Title: Machine Learning (2021/2022)

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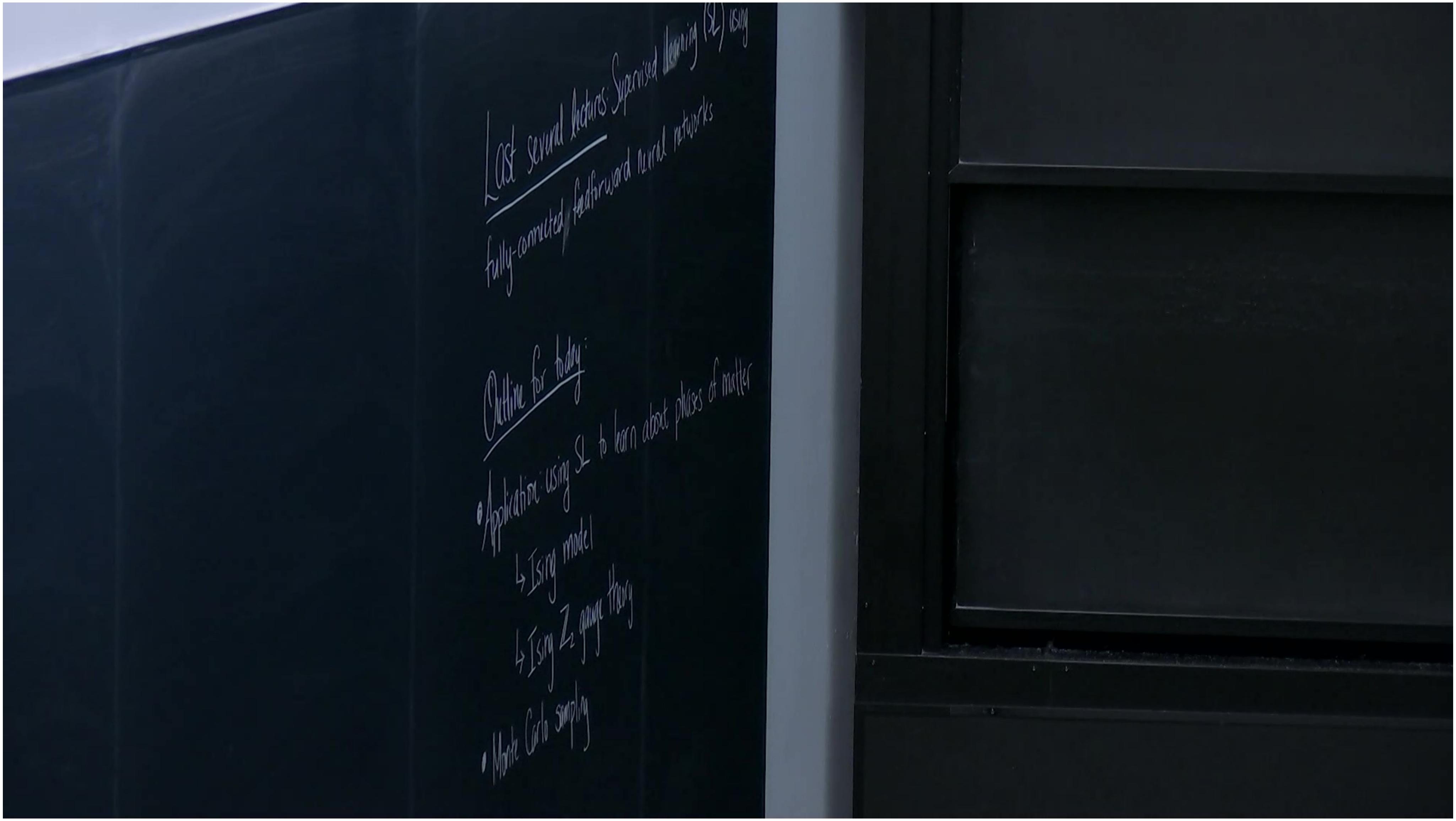
Collection: Machine Learning (2021/2022)

Date: April 18, 2022 - 11:30 AM

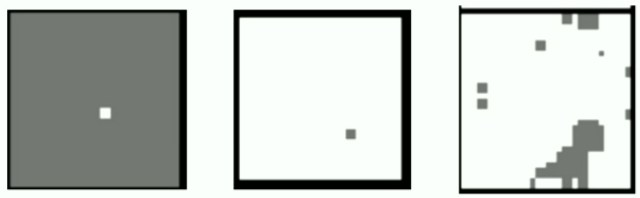
URL: https://pirsa.org/22040070

Abstract: This course is designed to introduce modern machine learning techniques for studying classical and quantum many-body problems encountered in condensed matter, quantum information, and related fields of physics. Lectures will focus on introducing machine learning algorithms and discussing how they can be applied to solve problem in statistical physics. Tutorials and homework assignments will concentrate on developing programming skills to study the problems presented in lecture.

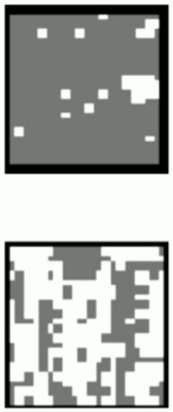

Last several lectures: Supervised learning (SL) using
fully-connected feedforward neural networks
Outline for today:
• Application: using SL to learn about phases of matter
↳ Ising model
↳ Ising Z_2 gauge theory
• Monte Carlo sampling

Ising model classification

Ferromagnetic phase:



Paramagnetic phase:



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Carrasquilla and Melko, arXiv:1605.01735

