Title: Machine Learning (2021/2022)

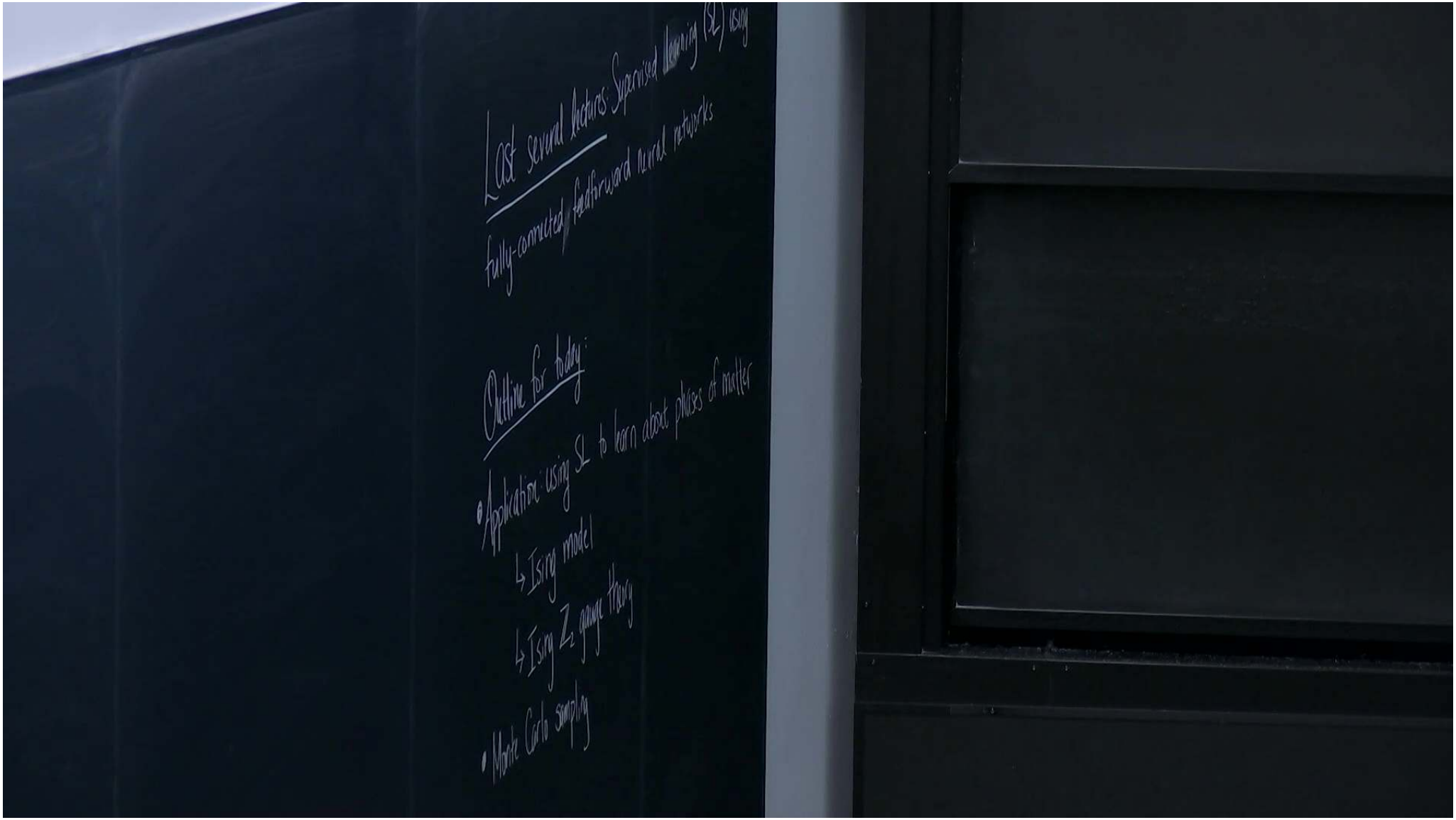
Speakers: Lauren Hayward

Collection: Machine Learning (2021/2022)

Date: April 18, 2022 - 11:30 AM

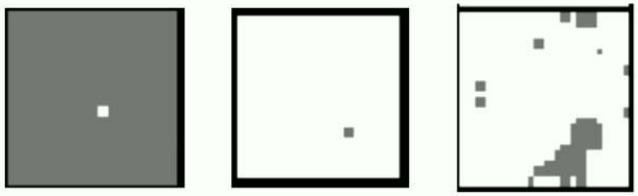
URL: https://pirsa.org/22040070

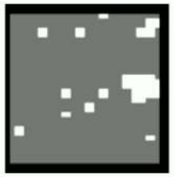
Abstract: This course is designed to introduce modern machine learning techniques for studying classical and quantum many-body problems encountered in condensed matter, quantum information, and related fields of physics. Lectures will focus on introducing machine learning algorithms and discussing how they can be applied to solve problem in statistical physics. Tutorials and homework assignments will concentrate on developing programming skills to study the problems presented in lecture.


Last several lectures: Supervised learning (SL) using
fully-connected feedforward neural networks
Outline for today:
• Application: using SL to learn about phases of matter
↳ Ising model
↳ Ising Z_2 gauge theory
• Monte Carlo sampling

Ising model classification

Ferromagnetic phase:





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Paramagnetic phase:

