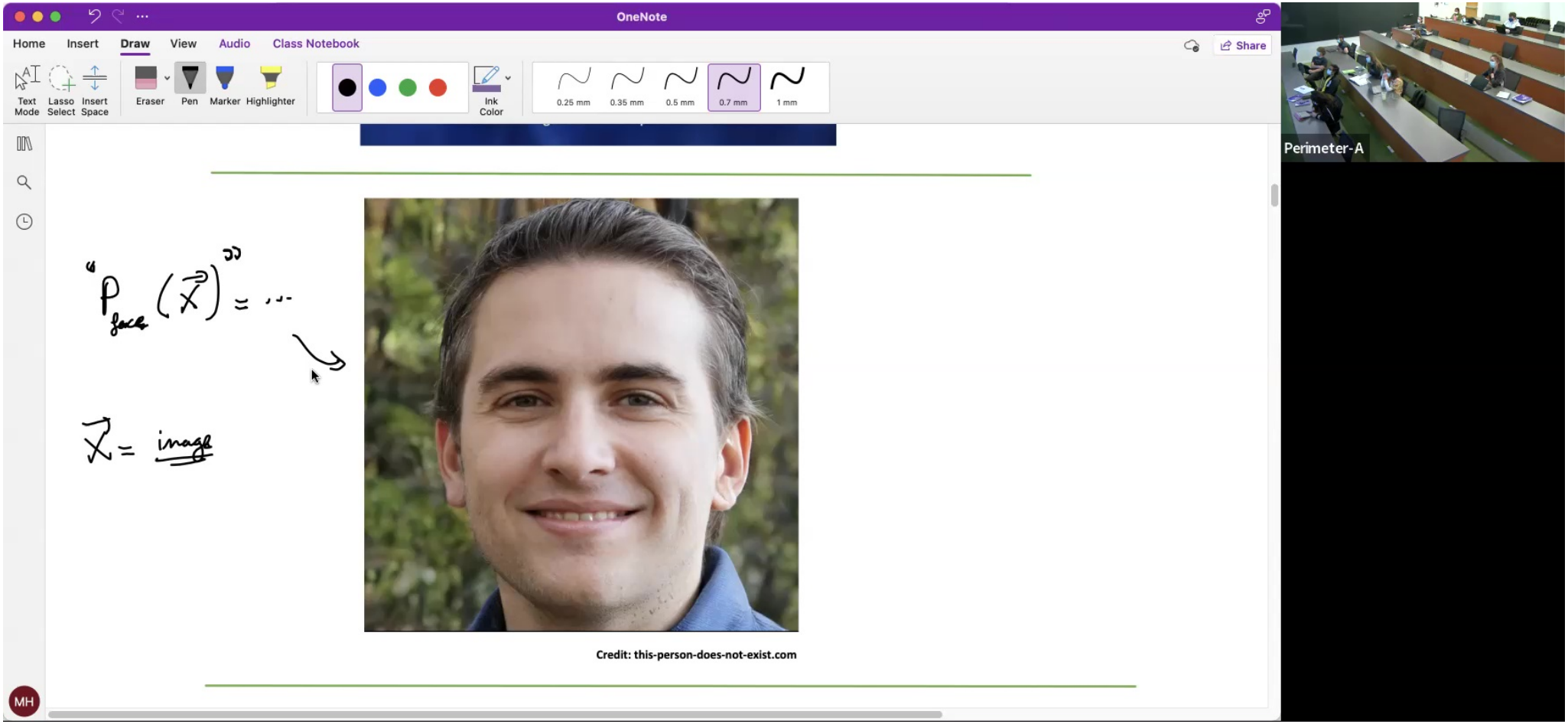
Model
Training
Sample new data points
Training data
Generated data
Perimeter-A
MH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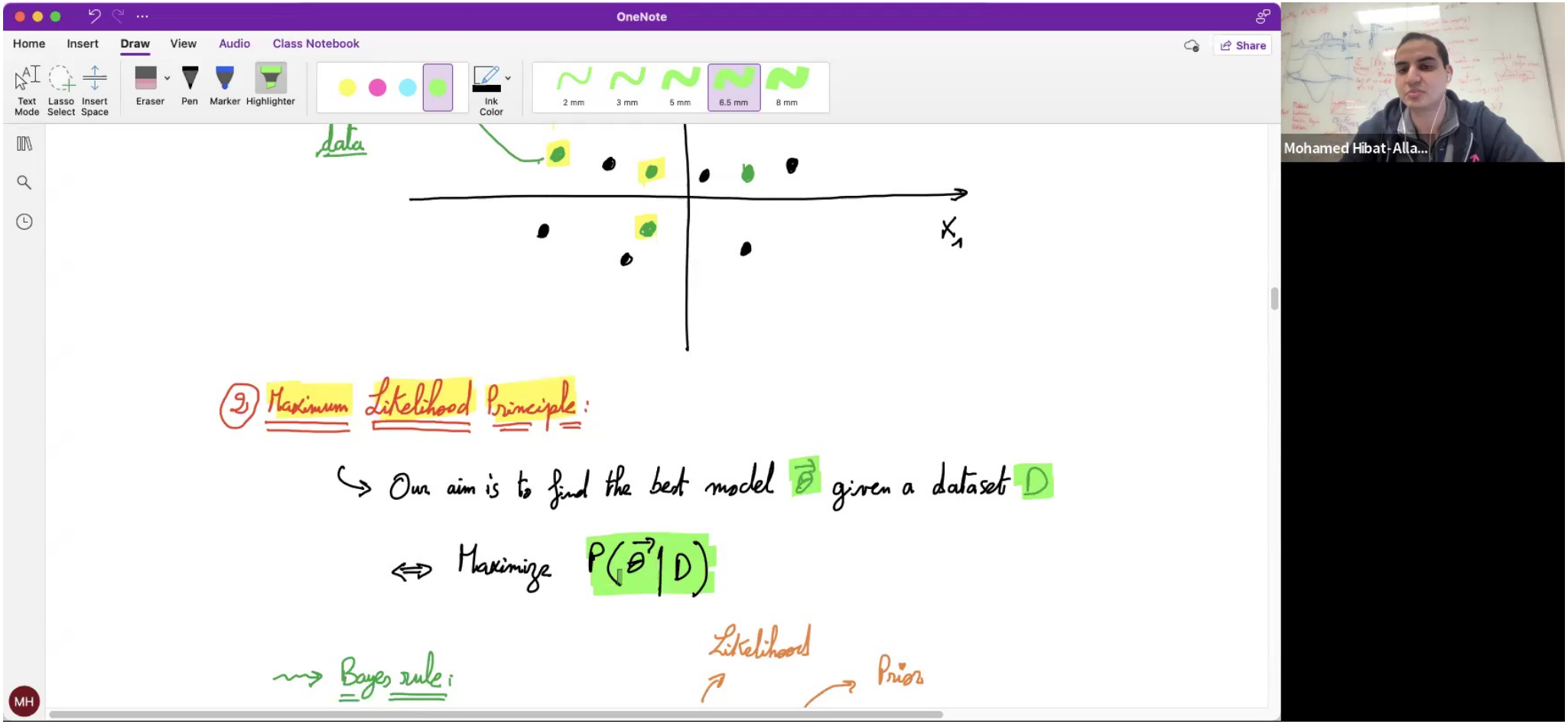
Perimeter-A
" $p_{face}(\vec{x})$ "
$\vec{x}$ = image
Credit: this-person-does-not-exist.com
MH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data
X_1
② Maximum Likelihood Principle :
↳ Our aim is to find the best model $\vec{\theta}$ given a dataset D
⇔ Maximize $P(\vec{\theta}|D)$
⇝ Bayes rule :
Likelihood
Prior
MH

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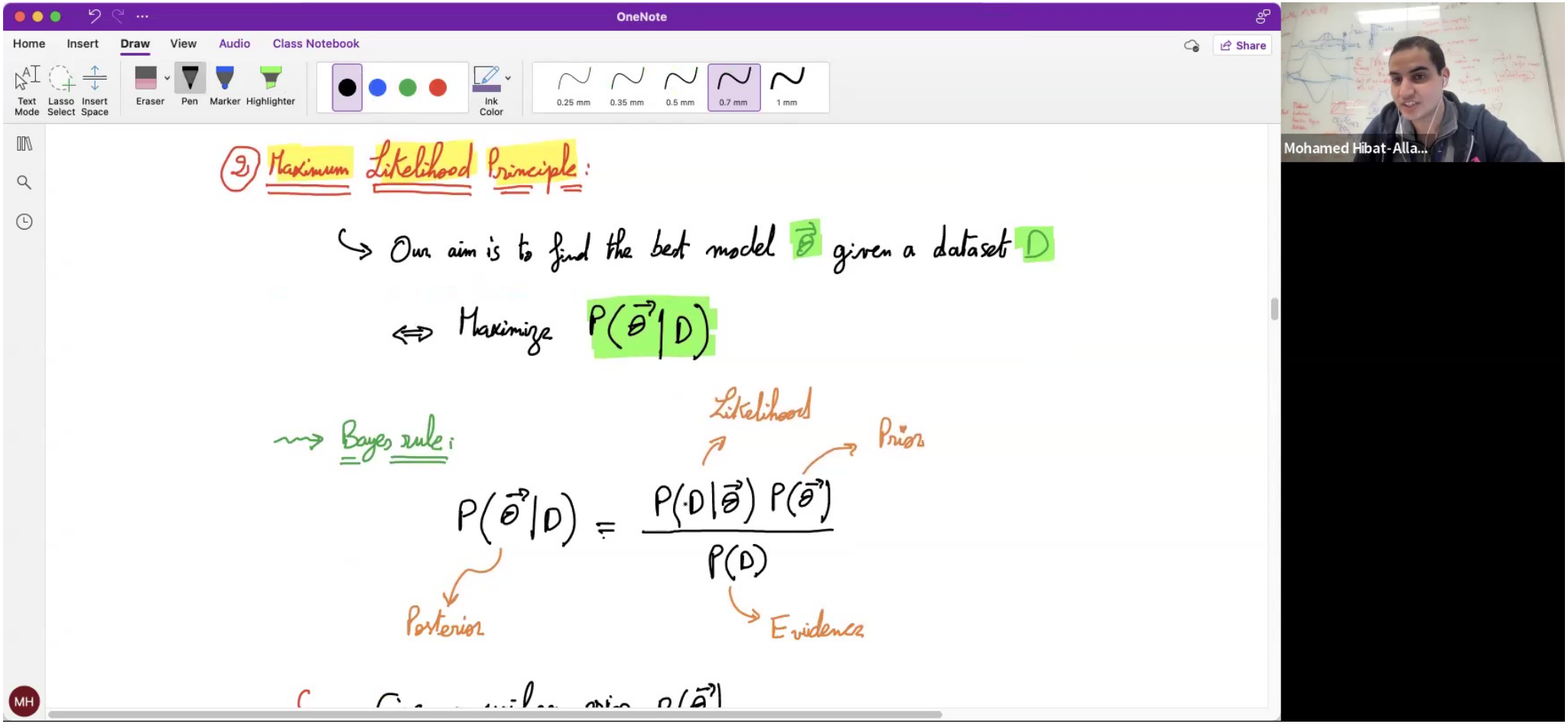
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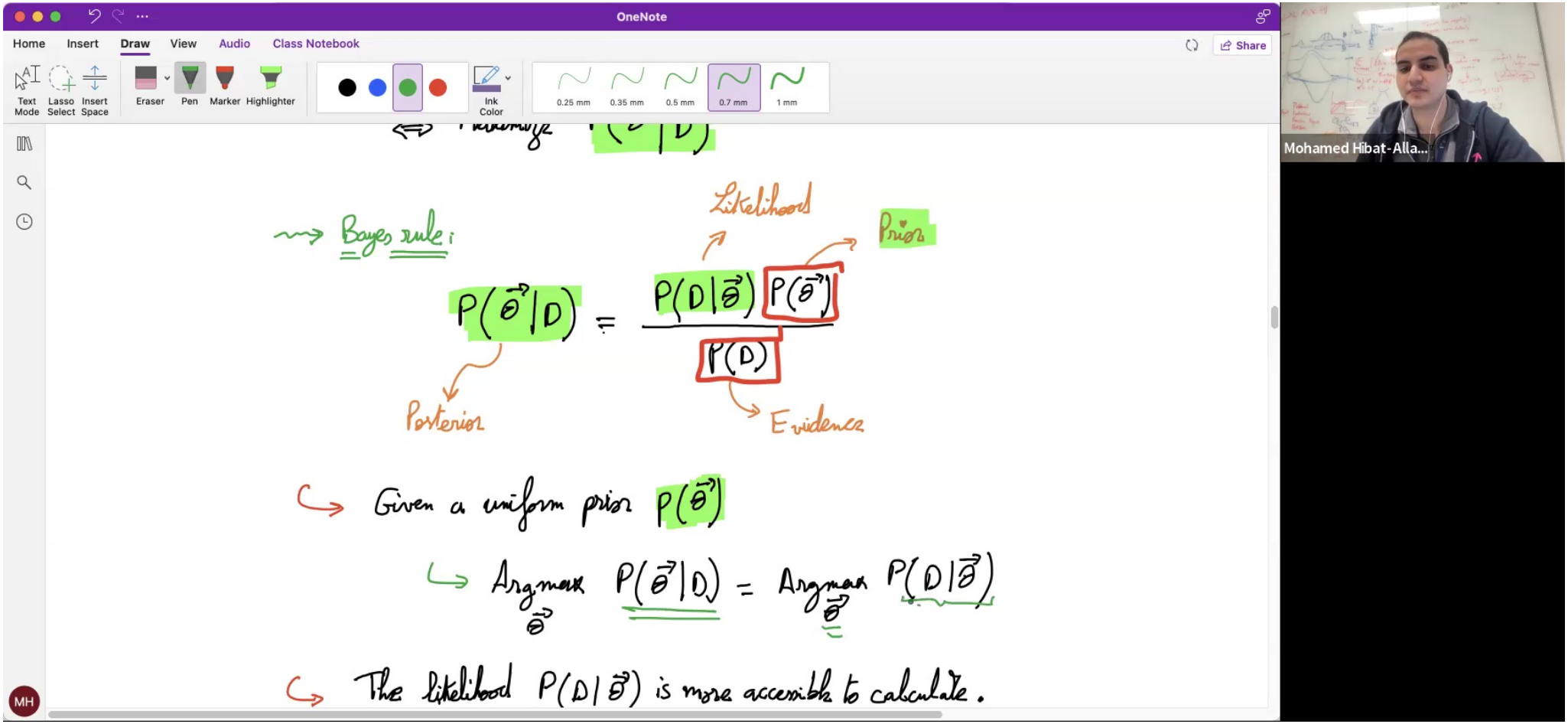
2) Maximum Likelihood Principle :
↳ Our aim is to find the best model $\vec{\theta}$ given a dataset D
⇔ Maximize $P(\vec{\theta}|D)$
⇝ Bayes rule :
Likelihood
Prior
$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})\,P(\vec{\theta})}{P(D)}$
Posterior
Evidence
MH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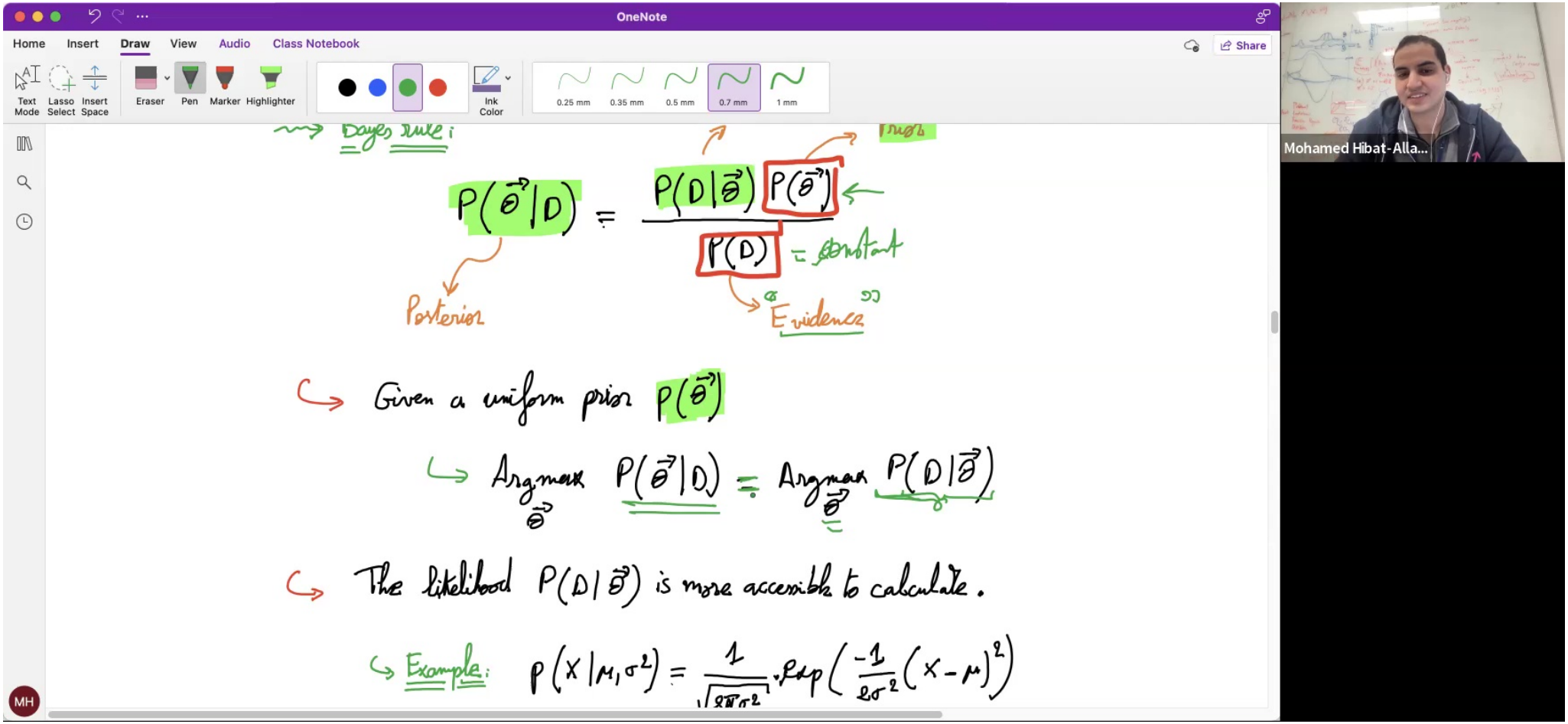
Bayes rule :
Likelihood
Prior
$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})\,P(\vec{\theta})}{P(D)}$
Posterior
Evidence
Given a uniform prior $P(\vec{\theta})$
$\text{Argmax}_{\vec{\theta}}\, P(\vec{\theta}|D) = \text{Argmax}_{\vec{\theta}}\, P(D|\vec{\theta})$
The likelihood $P(D|\vec{\theta})$ is more accessible to calculate.
MH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Bayes rule:
$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})\,P(\vec{\theta})}{P(D)}$
Prior
Posterior
Evidence
= constant
Given a uniform prior $P(\vec{\theta})$
$\text{Argmax}_{\vec{\theta}}\ P(\vec{\theta}|D) = \text{Argmax}_{\vec{\theta}}\ P(D|\vec{\theta})$
The likelihood $P(D|\vec{\theta})$ is more accessible to calculate.
Example: $P(X|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp\left(\frac{-1}{2\sigma^2}(X-\mu)^2\right)$
MH

