Title: Quantum Gravity

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Collection: Quantum Gravity (2021-2022)

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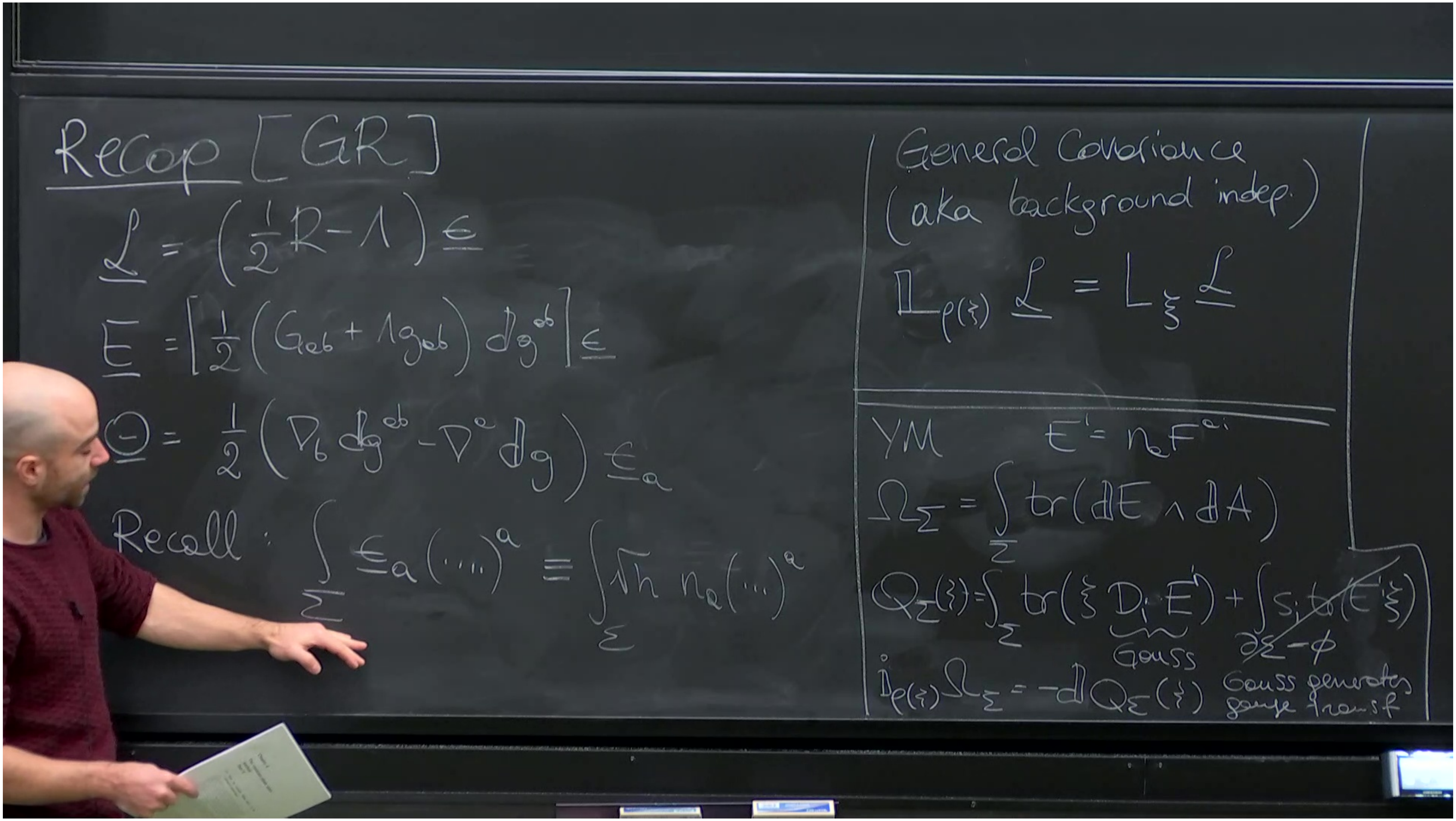
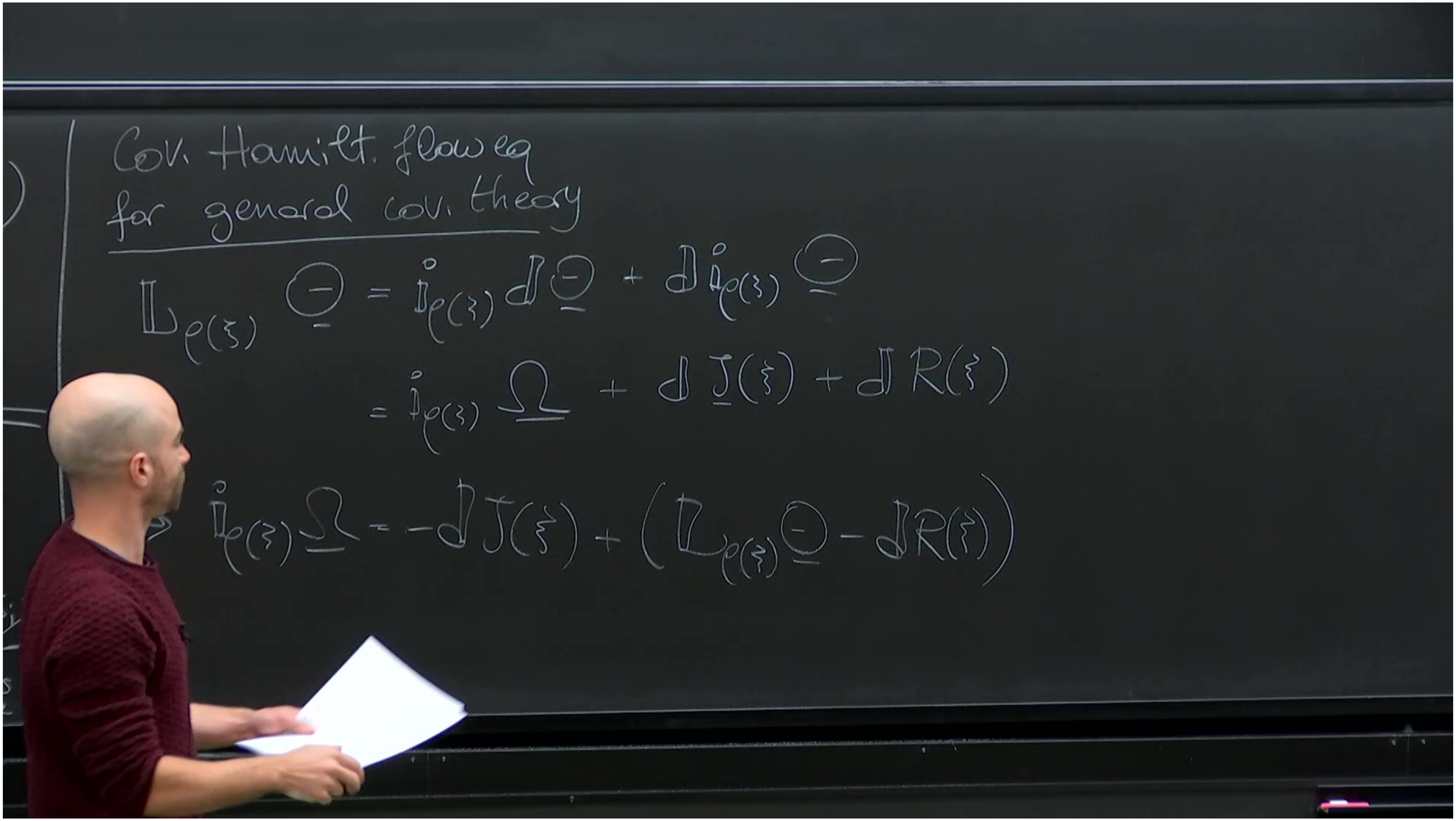
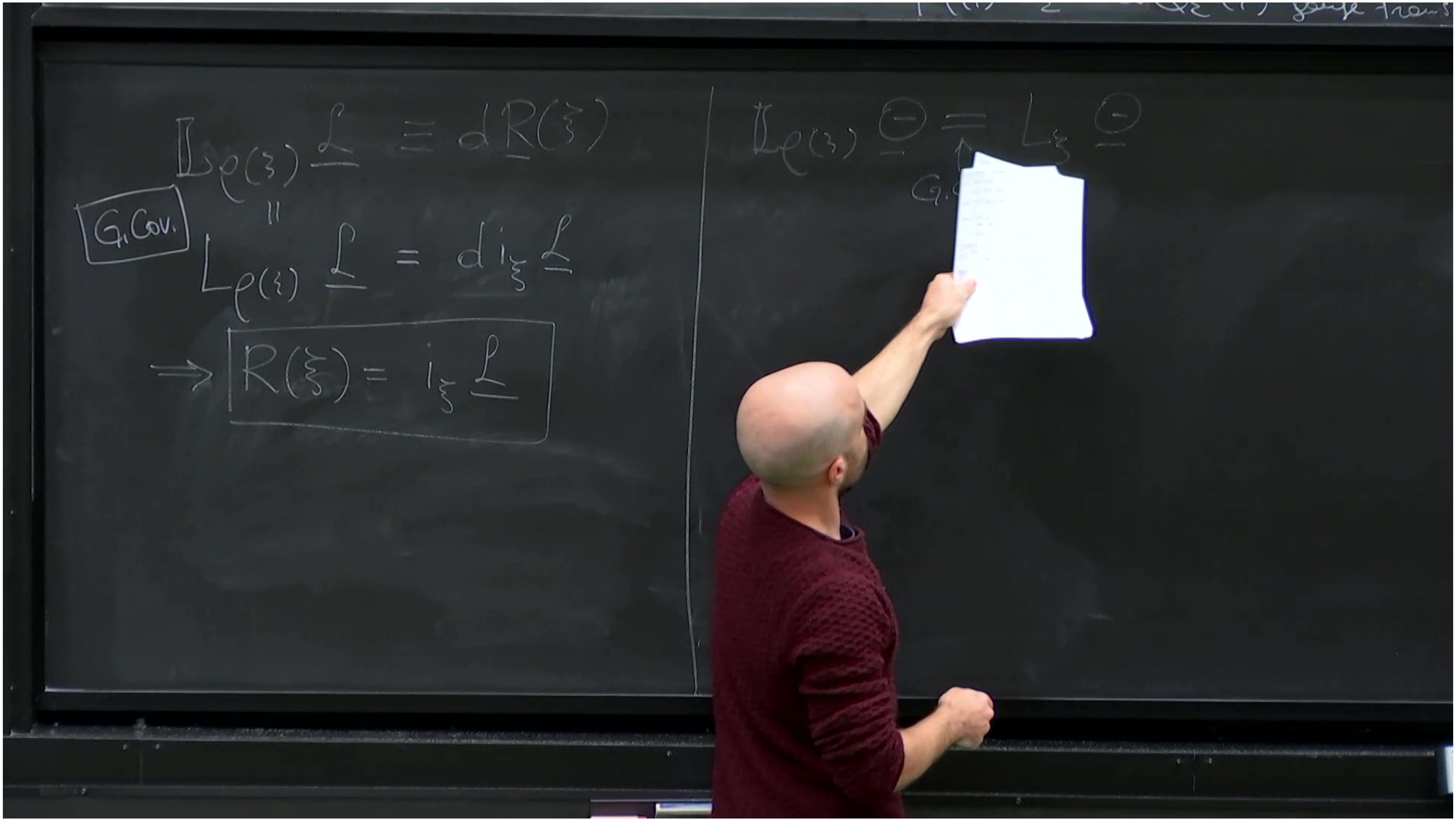
Abstract: Topics will include (but are not limited to): Canonical formulation of constrained systems, The Dirac program, First order formalism of gravity, Loop Quantum Gravity, Spinfoam models, Research at PI and other approaches to quantum gravity.