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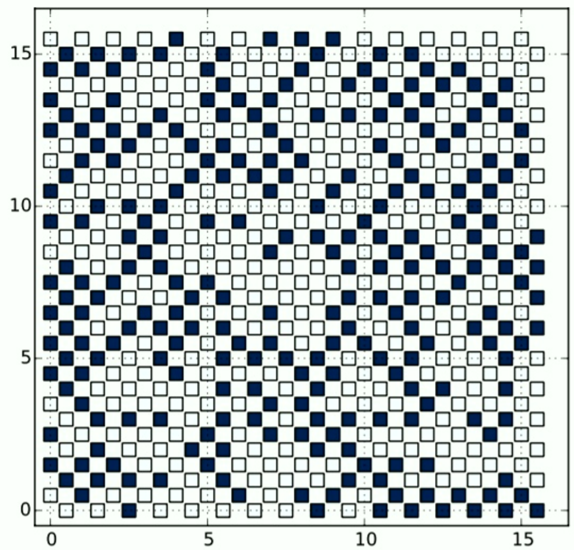
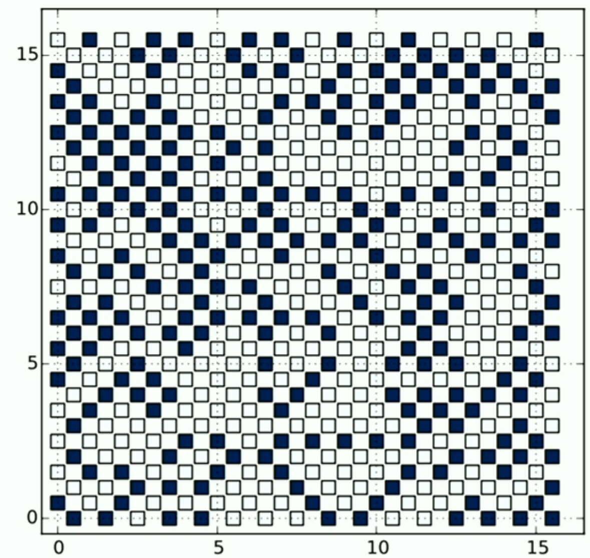
Ising gauge theory classification





Carrasquilla and Melko, arXiv:1605.01735

Distinguishing classical phases of matter using SL

see Carrasquilla & Melko, arXiv: 1605.01735

Classical Ising Model