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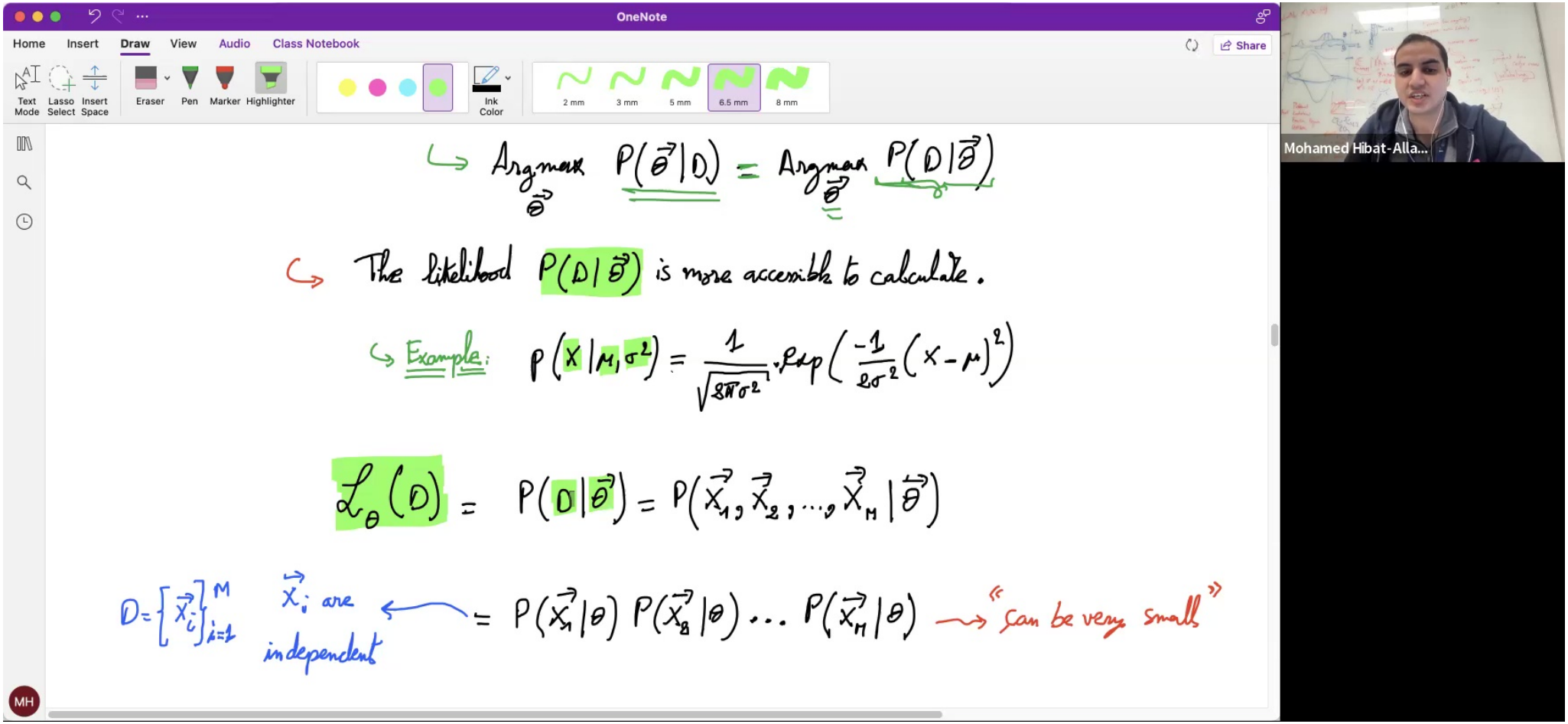
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$\hookrightarrow \underset{\vec{\theta}}{\text{Argmax}}\, P(\vec{\theta}|D) = \underset{\vec{\theta}}{\text{Argmax}}\, P(D|\vec{\theta})$
$\hookrightarrow$ The likelihood $P(D|\vec{\theta})$ is more accessible to calculate.
$\hookrightarrow$ Example: $P(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}(x-\mu)^2\right)$
$\mathcal{L}_\theta(D) = P(D|\vec{\theta}) = P(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_M|\vec{\theta})$
$D = \{\vec{x}_i\}_{i=1}^{M}$
$\vec{x}_i$ are independent
$= P(\vec{x}_1|\theta)\, P(\vec{x}_2|\theta) \dots P(\vec{x}_M|\theta)$
"can be very small"
MH
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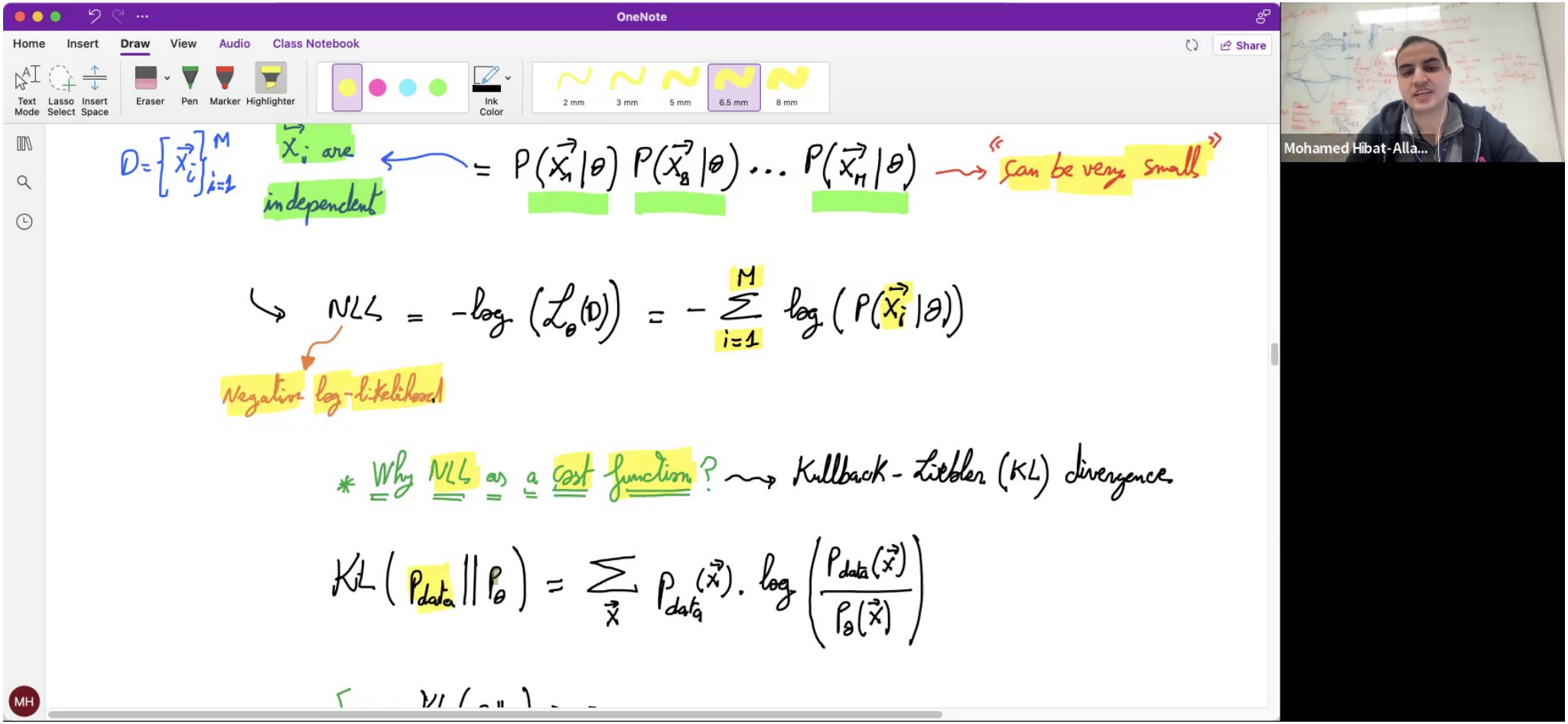
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$D = \{\vec{x}_i\}_{i=1}^{M}$
$\vec{x}_i$ are independent
$= P(\vec{x}_1|\theta)\, P(\vec{x}_2|\theta) \dots P(\vec{x}_M|\theta)$
« can be very small »
$\hookrightarrow NLL = -\log(\mathcal{L}_\theta(D)) = -\sum_{i=1}^{M} \log(P(\vec{x}_i|\theta))$
Negative log-likelihood
* Why NLL as a cost function ? ⇝ Kullback-Leibler (KL) divergence
$KL(P_{data} \| P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \cdot \log\left(\frac{P_{data}(\vec{x})}{P_\theta(\vec{x})}\right)$
MH
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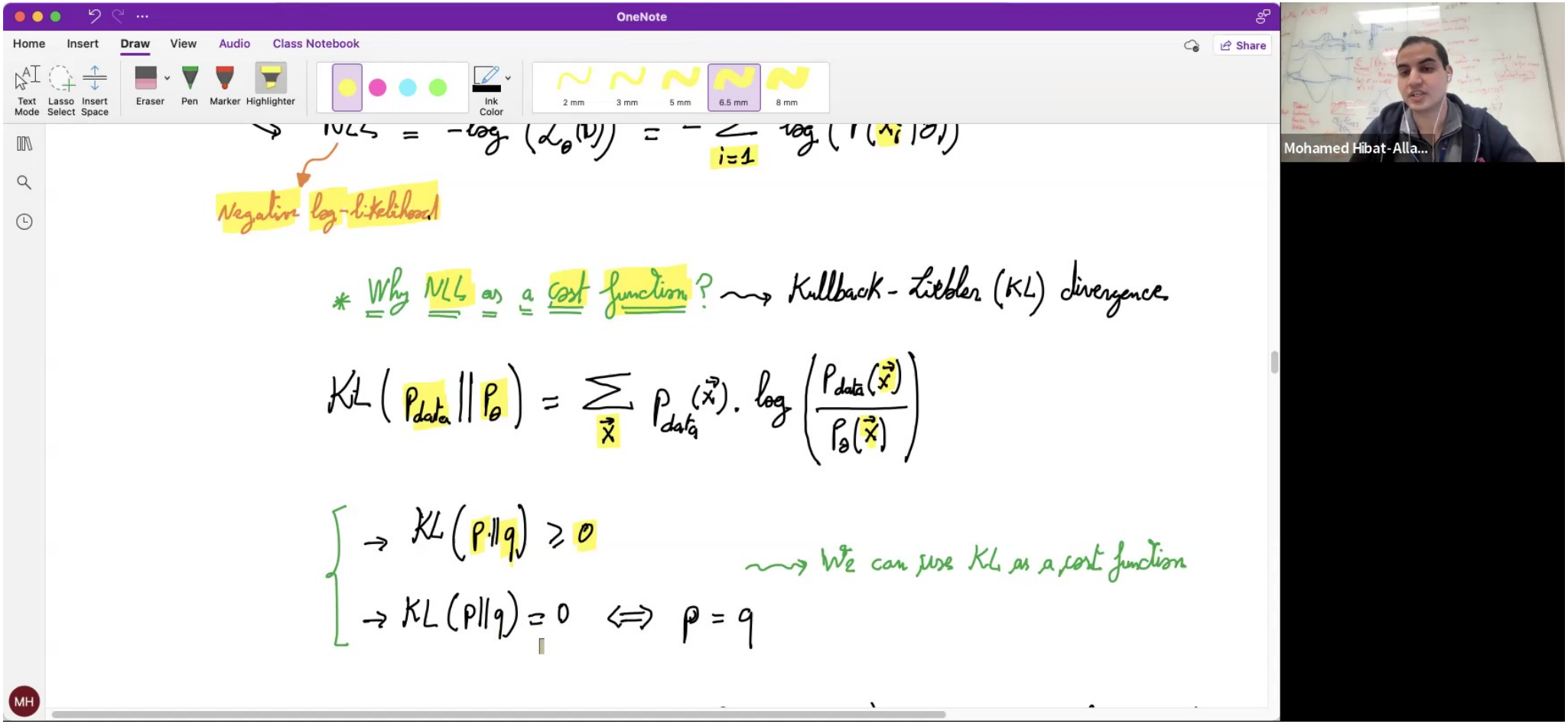
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$\mathrm{NLL} = -\log(\mathcal{L}_\theta(\mathcal{D})) = -\sum_{i=1} \log(P(x_i|\theta))$
Negative log-likelihood
* Why NLL as a cost function ? ⇝ Kullback-Leibler (KL) divergence
$KL(P_{data} \| P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \cdot \log\left(\frac{P_{data}(\vec{x})}{P_\theta(\vec{x})}\right)$
→ $KL(p\|q) \geq 0$
→ $KL(p\|q) = 0 \iff p = q$
⇝ We can use KL as a cost function
MH