Recap [GR]
$\underline{L} = \left(\frac{1}{2}R - \Lambda\right)\underline{\epsilon}$
$\underline{E} = \left[\frac{1}{2}\left(G_{ab} + \Lambda g_{ab}\right) \delta g^{ab}\right]\underline{\epsilon}$
$\underline{\Theta} = \frac{1}{2}\left(\nabla_b \delta g^{ab} - \nabla^a \delta g\right)\underline{\epsilon}_a$
Recall: $\int_\Sigma \underline{\epsilon}_a (\dots)^a = \int_\Sigma \sqrt{h}\, n_a (\dots)^a$
General Covariance (aka background indep.)
$\mathbb{L}_{\rho(\xi)} \underline{L} = L_\xi \underline{L}$
YM $E^i = n_a F^{ai}$
$\Omega_\Sigma = \int_\Sigma tr(\delta E \wedge \delta A)$
$Q_\Sigma(\xi) = \int_\Sigma tr(\xi \underbrace{D_i E^i}_{Gauss}) + \cancel{\int_{\partial\Sigma} s_i\, tr(E^i \xi)}$
$\partial\Sigma = \phi$
$\mathbb{I}_{\rho(\xi)} \Omega_\Sigma = -\delta Q_\Sigma(\xi)$
Gauss generates gauge transf


