Title: Quantum Gravity

Speakers: Aldo Riello

Collection: Quantum Gravity (2021-2022)

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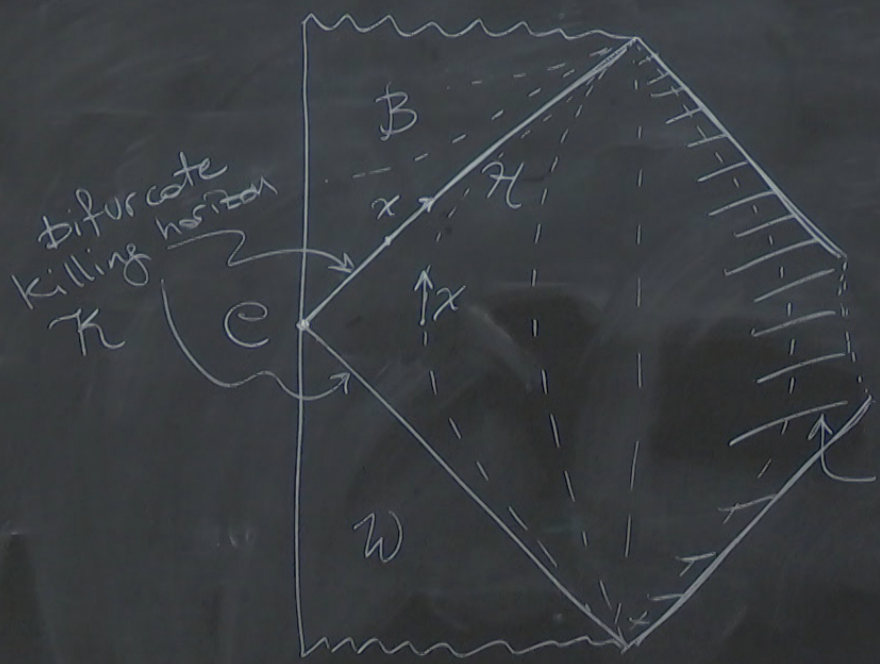
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.

Recap

$\mathcal{F}_{BH} \subset \mathcal{F}$

↑ stationary BH sol.s

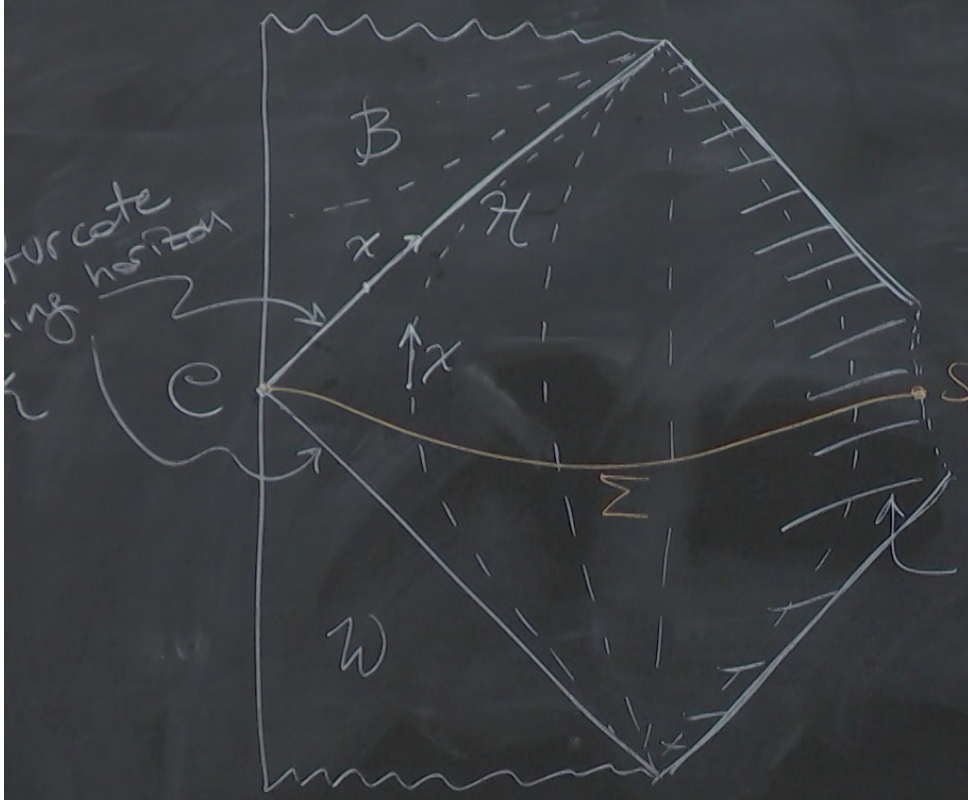
$\kappa \neq 0$



$\chi^a = t^a + \omega_H \phi^a$

asymptotically:
t^a ~ time transl
ϕ^a ~ rotation

asymptotically:
Minkowski
+ corrections $\left(\frac{1}{r}\right)$



$\chi^a = t^a + \omega_H \phi^a$

↑ Horizon's ang. vel.