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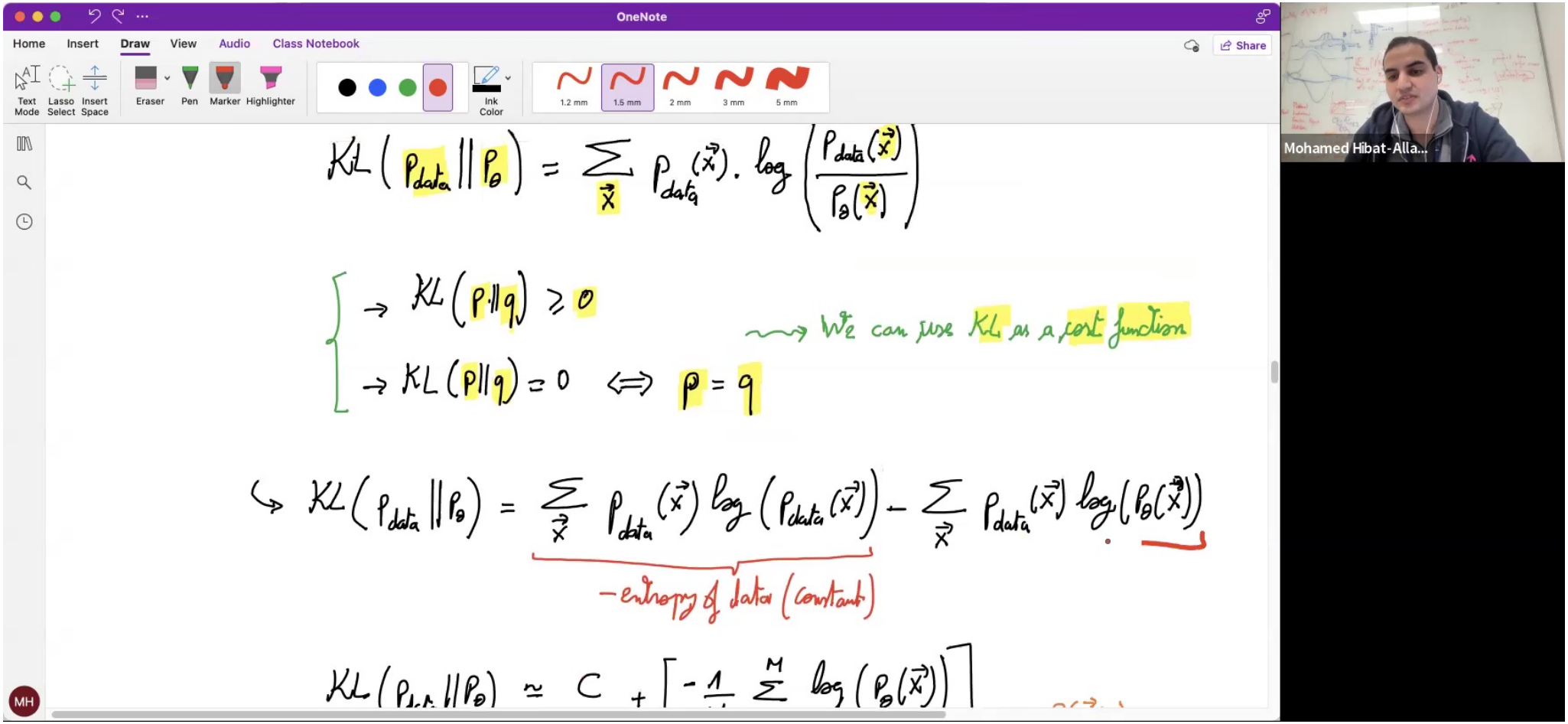
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$KL(P_{data} \| P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \cdot \log\left(\frac{P_{data}(\vec{x})}{P_\theta(\vec{x})}\right)$
$\rightarrow KL(P\|q) \geq 0$
$\rightarrow KL(P\|q) = 0 \iff P = q$
We can use KL as a cost function
$\hookrightarrow KL(P_{data} \| P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \log\left(P_{data}(\vec{x})\right) - \sum_{\vec{x}} P_{data}(\vec{x}) \log\left(P_\theta(\vec{x})\right)$
- entropy of data (constant)
$KL(P_{data} \| P_\theta) \simeq C + \left[-\frac{1}{M} \sum^{M} \log\left(P_\theta(\vec{x})\right)\right]$
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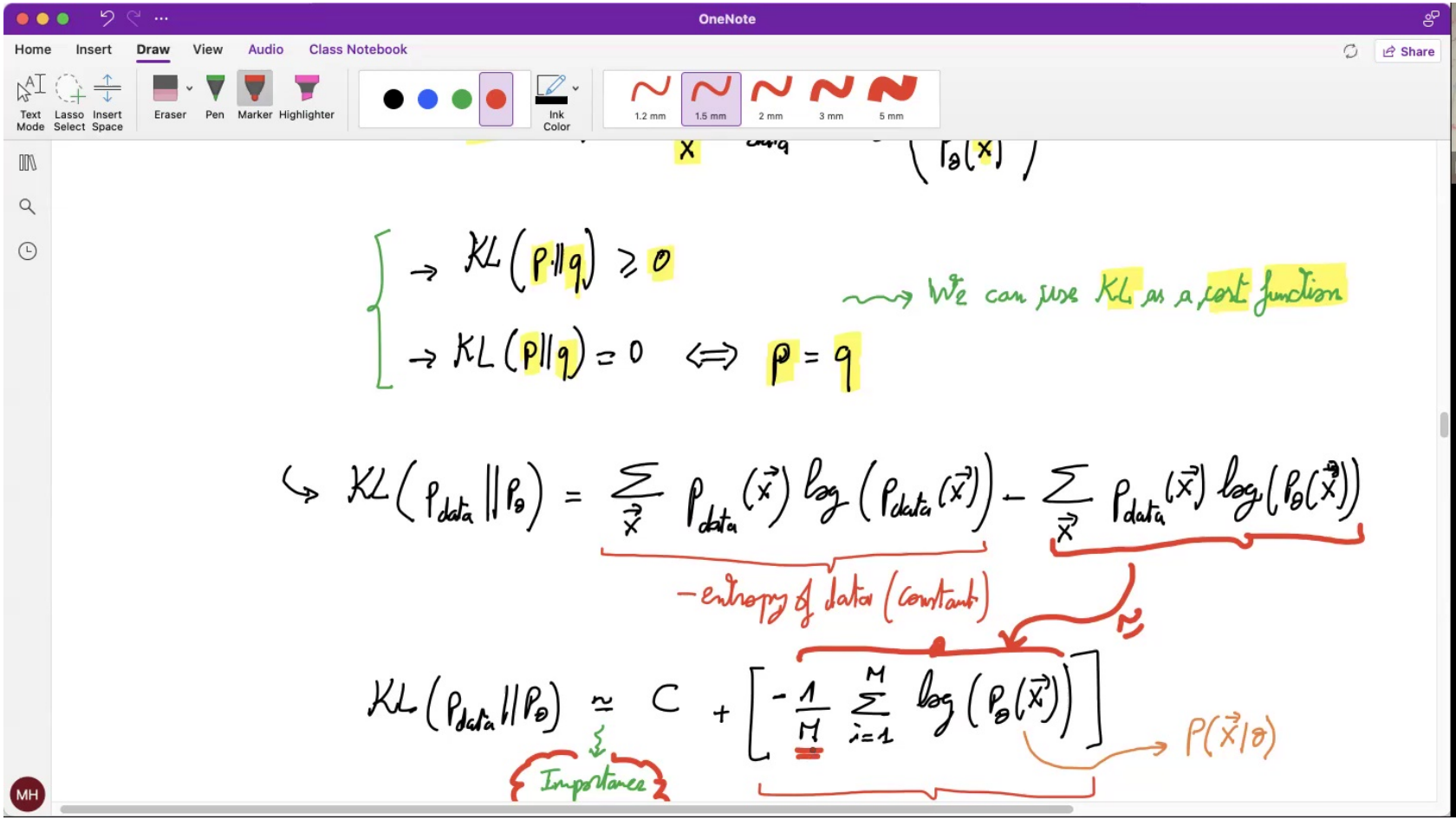
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$\rightarrow KL(p\|q) \geq 0$
$\rightarrow KL(p\|q) = 0 \iff p = q$
We can use KL as a cost function
$\hookrightarrow KL(p_{data}\|p_\theta) = \sum_{\vec{x}} p_{data}(\vec{x}) \log(p_{data}(\vec{x})) - \sum_{\vec{x}} p_{data}(\vec{x}) \log(p_\theta(\vec{x}))$
- entropy of data (constant)
$KL(p_{data}\|p_\theta) \simeq C + \left[-\frac{1}{M}\sum_{i=1}^{M} \log(p_\theta(\vec{x}))\right]$
Importance
$P(\vec{x}|\theta)$
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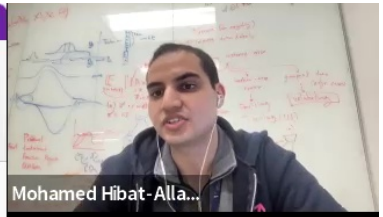
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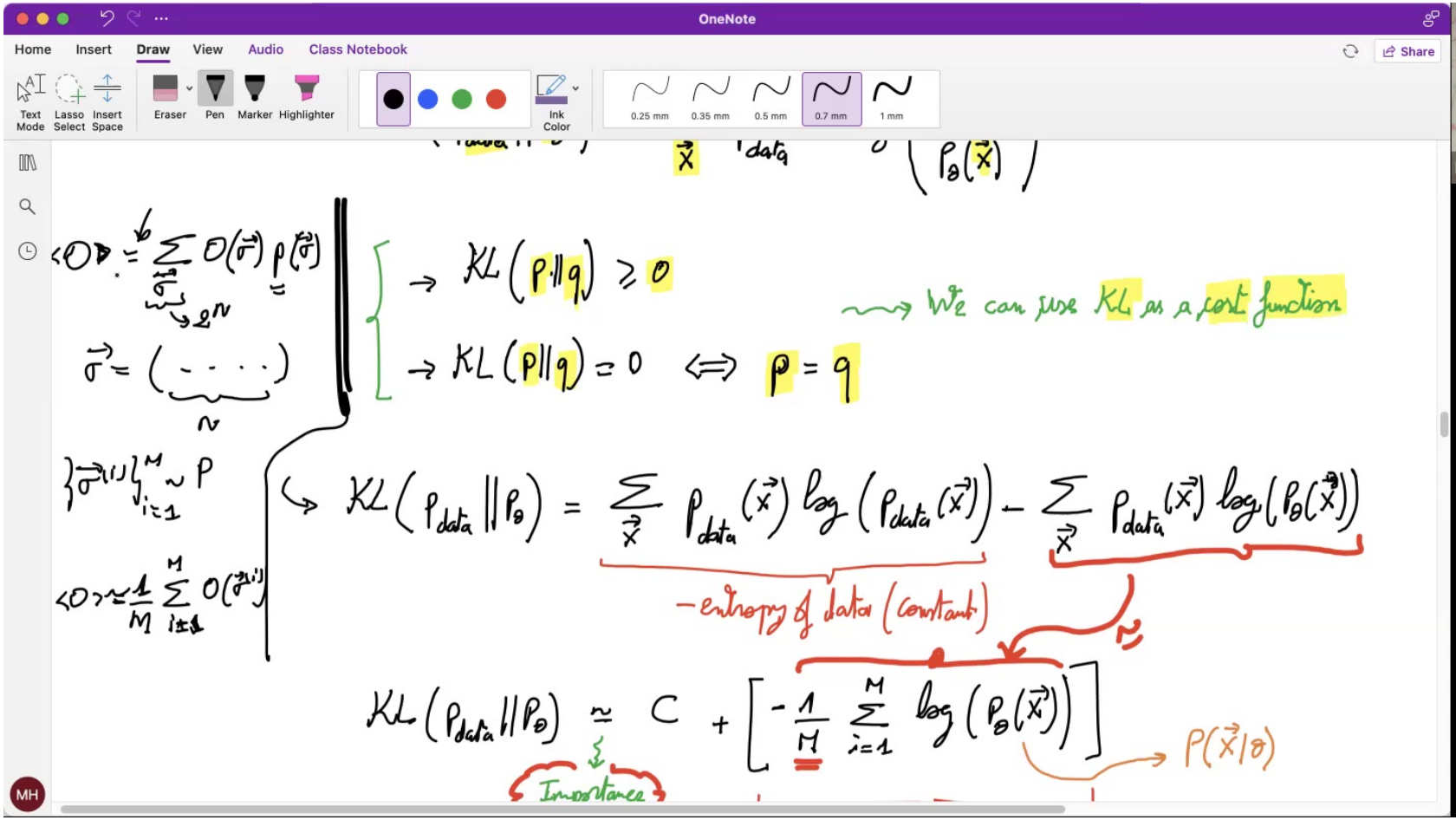
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$\langle O \rangle = \sum_{\vec{\sigma}} O(\vec{\sigma})\, p(\vec{\sigma})$
$\vec{\sigma} = (\cdots)$
N
$\{\vec{\sigma}^{(i)}\}_{i=1}^{M} \sim p$
$\langle O \rangle \simeq \frac{1}{M} \sum_{i=1}^{M} O(\vec{\sigma}^{(i)})$
$KL(p\|q) \geq 0$
$KL(p\|q) = 0 \iff p = q$
We can use KL as a cost function
$KL(p_{data}\|p_\theta) = \sum_{\vec{x}} p_{data}(\vec{x}) \log\left(p_{data}(\vec{x})\right) - \sum_{\vec{x}} p_{data}(\vec{x}) \log\left(p_\theta(\vec{x})\right)$
- entropy of data (constant)
$KL(p_{data}\|p_\theta) \simeq C + \left[-\frac{1}{M} \sum_{i=1}^{M} \log\left(p_\theta(\vec{x})\right)\right]$
Importance
$P(\vec{x}|\theta)$
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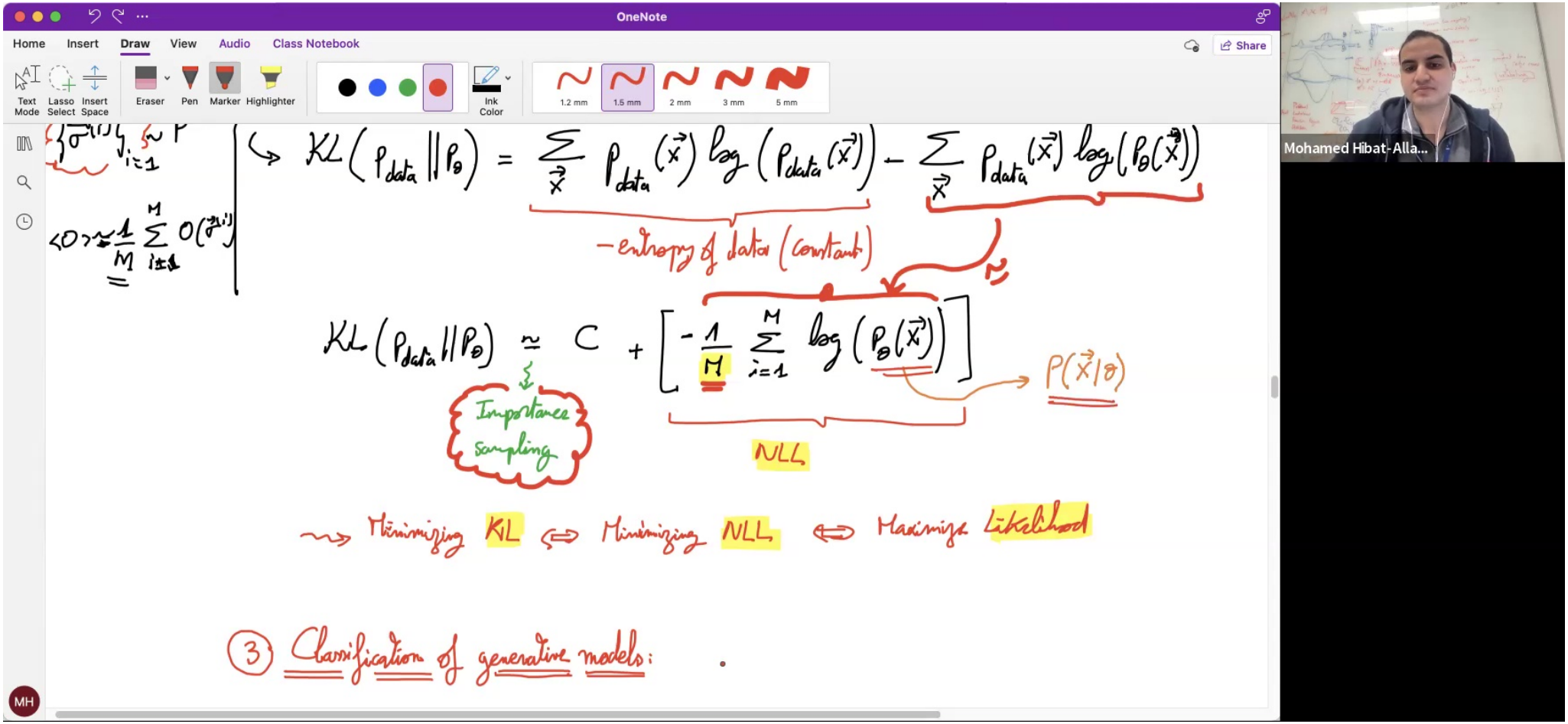
MH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$KL(P_{data} \| P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \log\left(P_{data}(\vec{x})\right) - \sum_{\vec{x}} P_{data}(\vec{x}) \log\left(P_\theta(\vec{x})\right)$
- entropy of data (constant)
$KL(P_{data} \| P_\theta) \simeq C + \left[-\frac{1}{M} \sum_{i=1}^{M} \log\left(P_\theta(\vec{x})\right)\right]$
$P(\vec{x}|\theta)$
Importance sampling
NLL
Minimizing KL ⇔ Minimizing NLL ⇔ Maximize Likelihood
3) Classification of generative models:
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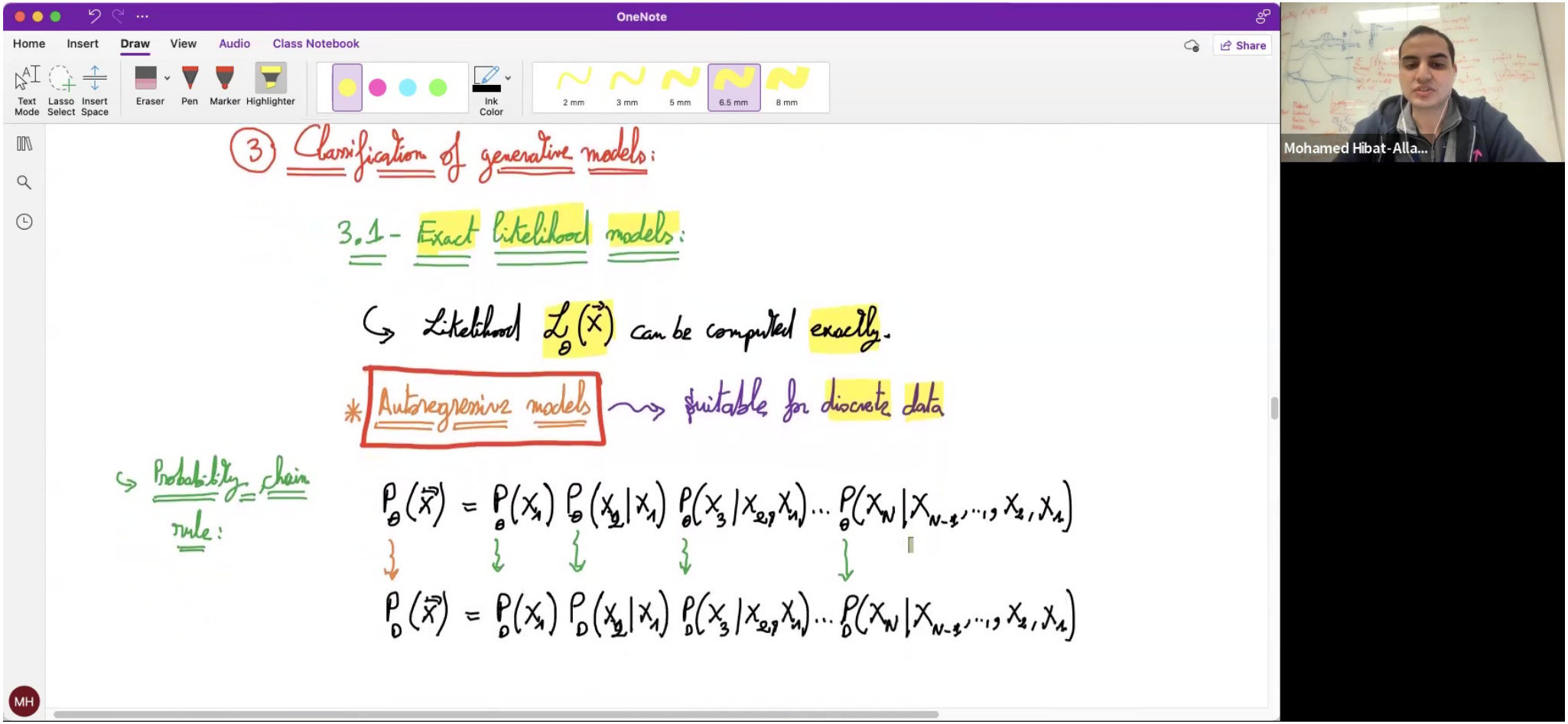
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3) Classification of generative models:
3.1- Exact likelihood models:
↳ Likelihood $\mathcal{L}_\theta(\vec{x})$ can be computed exactly.
* Autoregressive models ⇝ suitable for discrete data
↳ Probability chain rule:
$P_\theta(\vec{x}) = P_\theta(x_1)\, P_\theta(x_2|x_1)\, P_\theta(x_3|x_2,x_1) \dots P_\theta(x_N|x_{N-1},\dots,x_2,x_1)$
$P_D(\vec{x}) = P_D(x_1)\, P_D(x_2|x_1)\, P_D(x_3|x_2,x_1) \dots P_D(x_N|x_{N-1},\dots,x_2,x_1)$
MH