Cov. Hamilt. flow eq
for general cov. theory
$\mathbb{L}_{\rho(\xi)} \underline{\Theta} = \mathring{\mathbb{i}}_{\rho(\xi)} \mathbb{d} \underline{\Theta} + \mathbb{d} \mathring{\mathbb{i}}_{\rho(\xi)} \underline{\Theta}$
$= \mathring{\mathbb{i}}_{\rho(\xi)} \underline{\Omega} + \mathbb{d} \underline{J}(\xi) + \mathbb{d} R(\xi)$
$\Rightarrow \mathring{\mathbb{i}}_{\rho(\xi)} \underline{\Omega} = -\mathbb{d} J(\xi) + \left(\mathbb{L}_{\rho(\xi)} \underline{\Theta} - \mathbb{d} R(\xi) \right)$


$\mathbb{L}_{\rho(\xi)} \underline{\mathcal{L}} \equiv d\underline{R}(\xi)$
G. Cov.
$L_{\rho(\xi)} \underline{\mathcal{L}} = d i_\xi \underline{\mathcal{L}}$
$\Rightarrow \boxed{R(\xi) = i_\xi \underline{\mathcal{L}}}$
$\mathbb{L}_{\rho(\xi)} \underline{\Theta} = L_\xi \underline{\Theta}$

G. Cov.

$$\mathbb{L}_{\rho(\xi)} \underline{L} \equiv d \underline{R}(\xi)$$

$$\parallel$$

$$L_{\rho(\xi)} \underline{L} = d\, i_\xi \underline{L}$$

$$\Rightarrow \boxed{R(\xi) = i_\xi \underline{L}}$$

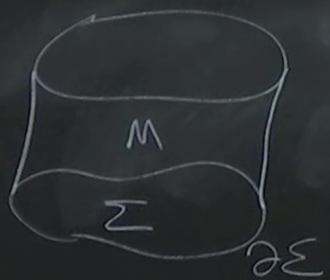
$$\mathbb{L}_{\rho(\xi)} \underline{\Theta} \underset{\uparrow \, \text{G. Cov.}}{=} L_\xi \underline{\Theta}$$

$$= i_\xi d \underline{\Theta} + d\, i_\xi \underline{\Theta}$$

$$\mathbb{d} R(\xi) = i_\xi \mathbb{d} \underline{L} = i_\xi \underline{E} + i_\xi d \underline{\Theta}$$

$$\Rightarrow \mathbb{L}_{\rho(\xi)} \Theta - \mathbb{d} R(\xi) = d\, i_\xi \underline{\Theta} - i_\xi \underline{E}$$

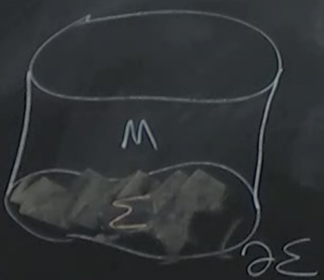
$$\mathring{\mathbb{I}}_{\rho(\xi)}\Omega = -\mathbb{d}J(\xi) - i_\xi E + d\, i_\xi \Theta$$



$$\int_\Sigma (\dots)$$

$$\mathring{\mathbb{I}}_{\rho(\xi)}\Omega_\Sigma = -\mathbb{d}Q_\Sigma(\xi) - \int_\Sigma i_\xi E + \int_{\partial\Sigma} i_\xi \Theta$$