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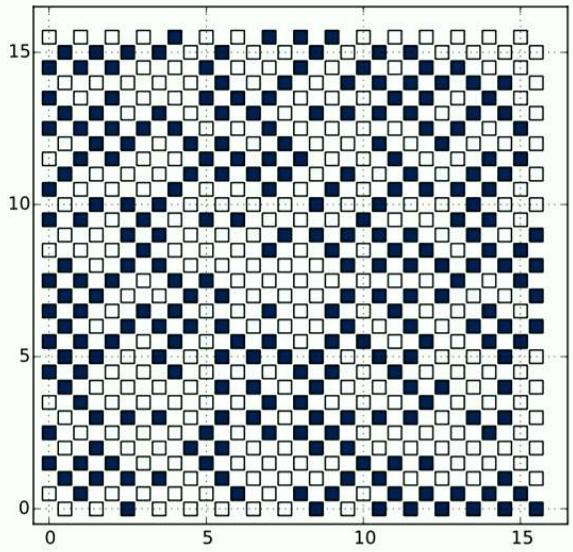
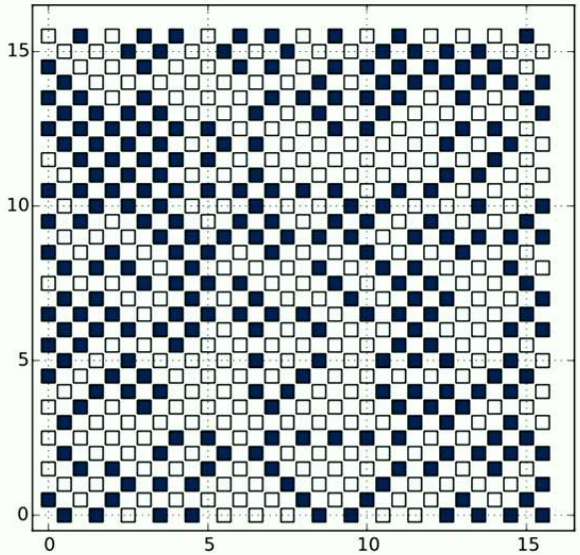
Carrasquilla and Melko, arXiv:1605.01735

PERIMETER SCHOLARS INTERNATIONAL

Ising gauge theory classification





Carrasquilla and Melko, arXiv:1605.01735

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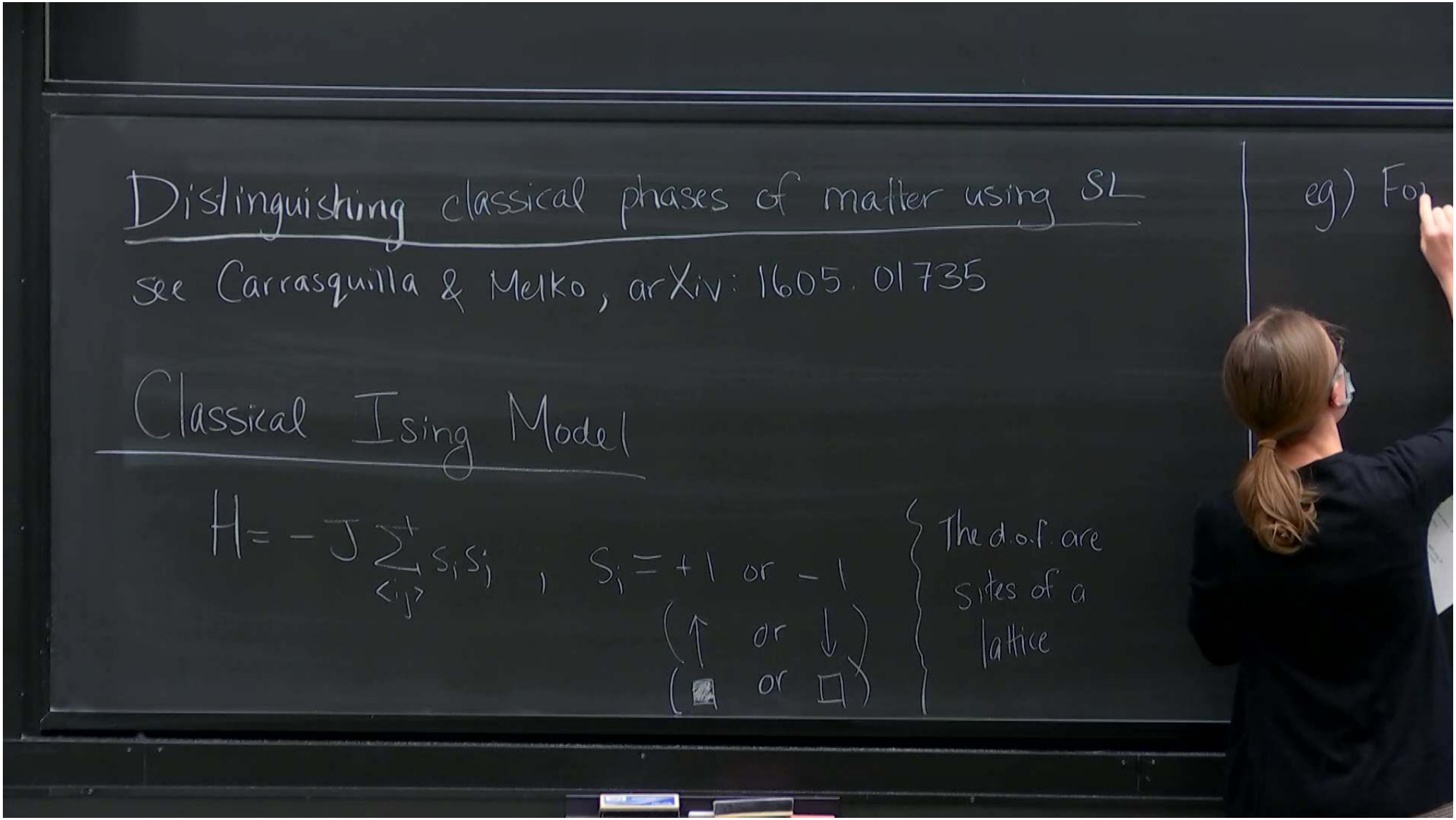
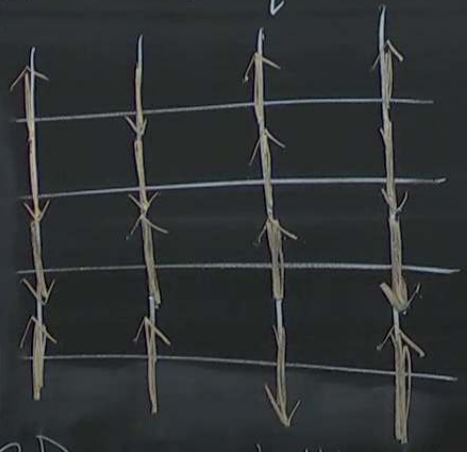

Distinguishing classical phases of matter using SL
see Carrasquilla & Melko, arXiv: 1605.01735
Classical Ising Model
$H = -J \sum_{\langle ij \rangle} s_i s_j$, $s_i = +1$ or -1
(↑ or ↓)
(or □)
The d.o.f. are sites of a lattice
eg) Fo