asymptotically:

t^a ~ time transl

ϕ^a ~ rotation

S_∞ ← sphere of const (∞-large) radius

asymptotically:

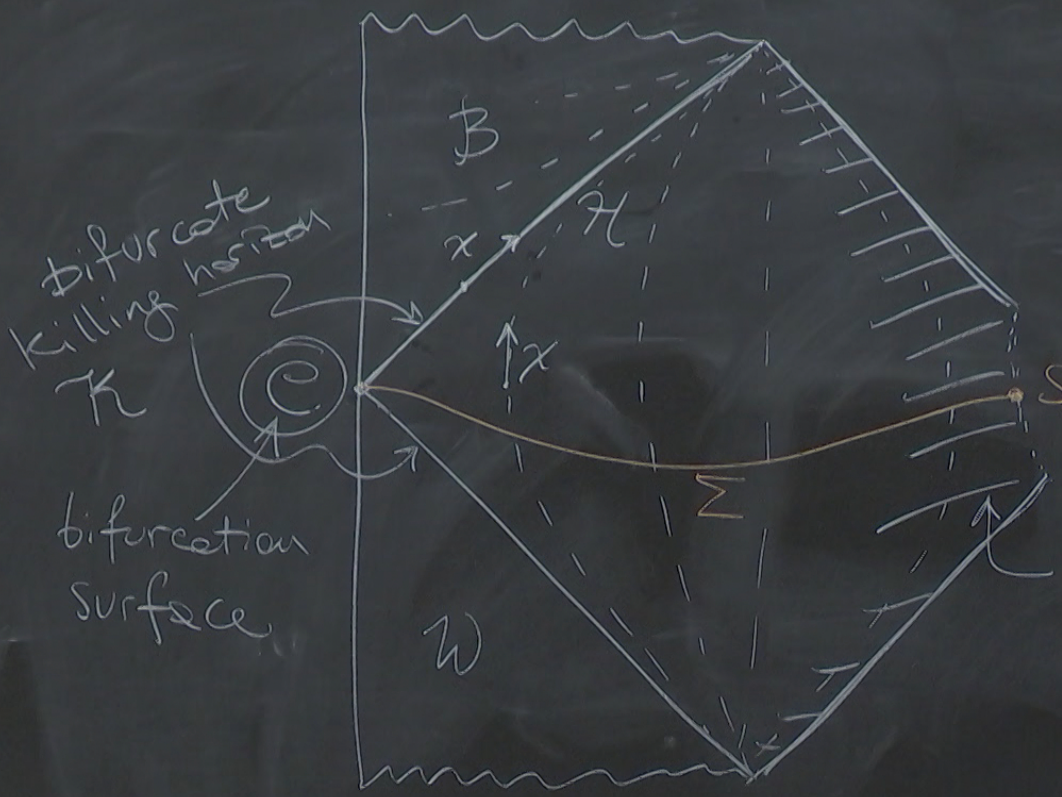
Minkowski

+ corrections $\left(\frac{1}{r}\right)$

$\subset \overline{F}$

stionary BH sol.s

$\neq 0$



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↑ Hori

asymptotically:
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Minkowski
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Properties of χ @ $\mathcal{K}$

- χ^a normal to $\mathcal{K}$ (null!)
- $\nabla_a(\chi^b\chi_b) = -2\kappa\chi_a$

  κ ↑ SURFACE GRAV. IS CONST on $\mathcal{K}$

  ↓

  $\chi^b\nabla_b\chi^a = \kappa\chi^a$ (Zeroth law)

- at $\mathcal{C}$: $\chi^a = 0$, $\nabla_a\chi_b = \kappa\,\varepsilon_{ab}$

  $\varepsilon_{ab}\varepsilon^{ab} = -2$

$\chi = \chi(g_{ab})$

$\chi^a = t^a + \omega_H \phi^a$

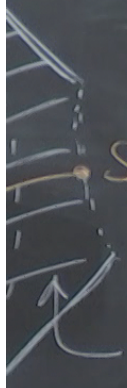
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$\chi = \chi(g_{ab})$

↳ $d\chi \neq 0$

if $\xi = \chi$, $\delta\chi \neq 0$

$$\mathring{\mathbb{I}}_{\varphi(\chi)} \Omega_\Sigma = -\left(\delta Q_\Sigma\right)(\chi) - \int_\Sigma i_\chi \underline{E} + \int_{\partial\Sigma} i_\xi \underline{\Theta}$$

↑ I first compute the variation, and only @ the end I plug in $\xi = \chi$

Goal: **1st law**

$$\kappa\, \delta A_{\mathcal{C}} \approx \delta M - \omega_H \delta J_{ang}$$

① ≈ 0 on-shell

② $\rho(\xi) = \int_M (\mathcal{L}_\xi g_{ab}) \frac{\delta}{\delta g_{ab}}$

$\equiv 0$ for $\xi = \chi(g_{ab})$ (killing)

② $\equiv 0$

③ $\int_{\partial\Sigma} i_\chi \underline{\underline{\Theta}} = \int_{\mathcal{C}} i_\chi \underline{\underline{\Theta}} + \int_{S_\infty} i_t \underline{\underline{\Theta}} + \omega_+ \int_{S_\infty} i_\phi \underline{\underline{\Theta}}$