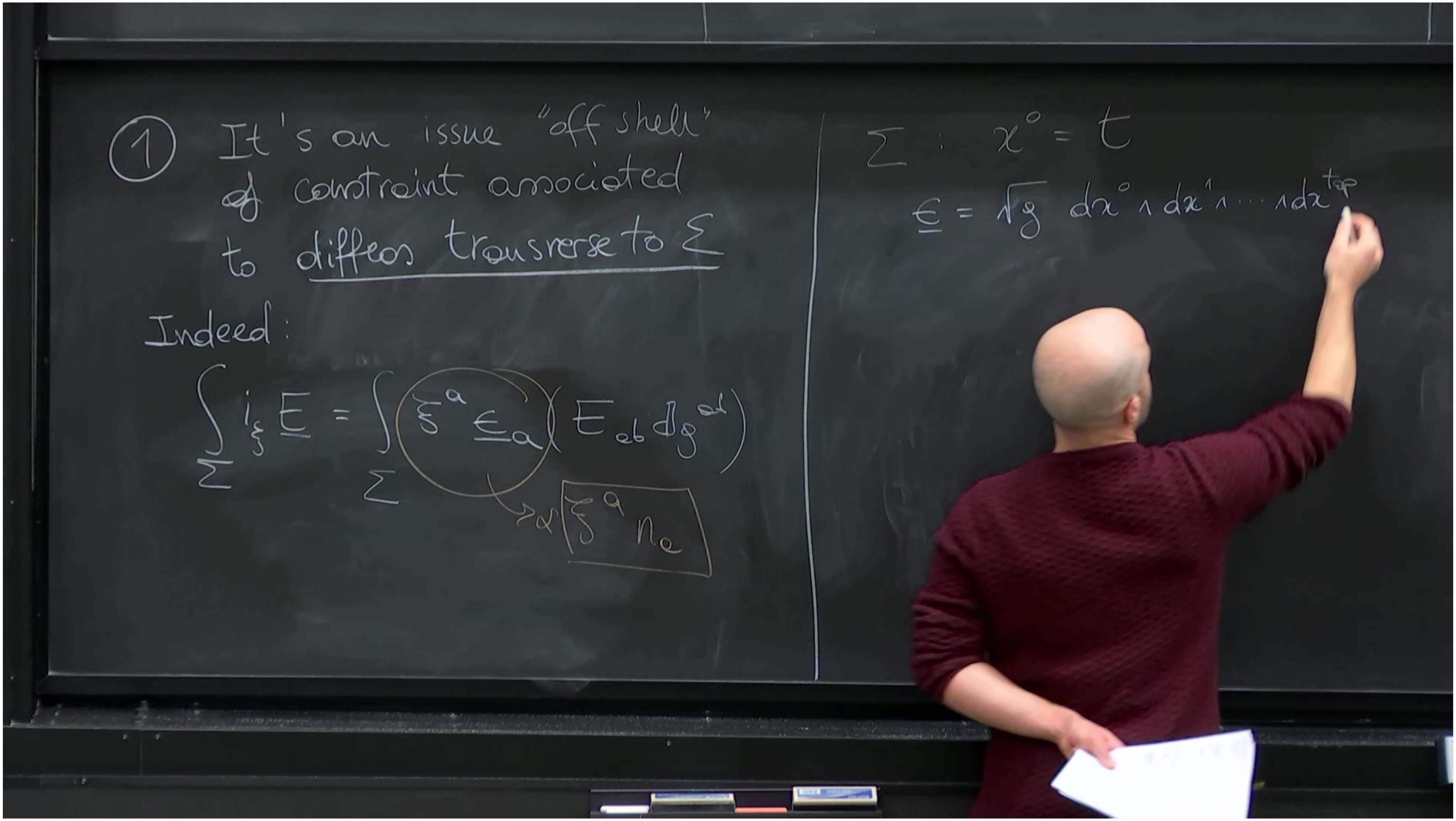
$$\mathring{I}_{\rho(\xi)}\Omega = -\delta J(\xi) - i_\xi E + d\, i_\xi \Theta$$



$$\int_\Sigma (\ldots)$$

OBSTRUCTION TO THE "NAIVE" STMNT THAT NOETHER CHARGE GENERATES GAUGE TR.

$$\boxed{\mathring{I}_{\rho(\xi)}\Omega_\Sigma = -\delta Q_\Sigma(\xi) - \underbrace{\int_\Sigma i_\xi E}_{\text{1 bulk}} + \underbrace{\int_{\partial\Sigma} i_\xi \Theta}_{\text{2 bdry}}}$$


① It's an issue "off shell" of constraint associated to diffeos transverse to Σ
Indeed:
$\int_\Sigma i_\xi \underline{E} = \int_\Sigma \xi^a \underline{\epsilon}_a (E_{ab} \delta g^{ab})$
$\to \boxed{\xi^a n_a}$
$\Sigma : x^0 = t$
$\underline{\epsilon} = \sqrt{g}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^{top}$

Pf in coords

$\Sigma \; : \; x^0 = t$

$\underline{\epsilon} = \sqrt{g}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^d$

$i_\xi \underline{\epsilon} \sim \sqrt{g} \sum_a (\pm) \xi^a \, dx^0 \wedge \cdots \wedge \widehat{dx^a} \wedge \cdots \wedge dx^d$

pull back to $\Sigma \Rightarrow dx^0 \equiv 0$

therefore only surviving term

is $\propto \xi^0 \, dx^1 \wedge \cdots \wedge dx^d$

Pf in coords

$\Sigma : x^0 = t \quad , \quad n_a = \delta_a^0$

$\underline{\epsilon} = \sqrt{g}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^d$

$i_\xi \underline{\epsilon} \sim \sqrt{g} \sum_a (\pm) \xi^a \, dx^0 \wedge \cdots \wedge \widehat{dx^a} \wedge \cdots \wedge dx^d$