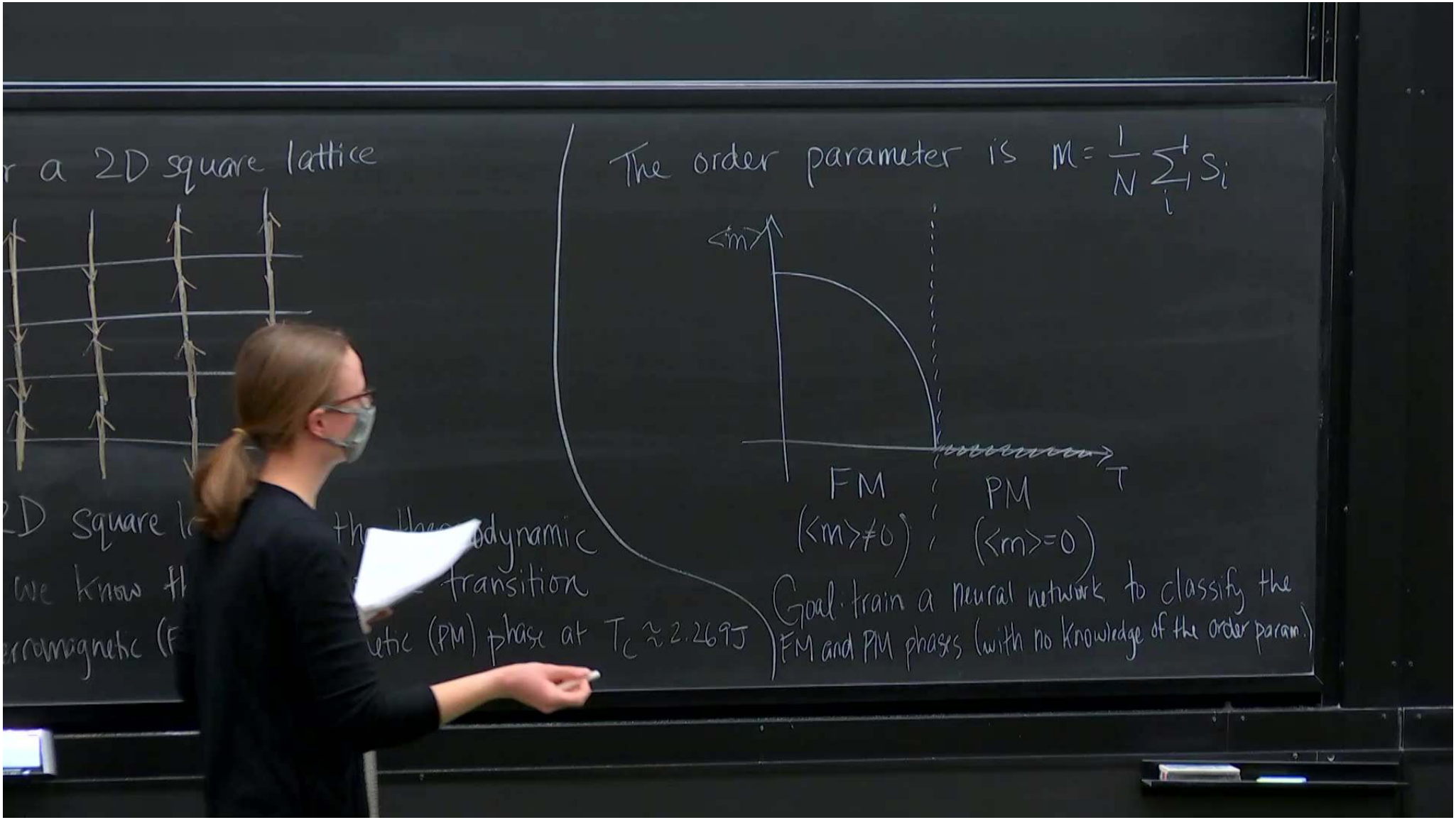
sing SL

d.o.f are
tes of a
lattice

eg) For a 2D square lattice



For a 2D square lattice in the thermodynamic limit, we know there is a phase transition from a ferromagnetic (FM) to paramagnetic (PM) phase at $T_c \approx 2.269J$


r a 2D square lattice
The order parameter is $M = \frac{1}{N}\sum_i^t S_i$
<m>
FM
(<m>≠0)
PM
(<m>=0)
T
odynamic
transition
etic (PM) phase at $T_C \approx 2.269J$
Goal: train a neural network to classify the
FM and PM phases (with no knowledge of the order param.)