$$H = -J \sum_{\langle ij \rangle} s_i s_j \,, \quad s_i = +1 \text{ or } -1$$

$$(\uparrow \text{ or } \downarrow)$$

$$(\blacksquare \text{ or } \square)$$

The d.o.f. are sites of a lattice

eg) Fo

sing SL

d.o.f. are
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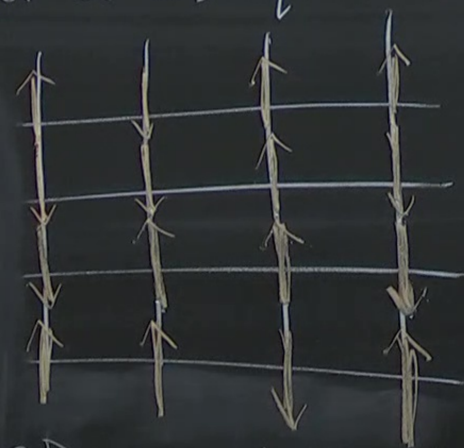
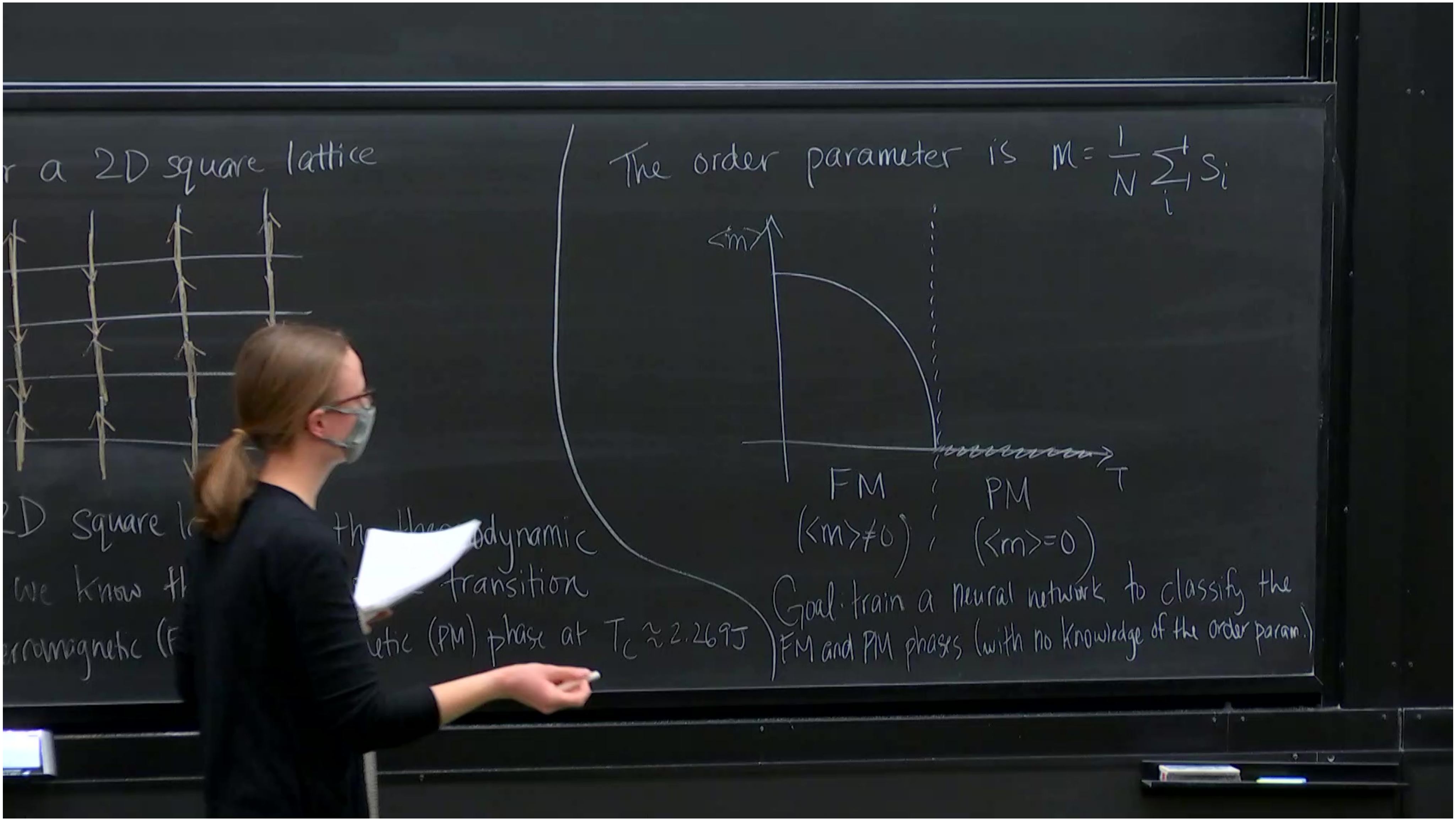
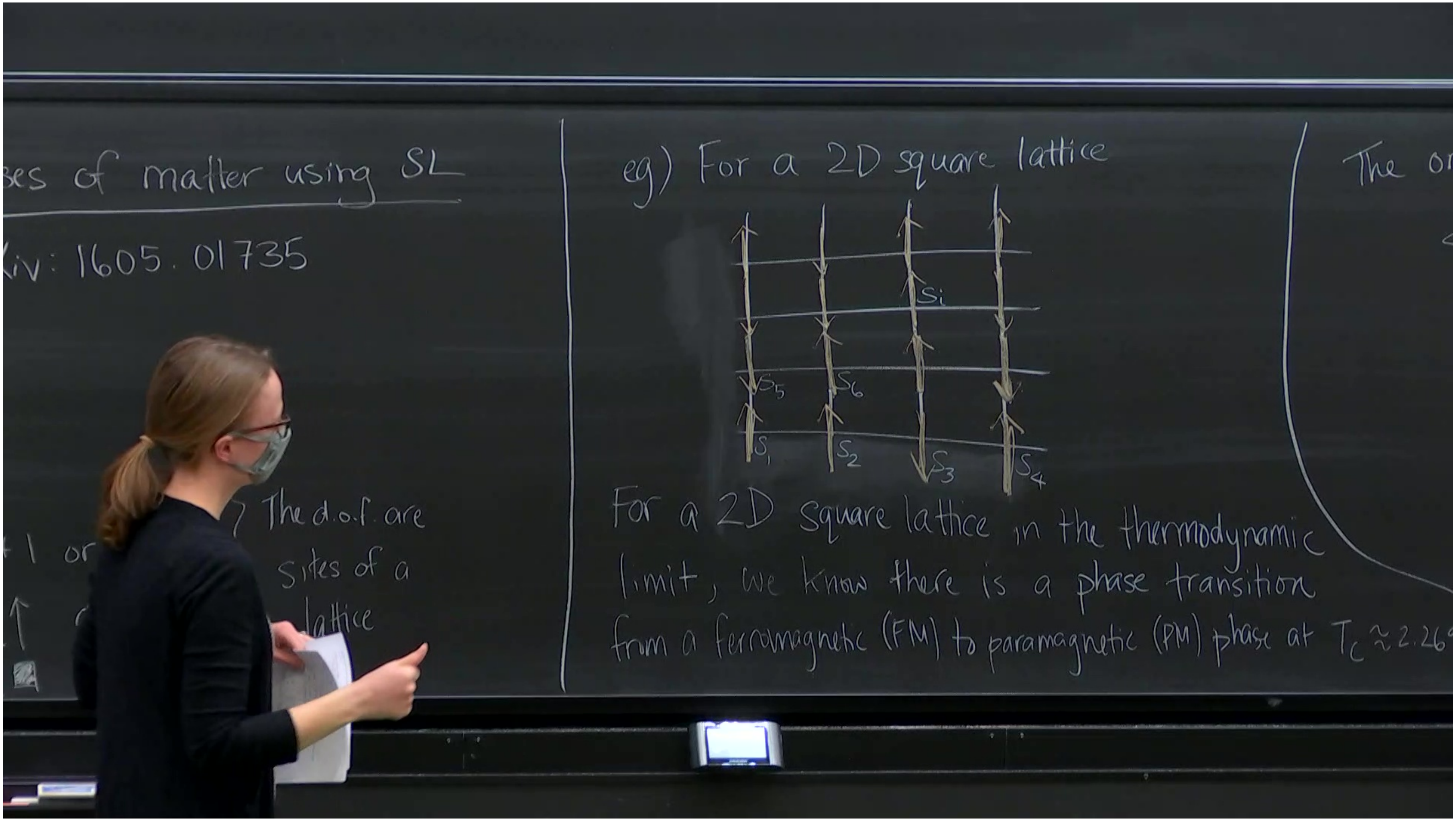
attice