$\chi^a \underline{\underline{\Theta}}_a$ $\equiv 0$ & $\chi|_{\mathcal{C}} = 0$ $\equiv 0$ $\phi^a \parallel S_\infty$

$\frac{\delta}{\delta g_{ab}}$

$(\mathcal{L}_\xi g_{ab})$

for $\xi = \chi(g_{ab})$
(Killing)

$\equiv 0$ $\phi^a \parallel S_\infty$

ASYMPTOTICALLY FLAT B. COND.

result

$$\int_C i_\chi \Theta + \int_{S_\infty} i_t \Theta + \omega_+ \int_{S_\infty} i_\phi \Theta = \int_{S_\infty} i_t \Theta = \frac{1}{8\pi} dM$$

$\equiv 0$ bc $\chi|_C = 0$

if $\xi = \chi$, $\delta\chi \neq 0$

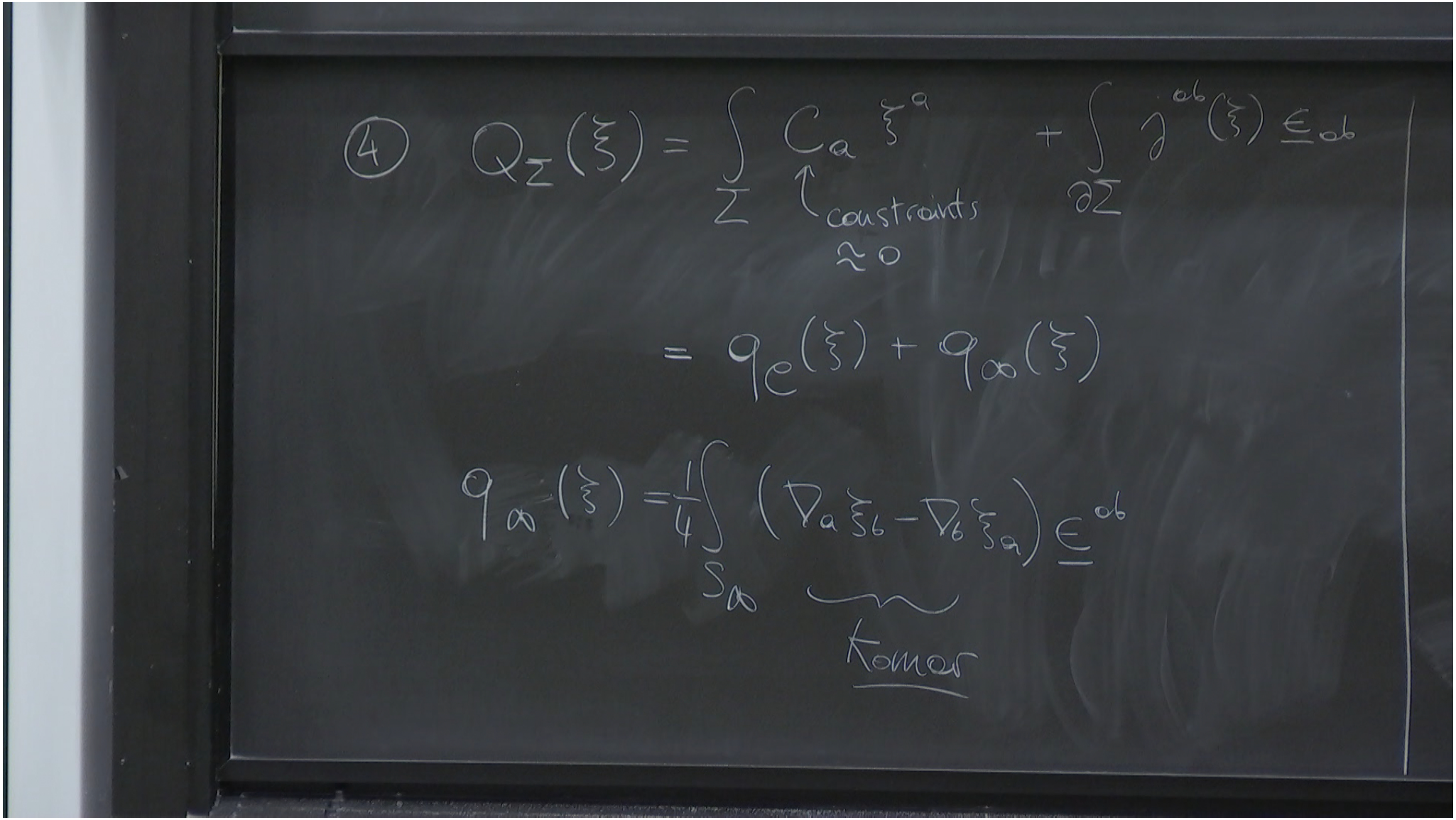
$$\mathring{\mathbb{I}}_{\varphi(\chi)} \Omega_\Sigma = -\left(\delta Q_\Sigma\right)(\chi) - \int_\Sigma i_\chi \underline{E} + \int_{\partial\Sigma} i_\chi \underline{\Theta}$$