ses of matter using SL

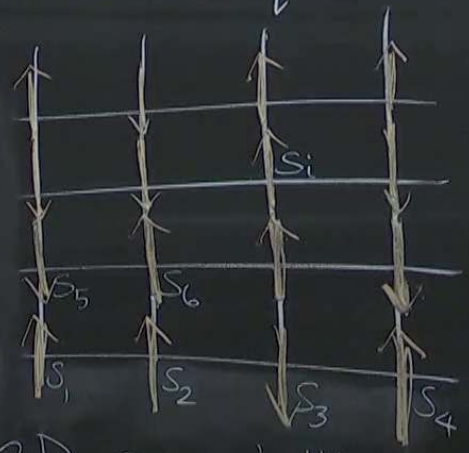
iv: 1605.01735

? The d.o.f are

sites of a

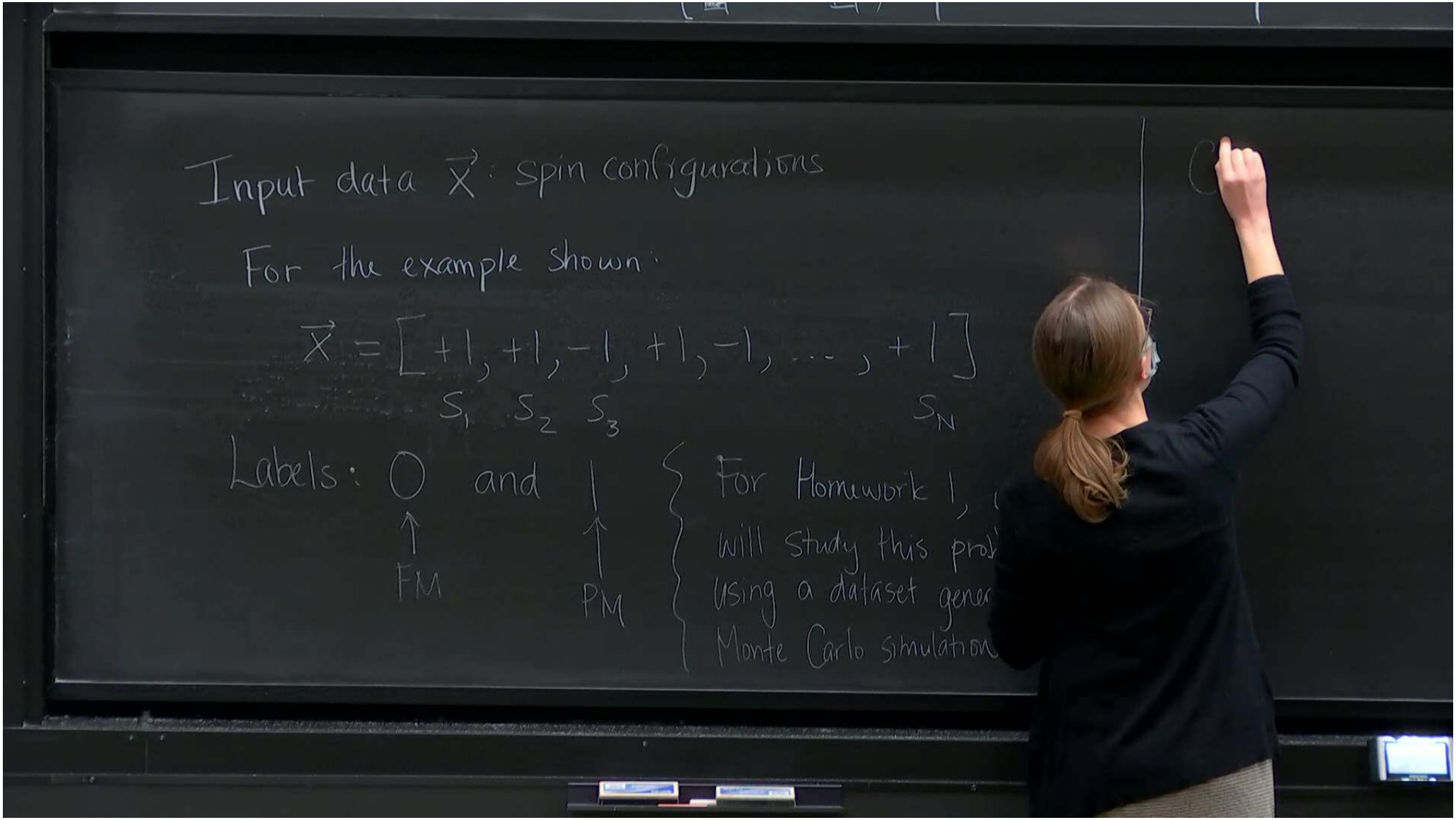
lattice

eg) For a 2D square lattice



For a 2D square lattice in the thermodynamic limit, we know there is a phase transition from a ferromagnetic (FM) to paramagnetic (PM) phase at $T_c \approx 2.26$

The o


Input data $\vec{X}$: spin configurations
For the example shown:
$\vec{X} = [+1, +1, -1, +1, -1, \ldots, +1]$
S_1 S_2 S_3 S_N
Labels: 0 and 1
FM
PM
For Homework 1,
will study this prob
using a dataset gener
Monte Carlo simulation

+ 1]

S_N

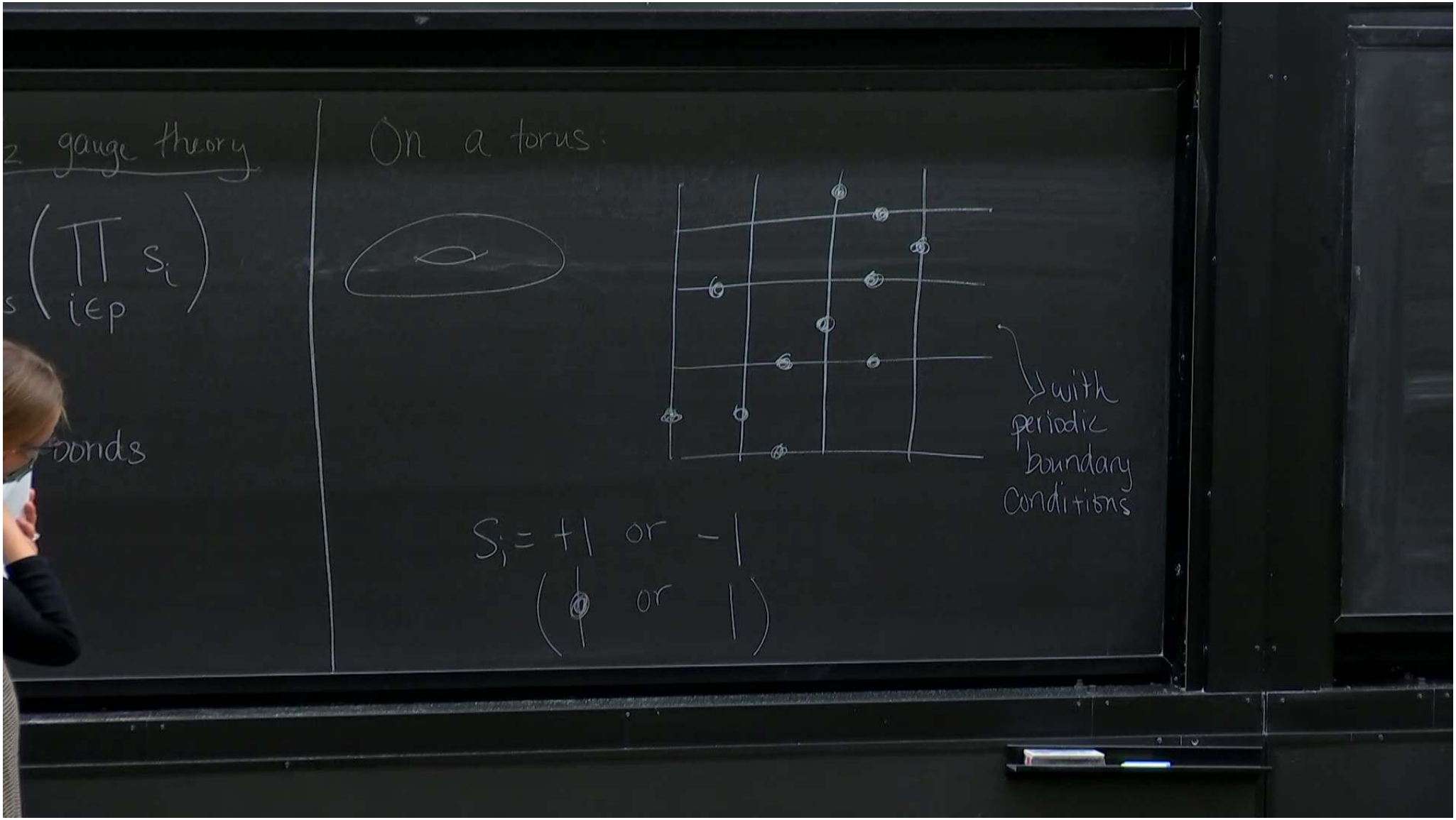
ork 1, you
his problem
set generated from
simulation