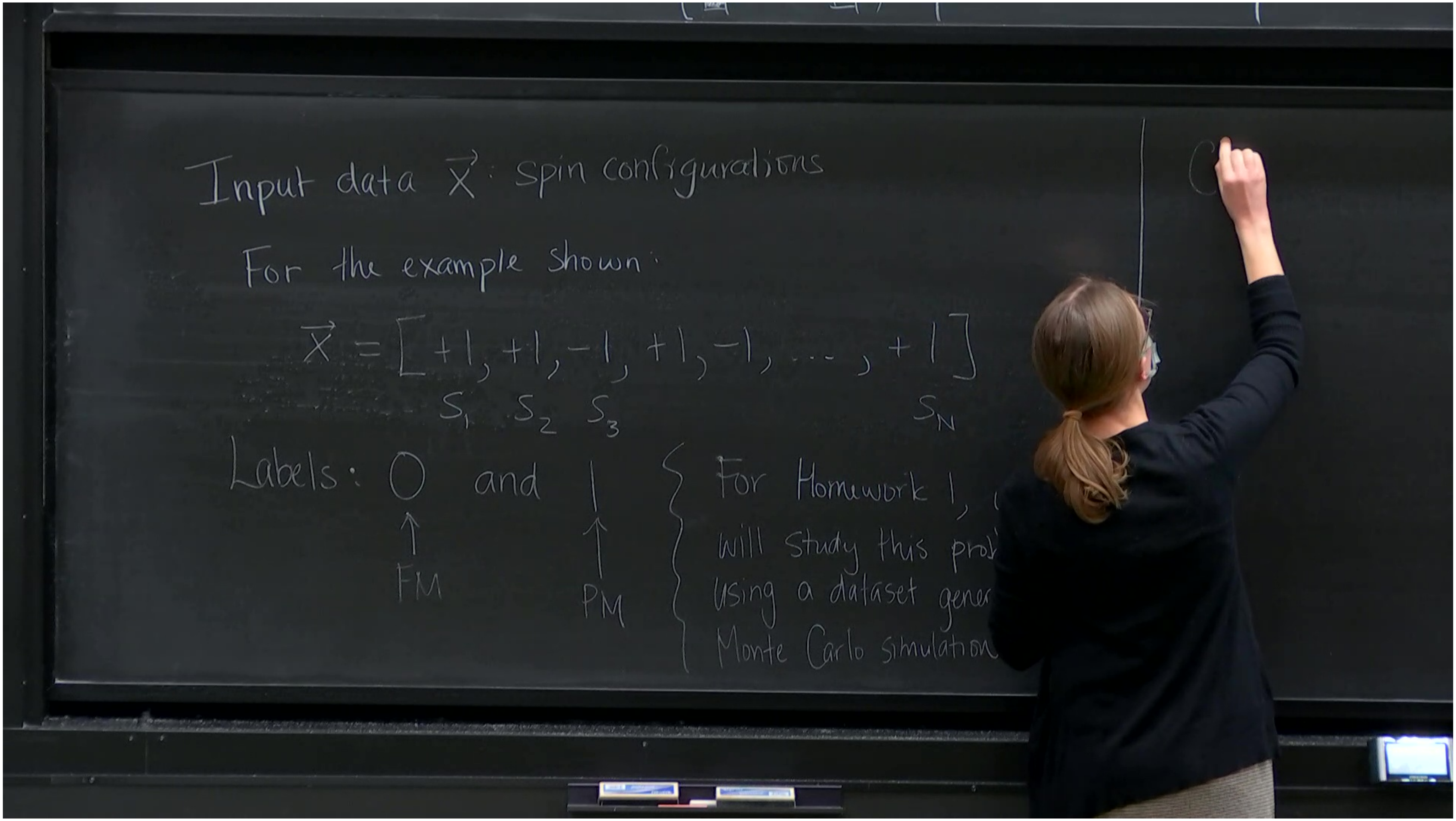
eg) For a 2D square lattice



For a 2D square lattice in the thermodynamic limit, we know there is a phase transition from a ferromagnetic (FM) to paramagnetic (PM) phase at $T_c \simeq 2.269J$


a 2D square lattice
The order parameter is $m = \frac{1}{N}\sum_i S_i$
$\langle m \rangle$
T
FM
$(\langle m \rangle \neq 0)$
PM
$(\langle m \rangle = 0)$
square
dynamic
we know
transition
phase at $T_c \approx 2.269J$
Goal: train a neural network to classify the
FM and PM phases (with no knowledge of the order param.)


ses of matter using SL
xiv: 1605.01735
The d.o.f. are
sites of a
lattice
eg) For a 2D square lattice
S_i
S_5 S_6
S_1 S_2 S_3 S_4
For a 2D square lattice in the thermodynamic
limit, we know there is a phase transition
from a ferromagnetic (FM) to paramagnetic (PM) phase at $T_c \approx 2.26$
The or


Input data $\vec{x}$: spin configurations
For the example shown:
$\vec{x} = [+1, +1, -1, +1, -1, \ldots, +1]$
S_1 S_2 S_3 S_N
Labels: 0 and 1
FM
PM
For Homework 1,
will study this prob
using a dataset gener
Monte Carlo simulation

+ 1]

S_N

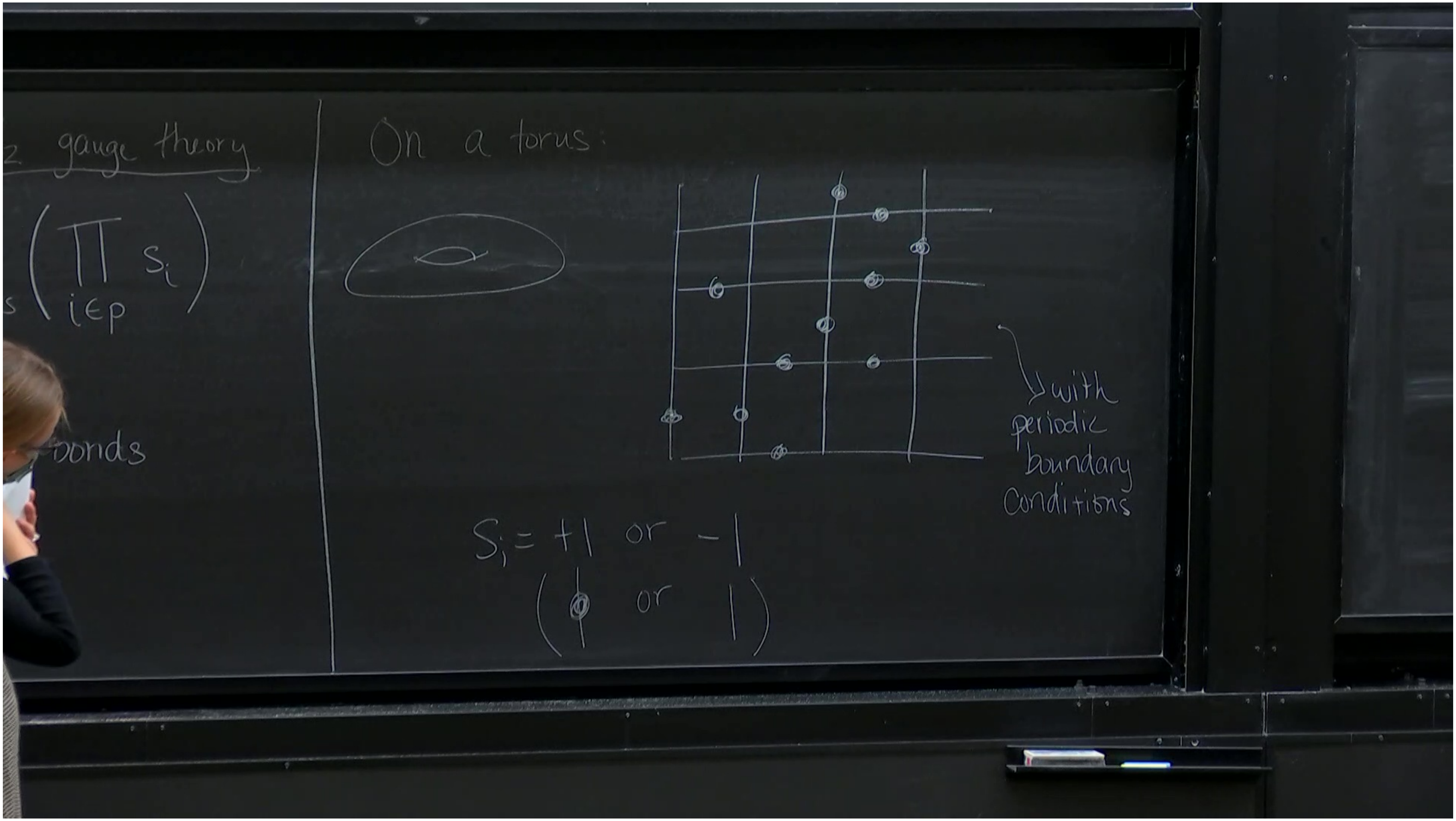
ork 1, you
this problem
set generated from
simulation