(2) (4) (1) (3)

↑ I first compute the variation, and only @ the end I plug in $\xi = \chi$

Goal: **1st law**

$$\kappa\, \delta A_{\mathcal{C}} \approx \delta M - \omega_H\, \delta J_{ang}$$


④ $Q_\Sigma(\xi) = \int_\Sigma C_a \xi^a + \int_{\partial\Sigma} j^{ab}(\xi)\,\underline{\epsilon}_{ab}$
constraints
≈ 0
$= q_e(\xi) + q_\infty(\xi)$
$q_\infty(\xi) = \frac{1}{4}\int_{S_\infty} (\nabla_a \xi_b - \nabla_b \xi_a)\,\underline{\epsilon}^{ab}$
Komar

$\xi) \in \mathscr{O}$

$$(\mathbb{I}q_{\infty})(\chi) = (\mathbb{I}q_{\infty})(t) + \omega_{+}(\mathbb{I}q_{\infty})(\phi)$$

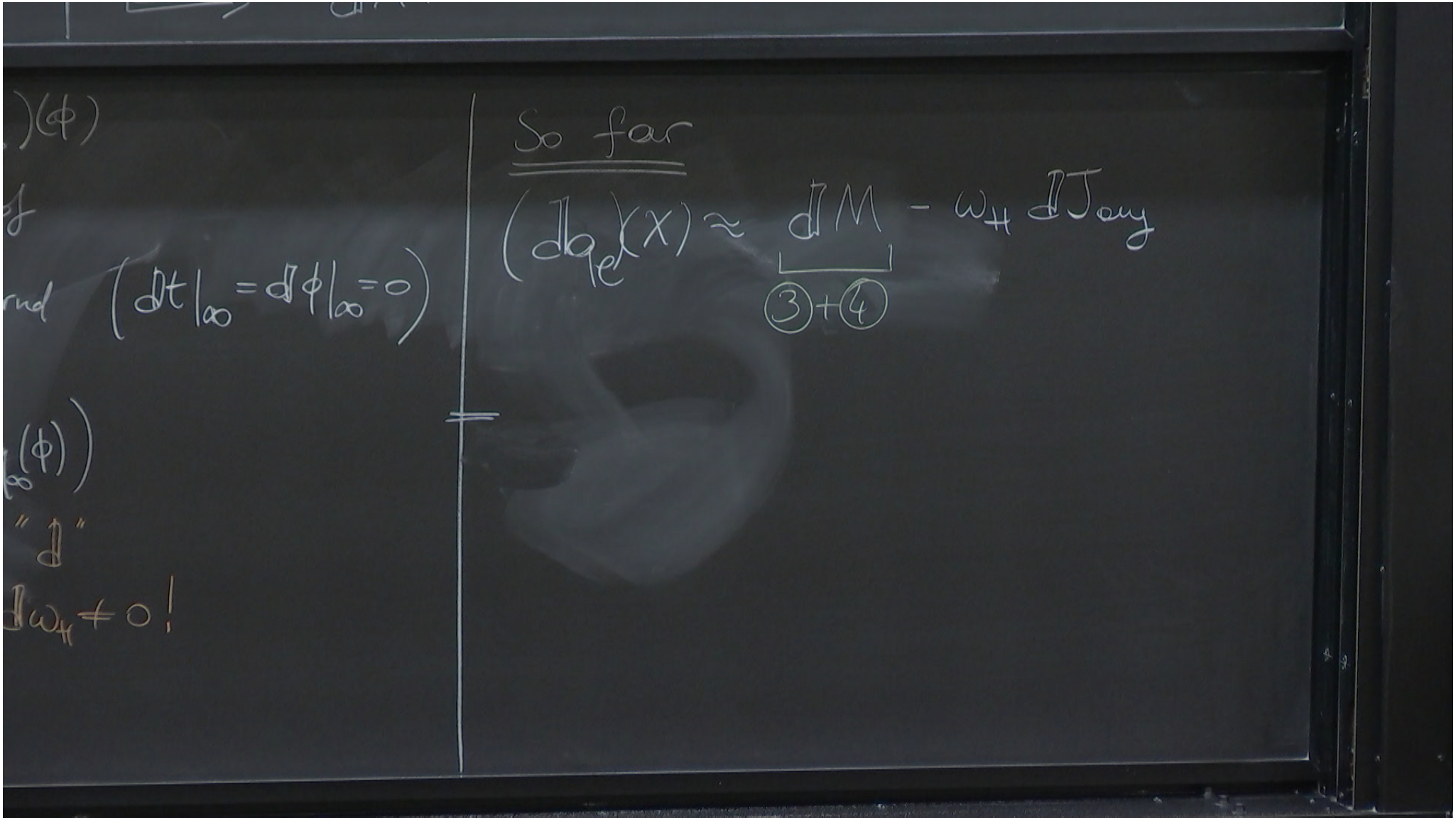
but t^a & ϕ^a at ∞ are those of the (asymptotic) Minkowski bckgrnd $\left(\mathbb{I}t|_{\infty} = \mathbb{I}\phi|_{\infty} = 0\right)$