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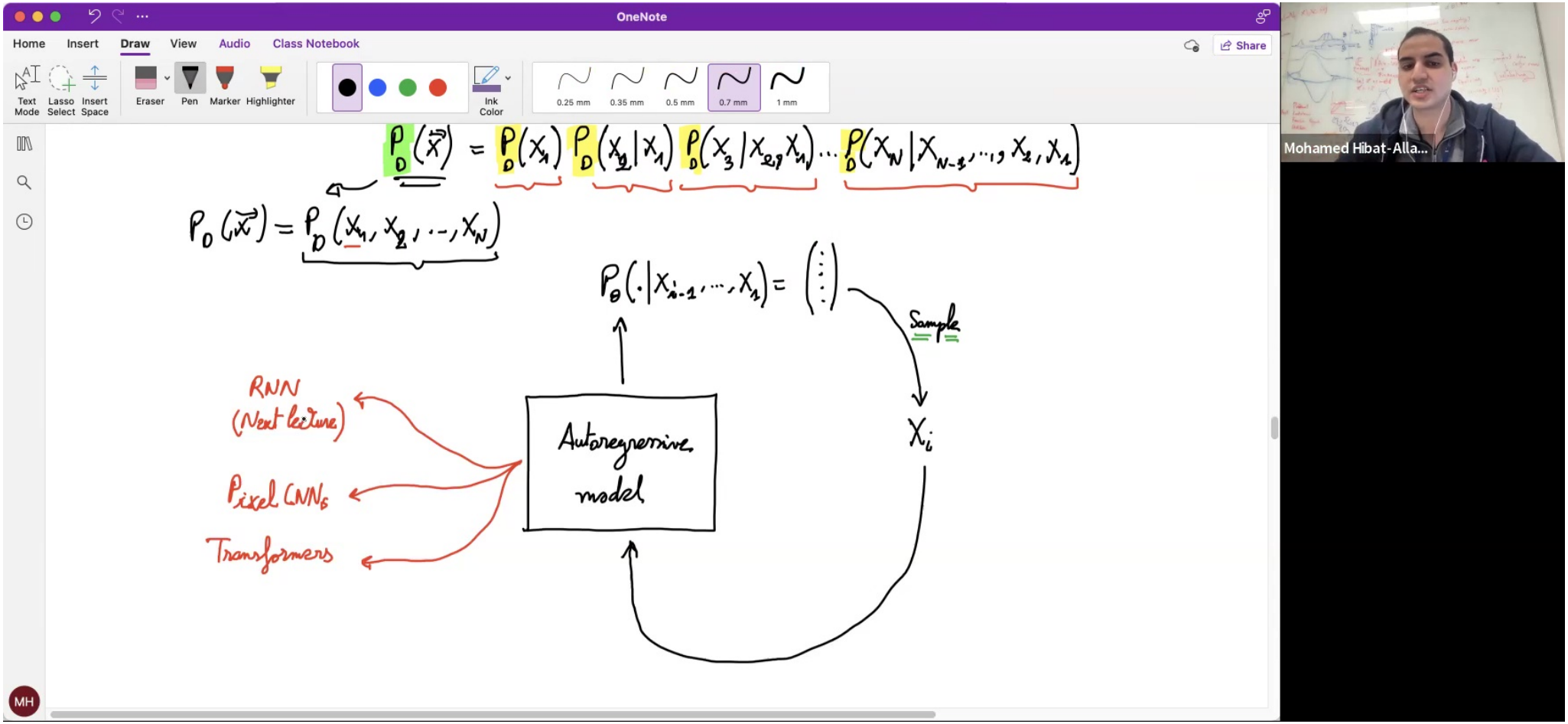
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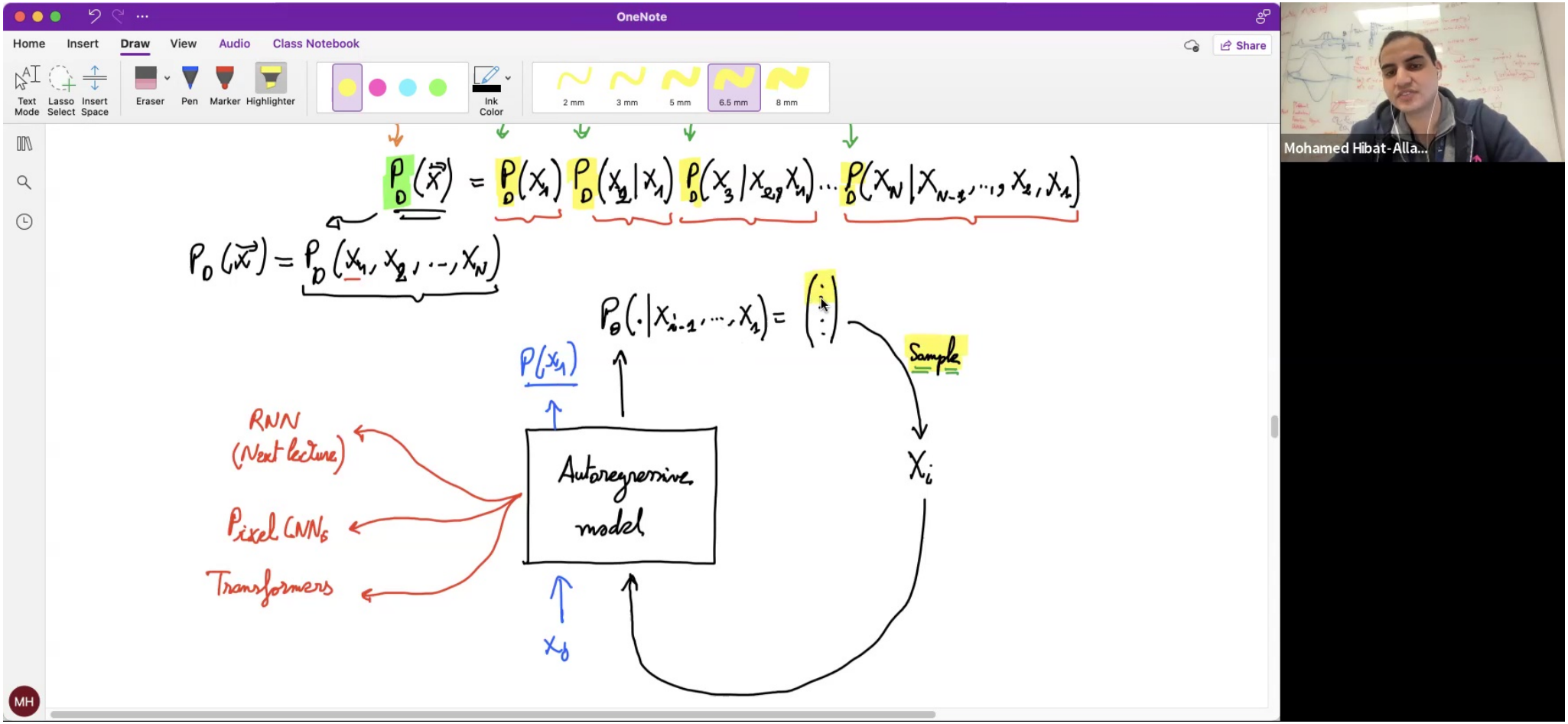
$p_D(\vec{x}) = p_D(x_1)\, p_D(x_2|x_1)\, p_D(x_3|x_2,x_1) \cdots p_D(x_N|x_{N-1},\dots,x_2,x_1)$
$p_D(\vec{x}) = p_D(x_1, x_2, \dots, x_N)$
$p_\theta(\cdot|x_{i-1},\dots,x_1) = \begin{pmatrix}\vdots\end{pmatrix}$
Sample
x_i
Autoregressive model
RNN (Next lecture)
PixelCNNs
Transformers
MH