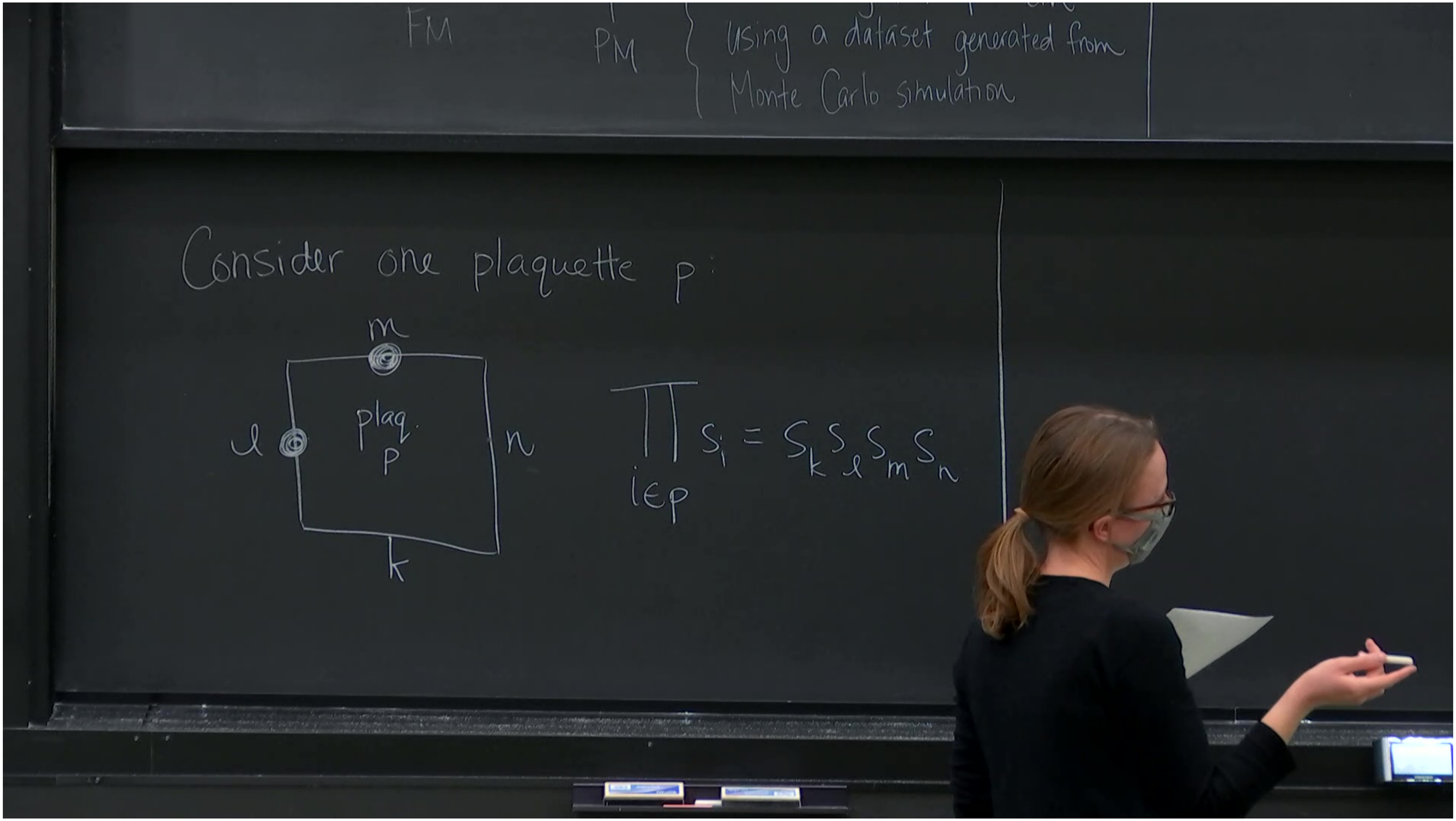
Classical Ising $\mathbb{Z}_2$ gauge theory

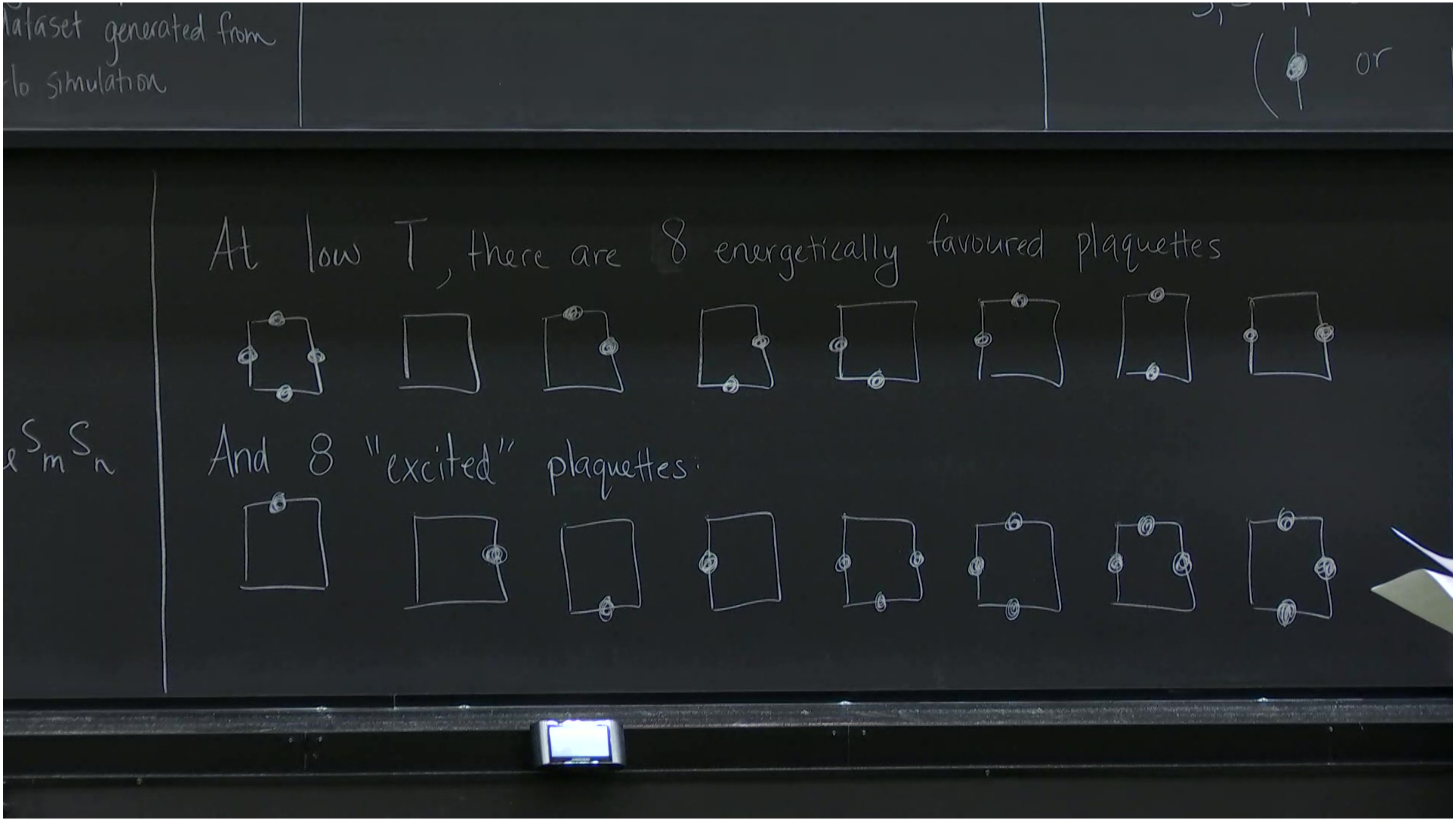
$$H = -J \sum_{\substack{\text{plaquettes} \\ p}} \left(\prod_{i \in p} s_i \right)$$