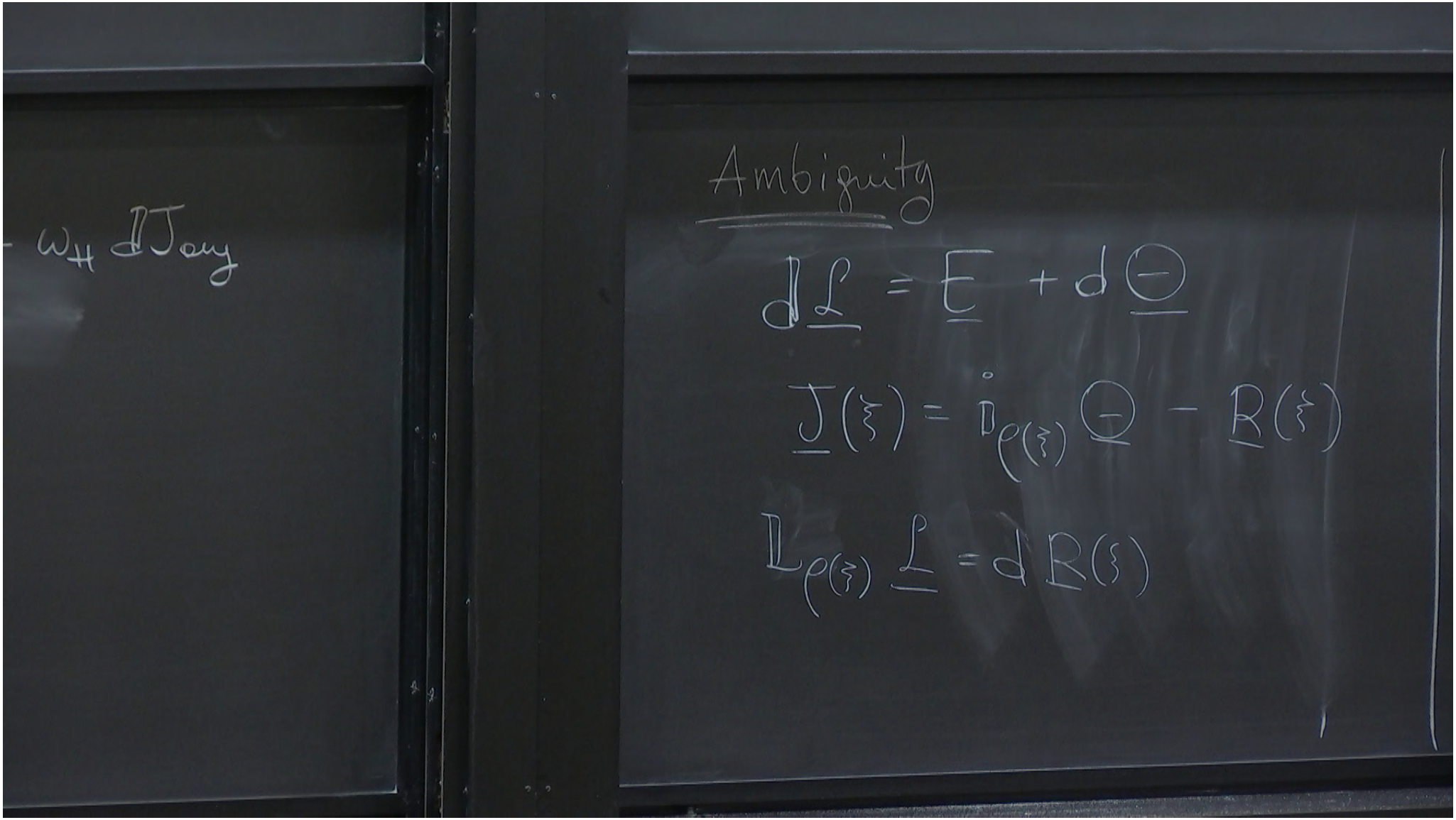
$$\hookrightarrow = \mathbb{I}(q_{\infty}(t)) + \omega_{+}(\mathbb{I}q_{\infty}(\phi))$$

(outside "$\mathbb{I}$" even if $\mathbb{I}\omega_{+} \neq 0$!)

Result: $q_{\infty}(\phi) = J_{ang}$

$q_{\infty}(t) = -\frac{1}{2}M$


)(φ)
rnd (δt|∞ = δφ|∞ = 0)
(φ))
"δ"
δω4 ≠ 0!
So far
(δaeff(X) ≈ δM - ω4 δJany
③+④