Mohamed Hibat-Alla...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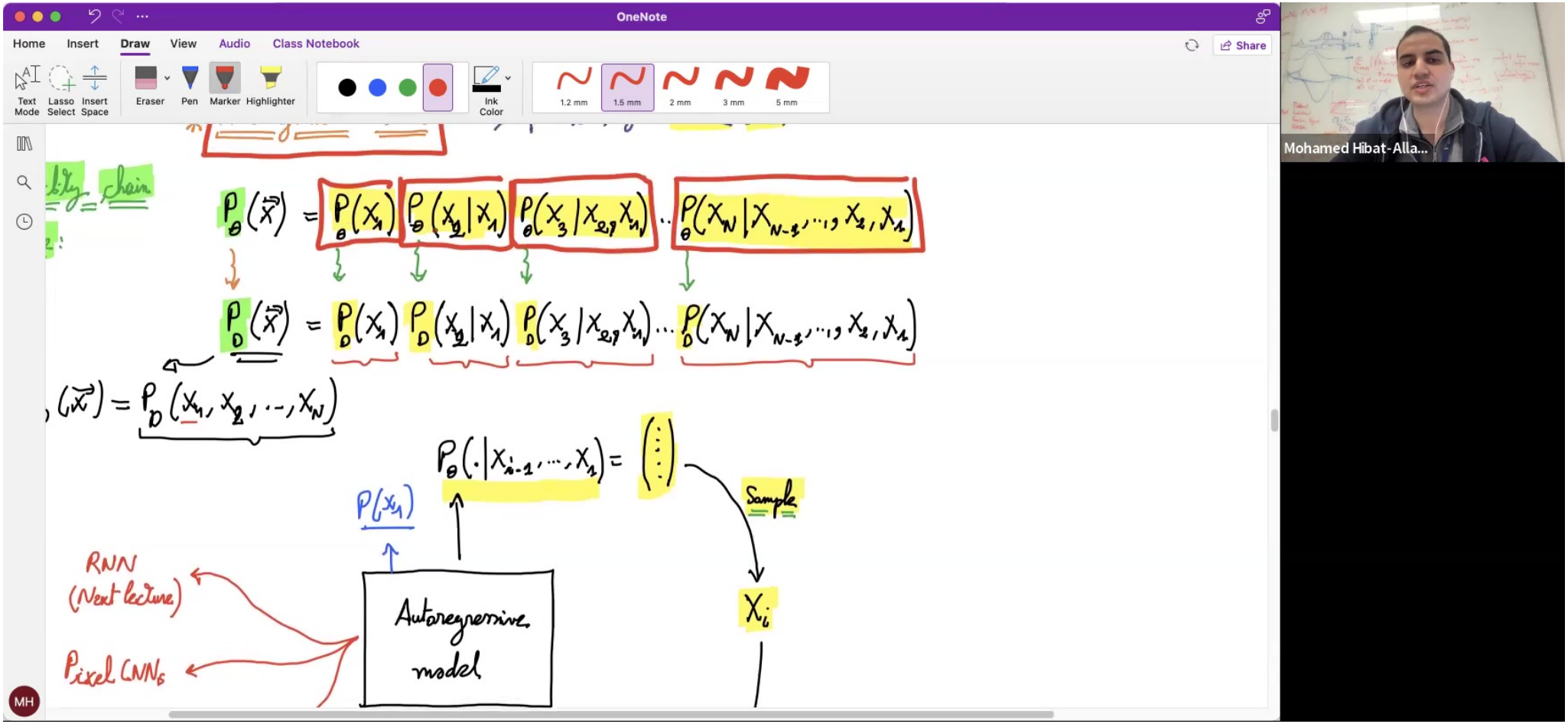
$p_D(\vec{x}) = p_D(x_1)\, p_D(x_2|x_1)\, p_D(x_3|x_2,x_1) \cdots p_D(x_N|x_{N-1},\dots,x_2,x_1)$
$p_\theta(\vec{x}) = p_\theta(x_1, x_2, \dots, x_N)$
$p_\theta(\cdot|x_{i-1},\dots,x_1) =$
Sample
$p(x_1)$
RNN
(Next lecture)
PixelCNNs
Transformers
Autoregressive model
x_i
x_0
Mohamed Hibat-Alla...
MH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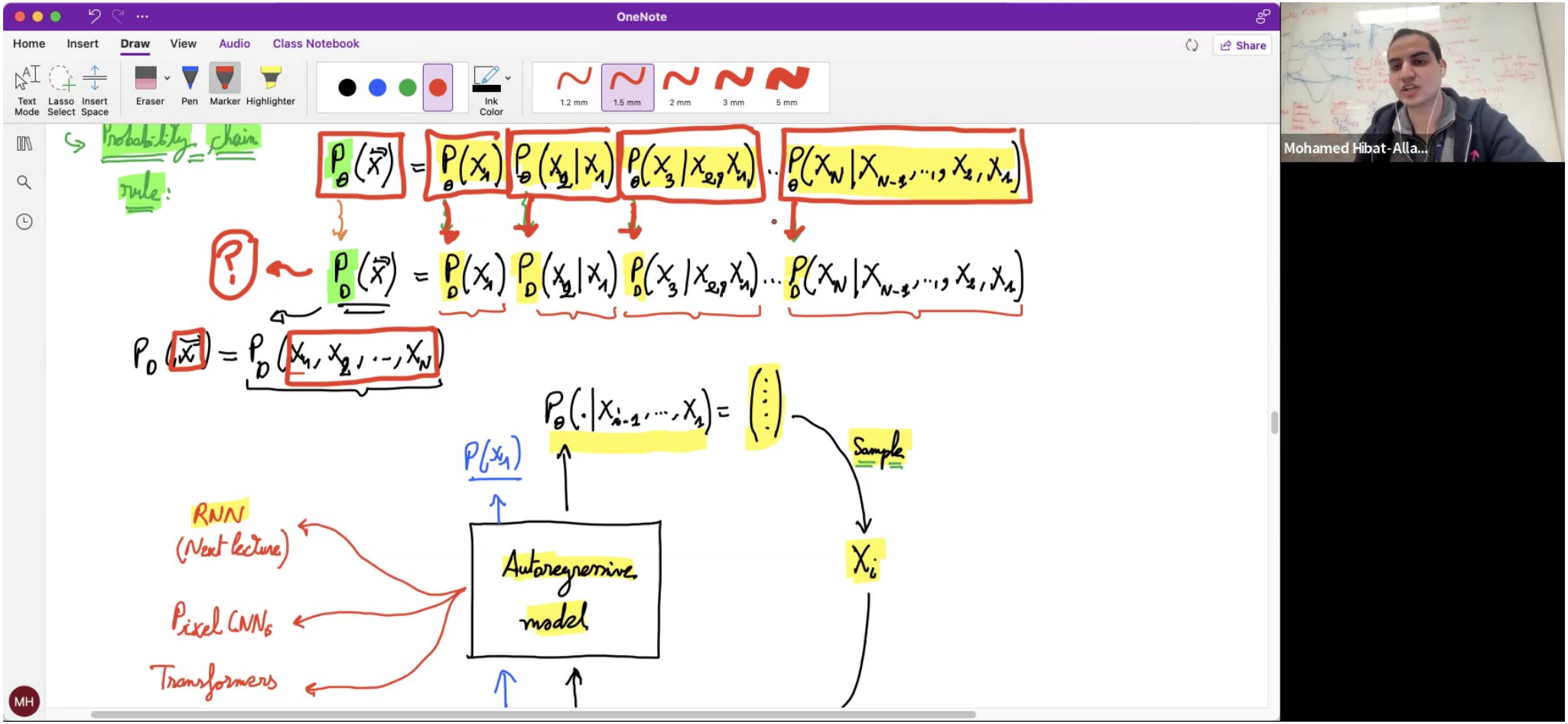
$p_\theta(\vec{x}) = p_\theta(x_1)\, p_\theta(x_2|x_1)\, p_\theta(x_3|x_2,x_1) \cdots p_\theta(x_N|x_{N-1},\dots,x_2,x_1)$
$p_D(\vec{x}) = p_D(x_1)\, p_D(x_2|x_1)\, p_D(x_3|x_2,x_1) \cdots p_D(x_N|x_{N-1},\dots,x_2,x_1)$
$p_D(\vec{x}) = p_D(x_1, x_2, \dots, x_N)$
$p_\theta(\cdot|x_{i-1},\dots,x_1) = (\dots)$
Sample
x_i
$p(x_1)$
RNN
(Next lecture)
PixelCNNs
Autoregressive model
MH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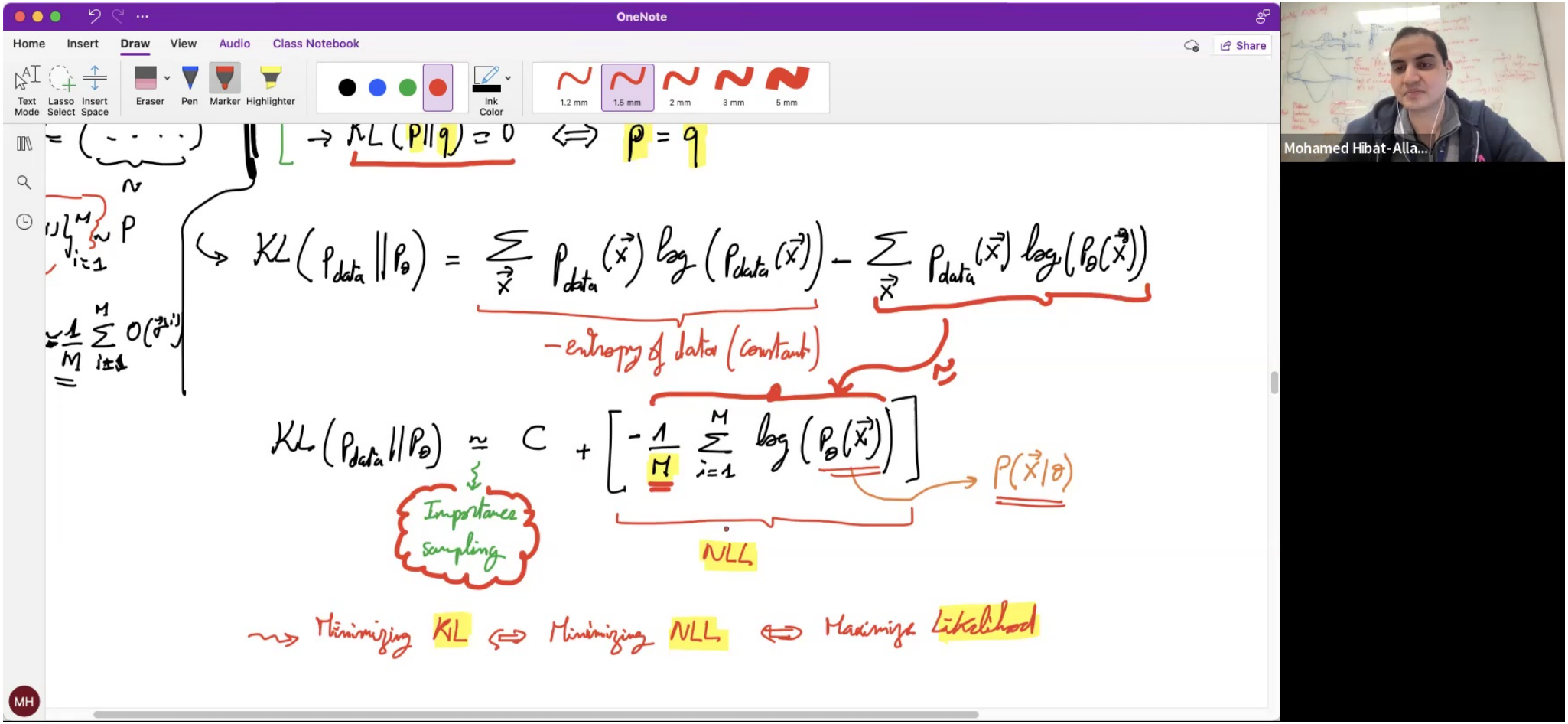
Probability chain rule:
RNN
(Next lecture)
PixelCNNs
Transformers
Autoregressive model
Sample
MH
Mohamed Hibat-Alla...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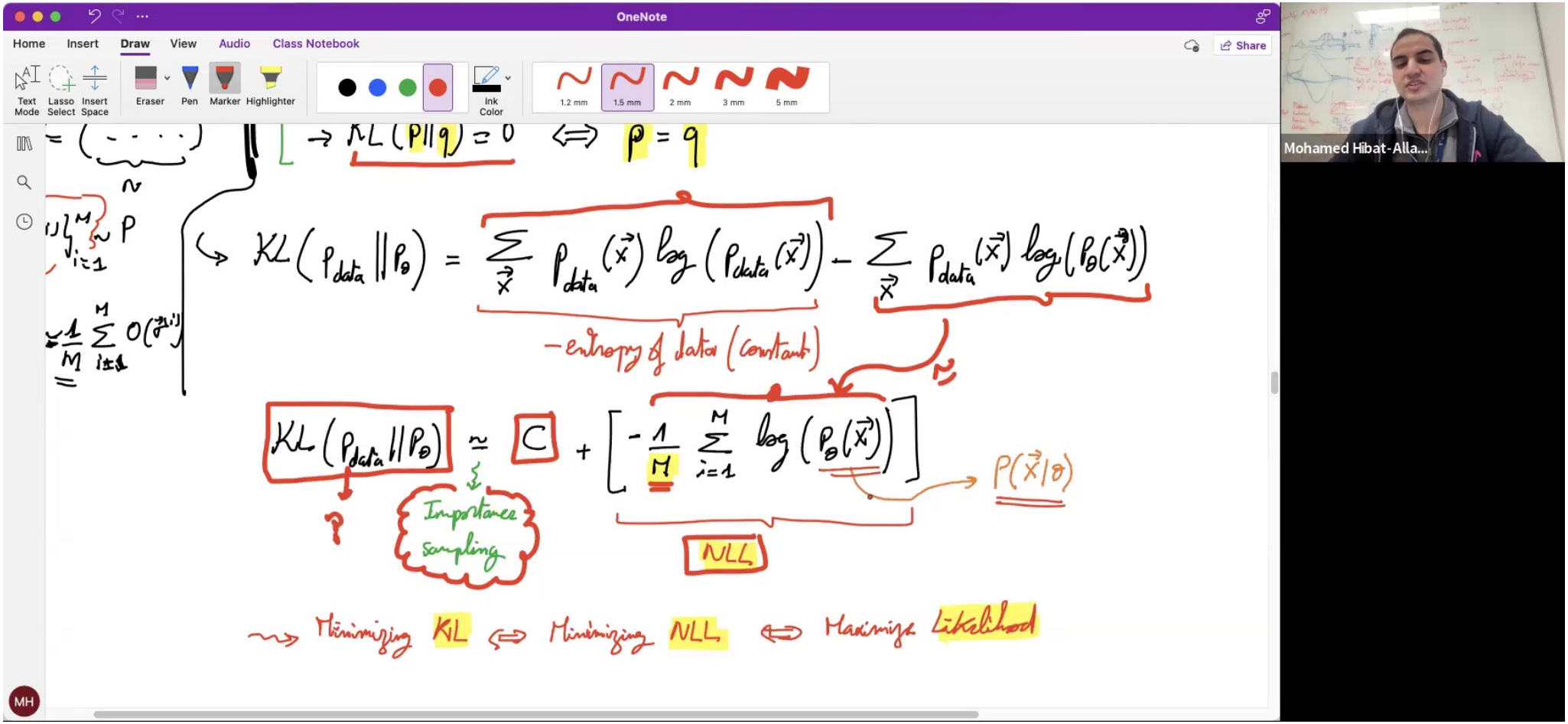
$KL(P\|q) \geq 0 \Leftrightarrow P = q$
$KL(P_{data}\|P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \log(P_{data}(\vec{x})) - \sum_{\vec{x}} P_{data}(\vec{x}) \log(P_\theta(\vec{x}))$
-entropy of data (constant)
$KL(P_{data}\|P_\theta) \simeq C + \left[-\frac{1}{M}\sum_{i=1}^{M} \log(P_\theta(\vec{x}^{(i)}))\right]$
Importance sampling
NLL
$P(\vec{x}|\theta)$
Minimizing KL $\Leftrightarrow$ Minimizing NLL $\Leftrightarrow$ Maximize Likelihood
Mohamed Hibat-Alla...
MH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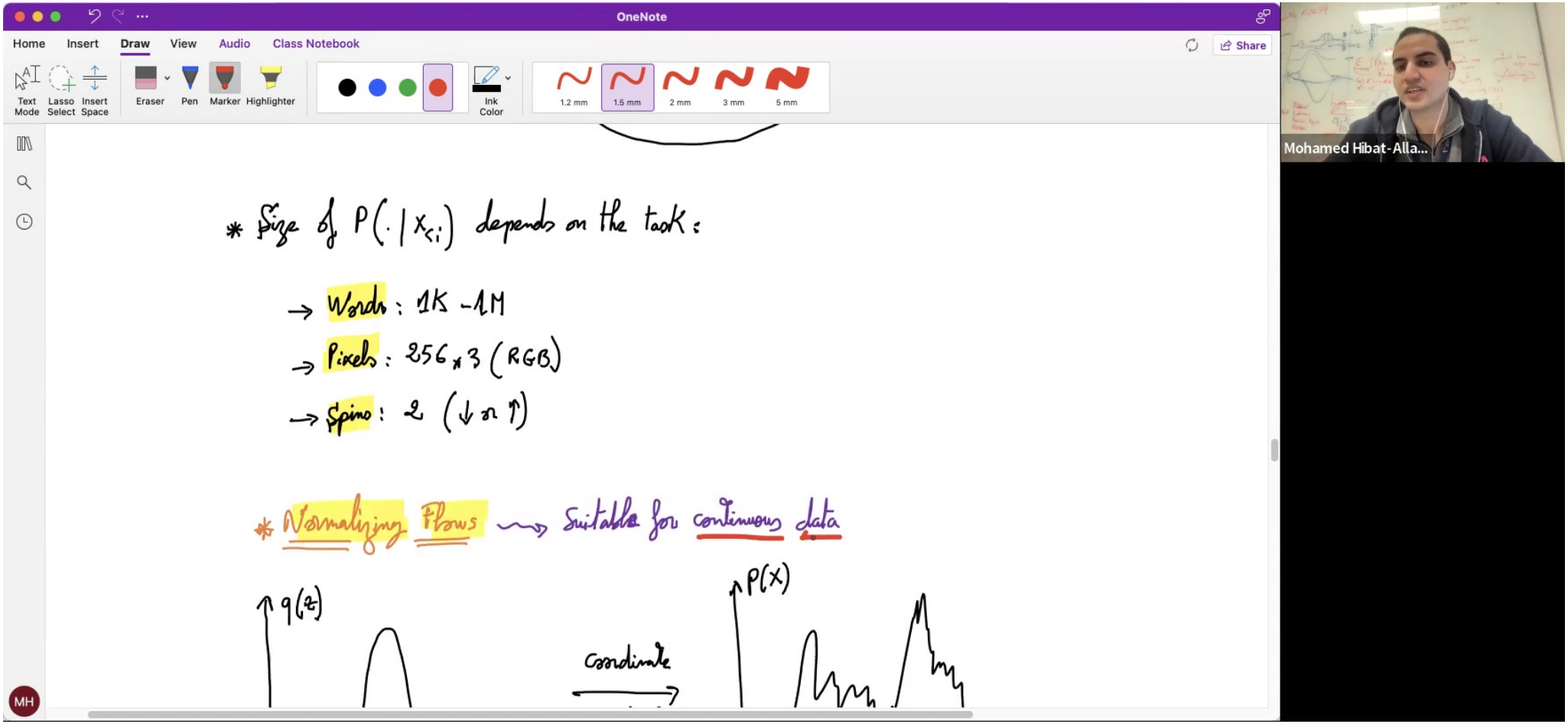
$KL(P\|q) \geq 0 \Leftrightarrow P = q$
$KL(P_{data}\|P_\theta) = \sum_{\vec{x}} P_{data}(\vec{x}) \log(P_{data}(\vec{x})) - \sum_{\vec{x}} P_{data}(\vec{x}) \log(P_\theta(\vec{x}))$
- entropy of data (constant)
$KL(P_{data}\|P_\theta) \simeq C + \left[-\frac{1}{M}\sum_{i=1}^{M} \log(P_\theta(\vec{x}))\right]$
Importance sampling
NLL
$P(\vec{x}|\theta)$
Minimizing KL $\Leftrightarrow$ Minimizing NLL $\Leftrightarrow$ Maximize Likelihood
MH
Mohamed Hibat-Alla...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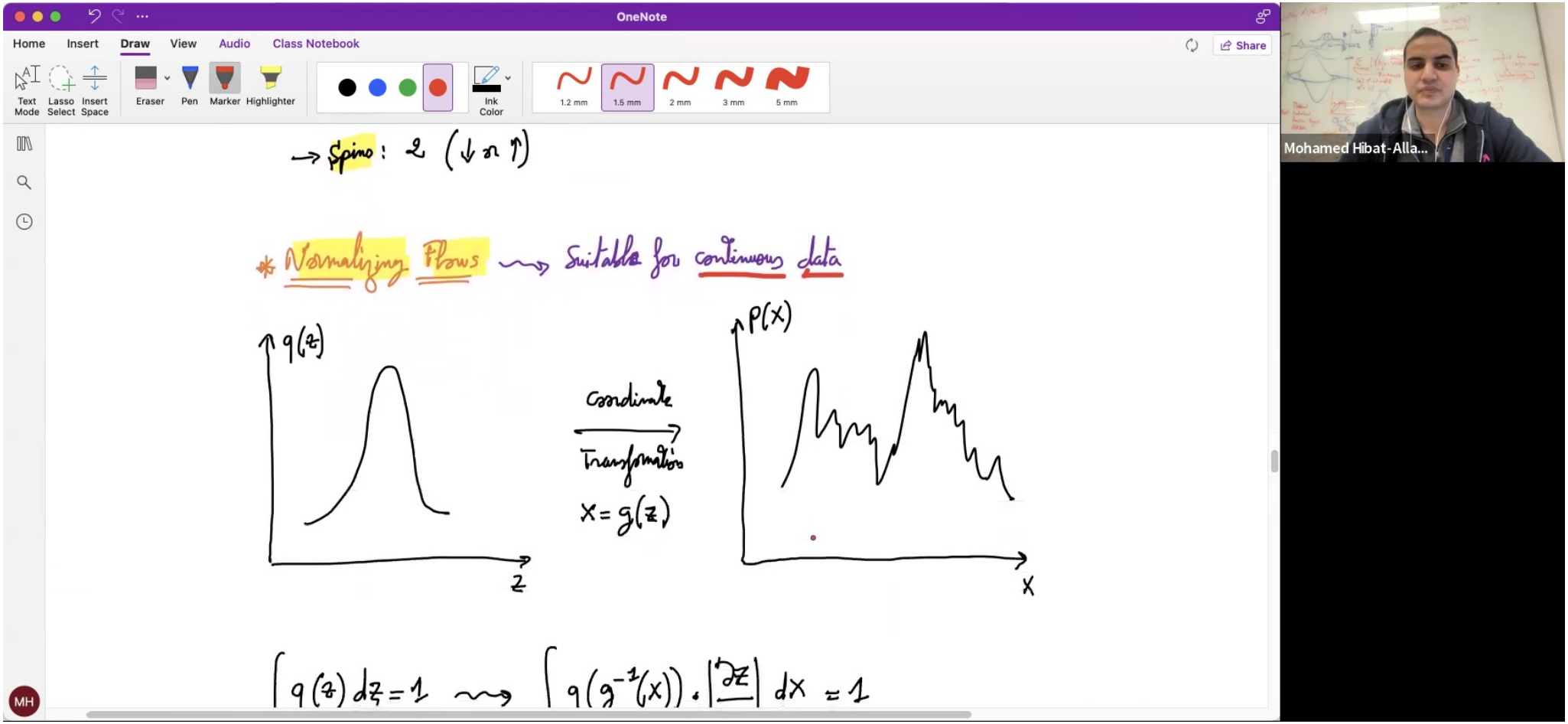
* Size of $P(\cdot | x_{<i})$ depends on the task:
→ Words: 1K - 1M
→ Pixels: 256 x 3 (RGB)
→ Spins: 2 (↓ or ↑)
* Normalizing Flows ⇝ Suitable for continuous data
q(z)
Coordinate
P(x)
MH

Mohamed Hibat-Alla...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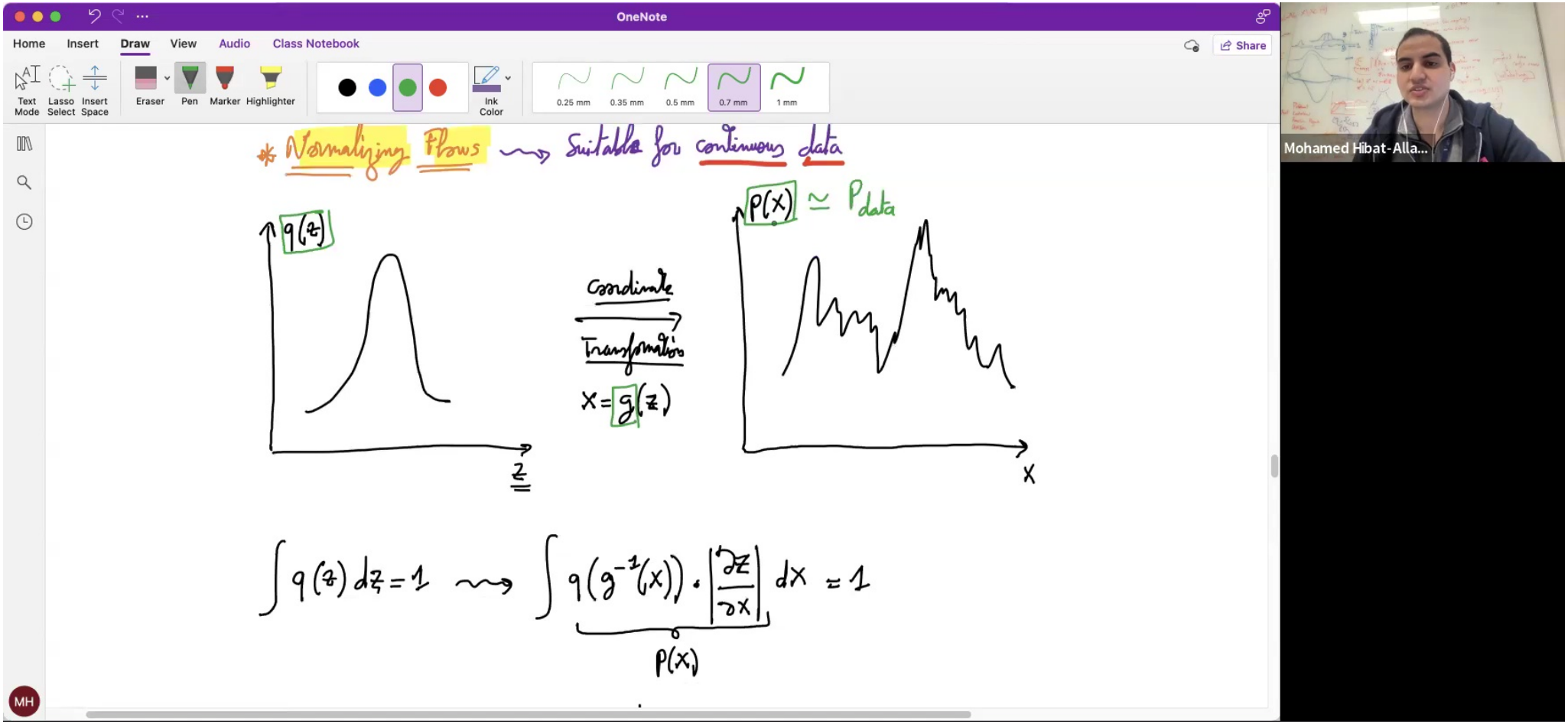
→ Spins: 2 (↓ or ↑)
* Normalizing Flows → Suitable for continuous data
q(z)
z
Coordinate Transformation
X = g(z)
p(x)
x
∫ q(z) dz = 1 → ∫ q(g⁻¹(x)) · |∂z| dx = 1
MH