Classical Ising Z_2 gauge theory

$$H = -J \sum_{\substack{\text{plaquettes} \\ p}} \left(\prod_{i \in p} s_i \right)$$

with $s_i = +1$ or -1

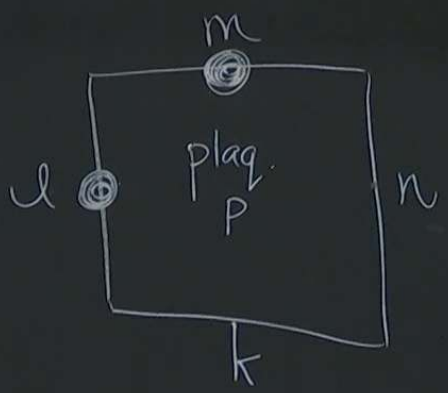
The d.o.f. are on the bonds of the lattice


gauge theory
$\left(\prod_{i \in p} s_i\right)$
bonds
On a torus:
with periodic boundary conditions
$s_i = +1$ or -1
(0 or 1)

FM

PM { using a dataset generated from Monte Carlo simulation

Consider one plaquette p:



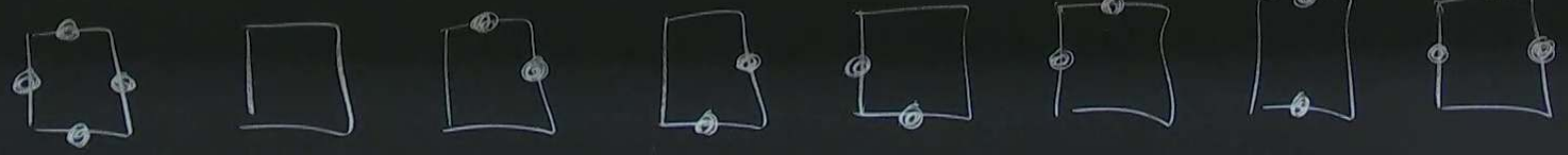
$$\prod_{i \in p} s_i = s_k s_l s_m s_n$$

dataset generated from
Monte Carlo simulation

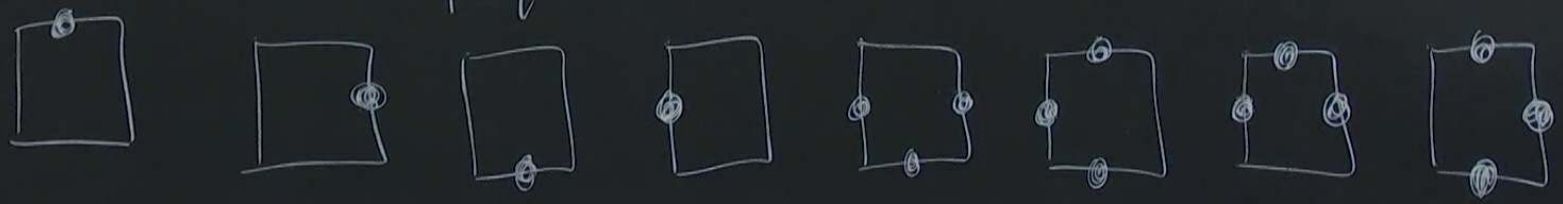
(↑ or

$e^{S_m S_n}$

At low T, there are 8 energetically favoured plaquettes

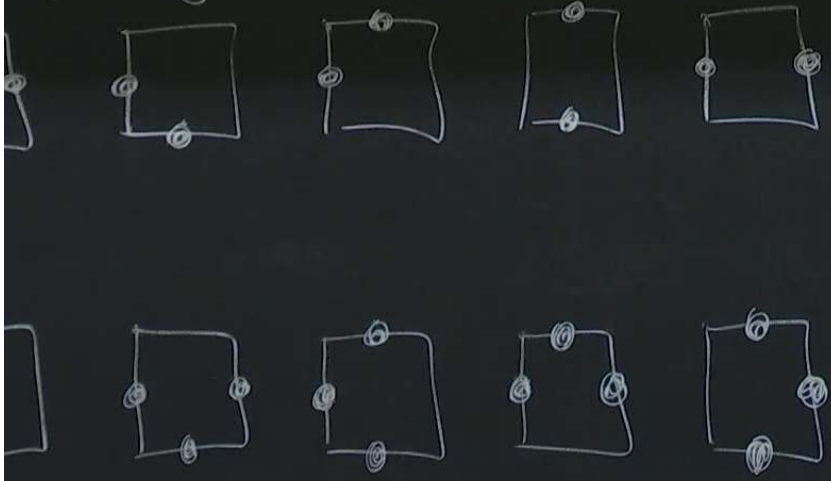


And 8 "excited" plaquettes:

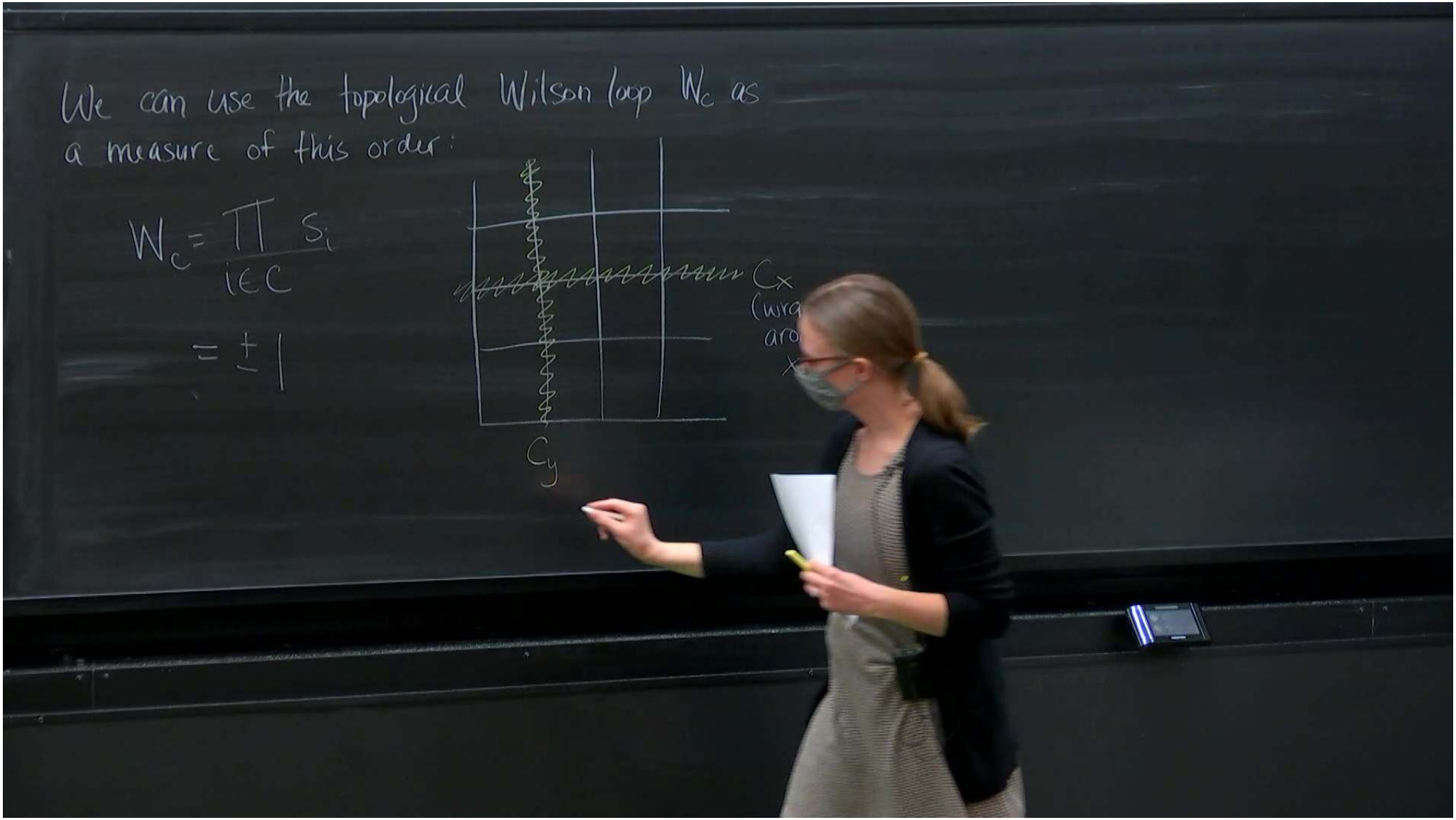


(or)

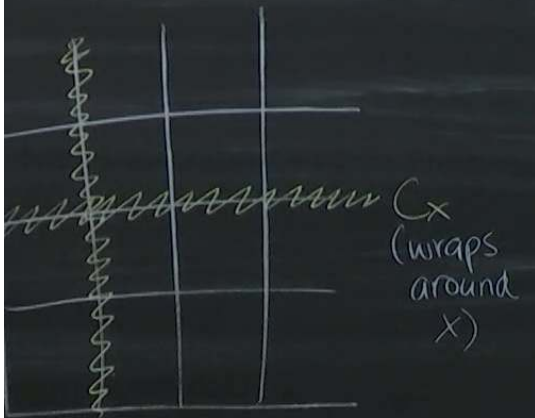
ergetically favoured plaquettes



There is a topologically-ordered phase exactly at T=0 in the thermodynamic limit, no order for T>0


We can use the topological Wilson loop W_c as a measure of this order:
$W_c = \prod_{i \in C} s_i$
$= \pm 1$
C_x
C_y

Wilson loop W_c as



In the topologically-ordered groundstate, W_{C_x} and W_{C_y} are constant for all choices of C_x and C_y

Four possibilities:

W_{C_x}	W_{C_y}
+1	+1
+1	−1
−1	+1
−1	−1

Monte Carlo (MC) Sampling

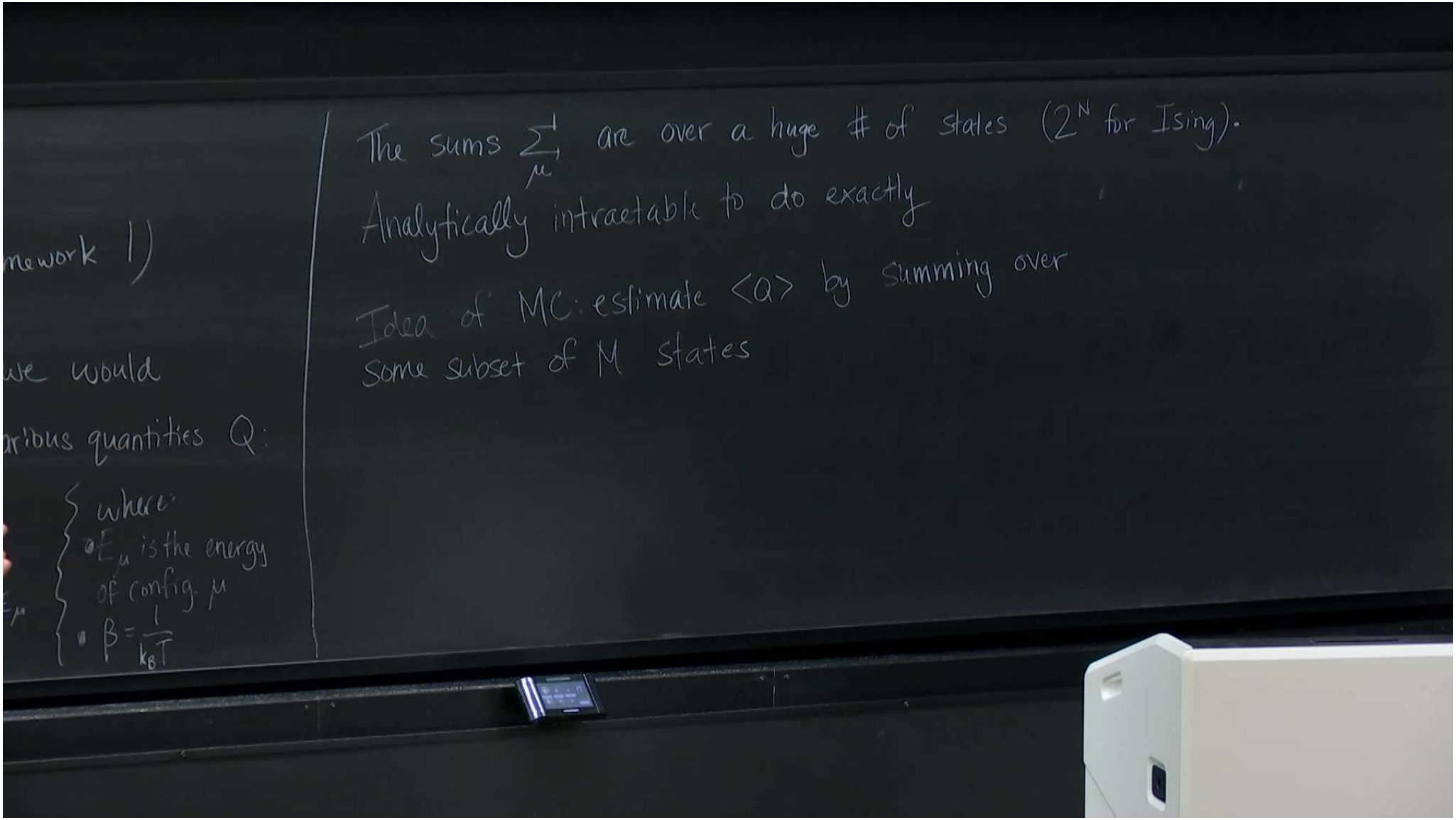
(Used to generate the data samples for Homework 1)

Consider a classical system where we would like to calculate expectation values of various quantities Q:

$$\langle Q \rangle = \sum_{\mu}^{\dagger} Q_\mu P_\mu$$

For the Boltzmann dist.: $P_\mu = \frac{1}{Z} e^{-\beta E_\mu}$, $Z = \sum_{\mu}^{\dagger} e^{-\beta E_\mu}$

where:
- E_μ is the energy of config. μ
- $\beta = \frac{1}{k_B T}$


nework 1)
we would
arious quantities Q:
where:
• E_μ is the energy of config. μ
• $\beta = \frac{1}{k_B T}$
The sums $\sum_\mu$ are over a huge # of states (2^N for Ising).
Analytically intractable to do exactly
Idea of MC: estimate $\langle Q \rangle$ by summing over some subset of M states

$\langle Q \rangle = \sum_\mu Q_\mu P_\mu$

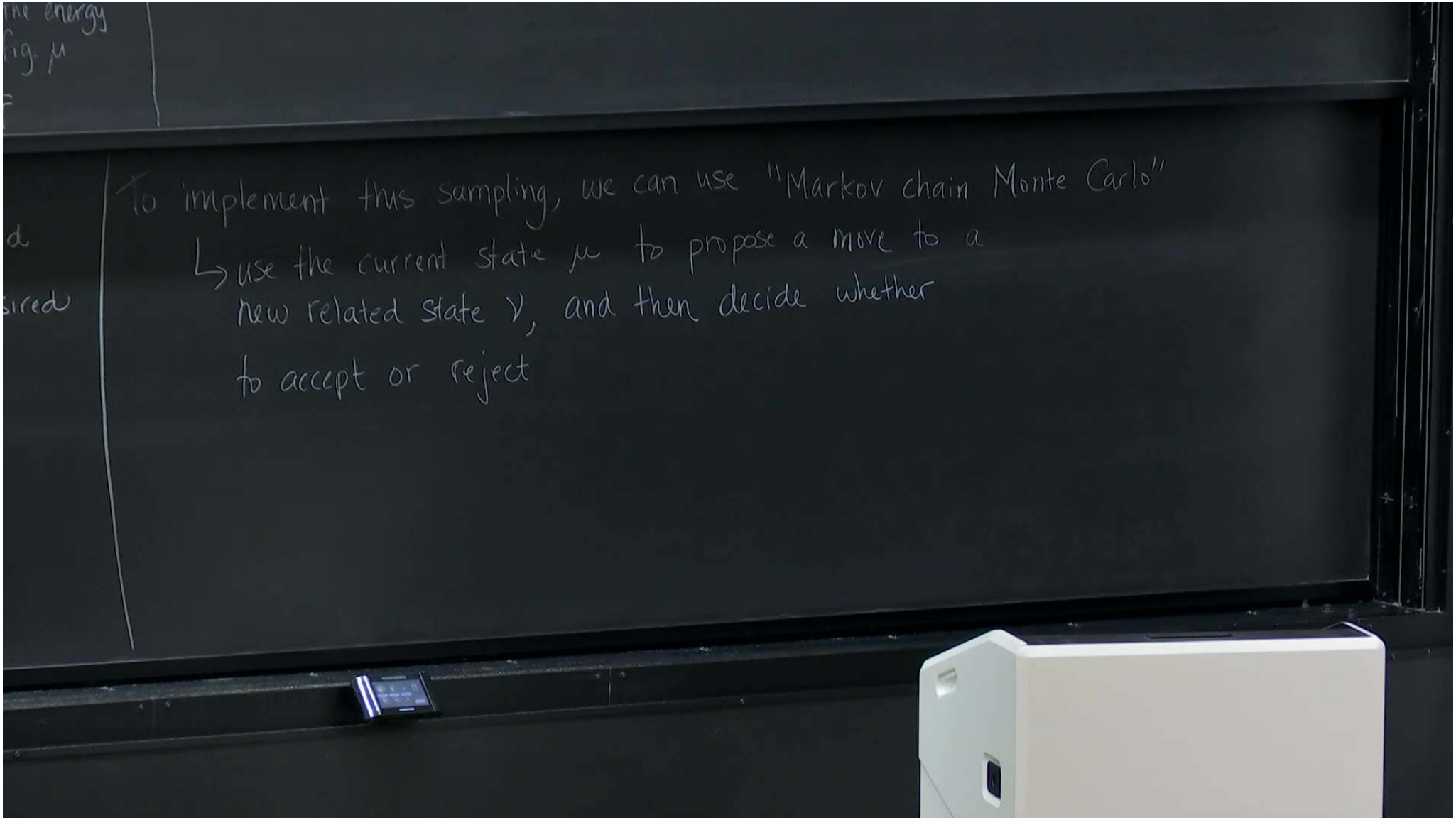
For the Boltzmann dist.: $P_\mu = \frac{1}{Z} e^{-\beta E_\mu}$, $Z = \sum_\mu e^{-\beta E_\mu}$