$\omega_{+} \delta J_{any}$
Ambiguity
$\delta \underline{L} = \underline{E} + d\underline{\Theta}$
$\underline{J}(\xi) = \dot{} \, \underline{\Theta}_{\rho(\xi)} - \underline{R}(\xi)$
$\mathbb{L}_{\rho(\xi)} \underline{L} = d\underline{R}(\xi)$

$$\underline{\ell} \longrightarrow \underline{\ell} + d\underline{\ell}$$

$$\underline{\Theta} \longrightarrow \underline{\Theta} + \nabla\underline{\ell} + d\underline{\beta}$$

$$\underline{R}(\xi) \longrightarrow \mathbb{D}_{\rho(\xi)}\underline{\ell} + d\underline{r}(\xi)$$

$$(\xi)$$

$$\underline{J}(\xi) \longrightarrow \underline{J}(\xi) + d\left(\overset{\circ}{\mathbb{I}}_{\rho(\xi)}\underline{\beta} - \underline{\underline{r}}(\xi)\right)$$

$$\underline{\Omega} \longrightarrow \underline{\Omega} + d\,\nabla\underline{\beta}$$

$$(d q_\epsilon)(\chi) = ?$$

$$q_\epsilon(\chi) = \frac{1}{4}\int_C (\nabla_a \chi_b - \nabla_b \chi_a)\,\underline{\epsilon}^{ab}$$

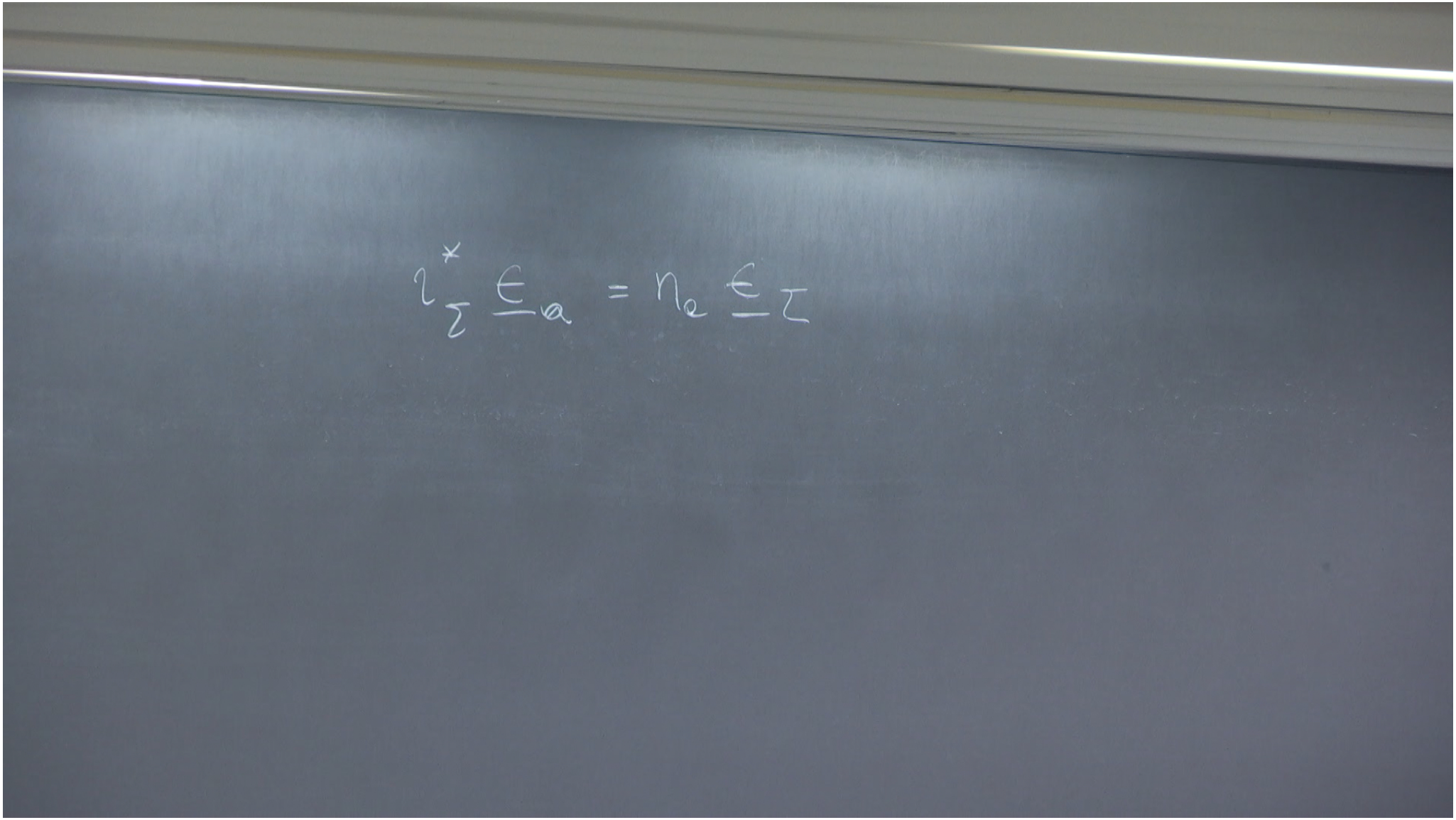
$$\uparrow \quad \tilde{\epsilon}^{ab}\,\underline{\epsilon}^{ab} = \varepsilon^{ab}\,\underline{\epsilon}_C$$