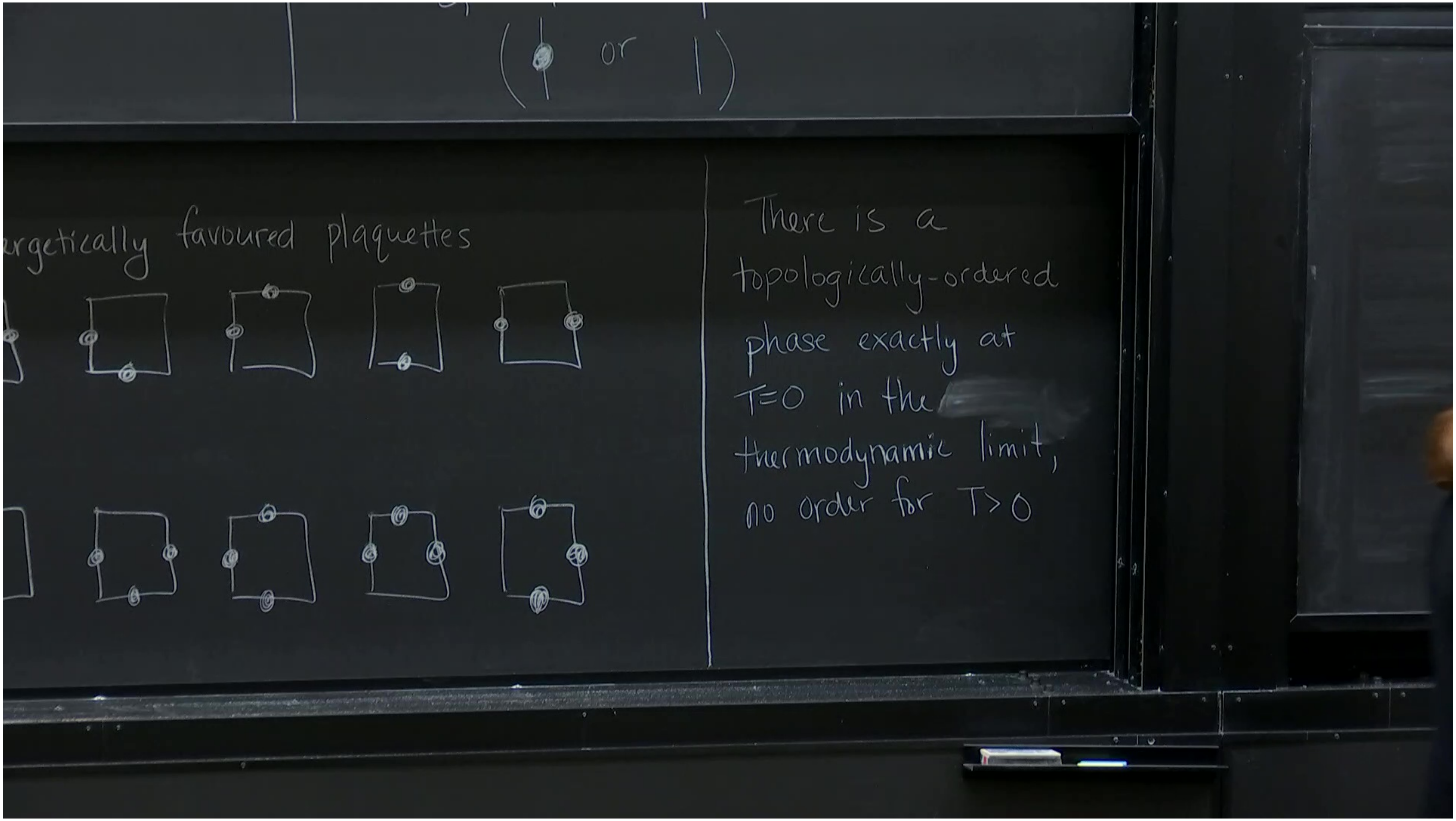
with $s_i = +1$ or -1

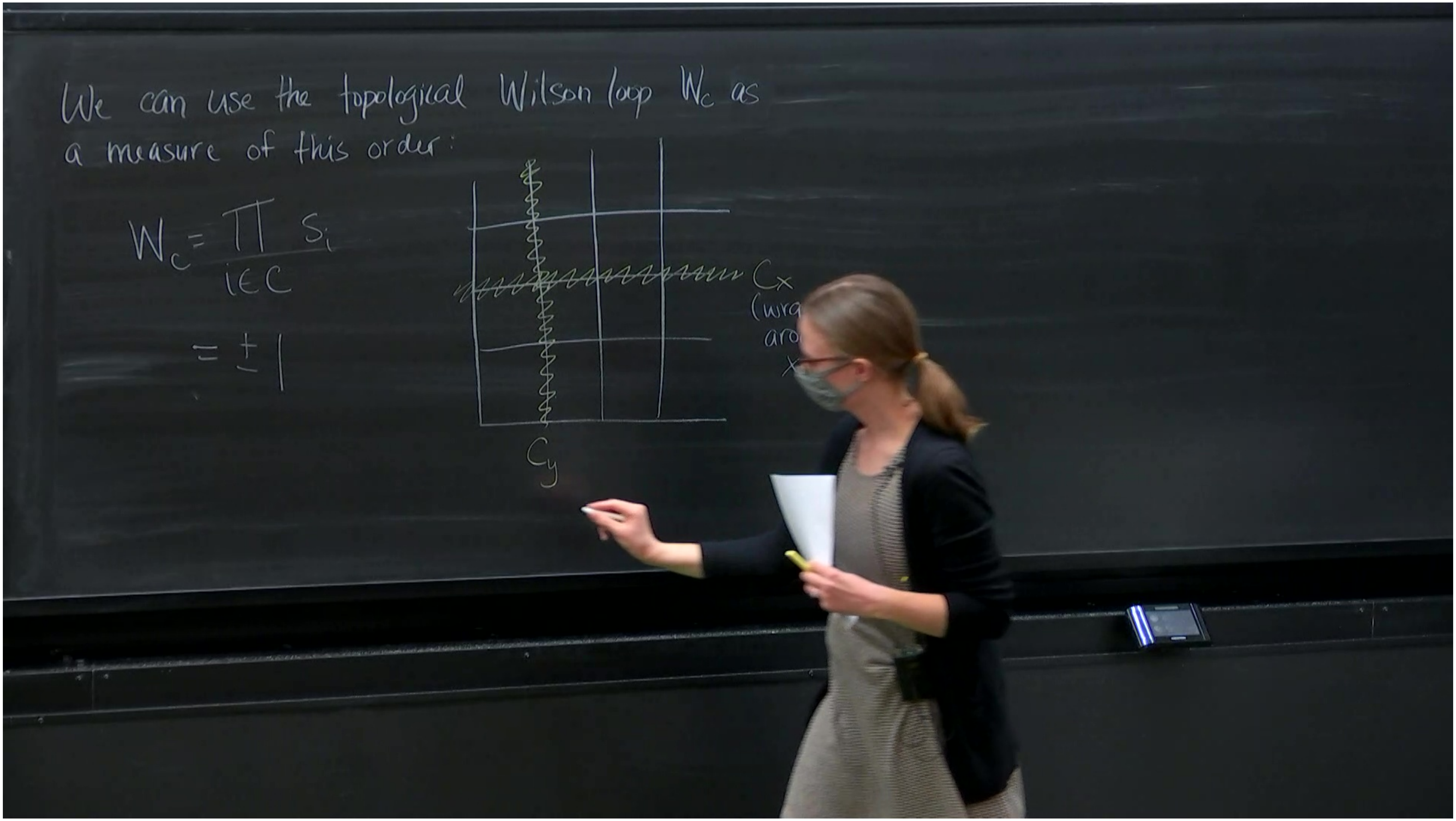
The d.o.f. are on the bonds of the lattice