Mohamed Hibat-Alla...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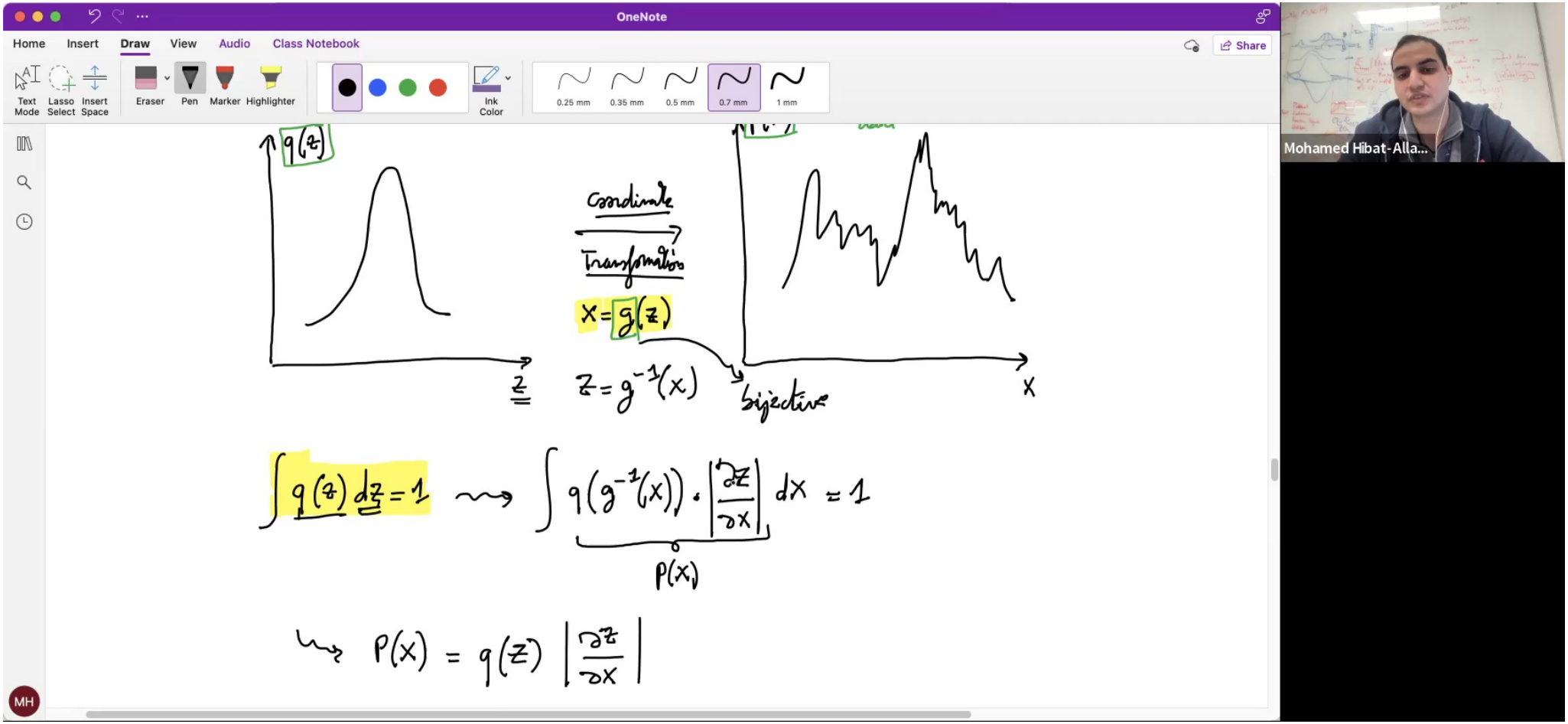
* Normalizing Flows ⇝ Suitable for continuous data
q(z)
z
coordinate Transformation
x = g(z)
p(x) ≃ p_data
x
∫ q(z) dz = 1 ⇝ ∫ q(g⁻¹(x)) · |∂z/∂x| dx = 1
p(x)
MH
Mohamed Hibat-Alla...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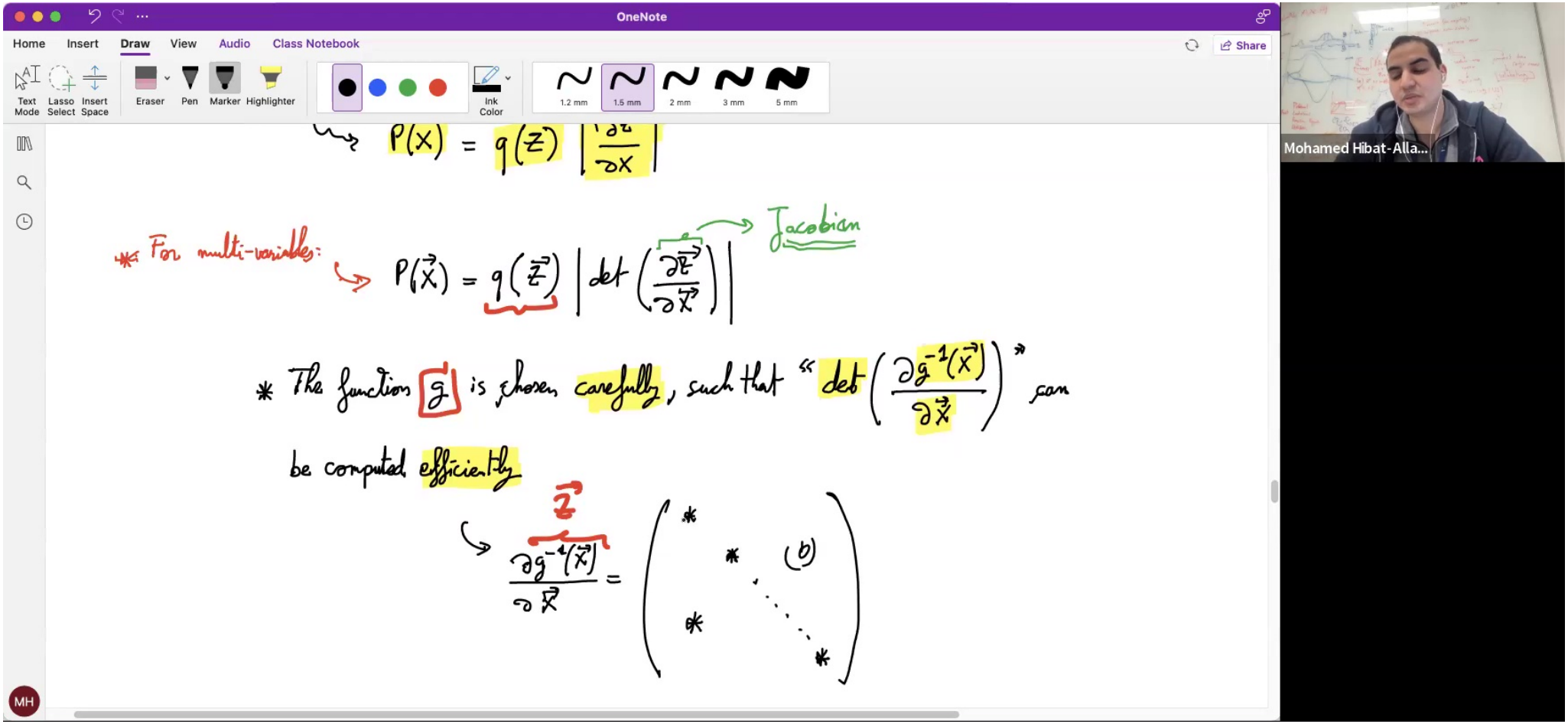
Mohamed Hibat-Alla...
MH
q(z)
z
Coordinate Transformation
X = g(z)
z = g⁻¹(X)
bijective
X
∫ q(z) dz = 1
∫ q(g⁻¹(X)) · |∂z/∂X| dX = 1
P(X)
P(X) = q(z) |∂z/∂X|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$P(x) = q(z) \left| \frac{\partial z}{\partial x} \right|$
* For multi-variables:
$P(\vec{x}) = q(\vec{z}) \left| \det \left(\frac{\partial \vec{z}}{\partial \vec{x}} \right) \right|$
Jacobian
* The function g is chosen carefully, such that "$\det \left(\frac{\partial g^{-1}(\vec{x})}{\partial \vec{x}} \right)$" can be computed efficiently
$\frac{\partial g^{-1}(\vec{x})}{\partial \vec{x}} = \begin{pmatrix} * & & & \\ & * & & (0) \\ & & \ddots & \\ & * & & * \end{pmatrix}$
MH

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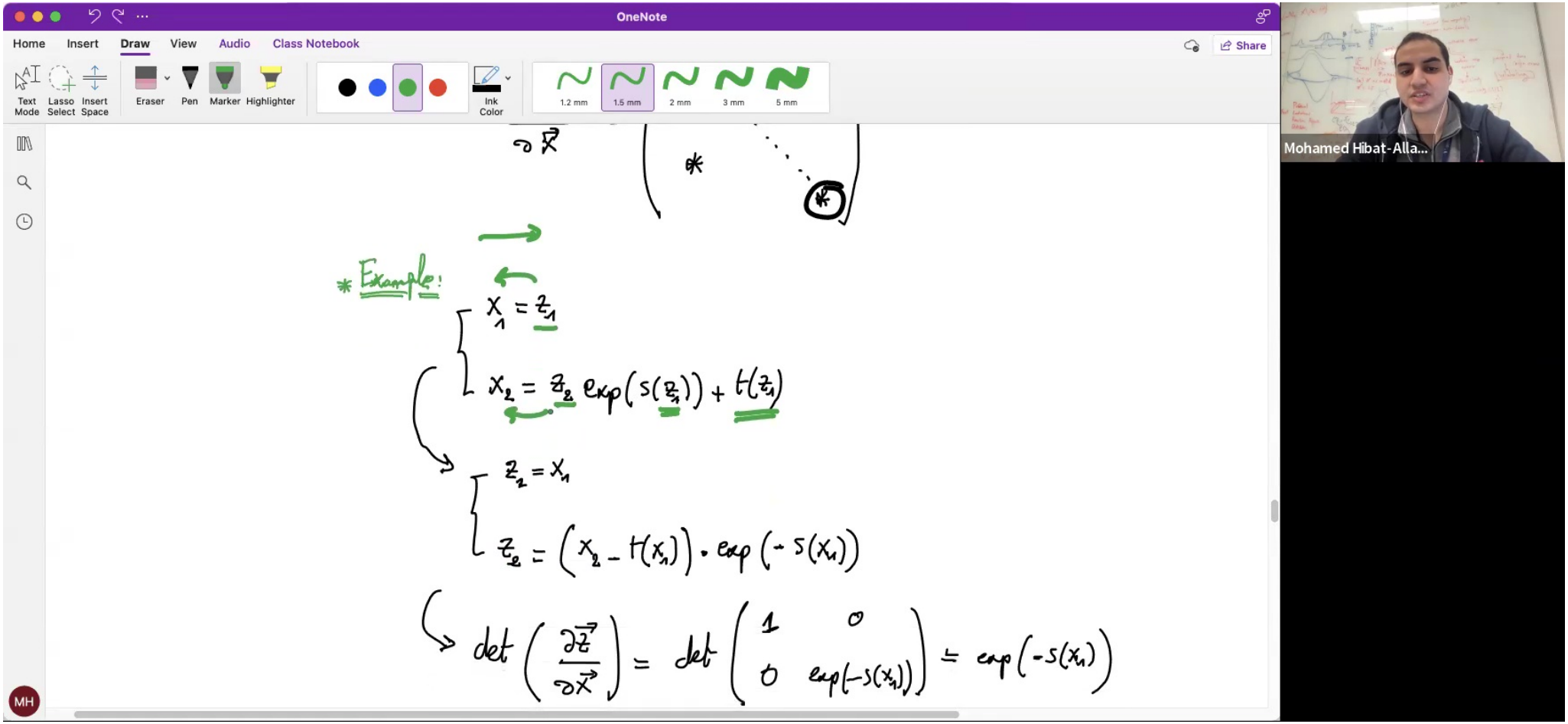
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Example:
$x_1 = z_1$
$x_2 = z_2 \exp(s(z_1)) + t(z_1)$
$z_2 = x_1$
$z_2 = (x_2 - t(x_1)) \cdot \exp(-s(x_1))$
$\det\left(\frac{\partial \vec{z}}{\partial \vec{x}}\right) = \det\begin{pmatrix} 1 & 0 \\ 0 & \exp(-s(x_1)) \end{pmatrix} = \exp(-s(x_1))$
MH

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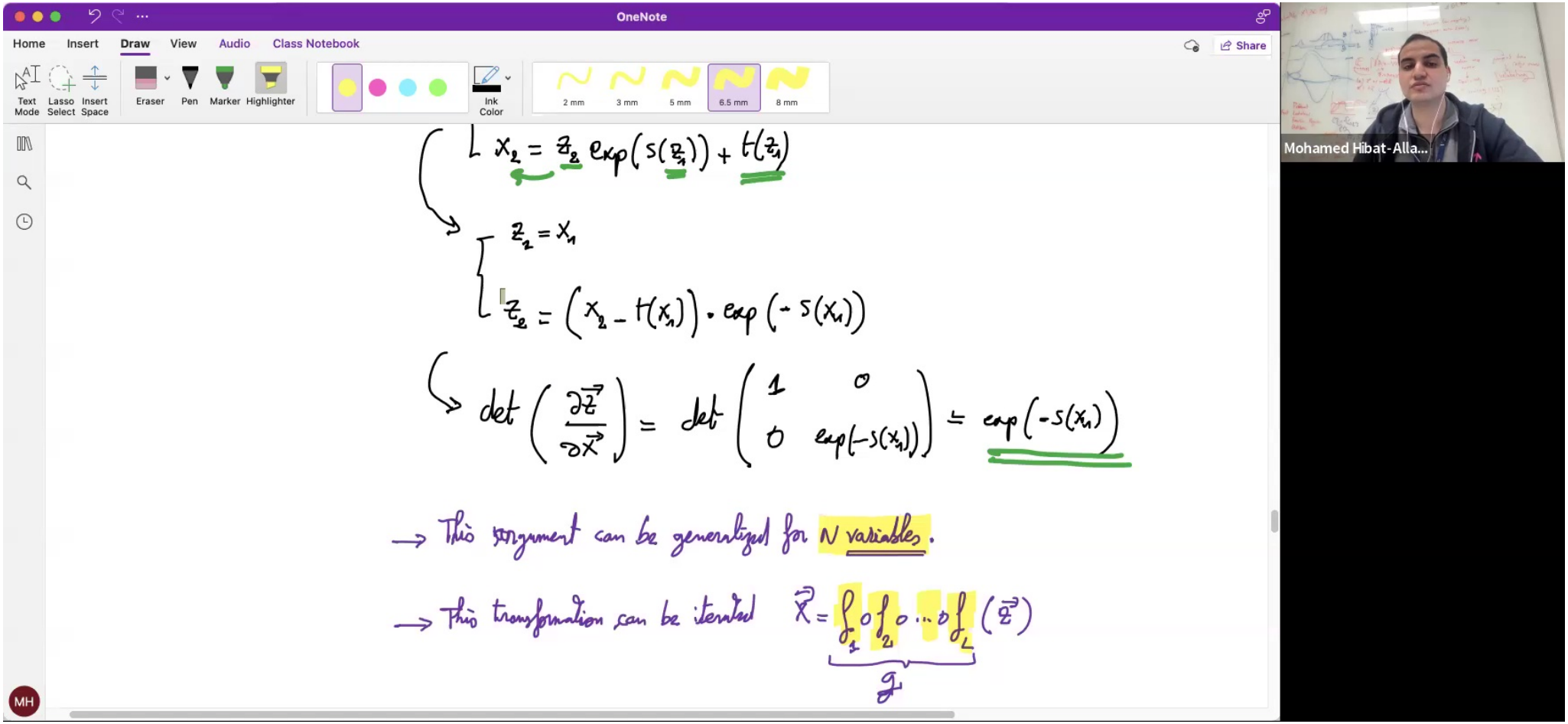
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$x_2 = z_2 \exp(s(z_1)) + t(z_1)$
$z_1 = x_1$
$z_2 = (x_2 - t(x_1)) \cdot \exp(-s(x_1))$
$\det\left(\frac{\partial \vec{z}}{\partial \vec{x}}\right) = \det\begin{pmatrix} 1 & 0 \\ 0 & \exp(-s(x_1)) \end{pmatrix} = \exp(-s(x_1))$
→ This argument can be generalized for N variables.
→ This transformation can be iterated $\vec{x} = f_1 \circ f_2 \circ \dots \circ f_L(\vec{z})$
g
MH