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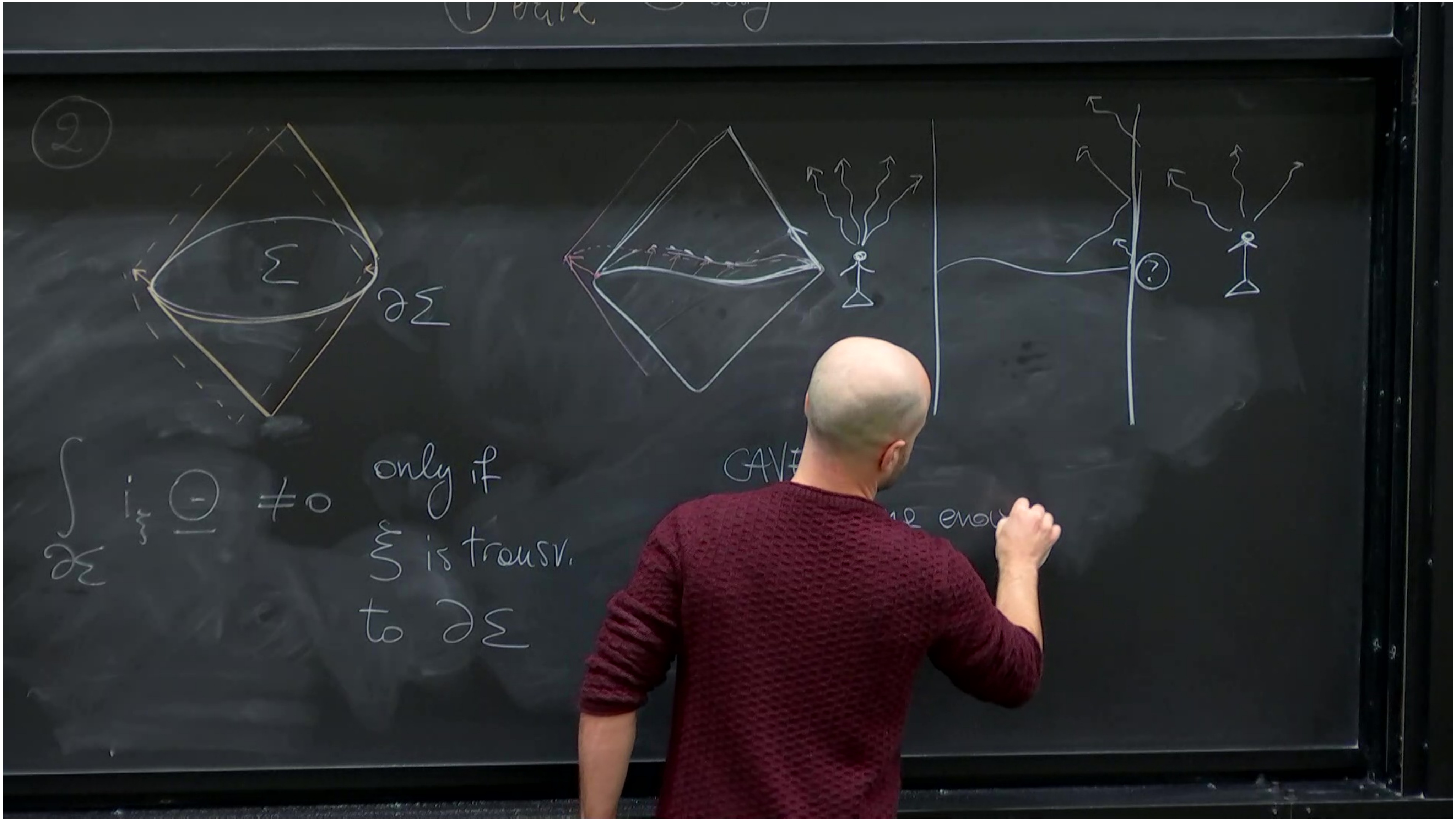
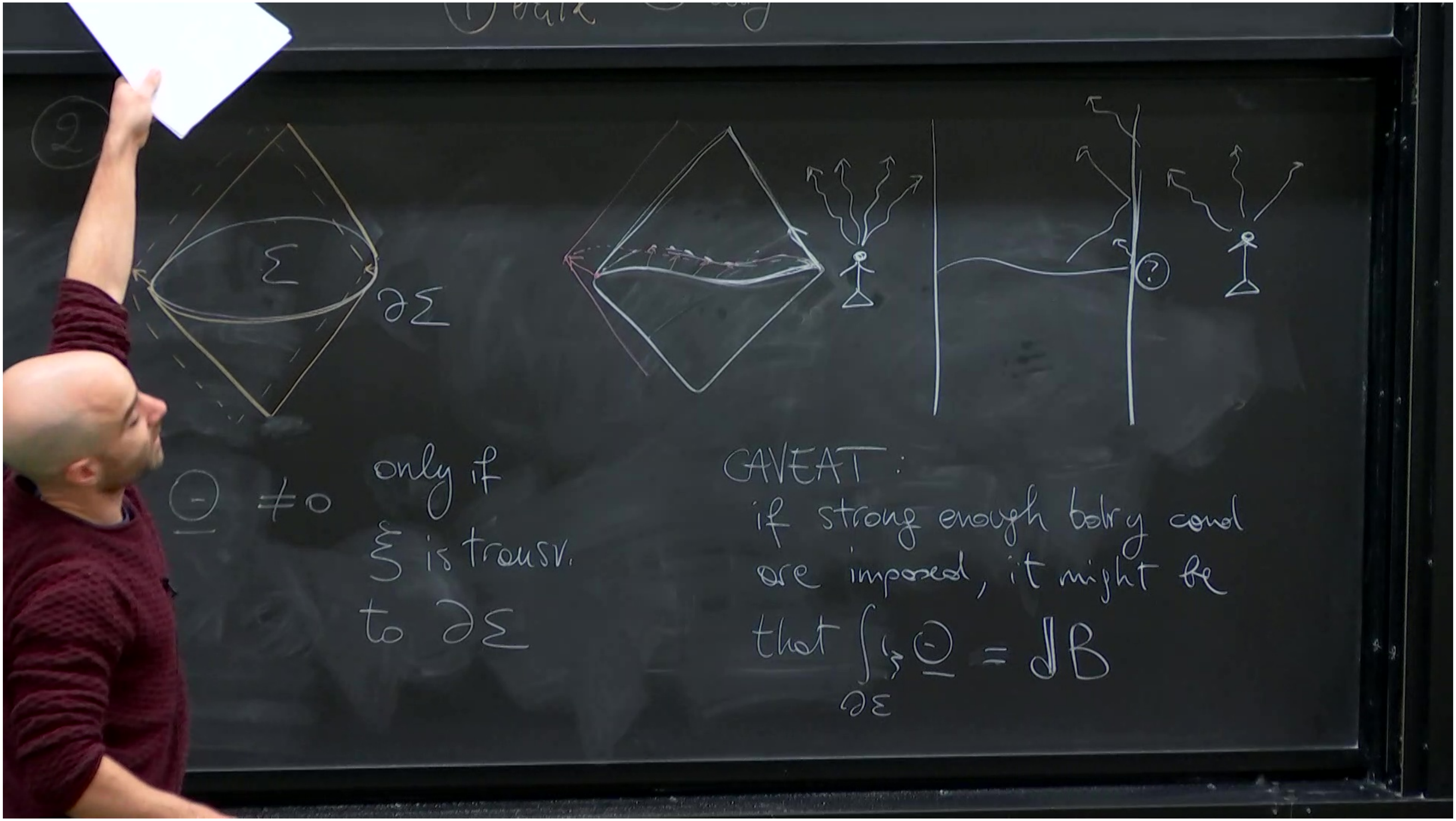