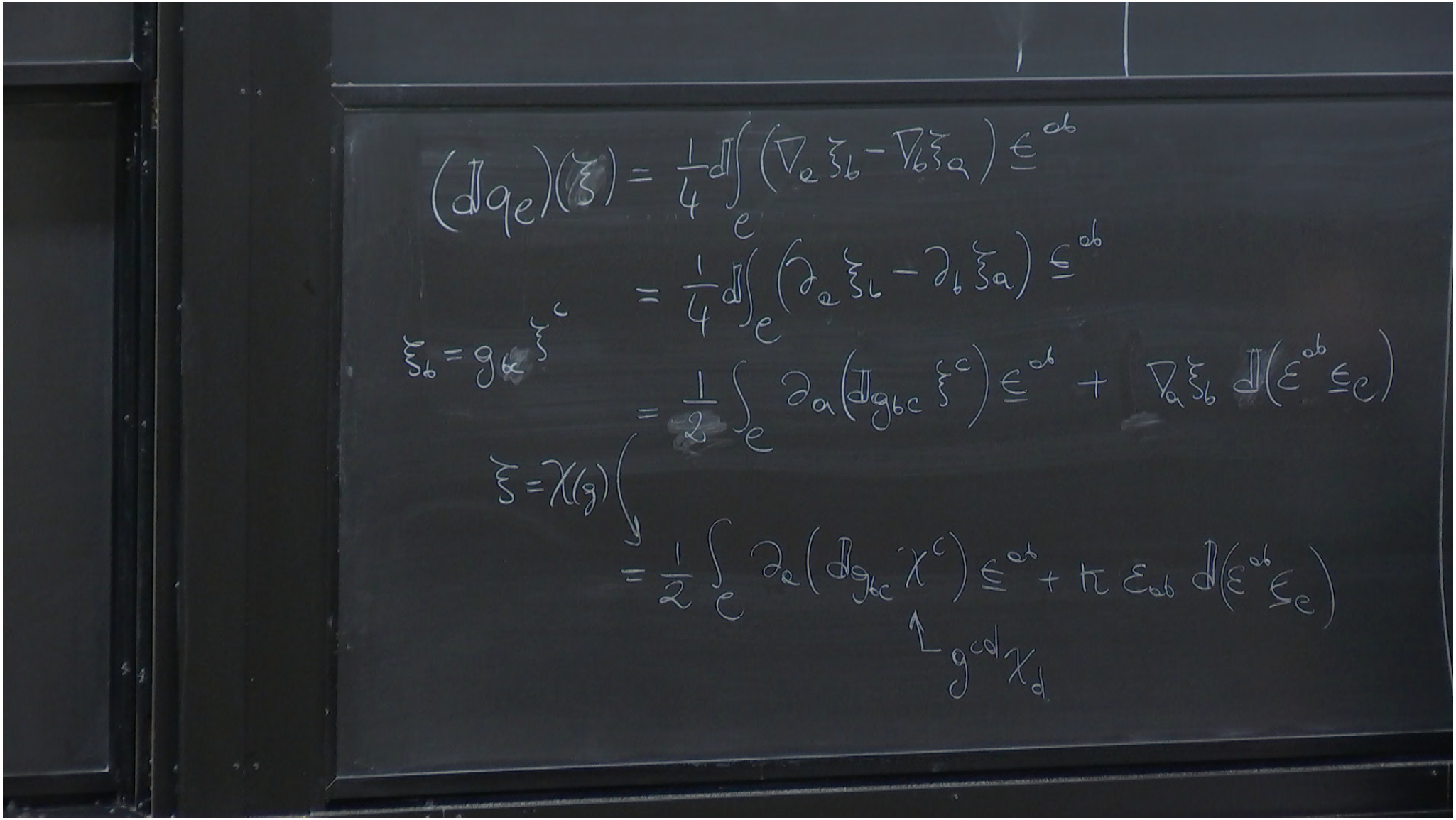
$$= \frac{1}{2}\int_C \kappa\ \varepsilon_{ab}\,\varepsilon^{ab}\,\underline{\epsilon}_C$$