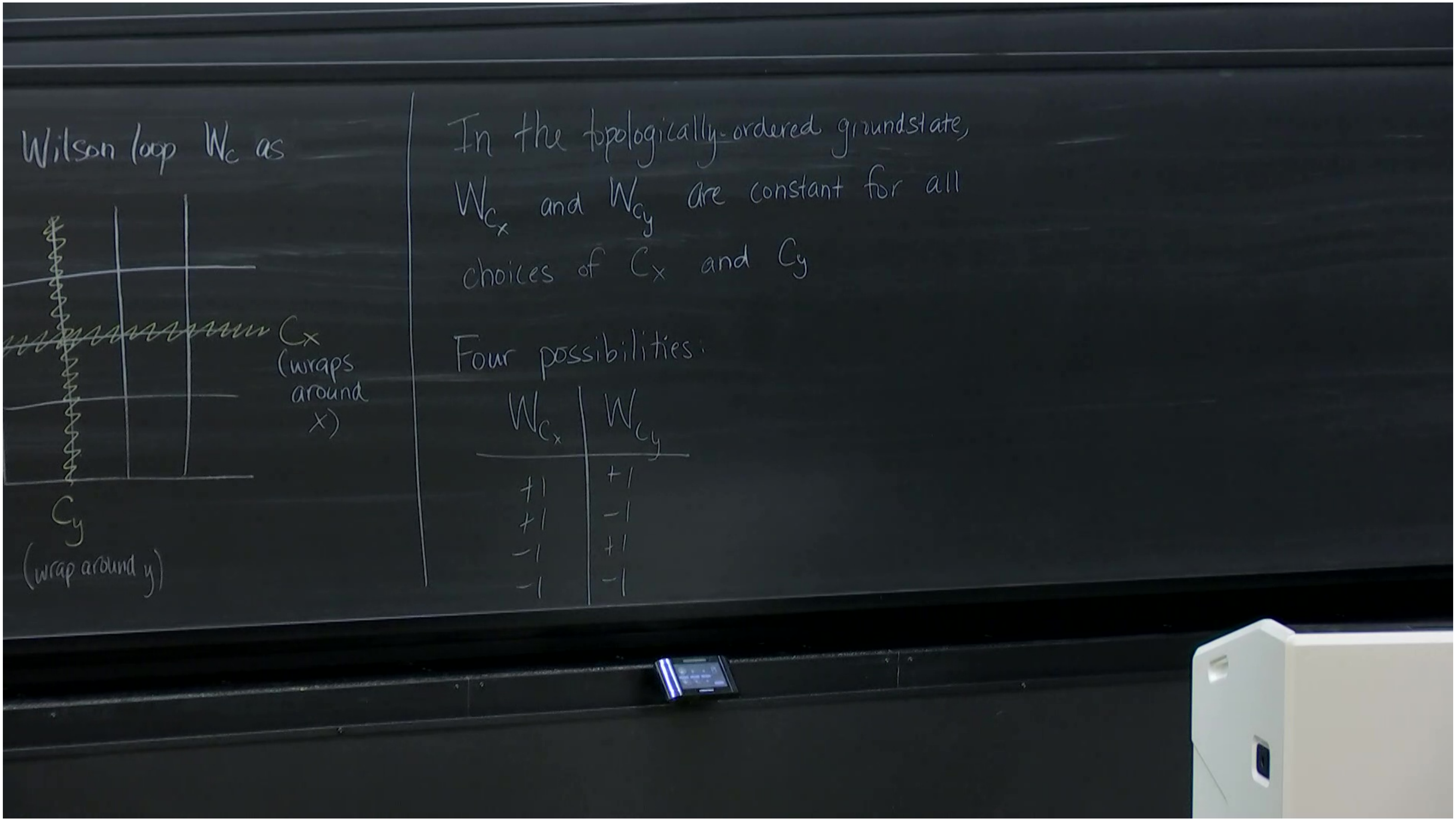

gauge theory
On a torus:
with periodic boundary conditions
S_i = +1 or -1
or
bonds