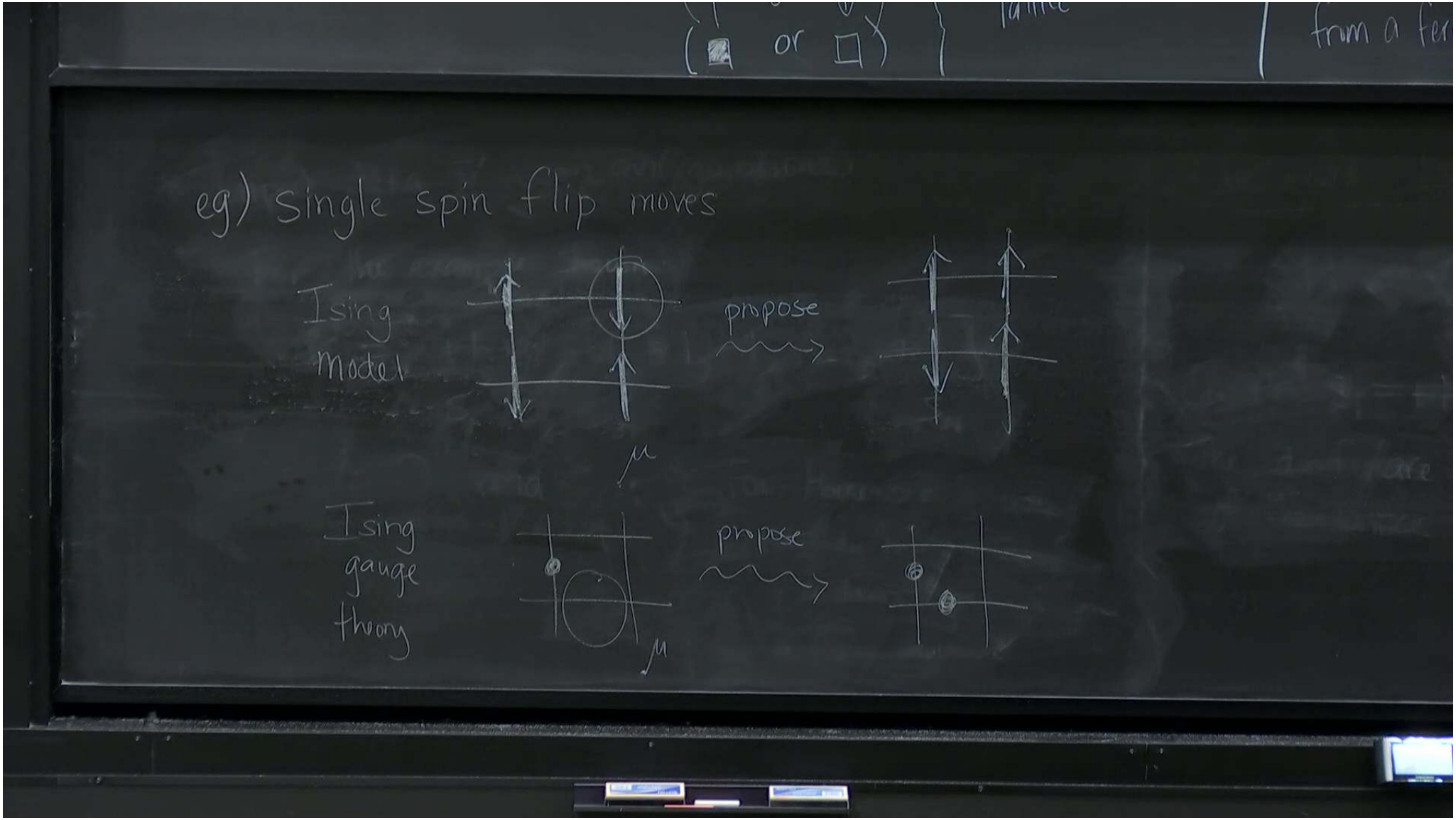
- E_μ is the energy of config. μ
- $\beta = \frac{1}{k_B T}$

In particular, if each μ_i $(1 \leq i \leq M)$ is selected from the set of all states according to the desired prob. dist P_{μ_i}, then:

$$\langle Q \rangle \simeq Q_M = \frac{1}{M} \sum_{i=1}^{M} Q_{\mu_i}$$

This estimate makes use of "importance sampling" (the more probable states are more likely to be selected)


the energy
fig. μ
d
sired
To implement this sampling, we can use "Markov chain Monte Carlo"
↳ use the current state μ to propose a move to a
new related state ν, and then decide whether
to accept or reject


or
from a fer
eg) single spin flip moves
Ising model
propose
μ
Ising gauge theory
propose
μ