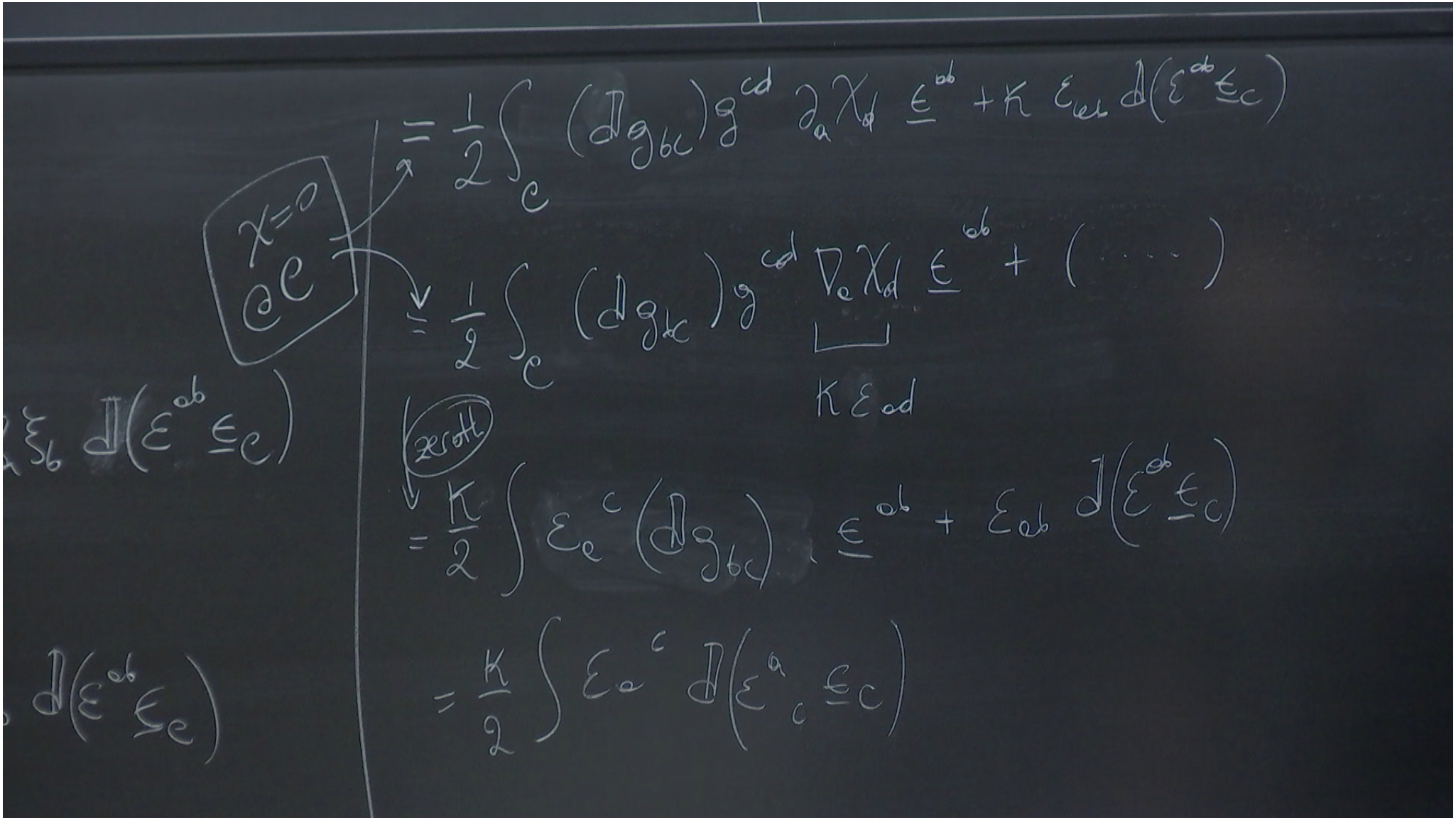
zeroth law

$$= \kappa\, A_C$$

(Noether charge $q_\epsilon(\chi)$ is the entropy $\sim A_C$)


$\lambda^*_\Sigma \underline{\epsilon}_a = n_a \underline{\epsilon}_\Sigma$


$(\mathbb{d}q_e)(\xi) = \frac{1}{4}\mathbb{d}\int_e (\nabla_a \xi_b - \nabla_b \xi_a)\,\underline{\epsilon}^{ab}$
$= \frac{1}{4}\mathbb{d}\int_e (\partial_a \xi_b - \partial_b \xi_a)\,\underline{\epsilon}^{ab}$
$\xi_b = g_{bc}\xi^c$
$= \frac{1}{2}\int_e \partial_a(\mathbb{d} g_{bc}\xi^c)\,\underline{\epsilon}^{ab} + \nabla_a \xi_b\, \mathbb{d}(\epsilon^{ab}\underline{\epsilon}_e)$
$\xi = \chi(g)$
$= \frac{1}{2}\int_e \partial_a(\mathbb{d} g_{bc}\chi^c)\,\underline{\epsilon}^{ab} + \pi\,\varepsilon_{ab}\,\mathbb{d}(\epsilon^{ab}\underline{\epsilon}_e)$
$\uparrow g^{cd}\chi_d$


$\chi = 0$
on $\mathcal{C}$
$\xi_b \, d(\epsilon^{ab} \underline{\epsilon}_c)$
$d(\epsilon^{ab} \underline{\epsilon}_c)$
$= \frac{1}{2} \int_{\mathcal{C}} (d g_{bc}) g^{cd} \partial_a \chi_d \, \underline{\epsilon}^{ab} + \kappa \, \epsilon_{ab} \, d(\epsilon^{ab} \underline{\epsilon}_c)$
$= \frac{1}{2} \int_{\mathcal{C}} (d g_{bc}) g^{cd} \nabla_a \chi_d \, \underline{\epsilon}^{ab} + (\cdots)$
$\kappa \, \epsilon_{ad}$
zeroth
$= \frac{\kappa}{2} \int \epsilon_a{}^c (d g_{bc}) \, \underline{\epsilon}^{ab} + \epsilon_{ab} \, d(\epsilon^{ab} \underline{\epsilon}_c)$
$= \frac{\kappa}{2} \int \epsilon_a{}^c \, d(\epsilon^a{}_c \underline{\epsilon}_c)$