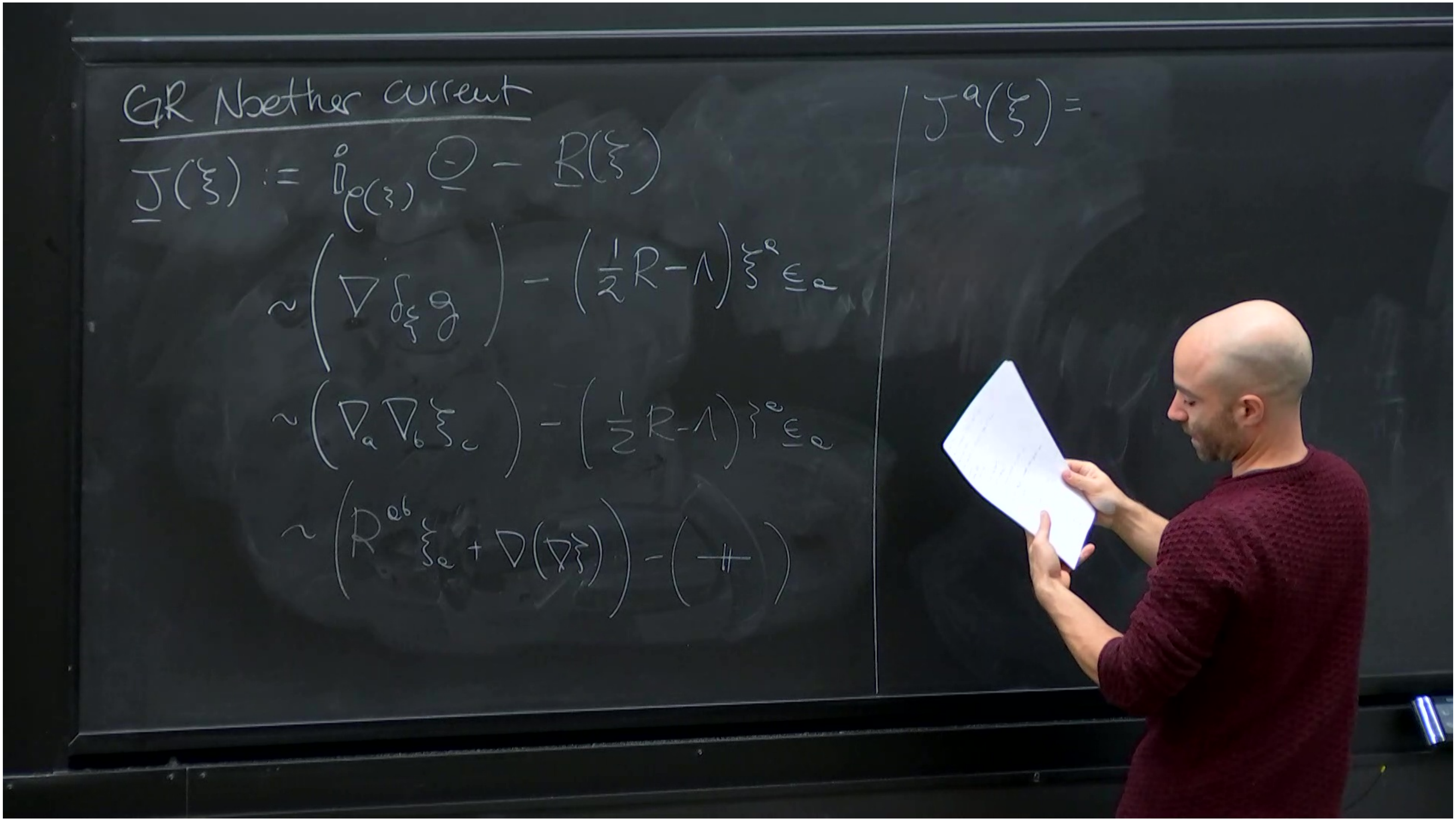
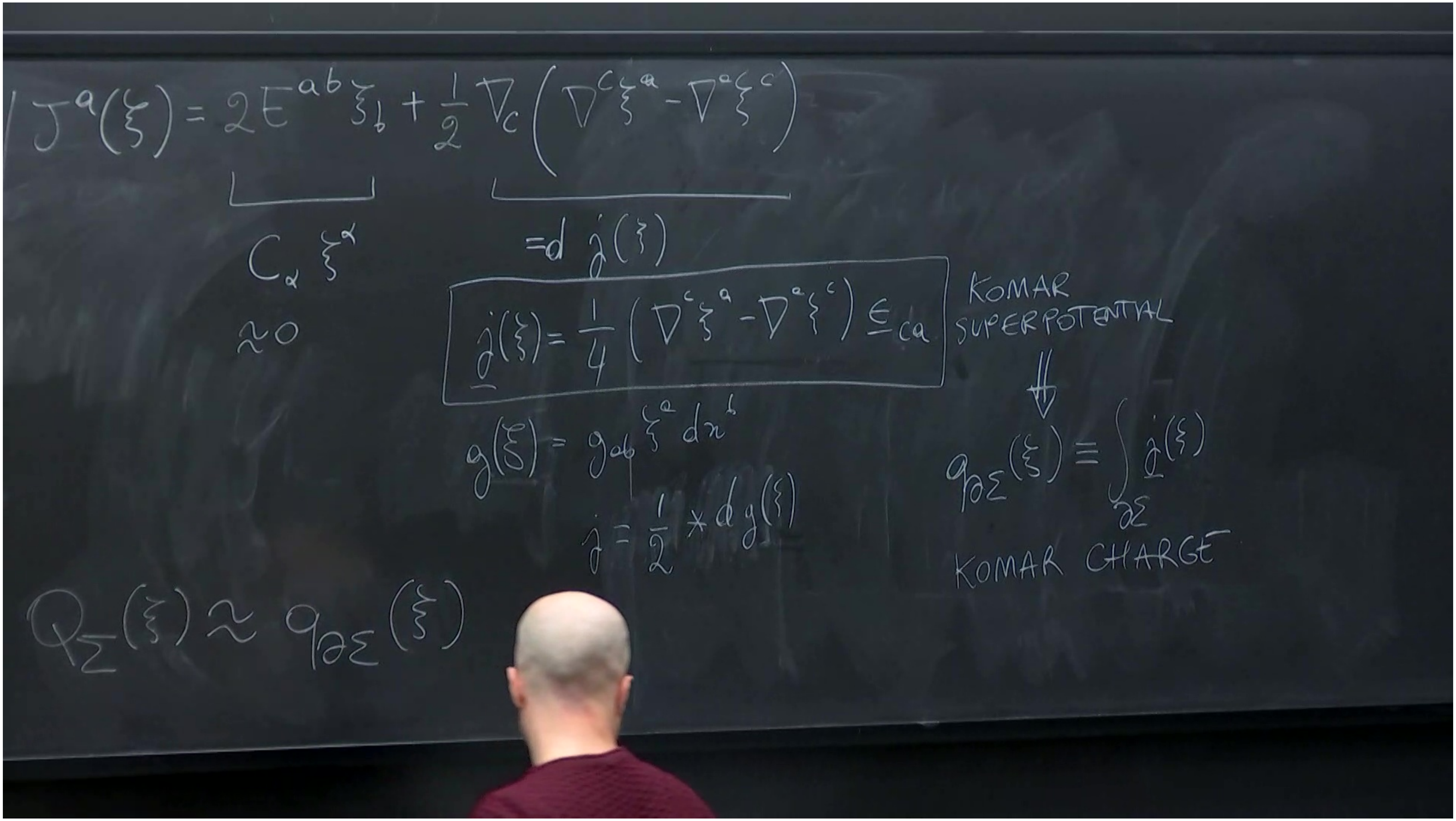
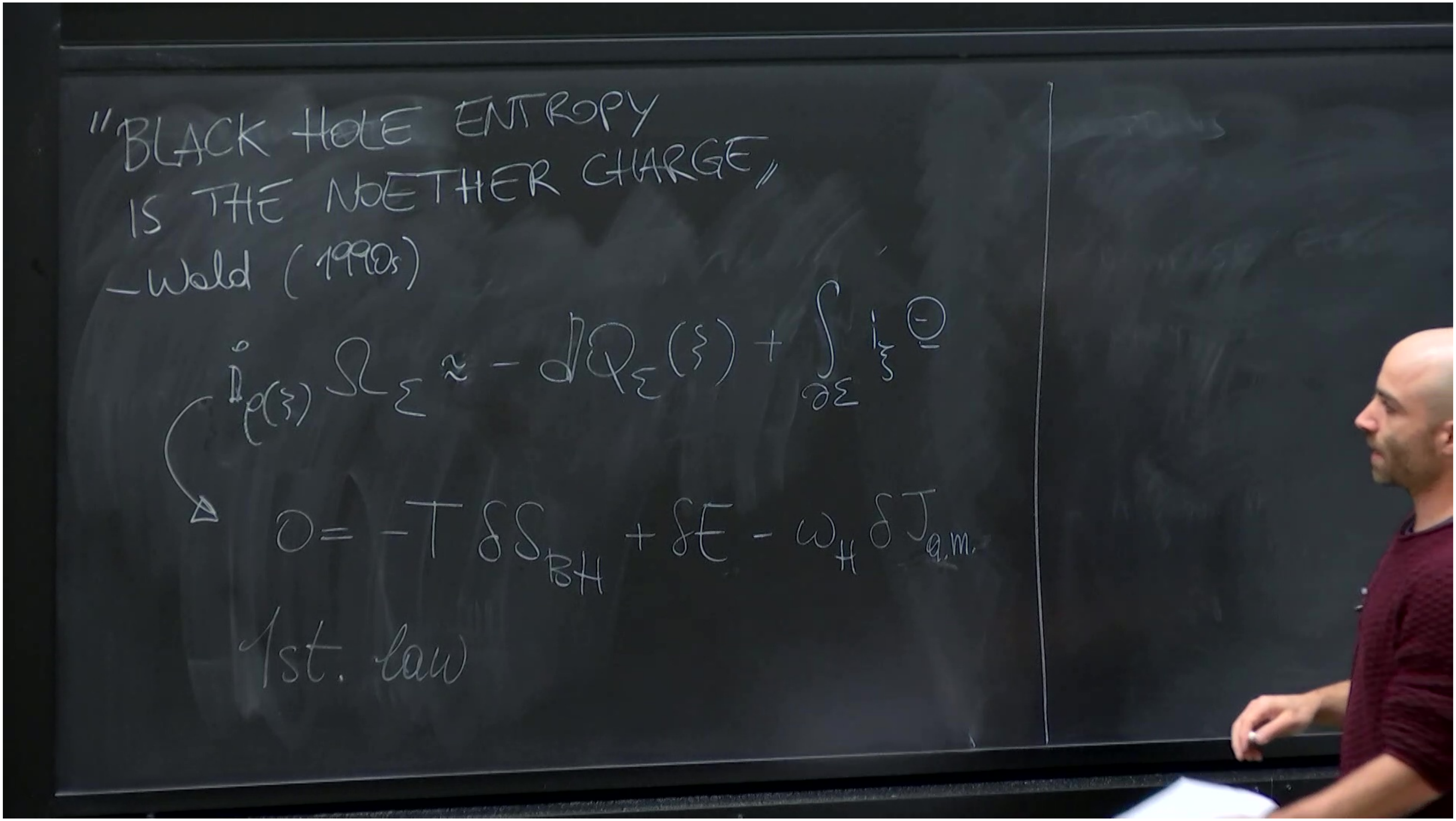