FM
PM
Using a dataset generated from
Monte Carlo simulation
Consider one plaquette p:
m
l
plaq. p
n
k
$\prod_{i \in p} s_i = s_k s_l s_m s_n$


dataset generated from
lo simulation
or
At low T, there are 8 energetically favoured plaquettes
And 8 "excited" plaquettes:


or
ergetically favoured plaquettes
There is a
topologically-ordered
phase exactly at
T=0 in the
thermodynamic limit;
no order for T>0


We can use the topological Wilson loop W_c as a measure of this order:
$W_c = \prod_{i \in C} s_i$
$= \pm 1$
C_x
C_y


Wilson loop W_c as
C_x
(wraps around x)
C_y
(wrap around y)
In the topologically-ordered groundstate, W_{C_x} and W_{C_y} are constant for all choices of C_x and C_y
Four possibilities:
W_{C_x} | W_{C_y}
+1 | +1
+1 | −1
−1 | +1
−1 | −1

Monte Carlo (MC) Sampling

(Used to generate the data samples for Homework 1)

Consider a classical system where we would like to calculate expectation values of various quantities Q:

$$\langle Q \rangle = \sum_{\mu} Q_{\mu} \, p_{\mu}$$

For the Boltzmann dist.: $p_{\mu} = \frac{1}{Z} e^{-\beta E_{\mu}}$, $Z = \sum_{\mu} e^{-\beta E_{\mu}}$

where:

- E_{μ} is the energy of config. μ
- $\beta = \frac{1}{k_B T}$

mework 1)

we would

arious quantities Q:

where:
- E_μ is the energy of config. μ
- $\beta = \frac{1}{k_B T}$

The sums $\sum_\mu$ are over a huge # of states (2^N for Ising).

Analytically intractable to do exactly

Idea of MC: estimate $\langle Q \rangle$ by summing over some subset of M states

$$\langle Q \rangle = \sum_{\mu} Q_{\mu} p_{\mu}$$

For the Boltzmann dist.: $p_\mu = \frac{1}{Z} e^{-\beta E_\mu}$, $Z = \sum_\mu e^{-\beta E_\mu}$

- E_μ is the energy of config. μ
- $\beta = \frac{1}{k_B T}$

In particular, if each μ_i $(1 \leq i \leq M)$ is selected from the set of all states according to the desired prob. dist p_{μ_i}, then:

$$\langle Q \rangle \simeq Q_M = \frac{1}{M} \sum_{i=1}^{M} Q_{\mu_i}$$

This estimate makes use of "importance sampling" (the more probable states are more likely to be selected)

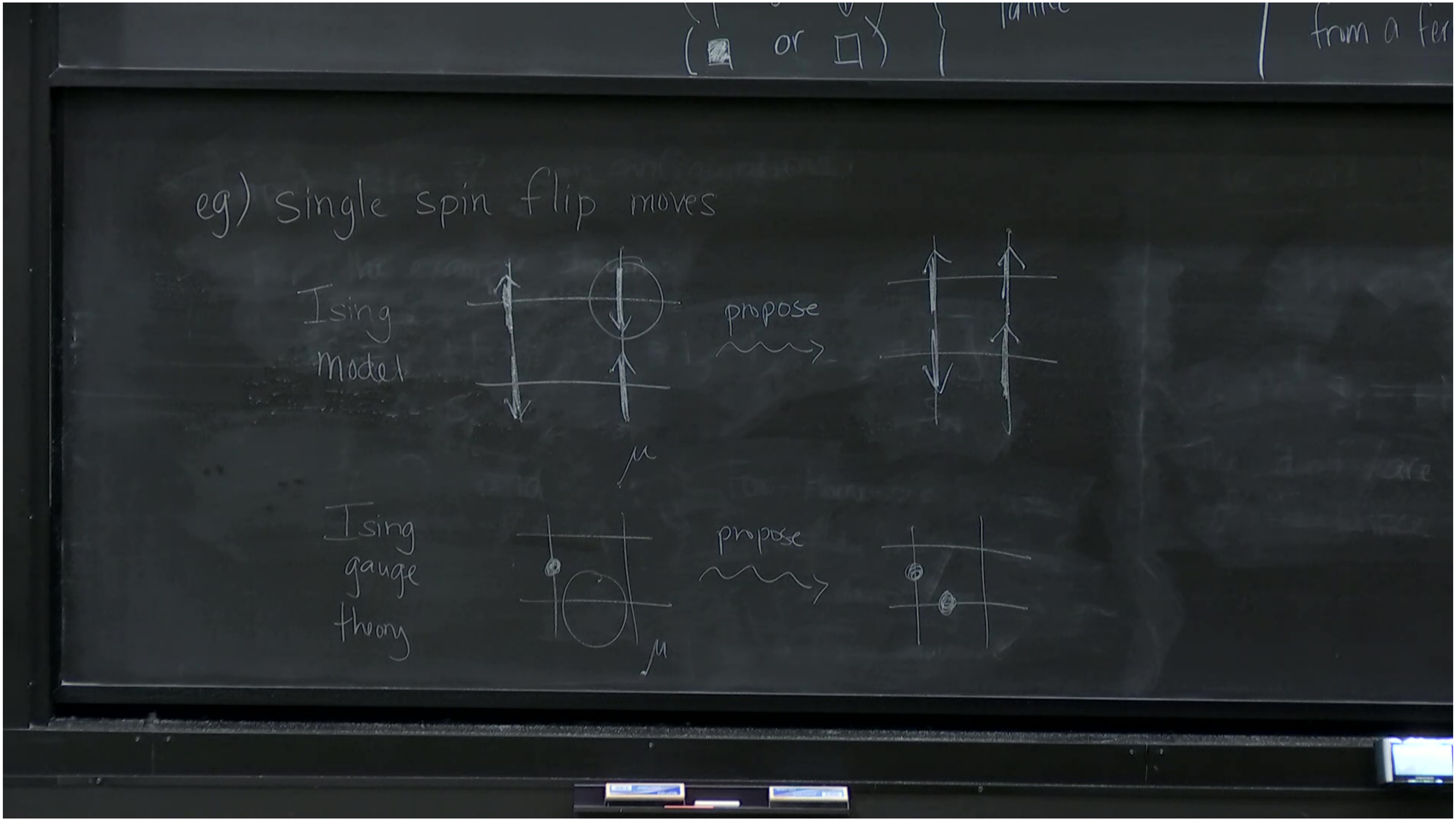
the energy
fig. μ

d

sired

To implement this sampling, we can use "Markov chain Monte Carlo"

↳ use the current state μ to propose a move to a new related state ν, and then decide whether to accept or reject


or
from a fer
eg) single spin flip moves
Ising model
propose
μ
Ising gauge theory
propose
μ