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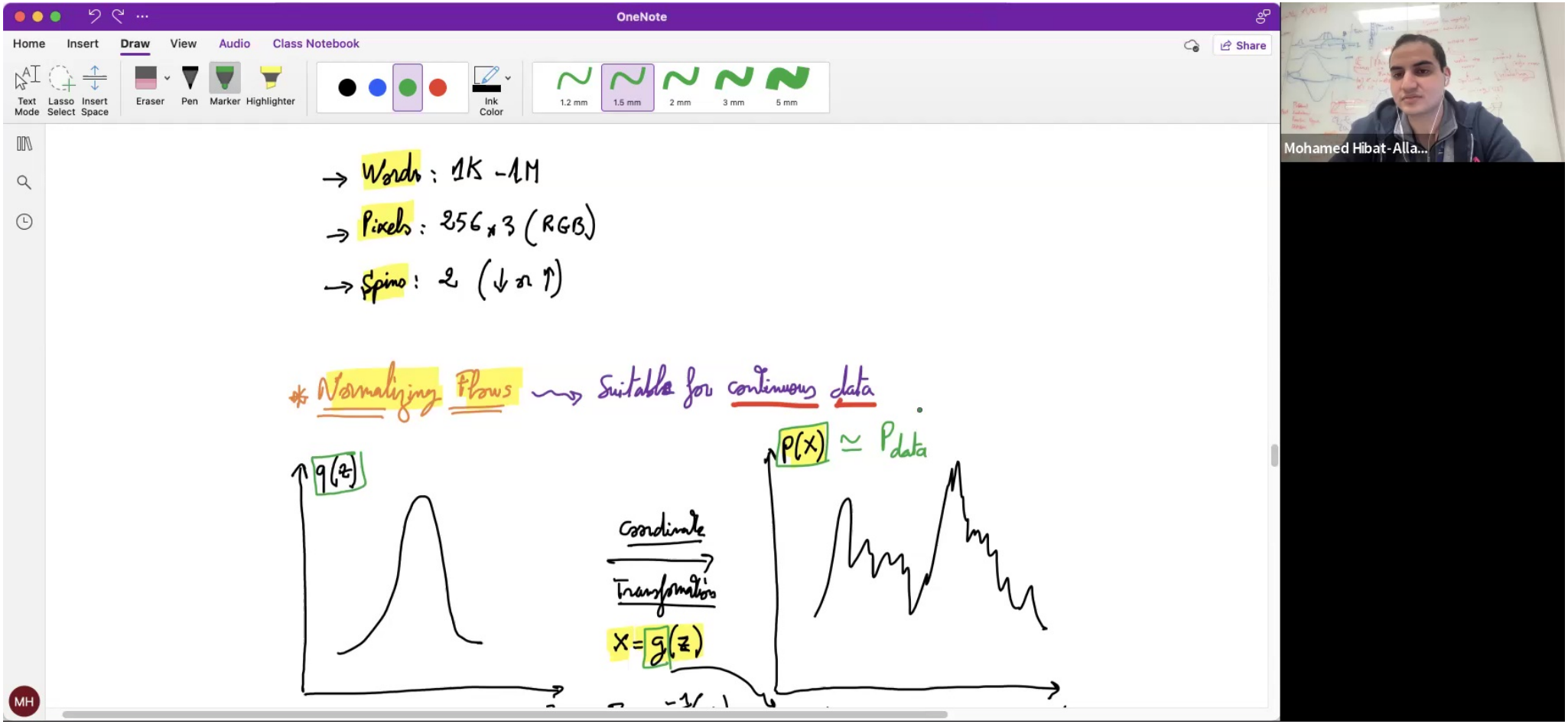
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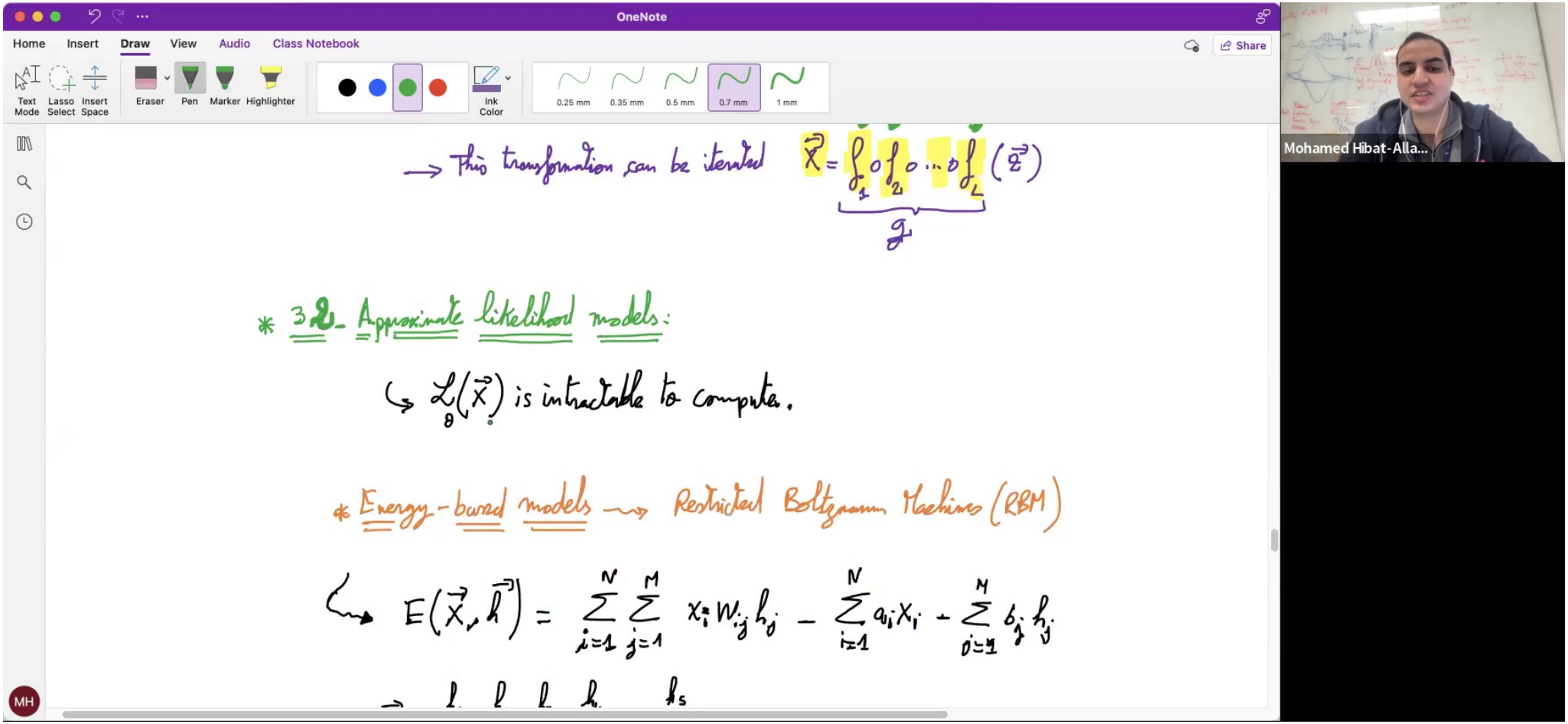
→ Words : 1K - 1M
→ Pixels : 256 x 3 (RGB)
→ Spins : 2 (↓ or ↑)
* Normalizing Flows ⇝ Suitable for continuous data
q(z)
Coordinate Transformation
X = g(z)
p(x) ≃ p_data
MH
Mohamed Hibat-Alla...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→ This transformation can be iterated $\vec{x} = \underbrace{f_1 \circ f_2 \circ \dots \circ f_L}_{g}(\vec{z})$
* 3.2- Approximate likelihood models:
↳ $\mathcal{L}_\theta(\vec{x})$ is intractable to compute.
* Energy-based models → Restricted Boltzmann Machines (RBM)
↳ $E(\vec{x}, \vec{h}) = \sum_{i=1}^{N}\sum_{j=1}^{M} x_i W_{ij} h_j - \sum_{i=1}^{N} a_i x_i - \sum_{j=1}^{M} b_j h_j$
→ h_1 h_2 h_3 h_4 h_5
MH
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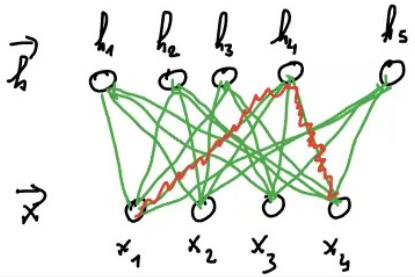
* 3.2- Approximate likelihood models:

↳ $\mathcal{L}_\theta(\vec{x})$ is intractable to compute.

* Energy-based models ⇝ Restricted Boltzmann Machines (RBM)

↳ $E(\vec{x}, \vec{h}) = \sum_{i=1}^{N}\sum_{j=1}^{M} x_i w_{ij} h_j - \sum_{i=1}^{N} a_i x_i - \sum_{j=1}^{M} b_j h_j$



⇝ Correlation between x_1 and x_4 is introduced through h_4.

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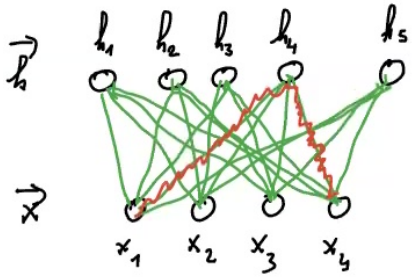
* Energy-based models ⇝ Restricted Boltzmann Machines (RBM)

$$E(\vec{x}, \vec{h}) = \sum_{i=1}^{N} \sum_{j=1}^{M} x_i W_{ij} h_j - \sum_{i=1}^{N} a_i x_i - \sum_{j=1}^{M} b_j h_j$$

$\vec{x} = (\ ) \updownarrow N$

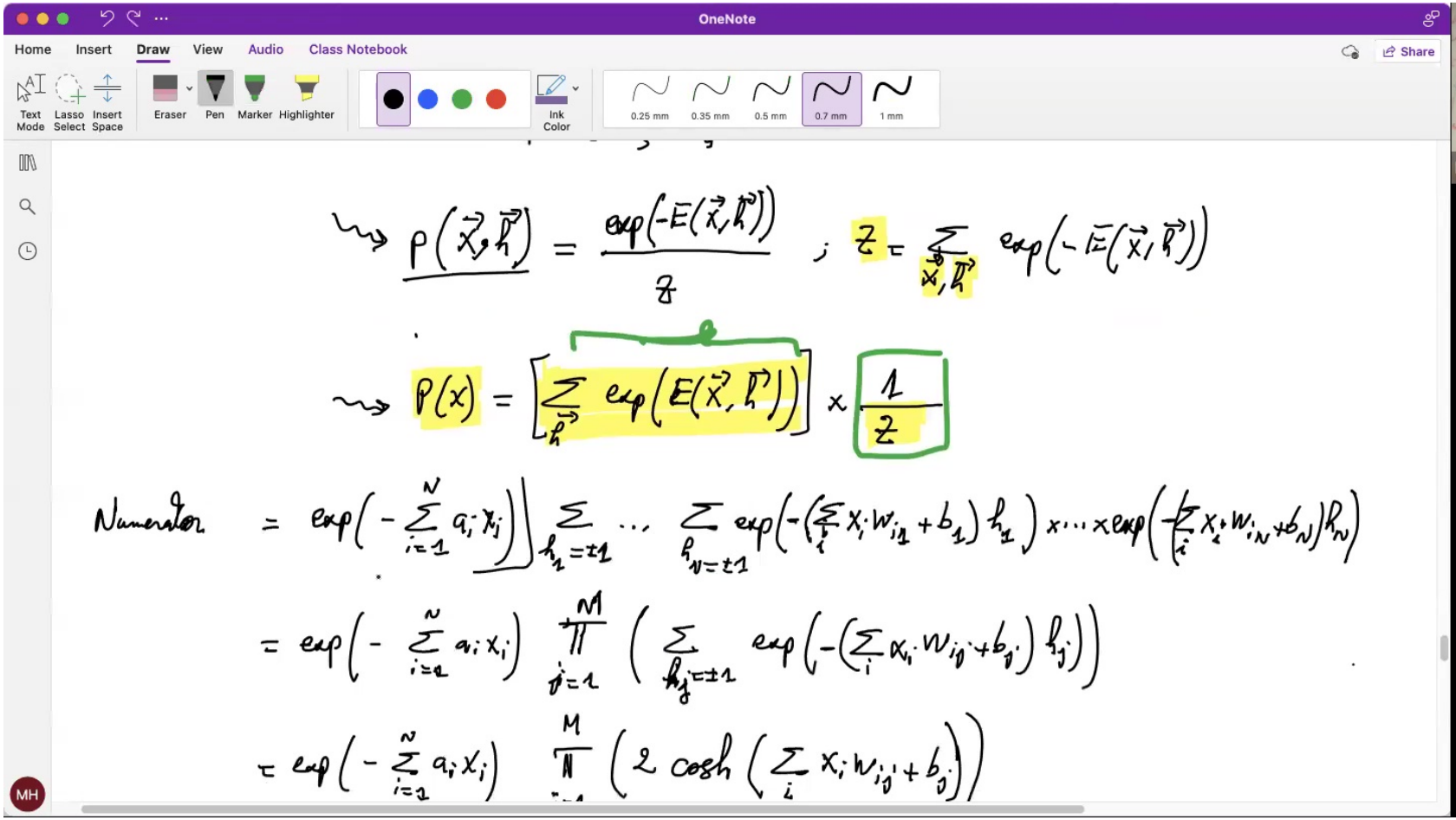
$\vec{h} = (\ ) \updownarrow M$



⇝ Correlation between x_1 and x_4 is introduced through h_4.

$$\rightsquigarrow p(\vec{x}, \vec{h}) = \frac{\exp(-E(\vec{x}, \vec{h}))}{Z} \quad ; \quad Z = \sum_{\vec{x}, \vec{h}} \exp(-E(\vec{x}, \vec{h}))$$

MH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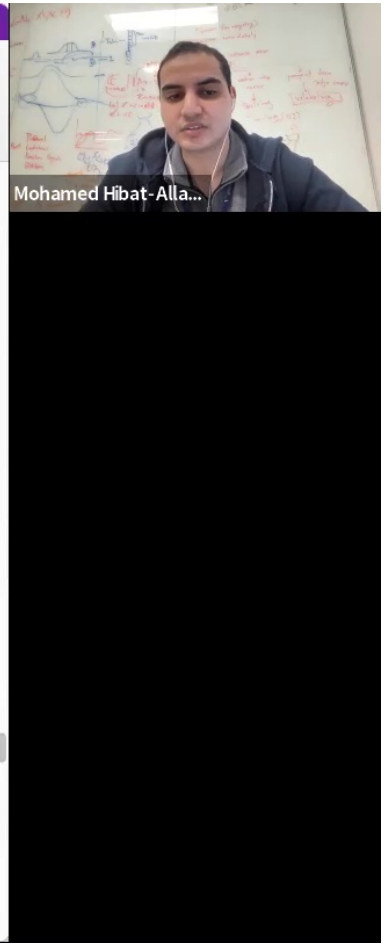
$\rightsquigarrow p(\vec{x},\vec{h}) = \frac{\exp(-E(\vec{x},\vec{h}))}{Z}$; $Z = \sum_{\vec{x},\vec{h}} \exp(-E(\vec{x},\vec{h}))$
$\rightsquigarrow p(x) = \left[\sum_{\vec{h}} \exp(E(\vec{x},\vec{h}))\right] \times \frac{1}{Z}$
Numerator $= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \sum_{h_1=\pm 1} \cdots \sum_{h_N=\pm 1} \exp\left(-\left(\sum_i x_i w_{i1} + b_1\right) h_1\right) \times \cdots \times \exp\left(-\left(\sum_i x_i w_{iN} + b_N\right) h_N\right)$
$= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \prod_{j=1}^{M} \left(\sum_{h_j=\pm 1} \exp\left(-\left(\sum_i x_i w_{ij} + b_j\right) h_j\right)\right)$
$= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \prod_{j=1}^{M} \left(2\cosh\left(\sum_i x_i w_{ij} + b_j\right)\right)$
MH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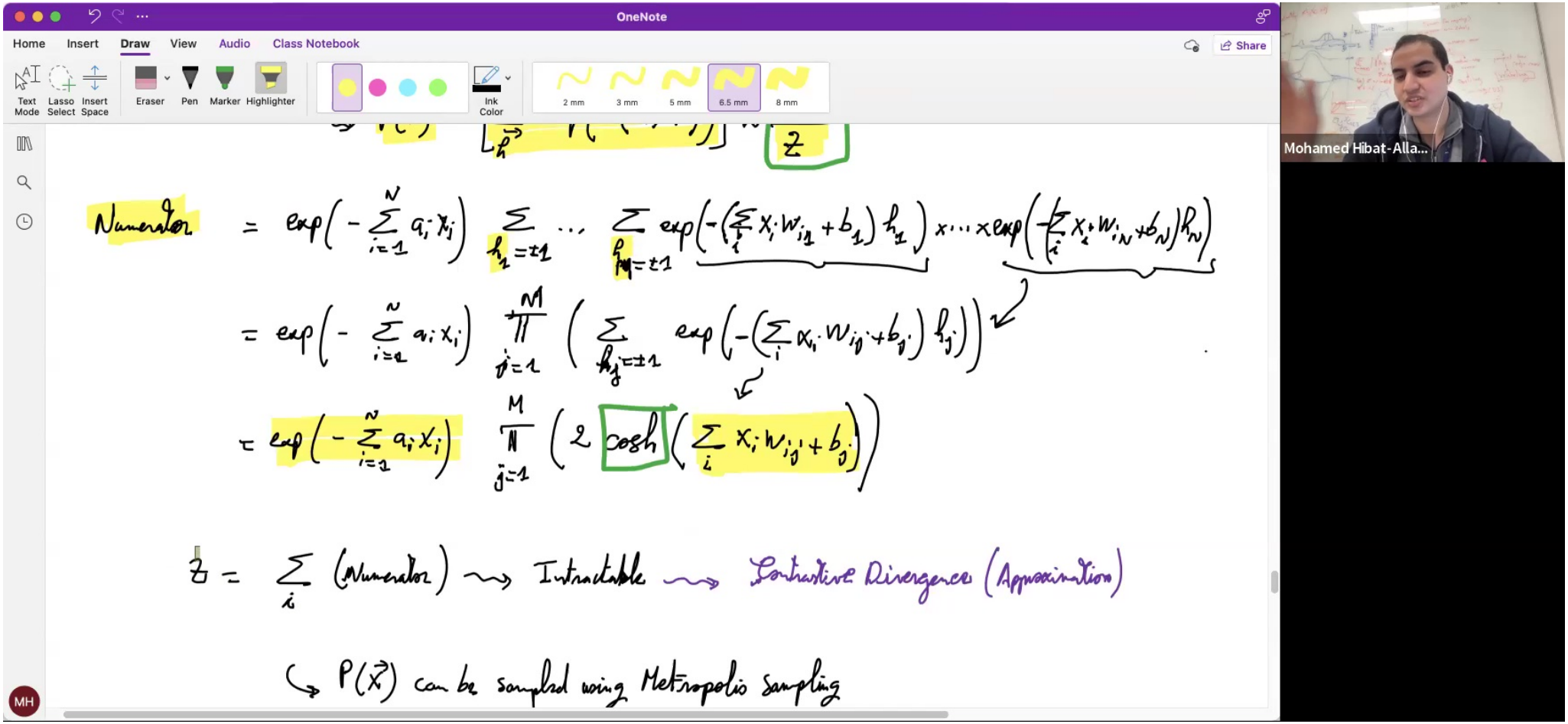
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Numerator
Z = Σ (Numerator) ⇝ Intractable ⇝ Contrastive Divergence (Approximation)
↳ P(x⃗) can be sampled using Metropolis sampling
MH