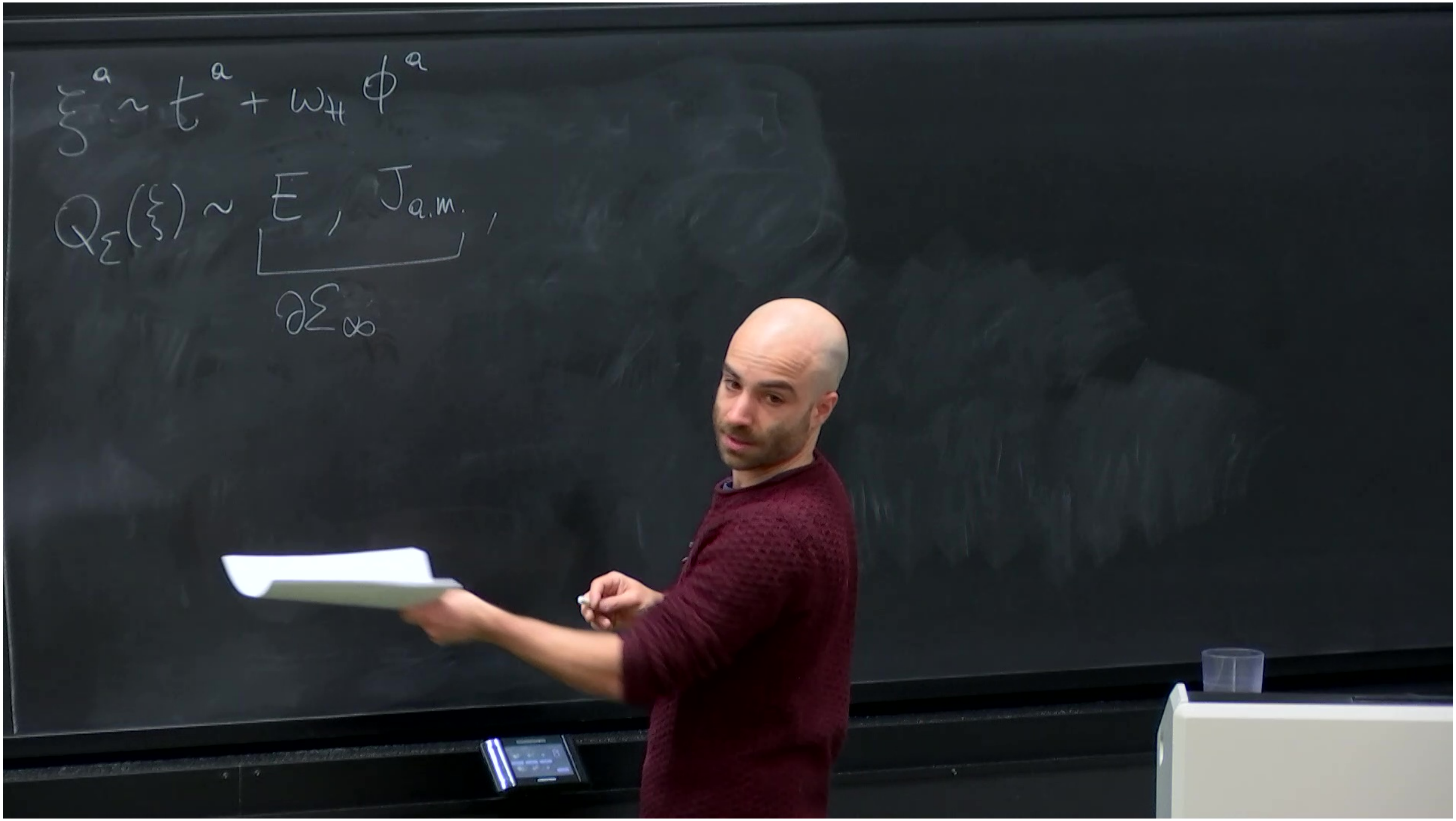
2
Σ
∂Σ
$\int_{\partial\Sigma} i_\xi \Theta \neq 0$
only if
ξ is transv.
to ∂Σ
?


2
Σ
∂Σ
?
Θ ≠ 0
only if
ξ is transv.
to ∂Σ
CAVEAT :
if strong enough bdry cond
are imposed, it might be
that ∫∂Σ i_ξ Θ = δB


GR Noether current
$\underline{J}(\xi) := i_{\rho(\xi)}\underline{\Theta} - \underline{B}(\xi)$
$\sim \left(\nabla \delta_\xi g\right) - \left(\frac{1}{2}R - \Lambda\right)\xi^e \underline{\epsilon}_e$
$\sim \left(\nabla_a \nabla_b \xi_c\right) - \left(\frac{1}{2}R - \Lambda\right)\xi^e \underline{\epsilon}_e$
$\sim \left(R^{eb}\xi_e + \nabla(\nabla\xi)\right) - \left(\quad \right)$
$J^a(\xi) =$


$J^a(\xi) = \underbrace{2E^{ab}\xi_b}_{C_\alpha \xi^\alpha \approx 0} + \underbrace{\frac{1}{2}\nabla_c\left(\nabla^c\xi^a - \nabla^a\xi^c\right)}_{= d\, \underline{j}(\xi)}$
$\boxed{\underline{j}(\xi) = \frac{1}{4}\left(\nabla^c\xi^a - \nabla^a\xi^c\right)\underline{\epsilon}_{ca}}$
KOMAR SUPERPOTENTIAL
$\Downarrow$
$q_{\partial\Sigma}(\xi) \equiv \int_{\partial\Sigma} \underline{j}(\xi)$
KOMAR CHARGE
$g(\xi) = g_{ab}\xi^a dx^b$
$j = \frac{1}{2} * d\, g(\xi)$
$Q_\Sigma(\xi) \approx q_{\partial\Sigma}(\xi)$


"BLACK HOLE ENTROPY
IS THE NOETHER CHARGE"
– Wald (1990s)
$\mathbb{I}_{\rho(\xi)} \Omega_{\Sigma} \approx - dQ_{\Sigma}(\xi) + \int_{\partial\Sigma} i_{\xi} \Theta$
$0 = -T\,\delta S_{BH} + \delta E - \omega_H\,\delta J_{a.m.}$
1st. law


$\xi^a \sim t^a + \omega_H \phi^a$
$Q_\Sigma(\xi) \sim \underbrace{E , J_{a.m.}}_{\partial\Sigma_\infty} ,$