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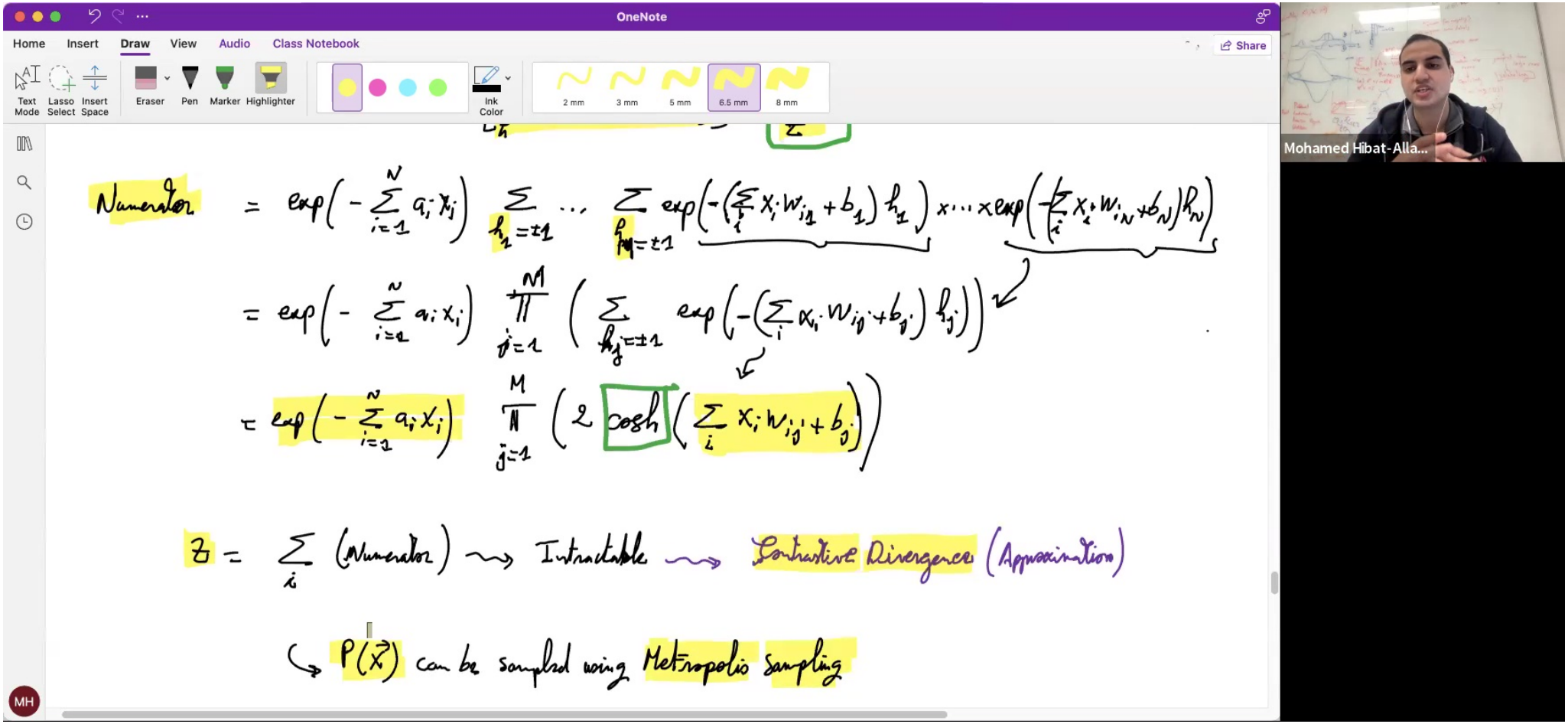
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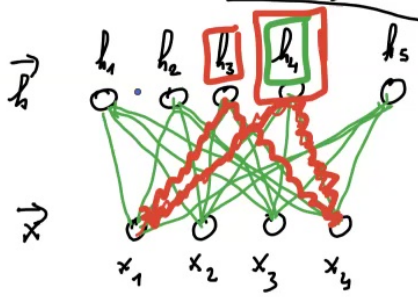
Numerator
Contrastive Divergence (Approximation)
Intractable
Metropolis sampling
Mohamed Hibat-Alla...
MH

* Energy-based models ⇝ Restricted Boltzmann Machines (RBM)

$$E(\vec{x}, \vec{h}) = \sum_{i=1}^{N}\sum_{j=1}^{M} x_i w_{ij} h_j - \sum_{i=1}^{N} a_i x_i - \sum_{j=1}^{M} b_j h_j$$

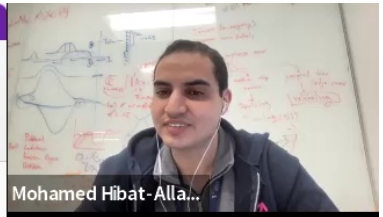
$\vec{x} = ( )^{N}$

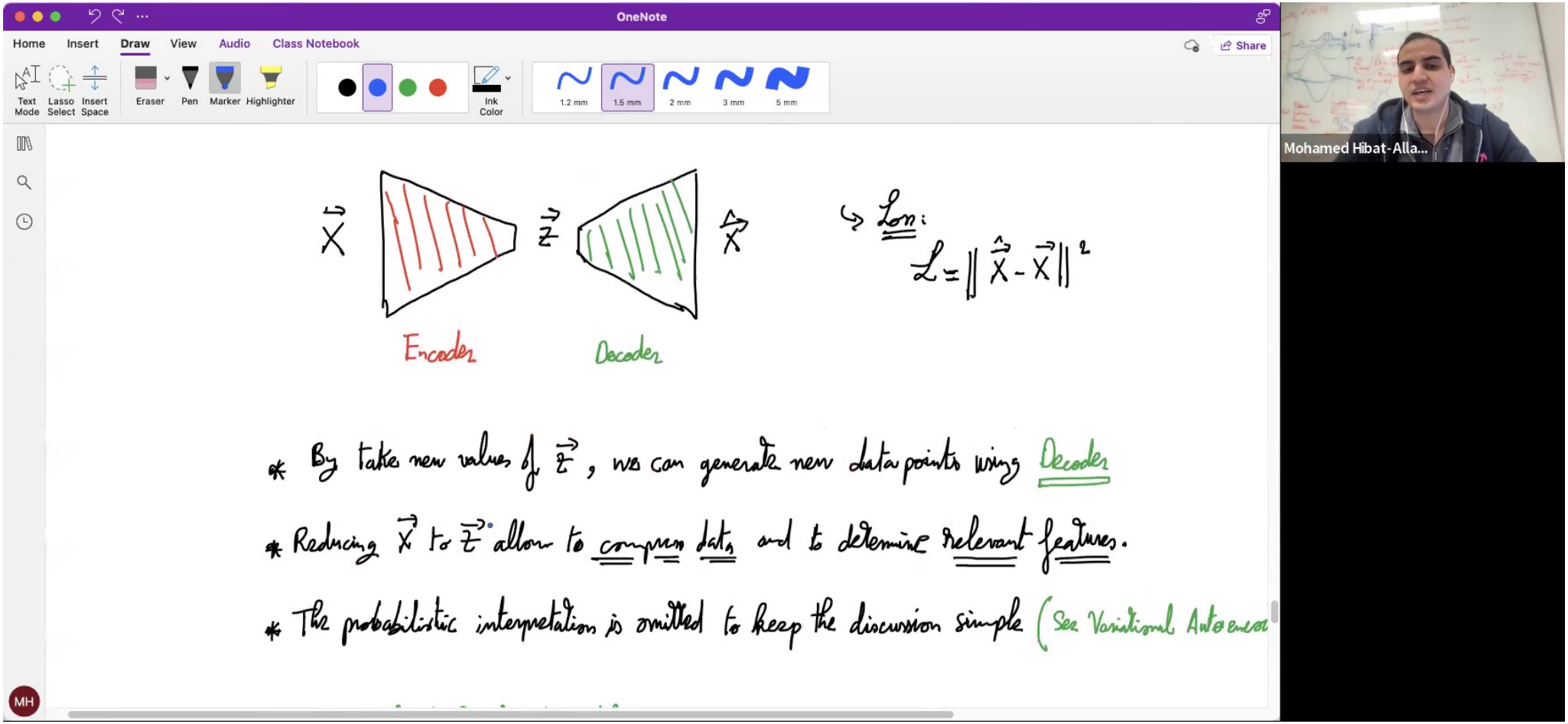
$\vec{h} = ( )^{M}$



⇝ Correlation between x_1 and x_4 is introduced through h_4.

$$\rightsquigarrow p(\vec{x}, \vec{h}) = \frac{\exp(-E(\vec{x}, \vec{h}))}{Z} \quad ; \quad Z = \sum_{\vec{x}, \vec{h}} \exp(-E(\vec{x}, \vec{h}))$$

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Encoder
Decoder
↳ Loss:
$\mathcal{L} = \| \hat{\vec{X}} - \vec{X} \|^2$
* By taking new values of $\vec{z}$, we can generate new data points using Decoder
* Reducing $\vec{X}$ to $\vec{z}$ allow to compress data and to determine relevant features.
* The probabilistic interpretation is omitted to keep the discussion simple (See Variational Autoencoder
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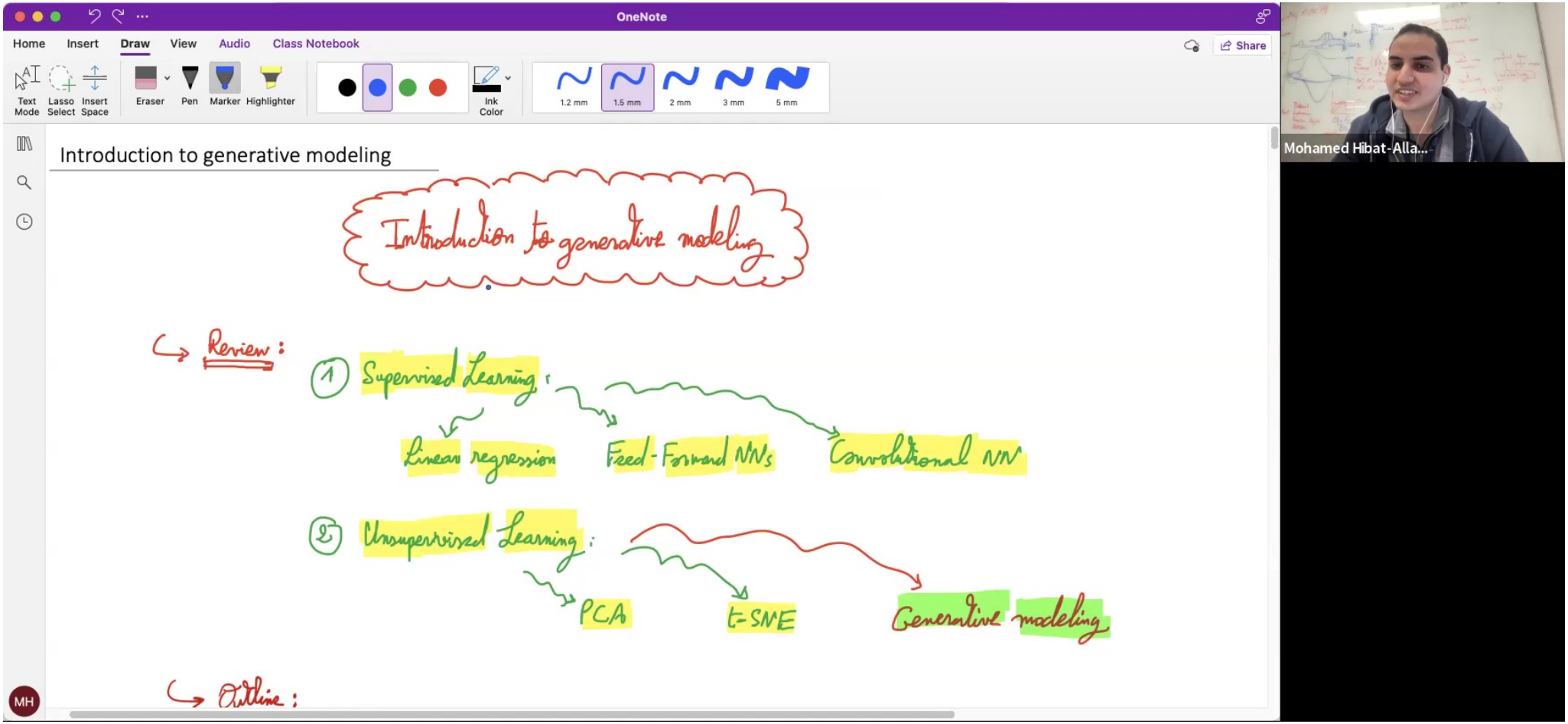
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Introduction to generative modeling
Introduction to generative modeling
Review :
1 Supervised Learning :
Linear regression
Feed-Forward NNs
Convolutional NN
2 Unsupervised Learning :
PCA
t-SNE
Generative modeling
Outline :
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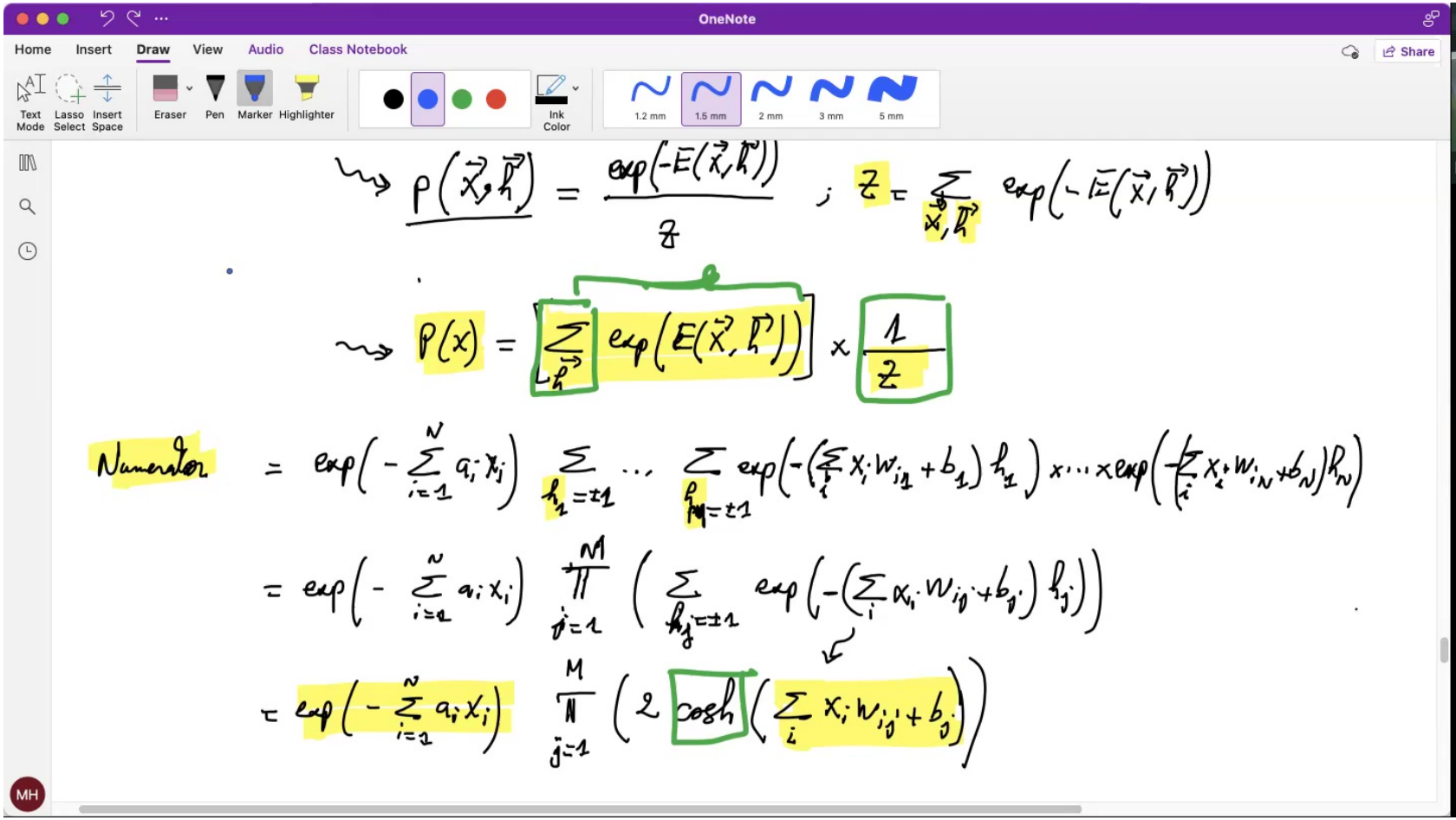
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$\rightsquigarrow P(\vec{x},\vec{h}) = \frac{\exp(-E(\vec{x},\vec{h}))}{Z}$; $Z = \sum_{\vec{x},\vec{h}} \exp(-E(\vec{x},\vec{h}))$
$\rightsquigarrow P(x) = \left[\sum_{\vec{h}} \exp(E(\vec{x},\vec{h}))\right] \times \frac{1}{Z}$
Numerator
$= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \sum_{h_1=\pm 1} \cdots \sum_{h_M=\pm 1} \exp\left(-\left(\sum_i x_i w_{i1} + b_1\right) h_1\right) \times \cdots \times \exp\left(-\left(\sum_i x_i w_{iN} + b_N\right) h_N\right)$
$= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \prod_{j=1}^{M} \left(\sum_{h_j=\pm 1} \exp\left(-\left(\sum_i x_i w_{ij} + b_j\right) h_j\right)\right)$
$= \exp\left(-\sum_{i=1}^{N} a_i x_i\right) \prod_{j=1}^{M} \left(2 \cosh\left(\sum_i x_i w_{ij} + b_j\right)\right)$
MH


Perimeter-A